Nucleon Spectroscopy with Multi-Particle Operators

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Correlation Matrix Techniques

- Begin by constructing an $N \times N$ basis of cross correlation functions

$$\mathcal{G}_{ij}^{\pm}(\vec{p}, t) = \sum_{\vec{x}} \mathrm{e}^{-i\vec{p}\cdot\vec{x}}\, \mathrm{Tr}_{\mathrm{sp}} \left[\Gamma_{\pm} \langle\, \Omega \,|\, \chi_i(\vec{x}, t)\, \overline{\chi}_j(\vec{0}, t_{src}) \,|\, \Omega \,\rangle \right]$$
$$= \sum_{\alpha} \lambda_i^{\alpha} \bar{\lambda}_j^{\alpha} \mathrm{e}^{-m_{\alpha} t}$$

- α enumerates the energy eigenstates of mass m_α and parity $\pm$ that we have projected with $\Gamma_{\pm} = (\gamma_0 \pm 1)/2$, λ_i^{α} and $\bar{\lambda}_j^{\alpha}$ are the couplings of our creation and annihiliation operators $\overline{\chi}_j$ and χ_i at the source and sink respectively.

Correlation Matrix Techniques (cont.)

- We then search for a linear combinations of operators

$$\phi^\alpha = \sum_i \chi_i v_i^\alpha \qquad \text{and} \qquad \bar{\phi}_j^\alpha = \sum_j \bar{\chi}_j u_j^\alpha$$

such that ϕ and $\bar{\phi}$ couple to a single energy eigenstate.

- One can then see from our cross correlation matrix equation that

$$\mathcal{G}_{ij}(t_0 + \Delta t) u_j^\alpha = \mathrm{e}^{-m_\alpha \Delta t} \mathcal{G}_{ij}(t_0) u_j^\alpha$$

- Hence the required values for u_j^α and v_i^α can be obtained from solving the eigenvalue equations

$$\left[\mathcal{G}^{-1}(t_0)\,\mathcal{G}(t_0 + \Delta t)\right]_{ij} u_j^\alpha = c^\alpha u_i^\alpha$$
$$v_i^\alpha \left[\mathcal{G}(t_0 + \Delta t)\,\mathcal{G}^{-1}(t_0)\right]_{ij} = c^\alpha v_j^\alpha,$$

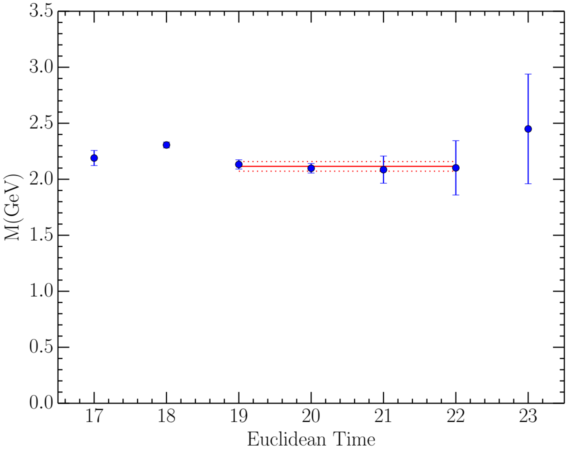
Correlation Matrix Techniques (cont.)

- As our correlation matrix is diagonalised at t_0 and $t_0 + \Delta t$ by the eigenvectors u_j^α and v_i^α we can obtain the eigenstate projected correlator

$$\mathcal{G}_\pm^\alpha = v_i^\alpha \mathcal{G}_{ij}^\pm u_j^\alpha$$

which is then use to extract a mass.

Typical mass fit



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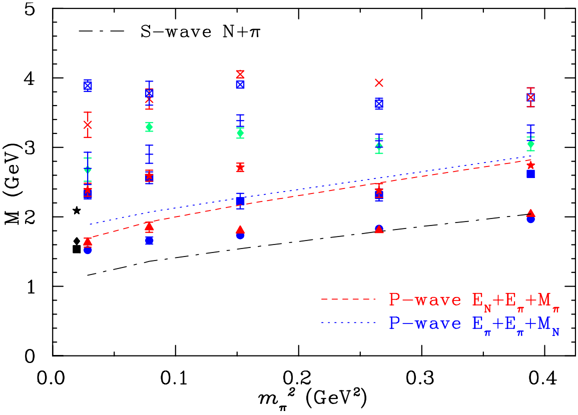
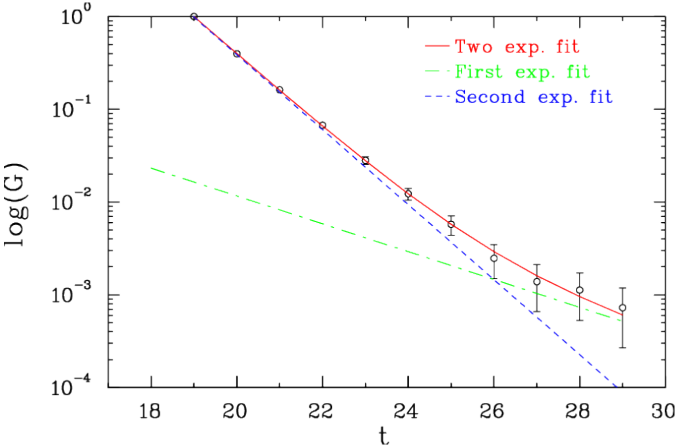
3-Quark Operator Results



$$\chi_1 = \epsilon^{abc}(u^{aT} C\gamma_5 d^b)u^c \quad \text{and} \quad \chi_2 = \epsilon^{abc}(u^{aT} C d^b)\gamma_5 u^c$$

M. S. Mahbub *et al.* [CSSM Lattice Collaboration], Phys. Rev. D **87**, 011501 (2013).

3-Quark Operator Results (cont.)



M. S. Mahbub, W. Kamleh, D. B. Leinweber and A. G. Williams, Annals Phys. **342**, 270 (2014)

3-Quark Operator Results (cont.)

t_0	$\triangle t$	$t_{\max}$	M_1	M_2	λ_1	λ_2	χ^2/dof
18	1	28	1.54(25)	2.45(41)	1.83(1.95)	6.22(1.23)	0.50
18	2	28	1.53(39)	2.36(50)	1.60(2.83)	6.19(2.02)	0.48
18	3	28	1.56(43)	2.37(60)	1.75(3.38)	6.02(2.48)	0.48
18	1	29	1.49(30)	2.38(40)	1.48(2.02)	6.43(1.28)	0.47
18	2	29	1.43(49)	2.26(41)	1.00(2.53)	6.60(1.77)	0.36
18	3	29	1.45(56)	2.25(49)	1.05(3.04)	6.52(2.20)	0.35
19	1	28	0.91(85)	1.95(11)	0.12(0.77)	16.25(0.97)	0.11
19	2	28	1.06(99)	1.97(20)	0.25(2.54)	16.31(1.58)	0.16
19	1	29	0.71(68)	1.93(06)	0.04(0.20)	16.05(0.92)	0.10
19	2	29	0.78(85)	1.93(08)	0.06(0.40)	16.09(1.03)	0.10

$\rightarrow$ no prediction

Toy Model

- Consider a simple 2-component toy model with QCD eigen-states given by

$$\begin{aligned} |a\rangle &= \cos\theta\,|1\rangle + \sin\theta\,|2\rangle \\ |b\rangle &= -\sin\theta\,|1\rangle + \cos\theta\,|2\rangle \end{aligned}$$

  where $|1\rangle$ and $|2\rangle$ denote a single-hadron and meson-baryon type component respectively, while θ is some arbitrary mixing.
- Now suppose we have a three quark operator χ_3 that has substantial overlap with $|1\rangle$ but not $|2\rangle$

$$\langle\Omega\,|\,\chi_3\,|\,1\rangle \propto C \quad \text{and} \quad \langle\Omega\,|\,\chi_3\,|\,2\rangle \ll C\,.$$

- So $\bar{\chi}_3$ acting on the vacuum creates

$$\big|\,1\big\rangle = \cos\theta\,\big|\,a\big\rangle - \sin\theta\,\big|\,b\big\rangle\,.$$

Toy Model (cont.)

- No operator sensitive to $|2\rangle \to$ no way to disentangle energy-eigenstates.
- Concern of not being able to see states with high $|2\rangle$ component and contamination of extracted state.
- In our work we therefore utilize 5-quark operators which are expected to have higher overlap with meson-baryon type states.

Toy Model (cont.)

- It is now known (from meson studies for example) that scattering states can be extracted by explicitly projecting the momentum of interest of each state. Rather than performing this projection, the question we endeavour to address is what role do five-quark operators (without explicitly projected momentum) have on the mass spectrum?

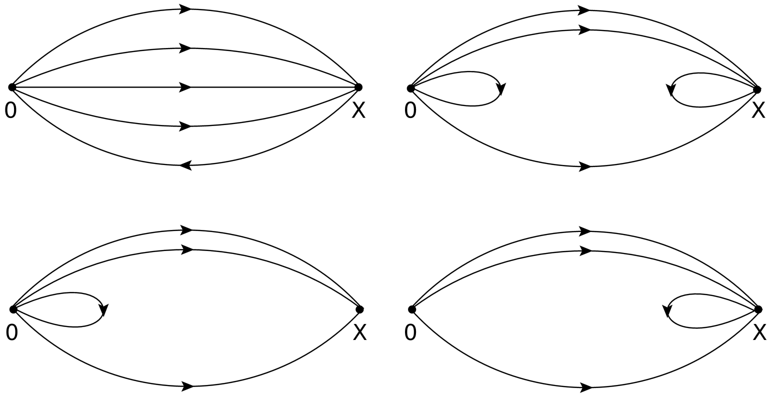
5-quark operators

- Using the Clebsch-Gordan coefficients we can therefore write down five quark operators

$$
\begin{aligned}
\chi_5(x) &= \sqrt{\frac{2}{3}}\left|n\pi^+\right\rangle - \sqrt{\frac{1}{3}}\left|p_3\pi^0\right\rangle \\
&= \frac{1}{2\sqrt{3}}\,\epsilon^{abc}\Big\{2\big(u^{Ta}(x)\,\Gamma_1\,d^b(x)\big)\,\Gamma_2 d^c(x)\left[\bar{d}^e(x)\,\gamma_5\,u^e(x)\right] \\
&\quad -\big(u^{Ta}(x)\,\Gamma_1\,d^b(x)\big)\,\Gamma_2 u^c(x)\left[\bar{d}^e(x)\,\gamma_5\,d^e(x)\right] \\
&\quad +\big(u^{Ta}(x)\,\Gamma_1\,d^b(x)\big)\,\Gamma_2 u^c(x)\left[\bar{u}(x)^e\,\gamma_5\,u^e(x)\right]\Big\},
\end{aligned}
$$

where χ_5 and χ_5' correspond to $(\Gamma_1, \Gamma_2) = (C\gamma_5, \mathrm{I})$ and $(\Gamma_1, \Gamma_2) = (C, \gamma_5)$ respectively.

5-quark operators (cont.)



- Now need to calculate the more computationally intense loop propagators $S(x, x)$.

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Methodology

- Proceed by generating an ensemble of random independent Z_4 noise vectors $\eta_1 \ldots \eta_N$ performing full dilution in spin, colour, and time as a means of variance reduction

$$\eta^a_\alpha(\vec{x}, t) = \sum_{b,\beta,t'} \eta^{ab,t'}_{\alpha\beta}(\vec{x}, t).$$

  where

$$\eta^{ab,t'}_{\alpha\beta}(\vec{x}, t) = \delta_{\alpha\beta}\delta^{ab}\delta_{tt'}\eta^a_\alpha(\vec{x}, t). \qquad \text{(No summation)}.$$

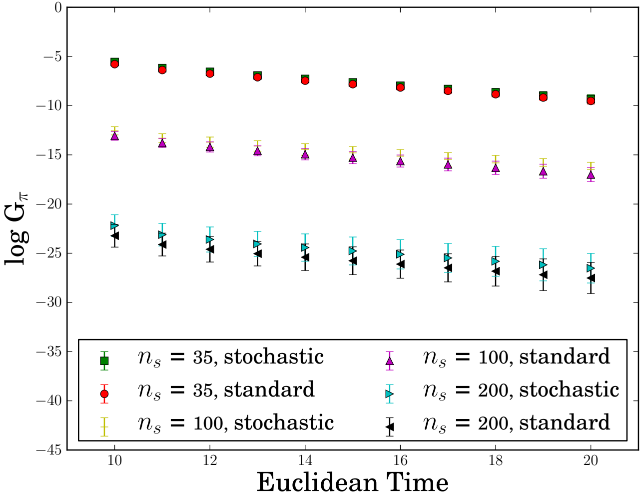
- The stochastic estimate of $S(y, x)$ for a single noise vector is then given by

$$S^{ca}_{\gamma\alpha}(\vec{y}, \vec{x}) = \sum_{b,\beta,t'} \chi^{cb,t'}_{\gamma\beta}(\vec{y}, t)\eta^{\dagger ab,t'}_{\alpha\beta}(\vec{x}, t).$$

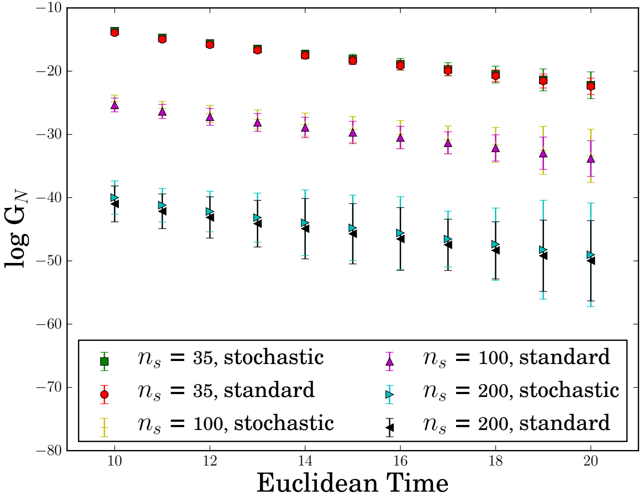
Test Results

- We now test the robustness of method by calculating correlators with stochastically estimated propagators and comparing them with correlators that use standard $S(x, 0)$ propagators.
- Replace only one of the propagators present with a stochastic one.
- Smearing of stochastically estimated propagators can be done post inversion.

Pion Correlator



Nucleon Correlator



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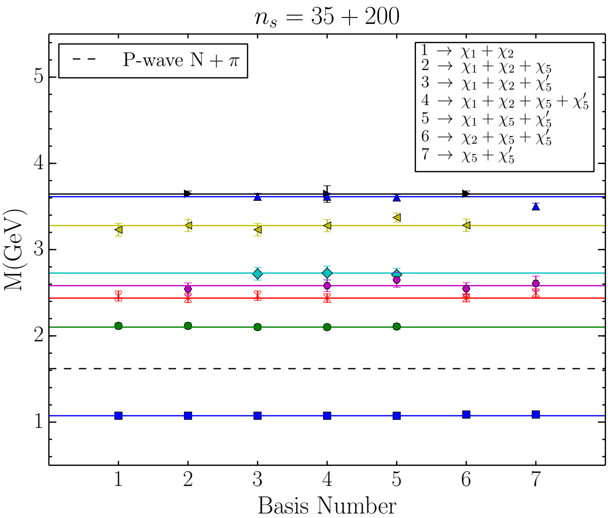
Configuration Details

- PACS-CS $2+1$ flavour dynamical-fermion configurations made available through the ILDG
- Non-perturbatively $\mathcal{O}(a)$-improved Wilson fermion action, and the Iwasaki gauge action.
- Lattice size is $32^3 \times 64$ with a spacing of $0.0907\ \mathrm{fm}$ providing a volume of $\approx (2.90\ \mathrm{fm})^3$.
- $\beta = 1.90$, the light quark mass is set by the hopping parameter $\kappa_{ud} = 0.13770$ which gives a pion mass of $m_\pi = 293\ \mathrm{MeV}$, while the strange quark mass is set by $\kappa_s = 0.13640$.
- Make use of 720 configurations.

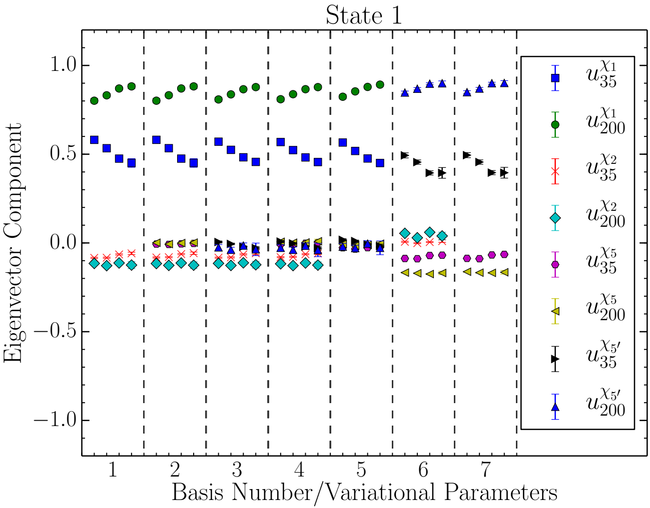
Table of Operators

Basis Number	Operators Used
1	χ_1, χ_2
2	χ_1, χ_2, χ_5
3	χ_1, χ_2, χ_5'
4	χ_1, χ_2, χ_5, χ_5'
5	χ_1, χ_5, χ_5'
6	χ_2, χ_5, χ_5'
7	χ_5, χ_5'

Correlation Matrix Results



Correlation Matrix Results (cont.)



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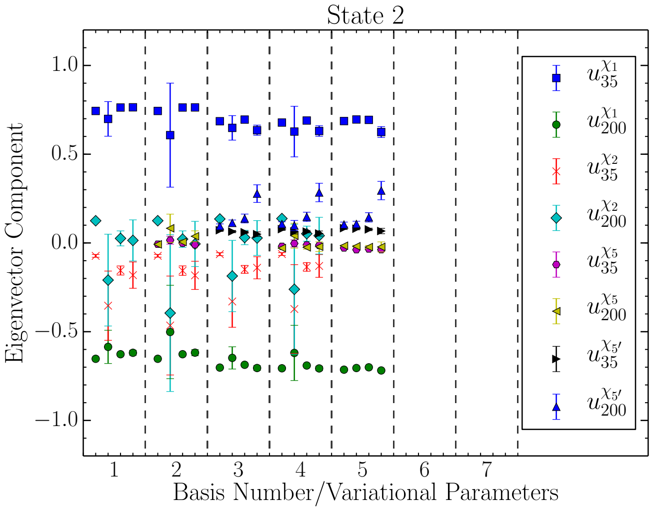

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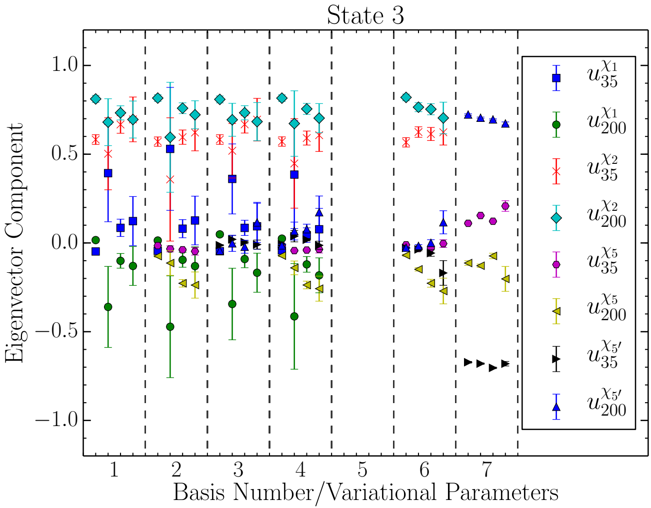
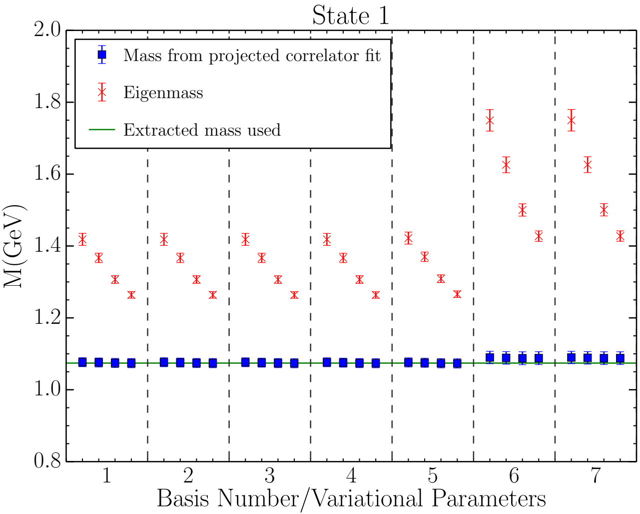


Correlation Matrix Results (cont.)



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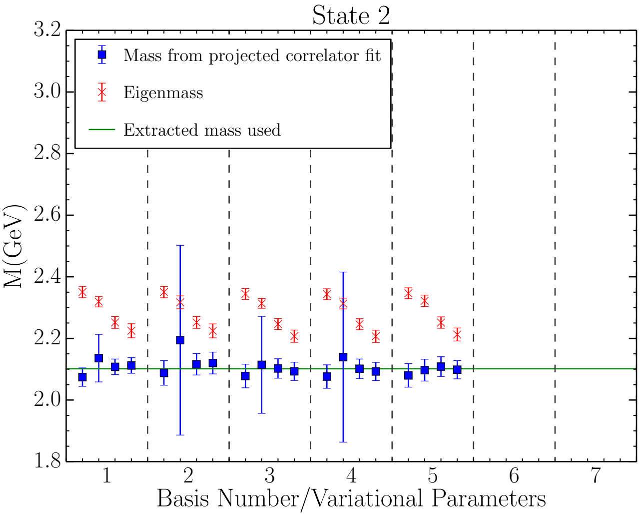

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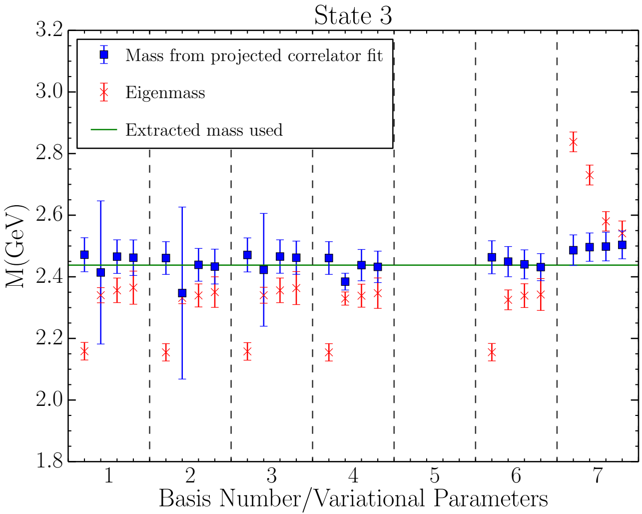

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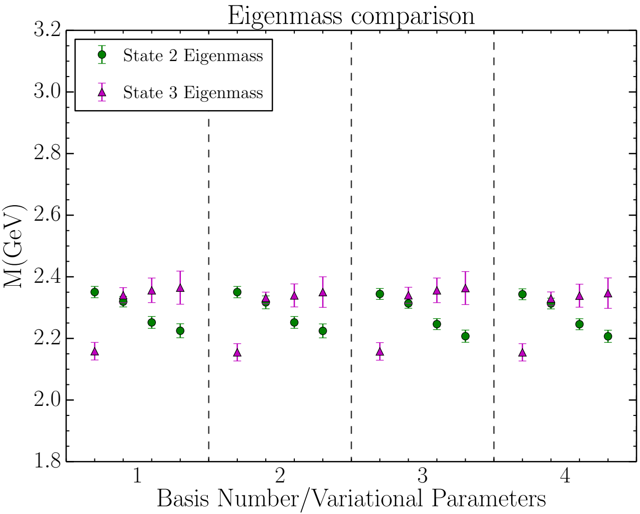

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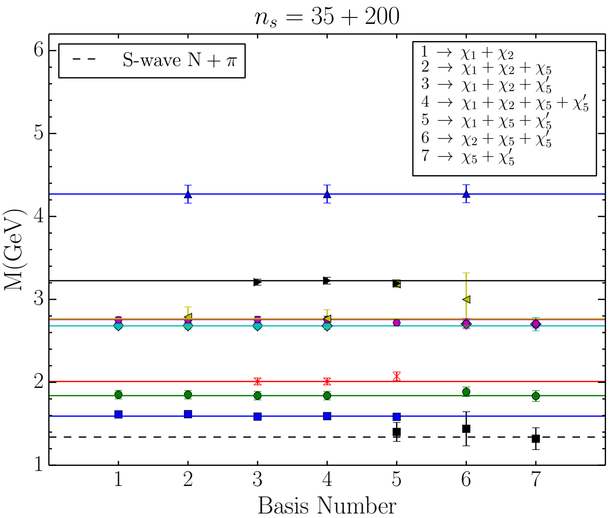

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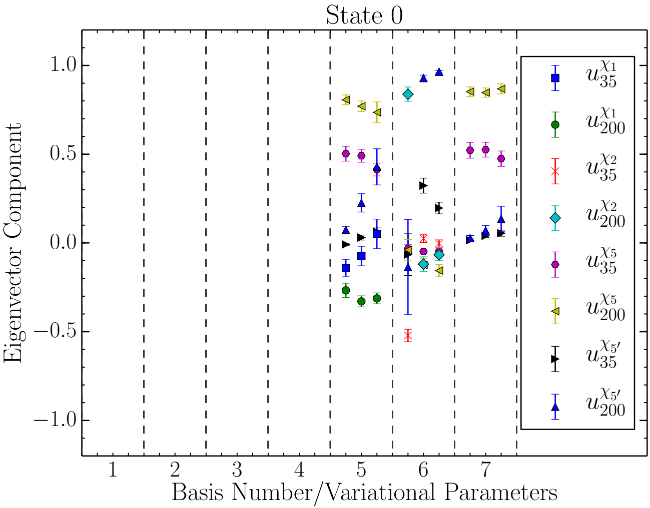
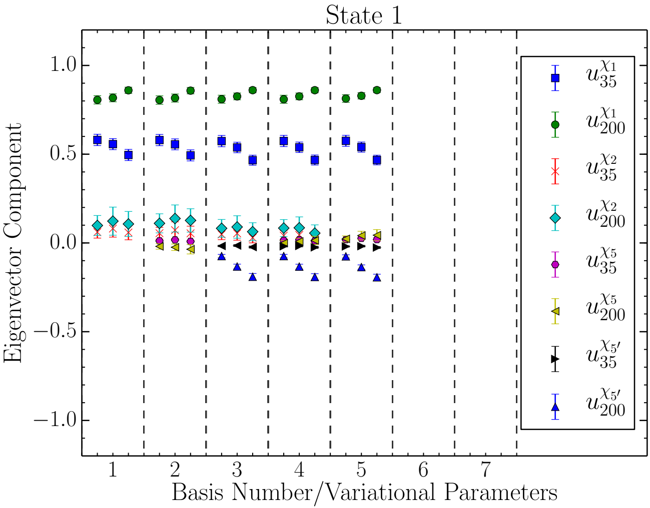


Correlation Matrix Results (cont.)



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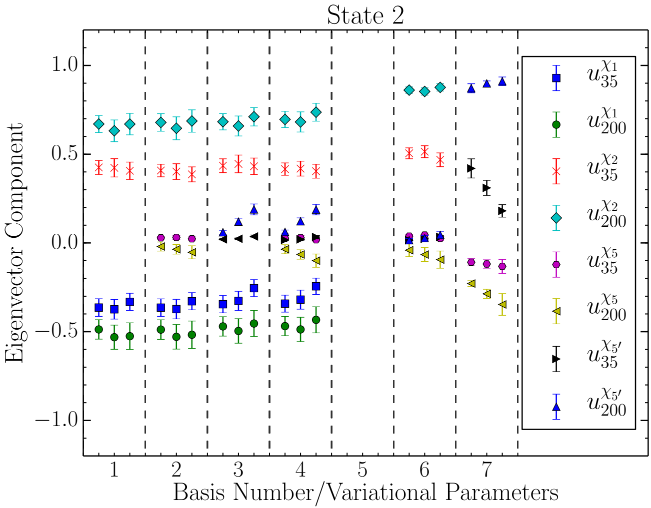

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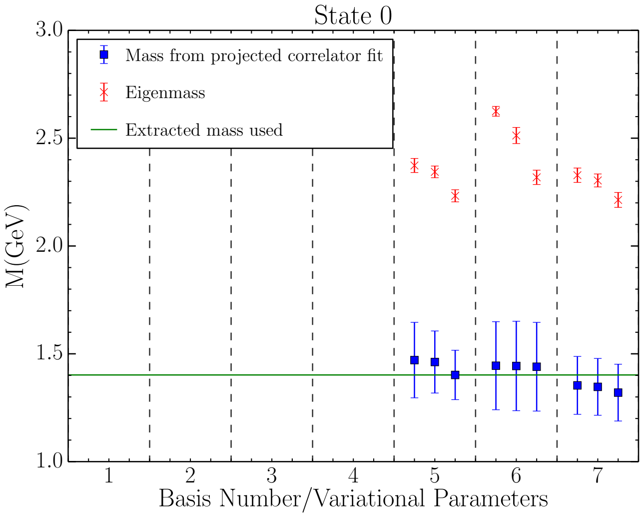

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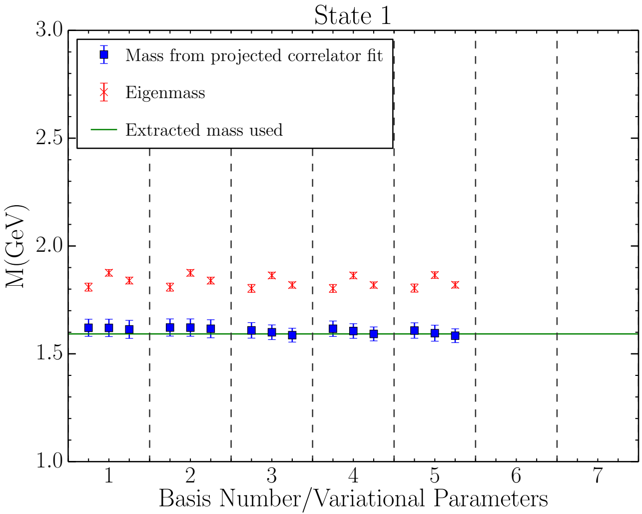

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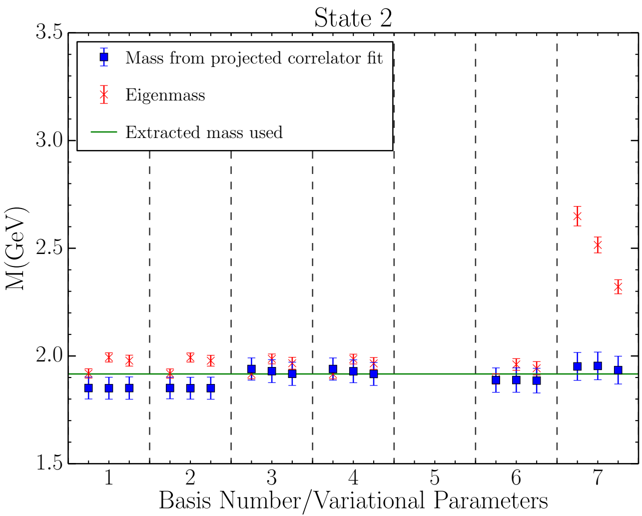

Summary

- Developed method to smear stochastically estimated loop propagators.
- Introduced five-quark operators and performed correlation matrix analyses with them.
- χ_5' seems to be important in accessing energies in the region of scattering states.
- Fitting a single state ansatz to eigenstate projected correlators enables reliable extraction of energies across qualitatively different variational bases. In particular, using the techniques described herein, one doesn't need access to low-lying states in order to reliably extract energies closely related to the resonances of Nature.

Future Work

- Further work will include explicitly specifying the momentum of particles present in scattering states and analysis in other channels.

Thanks for Listening!