A Perspective on Dyson-Schwinger equation

toy model of Pion

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Outline

- Introduction
- Pion Distribution Amplitude
- Pion gamma Transition Form Factor
- Conclusions

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Introduction

The Challenge of QCD

- QCD is the only known example in nature of a fundamental quantum field theory that is innately nonperturbative; Solving QCD will have profound implications for our understanding of the natural world.

- QCD is characterized by two emergent phenomena:

  confinement and dynamical chiral symmetry breaking(DCSB);
  a world without DCSB would be profoundly different.

- Must discover the origin of confinement, its relationship to DCSB and understand how these phenomenon influence hadronic observables

Introduction

Nature's strong messenger--Pion

- 1947-Pion discovered by Cecil Frank Powell; almost massless, Goldstone mode associated with dynamical chiral symmetry breaking in QCD; strong couplings to baryons.
- **Measure π γ transition form factor in space like region**

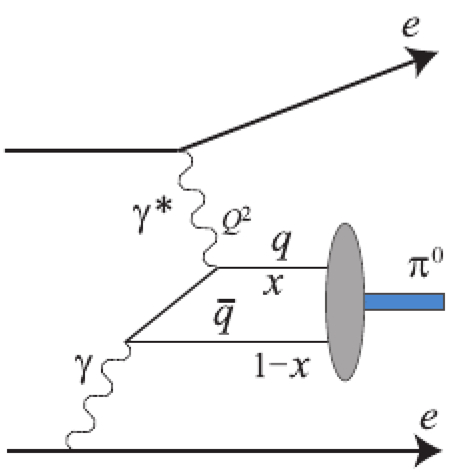

FIG. 1: A Feynman diagram for $\gamma\gamma^* \to q\bar{q} \to \pi^0$ in e^+e^- collisions

CELLO (1991) 0.7-2.2 GeV^2
CLEO (1998) 1.6-8.0 GeV^2
BaBar (2009) 4-40 GeV^2
Belle (2010) 4-40 GeV^2

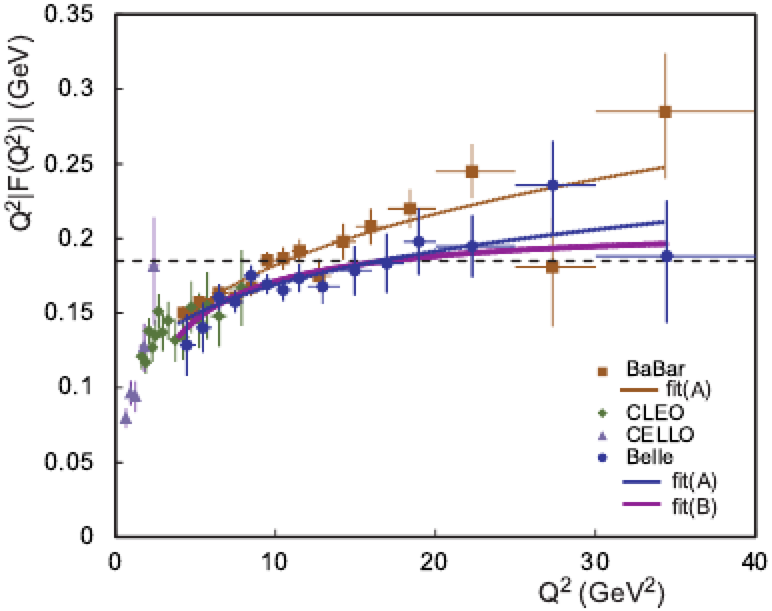

FIG. 24: Comparison of the results for the product $Q^2|F(Q^2)|$ for the π^0 from different experiments. The error bars are a quadratic sum of statistical and systematic uncertainties. For the Belle and BaBar results, only a Q^2-dependent systematic-error component is included. The two curves denoted fit(A) use the BaBar parameterization while the curve denoted fit(B) uses Eq.(23) (see the text). The dashed line shows the asymptotic prediction from pQCD (∼ 0.185 GeV).

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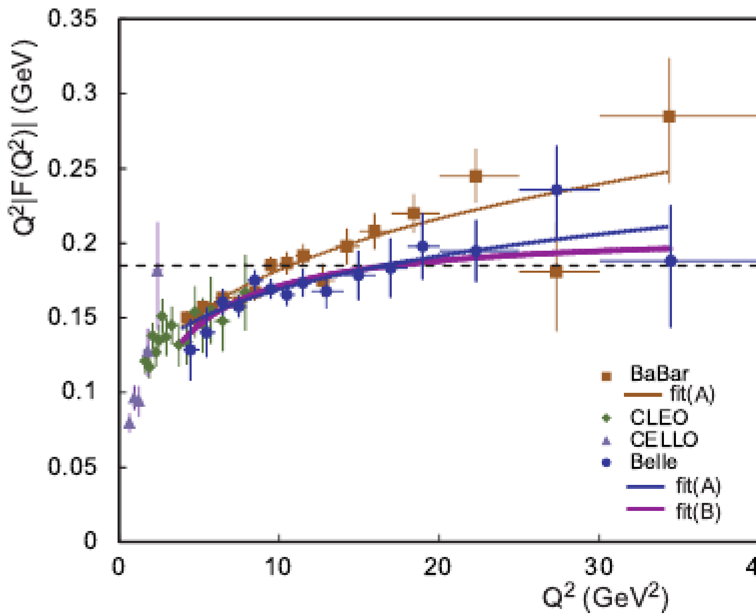
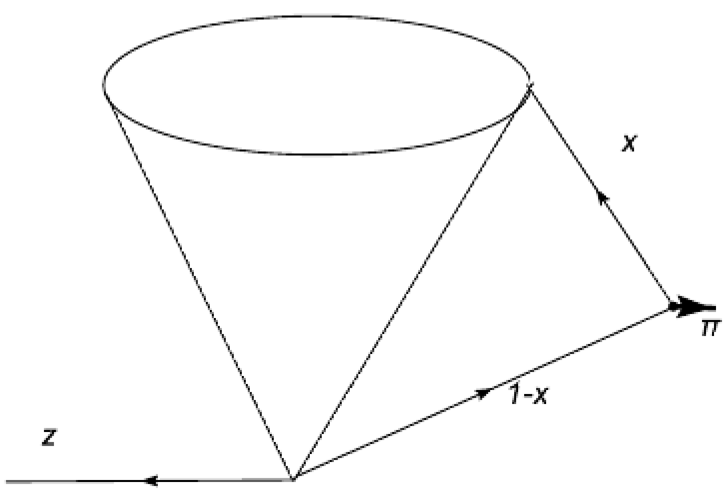


FIG. 24: Comparison of the results for the product $Q^2|F(Q^2)|$ for the π^0 from different experiments. The error bars are a quadratic sum of statistical and systematic uncertainties. For the Belle and BaBar results, only a Q^2-dependent systematic-error component is included. The two curves denoted fit(A) use the BaBar parameterization while the curve denoted fit(B) uses Eq.(23) (see the text). The dashed line shows the asymptotic prediction from pQCD ($\sim$ 0.185 GeV).

✯ Pion gamma transition form factor at lowest-order pQCD and leading twist 2

$$Q^2F(Q^2) = 2f_\pi \frac{1}{3}\int_0^1 dx \frac{\varphi_\pi^{(2)}(x)}{1-x}$$



Introduction

Nature's strong messenger--Pion

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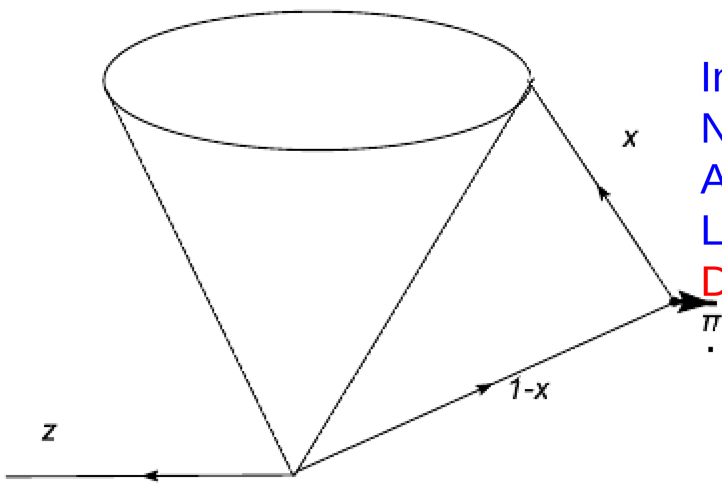
$$\langle 0|\bar{q}(-x)\gamma_5\gamma_\mu L[-x,x]q(x)|\pi(P)\rangle = f_\pi P_\mu \int_0^1 du e^{-i(2u-1)x\cdot P}\varphi_\pi^{(2)}(u) + \text{high twist}$$

☆ Pion gamma transition form factor at lowest-order pQCD and leading twist 2

$$Q^2F(Q^2) = 2f_\pi \frac{1}{3}\int_0^1 dx \frac{\varphi_\pi^{(2)}(x)}{1-x}$$

Distribution amplitudes are nonperturbative quantities



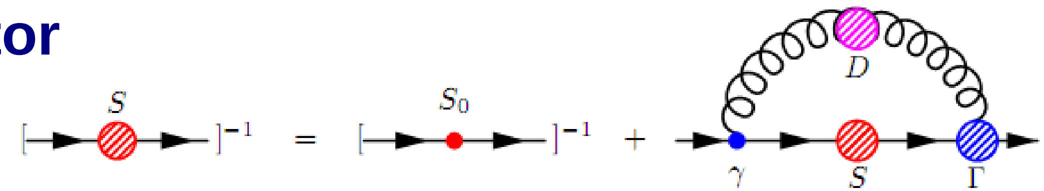
QCD Sum Rules (Chernyak, Zhitnitsky 1982)
Nonlocal QCD Sum Rules (Bakulev, Mikhailov, BMS...)
Instanton-vacuum (Dorokhov, Polyakov...)
NJL (Arriola et al...)
AdS/QCD (Brodsky et al...)
Lattice QCD (Braun et al; Donnellan et al; Arthur et al...)
Dyson-Schwinger equations (Chang et al...)
......

Meeting Challenge—QCDs Dyson-Schwinger Equations

➢ The equations of motion of QCD

- Each Green function satisfies integral equation involving other functions...an infinite tower of coupled integral equations
- The symmetric preserving truncation is necessary

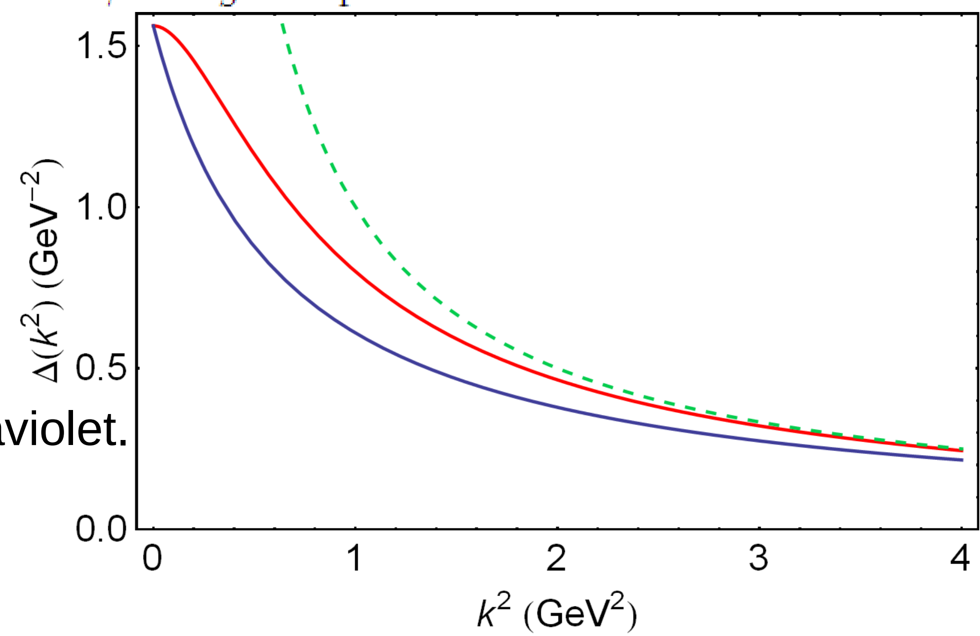
➢ The most important DSE is QCDs gap equation---quark propagator



Modeling gluon propagator

Si-xue Qin,et al
Phys. Rev. C. 84 (2011) 042202
Phys. Rev. C. 85 (2012) 035202

Landau Gauge;
Infrared constant;
Massive-like infrared and Massless in ultraviolet.



Introduction

Meeting Challenge—QCDs Dyson-Schwinger Equations

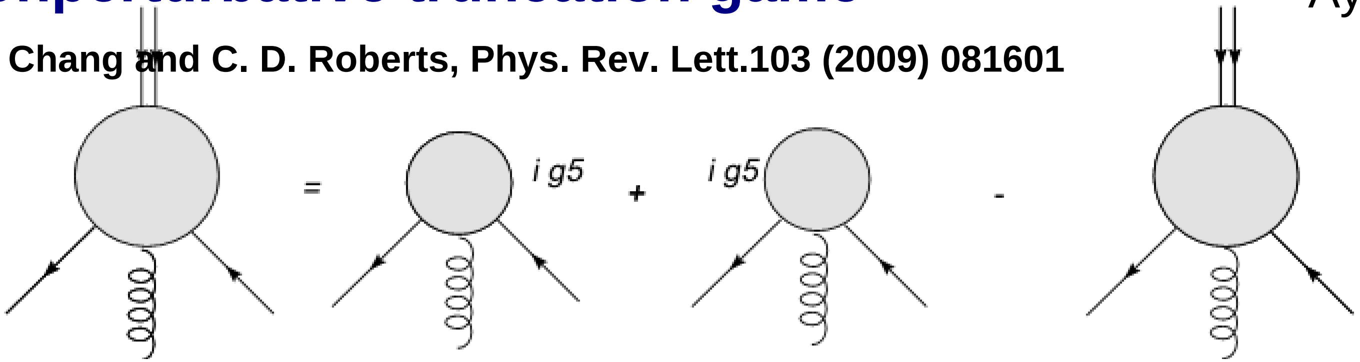
- **Model quark-gluon vertex**

J. S. Ball and Ting-Wai Chiu, Phys. Rev. D 22 (1980) 2542
Lei Chang, Yu-xin Liu and C. D. Roberts, Phys. Rev. Lett.106 (2011) 072001
Lei Chang and C. D. Roberts, Phys. Rev. C 85 (2012) 052201
Si-xue Qin, Lei Chang, Yu-xin Liu, C. D. Roberts, S. Schmidt, Phys. Lett. B 722, 384 (2013)

See Sixue Qin's talk
Ayse Kizilersu's talk

- **Nonperturbative truncation game**

Lei Chang and C. D. Roberts, Phys. Rev. Lett.103 (2009) 081601



$$P_\mu \Lambda^{us}_{5\mu\beta}(k,q;P) = \Gamma^u_\beta(q_+,k_+)i\gamma_5 + i\gamma_5\Gamma^s_\beta(q_-,k_-) - i(m_u+m_s)\Lambda^{us}_{5\beta}(k,q;P)$$

- **Necessary and sufficient to ensure axial-vector identity satisfied.**
- **Kernel's Ward-Takahashi identity provides means by which to construct a symmetry preserving kernel of the Bethe-Salpeter equation that is matched to any reasonable Ansatz for the dressed-quark-gluon vertex**
- **Solution is not unique, the separate ansatz is useful.**

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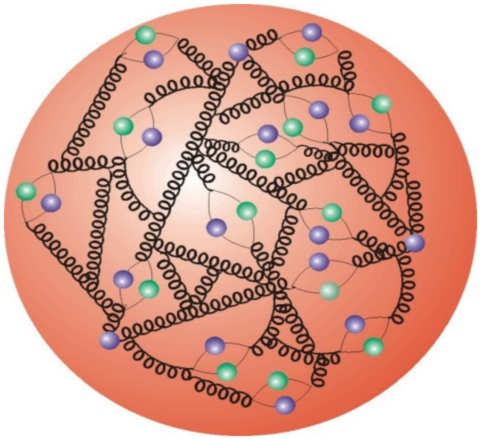
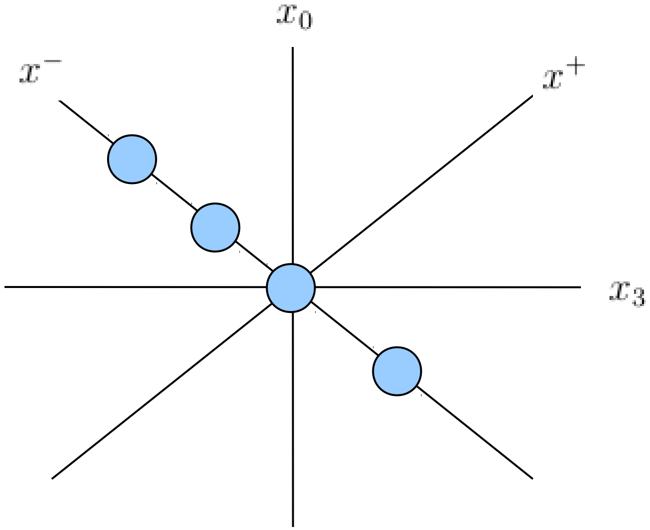
Pion's valence quark Distribution Amplitude

Quark and gluon fields are distributed along the light-cone x^- direction.





Valence quark picture

Definitive of a hadron – it's how we tell a proton from a neutron

Expresses charge; flavour; baryon number; and other Poincaré-invariant macroscopic quantum numbers

Parton physics involves time-dependent dynamics

- **Exact expression in QCD for the pion's valence-quark PDA**

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\,\gamma_5\gamma\cdot n\, S(k)\Gamma_\pi(k;P)S(k-P)$$

Fact:

Solving quark and amplitude in Euclidean Space;

Limited information of Quark propagator at k^2>=0;

Limited information of pion Amplitude at k^2>0 and **-1<k.P/sqrt[k^2 P^2]<1**

Physical require:

|k.P/sqrt[k^2 P^2]|<Infinity

For more elusive we need to know of amplitude with one quark leg moment square fixed but another goes to infinity

Analytical representation of BS amplitude

General Nakanishi integral representation

$$\begin{aligned}\Gamma_\pi(k;P) &= \gamma_5\{\gamma\cdot PF(k;P)+\gamma\cdot kG(k;P)+iE(k;P)+\sigma_{\mu\nu}k_\mu P_\nu H(k;P)\}\\ &= \gamma_5\left\{\int_{-1}^{1}dz\int_0^\infty d\gamma\frac{\gamma\cdot Pf(\gamma,z)+\gamma\cdot kg(\gamma,z)+ie(\gamma,z)+\sigma_{\mu\nu}k_\mu P_\nu h(\gamma,z)}{(k^2+zk\cdot P+M^2+\gamma)^2}\right\}\end{aligned}$$

Approximation-I: reduce to 1 dimension

$$e(z,\gamma)\to\rho(z)\delta(\gamma)$$

Lose k_\perp nontrivial dependence;
Integral properties appropriate correct

Approximation-II: numerical fighting

In practice we fit weight function rho(z) by the limited knowledge of Bethe-Salpeter amplitude;
There are some uncertainties in the approach of extracting rho(z)
Uncertainties would provide error estimates for the PDA and transition form factor

Relate weight function to PDA

An example to illustrate the relation

Quark propagator:

$$S^{-1}(k) = Z_2(i\gamma \cdot k + M)$$

Bethe-Salpeter Amplitude:

$$\Gamma_\pi(k;P) = i\gamma_5 \alpha Z_2 \frac{M}{f_\pi} \int_{-1}^{1} dz \rho(z) \frac{M^2 k \cdot P}{(k^2 + zk \cdot P + M^2)^2} + \text{regularization at bound points}$$

rho(z) is weight function accounting relative motion between the partons
rho(z) is kind of odd function;
rho(z) take no any singularity;

Twist-2 PDA:

$$f_\pi \varphi(x) = Z_2 \mathrm{tr}_{\mathrm{DC}} \int \frac{d^4k}{(2\pi)^4} \delta(n \cdot k - xn \cdot P) \gamma_5 \gamma \cdot n S(k) \Gamma_\pi(k - \frac{P}{2}; P) S(k - P)$$

Corresponding relation between rho(z) and PDA
Chiral limit would be considered for simplicity
Z2 wave function renormalization constant

Solving Euclidean PDA

Method-I: local moment

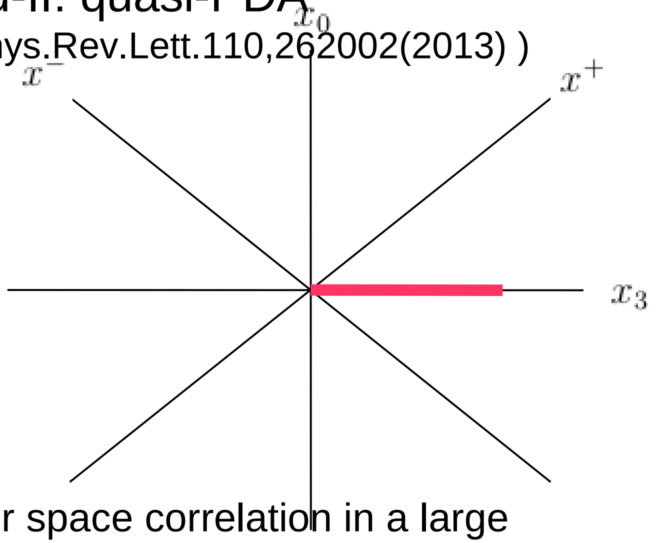
$$\int_0^1 dx x^m \varphi(x) \sim \int_{dk} (n \cdot k)^m \mathrm{tr}[\ldots]$$

Limit number of moment can be calculated on Lattice which can not give enough information about PDA;

Within DSEs approach the moments can be calculated to any large value if one performed analytical representation of quark propagator and BS amplitude; PDA can be extracted with enough information of moments.

Method-II: quasi-PDA

(X. Ji, Phys.Rev.Lett.110,262002(2013))



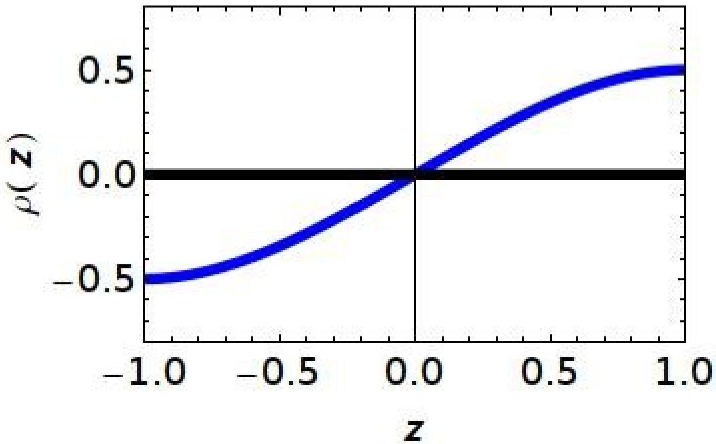
Consider space correlation in a large momentum P in the z-direction

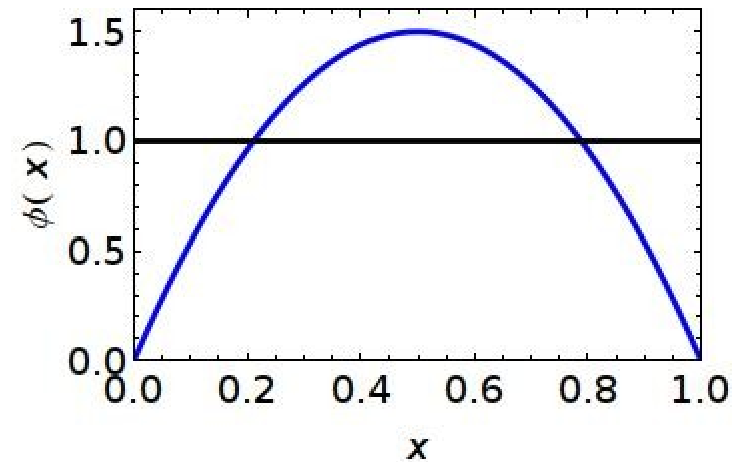
Quark and gluon fields distributed in the z-direction; The matrix element depends on the momentum P; One reaches the stand distribution as P \to infinity;

It is possible to perform finite P calculation on Lattice(matching condition between finite P and stand case);

Relate weight function to PDA

$$f_\pi \varphi^{(2)}(x) = Z_2 \mathrm{tr}_{\mathrm{DC}} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\gamma_5 \gamma\cdot n S(k)\Gamma_\pi(k-\frac{P}{2};P)S(k-P)$$





Blue line

$$\frac{\mathrm{d}\rho^{\text{up limit}}(z)}{\mathrm{d}z} = \frac{3}{4}(1-z^2) \Longrightarrow \varphi(x) = 6x(1-x)$$

QCD's conformal limit

Black line

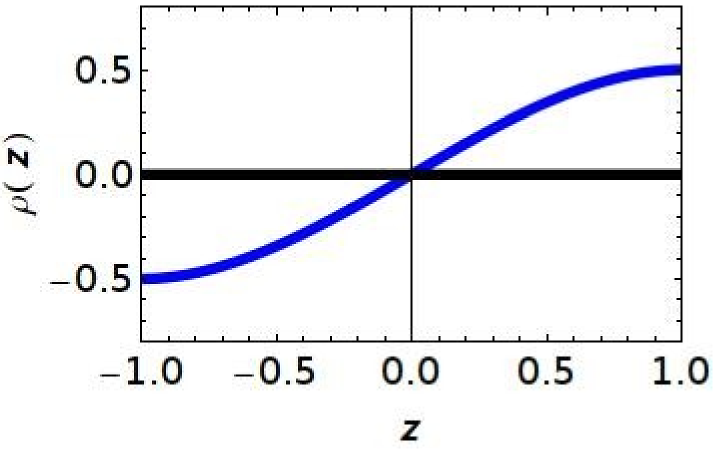
$$\frac{\mathrm{d}\rho^{\text{down limit}}(z)}{\mathrm{d}z} = \frac{1}{2}(\delta(1+z)+\delta(1-z)) \Longrightarrow \varphi(x) = 1$$

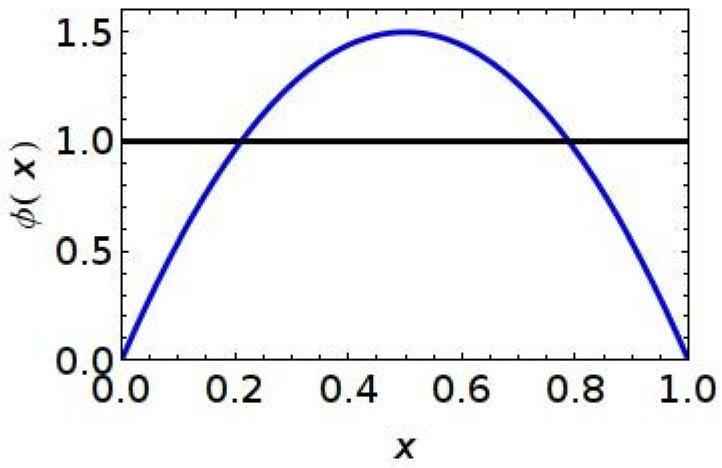
Point-particle-like characteristics:

One assigns equal probability to two distinct configurations: valence-quark with all the pion's momentum and valence-antiquark with none or antiquark with all the momentum and quark with none.

Relate weight function to PDA

$$f_\pi \varphi^{(2)}(x) = Z_2 \mathrm{tr}_{\mathrm{DC}} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\gamma_5 \gamma\cdot n S(k)\Gamma_\pi(k - \frac{P}{2};P)S(k-P)$$





$$\rho(z) \to \rho(z;\mu)$$

rho(z) should be scale dependent

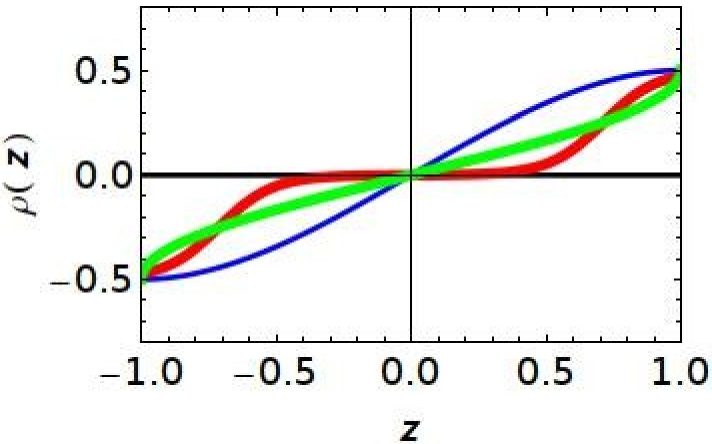
Down limit produce a very low hadronic scale PDA;

Up limit provide a appropriate conformal limit;

We estimate the true weight function would lay between these two limit.

Default scale is 1GeV!

Modeling weight function



Model-I: A smooth approximation to the step function

Red Line $$\rho_I(z) = \frac{1}{2}\left(\frac{1}{1+e^{\frac{-z+z_0}{t}}} - \frac{1}{1+e^{\frac{z+z_0}{t}}}\right)$$

Model-II: An integrable singularity form at boundary

Green Line $$\rho_{II}(z) = \mathrm{ArcSin}[z]$$

Model-I: the Bethe-Salpeter amplitude of Pion decreases as

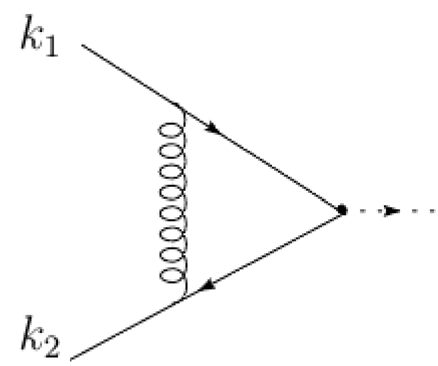
$$\Gamma_\pi(k_1^2, k_2^2) \to \frac{1}{k_1^2}$$

as $k_1^2 \to \infty$ with k_2^2 fixed finite, or $k_1^2 \to \infty$ with k_1^2/k_2^2 fixed finite.

Model-II: the Bethe-Salpeter amplitude of Pion decreases as

$$\Gamma_\pi(k_1^2, k_2^2) \to \frac{1}{\sqrt{k_1^2}}, \frac{1}{k_1^2}$$

as $k_1^2 \to \infty$ with k_2^2 fixed finite and as $k_1^2 \to \infty$ with k_1^2/k_2^2 fixed finite.

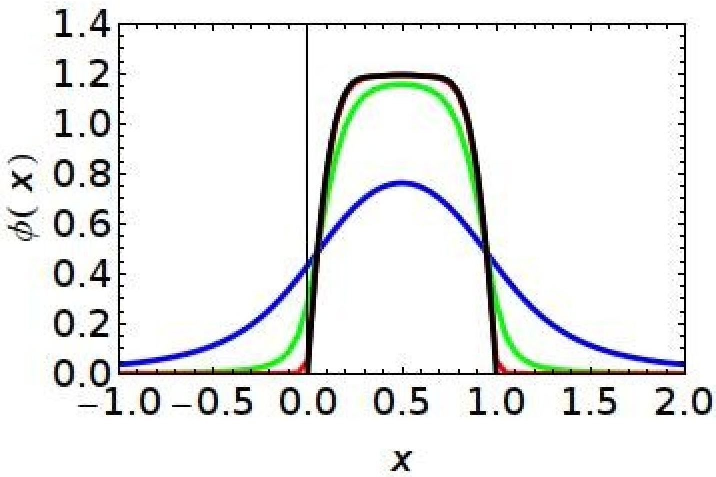


M=0.306GeV to produce
Fpion=0.0924GeV

A Euclidean quasi-distribution

Model-I illustration

Quasi-PDA at finite P3

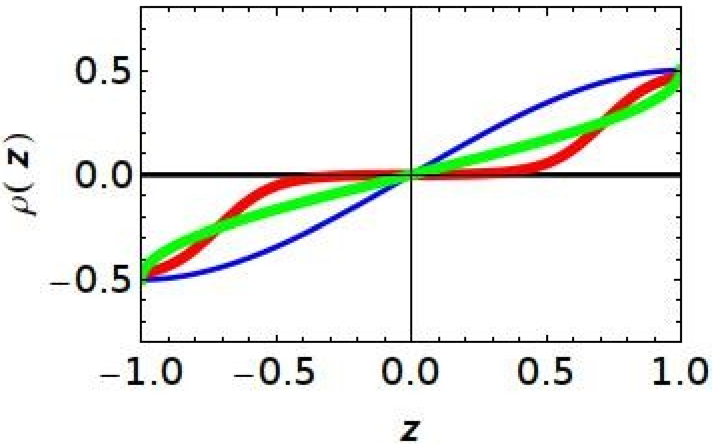


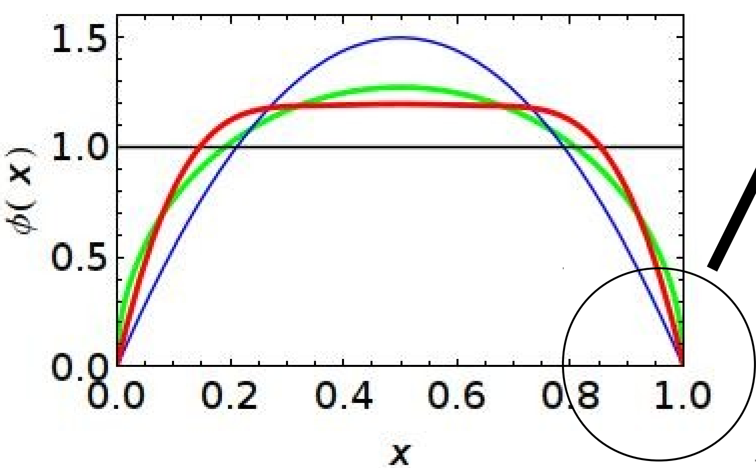
Blue Line: P3/M=2
Green Line: P3/M=10
Red Line: P3/M=100
Black Line: P3/M=Infinity

Open Question: could we read stand PDA based on finite P3 quasi-PDA?
Perturbative one-loop matching condition?
Nonperturbative matching condition?

PDAs
Model-I vs Model-II

M=0.306GeV to produce Fpion=0.0924GeV

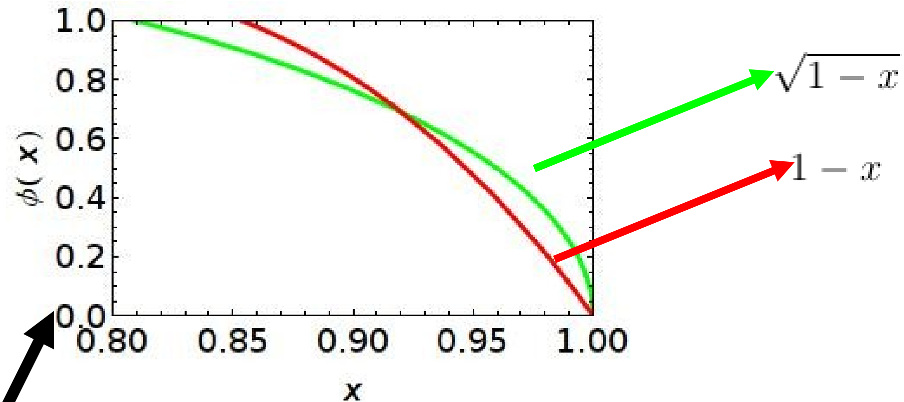




1,Second moment $\int dx\varphi(x)(2x-1)^2$

NJL-like	Model-I	Model-II	Asymptotic
1/3	1/4	1/4	1/5

2,x \to 1 limit



3,Inverse moment $\int dx \frac{\varphi(x)}{x}$

NJL-like	Model-I	Model-II	Asymptotic
Infinity	3.45	4	3

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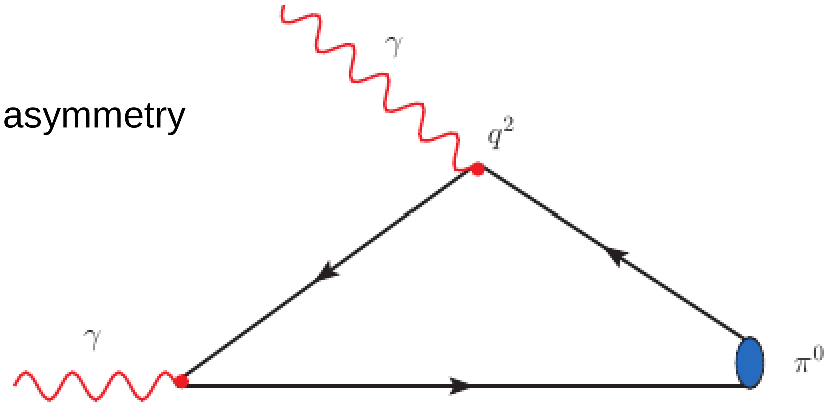
Dressed triangle diagram

- Nonperturbative expression of pion gamma transition form factor

$$\frac{1}{4\pi^2 f_\pi}\epsilon_{\mu\nu\rho\sigma}k_1^\rho k_2^\sigma G(Q_1^2,\omega^2) =$$
$$-2\mathrm{tr}_D\int \Gamma_\mu(k+k_2, k-k_1+k_2)S(k-k_1+k_2)\Gamma_\nu(k-k_1+k_2)\Lambda_\pi(k-\frac{k_1}{2}+\frac{k_2}{2};-k_1-k_2)$$

with $\omega^2 = \frac{Q_1^2-Q_2^2}{Q_1^2+Q_2^2}$. Photon asymmetry



- Chiral anomaly(Saturated with a triangle diagram)

$$P_\mu i\Gamma_\mu(k;P) = S^{-1}(k+\frac{P}{2}) - S^{-1}(k-\frac{P}{2})$$
$$P_\mu i\chi_{5\mu}(k;P) = S(k+\frac{P}{2})\gamma_5 + \gamma_5 S(k-\frac{P}{2})$$
$$S(k) = \frac{1}{i\gamma\cdot kA(k^2)+B(k^2)} = -i\gamma\cdot k\sigma_A(k^2)+\sigma_B(k^2)$$

$$\Gamma_\mu(k;P) = \gamma_\mu\frac{A(k_+)+A(k_-)}{2} + 2t_\mu\gamma\cdot t\frac{A(k_+)-A(k_-)}{k_+^2-k_-^2}+\ldots$$

$$\Longrightarrow G(0,\omega)=1$$

$$F(k;P) \to \sigma_A(k^2)$$
$$G(k;P) \to 2k\cdot P\sigma_A'(k^2)$$

Asymptotic Pion Transition Form Factor

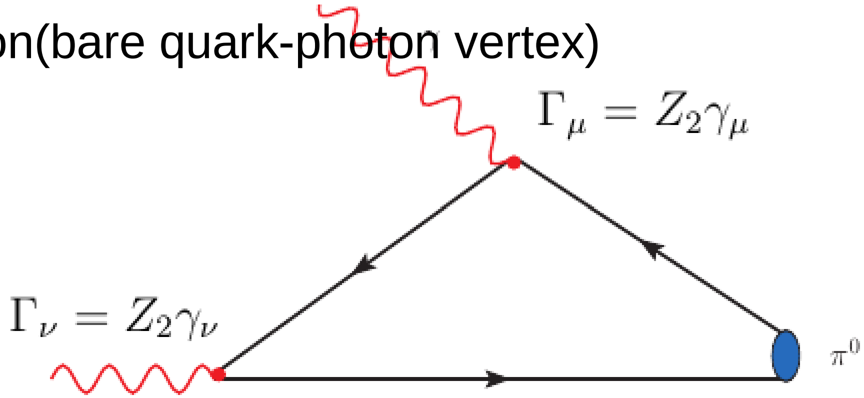
- Nonperturbative expression of pion gamma transition form factor

$$\frac{1}{4\pi^2 f_\pi}\epsilon_{\mu\nu\rho\sigma}k_1^\rho k_2^\sigma G(Q_1^2,\omega^2) =$$
$$-2\mathrm{tr}_D\int \Gamma_\mu(k+k_2, k-k_1+k_2)S(k-k_1+k_2)\Gamma_\nu(k-k_1+k_2)\Lambda_\pi(k-\frac{k_1}{2}+\frac{k_2}{2};-k_1-k_2)$$

with $\omega^2 = \frac{Q_1^2-Q_2^2}{Q_1^2+Q_2^2}$. Photon asymmetry

- UV leading order contribution(bare quark-photon vertex)

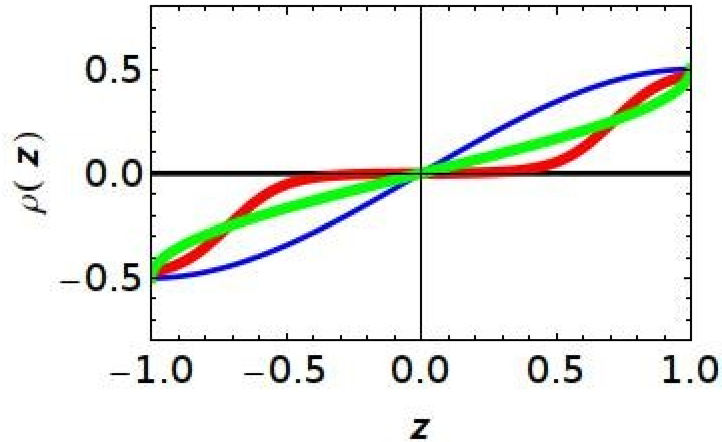
Exactly !

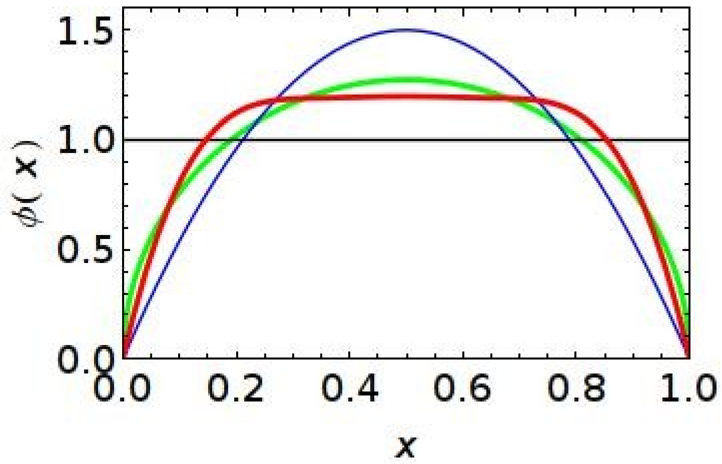


$$\frac{1}{4\pi^2 f_\pi}G(Q_1\to\infty,\omega^2=1)\to 8f_\pi\int_{-1}^{1}\mathrm{d}z\frac{\log\left[\frac{1+z}{2}\right]}{3(z-1)}\frac{\mathrm{d}\rho(z)}{\mathrm{d}z} = 2f_\pi\int_0^1\mathrm{d}x\frac{\varphi(x)}{3(1-x)}$$

rho(z) and PDA fixed at 2GeV

Transition Form Factor

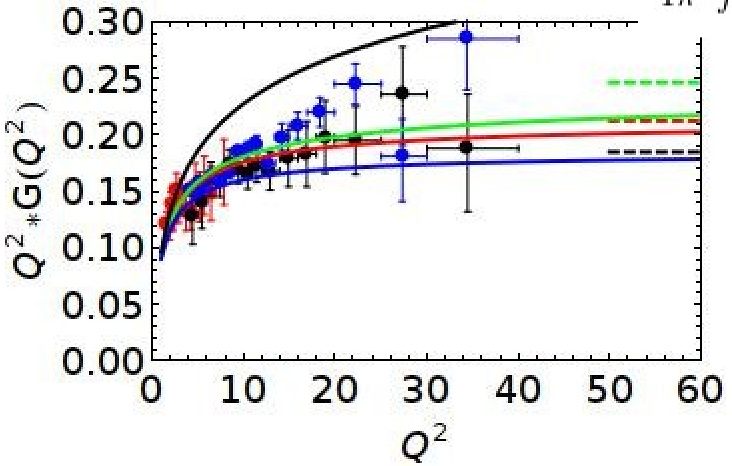




Triangular diagram calculation

Large Q limit

$$\frac{1}{4\pi^2 f_\pi} Q^2 G(Q^2 \to \infty) = 2 f_\pi \int_0^1 dx \frac{\varphi(x)}{3(1-x)}$$



Evolution of Bethe-Salpeter Amplitude

Model-I illustration

With the input of weight function

$$\rho_I(z) = \frac{1}{2}\left(\frac{1}{1+\mathrm{e}^{\frac{-z+z_0}{t}}} - \frac{1}{1+\mathrm{e}^{\frac{z+z_0}{t}}}\right)$$

the PDA can be well expressed by

$$\varphi(x) = 6x(1-x)\left(1 + a_2 C_2^{3/2}(2x-1) + a_4 C_4^{3/2}(2x-1) + a_6 C_6^{3/2}(2x-1) + a_8 C_8^{3/2}(2x-1)\right)$$

We find the following modified weight function

$$\rho_{I,\mathrm{m}}(z) = \frac{3}{4}(1-z^2)\left(1 + 6a_2 C_2^{3/2}(z) + 15a_4 C_4^{3/2}(z) + 28a_6 C_6^{3/2}(z) + 45a_8 C_8^{3/2}(z)\right)$$

can produce the above PDA exactly.

The coefficients in PDA can be evoluted to any scale as

$$a_n(\mu) = a_n(\mu_0)\left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{-\frac{\gamma_n}{\beta_0}}$$

with $\beta_0 = 11 - \frac{2}{3}N_f$, $\gamma_j = \frac{4}{3}\left(3 + \frac{2}{(j+1)(j+2)} - 4\sum_{k=1}^{j+1}\frac{1}{k}\right)$ and $\alpha[\mu]$ is the running coupling

$$\rho(z)$$

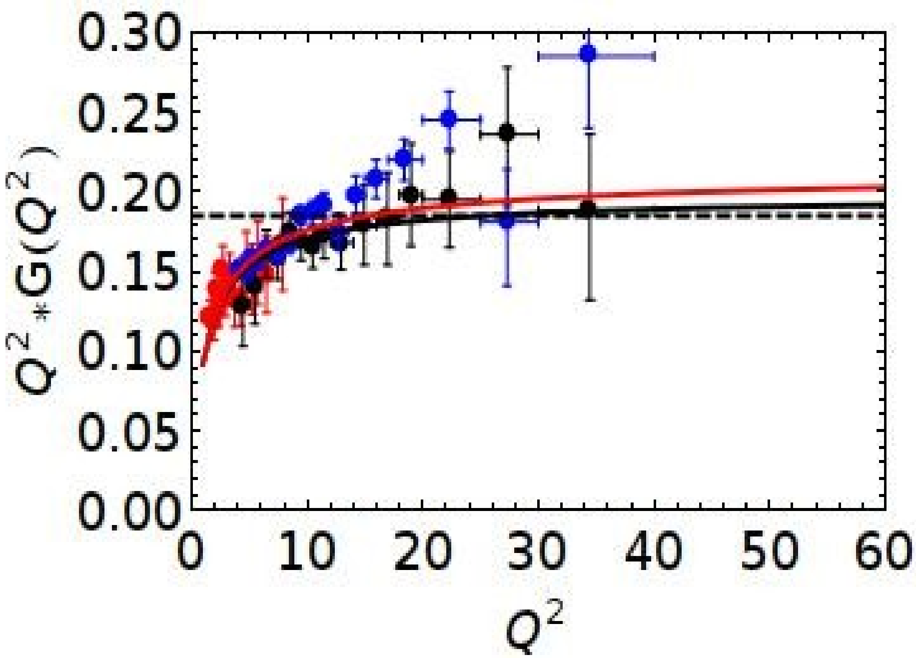
$$\Big\downarrow$$

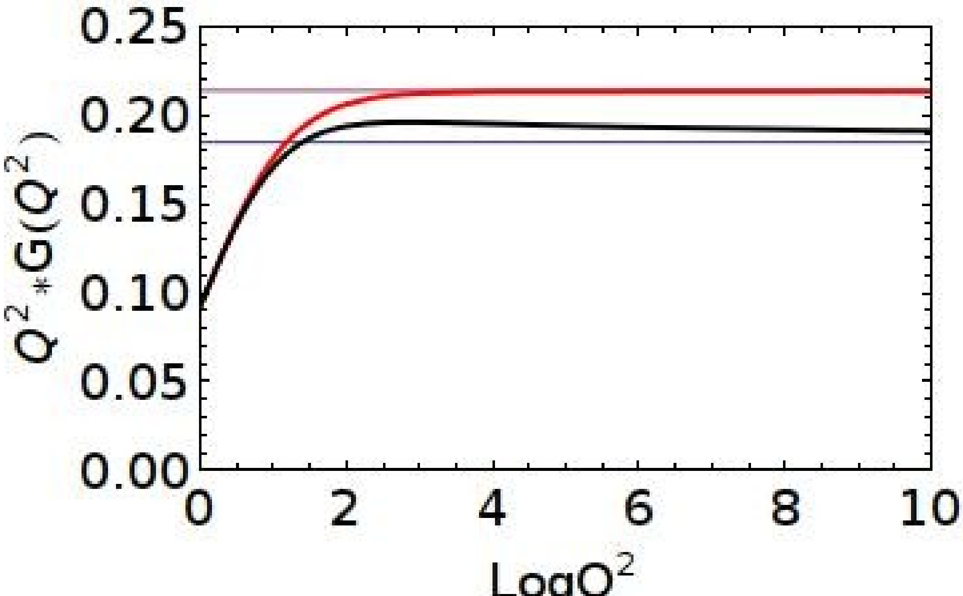
$$\rho(z,\mu)$$

$$\rho(z,\mu^2=Q^2)$$

Evolution of Transition Form Factor

Model-I illustration



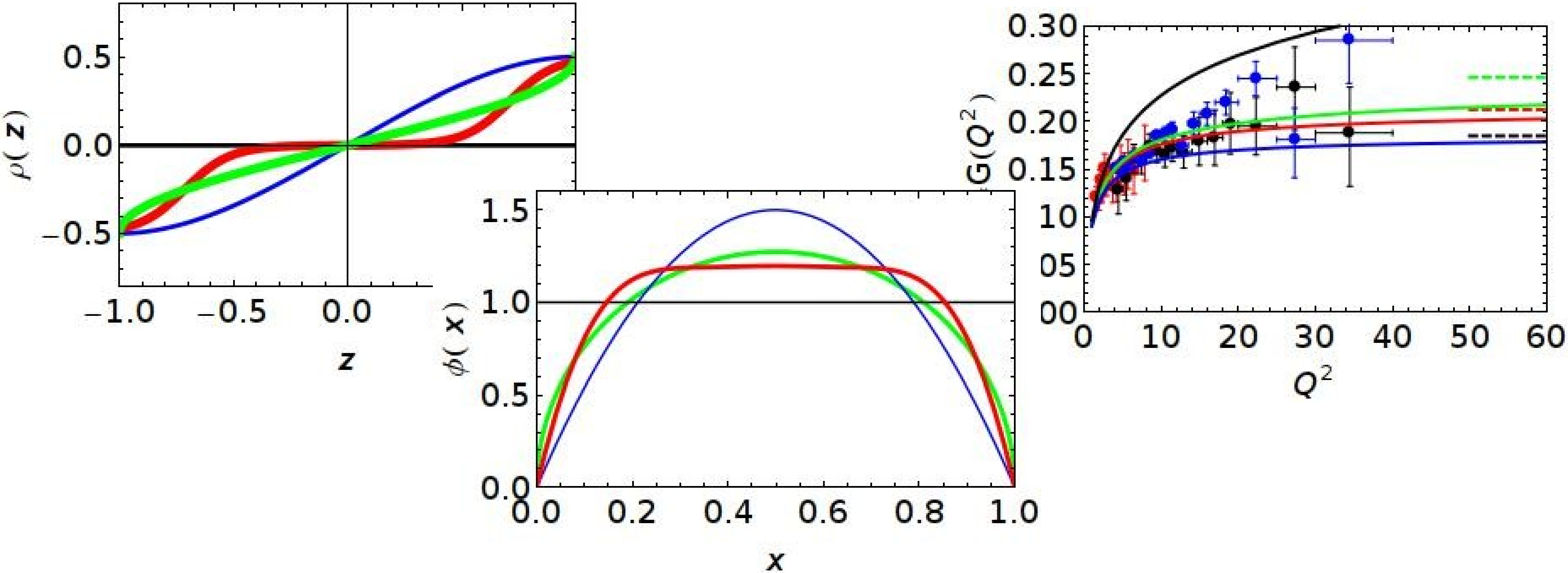


Red Line: Before evolution
Black Line: After evolution

- At Q2=60GeV^2 the evolution suppress form factor about 6%;
- Without evolution the form factor reaches its limit(1GeV scale PDA) from below ;
- Including evolution the form factor reaches asymptotic limit(6x(1-x)PDA) from above very slowly.

Outline

- Introduction
- Pion Distribution Amplitude
- Pion gamma Transition Form Factor
- Conclusions



- We study the relation between pion BS amplitude and its twist-2 PDA;

- The relation of PDA and pion gamma transition form factor can be read from the BS amplitude;

- A practical method to include QCD evolution in BS amplitude;

Thank you