SPECTRUM OF STATES WITH ONE CURRENT ACTING ON THE ADJOINT VACUUM OF MASSLESS QCD2

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ABSTRACT

Consider a "one current" state, obtained by application of a color current on the "adjoint" vacuum, in QCD_2, with quarks in fundamental representation.

The quarks are taken to be massless. Then theory on the light-front can be "currentized", namely formulated in terms of currents only.

Adjoint vacuum obtained by applying a current derivative, at zero momentum, on the singlet vacuum.

In general the "one current" states are not eigenstates of $M^2 = 2P^+P^-$, apart from the large N_f limit.

Problems with infra-red regularizations are pointed out.

Connection to fermionic structure is made.

INTRODUCTION

QCD2 in terms of only colored currents ("currentization"), turns out to be very natural once the system is quantized on the light-front.

Both the momentum and the Hamiltonian, and hence also M^2, are expressed in terms of the light-cone colored currents.

In fact only the left (or right) currents are needed.

The currentization was shown to hold for multiflavor fundamental and adjoint quarks.

In fact, it can be applied whenever the free fermions energy momentum tensor can be written in term of a Sugawara form.

The light-cone momentum and Hamiltonian of any CFT that posseses an affine Lie algebra and is coupled to non Abelian gauge fields associated with the same algebra, can also be described in terms of holomorphic currents.

The Fock space of physical states is obtained by applying current creation operators on the vacuum .

The lowest physical states constructed by applying current creation operators on the **singlet vacuum**, are those built from two currents.

A 't Hooft like equation for the wave function of these two currents states was obtained, and solved for the lowest massive state.

A. Armoni, Y. Frishman, J.Sonnenschein, Nucl. Phys. B **596** 459 (2001), hep-th/0011043.

Excellent agreement with the DLCQ results.

Gross, Hashimoto and Klebanov 1998

Antonucchio and Pinsky 1998

Trittmann 2000

Turns out that one can also construct states by using only one current creation operators, applied on the **adjoint vacuum**.

The latter is obtained by acting on the singlet vacuum with fermionic zero modes.

In the case of adjoint fermions by a single adjoint zero mode, and for fundamental fermions by quark anti-quark zero modes.

In a scheme where only currents are being used one should be able to express the adjoint vacuum also in terms of currents, which we do, as mentioned before, for the case of fundamental quarks.

The outcome of our analysis is that in the large N_f limit the state of one current applied on the adjoint vacuum, is indeed an eigenstate of the mass operator with a mass of $\sqrt{\frac{e^2 N_f}{\pi}}$.

In the large N_c limit this is not the case. Acting with M^2 on this state, the dominant daughter state is a two current state.

We further analyze the fermionic structure of these states, especially in the large N_c limit, to connect to the 't Hooft analysis.

En route to these results we are faced with technical obstacles, due to IR divergences.

In certain parts of the computations we were able to regularize the IR divergences, but in others we left it as an open problem.

Assuming that a regularization scheme can be found, we have fully determined the normalizations of the relevant states.

QCD2 CURRENTIZATION

Multi-flavor massless fermions in the fundamental representation of $SU(N_c)$

$$S = \int d^2x \text{ tr } (-\frac{1}{2e^2}F_{\mu\nu}^2 + i\bar{\Psi} \not{D}\Psi)$$

$$\Psi = \Psi_a^i,\ i = 1 \ldots N_c,\ a = 1 \ldots N_f,$$

$$D_\mu = \partial_\mu + iA_\mu.$$

An alternative description is achieved by bosonizing the theory. The bosonization of multi-flavor massive QCD_2 is complicated since one has to translate the fermions into bosonic variables which are group elements of $U(N_F \times N_c)$

Y. Frishman and J. Sonnenschein,
"Bosonization of Colored Flavored Fermions and QCD in Two-Dimensions,"
Nucl. Phys. B **294** (1987) 801.

For massless fermions one can use bosonization in the $SU(N_c) \times SU(N_f) \times U_B(1)$ scheme. This scheme is more convenient since it decouples the color and flavor degrees of freedom.

For a Review see

Y. Frishman and J. Sonnenschein,
"Bosonization and QCD in Two-Dimensions",
Phys.Rept.223 (1993) 309, hep-th/9207017

The bosonized form of the action in this scheme is

$$S_b = N_f S_{WZW}(h) + N_c S_{WZW}(g) +$$
$$\int d^2x\, \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \int d^2x \,\mathrm{tr}\, \frac{1}{2e^2} F_{\mu\nu}F^{\mu\nu} +$$
$$\frac{N_f}{2\pi}\int d^2x \mathrm{tr}\, (ih^\dagger\partial_+ hA_- + ih\partial_- h^\dagger A_+ -$$
$$A_+ hA_- h^\dagger + A_+A_-)$$

$h \in SU(N_c)$, $g \in SU(N_f)$, ϕ is the bosonic field for the baryon number and S_{WZW} stands for the Wess-Zumino-Witten action

$$S_{WZW}(g) = \frac{1}{8\pi} \int_\Sigma d^2x \ \mathrm{tr}\ (\partial_\mu g \partial^\mu g^{-1}) +$$
$$\frac{1}{12\pi} \int_B d^3y \epsilon^{ijk}\ \mathrm{tr}\ (g^{-1}\partial_i g)(g^{-1}\partial_j g)(g^{-1}\partial_k g)$$

B is a three dimensional volume whose boundary is the two dimensional surface Σ, which in our case is the 1+1 Minkowsky space.

The flavored sectors are indeed decoupled from the colored one. Moreover the former ones are entirely massless. Since we are interested in the massive spectrum of the theory we will set aside the g and ϕ fields and analyze only the colored field h.

The residual interaction of the zero modes of the g, h and ϕ fields, will not be important to our discussion.

Choose the light cone gauge $A_- = 0$, and quantize the system on the light-front $x_- = 0$. Integrating A_+ we get an effective non-local action

$$S = N_f S_{WZW}(h) - \frac{1}{2}e^2 \int d^2x \,\mathrm{tr}\,(\frac{1}{\partial_-}J^+)^2$$

$$J^+ = \frac{iN_f}{2\pi} h\partial_- h^\dagger.$$

The light-front momentum and energy take now simplified forms.

The momentum takes the Sugawara form

$$P^+ = \frac{1}{N_c + N_f} \times$$

$$\int dx^- : J^a(x^-, x^+ = 0) J^a(x^-, x^+ = 0) :$$

The energy

$$P^- = -\frac{e^2}{2\pi} \times$$

$$\int dx^- : J^a(x^-, x^+ = 0) \frac{1}{\partial_-^2} J^a(x^-, x^+ = 0) :$$

$J = \sqrt{\pi} J^+$.

Evaluate mass

$$2P^+ P^- \left|\psi\right\rangle = M^2 \left|\psi\right\rangle.$$

Note:

1.

The equations that determine the spectrum are entirely expressed in terms of currents.

2.

Both P^+ and P^- depend only on J^+ and not on J^-.

3.

The only condition for currentizing a theory of fermions coupled to non Abelian gauge fields, is that T^{++} of the free fermionic theory could be rewritten in terms of a Sugawara form.

4.

In particular this can be done for adjoint fermions and for fermions in the symmetric and antisymmetric "two box" representations.

In momentum space

$$J(p^+) = \int \frac{dx^-}{\sqrt{2\pi}} e^{-ip^+x^-} J(x^-, x^+ = 0)$$

Normal ordering in the expression of P^+ and P^- are naturally with respect to p^+, where $p^+ < 0$ denotes a creation operator. To simplify the notation we write from now on p instead of p^+.

$$P^{+} = \frac{2}{N_f + N_c} \int_0^{\infty} dp J^b(-p) J^b(p)$$

$$P^{-} = \frac{e^2}{2\pi} \int_0^{\infty} dp \Phi(p) J^b(-p) J^b(p)$$

$$\Phi(p) = \frac{1}{2}\left(\frac{1}{(p+i\epsilon)^2} + \frac{1}{(p-i\epsilon)^2}\right)$$

$\Phi(p)$ is $(-\frac{\sqrt{\pi}}{2})$ times the Fourier transform of the 'potential' $|x-y|$.

In computing the eigenvalues of $M^2 = 2P^+P^-$ we will need the algebra

$$[J^a(p), J^b(p')] =$$

$$\frac{1}{2} N_f \; p \; \delta^{ab} \delta(p+p') + i f^{abc} J^c(p+p')$$

The vacuum $|0\rangle$ obeys

$$\forall p > 0, \;\; J(p) \, |0\rangle = 0$$

Physical states are built by applying the current creation operators on the vacuum

$$|\psi\rangle = \mathrm{tr} \; J(-p_1) \ldots J(-p_n) \, |0\rangle$$

Note that this basis is not orthogonal.

THE ONE-CURRENT STATE

Define "adjoint vacuum"

$$|0,\ ij\rangle = \lim_{\epsilon\longrightarrow 0} b^{\dagger i}_{\beta}(\epsilon) d^{\dagger\beta}_{j}(\epsilon)\,|0\rangle$$

$b^{\dagger i}_{\beta}$ and $d^{\dagger\beta}_{j}$ are the creation operators of a quark and anti-quark respectively.

We can represent the action of the above adjoint zero mode on the vacuum by the derivative of a creation current taken at zero momentum.

$$J'^{i}_{j}(k)\,|0\rangle_{k=0^-} =$$

$$\sqrt{\frac{\pi}{2}}\frac{d}{dk}\int_0^{\infty} dp \int_0^{\infty} dq \delta(k+p+q) b^{\dagger i}_{\beta}(p) d^{\dagger\beta}_{j}(q)\,|0\rangle_{k=0^-}$$

$$= -\sqrt{\frac{\pi}{2}} b^{\dagger i}_{\beta}(\epsilon) d^{\dagger\beta}_{j}(\epsilon)\,|0\rangle_{\epsilon\longrightarrow 0}.$$

As the currents are traceless, we have to subtract the trace part for $i = j$. The latter can be neglected for large N_c.

Comments:

1. The adjoint vacuum can also have flavor quantum numbers. In fact, when using different flavor indices on the fermion and anti-fermion [and not summing], we can get a vacuum that is also adjoint in flavor. This will result in a "current-ball" being an adjoint flavor multiplet as well.

2. For the case of $N_f = N_c$, the state constructed from the bosonic adjoint vacuum is degenerate with the one constructed from the fermionic adjoint vacuum.

Denote

$$Z^a \equiv -\sqrt{\frac{2}{\pi}}(J^a)'(0)$$

The "one current" state we have in mind is

$$|k\rangle = J^b(-k)Z^b|0\rangle.$$

This state is obviously a global color singlet, but in our Light Cone gauge $A_- = 0$ it is also a local color singlet, as the appropriate line integral vanishes.

Now

$$\sqrt{\frac{\pi}{2}}\left[J^a(p), Z^b\right] = \frac{1}{2}N_f\delta^{ab}\delta(p) - if^{abc}(J^c)'(p)$$

From which, for $p > 0$

$$J^a(p)Z^b|0\rangle = 0$$

Hence the state $Z^b|0\rangle$ is annihilated by all the annihilation currents, and so it is indeed a colored vacuum.

From

$$\left[P^+, J^b(-k)\right] = kJ^b(-k)$$

we get that our state $|k\rangle$ is indeed of momentum k.

EVALUATION OF M^2

$$\left[\int_0^\infty dp\phi(p)J^a(-p)J^a(p), J^b(-k)\right] =$$

$$\frac{1}{2}N_f\frac{1}{k}J^b(-k)+$$

$$if^{abc}\int_0^k dp\phi(p)J^a(-p)J^c(p-k)+$$

$$if^{abc}\int_k^\infty dp\,(\phi(p)-\phi(p-k))\times$$

$$J^a(-p)J^c(p-k)$$

In P^- (and in P^+) we ignore contributions from zero - mode states, that is, we cut the integrals at ϵ, and then take the limit.

$$P^- J^b(-k) Z^b|0\rangle = \left[P^-, J^b(-k)\right] Z^b|0\rangle$$

as the Hamiltonian annihilates the color vacuum as well.

Using the commutator of the Hamiltonian with a current, which we evaluated before, we get

$$\frac{\pi}{e^2} P^- J^b(-k) Z^b|0\rangle = \frac{1}{2} N_f \frac{1}{k} J^b(-k) Z^b|0\rangle +$$
$$i f^{abc} \int_0^k dp \phi(p) J^a(-p) J^c(p-k) Z^b|0\rangle$$

$$M^2 J^b(-k)Z^b|0\rangle = 2P^-P^+J^b(-k)Z^b|0\rangle$$
$$= 2kP^-J^b(-k)Z^b|0\rangle$$
$$= (\frac{e^2N_f}{\pi})J^b(-k)Z^b|0\rangle +$$
$$(\frac{2e^2}{\pi}k)if^{abc}\int_0^k \phi(p)J^a(-p)J^c(p-k)Z^b|0\rangle$$

So, in the large N_f limit, the state $J^b(-k)Z^b|0\rangle$ is an eigenstate, with eigenvalue $\frac{e^2N_f}{\pi}$.

To see the exact dependence of the two terms in the equation above (the one and two current states) on N_f and N_c, we should normalize them.

$$\langle 0| Z^a J^a(k) J^b(-k) Z^b |0\rangle =$$
$$\frac{1}{2} N_f k\delta(0) \langle 0| Z^b Z^b |0\rangle + N_c \langle 0| Z^b Z^b |0\rangle$$

The second term in the last line can be neglected compared to the first, as it is a constant compared with $\delta(0)$, the space volume divided by 2π.

$$\langle 0|Z^b Z^b|0\rangle = (N_c^2 - 1)\langle 0|Z^1 Z^1|0\rangle$$

and the factor $k\delta(0)$ is the normalization of a plane wave of momentum k.

The normalized state, for $N_c >> 1$,

$$\frac{1}{N_c\sqrt{\frac{1}{2}N_f}} J^b(-k) Z^b|0\rangle \qquad (1)$$

relative to $\langle 0|Z^1 Z^1|0\rangle$.

The normalization of the second term is more complicated. A lengthy but straightforward calculation gives

$$\left\| \left(i f^{def} k \int_0^k dq \Phi(q) J^d(-q) J^f(q-k) \right) Z^e \,|0\rangle \right\|^2 =$$

$$N_c \left(N_c^2 - 1 \right) \left(\frac{1}{2} N_f \right)^2 k\delta(0) \left\langle 0 \left| Z^1 Z^1 \right| 0 \right\rangle \times (Ints)$$

$$(Ints) =$$

$$k \int_0^k dp p(k-p)\Phi(p) \left(\Phi(p) - \Phi(k-p)\right)$$

$$-k\frac{N_c}{N_f} \int_0^k dp \Phi(p) \int_0^{k-p} dq q \Phi(q)$$

We have written only the terms proportional to $\delta(0)$.

Useful formulae for the evaluation, involving sums of products of structure functions of $SU(N)$, are given in the Appendix.

The various momentum integrals (including the ones for the non dominant terms) are divergent for $\epsilon \rightarrow 0$, thus they should be regulated. We leave this problem for now, and assume henceforth that they are regulated and finite. For simplicity the integrals (including the factor k) appearing in the two dominant terms will be notated R_1 and $-R_2$

in the following expressions. Note that we have $\frac{1}{\epsilon^2}$ and $\frac{1}{\epsilon}$ divergences and also $\ln(\frac{k^2}{\epsilon^2})$. It seems that these are cancelled in R_2.

Define now the normalized states

$$|S_1\rangle = C_1 \left(J^b(-k) Z^b \,|0\rangle \right)$$

$$|S_2\rangle = iC_2 k f^{abc} \int_0^k dp \Phi(p) J^a(-p) J^c(p-k) Z^b \,|0\rangle$$

where

$$C_1 = \frac{1}{N_c \sqrt{\frac{1}{2} N_f}} \qquad C_2 = \frac{\frac{2}{N_f \sqrt{N_c^3}}}{\sqrt{R_1 + R_2 \frac{N_c}{N_f}}}$$

The mass eigenvalue equation

$$M^2|S_1\rangle = \frac{e^2}{\pi}N_f|S_1\rangle + \frac{e^2 N_c}{\pi}\sqrt{2}\sqrt{R_1\frac{N_f}{N_c} + R_2}|S_2\rangle$$

In the large flavor limit, our state $|S_1\rangle$ is an eigenstate with mass

$$M = \sqrt{\frac{e^2 N_f}{\pi}}.$$

In the large color limit, however, we actually get that the second term dominates by a factor of N_c. Moreover, while the first term goes to zero in the large N_c limit, due to the factor of e^2, the second term survives.

FERMIONIC STRUCTURE

Consider

$$J^i_j(-k)b^{\dagger j}_\beta(\epsilon)d^{\dagger\beta}_i(\epsilon)\,|0\rangle$$

with

$$J^i_j(-k) = \frac{1}{2\pi}\int_0^\infty dp$$

$$[b^{\dagger i}_\beta(p+k)b^\beta_j(p)+\theta(k-p)b^{\dagger i}_\beta(k-p)d^{\dagger\beta}_j(p)$$
$$-d^{\dagger\beta}_j(p+k)d^i_\beta(p)]$$

The 4-quarks part has a coefficient which is independent of N_c.

As for the 2-quark part, it involves an anti-commutator of creation with annihilation, yielding a state which is a combination of

$$b^{\dagger j}_{\beta}(k) d^{\dagger \beta}_{j}(\epsilon) \quad and \qquad b^{\dagger j}_{\beta}(\epsilon) d^{\dagger \beta}_{j}(k)$$

with a coefficient that is proportional to N_c.

Thus for large N_c, we have a quark-antiquark combination of momenta $(k, 0)$ and $(0, k)$.

As 't Hooft found all meson states for large N_c, and each has a well defined momentum distribution, it is clear that our state is not a mass eigenstate of large N_c. This is of course part of our explicit calculation in the previous section.

DISCUSSION

Investigation of two dimensional massless multi flavor QCD in its "currentized" form.

Useful tool to solve those QCD_2 systems.

Spectrum of states that are constructed by applying a single current creation operator on the adjoint vacuum.

The later was shown to be given in terms of the derivative with respect to k, at $k = 0$, of the current acting on the singlet vacuum.

In general, and in particular also in the large N_c, these states are not eigenstates of M^2. However, in the large N_f limit they are eigenstates.

Previously

A. Armoni, Y. Frishman and J. Sonnenschein,
"Massless QCD(2) from current constituents," Nucl. Phys. B **596**, *459 (2001)* [arXiv:hep-th/0011043]

the spectrum of two current states on the singlet vacuum was derived.

It was also shown there that in the large N_f limit there is a continuum of states with mass above $2 \times e\sqrt{\frac{N_f}{\pi}}$. This indicates that there is a non-interacting "currentball" meson of mass $e\sqrt{\frac{N_f}{\pi}}$.

Now, in the same large N_f limit, we have indeed found an eigenstate of M^2 with exactly this mass.

The state we have found is a color singlet. In fact it is easy to see that in the large N_f limit, there are N_c^2-1 colored eigensates of M^2 with the same mass.

Interpretation: For large N_f, QCD_2 is transfered into a set of N_c^2-1 Abelian systems. Now in QED_2 it is well known that the Schwinger mechanism yields a massive state of mass $\frac{e}{\sqrt{\pi}}$. This Schwinger state is often considered as a bound state of an electron-positron. For large N_f, the M^2 eigenstates are therefore just the Schwinger states appearing in a multiplicity of $N_c^2 - 1$.

Open questions:

1.

The computation of the spectrum of the states requires a regularization prescription that we have only partially found.

2.

Diagonalize the "currentball" states created by applying any number of current creation operators on the various vacua.

3.

The question of what have we learned from the currentization procedure and from the two dimensional spectrum of states, about four dimensional QCD.

In particular a challenging question is to investigate the possibility of a Schwinger like mechanism also in four dimensions.

Summation Identities

The generators T^a of $SU(N)$, in the adjoint representation, are related to the structure constants f^{abc} as

$$(T^a)_{bc} = -if^{abc}$$

Thus

$$f^{abc}f^{abd} = Tr(T^cT^d) = N\delta^{cd}$$

and

$$f^{abc}f^{a'bc'}f^{aa'd}f^{cc'd} = Tr(T^bT^dT^bT^d)$$

$$= if^{bde}Tr(T^eT^bT^d) + Tr(T^bT^bT^dT^d)$$

$$= \frac{1}{2}N^2(N^2 - 1)$$

where we used

$$\sum_a T^a T^a = N I_{adj}$$

with I_{adj} the unit matrix in the adjoint representation.