Light Quark Confinement And The Trajectory Of The Pseudoscalar Meson

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Why?

SPIRES: Almost 70% of papers with *"excited meson(s)"* in the title published in the last 10 years.

What do we know about calculating excited states?

What do we *think* we know?

How many excited states can one expect to find when you have some vertex correlator?

Collaborators

ARGONNE: Craig Roberts, Arne Höll

PITTSBURG UNIV: Pieter Maris

GRAZ UNIV: Andreas Krassnigg

KENT STATE UNIV: Peter Tandy, (Mandar Bhag-wat)

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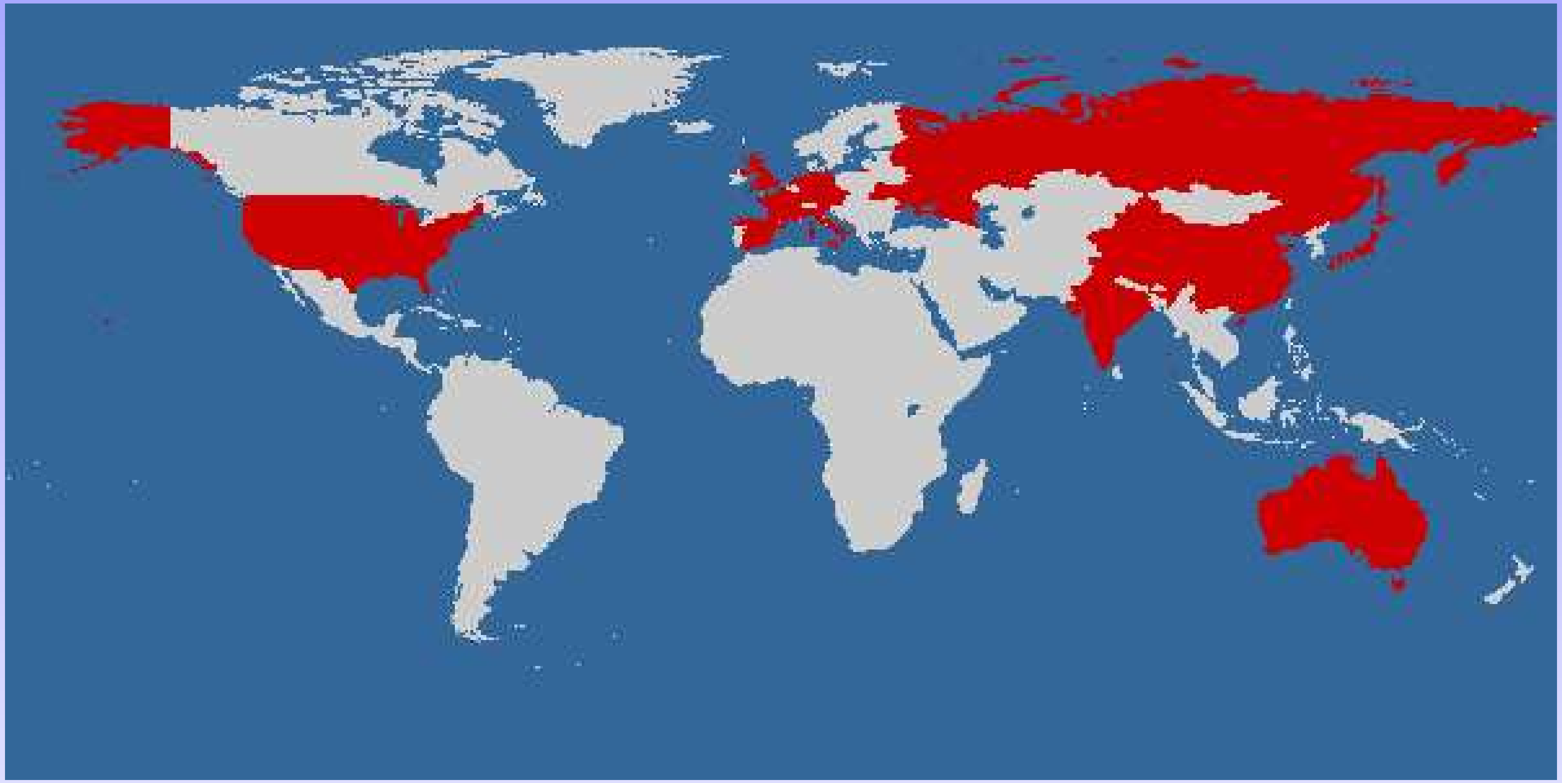
Recent DSE Publications



Last 25 papers listed on SPIRES with "*DSE*" in the title…

Dyson–Schwinger Equation

- Poincaré covariant framework.
- Equations of Motion for n-point functions
- Hadrons: poles in corresponding n-point functions
- Exact QCD results provable through the existence of a nonperturbative, symmetry preserving truncation.
- Pion is both a Goldstone mode and a bound state of strongly dressed quarks.

Maris, Roberts *Phys. Rev.* **C56** 3369

Maris, Tandy *Phys. Rev.* **C60** 055214

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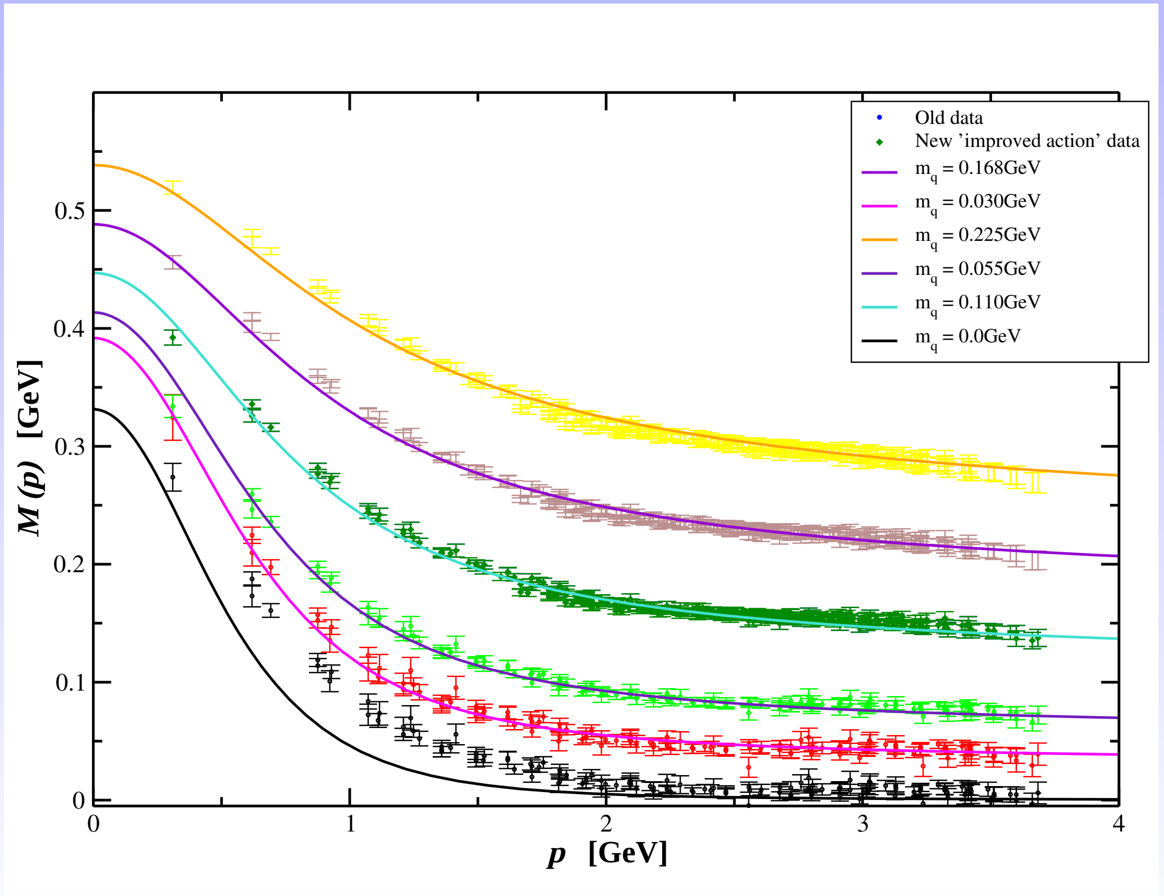
Dyson–Schwinger Equation

- Understanding of systematic error of the simplest truncation quantified.
- One parameter model provides persuasive results.

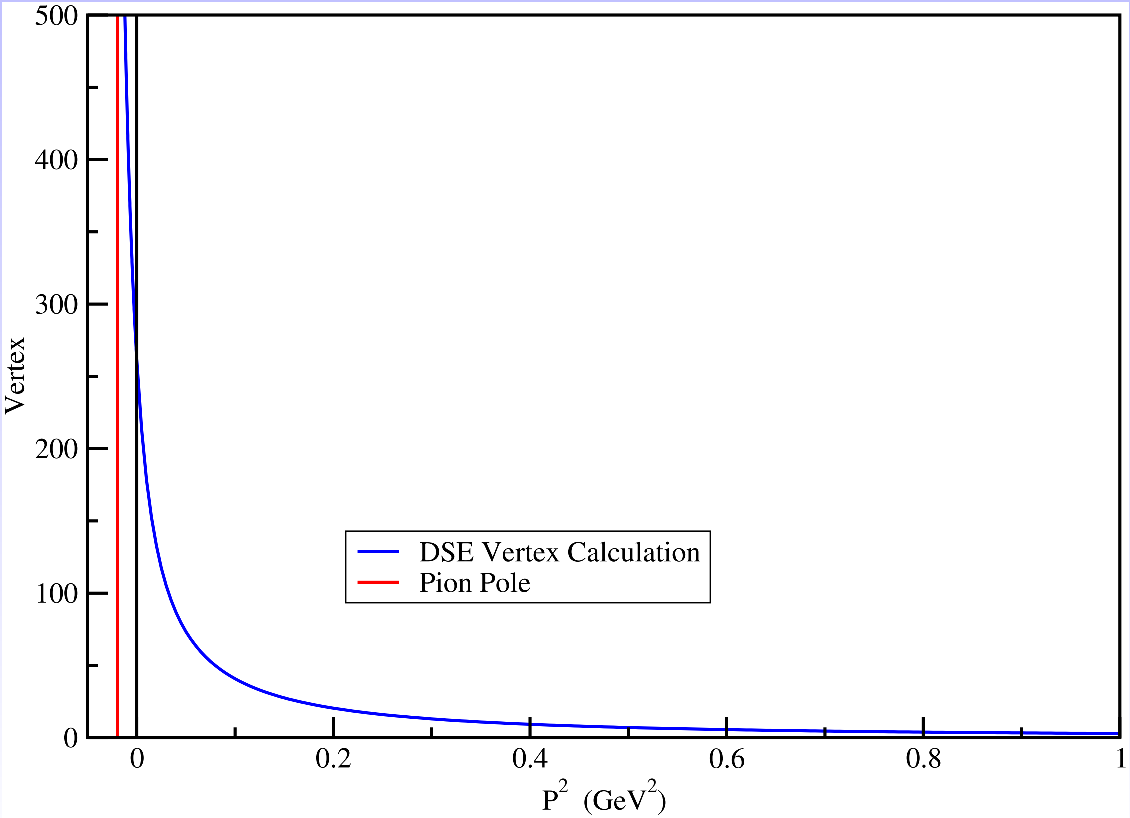
DSE: Bhagwat, Pichowsky, Roberts, Tandy nucl-th/0304003

Lattice: Bowman, Heller, Leinweber, Williams hep-lat/0209129



This Study: Pions

- Solve corresponding inhomogeneous vertex DSE.
- Numerical solution exists for both TIMELIKE and SPACELIKE momentum.
- Poles in timelike region → bound states



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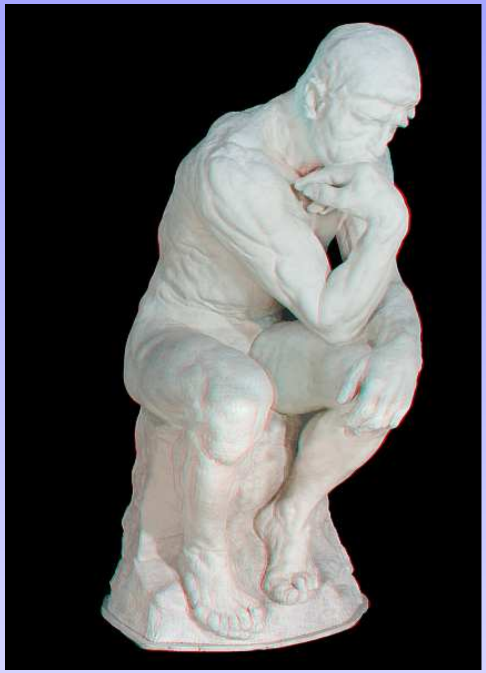
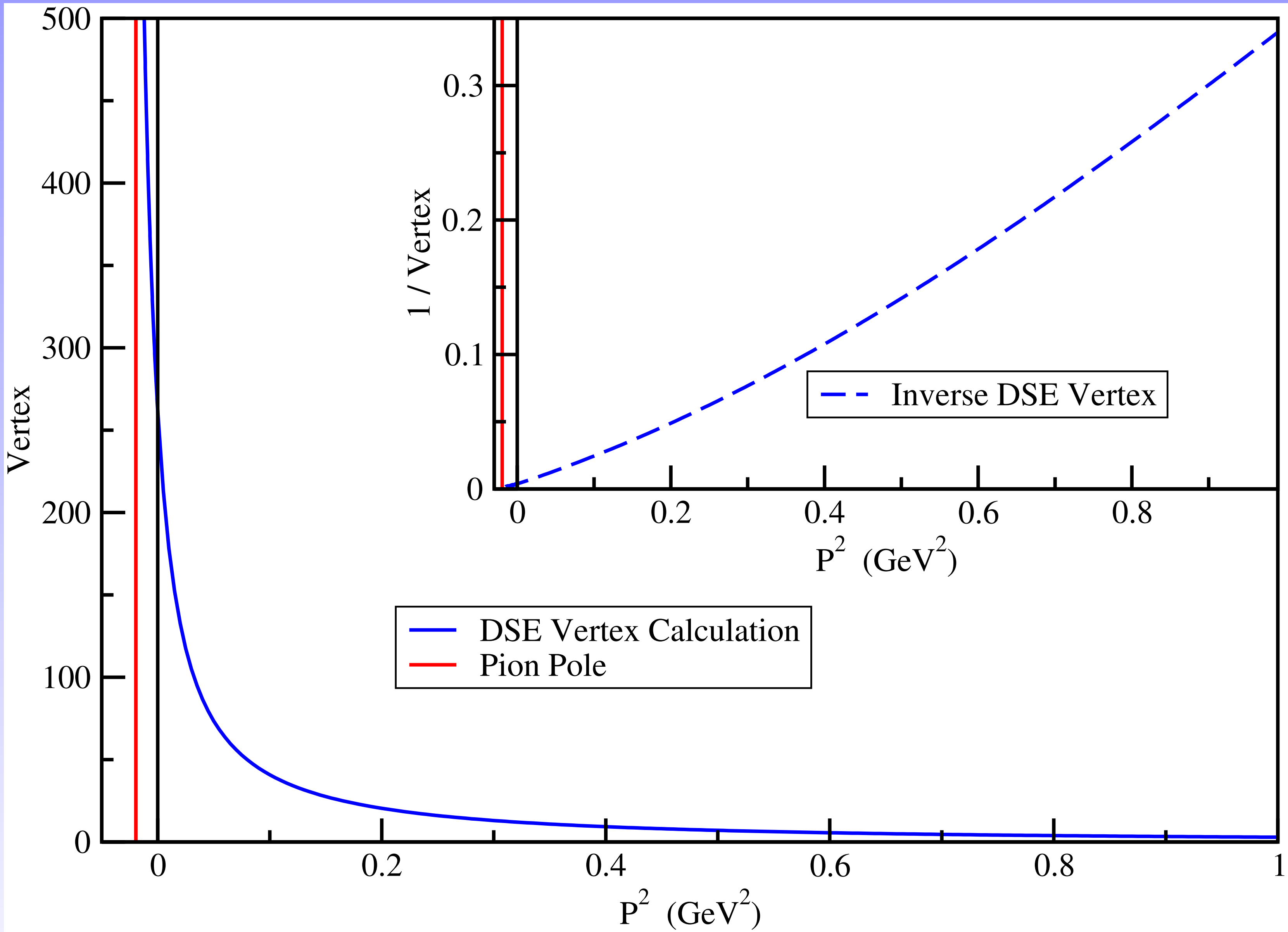
Numerical Reality



- ☹ Finding exact pole location is HARD
- ☺ Finding zeros is EASY

So. . . invert the data!


500
400
300
200
100
0
Vertex
0
0.2
0.4
0.6
0.8
1
P^2 (GeV^2)
DSE Vertex Calculation
Pion Pole
0.3
0.2
0.1
0
1 / Vertex
0
0.2
0.4
0.6
0.8
P^2 (GeV^2)
Inverse DSE Vertex

Simple Model Investigation

Naïvely the spectral model of a 3-point function is a sum of bound states.

Make a simple model for a vertex:

- A sum of monopoles

$$V(\vec{P}) = a + \sum_i \frac{a_i}{\vec{P}^2 + m_i^2}$$

- m_i : Mass of bound state i
- a_i : Residue of bound state i

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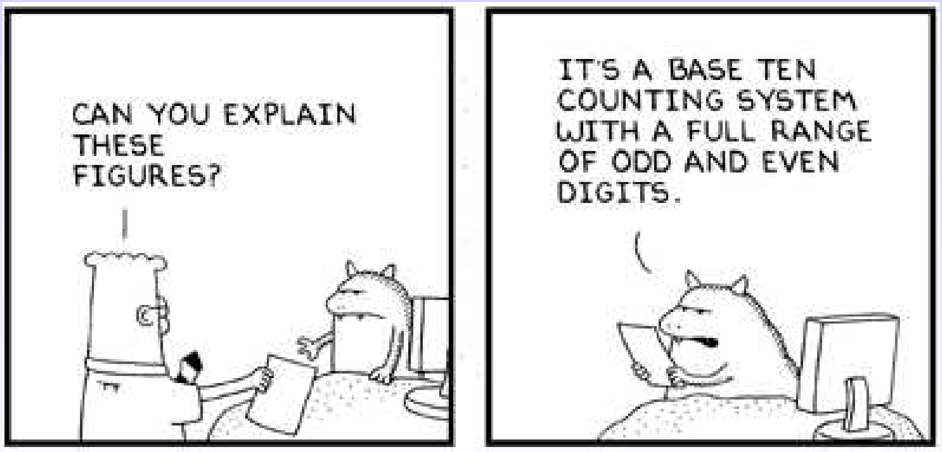
Use some physics

What do we know about m_i and c_i?

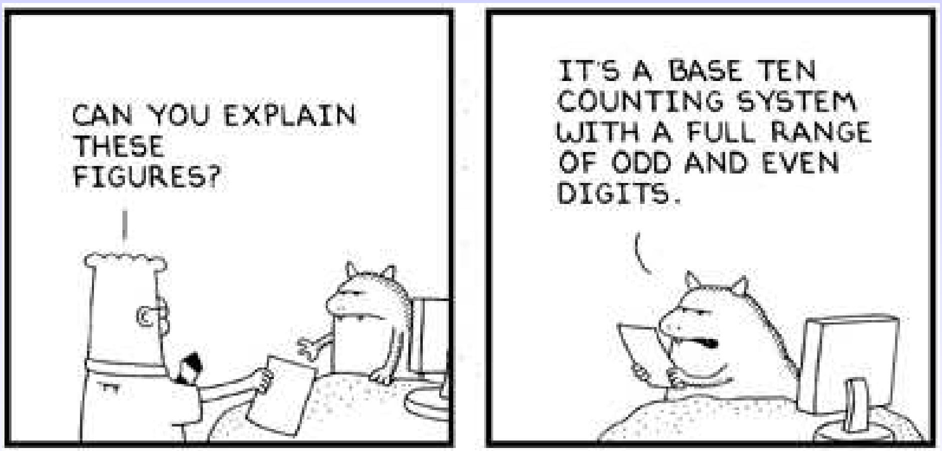


Use some physics

What do we know about m_i and c_i?

Mass

- Masses of the bound states
- PDG publishes these for the REAL WORLD



Use some physics

What do we know about m_i and c_i?

Mass

- Masses of the bound states
- PDG publishes these for the REAL WORLD

Residue

- Related to the decay constant of the bound state, i.e. f_π
- We **know** that these alternate in sign

`Höll, Krassnigg, Roberts nucl-th/0406030`

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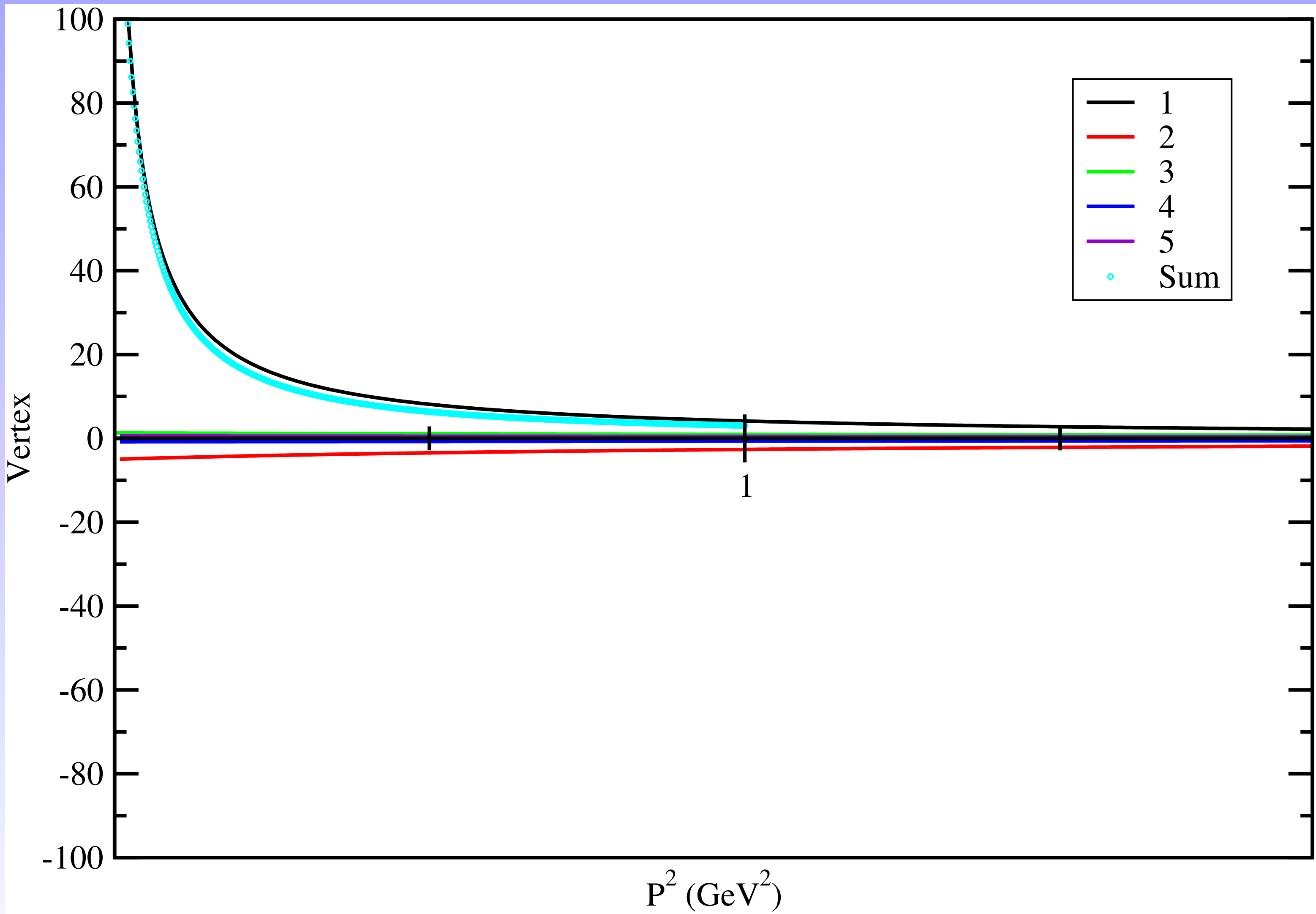
Visualise the problem

Mass	Residue
0.14	4.23
1.06	-5.6
1.72	3.82
2.05	-3.45
2.2	2.8

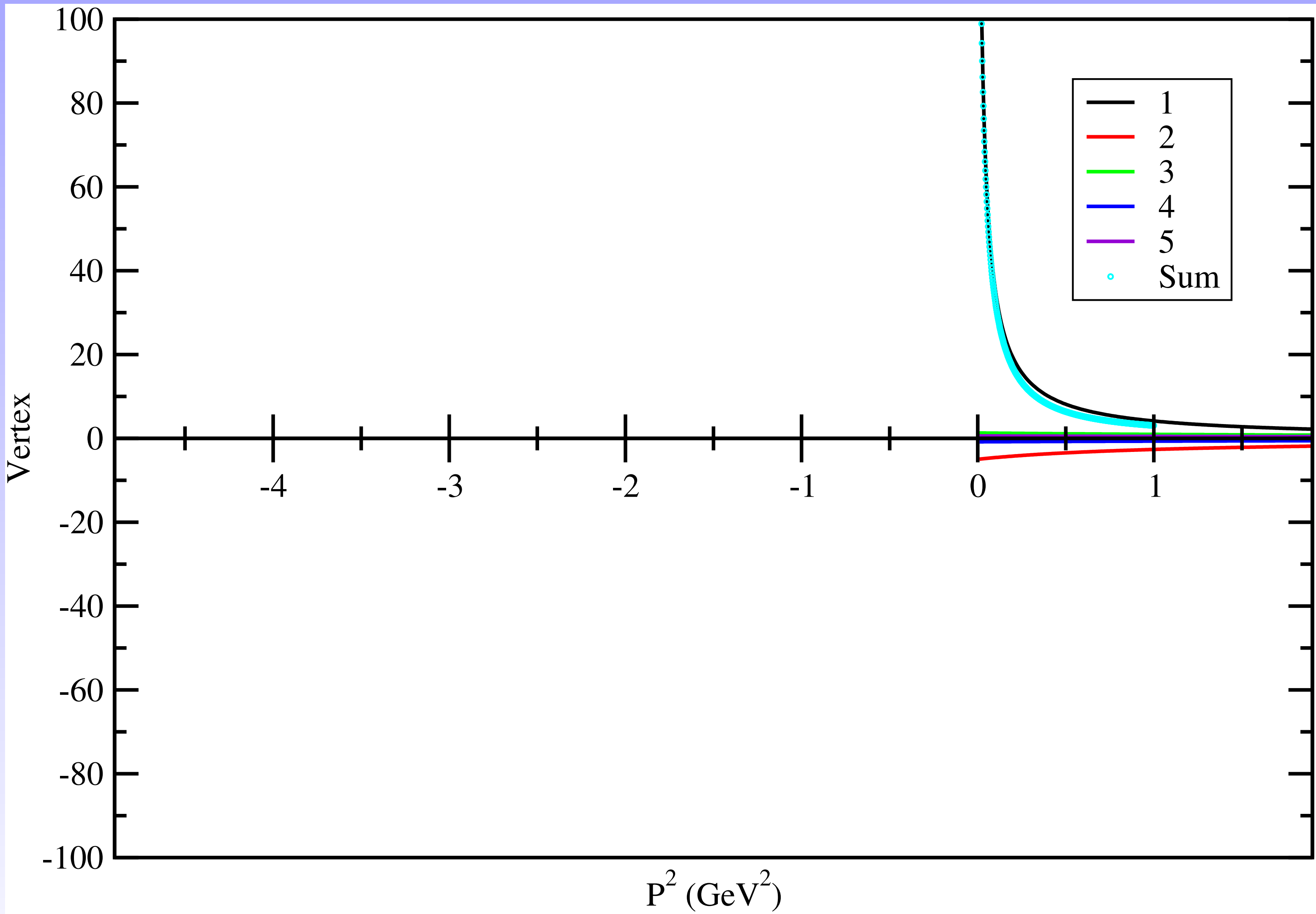
Masses are motivated by the PDG.

Residues are all of the same magnitude.

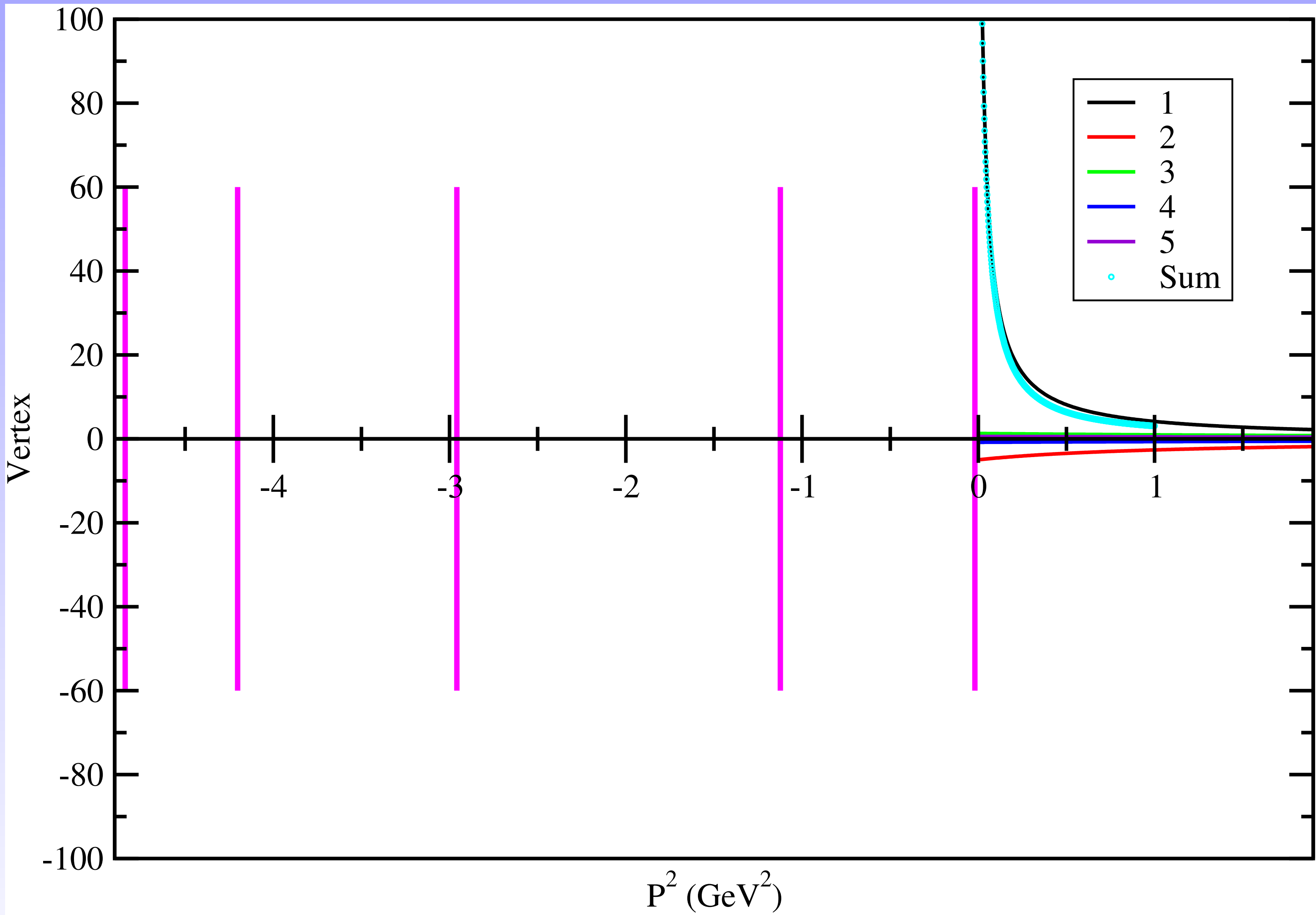
The Simple Model



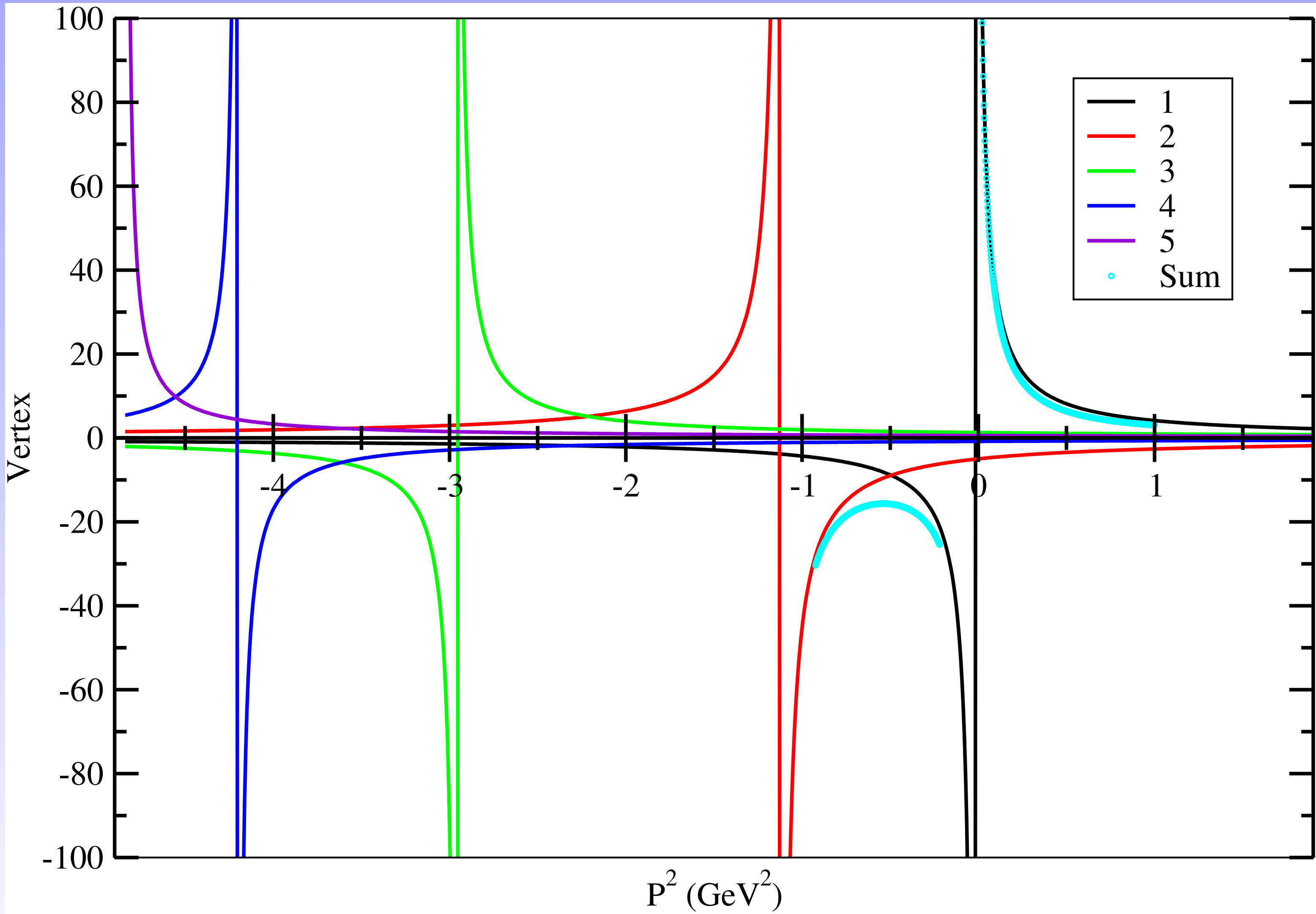
The Simple Model



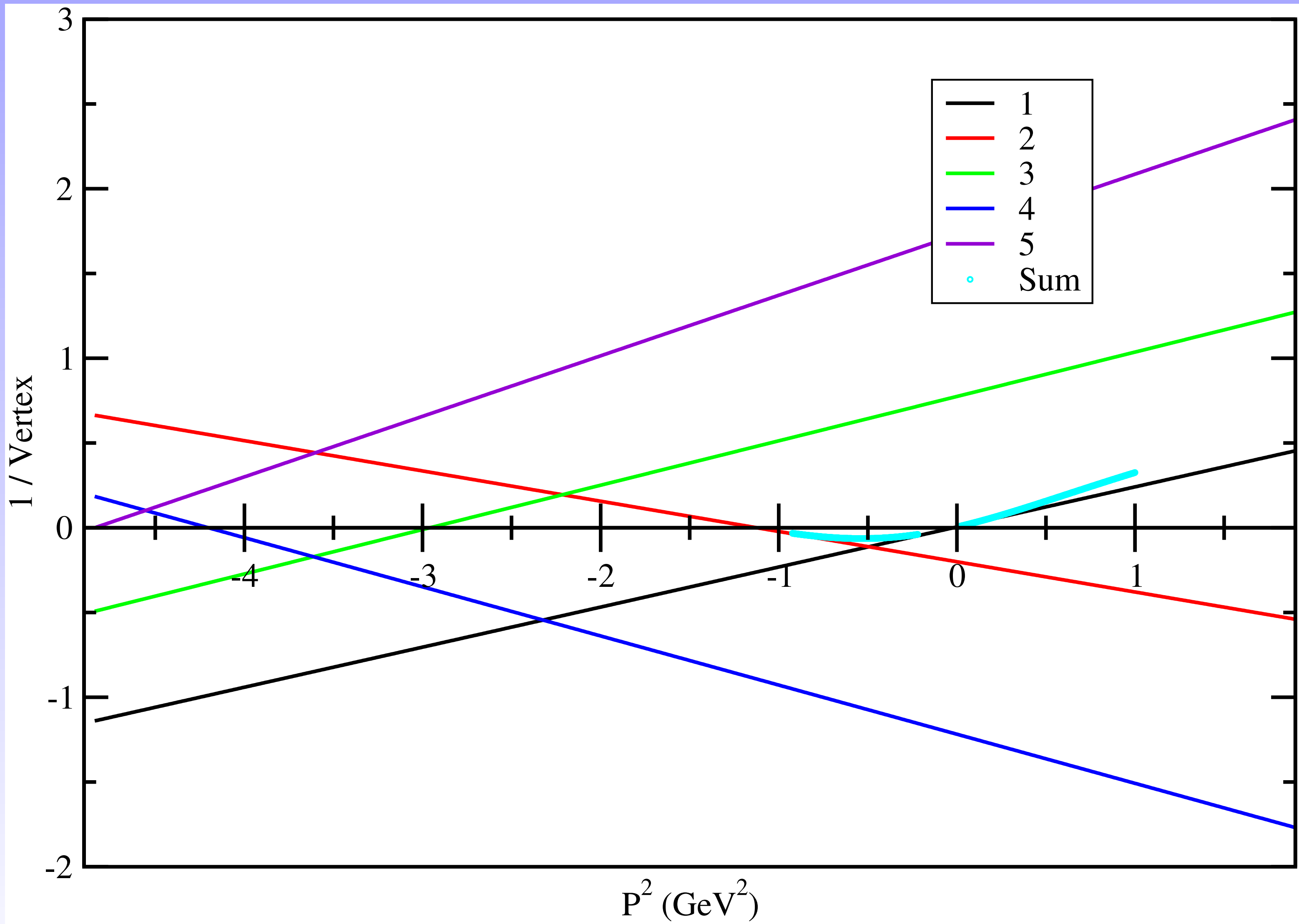
The Simple Model



The Simple Model



Invert for zeros



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Fitting the Data

Assume we know nothing about the source of the data...
Fit a Padé to data:

- We know the data has zeros
- Little else is known about the functional form i.e. what does the background look like?

So choose a generic form:

$$f(P^2) = \frac{a_0 + a_1 P^2 + a_2 P^4 + \ldots + a_n P^{2n}}{1 + a_{n+1} P^2 + a_{n+2} P^4 + \ldots + a_{2n} P^{2n}}$$

Interpreting the solution

The "data" has an unknown number of bound states contributing.

How reliable are the n-zeros we find?

- The contribution to the vertex from the bound states falls as $1/m^2$
$\Rightarrow$ Clear hierarchy
- At BEST the first $n-1$ solutions would be reliable
- The last solution would contain all the remaining physics

How?

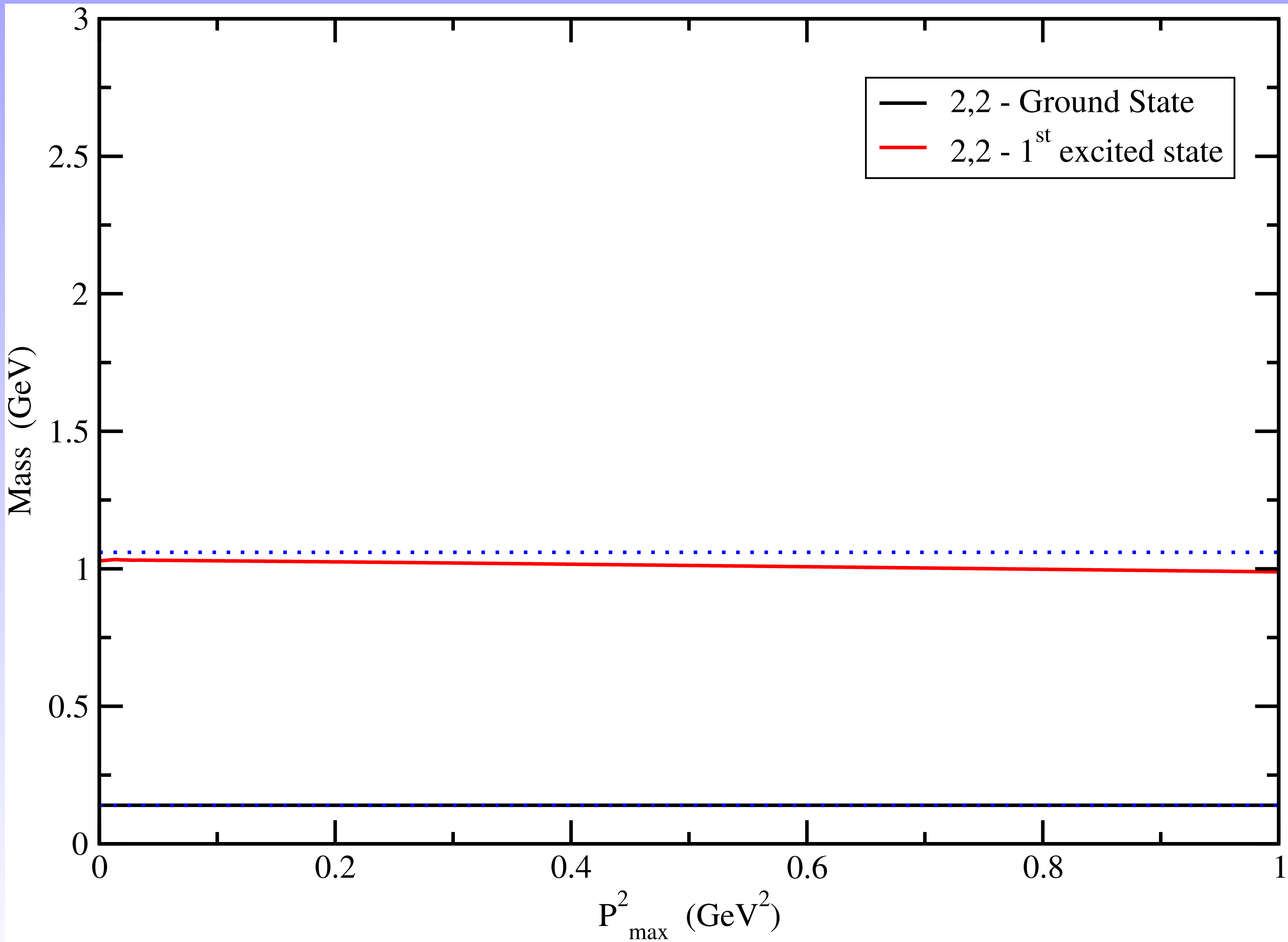
The fitting function:

$$f(P^2) = \frac{a_0 + a_1 P^2 + a_2 P^4 + \ldots + a_n P^{2n}}{1 + a_{n+1} P^2 + a_{n+2} P^4 + \ldots + a_{2n} P^{2n}}$$

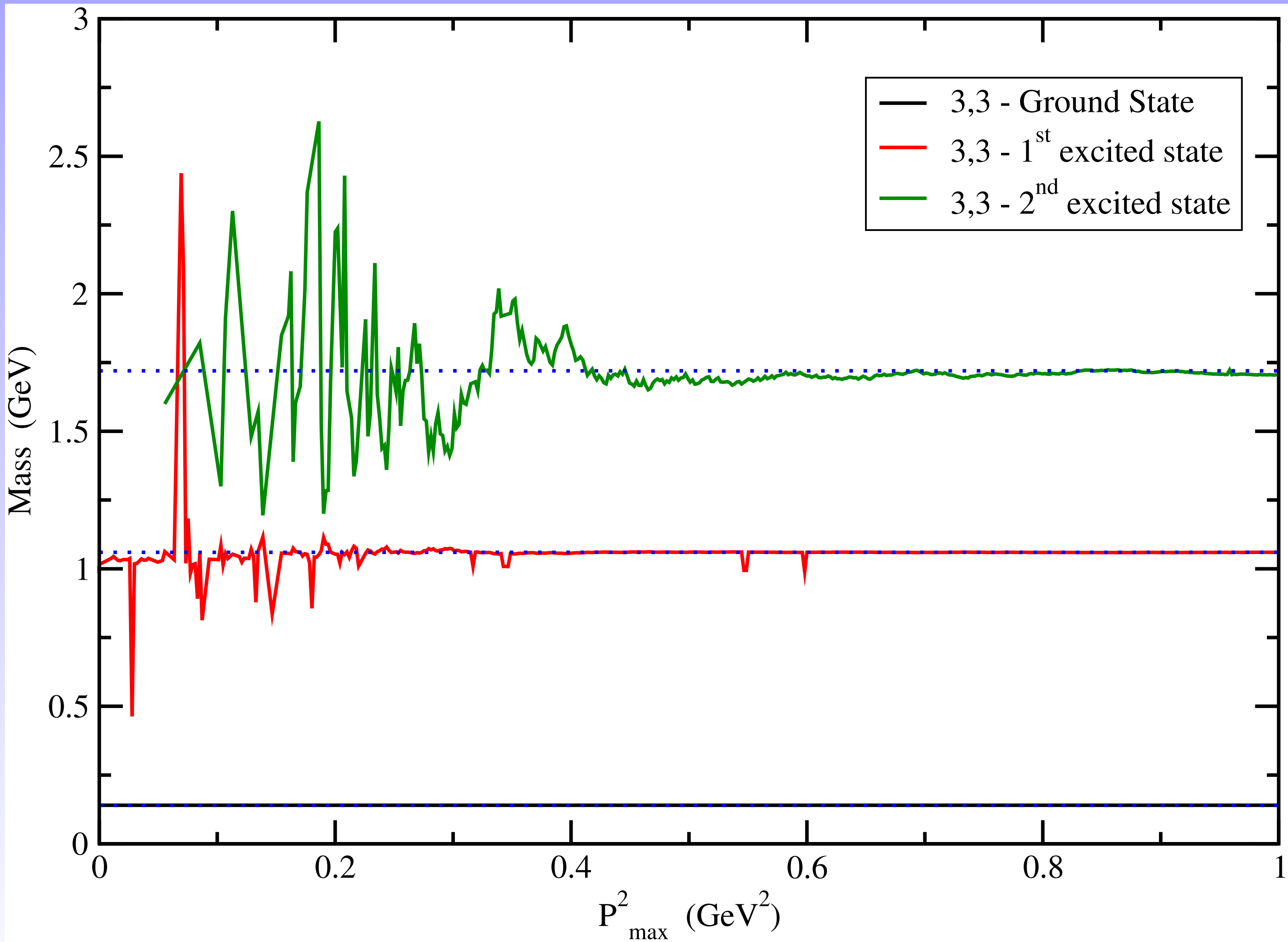
Have data for $P^2 \in (0, 1]$ GeV^2.

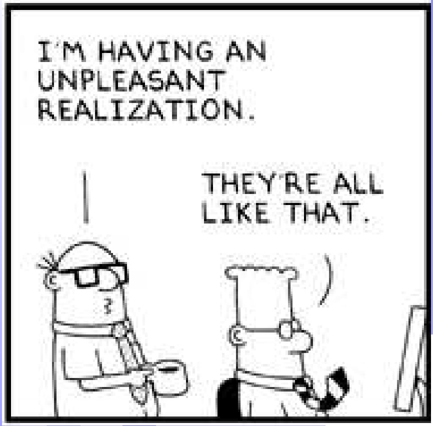
1. Fit data in range $P^2 = (0, P^2_{max}]$.
2. Increase P^2_{max} and repeat fit.

Results — Masses

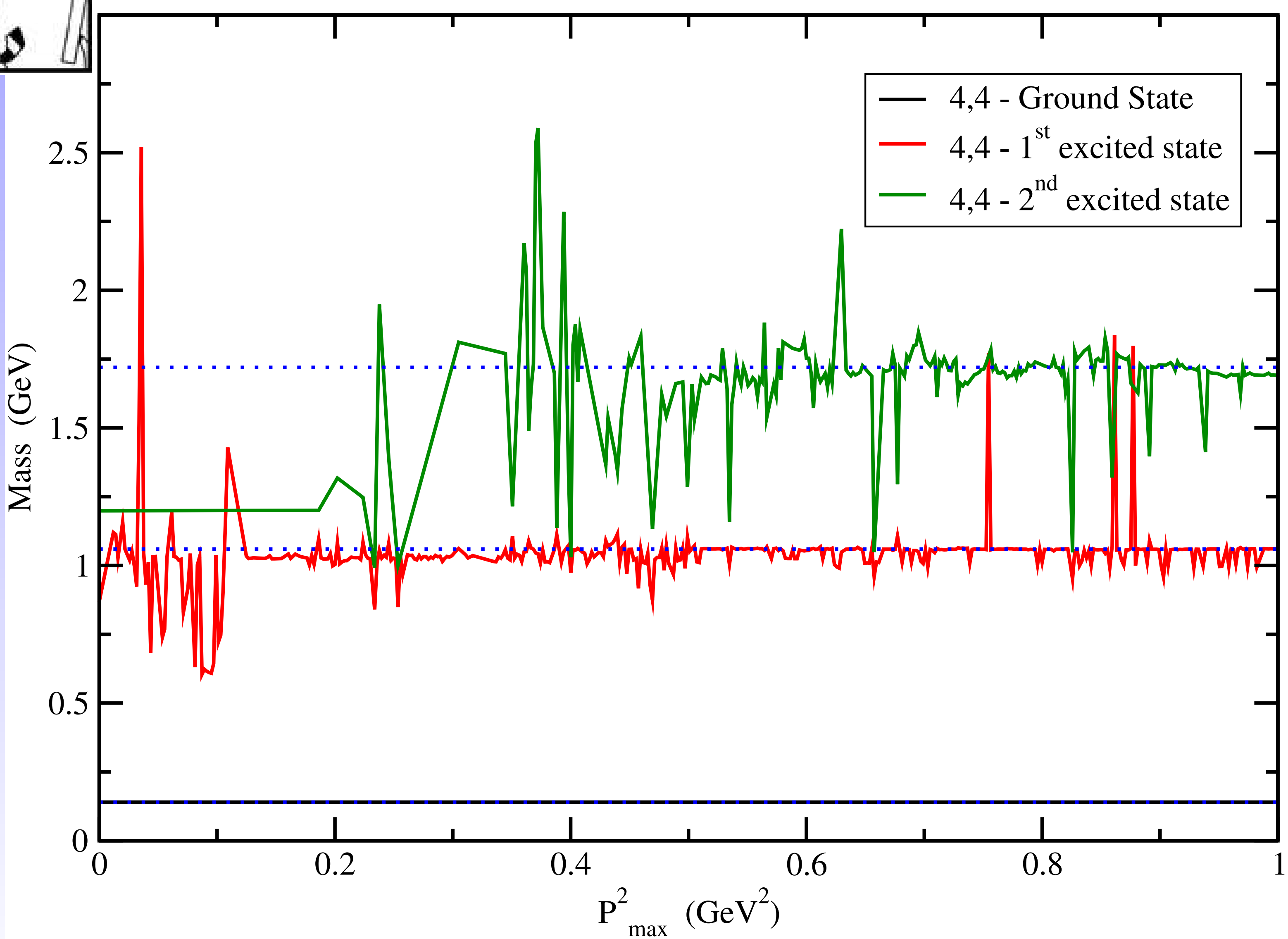


Results — Masses





Results — Masses



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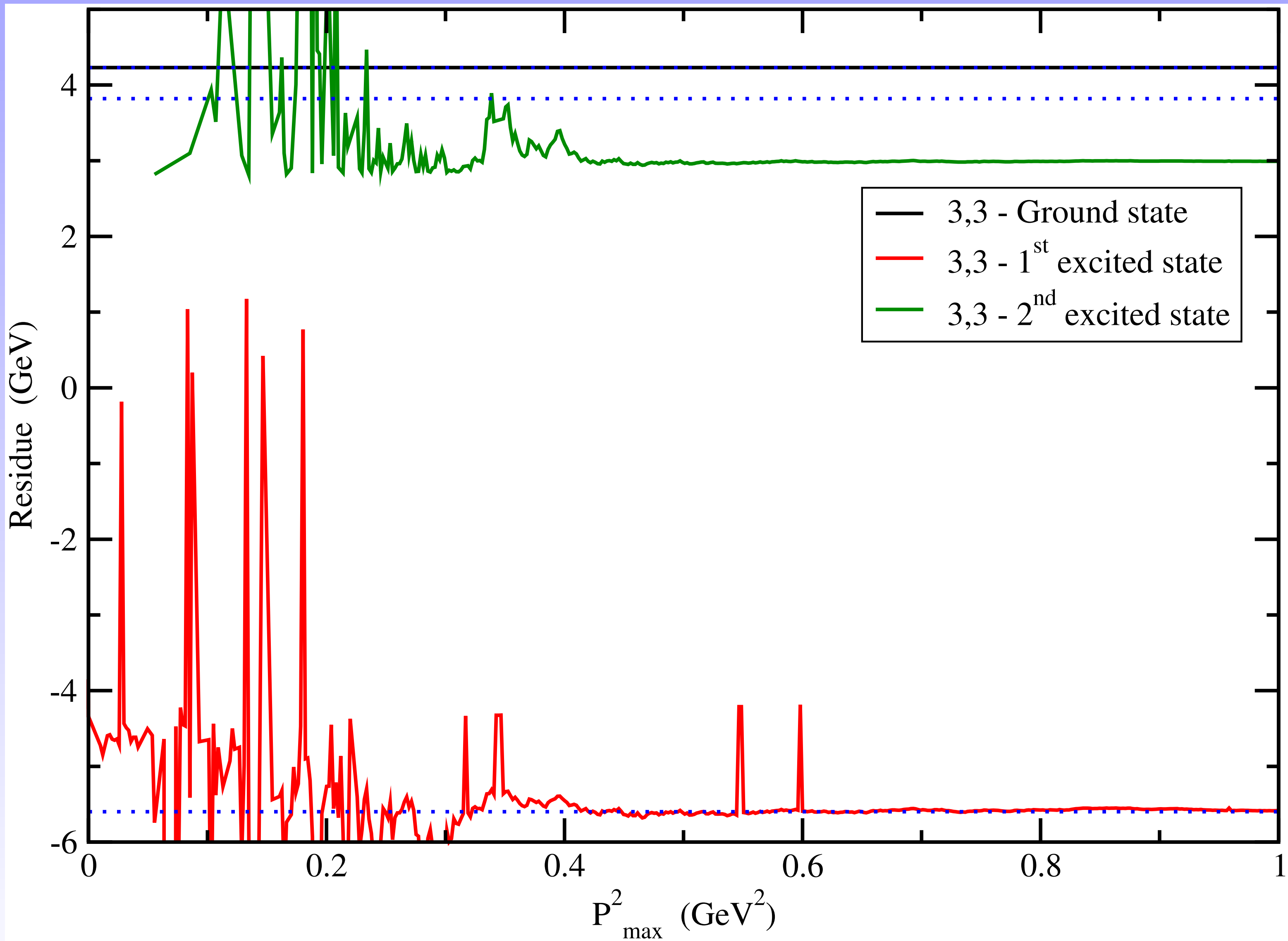
Results — Residues



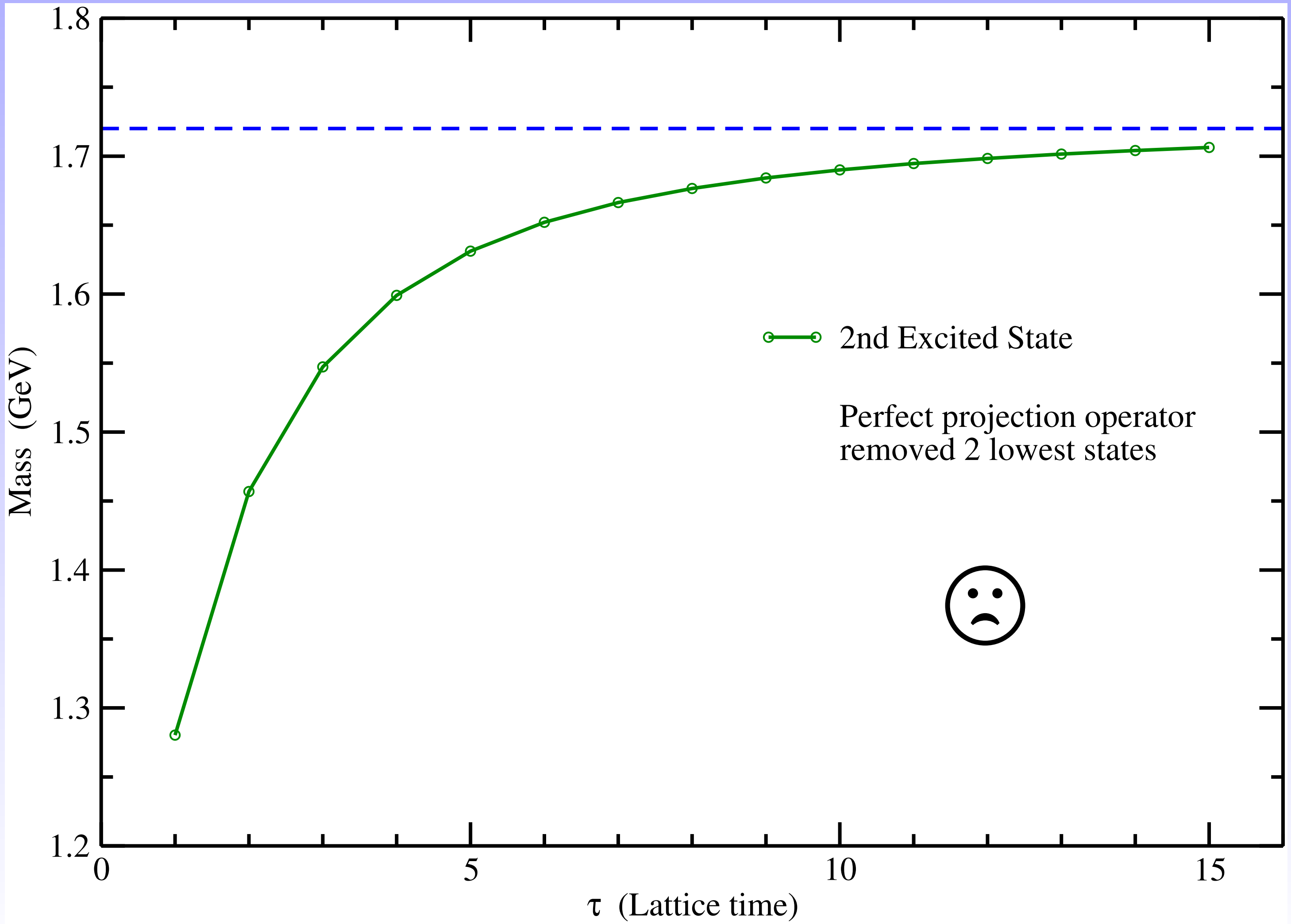
The Glass is Half Full

Optimistically there is a chance to get some information about excited states with only spacelike data. . .

	Mass	Residue
Ground State:	☺	☺
First Excited State:	☺	☺
Second Excited State:	☺	☹

Alternate Solution

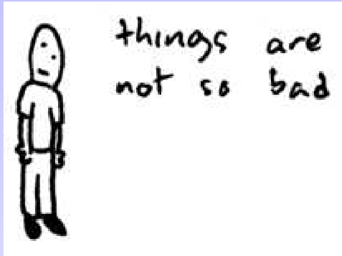
WISDOM: The best (only?) way to get access to higher excited states is through improved operators.



Return to DSE

Strengths

- Obeys the symmetries of QCD.
- FAST to calculate.
- Able to calculate in *both* Spacelike and Timelike regimes.
- Systematic errors well quantified.



Weaknesses

- Pion loops not included.
- Current implementation restricts maximum accessible meson mass.

Simplest(?) Situation

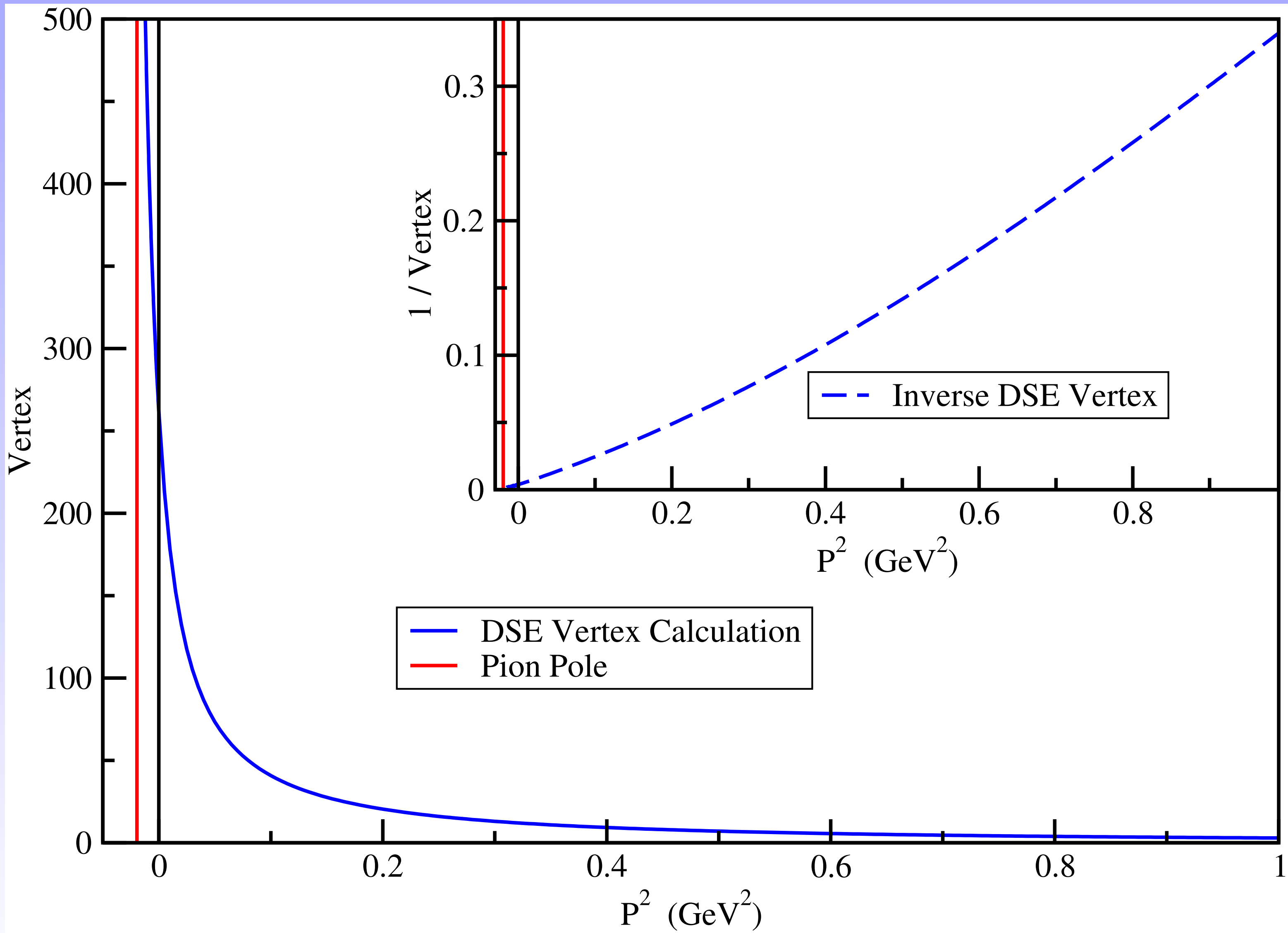
Pseudoscalar
i.e. π

Light Quarks
i.e. $m_q = m_{u,d}$

Spacelike momentum
i.e. $P^2 > 0\,\mathrm{GeV}^2$

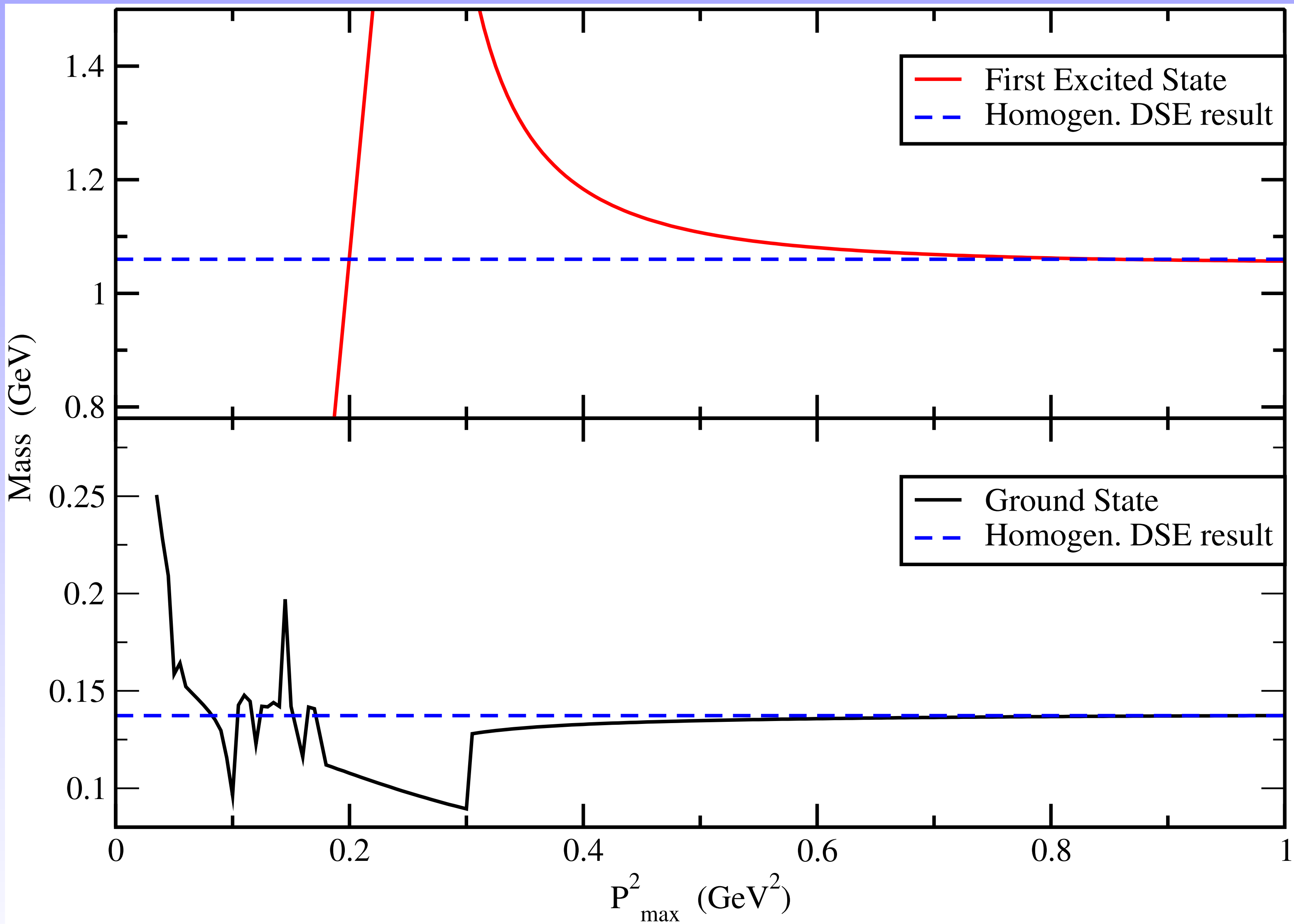
DSE Vertex Calculation

ONLY use spacelike data!



Extracted Masses

Consistent with homogeneous solution

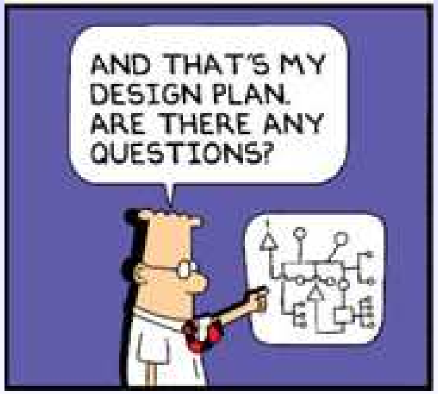


Conclusion

- Even without timelike information the extraction of the ground and first excited states is RELIABLE.

- The sign change in the decay constant is REPRODUCED *without* biasing the fit.

- With inclusion of timelike data higher excited states become ACCESSIBLE.



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