Aspects of Light-Front Hadron Physics
Where DSEs and Lattice-QCD Meet

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• **Parton Distribution Amplitudes** (pion, kaon). Close contact with lattice-QCD moments. Applications to high energy exclusive form factors, ultraviolet behavior.

• **Parton Distribution Functions** (pion). A work in progress.

• X. Ji’s **space-like correlator approach to PDFs**—a model investigation.
Lattice-QCD and DSE-based modeling

- Lattice: $\langle \mathcal{O} \rangle = \int D\bar{q}qG \mathcal{O}(\bar{q}, q, G) \ e^{-S[\bar{q}, q, G]}$
  - Euclidean metric, x-space, Monte-Carlo
  - Issues: lattice spacing and vol, sea and valence $m_q$, fermion Det
  - Large time limit $\Rightarrow$ nearest hadronic mass pole

- EOMs (DSEs): $0 = \int D\bar{q}qG \ \frac{\delta}{\delta q(x)} e^{-S[\bar{q}, q, G]+(\bar{\eta}, q)+(\bar{q}, \eta)+(J, G)}$
  - Euclidean metric, p-space, continuum integral eqns
  - Issues: truncation and phenomenology—not full QCD
  - Analytic contin. $\Rightarrow$ nearest hadronic mass pole
  - Can be quick to identify systematics, mechanisms, $\cdots$

Expect: qualitatively new insight where other methods can’t, eg high $Q^2$

Do not expect: final, precision-QCD results
DSE Modeling of Hadron Physics

- Most common: Rainbow-ladder truncation of QCD's eqns of motion. Approximation to full BSE kernel now being utilized.

- Constrain modeling by preserving AV-Ward-Takahashi Id, V-WTI. [Color singlet]

- Naturally implements DCSB, conserved vector current, Goldstone Thm, PCAC...

- RL truncation only good for ground state vector & pseudoscalar mesons, q-qq descriptions of baryons with AV and S diquarks.

- At the very least: DSE continuum QCD modeling suited for surveying the landscape quickly from large to small scales; finding out which underlying mechanisms are dominant. Applicable to all scales, high $Q^2$ form factors, etc

- Unifying DSE treatment of light front quantities (PDFs, GPDs, DA) with other aspects of hadron structure: masses, decays, charge form factors, transition form factors....

- Pion & kaon q-qbar Bethe-Salpeter wavefn is very well known

\[
\text{AV - WTI : } m_q \rightarrow 0, P \rightarrow 0 \Rightarrow \Gamma_{q\bar{q}}(k^2) = i\gamma_5 \frac{1}{4} \text{tr} S_0^{-1}(k) \frac{f_0}{f_\pi} + O(P)
\]

Modern Context for Rainbow-Ladder Kernel


Identified enough strength for physical DCSB

\[ \Rightarrow m_G(k^2) \quad \text{m}_G(0) \sim 0.38 \text{ GeV} \]

\[ K_{BSE}^{RL} = \frac{4\pi \hat{\alpha}_{\text{eff}}(q^2)}{m_G^2(q^2) + q^2} \]

\[ \Rightarrow \frac{\hat{\alpha}_{\text{eff}}(0.1)}{\pi} \approx 3 - 4 \]

BSE kernel from ab initio gauge sector DSE work now agrees satisfactorily with the kernel from fitting data: Binosi, Chang, Papavassiliou, Roberts, PLB742, 183 (2015)
Parton Distribution Amplitudes
Fit numerical DSE-BSE solns to Nakanishi forms to allow analytic Feyn Integral Methods

\[ \Gamma_\pi(q^2, q \cdot P) = \gamma_5 \left\{ E_\pi(q^2, q \cdot P) + P F_\pi(\ldots) + q \cdot P G_\pi(\ldots) + \sigma : qP H_\pi(\ldots) \right\} \]

Use Nakanishi Repn (or PTIR) (1965) :-

\[ \mathcal{F}(q^2; q \cdot P) = \int_{-1}^{1} d\alpha \int_{0}^{\infty} d\Lambda \left\{ \frac{\rho_{IR}(\alpha; \Lambda)}{(q^2 + \alpha q \cdot P + \Lambda^2)^{m+n}} + \frac{\rho_{UV}(\alpha; \Lambda)}{(q^2 + \alpha q \cdot P + \Lambda^2)^n} \right\} \]

\[ \rho_{IR}(\alpha; \Lambda) \rightarrow \rho_1(\alpha) \delta(\Lambda - \Lambda_{IR_1}) + \cdots 3 \]

\[ S(q) = \sum_{k=1}^{3} \left( \frac{z_k}{i q + m_k} + \frac{z_k^*}{i q + m_k^*} \right) \]

Works for u-, d-, s-, c-, b-quarks. Also for lattice-QCD propagators.

Pion Distribution Amplitude (leading twist)

\[
f_\pi \phi_\pi(x) = \int \frac{d\lambda}{2\pi} e^{-ixP.n\lambda} \langle 0 | \bar{q}(0) \gamma_5 \not{q}(\lambda n) | \pi(P) \rangle
\]

\[
f_\pi \langle x^m \rangle_\phi = \frac{Z_2 N_c}{P \cdot n} \text{tr} \int_k (\frac{k \cdot n}{P \cdot n})^m \gamma_5 \not{q} \left[ S(k) \Gamma_\pi \left( k - \frac{P}{2} ; P \right) S(k - P) \right]
\]

\[\mu = 2 \text{ GeV}\]

Broadening of PDA is an expression of DCSB --- long sought after in LF QFT

BS Wavefn
Pion Distribution Amplitude

ERBL (~1980):
\[ \phi_\pi(x; \mu) = 6x(1-x) \left\{ 1 + \sum_{n=2,4\ldots} a_n(\mu) C_n^{3/2}(2x-1) \right\} \]

\[ a_n(\mu) = a_n(\mu_0) \left[ \frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right] \gamma_n^{(0)}/\beta_0 \]

Evolution to higher scales is EXTREMELY SLOW
Not much change up to LHC energy

Conformal limit:
\[ a_n(\mu \to \infty) = 0 \]

Efficient representation of DSE results:
\[ \phi_\pi(x; \mu) = N_\alpha x^\alpha (1-x)^\alpha \left\{ 1 + \sum_{n=2}^{\infty} \tilde{a}_n(\mu) C_n^{\alpha+1/2}(2x-1) \right\} \]

\[ \phi_K(x; \mu) = N_\alpha x^\alpha (1-x)^\alpha \left\{ 1 + \sum_{n=2,4\ldots} \tilde{a}_n(\mu) C_n^{\alpha+1/2}(2x-1) \right\} \]

\[ + N_\beta x^\beta (1-x)^\beta \left\{ \sum_{n=1,3\ldots} \tilde{a}_n(\mu) C_n^{\beta+1/2}(2x-1) \right\} \]
Low Order Truncation of ERBL-Gegenbauer Expn of PDA

\[ \phi_\pi(x; \mu) = 6x(1 - x) \left\{ 1 + \sum_{n=2,4} a_n(\mu) C_n^{3/2}(2x - 1) \right\} \]

DSE soln

\[ \{0, 1.\}, \{2, 0.233104\}, \{4, 0.112135\}, \{6, 0.0683202\}, \{8, 0.0469145\}, \{10, 0.0346469\}, \{12, 0.0268732\}, \{14, 0.0215933\}, \{16, 0.0178199\}, \{18, 0.0150159\}, \{20, 0.0128672\}, \{22, 0.0111788\}, \{24, 0.00982438\}, \{26, 0.00871886\}, \{28, 0.00780296\}, \{30, 0.00703438\}, \{32, 0.0063823\}, \{34, 0.00582279\}, \{36, 0.00534272\}, \{38, 0.00493277\}, \{40, 0.00447911\} \]

\[ \phi(x) \]

\[ \mu = 2 \text{ GeV} \]

A double-humped PDA is almost ruled out by V. Braun, I. Filyanov, Z. Phys. C44, 157 (1989)
One Lattice-QCD Moment Almost Determines Pion DA

Pion Distribution Amplitude from Lattice QCD
I. C. Cloët, L. Chang, C. D. Roberts, S. M. Schmidt, and P. C. Tandy

\[
\phi_{\pi}^{\text{LQCD}}(x; \mu = 2) = N x^\alpha (1 - x)^\alpha
\]
\[
\alpha = 0.35 + 0.32 - 0.24
\]

\[
\langle (2x - 1)^2 \rangle^{\text{LQCD}}_{\mu=2} = 0.27 \pm 0.04
\]

V. Braun et al., PRD 74, 074501 (2006)

DSE beyond RL
\[
\phi^{\text{asym}}(x) = \phi_{\pi}(x; \mu \to \infty)
\]

DSE RL

Lattice-QCD

\[
\mu = 2 \text{ GeV}
\]

\[
\phi_{\pi}(x)
\]

\[
0.0 \quad 0.25 \quad 0.50 \quad 0.75 \quad 1.0
\]

Cairns LHV July 2015
\[ \langle (2x - 1)^2 \rangle_{\mu=2 \text{ GeV}}^{LQCD} = 0.2361 (41) (39) \quad \text{V. Braun et al., arXiv:1503.03656 [hep-lat]} \]

DSE prediction: 0.251
Table 1: Meson PDA moments obtained using numerical simulations of lattice-
regularised QCD with $N_f = 2 + 1$ domain-wall fermions and nonperturbative
renormalisation of lattice operators [29]: linear extrapolation to physical pion
mass, MS-scheme at $\zeta = 2$ GeV, two lattice volumes. The first error is statisti-
cal, the second represents an estimate of systematic errors, including those from
the s-quark mass, discretisation and renormalisation.

<table>
<thead>
<tr>
<th>meson</th>
<th>$\langle (x - \bar{x})^n \rangle$</th>
<th>$16^3 \times 32$</th>
<th>$24^3 \times 64$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>n=2</td>
<td>0.25(1)(2)</td>
<td>0.28(1)(2)</td>
</tr>
<tr>
<td>$\rho_\parallel$</td>
<td>n=2</td>
<td>0.25(2)(2)</td>
<td>0.27(1)(2)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>n=2</td>
<td>0.25(2)(2)</td>
<td>0.25(2)(2)</td>
</tr>
<tr>
<td>$K$</td>
<td>n=1</td>
<td>0.035(2)(2)</td>
<td>0.036(1)(2)</td>
</tr>
<tr>
<td>$K^*$</td>
<td>n=1</td>
<td>0.037(1)(2)</td>
<td>0.043(2)(3)</td>
</tr>
<tr>
<td>$K_\parallel$</td>
<td>n=2</td>
<td>0.25(1)(2)</td>
<td>0.26(1)(2)</td>
</tr>
<tr>
<td>$K^*_\parallel$</td>
<td>n=2</td>
<td>0.25(1)(2)</td>
<td>0.25(2)(2)</td>
</tr>
</tbody>
</table>

$\varphi(x) = x^\alpha (1 - x)^\beta / B(\alpha, \beta)$.

$16^3 \times 32$: $\alpha_{us} = 0.56^{+0.21}_{-0.18}$, $\beta_{us} = 0.45^{+0.19}_{-0.16}$.

$24^3 \times 64$: $\alpha_{us} = 0.48^{+0.19}_{-0.16}$, $\beta_{us} = 0.38^{+0.17}_{-0.15}$.

DAs of light quark mesons look much the same—-with small
flavor breaking.
Kaon Distribution Amplitude

Size of SU(2)xSU(3) spin-flavor symmetry-breaking?

data, as strong interaction bound states whose decay is mediated only by the weak interaction, so that they have a relatively long lifetime, kaons have been instrumental in establishing the foundation and properties of the Standard Model; notably, the physics of CP violation. In this connection the nonleptonic decays of B mesons are crucial because, e.g., the transitions \( B^+ \rightarrow (\pi K)^\pm \) and \( B^0 \rightarrow \pi^+\pi^0 \) provide access to the imaginary part of the CKM matrix element \( V_{ub} \): \( \gamma = \text{Arg}(V_{ub}^*) \) [4]. Factorisation theorems have been derived and are applicable to such decays [5]. However, the formulae involve a certain class of so-called “non-factorisable” corrections because the parton distribution amplitudes (PDAs) of strange mesons are not symmetric with respect to quark and antiquark momenta. Therefore, any derived estimate of \( \gamma \) is only as accurate as the evaluation of both the difference between \( K \) and \( \pi \) PDAs and also their respective differences from the asymptotic distribution, \( \varphi_{\text{asy}}^s(u) = 6u(1 - u) \). Amplitudes of twist-two and -three are involved. With this motivation, we focus on the twist-two amplitudes herein.

Kaon Distribution Amplitude

\[ \mu = 2 \text{ GeV} \]

skewness implies only 14% flavor symm breaking due to DCSB

\[ \left\{ \frac{m_s - m_u}{m_s + m_u} \right\} \approx 66\% \]

### Kaon DA Moments

Table 1

Moments \((u^m_\Delta = 2u - 1)\) of the \(K\)-meson PDA computed using Eqs. (11) and (12), compared with selected results obtained elsewhere: Refs. [40,41], lattice-QCD; Ref. [10], analysis of lattice-QCD results in Ref. [41]; Refs. [42–46], compilation of results from QCD sum rules; and Ref. [47], holographic soft-wall Ansatz for the kaon’s light-front wave function. We also list values obtained with \(\varphi = \varphi_{\text{asy}}\) Eq. (14), and \(\varphi = \varphi_{\text{ms}}\) Eq. (16), because they represent lower and upper bounds, respectively, for concave distribution amplitudes.

<table>
<thead>
<tr>
<th>(m^m_\Delta)</th>
<th>(m = 1)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>RL</td>
<td>0.11</td>
<td>0.24</td>
<td>0.064</td>
<td>0.12</td>
<td>0.045</td>
<td>0.076</td>
</tr>
<tr>
<td>DB</td>
<td><strong>0.040</strong></td>
<td>0.23</td>
<td><strong>0.021</strong></td>
<td>0.11</td>
<td><strong>0.013</strong></td>
<td>0.063</td>
</tr>
<tr>
<td>DSE-QCD:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lattice-QCD:</td>
<td>[40]</td>
<td>0.027(2)</td>
<td>0.26(2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[41]</td>
<td>0.036(2)</td>
<td>0.26(2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[10]</td>
<td>0.036(2)</td>
<td>0.26(2)</td>
<td>0.020(2)</td>
<td>0.13(2)</td>
<td>0.014(2)</td>
</tr>
<tr>
<td>QCD Sum Rules:</td>
<td>[42–46]</td>
<td>0.035(8)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[47]</td>
<td>0.04(2)</td>
<td>0.24(1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\varphi = \varphi_{\text{ms}})</td>
<td>0.33</td>
<td>0.33</td>
<td>0.2</td>
<td>0.2</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>(\varphi = \varphi_{\text{asy}})</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>0.086</td>
<td>0</td>
<td>0.048</td>
</tr>
</tbody>
</table>

Shi Chao, L. Chang, C.D. Roberts, P.C. Tandy, PLB738, 512 (2014)
The Pion Charge Form Factor: Transition from npQCD to pQCD

\[ F_\pi(Q^2 = uv) = \int_0^1 dx \int_0^1 dy \frac{\phi^*_\pi(x; Q)}{\phi_\pi(y; Q)} [T_H(x, y; Q^2)] \phi_\pi(y; Q) + \text{NLO/higher twist} \]


\[ Q^2 \gg \Lambda_{\text{QCD}}^2 : Q^2 F_\pi(Q^2) \to 16\pi f_\pi^2 \alpha_s(Q^2) \omega_\phi^2(Q^2) + O(1/Q^2) \]

at \( Q^2 \sim 3 - 4 \text{ GeV}^2 \), \( \Rightarrow 0.1 \)

JLab expt, Theory \( \Rightarrow 0.45 \)

But, recent DSE theory \( \Rightarrow \phi_\pi(x; \mu = 2 \text{ GeV}) \Rightarrow \omega_\phi^2 \to 3.3 \)

Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang, I. C. Cloët, C. D. Roberts, S. M. Schmidt, and P. C. Tandy

UV-QCD is not Asymptotic QCD

\[ Q^2 \gg \Lambda_{QCD}^2 : \quad Q^2 F_\pi(Q^2) \to 16\pi f_\pi^2 \alpha_s(Q^2) \omega_\phi^2(Q^2) + O(1/Q^2) \]
Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang, I. C. Cloët, C. D. Roberts, S. M. Schmidt, and P. C. Tandy

\[ Q^2 F_\pi(Q^2) \] [GeV^2]

\[ Q^2 \]

\[ Q^2 [GeV^2] \]

VMD \( \rho \) pole

DSE 2013

uv-QCD \( \phi_{\pi}^{2 \text{GeV}}(x) \)

uv-QCD \( \phi_{\pi}^{10 \text{GeV}}(x) \)

confm-QCD \( \phi_{\pi \text{asym}}(x) \)

JLab data: G. Huber et al., PRC78, 045203 (2008)

JLab 2001, 6, 8

CERN '80s

JLab 12 GeV

Cairns LHV July 2015
Pion Form Factor: Running q Mass Fn Effect

With dynamical $M_q(p^2)$

With constituent $M_q$

60% reduction

JLab data: G. Huber et al., PRC78, 045203 (2008)
Pion Parton Distribution Functions
The Leading Order PDF

\[ q_f(x) = \frac{1}{4\pi} \int d\lambda \ e^{-i x P \cdot n \lambda} \langle \pi(P) \mid \bar{\psi}_f(\lambda n) \gamma \psi_f(0) \mid \pi(P) \rangle_c \]

RL DSE:

\[ \langle x^m \rangle^{RL}_\nu = \frac{-N_c}{2P \cdot n} \text{tr} \int_\ell \Gamma_\pi(\ell - \frac{P}{2}) \left[ \left( \frac{\ell \cdot n}{P \cdot n} \right)^m n \cdot \partial_\ell S(\ell) \right] \Gamma_\pi(\ell - \frac{P}{2}) S(\ell - P) \]

Method can easily exceed the Lattice – QCD practical limit: \( m = 3 \)
Estimate 1-Pion Loop Contribution to Pion PDF

\[ \pi^+ : \langle x^1 \rangle_\mu = \int_0^1 dx \, x \{ u + \bar{u}_{\text{sea}} + \bar{d} + d_{\text{sea}} + g(x) \} \approx 2 \langle x \, q_v(x) \rangle + 4 \langle x \, q_{\text{sea}}(x) \rangle + \langle x \, g(x) \rangle = 1 \]

\[ u = u_v + u_{\text{sea}}, \quad \bar{d} = \bar{d}_v + \bar{d}_{\text{sea}} \]

Empirical GRS/ASV \Rightarrow universal \ q_v(x), \ q_{\text{sea}}(x) \ at \ \mu = 0.630 \ GeV

\[ \Gamma_\pi = \sqrt{1 - \alpha^2} \, \Gamma_{q\bar{q}}^{RL} + \alpha \, \Gamma_{\pi q\bar{q}} \]

CPT: 18% effect

\[ r_{ch}^2 = (1 - \alpha^2) \, r_{RL}^2 + \alpha^2 \, r_{\pi - lp}^2 \]

DSE-RL: \ r_{RL}^2 = r_{ch}^2 \Rightarrow \alpha^2 = 18\%

PDF Consequence:

\[ q_v(x) = (1 - \alpha^2) \, q_v^{RL}(x) + q_v^{\pi - lp}(x) \]

with \ \langle q_v^{\pi - lp}(x) \rangle = \alpha^2 = 0.18
Analysis of Pion Parton Momentum Sum Rule

TABLE II: Momentum fraction sum rule from this work at scale $Q_0 = 0.630$ GeV corresponding to the ASV [13] compilation.

<table>
<thead>
<tr>
<th>$2q_{val}^{RL}$</th>
<th>$2q_{val}^{DSE}$</th>
<th>$4q_{sea}^{ASV}$</th>
<th>gluon</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle x \rangle_\pi$</td>
<td>0.770</td>
<td>0.649</td>
<td>0.0498</td>
<td>0.300</td>
</tr>
</tbody>
</table>

Modern empirical expt parameterization:
Aicher, Shafer, Vogelsang, (ASV) PRL 105, 252003 (2010)

Many Moments via Feyn PTIR--Easy

Modern empirical expt parameterization:
Aicher, Shafer, Vogelsang, (ASV) PRL 105, 252003 (2010)

\[ Q_0 = 0.446 \text{ GeV} \]
Spacelike Correlator Approximation for PDFs
To help lattice-QCD be more applicable to hadron PDFs and GPDs than just the first 3 moments?

Parton Physics on a Euclidean Lattice

Xiangdong Ji$^{1,2}$

Standard light-cone correlator, leading twist:

$$q_f(x) = \frac{1}{4\pi} \int d\lambda \ e^{-ixP\cdot n} \langle \pi(P) | \bar{\psi}_f(\lambda n) \gamma^\mu \gamma^\nu \psi_f(0) | \pi(P) \rangle_c$$

Ji: Take large $P_z$ limit of frame-dependent equal-time correlator:

$$\tilde{q}_f(x; P_z) = \frac{1}{4\pi} \int dz \ e^{-ixP_z z} \langle \pi(P) | \bar{\psi}_f(z) \gamma^z \psi_f(0) | \pi(P) \rangle_c$$

$$\rightarrow q_f(x) \ as \ P_z \rightarrow \infty \quad \text{How fast?}$$
Typical Hadron PDF $q(x)$: a sketch for pion
Pz Dependence of quasi-pdf of u-ubar “pion”

Evaluate $q(x)$ directly using Cauchy Residue Thm for $\int_{-\infty}^{\infty} dk^-$

$$q_A(x) = i N_c \text{tr} \int \frac{dk^+ dk^- d^2k_\perp}{(2\pi)^4} \delta(k^+ - xP^+) \text{tr}[\Gamma_{\pi} S (i\gamma^+) S \Gamma_{\pi} S]$$

Evaluate $\tilde{q}(x; P_z)$ directly using Cauchy Residue Thm for $\int_{-\infty}^{\infty} dk^0$

$$\tilde{q}_A(x) = i N_c \text{tr} \int \frac{dk^0 dk_z d^2k_\perp}{(2\pi)^4} \delta(k_z - xP_z) \text{tr}[\Gamma_{\pi} S (i\gamma^z) S \Gamma_{\pi} S]$$
Pz Dependence of quasi-pdf of u-ubar “pion”

quasi-pdf, valence toy model

$\pi_q(x; P_z)$

$P_z = 0.5$
$P_z = 1$
$P_z = 2$
$P_z = 3$
$P_z = 5$

True pdf ($P_z = \infty$)

---I.Cloet, Lei Chang, PCT, in progress (2015)…….
Pz Dependence of quasi-pdf of u-ubar “pion”

valence $\pi$ toy model, quasi-pdf moments

---I.Cloet, Lei Chang, PCT, in progress (2015).......

Cairns LHV July 2015
• **Parton Distribution Amplitudes** (pion, kaon). DSE approach shows good contact with available lattice-QCD moments. Flavor symmetry breaking in K DA made quantitative. Helps identify that the ultraviolet partonic behavior is just about within reach of JLab pion FF experiments. Expect soon a clarification of what ultraviolet “limit” the differing BaBar—Belle data should be compared to for the pion transition FF.

• **Parton Distribution Functions** (pion). Qualitative behavior of empirical data fits reproduced by DSE q-qbar + pion loop analysis. Further work in progress.

• X. Ji’s **space-like correlator approach to PDFs**—a model investigation. Spurious sea-quark contributions seem unavoidable if Pz < 2 GeV. For x > 0.8, need Pz > 4 GeV for confidence in the shape. Further work in progress.
Continuum QCD, Dyson-Schwinger Eqns and Hadron Physics

Collaborators:

- Craig Roberts, Argonne National Lab, USA
- Adnan Bashir, University of Michoacan, Morelia, Mexico
- Ian Cloet, Argonne National Lab, USA
- Hong-shi Zong, Nanjing Univ, China
- Lei Chang, Peking U, Argonne/Julich/Univ Adelaide, Australia
- Chao Shi, Nanjing Univ, [visiting Kent State U]
- Konstantin Khitrin, PhD student, Kent State Univ, USA
- Javier Cobos-Martinez, Univ of Sonora, Mexico
The End
Where Asym FF Could be Calculated, its Power Law was Correct:-

\[ \gamma^* \pi \gamma^* \text{ Asymptotic Limit} \]

Lepage and Brodsky, PRD22, 2157 (1980): LC-QCD/OPE ⇒

![Graph showing the asymptotic limit of the gamma* pi gamma* function with various curves representing different theoretical models.](image-url)
Pion Transition Form Factor

\[ F(Q^2) = \frac{2f_\pi}{Q^2} \omega_\phi(Q^2) + \text{NLO/higher twist} \]

\[ \omega_\phi(Q^2) = \frac{1}{3} \langle x^{-1} \rangle Q^2 \rightarrow 1, \quad Q^2 \rightarrow \infty \]

\[ \gamma^* \gamma - \pi \]

Leading (BL) term with \( \phi_\pi(x; \mu=Q) \)

Asympt/Conformal QCD limit (BL) with \( \phi_\pi^{\text{asy}}(x) \)