Advances In LF-QCD and New Perspectives on QCD from AdS/CFT

- Progress in hadron physics: Must confront the structure of hadrons at the amplitude level!
- AdS/CFT: Anti-deSitter Space/ Conformal Field Theory
- AdS/CFT provides a remarkably simple picture of the quark structure of the proton
- Non-perturbative derivation of constituent counting rules

Polchinski & Strassler

Hadron Dynamics at the Amplitude Level

- DIS studies have primarily focussed on probability distributions: integrated and unintegrated
- We need to determine hadron wavefunctions!
- Test QCD at the amplitude level: Phases, multi-parton correlations, spin, angular momentum
- Impact space: D. Soper, X.Ji, M.Burkardt

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Invariant under boosts. Independent of P^μ

The Light-Front Fock Expansion

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

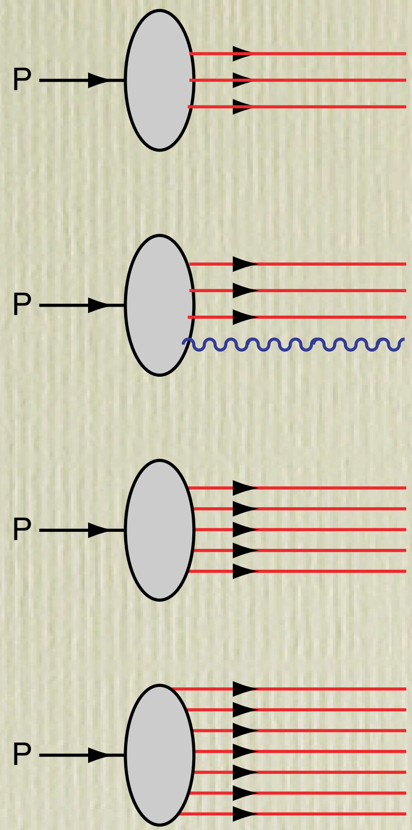
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Ashery E791 Measure pion LFWF in diffractive dijet production
Confirms color transparency !

Mueller, sjb Frankfurt, Miller, Strikman Bertsch et al

$$H^{QCD}_{LC}|\Psi_h\rangle = \mathcal{M}^2_h \; |\Psi_h\rangle$$

$$H^{QCD}_{LC} = P_\mu P^\mu = \quad P^- P^+ - \vec{P}^2_\perp$$

The hadron state $|\Psi_h\rangle$ is expanded in a Fock-state complete basis of non-interacting n-particle states $|n\rangle$ with an infinite number of components

$$\left|\Psi_h(P^+, \vec{P}_\perp)\right\rangle =$$

$$\sum_{n,\lambda_i} \int \; [dx_i \;\, d^2\vec{k}_{\perp i}] \psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\times \; |n : x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$$

$$\sum_n \int [dx_i \;\, d^2\vec{k}_{\perp i}] \; |\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)|^2 = 1$$

Hadron Distribution Amplitudes

$$\phi(x_i, Q) \equiv \Pi_{i=1}^{n-1} \int^Q d^2\vec{k}_\perp \ \psi_n(x_i, \vec{k}_{\perp i})$$

- Fundamental measure of valence wavefunction
- Gauge Independent (includes Wilson line)
- Evolution Equations, OPE
- Conformal Expansion
- Hadronic Input in Factorization Theorems

Lepage; SJB
Efremov, Radyuskin

Minkowski space !

Solving the LF Heisenberg Eqn.

- Discretized Light-Cone Quantization (DLCQ) Pauli, sjb
- Many 1+1 model field theories completely solved
- Transverse Lattice
- Bethe-Salpeter/Dyson Schwinger at fixed LF time
- Angular Structure of Solutions known
  Karmanov et al
- Use AdS/CFT model solutions as starting point!

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances, counting, rules for form factors and hard exclusive processes; nonpertubative derivation

- Goal: Use AdS/CFT to provide models of hadron structure: confinement at large distances, near conformal behavior at short distances

- Holographic Model: Initial “classical” approximation to QCD: Remarkable agreement with light hadron spectroscopy

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing $H^{LF}{}_{QCD}$; variational methods

Hadrons Fluctuate in Particle Number

- Proton Fock States

  $|uud>, |uudg>, |uuds\bar{s}>, |uudc\bar{c}>, |uudb\bar{b}> \cdots$

- Strange and Anti-Strange Quarks not Symmetric

  $$s(x) \neq \bar{s}(x)$$

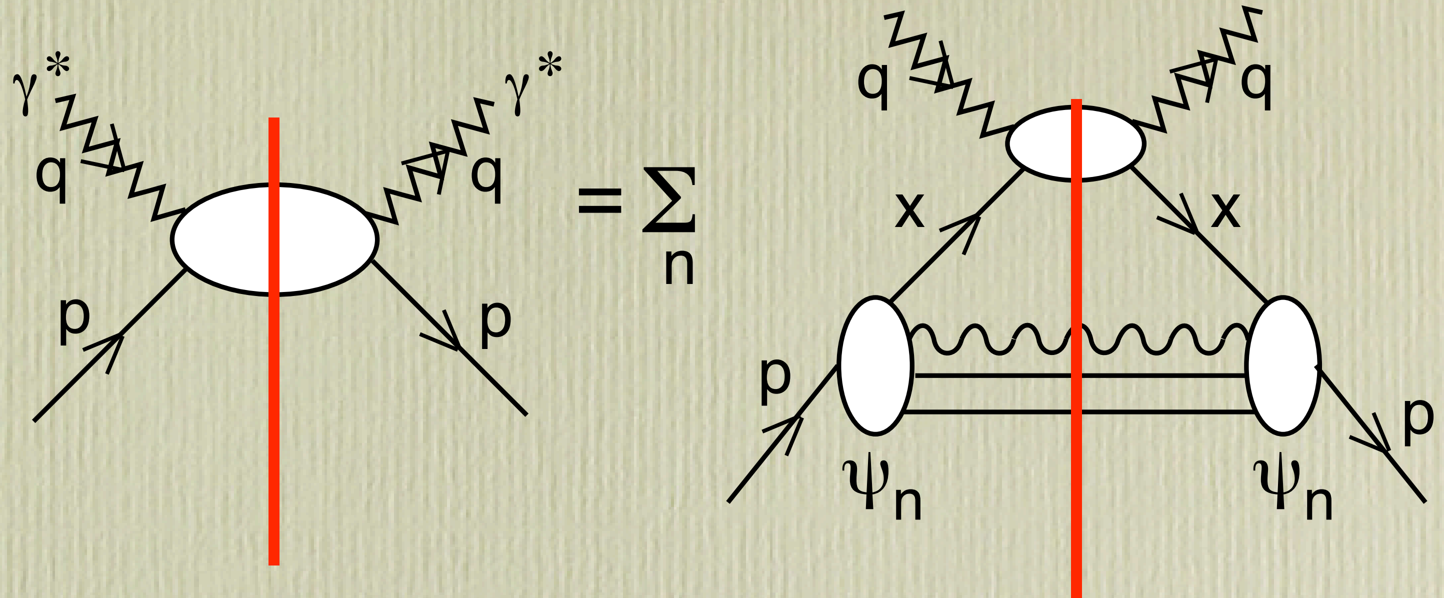
- "Intrinsic Charm": High momentum heavy quarks

- "Hidden Color": Deuteron not always p + n

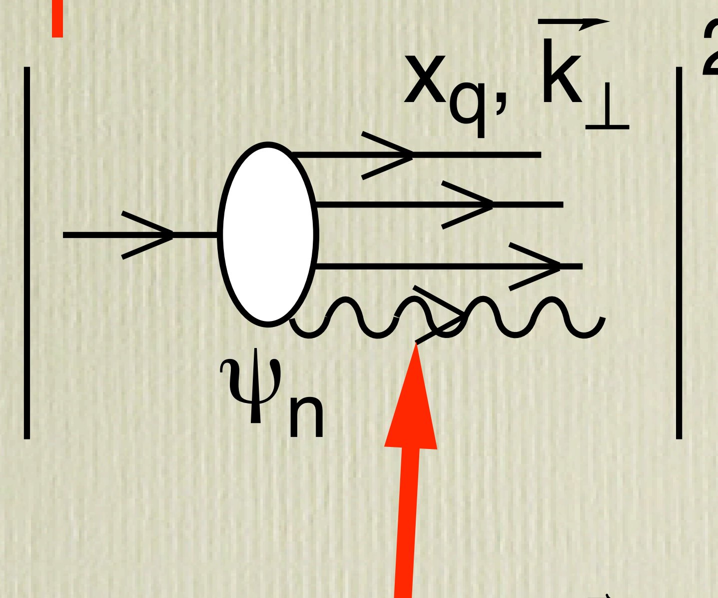
- Orbital Angular Momentum Fluctuations - Anomalous Magnetic Moment

- Light-Front Fock Expansions
- LFWFs boost invariant
- Direct connection to form factors, structure functions, distribution amplitudes, GPDs
- Higher Twist Correlations
- Orbital Angular Momentum, physical polarization in $A^+ = 0$ gauge
- Sum Rules
- Validated in QED, Bethe-Salpeter Eqn.

Deep Inelastic Lepton Proton Scattering



Imaginary Part of
Forward Virtual Compton Amplitude



$q(x,Q^2) = \sum_n \int^{k_\perp^2 \leq Q^2 \perp} d^2k_\perp \, |\Psi_n(x,k_\perp)|^2$

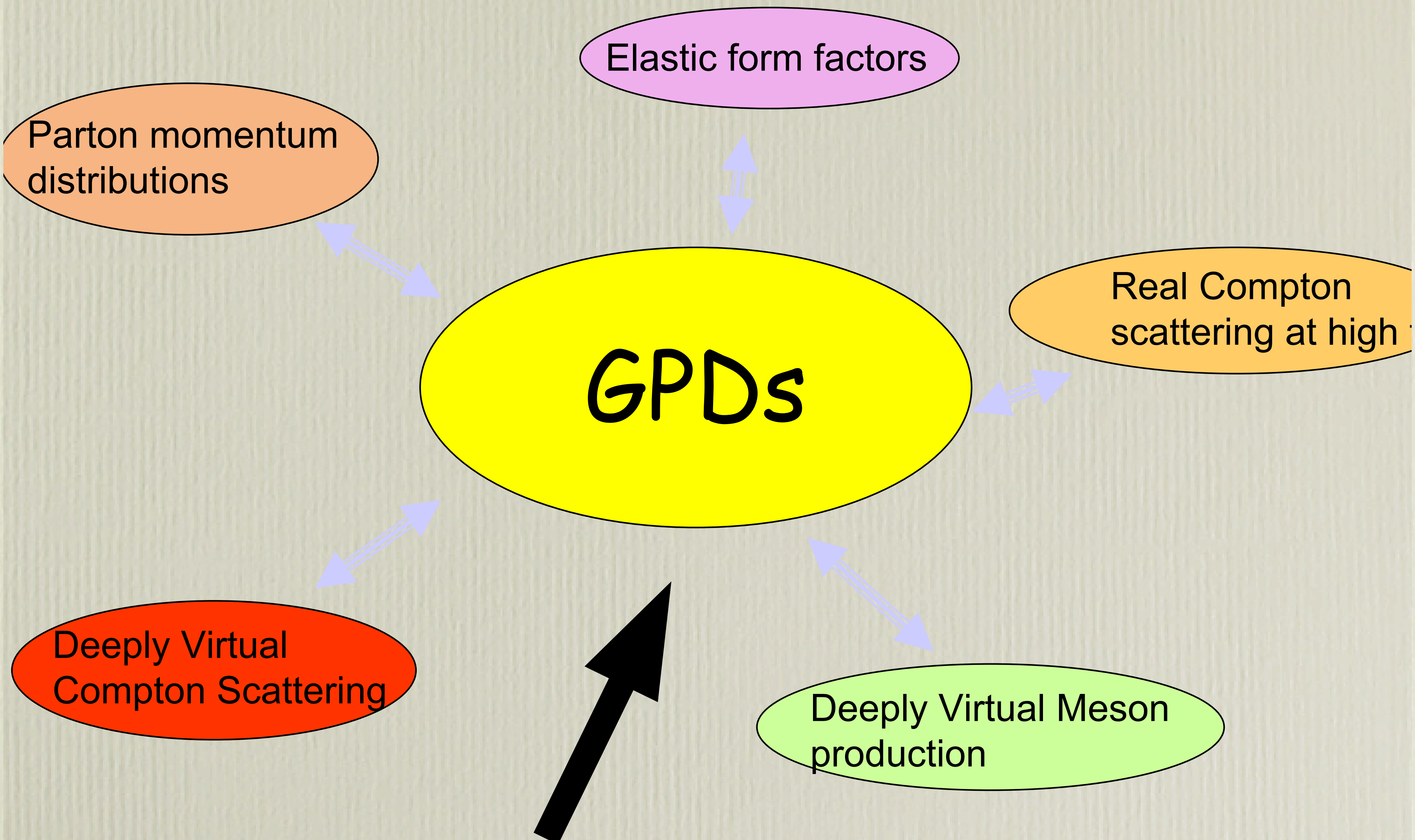
All spin,flavor distributions

$x = x_q$

Light-Front Wave Functions $\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

DDIS from rescattering BHMPS

A Unified Description of Hadron Structure



Light Front Wavefunctions

Exact Representation of Form Factors using LFWFs

Drell Yan, West, Drell SJB

Hadron form factors can be expressed as a sum of overlap integrals of light-front wave functions:

$$F(q^2) = \sum_n \int \left[dx_i\right] \left[d^2\vec{k}_{\perp i}\right] \sum_j e_j \psi_n^*(x_i, \vec{k}'_{\perp i}, \lambda_i) \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i), \tag{1}$$

where the variables of the light-cone Fock components in the final-state are given by

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\ \vec{q}_\perp, \tag{2}$$

for a struck constituent quark and

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\ \vec{q}_\perp, \tag{3}$$

for each spectator. The momentum transfer is $q^2 = -\vec{q}_\perp^{\,2} = -2P \cdot q = -Q^2$. The measure of the phase-space integration is

$$\left[dx_i\right] = \prod_{i=1}^n \frac{dx_i}{\sqrt{x_i}}\, \delta\left(1 - \sum_{j=1}^n x_j\right), \tag{4}$$

$$\left[d^2\vec{k}_{\perp i}\right] = (16\pi^3) \prod_{i=1}^n \frac{d^2\vec{k}_{\perp i}}{16\pi^3} \delta^{(2)}\left(\sum_{\ell=1}^n \vec{k}_{\perp \ell}\right). \tag{5}$$

- Essential check: Vanishing of "anomalous gravitomagnetic moment": $B(0) = 0$

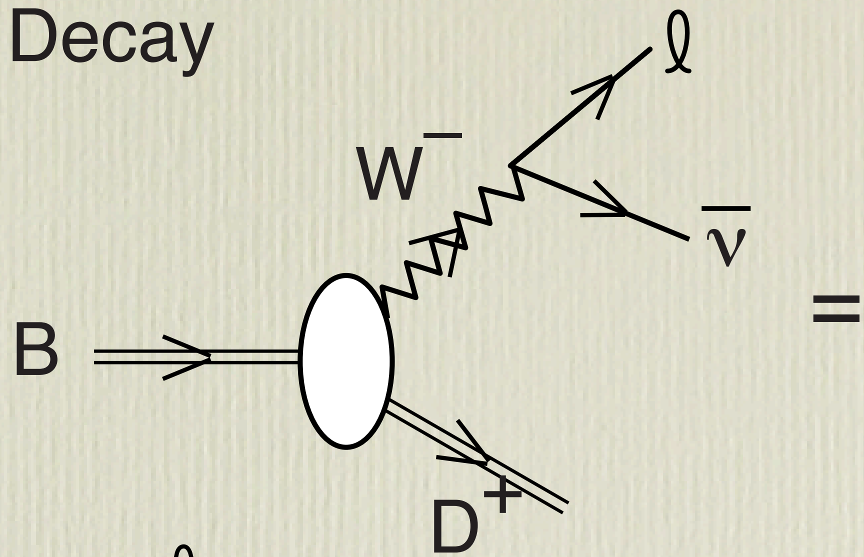
Connection to Ji's sum rule

- Exact property of LFWFS, Fock-state by Fock-state

Hwang, Schmidt, sjb

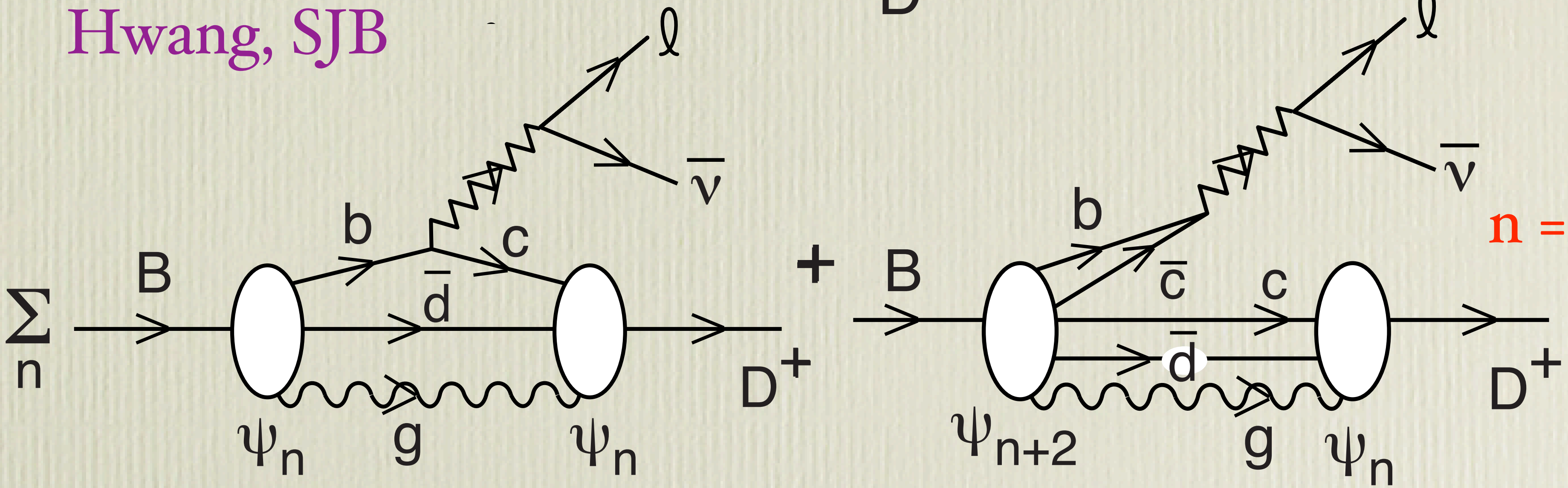
Weak Exclusive Decay

$\langle D \,|\, J^{+}(0) \,|\, B \rangle$



Exact Formula

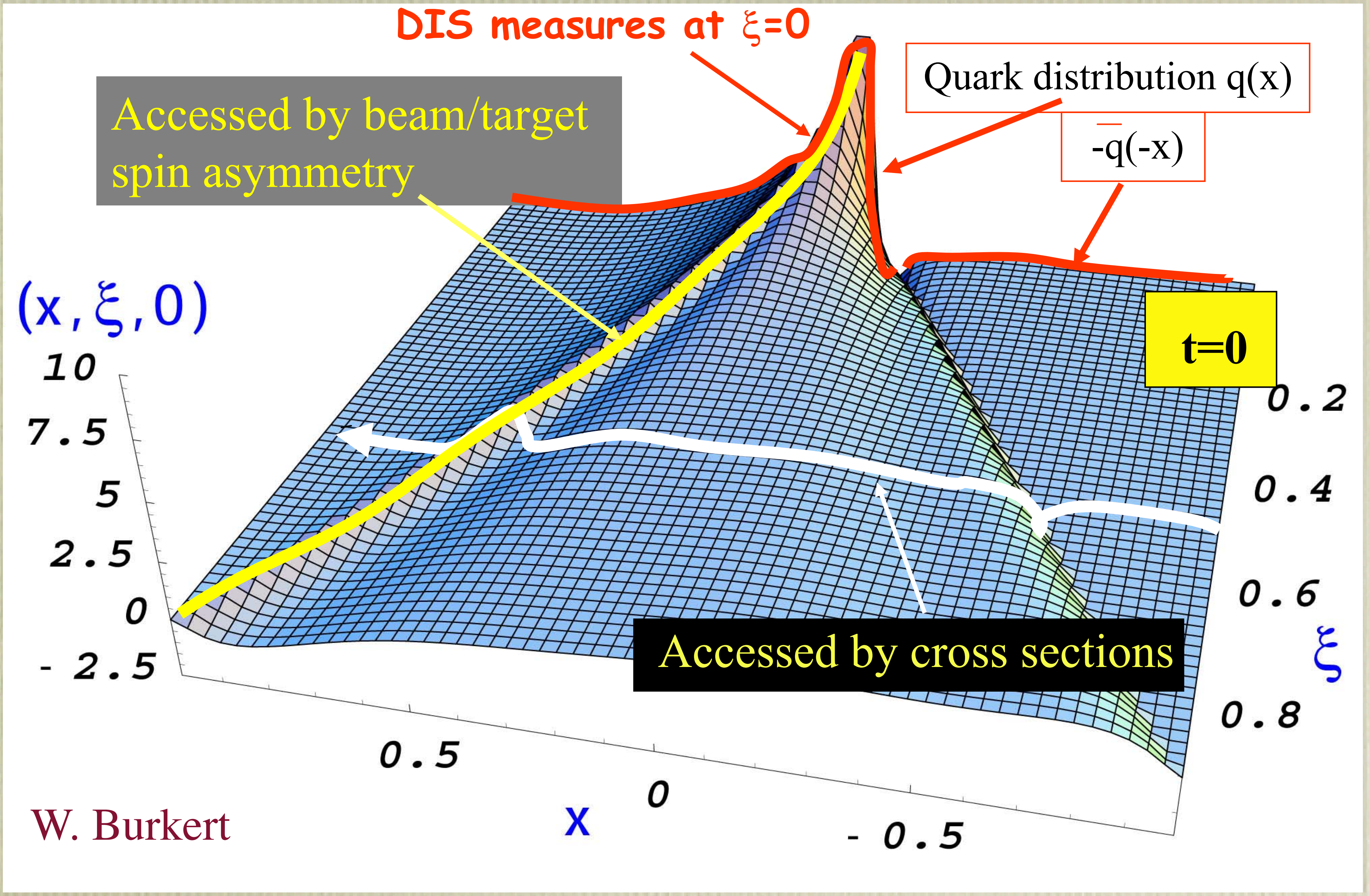
Hwang, SJB



$n = n' + 2$

Annihilation amplitude needed for Lorentz Invariance

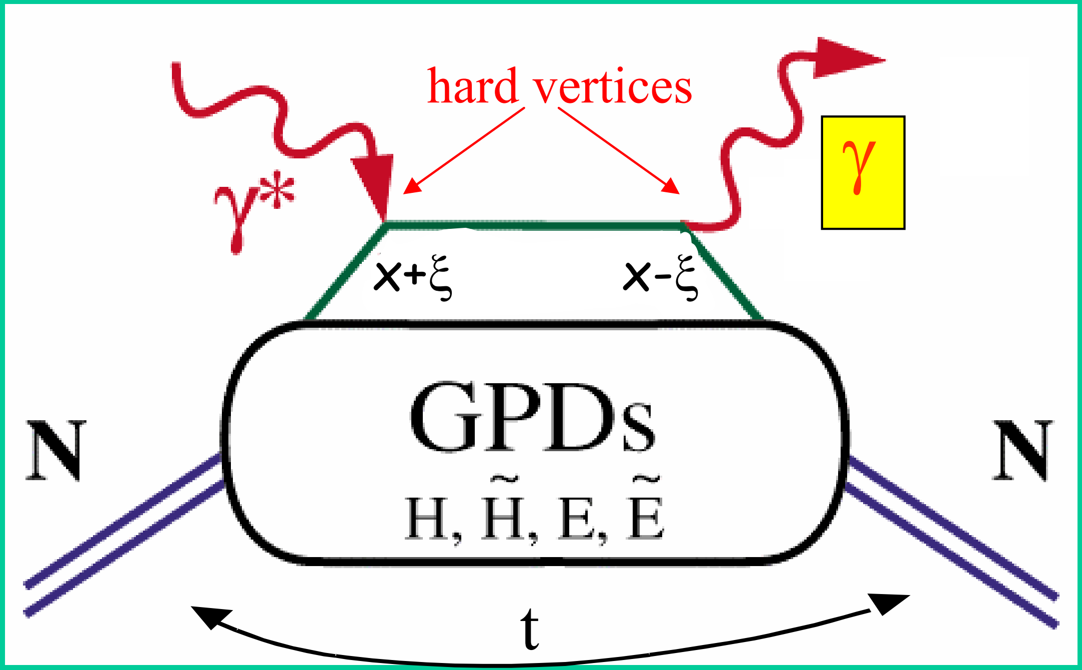
Access GPDs through x-section & asymmetries



GPDs & Deeply Virtual Exclusive Processes

"handbag" mechanism

Deeply Virtual Compton Scattering (DVCS)



x – longitudinal quark momentum fraction

2ξ – longitudinal momentum transfer

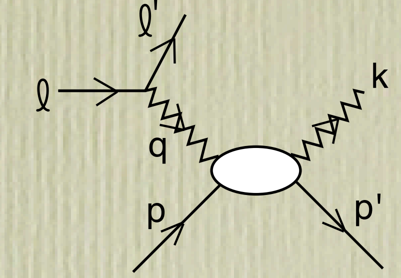
$\sqrt{-t}$ – Fourier conjugate to transverse impact parameter

$H(x,\xi,t), E(x,\xi,t), \ldots$

$$\xi = \frac{x_B}{2-x_B}$$

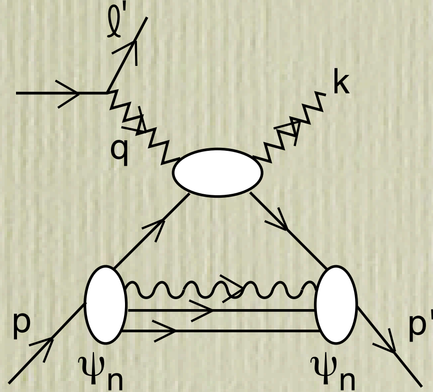
$\langle p' \lambda' | J^{\mu}(z) J^{\nu}(0) | p \lambda \rangle$

Large $- q^2 = Q^2$



$\gamma^* p \to \gamma p'$

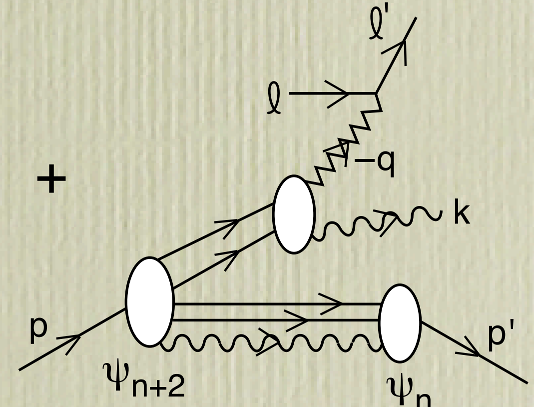
$\Rightarrow \sum_n$



Given LFWFs, compute all GPDs !

Deeply Virtual Compton Scattering

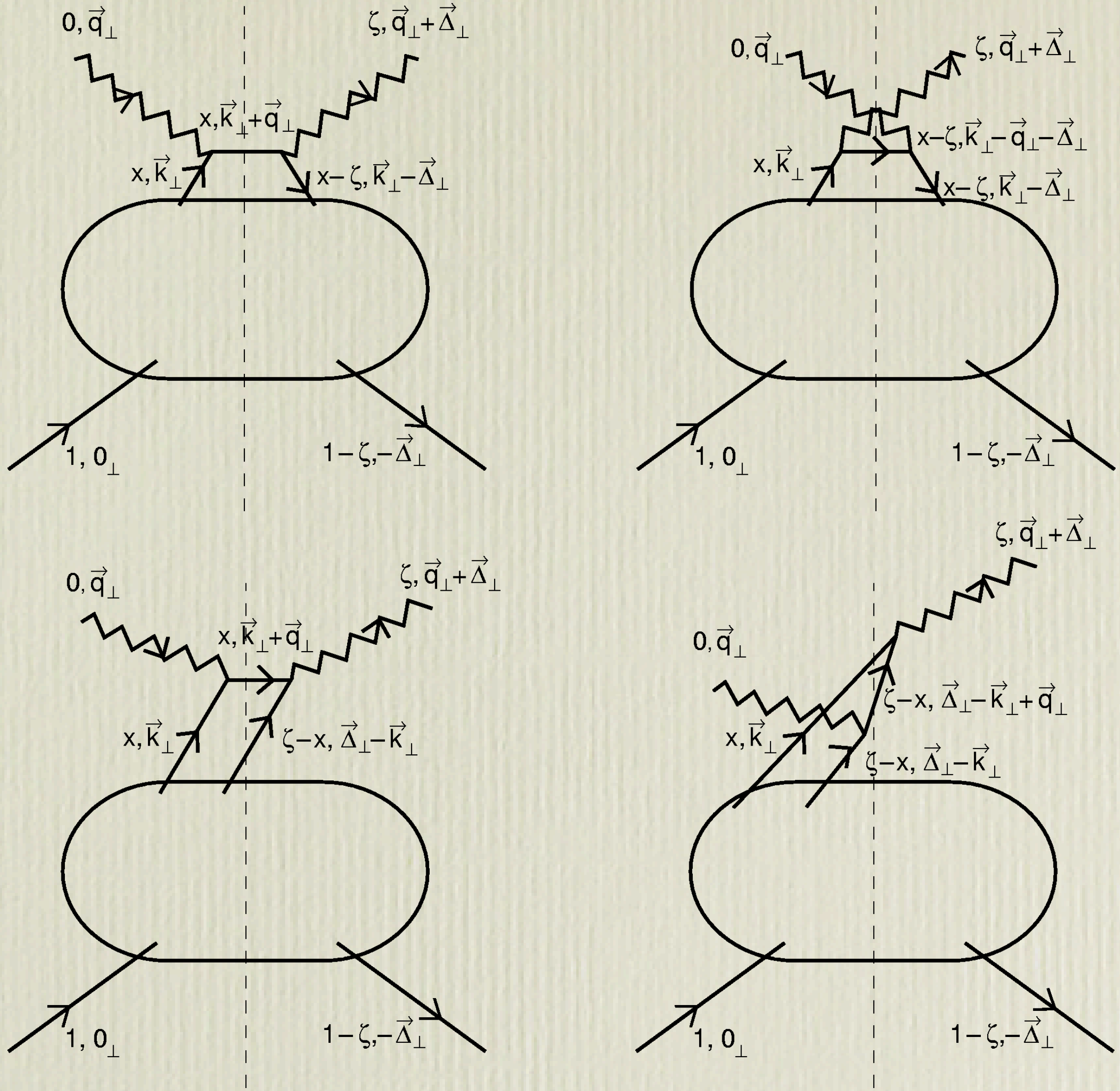
+



n = n' + 2

ERBL Evolution

Required for Lorentz Invariance



Light-cone wavefunction representation of deeply virtual Compton scattering ☆

Stanley J. Brodsky [a], Markus Diehl [a,1], Dae Sung Hwang [b]

Advances in LFQCD- AdS/CFT

Example of LFWF representation of GPDs (n => n)

Diehl,Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\,\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\,\mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1-\sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times\, \delta(x-x_1)\,\psi^{\uparrow *}_{(n)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\,\psi^{\downarrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i),$$

where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1-\zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i=2,\ldots,n.$$

Example of LFWF representation of GPDs (n+1 => n-1)

Diehl,Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}}\frac{\Delta^1 - i\Delta^2}{2M} E_{(n+1\to n-1)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{3-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n+1} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3} 16\pi^3 \delta\left(1-\sum_{j=1}^{n+1} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n+1} \vec{k}_{\perp j}\right)$$

$$\times 16\pi^3 \delta(x_{n+1}+x_1-\zeta)\delta^{(2)}(\vec{k}_{\perp n+1}+\vec{k}_{\perp 1}-\vec{\Delta}_\perp)$$

$$\times \delta(x-x_1)\psi^{\uparrow *}_{(n-1)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\psi^{\downarrow}_{(n+1)}(x_i,\vec{k}_{\perp i},\lambda_i)\delta_{\lambda_1 -\lambda_{n+1}},$$

where $i = 2, \ldots, n$ label the $n-1$ spectator partons which appear in the final-state hadron wavefunction with

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp.$$

Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$
$$\widetilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int^1 dx \sum_q \left[H^q(x,\xi,t)\right] = F_1(t) \text{ Dirac f.f.}$$
$$\int^1 dx \sum_q \left[E^q(x,\xi,t)\right] = F_2(t) \text{ Pauli f.f.}$$
$$\int_{-1}^1 dx\, \widetilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^1 dx\, \widetilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

$$H^q, E^q, \widetilde{H}^q, \widetilde{E}^q(x,\xi,t)$$

Verified using LFWFs

Diehl,Hwang, sjb

Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^1 x\,dx\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

Deeply Virtual Compton Scattering

$$\gamma^* p \to \gamma p', \gamma^* p \to \pi^+ n',$$

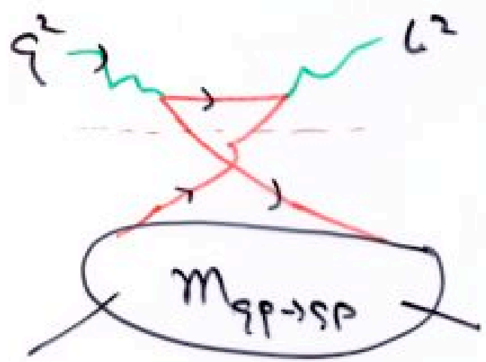
- Remarkable sensitivity to spin, flavor, dynamics
- Measure Real and Imaginary parts from Bethe-Heitler Interference; phase determined by Regge theory (Kuti-Weiskopf)

Close, Gunion, sjb

- J=0 fixed pole: test QCD contact interaction!
- Sum Rules connecting to form factors, Lz
- Evolution Equations (ERBL), PQCD constraints
- Convolutions of Light-front wavefunctions

Given $\mathcal{M}_{qp\to qp}(\hat{s},t)$

predict $T_{\mu\nu}(q^2, p\cdot q, t)$



gauge-invariant crossing symmetric

+ crossed graph

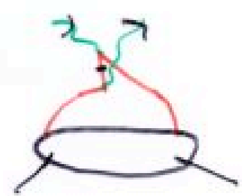
Measure Real part from interference with B.H.

In $T_{\mu\nu}(q^2, p\cdot q, t=0) = W_{\mu\nu}(q^2, p\cdot q)$

Bj-scaling structure functions

Extra J=0 pieces from "fixed pole"

instantaneous quark exchange



"J=0 Fixed Pole"

Two-photon contact Interaction -- Unique to gauge theory

Close, Gunion, SJB

J=0 Fixed pole

Close, Gunion, sjb

- Contact term (seagull for scalar quarks)
- Real phase
- $M = s^0\, F(t)$
- Independent of Q^2 at fixed t
- <1/x> Moment: Related to Feynman-Hellman Theorem

Test J=0 Fixed Pole: $s^2\, d\sigma/dt(\gamma\, p \to \gamma\, p) \simeq F^2_0(t)$

Measure at BaBar, Belle, CLEO, BES

Timelike $e^+e^- \to \pi^+\pi^-\gamma$

Measure "timelike annihilation" DVCS
$\gamma^* \to \pi^+\pi^-\gamma$

Interferes with Bremsstrahlung from the annihilating leptons.

Electron-Positron asymmetry measures interference of pion form factor and DVCS amplitude.

Re $M^\dagger(\gamma^* \to \pi^+\pi^-) \times M(\gamma^* \to \pi^+\pi^-\gamma)$

Extend to all hadron pairs.

Single spin asymmetries give conjugate phase.

Afanasev, Carlson, sjb

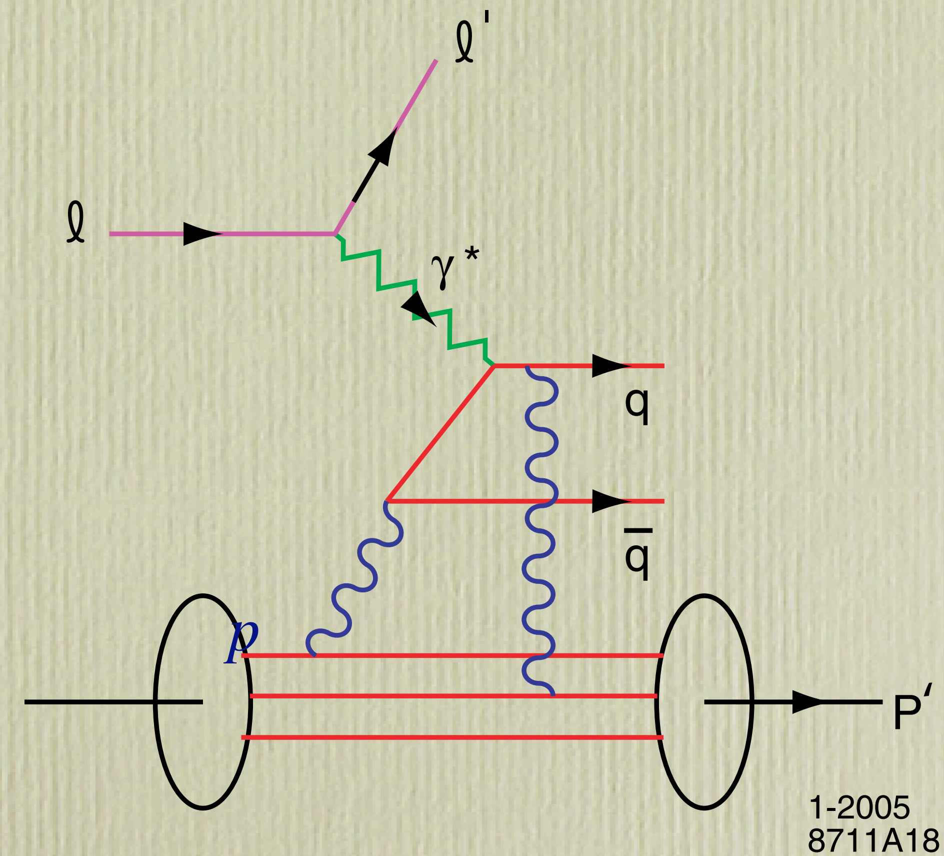
Include Corrections to Handbag Diagram in DVCS

- Wilson line: Rescattering Corrections to Deep Inelastic Scattering

Hoyer, Marchal, Peigne, Sannino, sjb

- Diffractive DIS
- Single Spin Asymmetries — Hwang, Schmidt, sjb; Collins; X.Ji
- Shadowing, Antishadowing — Schmidt, Yang, Lu, sjb
- All Leading Twist
- Gauge Invariant (Also in LCG)

Final-State Interaction Produces Diffractive DIS



Quark Rescattering

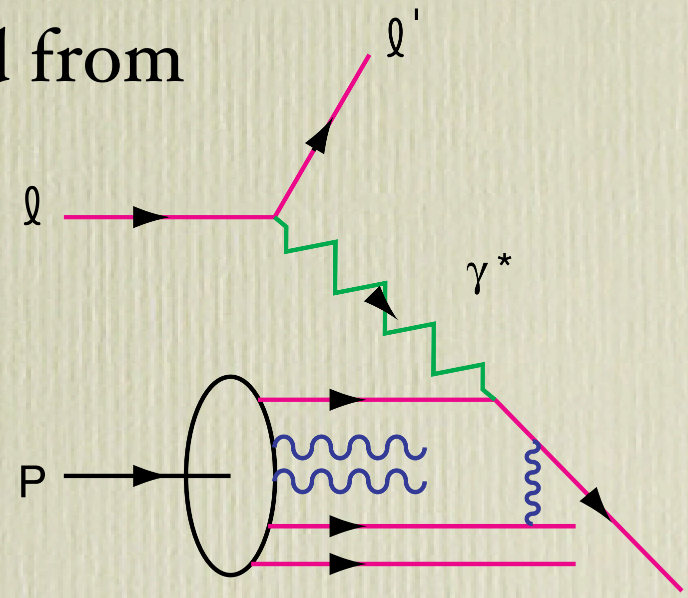
Hoyer, Marchal, Peigne, Sannino, SJB (BHMPS)

Enberg, Hoyer, Ingelman, SJB

Hwang, Schmidt, SJB

Leading Twist!

- Quarks Reinteract in Final State
- Analogous to Coulomb phases, but not unitary
- Observable effects: DDIS, SSI, shadowing, non-universal antishadowing
- Structure functions cannot be computed from LFWFs computed in isolation
- Recover Wilson line physics even in lcg

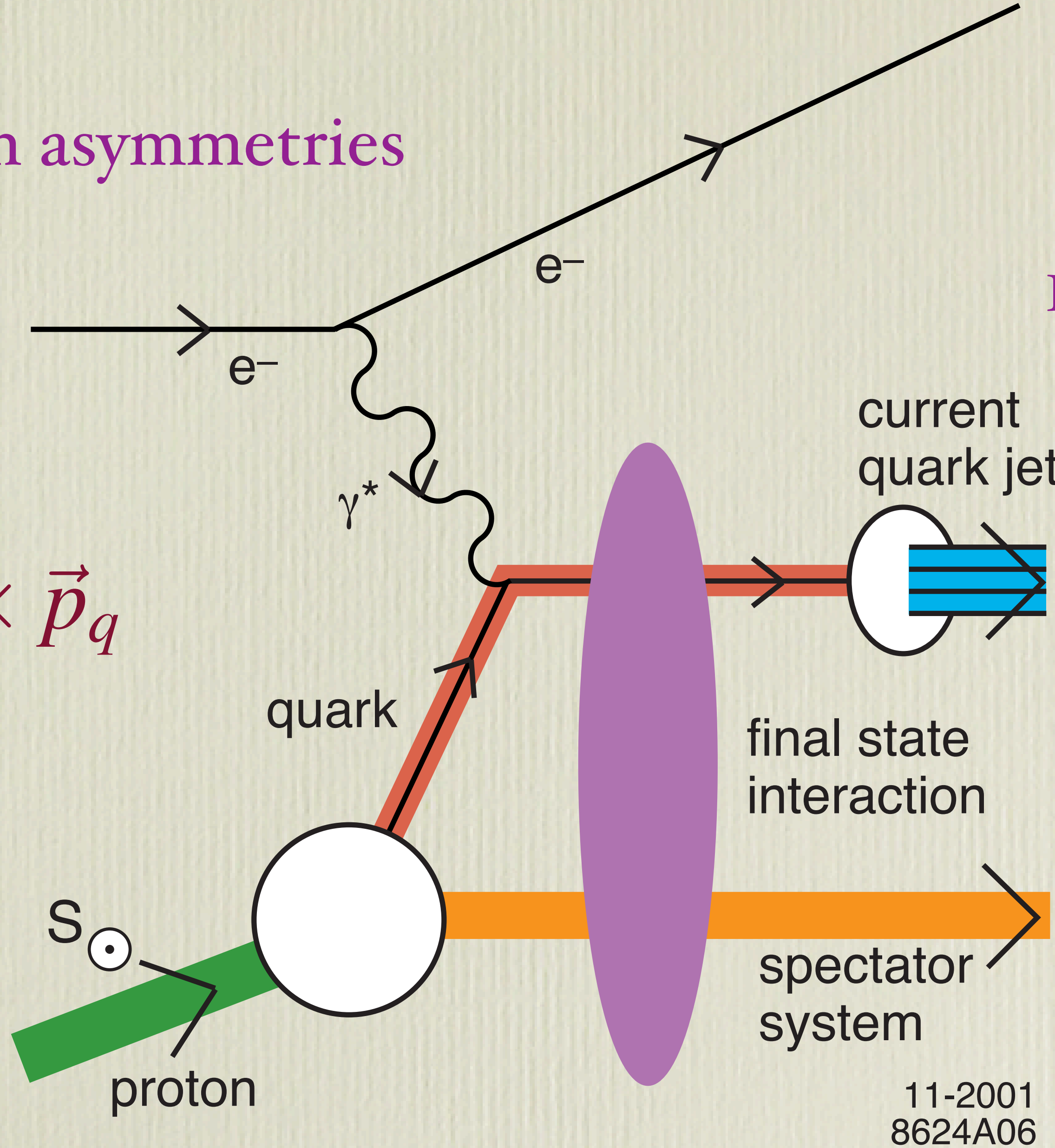


Single-spin asymmetries

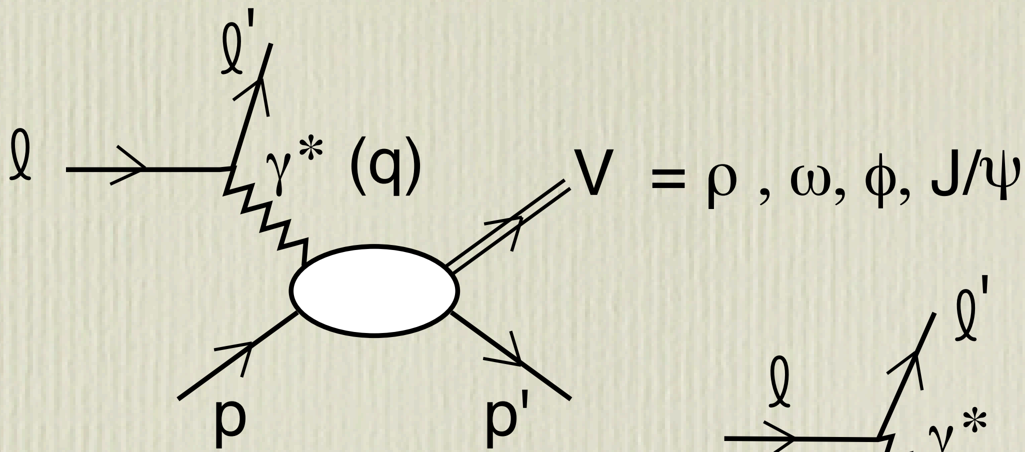
Sivers Effect

Hwang, Schmidt, sjb
Collins

$$\vec{S}_p \cdot \vec{q} \times \vec{p}_q$$



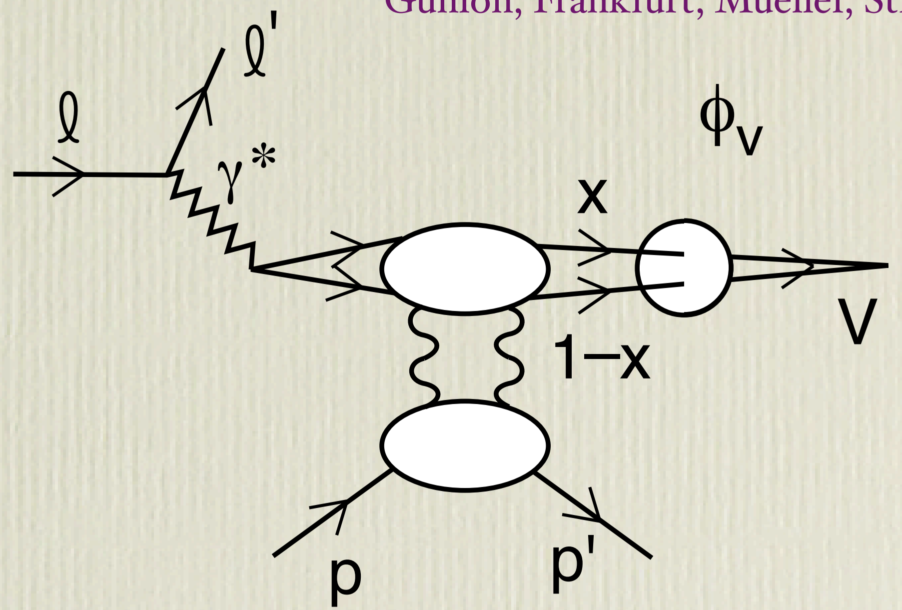
Vector Meson Leptoproduction

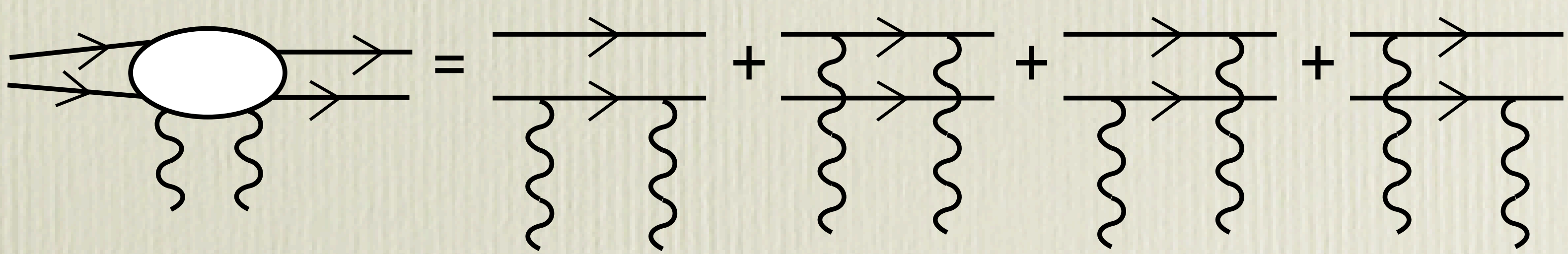


Gunion, Frankfurt, Mueller, Strikman, sjb

Large $-q^2 = Q^2$

⇨

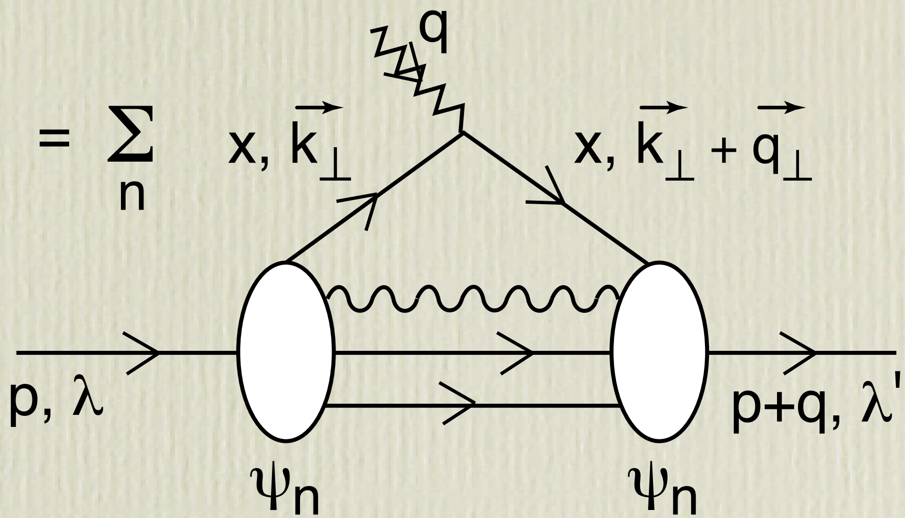




$$\phi(x,Q) = \int d^2k_\perp \Psi_{q\bar{q}}(x,\vec{k}_\perp)$$

Form Factors $\ell p \rightarrow \ell' p'$ $\langle p' \lambda' | J^{+}(0) | p \lambda \rangle$

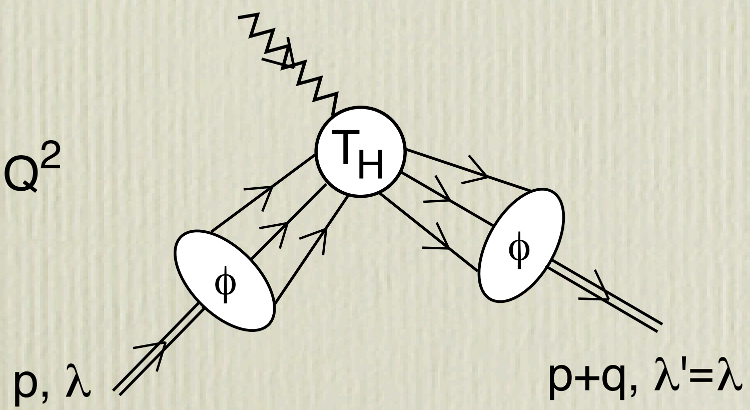
$$F_{\lambda\lambda'}(Q^2) = \sum_n$$



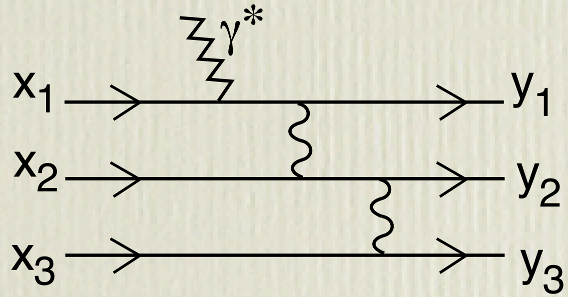
QCD Factorization

Lepage, Sjb
Efremov
Radyushkin

Large Q^2



$T_H = \sum$

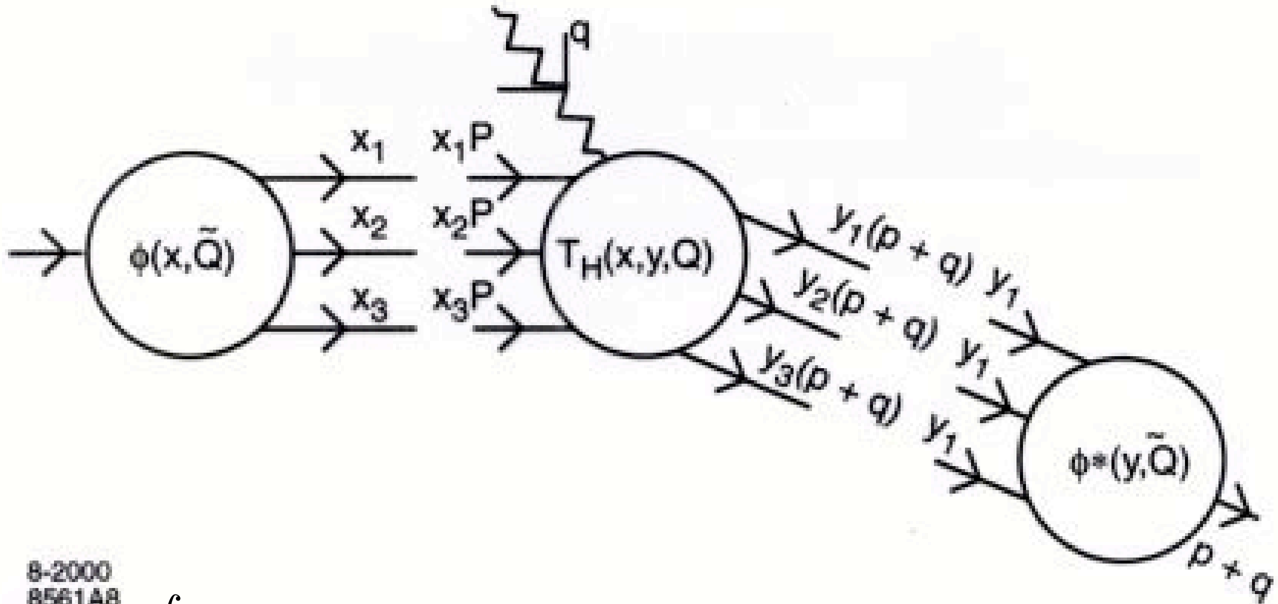


$+ \ \ldots$

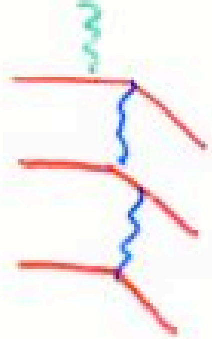
$$= \frac{\alpha_s^2}{Q^4} f(x_i, y_i,)$$

Scaling Laws from PQCD or AdS/CFT

Primary Test of QCD Factorization, Scaling



$$M = \int \prod dx_i dy_i \phi_F(x, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \phi_I(y_i, Q)$$



$\frac{\alpha_s^2(Q^*)}{Q^4}$

PQCD and Exclusive Processes

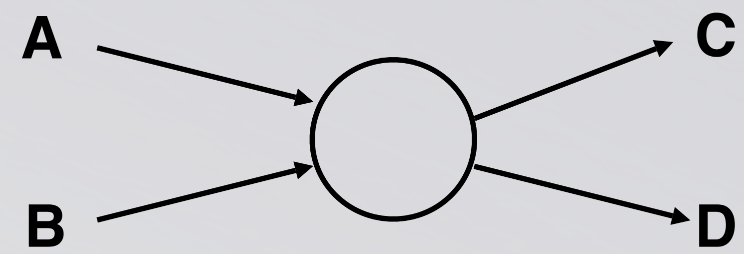
G.P. Lepage, sjb

$$M = \int \prod dx_i dy_i \phi_F(x, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \phi_I(y_i, Q)$$

- Iterate kernel of LFWFs when at high virtuality; distribution amplitude contains all physics below factorization scale
- Rigorous Factorization Formulae: Leading twist
- Underly Exclusive B-decay analyses
- Distribution amplitude: gauge invariant, OPE, evolution equations, conformal expansions
- BLM scale setting: sum nonconformal contributions in scale of running coupling

Scaling is a manifestation of asymptotically free hadron interactions

From dimensional arguments at high energies in binary reactions:



CONSTITUENT COUNTING RULE

Brodsky and Farrar, Phys. Rev. Lett. 31 (1973) 1153
Matveev et al., Lett. Nuovo Cimento, 7 (1973) 719

Counting Rules:

$$q(x) \sim (1-x)^{2n_{spect}-1} \text{ for } x \to 1$$

$$F(Q^2) \sim \left(\frac{1}{Q^2}\right)^{(n-1)}$$

$$\frac{d\sigma}{dt}(AB \to CD) \sim \frac{F(t/s)}{s^{(n_{participants}-2)}}$$

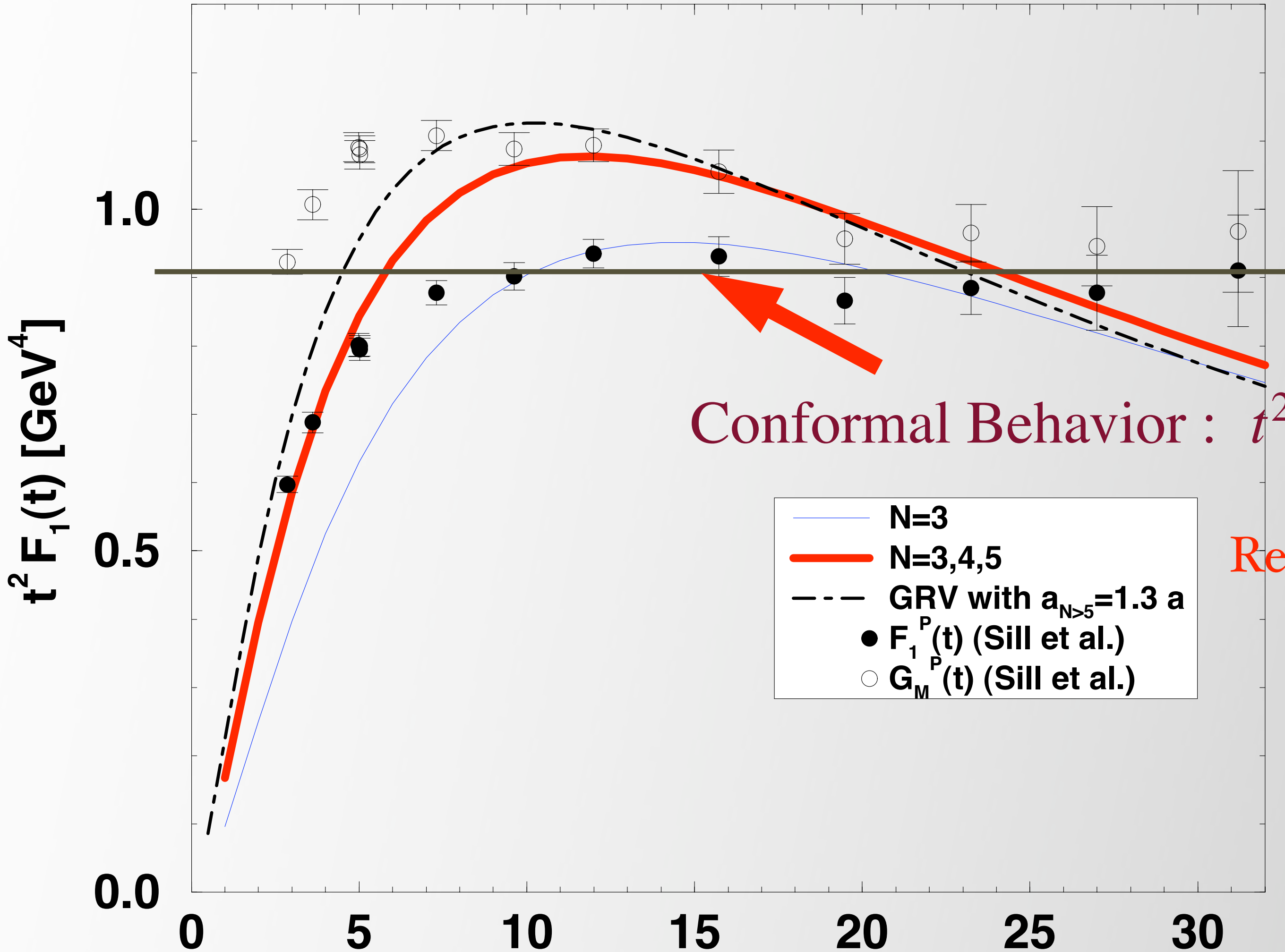
$$n_{participants} = n_A + n_B + n_C + n_D$$

$$\frac{d\sigma}{d^3p/E}(AB \to CX) \sim F(\hat{t}/\hat{s}) \times \frac{(1-x_R)^{(2n_{spectators}-1)}}{(p_T^2)^{(n_{participants}-2)}}$$

helicity conservation

Farrar, Jackson;
Lepage, sjb;
Burkardt, Schmidt, Sjb

Proton Form Factor



-t (GeV^2)

Conformal Behavior : $t^2 F_1(t) = \text{const}$

Remarkable scaling behavior

Non-perturbative model: Diehl, Kroll

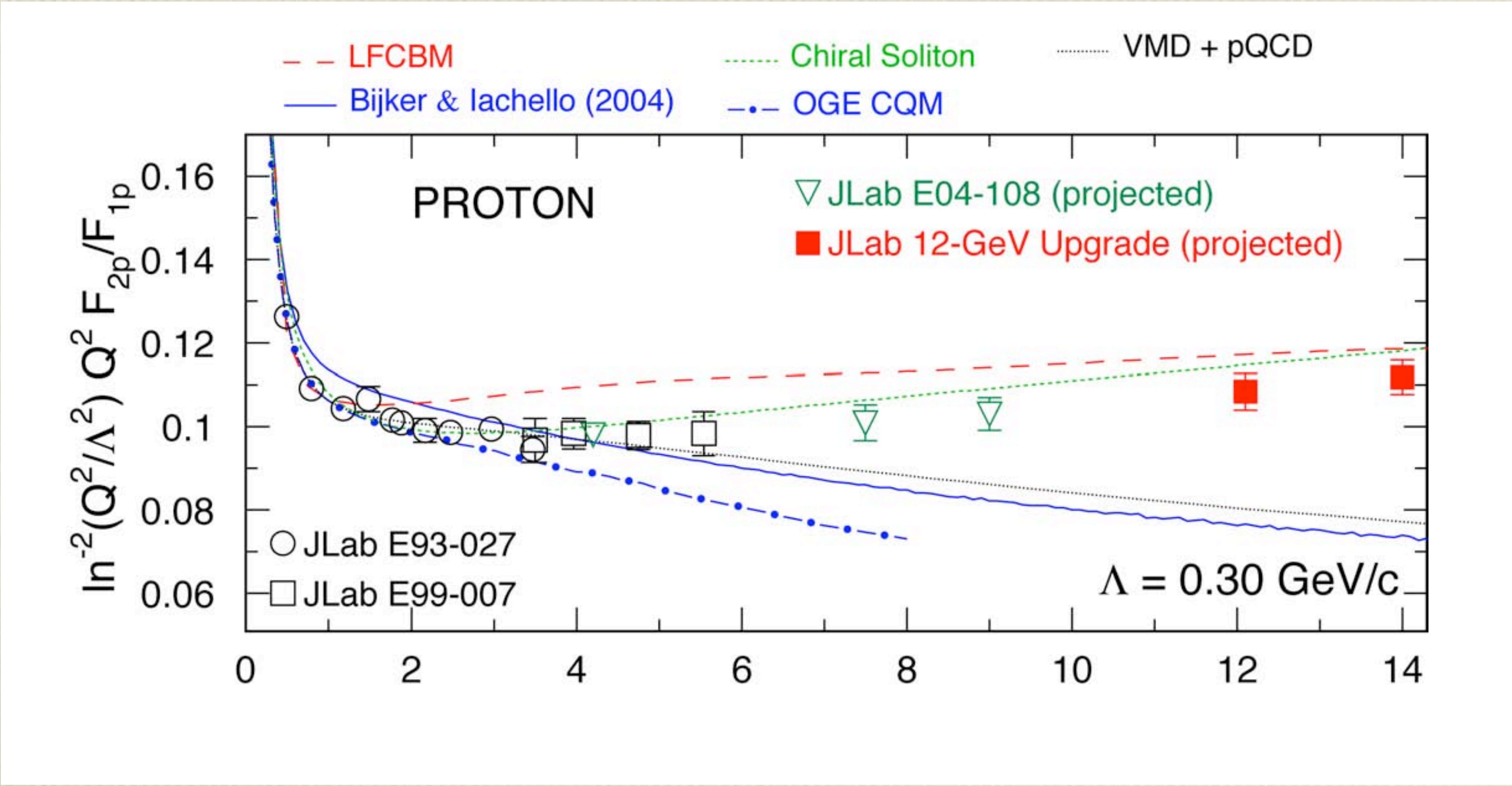
Advances in LFQCD- AdS/CFT

PQCD prediction:

Ji, Ma, Yuan

$$\frac{F_2(Q^2)}{F_1(Q^2)} \longrightarrow \frac{\Lambda^2_{QCD}\ln^2 Q^2}{Q^2}$$

Contribution from nonzero orbital angular momentum $L_z = \pm 1$



Breakdown of Rosenbluth Formula for G_E, G_M separation

- Two-photon exchange correction, elastic and inelastic nucleon channels significant; interference with one-photon exchange destroys Rosenbluth method

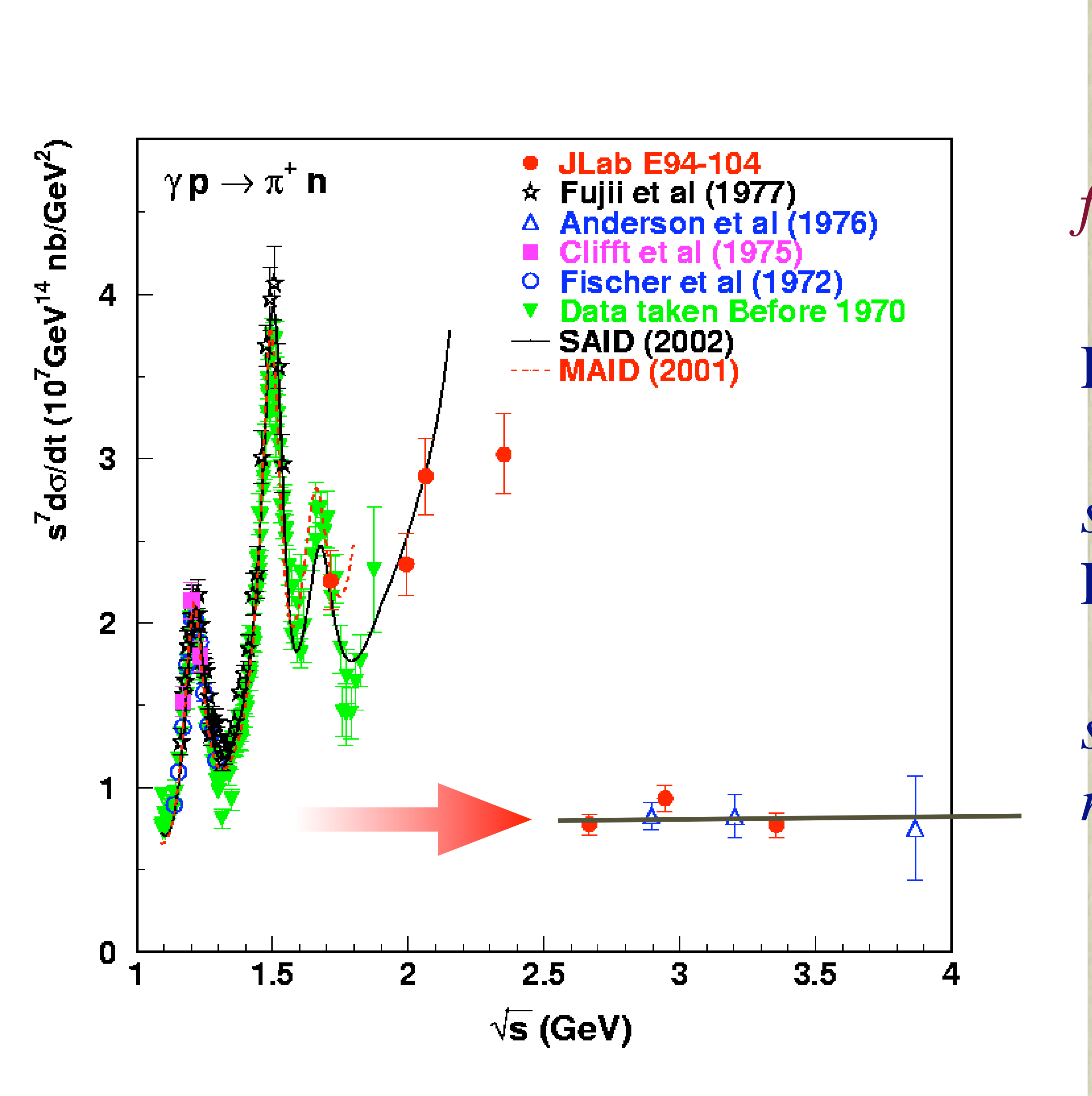
  Blunden, Melnitchouk; Afanasev, Chen,Carlson, Vanderhaegen, sjb

- Use J-Lab polarization transfer method

- Timelike form factors for radiative return; angular separation

- $e^+ e^-$ charge asymmetry from interference

Test of PQCD Scaling



$s^7 d\sigma/dt(\gamma p \to \pi^+ n) \sim const$

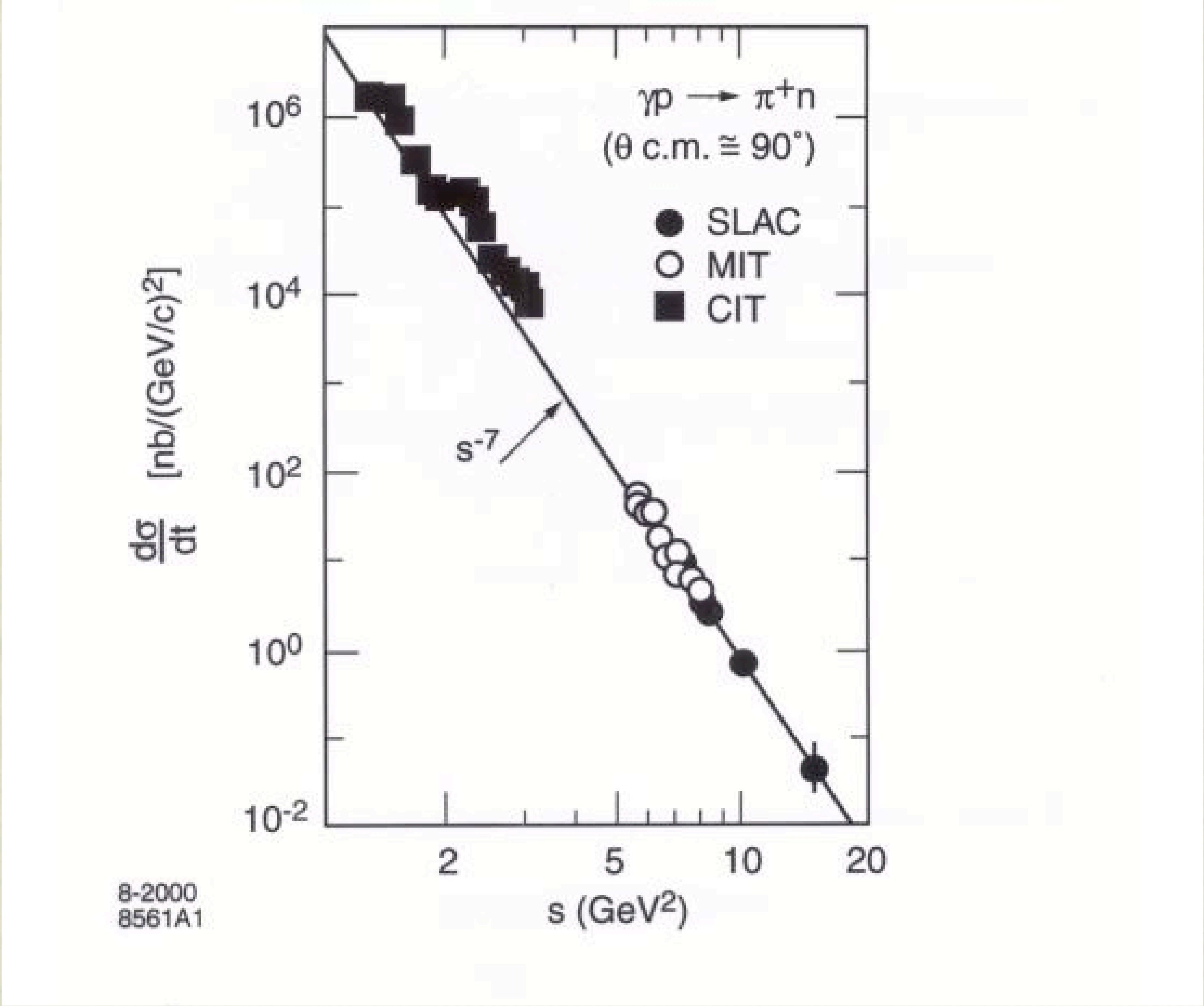
fixed θ_{CM} scaling

PQCD and AdS/CFT:

$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B \to C+D) =$$
$$F_{A+B\to C+D}(\theta_{CM})$$

$$s^7\frac{d\sigma}{dt}(\gamma p \to \pi^+ n) = F(\theta_{CM})$$
$$n_{tot} = 1+3+2+3 = 9$$

Possible substructure at strangeness and charm thresholds



$\gamma p \to \pi^+ n$
(θ c.m. ≅ 90°)
SLAC
MIT
CIT
s^{-7}
$\frac{d\sigma}{dt}$ [nb/(GeV/c)^2]
10^6
10^4
10^2
10^0
10^{-2}
2
5
10
20
s (GeV^2)
8-2000
8561A1

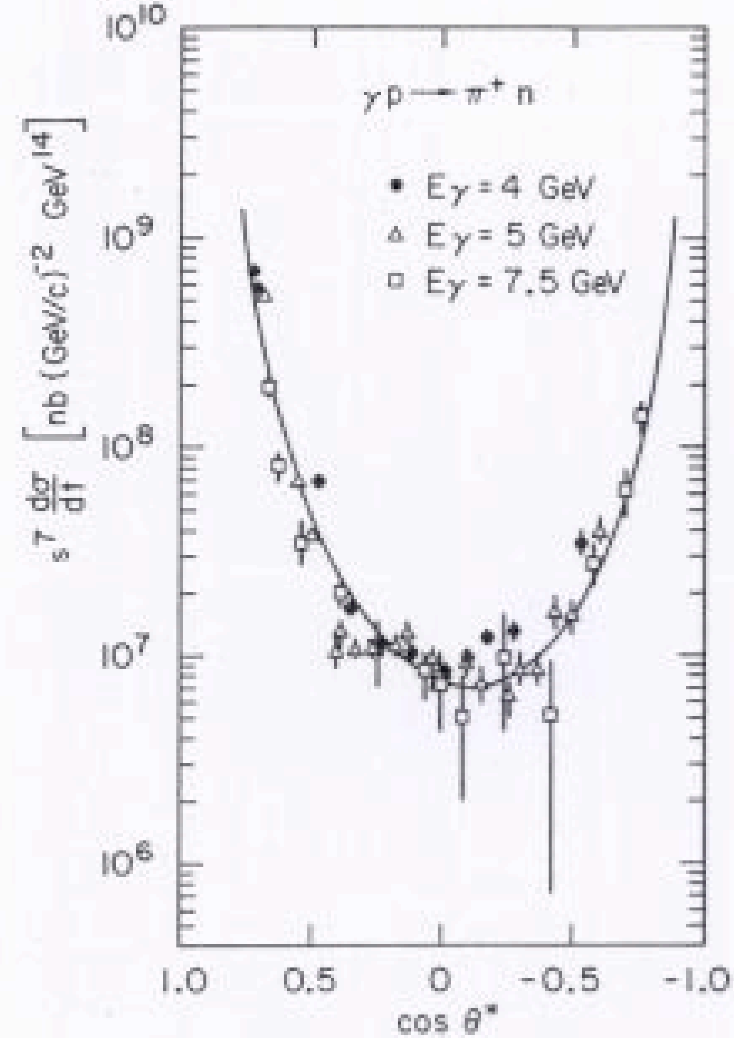


FIG. 6. $s^7 d\sigma/dt$ versus $\cos\theta^*$ for the reaction $\gamma p \to \pi^+ n$. The solid line shows the empirical function $(1-z)^{-5}(1+z)^{-4}$ where ($z=\cos\theta^*$), which is an empirical fit to the angular distribution.

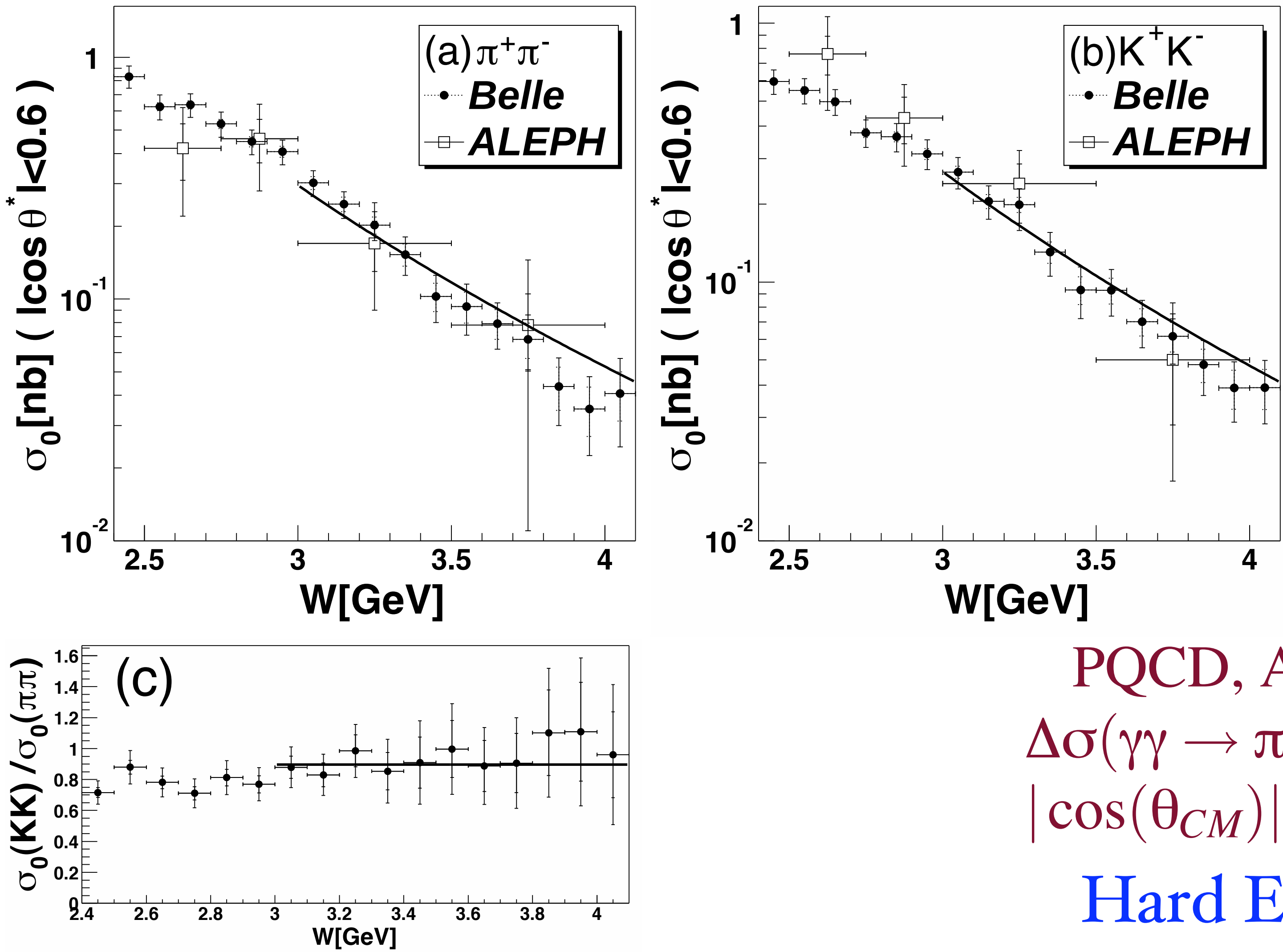


Fig. 5. Cross section for (a) $\gamma\gamma \to \pi^+\pi^-$, (b) $\gamma\gamma \to K^+K^-$ in the c.m. angular region $|\cos\theta^*| < 0.6$ together with a W^{-6} dependence line derived from the fit of $s|R_M|$. (c) shows the cross section ratio. The solid line is the result of the fit for the data above 3 GeV. The errors indicated by short ticks are statistical only.

PQCD, AdS/CFT:

$$\Delta\sigma(\gamma\gamma \to \pi^+\pi^-, K^+, K^-) \sim 1/W^6$$

$$|\cos(\theta_{CM})| < 0.6$$

Hard Exclusive Processes:
Fixed angle

$$\text{PQCD:}\quad \frac{d\sigma}{d|\cos\theta^*|}(\gamma\gamma\to M^+M^-)\approx\frac{16\pi\alpha^2}{s}\frac{|F_M(s)|^2}{\sin^4\theta^*},$$

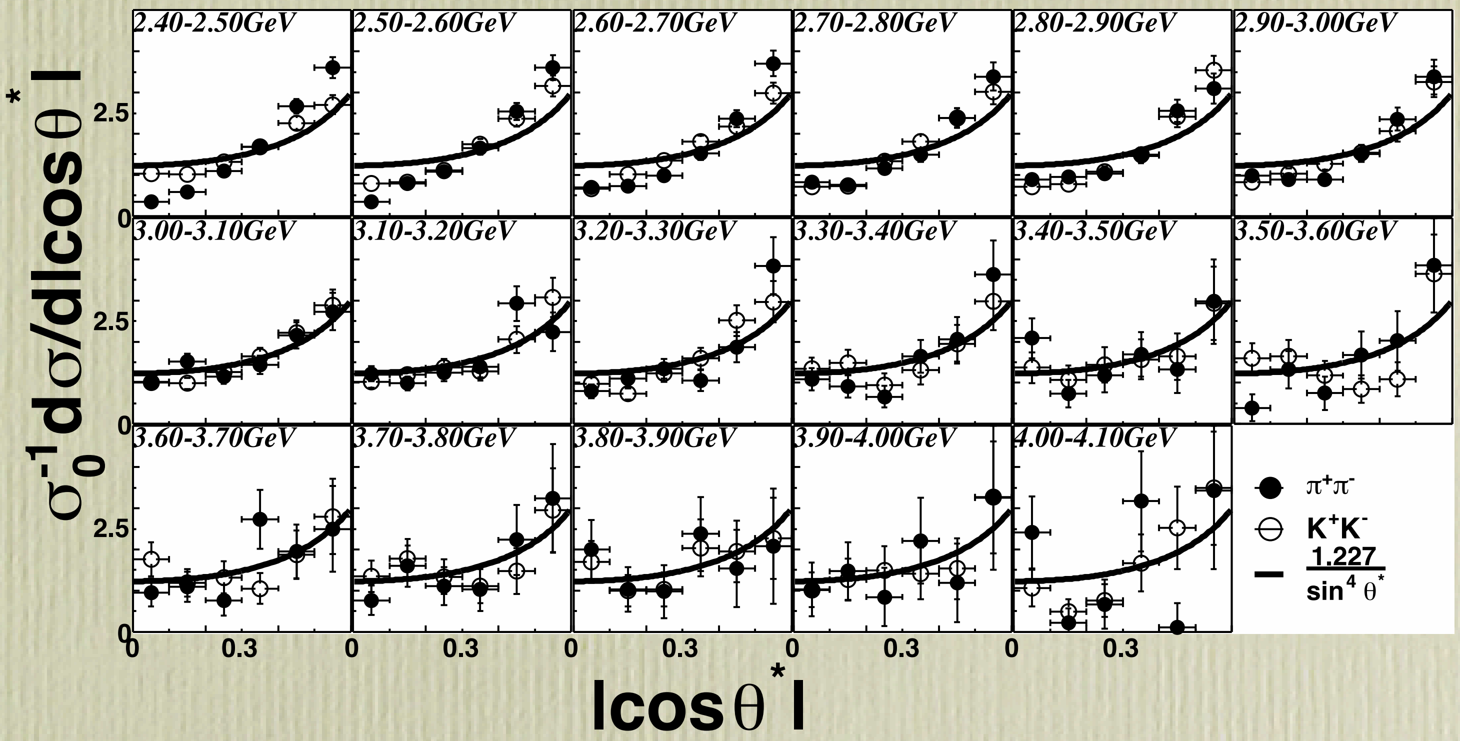
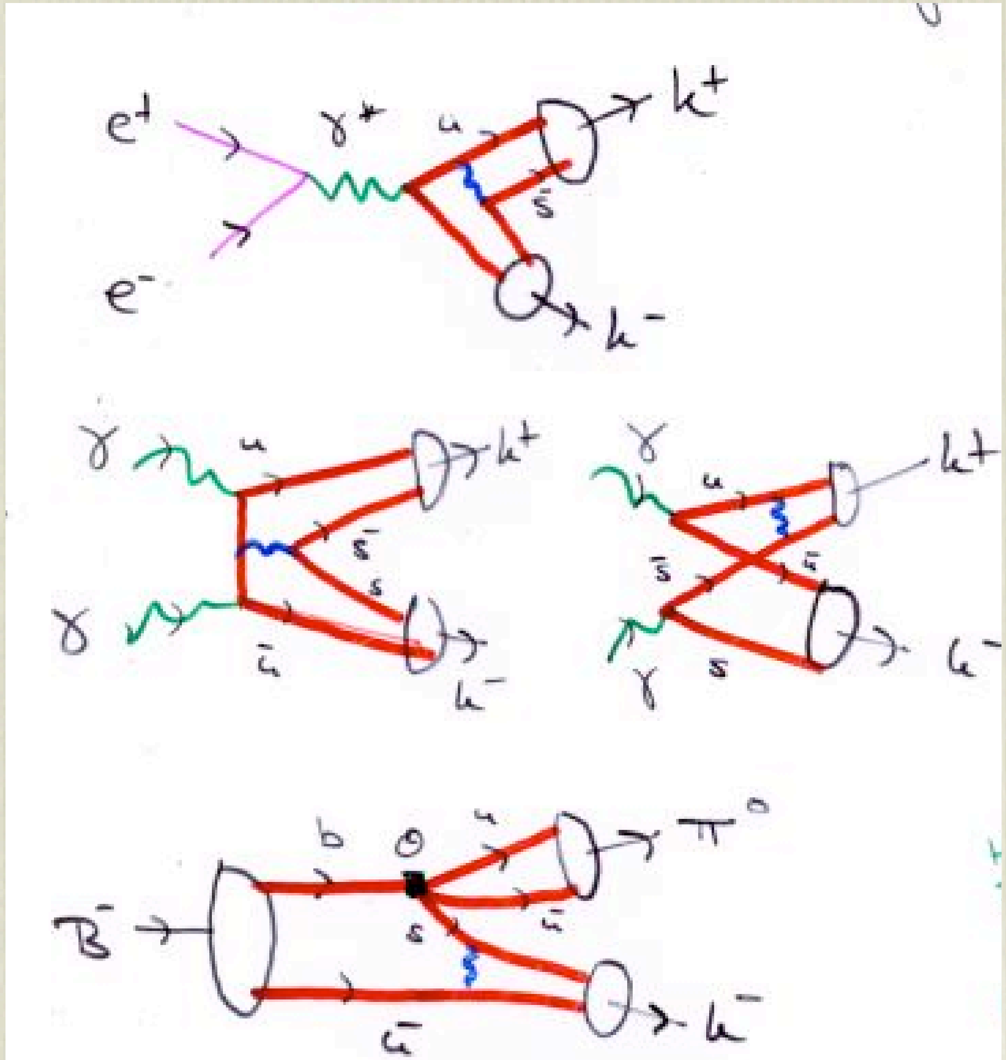


Fig. 4. Angular dependence of the cross section, $\sigma_0^{-1}d\sigma/d|\cos\theta^*|$, for the $\pi^+\pi^-$(closed circles) and K^+K^-(open circles) processes. The curves are $1.227\times\sin^{-4}\theta^*$. The errors are statistical only.

Measurement of the $\gamma\gamma\to\pi^+\pi^-$ and
$\gamma\gamma\to K^+K^-$ processes
at energies of 2.4–4.1 GeV

Belle Collaboration

Common Ingredients:
Universal LFWFS, Distribution Amplitudes



sjb, gpl

Critical Test of PQCD vs. “Handbag”

$$\frac{\Delta\sigma(\gamma\gamma \to \pi^0\pi^0)}{\Delta\sigma(\gamma\gamma \to \pi^+\pi^-)}$$

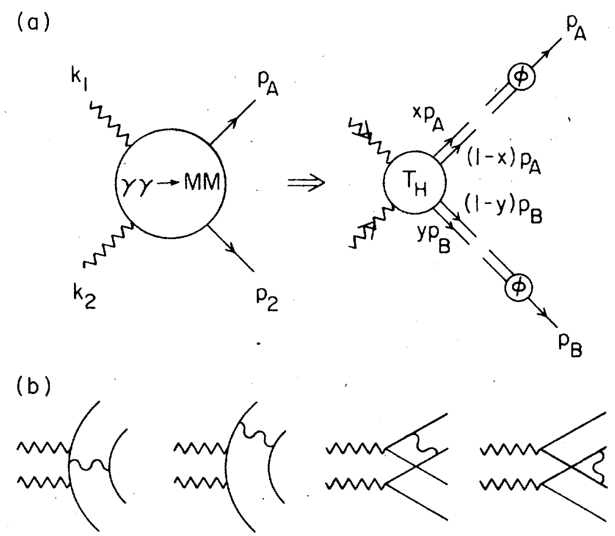


FIG. 1. (a) Factorized structure of the $\gamma\gamma\to M\overline{M}$ amplitude in QCD at large momentum transfer. The T_H amplitude is computed with quarks collinear with the outgoing mesons. (b) Diagram contributing to T_H $(\gamma\gamma\to M\overline{M})$ to lowest order in α_s.

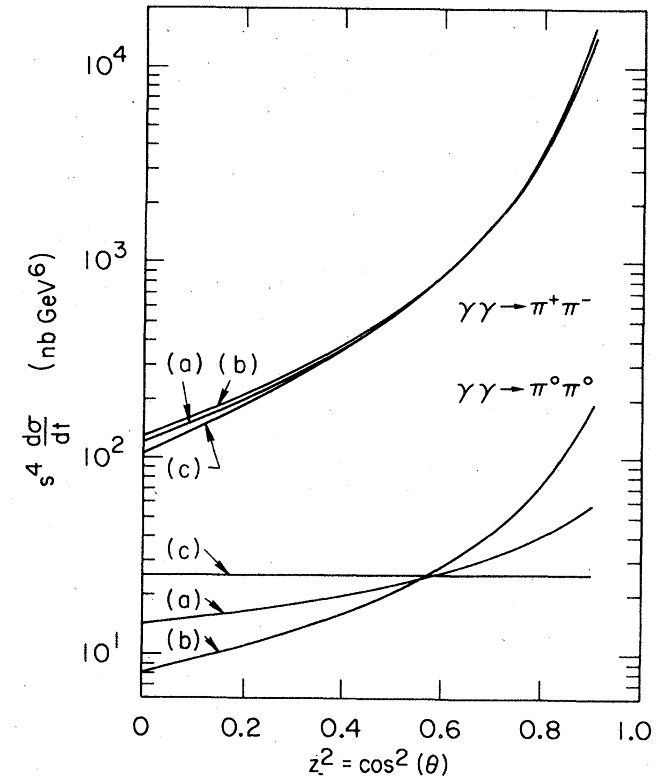
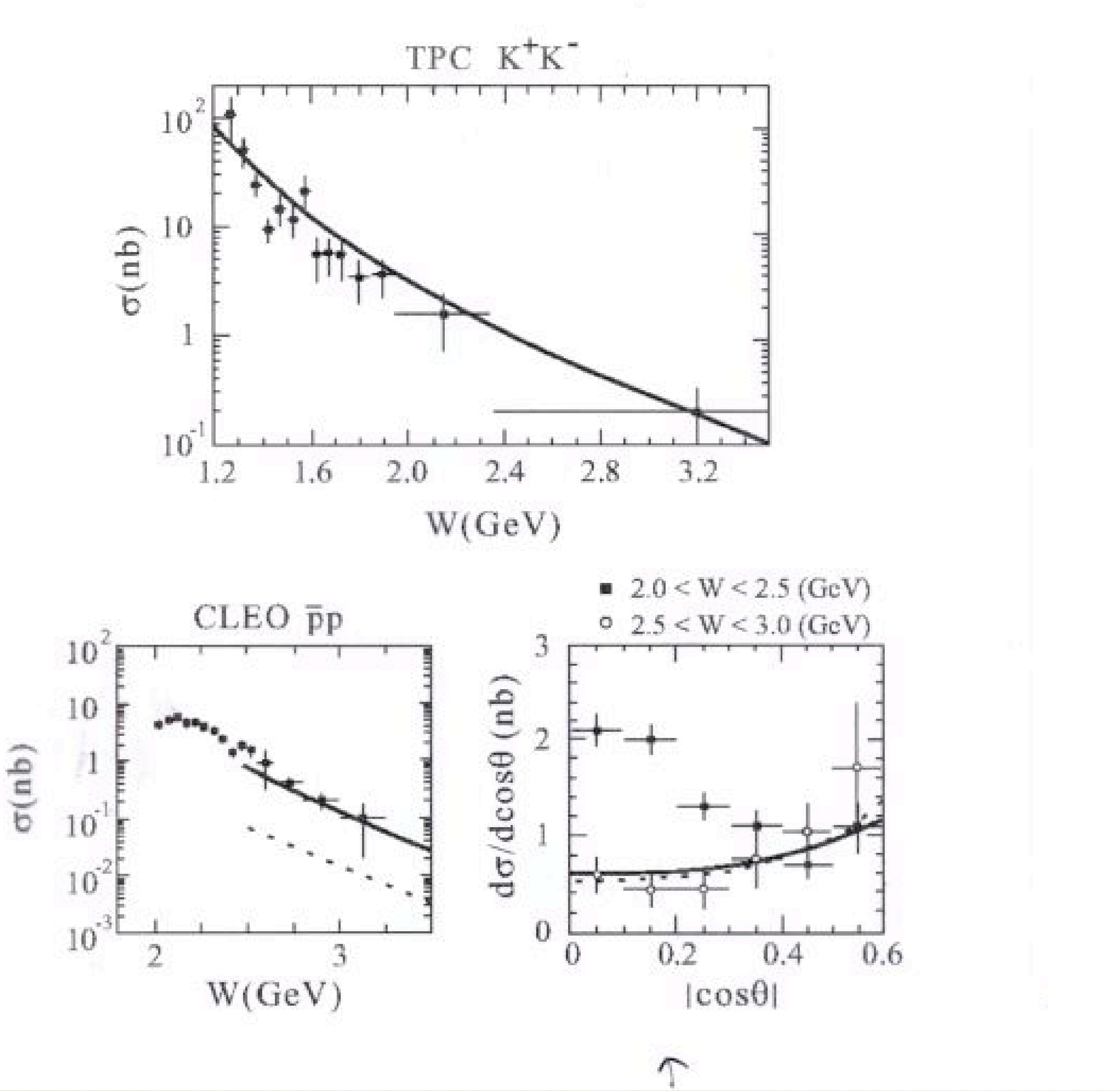


FIG. 3. QCD predictions for $\gamma\gamma\to\pi\pi$ to leading order in QCD. The results assume the pion-form-factor parametrization $F_\pi(s)\sim 0.4$ GeV$^2/s$. Curves (a), (b), and (c) correspond to the distribution amplitudes $\phi_M = x(1-x)$, $[x(1-x)]^{1/4}$, and $\delta(x-\frac{1}{2})$, respectively. Predictions for other helicity-zero mesons are obtained by multiplying with the scale constants given in Table I.

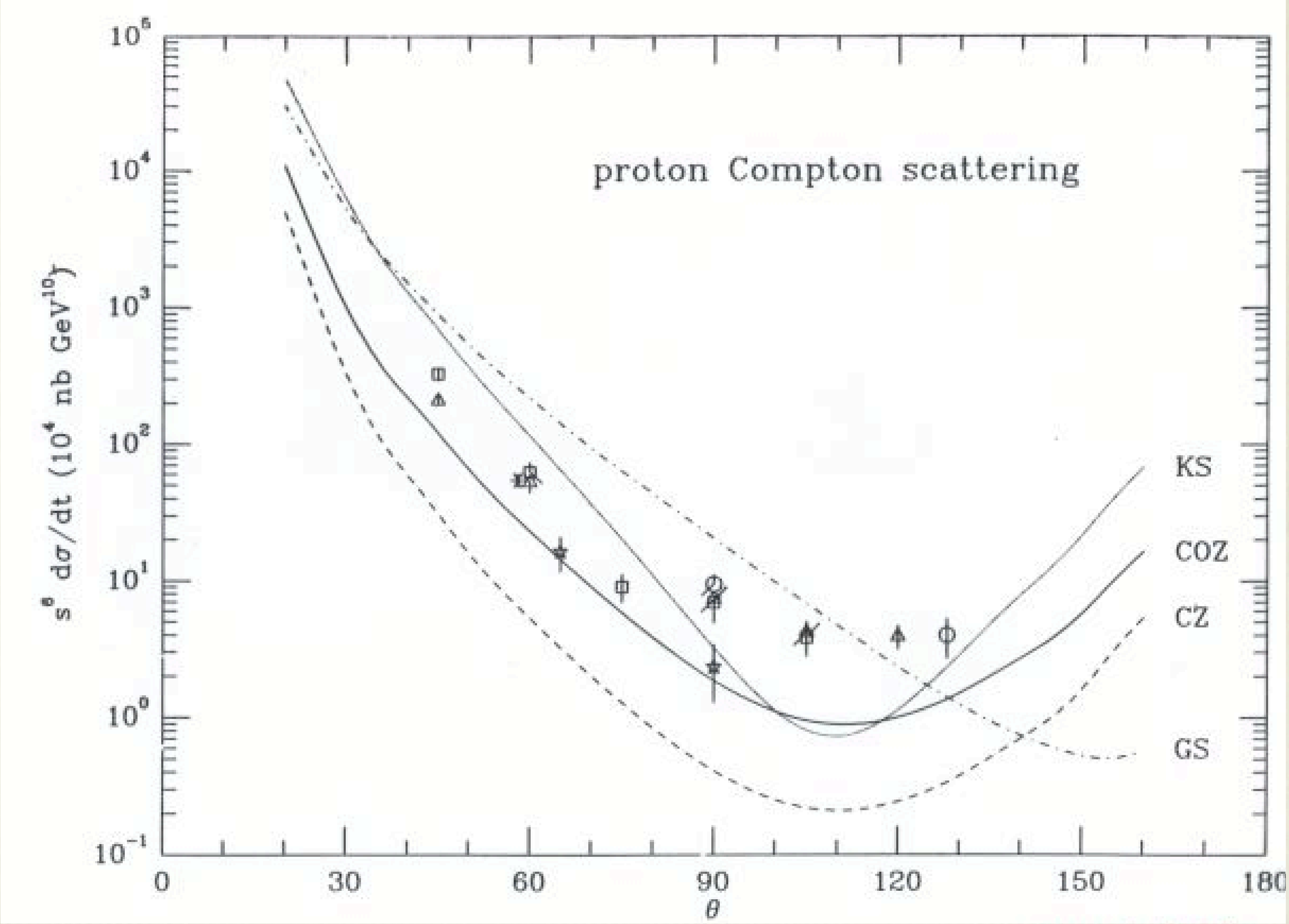
Handbag model (Diehl, Kroll et al) neglects $e_1 \times e_2$ cross terms



TPC K+K-
σ(nb)
W(GeV)
CLEO p̄p
σ(nb)
W(GeV)
2.0 < W < 2.5 (GeV)
2.5 < W < 3.0 (GeV)
dσ/dcosθ (nb)
|cosθ|

Two Regimes

Large-Angle Compton Scattering



Scaling of deuteron FFs

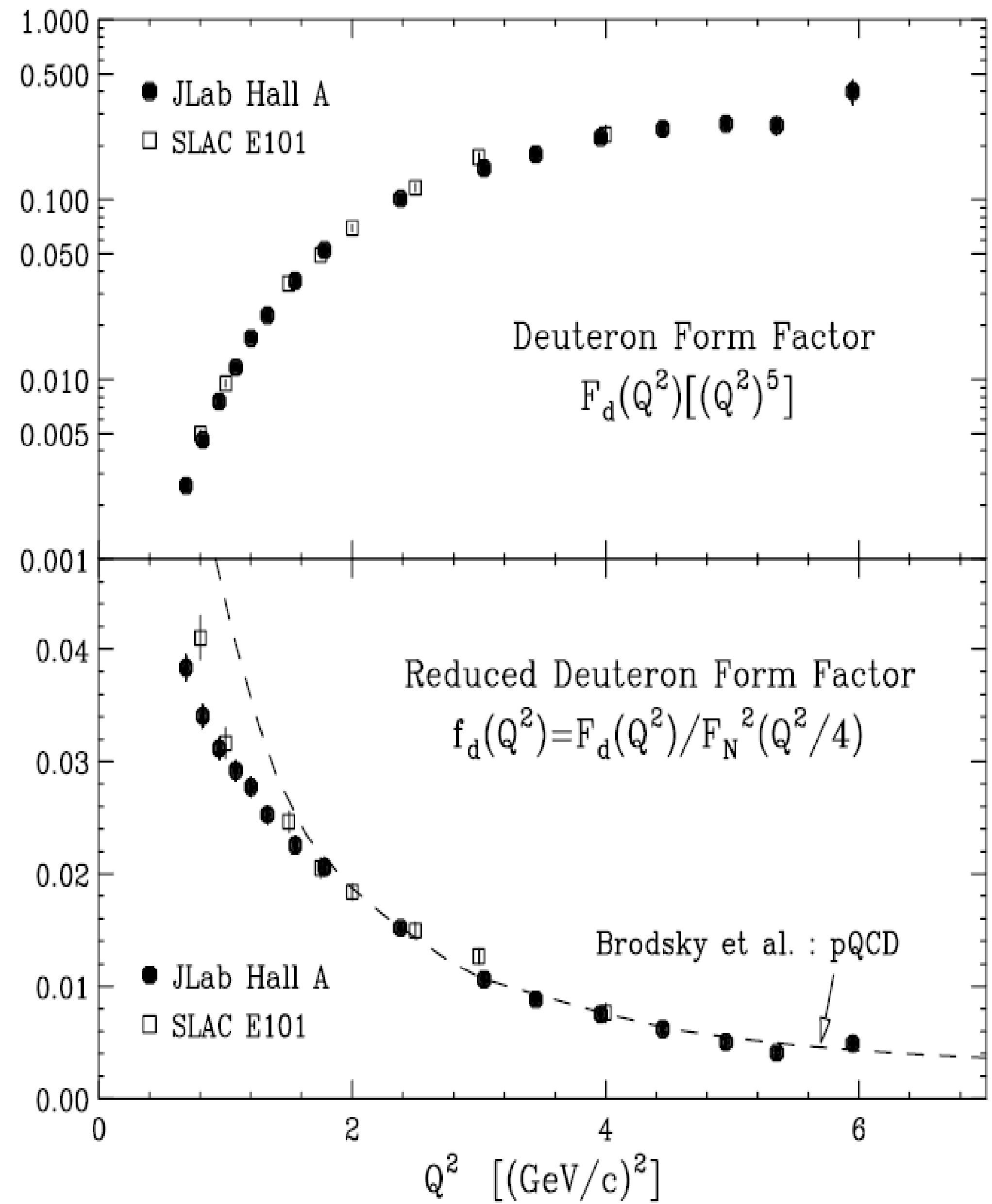
Alexa et al., PRL 82,1374 (1999)

CCR in elastic scattering

leading term: $FF \propto \left(Q^2\right)^{-n_d}$

the "deuteron FF": $F_d \equiv \sqrt{A} \propto Q^{-10}$

For Q^2 above ≈4 GeV^2 data are consistent with CCR



QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2}\right]^5 \sum_{m,n} d_{mn} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n^d - \gamma_m^d} \left[1 + O\left(\alpha_s(Q^2), \frac{m}{Q}\right)\right]$$

Chertok, Lepage, Ji, sjb

Define “Reduced” Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N{}^2(Q^2/4)} \,.$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-(2/5)\, C_F/\beta}$$

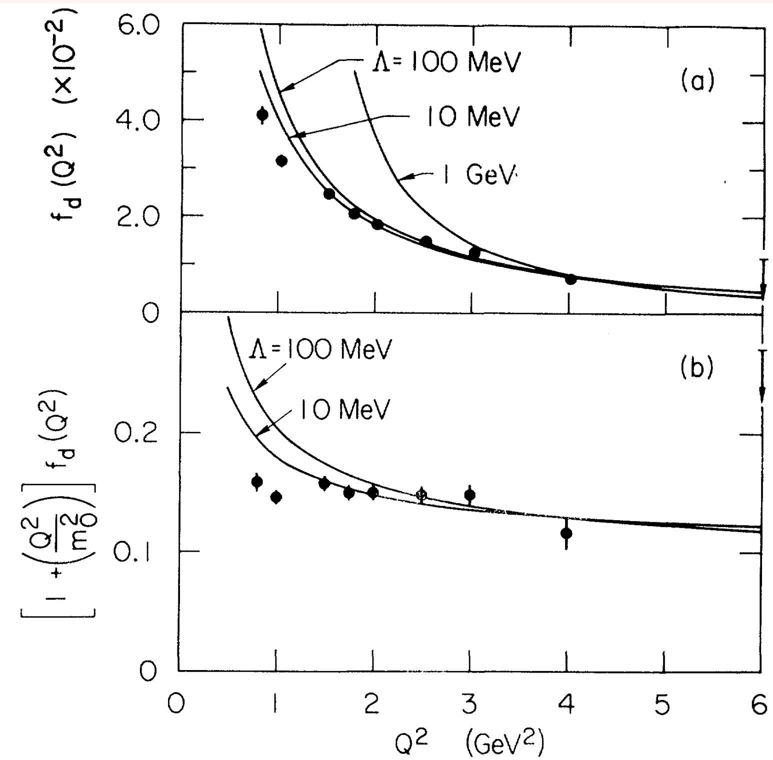
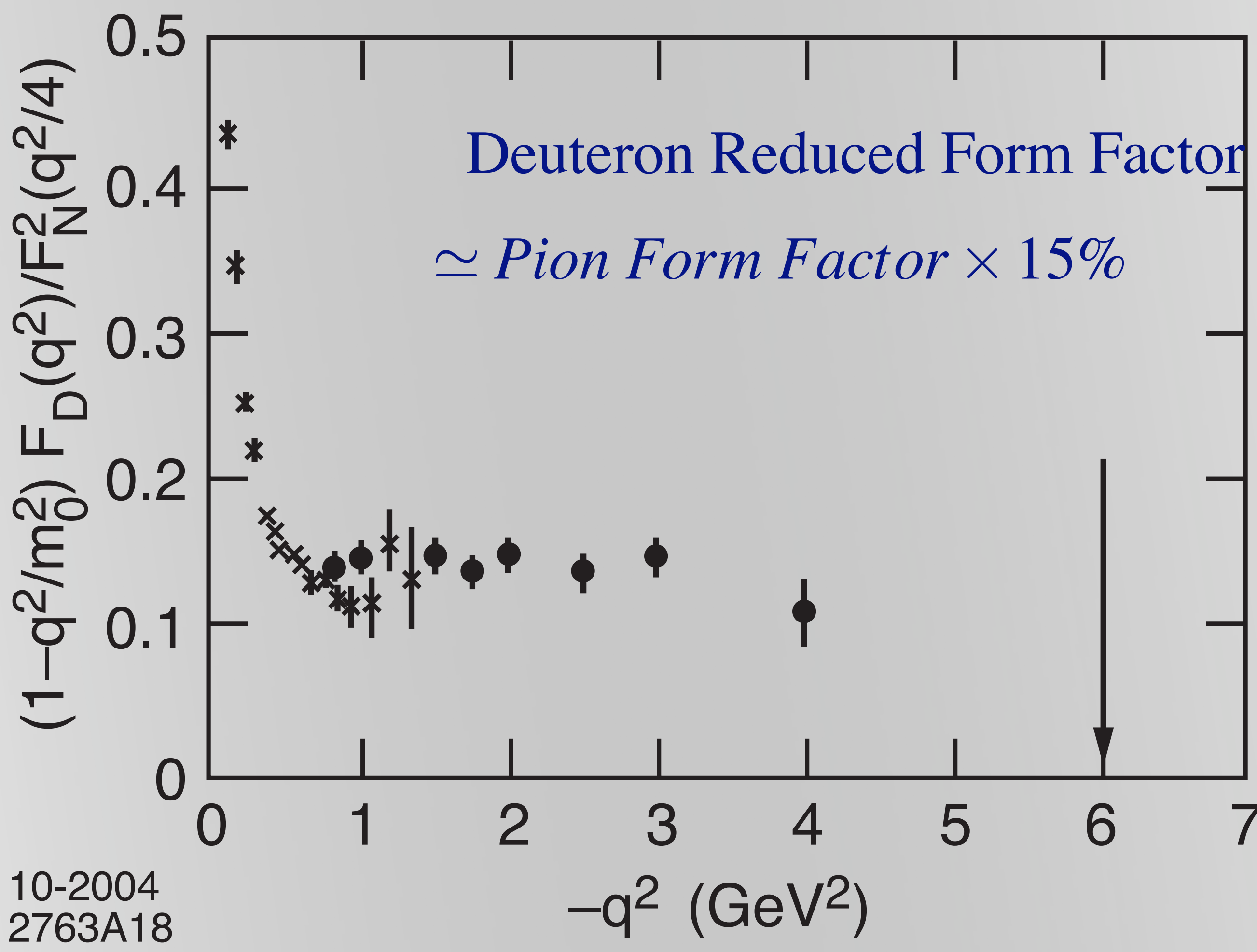


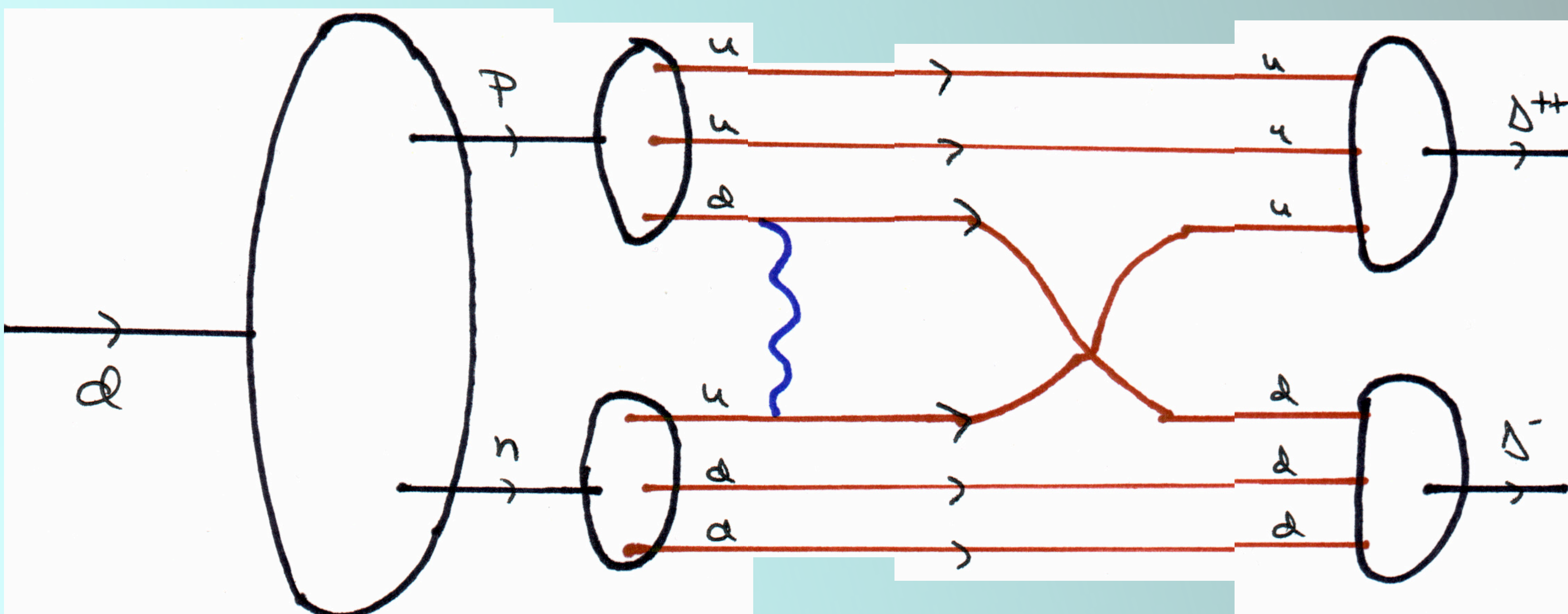
FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2)[\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71\ \mathrm{GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4\ \mathrm{GeV}^2$ data point. (b) Comparison of the prediction $[1 + (Q^2/m_0{}^2)] f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0{}^2 = 0.28\ \mathrm{GeV}^2$ is used (Ref. 8).



Resembles pion form factor times 15%

- 15% Hidden Color in the Deuteron

Structure of Deuteron in QCD



Hidden Color Fock State

Delta-Delta Fock State

Hidden Color in QCD

- Deuteron six quark wavefunction: Lepage, Ji, sjb
- 5 color-singlet combinations of 6 color-triplets -- one state is |n p>
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict $\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$ at high Q^2

Evolution Equation for Deuteron

- Distribution Amplitude 5 x 1 Column Matrix
- n p at large distances
- Equal weights at short distances
- Hidden Color: First principle prediction of QCD

G.P. Lepage, C. R. Ji, SJB

The evolution equation for six-quark systems in which the constituents have the light-cone longitudinal momentum fractions x_i $(i=1,2,\ldots,6)$ can be obtained from a generalization of the proton (three-quark) case.[2] A nontrivial extension is the calculation of the color factor, C_d, of six-quark systems[5] (see below). Since in leading order only pairwise interactions, with transverse momentum Q, occur between quarks, the evolution equation for the six-quark system becomes $\{[dy]=\delta(1-\sum_{i=1}^{6} y_i)\prod_{i=1}^{6} dy_i$, $C_F=(n_c^2-1)/2n_c=\frac{4}{3}$, $\beta=11-\frac{2}{3}n_f$, and n_f is the effective number of flavors$\}$

$$\prod_{k=1}^{6} x_k\left[\frac{\partial}{\partial\xi}+\frac{3C_F}{\beta}\right]\tilde{\Phi}(x_i,Q)=-\frac{C_d}{\beta}\int_0^1 [dy]\,V(x_i,y_i)\,\tilde{\Phi}(y_i,Q),$$

$$\xi(Q^2)=\frac{\beta}{4\pi}\int_{Q_0^2}^{Q^2}\frac{dk^2}{k^2}\alpha_s(k^2)\sim\ln\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right).$$

$$V(x_i,y_i)=2\prod_{k=1}^{6} x_k\sum_{i\neq j}^{6}\theta(y_i-x_i)\prod_{l\neq i,j}^{6}\delta(x_l-y_l)\frac{y_j}{x_j}\left(\frac{\delta_{h_i\bar{h}_j}}{x_i+x_j}+\frac{\Delta}{y_i-x_i}\right)$$

where $\delta_{h_i\bar{h}_j}=1$ (0) when the helicities of the constituents $\{i,j\}$ are antiparallel (parallel). The infrared singularity at $x_i=y_i$ is cancelled by the factor $\Delta\tilde{\Phi}(y_i,Q)=\tilde{\Phi}(y_i,Q)-\tilde{\Phi}(x_i,Q)$ since the deuteron is a color singlet.

$\gamma\ d \rightarrow p\ n$ at SLAC

Mirazita

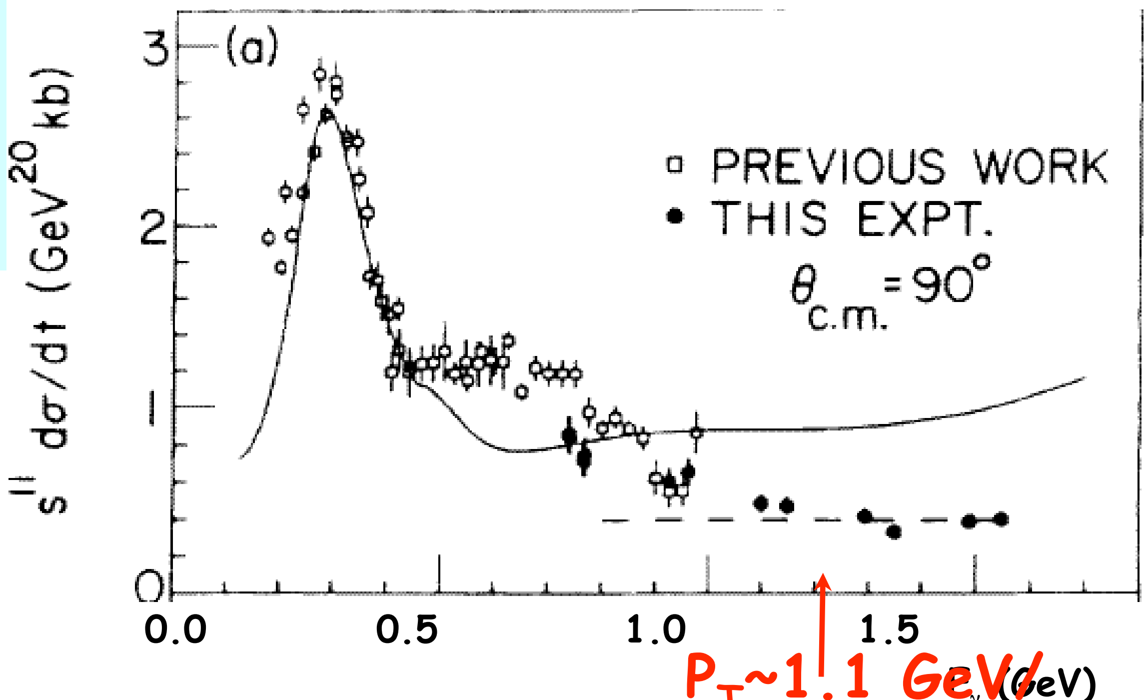
$n = 1+6+3+3 = 13$ ➡ $d\sigma/dt \approx s^{2-n}$ ➡ $s^{11} d\sigma/dt$ = cost

onset of scaling governed by proton transverse momentum

$P_T^2 = 1/2\ E_\gamma\ M_d\ \sin^2(\theta_{cm})$

- CCR scaling for $p_T > 1.1$ GeV
- power law fit $n = 10.5 \pm 0.7$

J. Napolitano et al., P.R.L. 61, (1988) 2530



$P_T \sim 1.1$ GeV/c

New extensive studies at SLAC and JLab

- Remarkable Test of Quark Counting Rules
- Deuteron Photo-Disintegration $\gamma d \to np$
- $$\frac{d\sigma}{dt} = \frac{F(t/s)}{s^{n_{tot}-2}}$$
- $$n_{tot} = 1 + 6 + 3 + 3 = 13$$

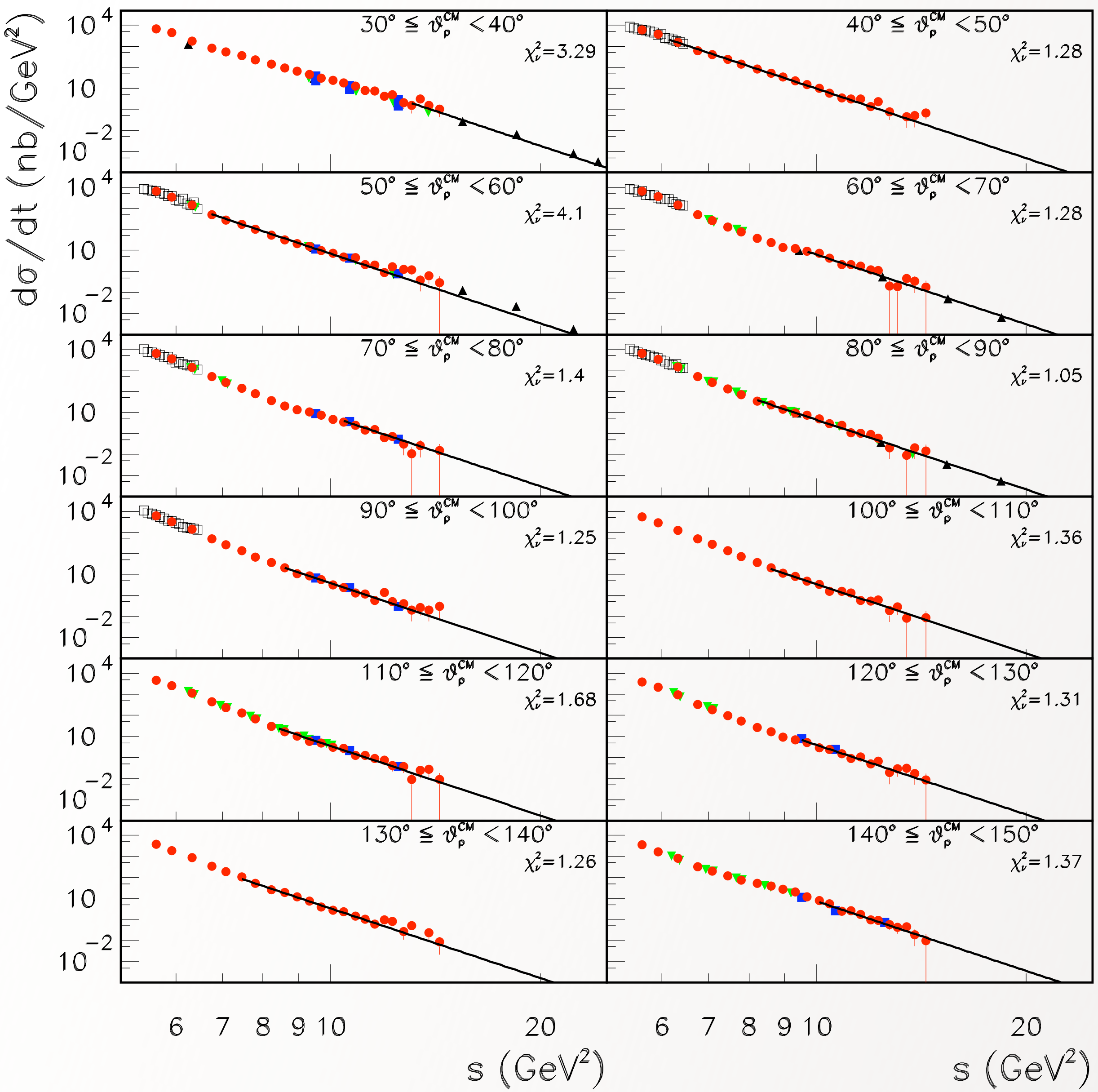
Scaling characteristic of
scale-invariant theory at short distances

Conformal symmetry

Hidden color: $$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$$

at high p_T

Deuteron Photodisintegration & Dimensional Counting Rules



PQCD and AdS/CFT:

$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B \to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

$$s^{11}\frac{d\sigma}{dt}(\gamma d \to np) = F(\theta_{CM})$$

$n_{tot} - 2 =$
$(1 + 6 + 3 + 3) - 2 = 11$

Check of CCR

Fit of $d\sigma/dt$ data for the central angles and $P_T \geq 1.1$ GeV/c with

$A\ s^{-11}$

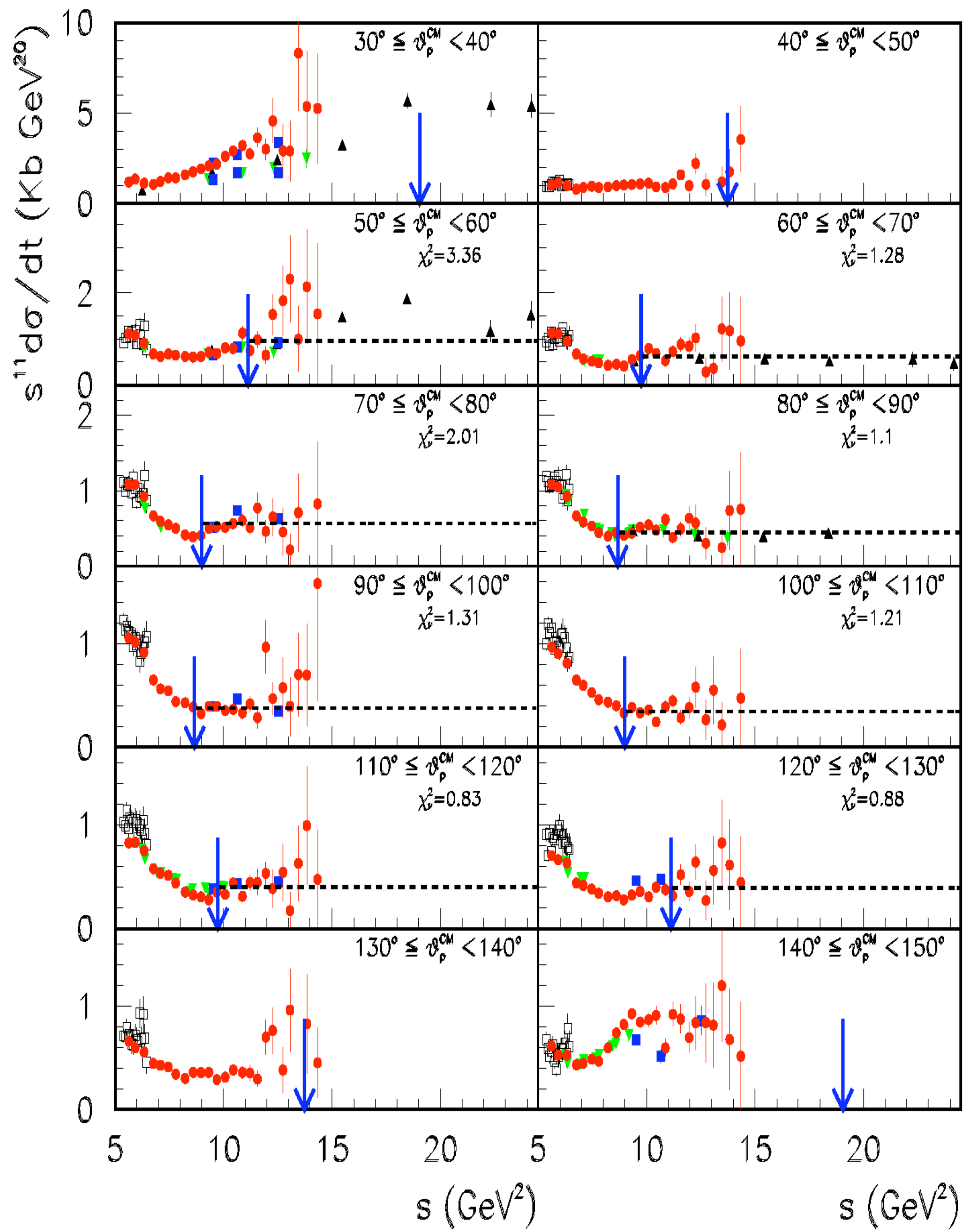
For all but two of the fits

$\chi^2 \leq 1.34$

- Better χ^2 at 55° and 75° if different data sets are renormalized to each other
- No data at $P_T \geq 1.1$ GeV/c at forward and backward angles
- Clear s^{-11} behaviour for last 3 points at 35°

Data consistent with CCR

P.Rossi et al, P.R.L. 94, 012301 (2005)



Key Quantity of Nuclear and Hadron Physics

Proton-Proton
Scattering

$$\frac{d\sigma}{dt}(pp \to pp) = \frac{F(t/s)}{s^{9.7\pm0.5}}$$

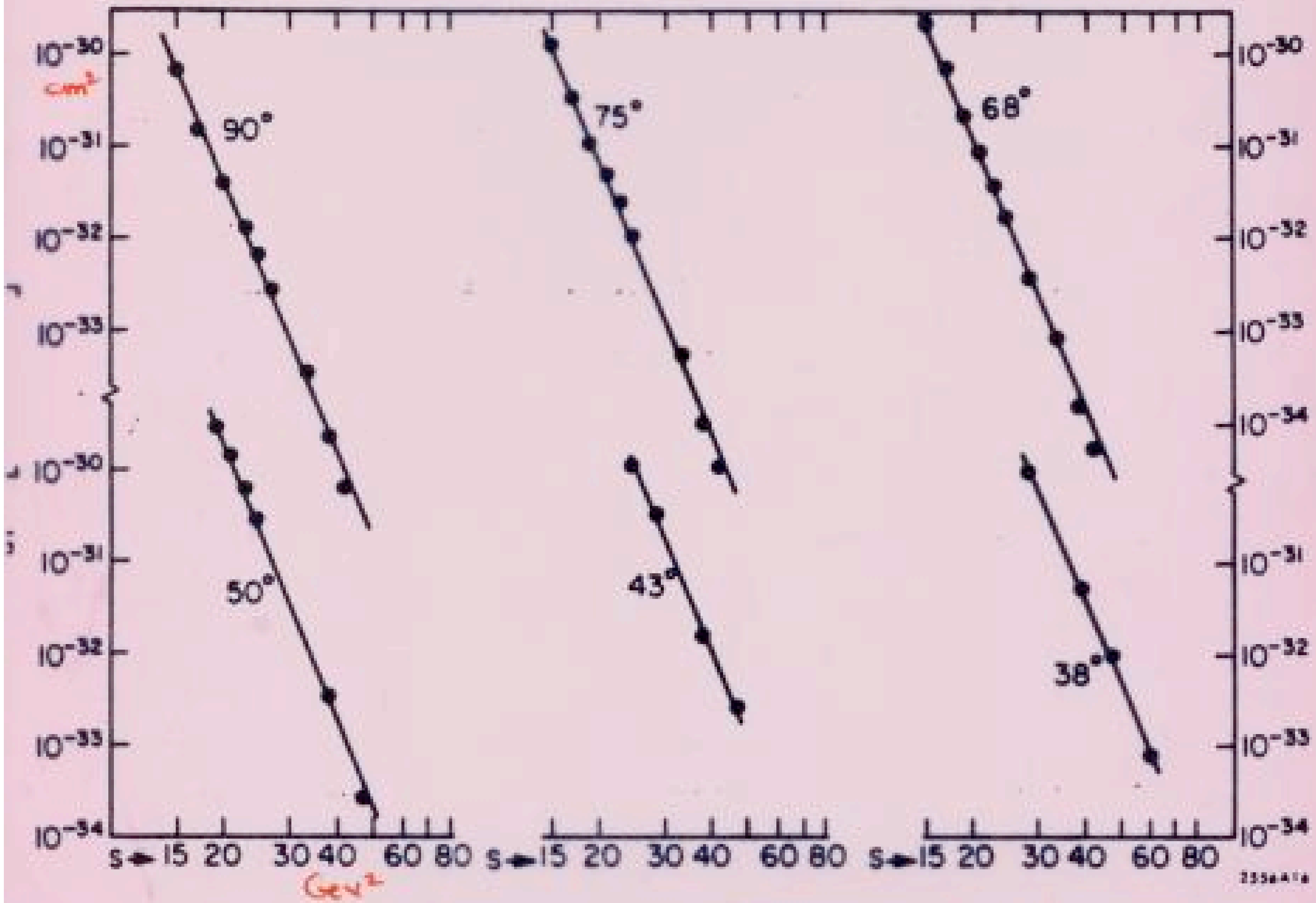


Figure 22. Test of fixed θ_{CM} scaling for elastic pp scattering. The data compilation is Landshoff and Polkinghorne.

Why do dimensional counting rules work so well?

- PQCD predicts log corrections from powers of α_s, logs, pinch contributions

- QCD coupling evaluated in IR regime.

- IR Fixed point! DSE: Alkofer, von Smekal et al.

- QED, EW -- define coupling from observable, predict other observable

Define QCD Coupling from Observable

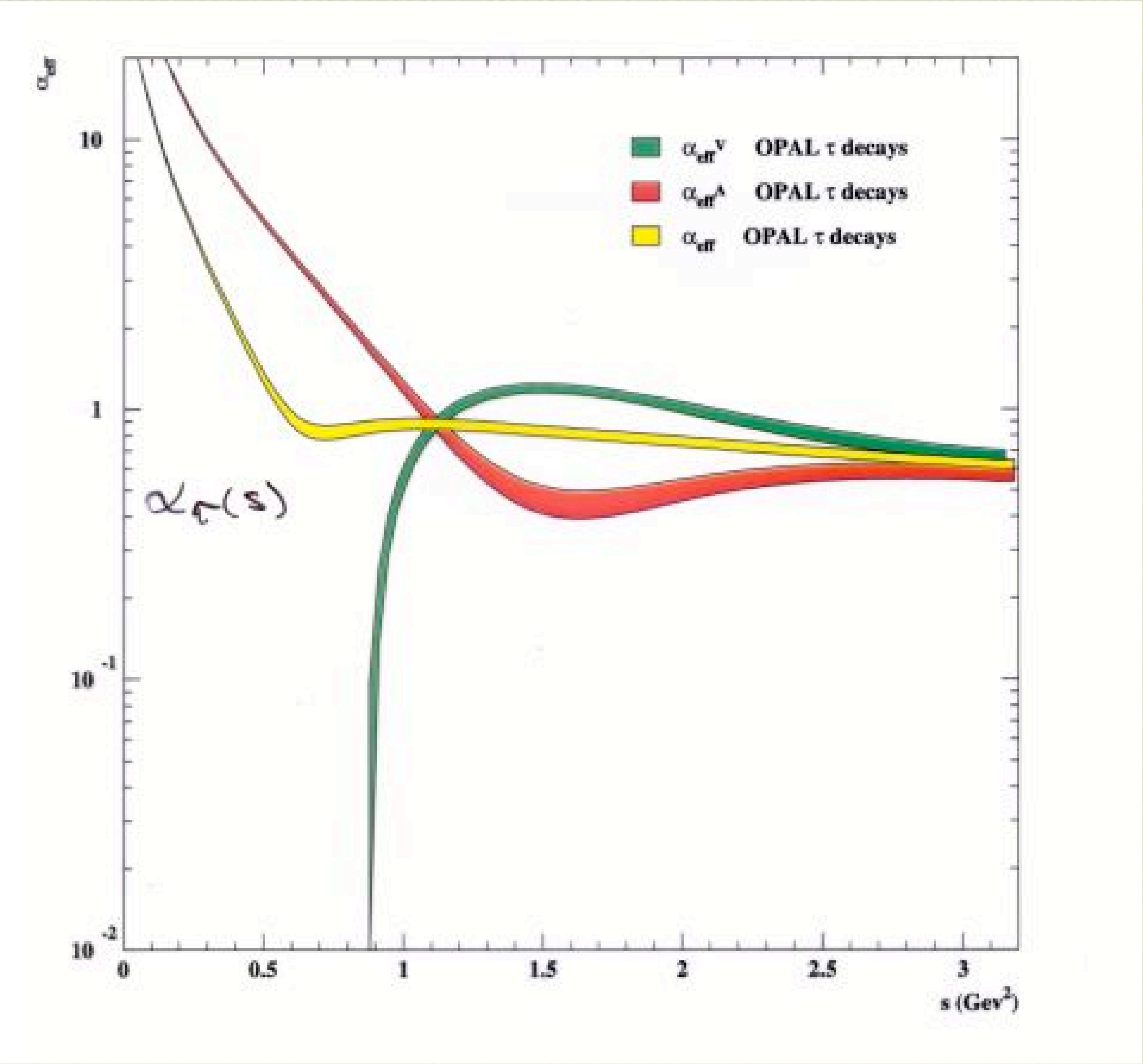
Grunberg

$$R_{e^+e^-\to X}(s) \equiv 3\Sigma_q e_q^2 \; [1 + \frac{\alpha_R(s)}{\pi}]$$

$$\Gamma(\tau \to Xe\nu)(m_\tau^2) \equiv \Gamma_0(\tau \to u\bar{d}e\nu)\times[1+\frac{\alpha_\tau(m_\tau^2)}{\pi}]$$

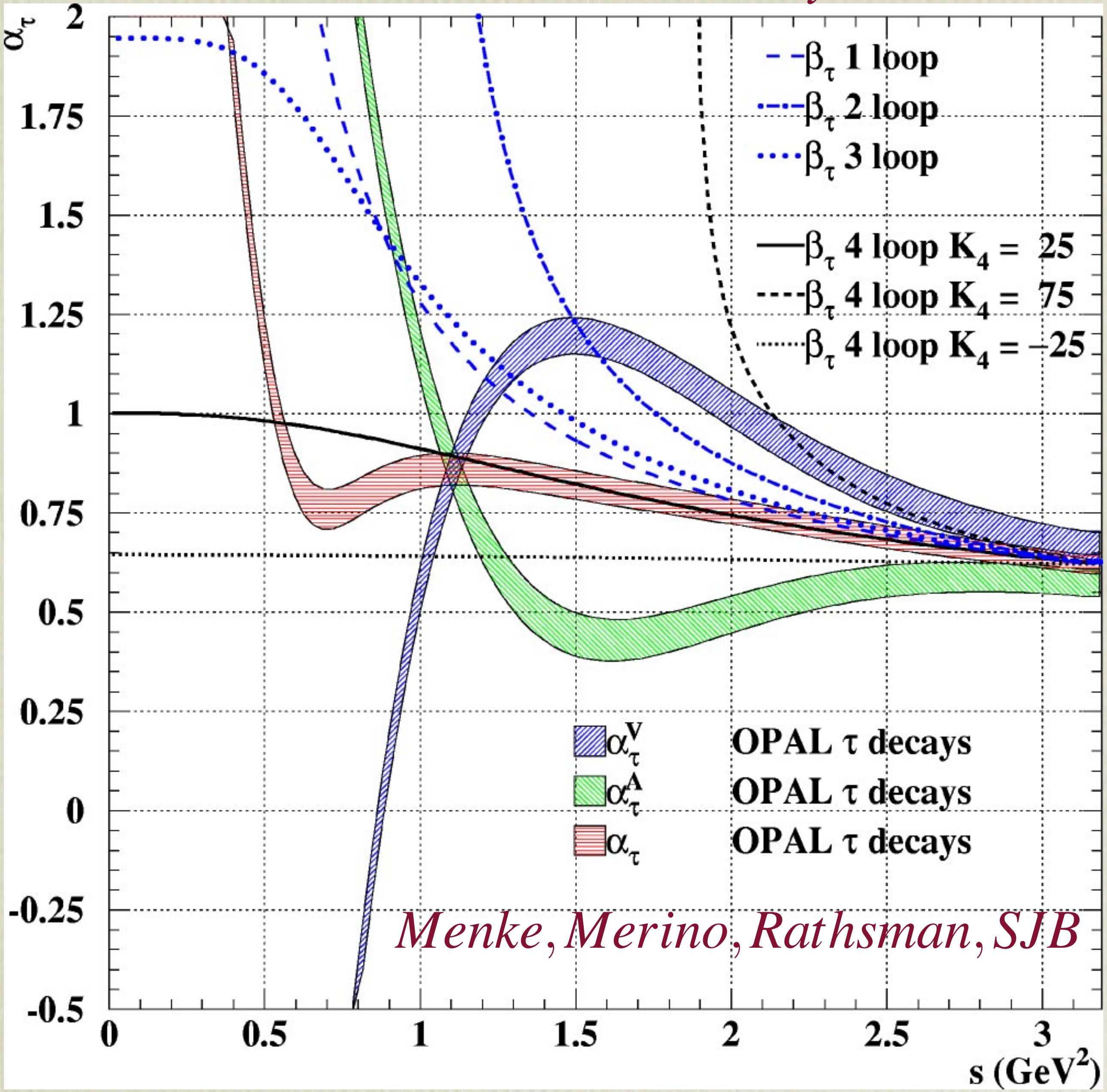
Relate observable to observable at commensurate scales

H.Lu, sjb



Menke, Merino, Rathsman, SJB

QCD Effective Coupling from *hadronic τ decay*



- Define effective charge (Grunberg)

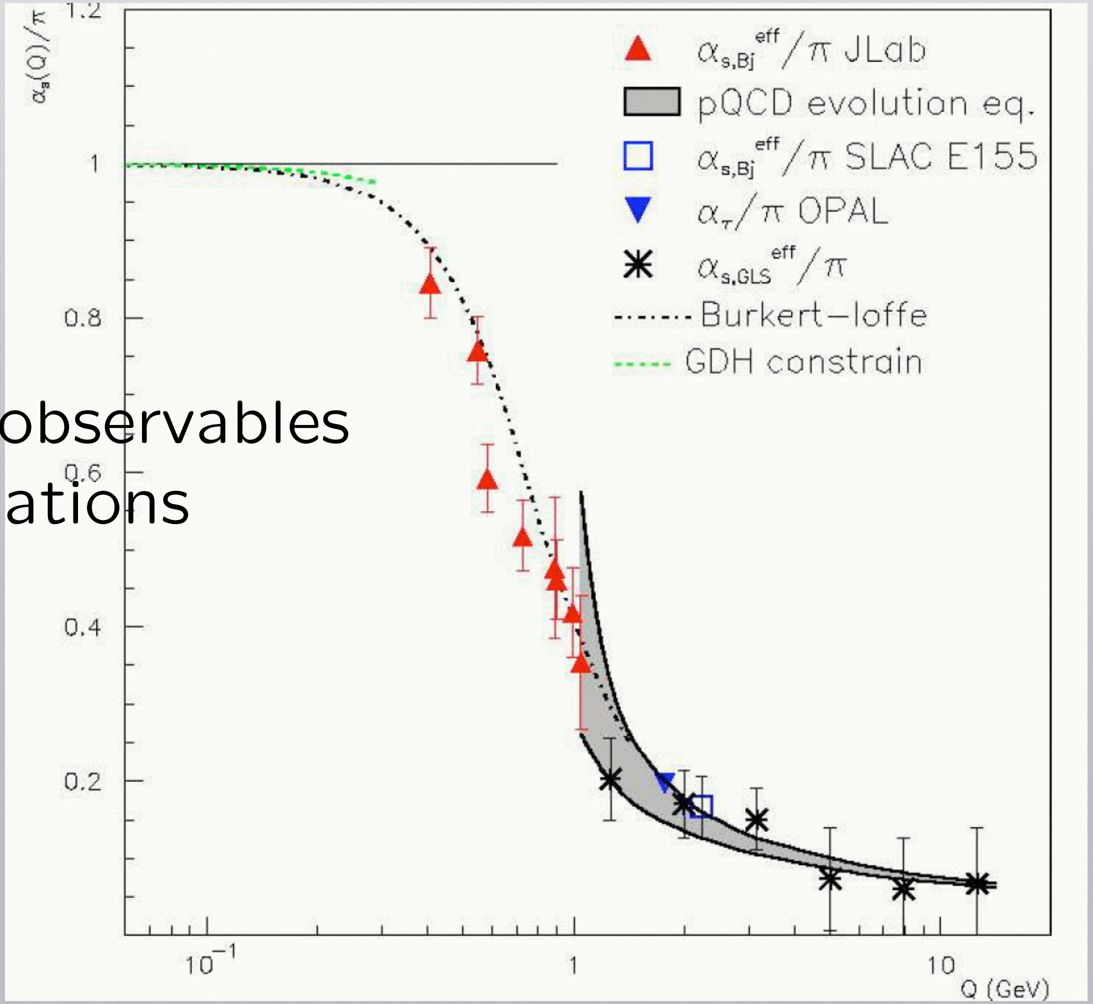
$$\int_0^1 dx[g_{1n}(x,Q^2) - g_{1p}(x,Q^2)] \equiv \frac{g_A}{6}\left(1 - \frac{\alpha_{g_1}(Q^2)}{\pi}\right)$$

- $\frac{d}{dQ^2}\alpha_{g_1}(Q^2) = \beta(Q^2)$: standard QCD evolution

- β_0, β_1 universal

- Connect $\alpha_{g_1}(Q^2)$ to other observables via Commensurate Scale Relations

- Eliminate $\alpha_{\overline{MS}}$



A. Deur, et al
Preliminary

Stan Brodsky, SLAC

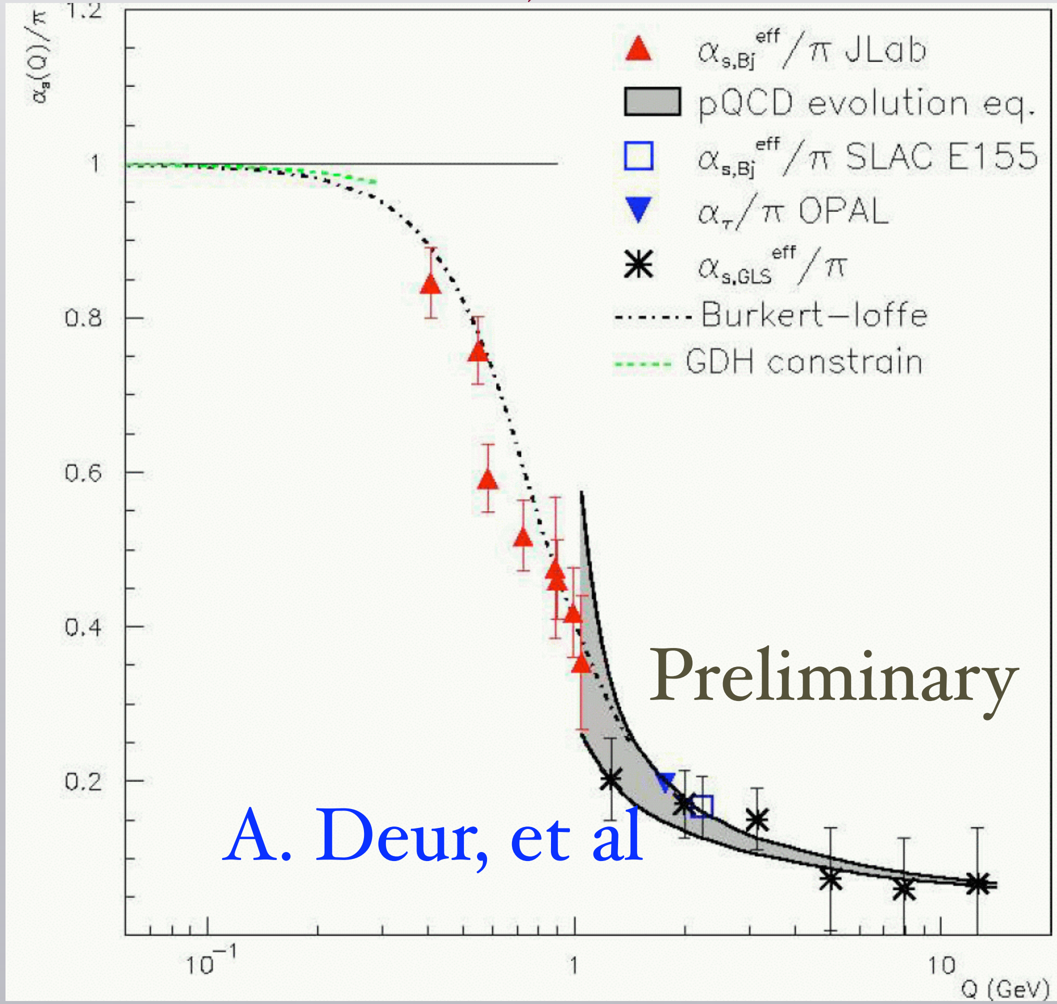
G.Gabadadze, H.J.Lu, A.Kataev, J.Rathsman, SJB

- Generalized Crewther Relation

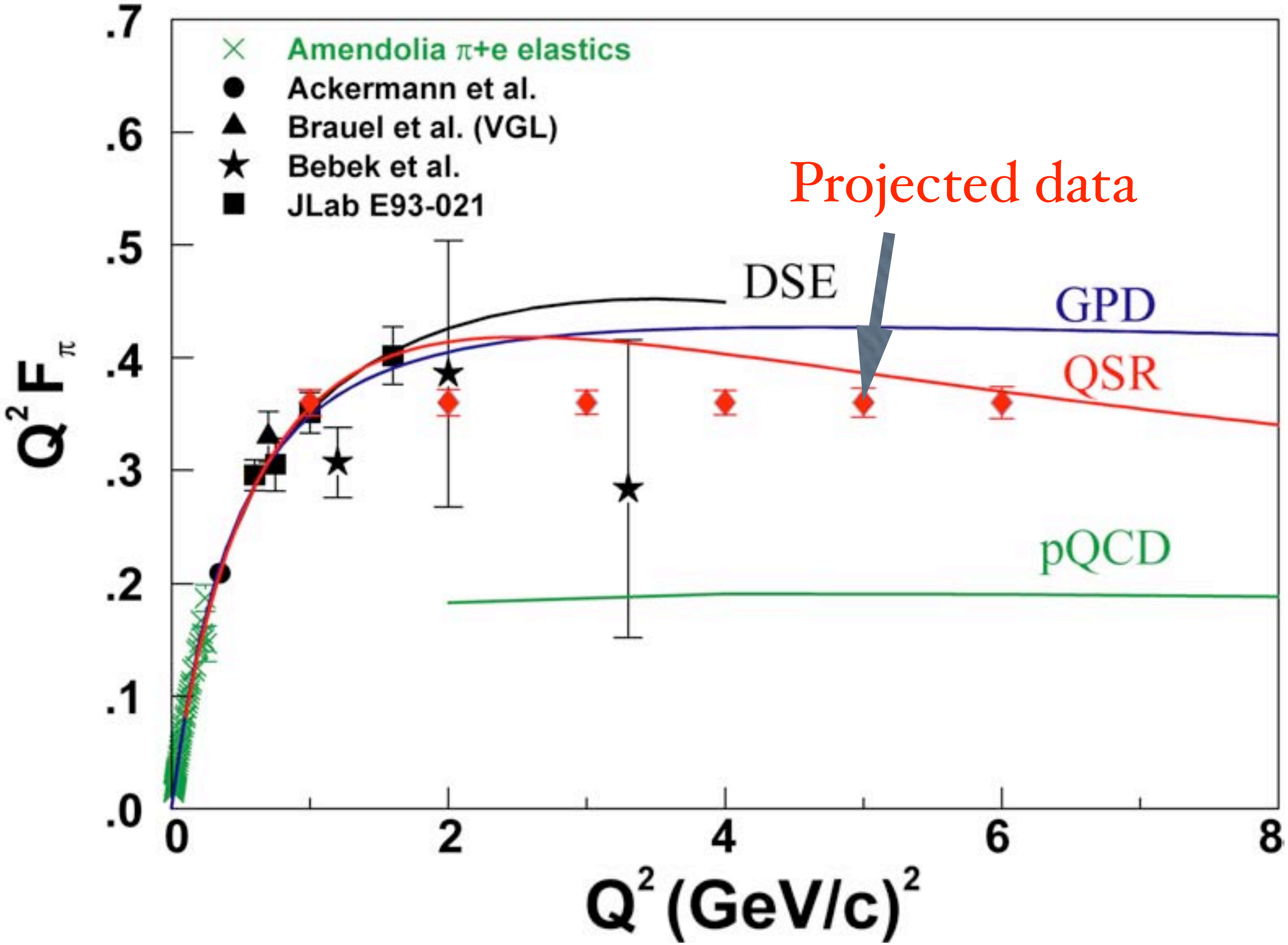
$$[1 - \frac{\alpha_{g_1}(Q^2)}{\pi}] \times [1 + \frac{\alpha_R(s^*)}{\pi}] = 1$$

at $s^* = CQ^2$.

- Exact at leading twist.
- No scale ambiguity!
- Extraordinary Test of QCD
- $\frac{\alpha_{g_1}(Q^2)}{\pi}$:
Analytic at quark thresholds.



Spacelike Pion Form Factor



Example: Normalization of pion form factor in PQCD

- Data requires extrapolation to $t = m_\pi^2$
- BLM scale, $\alpha_s(e^{-\frac{5}{3}} xyq^2) \sim \alpha_s(\frac{Q^2}{20})$ Ji, Pang, Robertson, sjb
- Higher order corrections
- resummation from AdS/CFT $\alpha_s \to \sqrt{\alpha_s}$
- triple gluon coupling at NLO
- IR Fixed point

Near-Conformal Behavior of LFWFs Lead to PQCD Scaling Laws

- Bjorken Scaling of DIS
- Counting Rules of Structure Functions at large x
- Dimensional Counting Rules for Exclusive Processes and Form Factors
- Conformal Relations between Observables
- No Renormalization Scale Ambiguity

- Measure behavior of proton LFWF at large x_{bj}

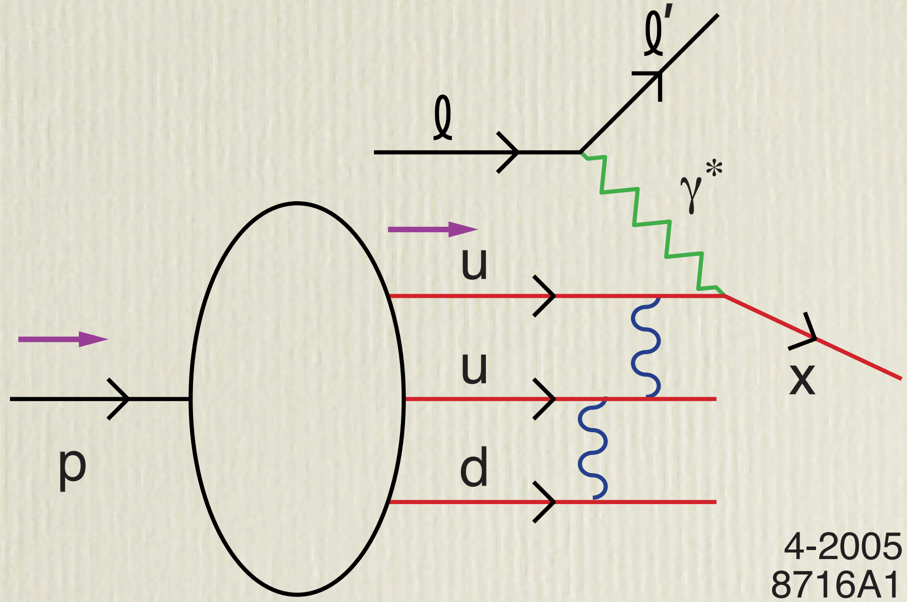
- Strong function of quark spin projection relative to proton spin projection

Farrar, Jackson
Gunion
Lepage, SJB
Burkardt, Schmidt, SJB
Ji, Ma, Yuan

PQCD:

$q(x) \sim (1-x)^3 \qquad S^z_q = S^z_p$

$q(x) \sim (1-x)^5 \qquad S^z_q = -S^z_p$



Traditional PQCD Method:
Iterate QCD Kernel

Conformal symmetry: Template for QCD

- Initial approximation to PQCD; correct for non-zero beta function
- Commensurate scale relations: relate observables at corresponding scales
- Infrared fixed-point for α_s
- Effective Charges: analytic at quark mass thresholds
- Eigensolutions of Evolution Equations

Conformal Symmetry

$\mathcal{L}_{classical}$ renormalizable theory

* Poincaré plus

* dilation $[x^\mu \to \lambda x^\mu]$

* conformal transformation $\left[\text{inversion}\left(x^\mu \to -\frac{x^\mu}{x^2}\right) \otimes \text{translation} \otimes \text{inversion}\right]$

broken by

masses

quantum loops : running coupling, ...

For $\beta = \frac{d\alpha_s(Q^2)}{d\ln Q^2} = 0$ (e.g. fixed point theory)

* PQCD $\Rightarrow$ conformal theory

all orders proof : G. Parisi, PL 39B, 643 (1972)

Theories with Conformal Symmetry

invariant under Poincare transformations $M^{\mu\nu}, P^{\nu}$
+ Conformal transformations D, K^{ν}

generators form group

$$\boxed{SO(4,2)} \qquad (d=4)$$

$SO(4,2)$ has representations on both

and $\begin{cases} \text{Minkowski space} & R^{(3,1)} \\ AdS_5 & \end{cases}$

Minkowski metric

$$ds^2 = dt^2 - d\vec{x}^2$$

AdS_5 metric

fifth dim.

$$ds^2 = \frac{r^2}{R^2}\left(dt^2 - d\vec{x}^2\right) - \frac{R^2}{r^2}\, dr^2$$

Dilatations

$$x^{\mu} \to \lambda x^{\mu} \quad ; \quad (x^{\mu}, r) \to \left(\lambda x^{\mu}, \frac{r}{\lambda}\right)$$

Duality between strongly coupled conformal theory and weakly coupled type IIB string theory

AdS/CFT Correspondence

Anti de Sitter — conformal field theory

Maldacena (1998)

Remarkable duality between

Supergravity string theory in 10 dimensions

and

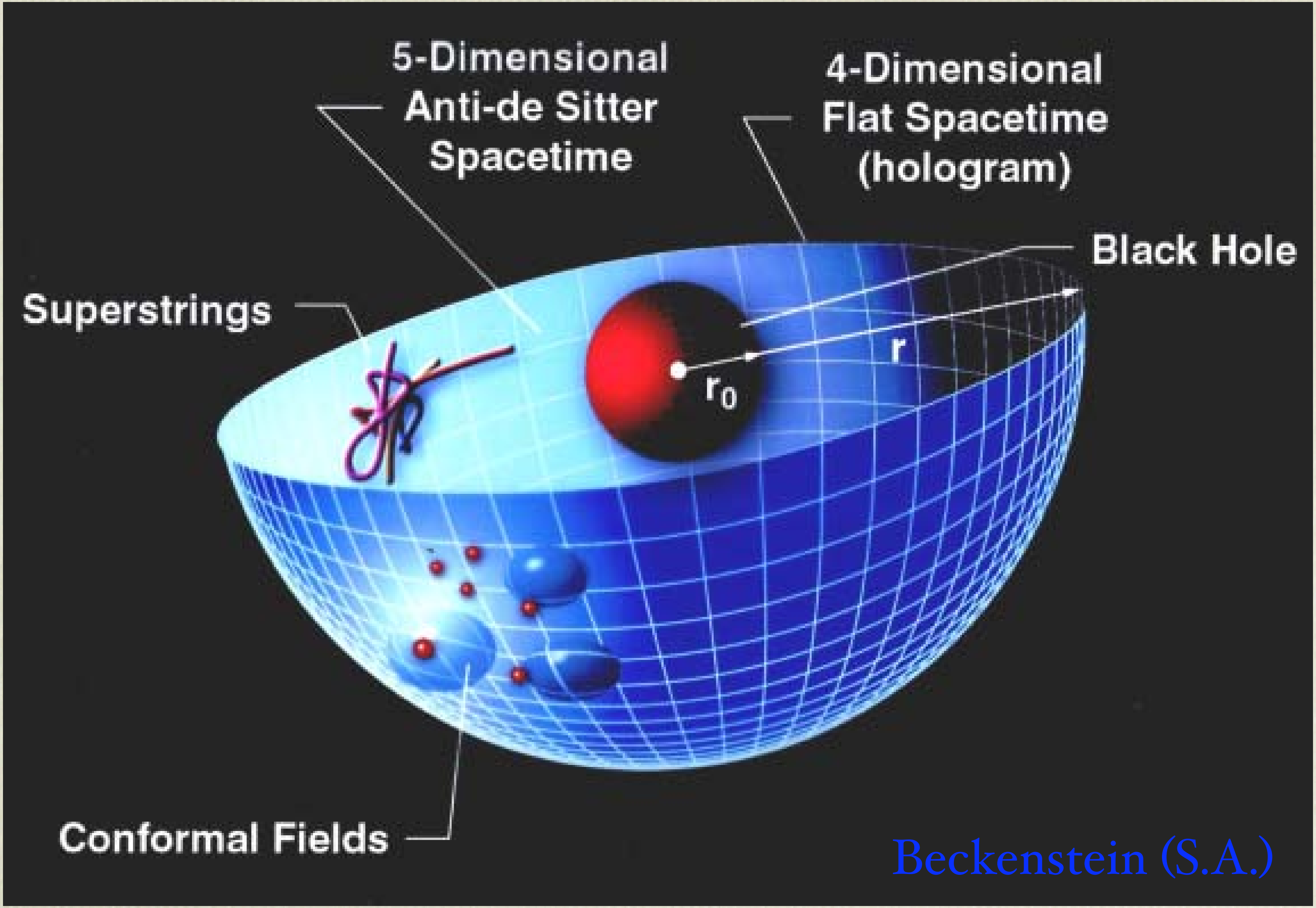
Supersymmetric Yang-Mills field theory in 3+1 Minkowski space-time

$$\boxed{AdS_5 \otimes S^5 \Leftrightarrow SO(4,2) \otimes (\mathcal{N}=4)}$$

5 dim. Sphere

Symmetries of Conformal Transformations + Poincare Invariance of Minkowski space

$\mathcal{N}=4$ SUSY



Beckenstein (S.A.)

References:

* J. M. Maldacena hep-th/9711200 /9803002
* J. Polchinski + M. J. Strassler
  hep-th/0109174, 0209211
* R. C. Brower + C. I. Tan
  hep-th/0207071
* S. J. Rey + J. T. Lee
  hep-th/9803001
* S. J. Brodsky + G. F. de Teramond
  hep-th/0310227
* O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri
  + Y. Oz hep-th/9905111

deTeramond, sjb

AdS/CFT

$z = R^2/r$

- Use mapping of SO(4,2) to AdS5
- Scale Transformations represented by wavefunction Ψ(r) in 5th dimension

$$x_\mu^2 \to \lambda^2 x_\mu^2 \quad \equiv \quad r \to \frac{r}{\lambda} \quad \equiv \quad z \to \lambda z$$

- Holographic model: Confinement at large distances and conformal symmetry at short distances

$$0 < z < z_0 = \frac{1}{\Lambda_{QCD}},\ r > r_0 = \Lambda_{QCD} R^2$$

- Match solutions at large r to conformal dimension of hadron wavefunction at short distances

$$\psi(r) \to r^{-\Delta} \text{ at large } r, \text{ small } z$$

- Truncated space simulates "bag" boundary conditions

$$\psi(z_0) = \psi(r_0) = 0 \qquad r = \frac{R^2}{z}$$

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu,\nu} dx^\mu dx^\nu - dz^2): \; x^\mu \to \lambda x^\mu, \; z \to \lambda z,$$

maps scale transformations into the holographic coordinate $r = R^2/z$.

- String mode in r is the extension of the hadron wf into the fifth dimension.
- Different values of r correspond to different scales at which the hadron is examined.
- Invariant separation between quarks:

$$x_\mu x^\mu \to \lambda^2 x^2, \quad r \to \frac{r}{\lambda}.$$

- The AdS boundary at $r \to \infty$ correspond to the $Q \to \infty$, UV zero separation limit.

AdS/CFT and QCD

Bottom-Up Approach

- Nonperturbative derivation of dimensional counting rules of hard exclusive glueball scattering for gauge theories with mass gap dual to string theories in warped space:
  Polchinski and Strassler, hep-th/0109174.

- Deep inelastic structure functions at small x:
  Polchinski and Strassler, hep-th/0209211.

- Derivation of power falloff of hadronic light-front Fock wave functions, including orbital angular momentum, matching short distance behavior with string modes at AdS boundary:
  Brodsky and de Téramond, hep-th/0310227.

- Low lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD:
  Boschi-Filho and Braga, hep-th/0209080; hep-th/0212207. de Téramond and Brodsky, hep-th/040907 hep-th/0501022; Erlich, Katz, Son and Stephanov, hep-ph/0501128; Hong, Yong and Strassler, hep-th/0501197; Da Rold and Pomarol, hep-ph/0501218.

- Gluonium spectrum (top-bottom):

  Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115.

- D3/D7 branes (top-bottom):

  Karch and Katz, hep-th/0205236; Karch, Katz and Weiner, hep-th/0211107; Kruczenski, Mateos, Myers and Winters, hep-th/0311270; Sakai and J. Sonnenschein, hep-th/0305049; Babington, Erdmenger, Evans, Guralnik and Kirsch, hep-th/0312263; Nunez, Paredes and Ramallo, hep-th/0311201; Hong, Yoon and Strassler, hep-th/0312071; hep-th/0409118; Kruczenski, Pando Zayas, Sonnenschein and Vaman, hep-th/0410035; Sakai and S. Sugimoto, hep-th/0412141; Paredes and Talavera, hep-th/0412260; Kirsh and Vaman, hep-th/0505164.

- Other aspects of high energy scattering in warped spaces:

  S. B. Giddings, hep-th/0203004; Andreev and Siegel, arXiv:hep-th/0410131; Kang and Nastase, hep-th/0410173; Nastase, hep-th/0501039; hep-th/0501068.

- Branes in Minkowski space:

  Siopsis, hep-th/0503245.

Glueball Spectrum

- AdS wave function with effective mass μ:

$$\left[z^2\,\partial_z^2 - (d-1)z\,\partial_z + z^2\,\mathcal{M}^2 - (\mu R)^2\right] f(z) = 0,$$

where $\Phi(x,z) = e^{-iP\cdot x}\, f(z)$ and $P_\mu P^\mu = \mathcal{M}^2$.

- Glueball interpolating operator with twist -dimension minus spin- two, and conformal dimension $\Delta = 4 + L$

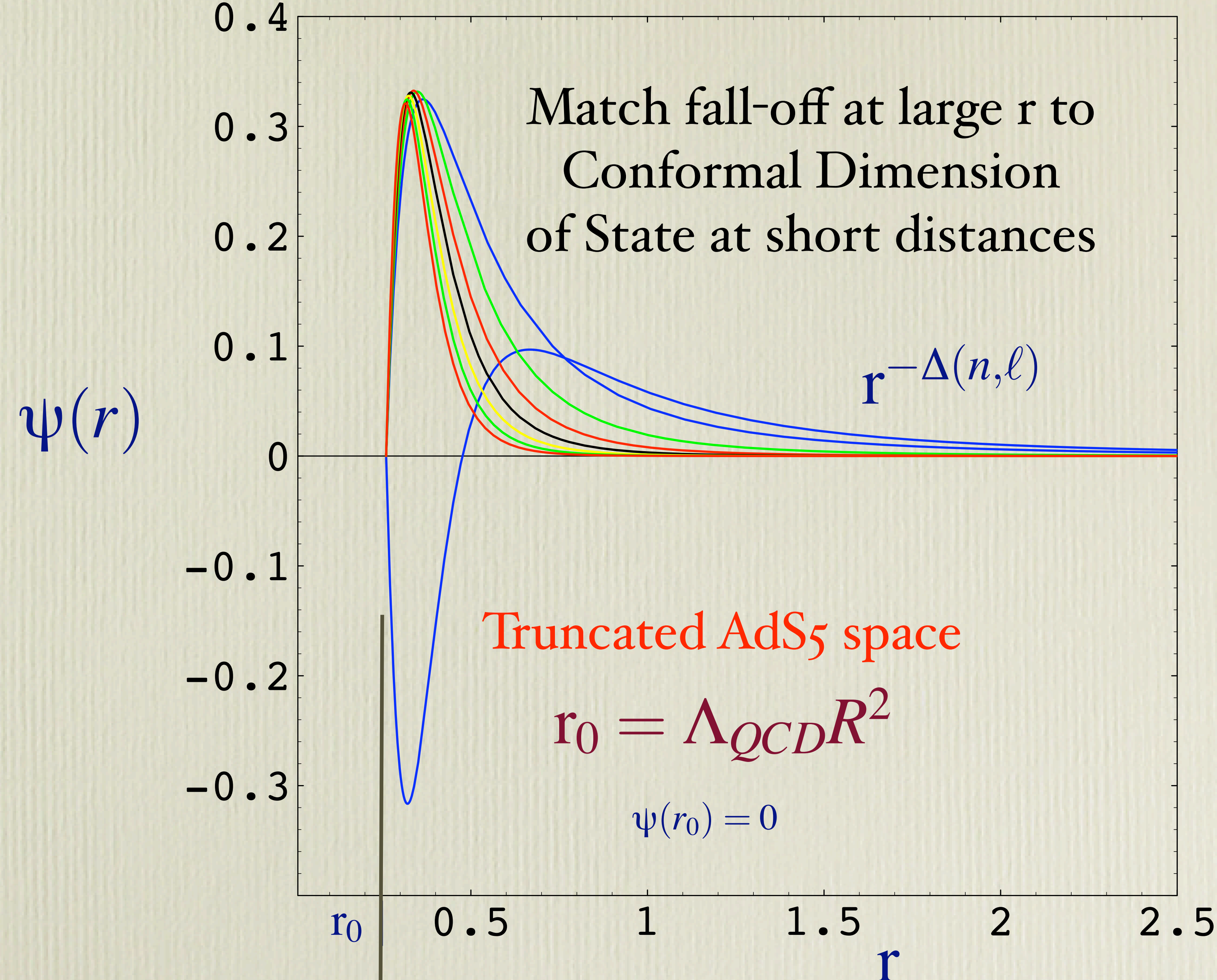
$$\mathcal{O}_{4+L} = F D_{\{\ell_1} \ldots D_{\ell_m\}} F,$$

where $L = \sum_{i=1}^{m} \ell_i$ is the total internal space-time orbital momentum.

- Normalizable scalar AdS mode (d = 4):

$$\Phi_{\alpha,k}(x,z) = C_{\alpha,k} e^{-iP\cdot x} z^2 J_\alpha\left(z\,\beta_{\alpha,a}\Lambda_{QCD}\right)$$

with $\alpha = 2 + L$ and scaling dimension $\Delta = 4 + L$.



Advances in LFQCD- AdS/CFT

4-d mass spectrum from boundary conditions on the normalizable string mode at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$

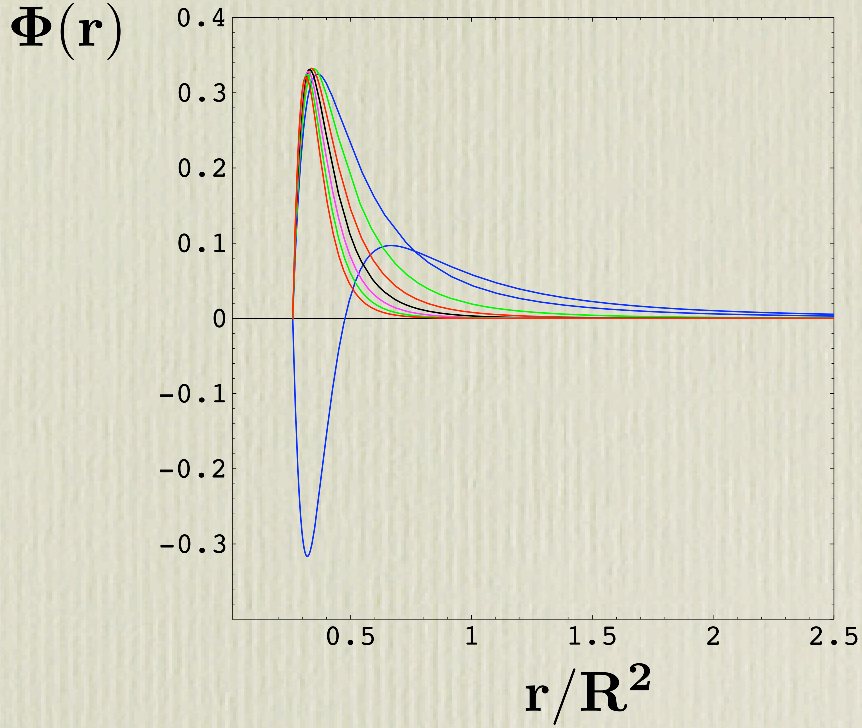


Fig: Gluonium orbital string modes for $\Lambda_{QCD} = r_o/R^2$ = 0.26 GeV.

$L = 0$ lowest glueball state Θ^{++}: $\mathcal{M} = 1.34\,\text{GeV}, \quad \Lambda_{QCD} = 0.26\,\text{GeV}$.

Lattice results: $N_C = 3, \;\; \mathcal{M} = 1.47 - 1.64\,\text{GeV}$

Meson Spectrum

- Wave eq. in AdS for a vector field Φ_μ with polarization along Poincaré coordinates:

$$\left[z^2\,\partial_z^2 - (d-1)z\,\partial_z + z^2\,\mathcal{M}^2 - (\mu R)^2 + d - 1\right] f_\mu(z) = 0,$$

where $\Phi_\mu(x,z) = e^{-iP\cdot x}\, f_\mu(z)$ and $P_\mu P^\mu = \mathcal{M}^2$ ($\Phi_z = 0$ gauge).

- Vector meson: twist-two, dimension $\Delta = 3 + L$

$$\mathcal{O}^\mu_{3+L} = \overline{\psi}\gamma^\mu D_{\{\ell_1}\ldots D_{\ell_m\}}\psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Normalizable AdS vector mode:

$$\Phi^\mu_{\alpha,k}(x,z) = C_{\alpha,k} e^{-iP\cdot x} z^2 J_\alpha\left(z\,\beta_{\alpha,k}\Lambda_{QCD}\right)\epsilon^\mu,$$

with $\alpha = 1 + L$ and $\Delta = 3 + L$.

- 4-d mass spectrum $\Phi^\mu(x, z_o) = 0$: $\mathcal{M}_{\nu,n} = \alpha_{\nu,n}\Lambda_{QCD}$.

- Pseudoscalar mesons: $\mathcal{O}_{3+L} = \overline{\psi}\gamma_5 D_{\{\ell_1}\ldots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).

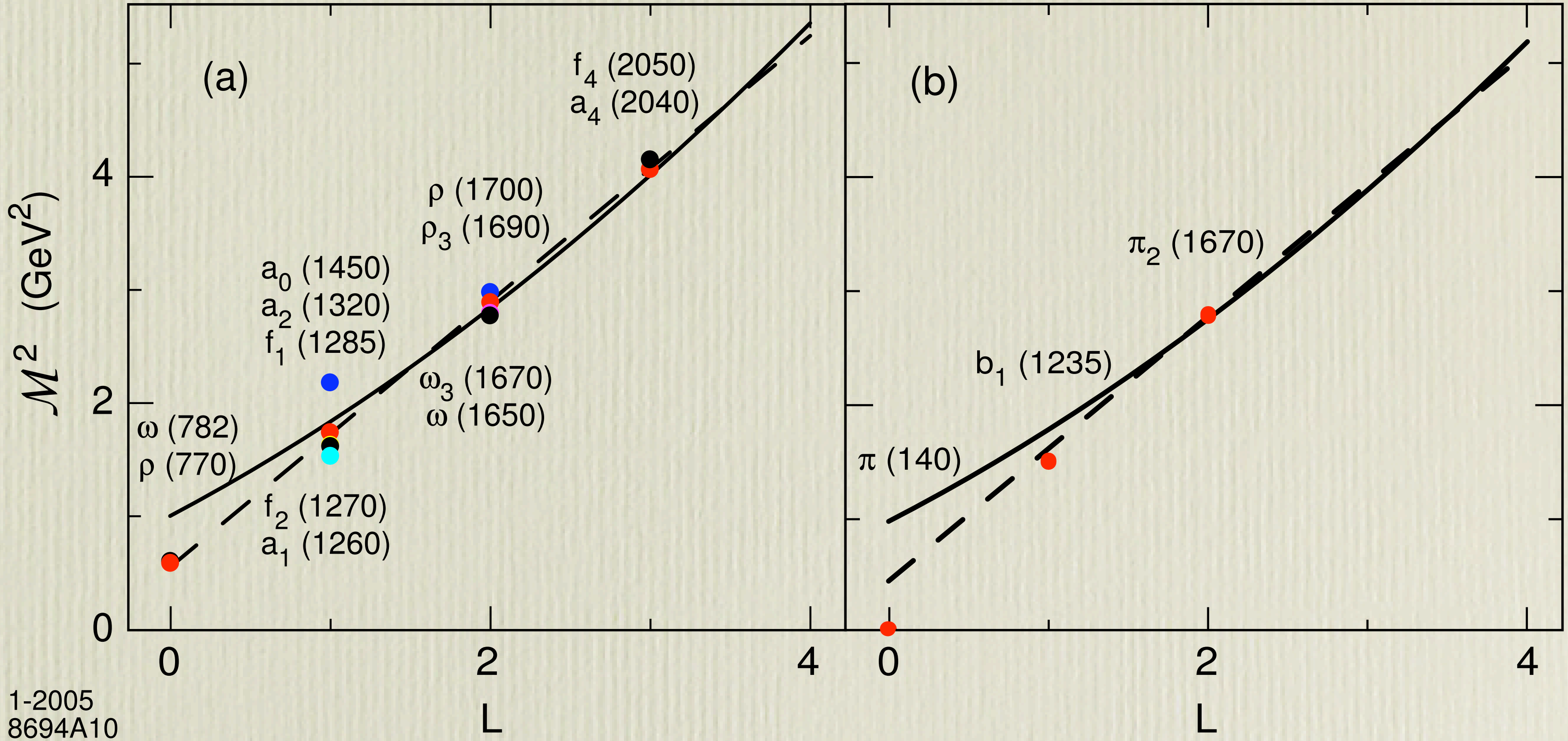


Fig: Light meson orbital spectrum: 4-dim states dual to vector fields in the bulk, $\Lambda_{QCD} = 0.26$ GeV

Baryon Spectrum

- Solve the full 10-dim Dirac, $\not{D}\hat{\Psi} = 0$, since baryons are charged under the $SU(4) \sim SO(6)$ R-symmetry of S^5 (string y-junction) - baryon number conservation?

- $\hat{\Psi}$ is expanded in terms of eigenfunctions $\eta_\kappa(y)$ of the Dirac operator on the compact space X with eigenvalues λ_κ:

$$\hat{\Psi}(x, z, y) = \sum_\kappa \Psi_\kappa(x, z)\eta_\kappa(y)$$

- From the 10-dim Dirac equation, $\not{D}\hat{\Psi} = 0$:

$$\left[z^2\, \partial_z^2 - d\, z\, \partial_z + z^2\mathcal{M}^2 - (\lambda_\kappa + \mu)^2 R^2 + \frac{d}{2}\left(\frac{d}{2} + 1\right) + (\lambda_\kappa + \mu)R\, \hat{\Gamma}\right] f(z) = 0,$$

$$i\not{D}_X \eta(y) = \lambda\, \eta(y),$$

where $\Psi(x, z) = e^{-iP\cdot x}\, f(z)$, $P_\mu P^\mu = \mathcal{M}^2$ and $\hat{\Gamma} u_\pm = \pm u_\pm$. For AdS_5, $\hat{\Gamma}$ is the four-dim chirality operator γ_5.

Henningson and Sfetsos; Muck and Viswanathan

- μ determined asymptotically by spectral comparison with orbital excitations in the boundary: $\mu = L/R$ and λ are the eigenvalues of the Dirac equation on S^{d+1}:

$$\lambda_\kappa R = \pm\left(\kappa + \frac{d}{2} + \frac{1}{2}\right), \quad \kappa = 0, 1, 2...$$

- Baryon: twist-three, dimension $\Delta = \frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Normalizable AdS fermion mode (lowest KK-mode $\kappa = 0$:

$$\Psi_{\alpha,k}(x,z) = C_{\alpha,k} e^{-iP\cdot x} z^{\frac{5}{2}} \Big[J_\alpha(z\beta_{\alpha,k}\Lambda_{QCD})\, \mu_+(P) + J_{\alpha+1}(z\beta_{\alpha,k}\Lambda_{QCD})\, \mu_-(P) \Big].$$

where $\mu^- = \frac{\gamma^\mu P_\mu}{P}\mu^+$, $\alpha = 2 + L$ and $\Delta = \frac{9}{2} + L$.

- 4-d mass spectrum $\Psi(x, z_o)^\pm = 0 \quad \Longrightarrow$ parallel Regge trajectories for baryons !

$$\mathcal{M}^+_{\nu,n} = \alpha_{\nu,n}\Lambda_{QCD}, \qquad \mathcal{M}^-_{\nu,n} = \alpha_{\nu+1,n}\Lambda_{QCD}$$

- Spin-$\frac{3}{2}$ Rarita-Schwinger eq. in AdS similar to spin-$\frac{1}{2}$ in the $\Psi_z = 0$ gauge for polarization along Minkowski coordinates, Ψ_μ. See: Volovich, hep-th/9809009.

- $SU(6)$ multiplet structure for N and Δ orbital states, including internal spin S and L.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

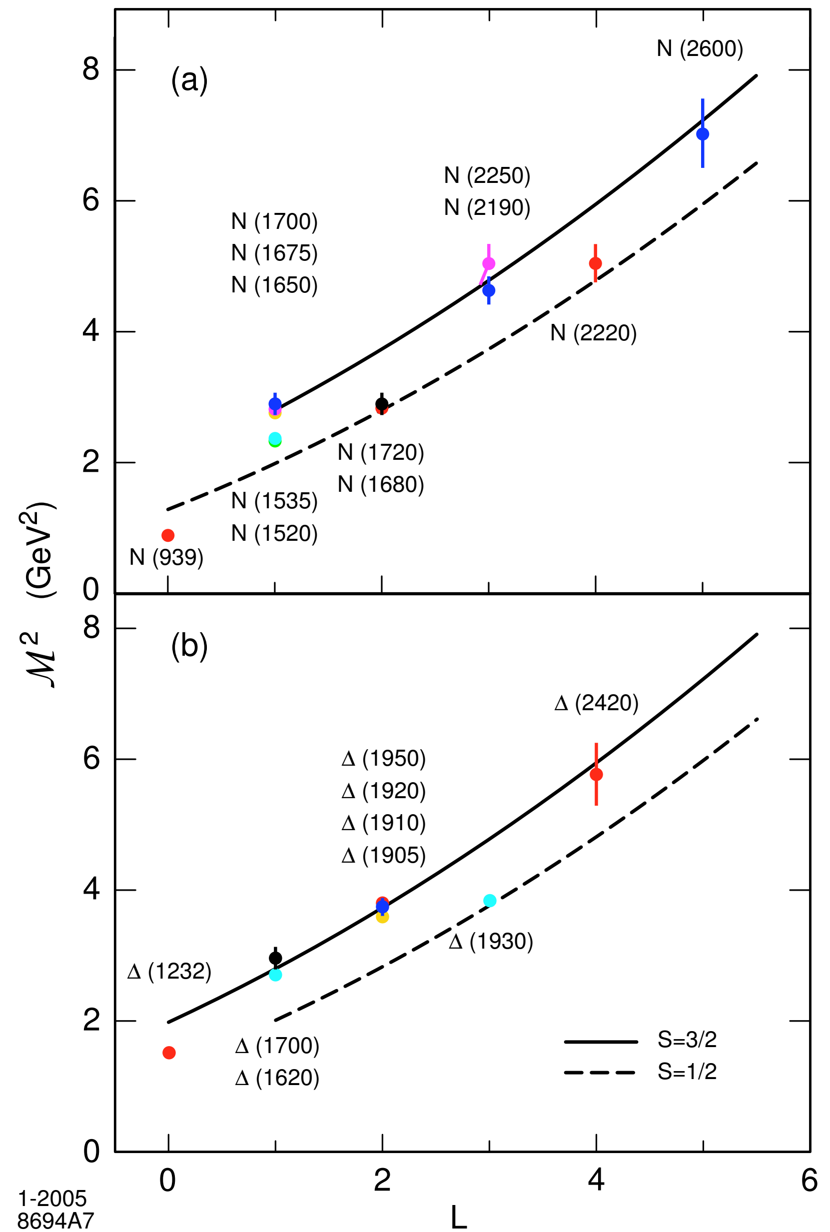


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.22 GeV

Hadronic Form Factor in Space and Time-Like Regions

SJB and GdT in preparation

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron Φ_I and Φ_F and the non-normalizable mode J, dual to the external source (hadron spin σ):

$$\begin{aligned} F(Q^2)_{I \to F} &= R^{3+2\sigma} \int_0^\infty \frac{dz}{z^{3+2\sigma}}\, e^{(3+2\sigma)A(z)} \Phi_F(z)\, J(Q,z)\, \Phi_I(z) \\ &\simeq R^{3+2\sigma} \int_0^{z_o} \frac{dz}{z^{3+2\sigma}}\, \Phi_F(z)\, J(Q,z)\, \Phi_I(z), \end{aligned}$$

- $J(Q,z)$ has the limiting value 1 at zero momentum transfer, $F(0)=1$, and has as boundary limit the external current, $A^\mu = \epsilon^\mu e^{iQ\cdot x} J(Q,z)$. Thus:

$$\lim_{Q \to 0} J(Q,z) = \lim_{z \to 0} J(Q,z) = 1.$$

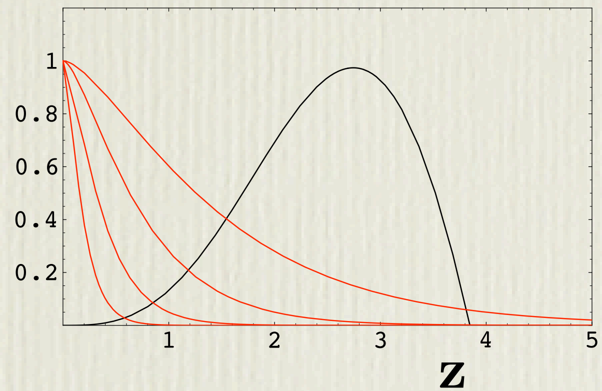
- Solution to the AdS Wave equation with boundary conditions at $Q=0$ and $z \to 0$:

$$J(Q,z) = zQK_1(zQ).$$

Polchinski and Strassler, hep-th/0209211; Hong, Yong and Strassler, hep-th/0409118.

- Propagation of external perturbation suppressed inside AdS.
- At large enough $Q \sim r/R^2$, the interaction occurs in the large-r conformal region. Important contribution to the FF integral from the boundary near $z \sim 1/Q$.

$$\mathbf{J}(\mathbf{Q}, \mathbf{z}), \quad \mathbf{\Phi}(\mathbf{z})$$

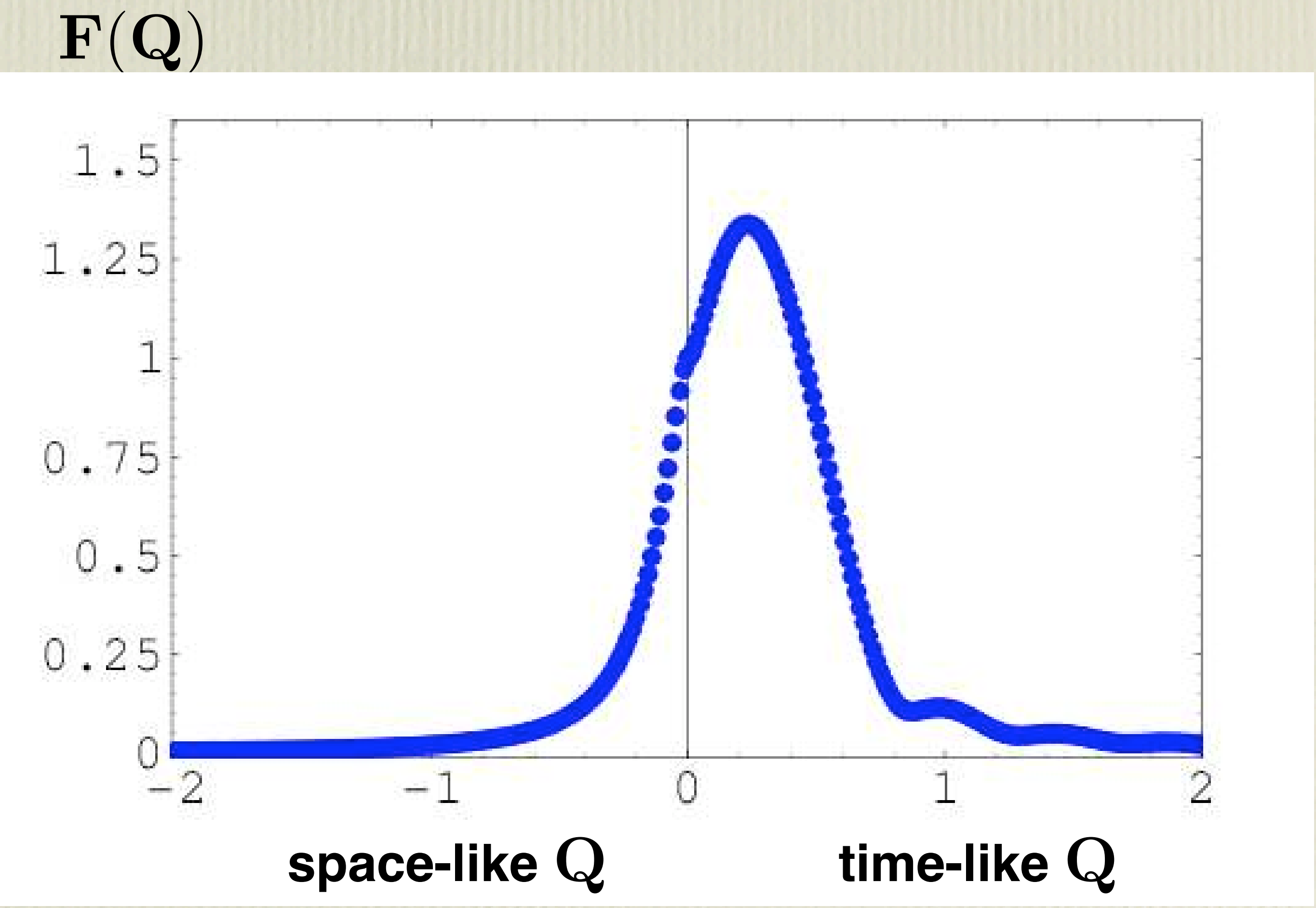


- Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Prediction for the pion form factor in the holographic model (numerical analysis):



AdS/CFT and Light-Front Wavefunctions

- Light-Front Wavefunctions can be determined by matching functional dependence in fifth dimension to scaling in impact space.

$$\left[z^2\,\partial_z^2 - (d-1)z\,\partial_z + z^2\,\mathcal{M}^2 - (\mu R)^2\right] f(z) = 0,$$

$$z \leftrightarrow b$$

- High transverse momentum behavior matches PQCD LFWF with orbital: Belitsky, Ji, Yuan

Holographic Model for Light-Front Wavefunctions

SJB and GdT in preparation

Define the transverse center of momentum $\vec{R}_\perp$ of a hadron in terms of the energy momentum tensor $T^{\mu\nu}$

$$\vec{R}_\perp = \frac{1}{P^+} \int dx^- \int d^2\vec{r}_\perp T^{++} \vec{r}_\perp .$$

Soper

In terms of partonic variables:

$$x_i \vec{r}_{\perp i} = \vec{R}_\perp + \vec{b}_{\perp i},$$

where $\vec{r}_{\perp i}$ are the physical coordinates and $\vec{b}_{\perp i}$ are frame-independent internal coordinates:

$$\vec{R}_\perp = \sum_i x_i \vec{r}_{\perp i}, \quad \sum_i \vec{b}_{\perp i} = 0.$$

- The normalizable string modes Φ_α obey the completeness relation:

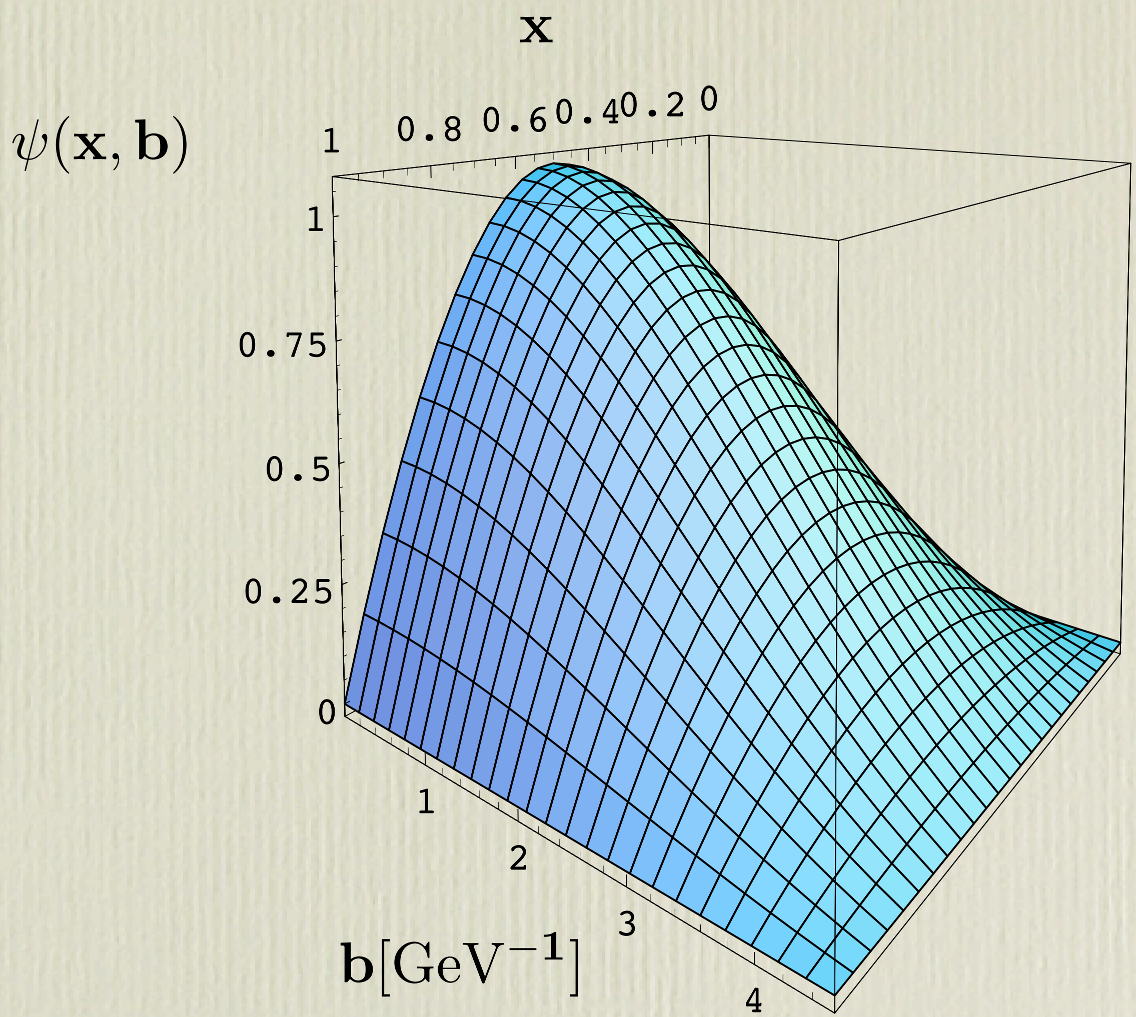
$$\sum_\alpha \Phi_\alpha(z)\Phi_\alpha(z') = \left(\frac{Re^{A(z)}}{z}\right)\delta(z - z').$$

- Mapping of string modes to impact space representation of LFWF, which also span a complete basis.

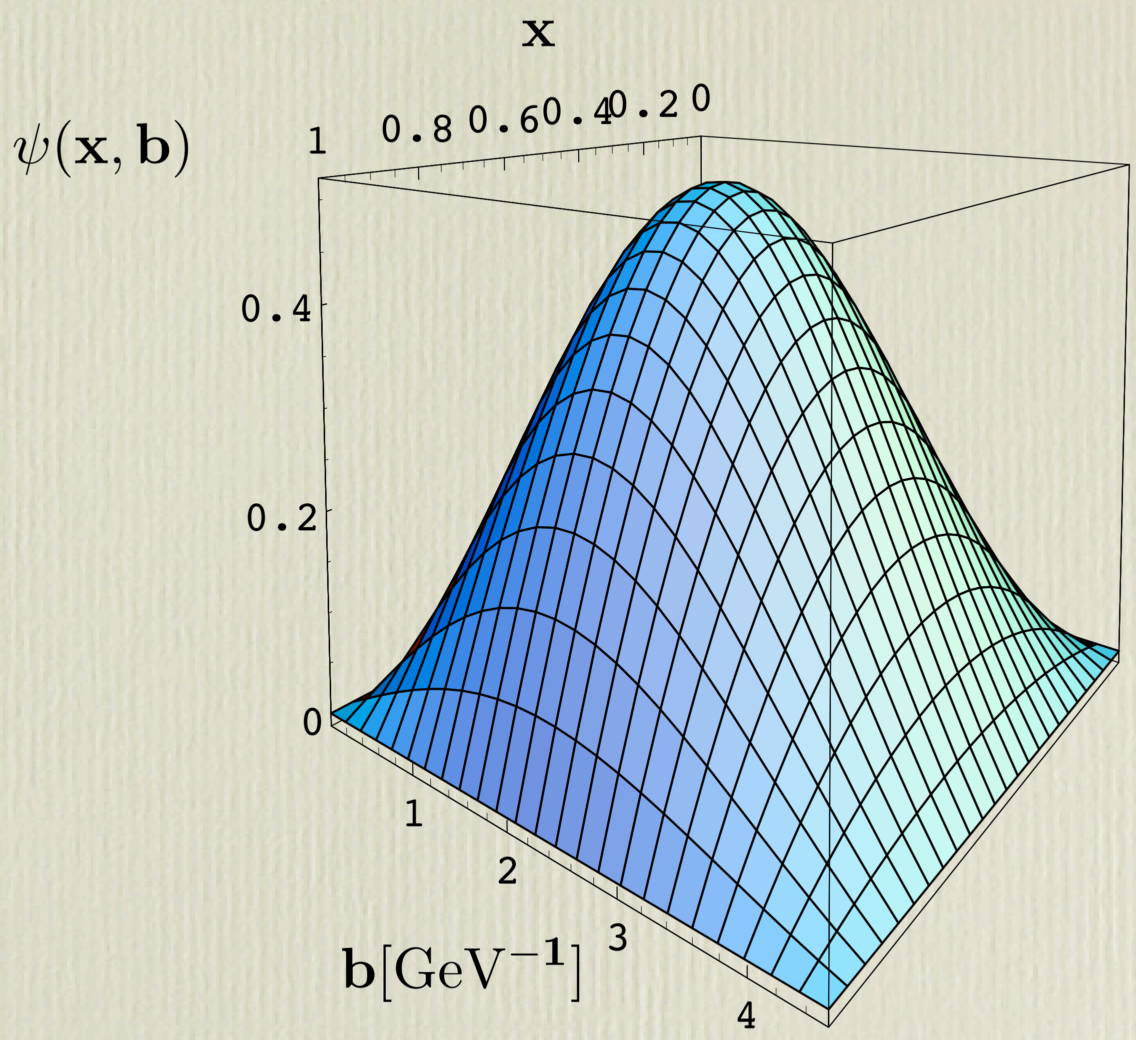
- Two-parton $n = 2$ LFWF including orbital angular momentum $\ell = 0, 1, 2\ldots$ and radial modes $k = 1, 2, 3, \ldots$ is to first approximation:

$$\psi_{n,\ell,k}(x, b) = B_{n,\ell,k}\, x(1 - x)\frac{J_{n+\ell-1}\left(b\beta_{n-1,k}\Lambda_{QCD}\right)}{b}, \quad (1)$$

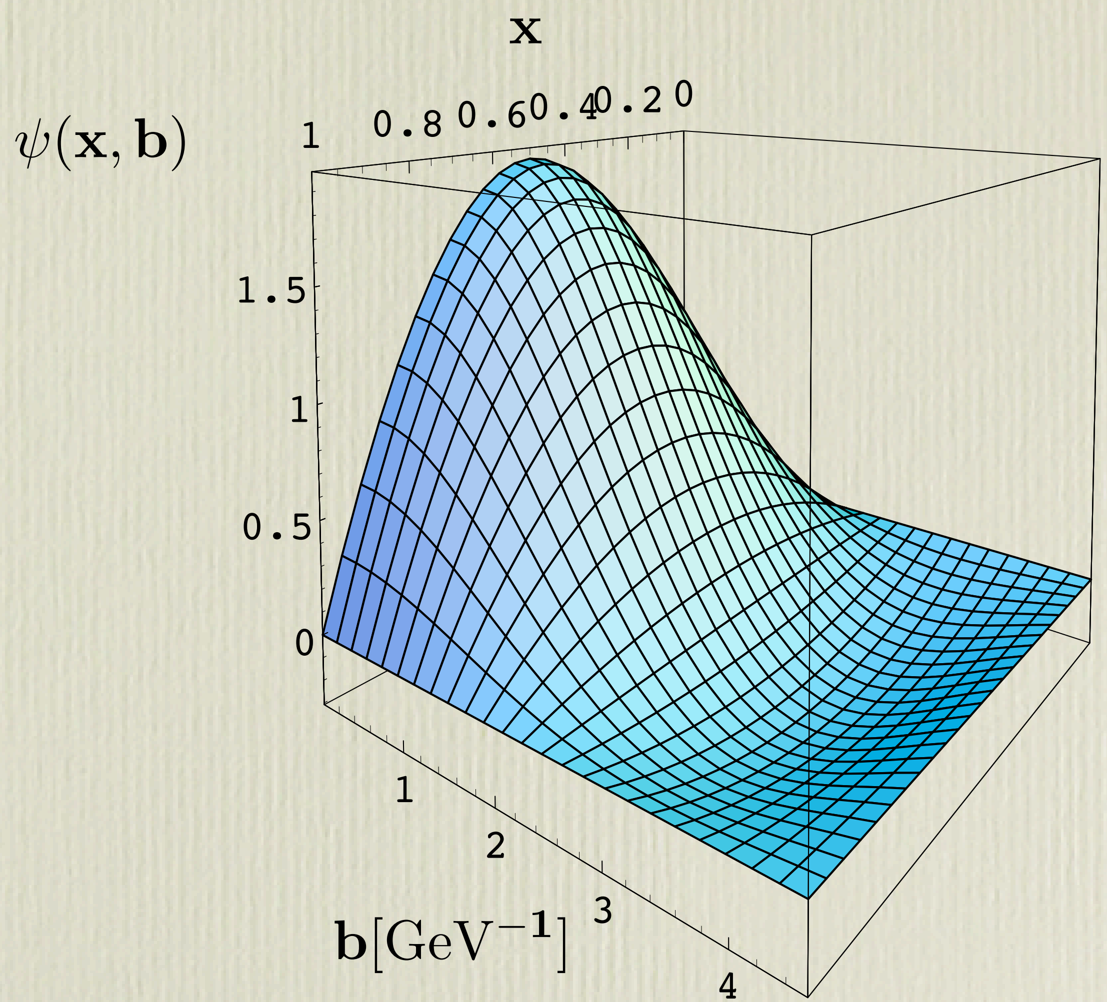
where $b = |\vec{b}_\perp|$.



Two-parton ground state LFWF in impact space $\psi(x,b)$ for a for $n=2,\ell=0,k=1$.



Two-parton first orbital exited state in impact space $\psi(x, b)$ for a for $n = 2, \ell = 1, k = 1$.



Two-parton first radial exited state LFWF in impact space $\psi(x,b)$ for $n=2, \ell=0, k=2$.

Features of HolographicModel

- Ratio of proton to Delta trajectories= ratio of zeroes of Bessel functions.
- Only one scale Λ_{QCD} determines hadron spectrum (slightly different for mesons and baryons)
- Only quark-antiquark, qqq, and g g hadrons appear at classical level
- Covariant version of bag model: confinement+conformal symmetry

Two approaches to evaluating LFWFs at Short Distances

$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$

$k_\perp^2 >> \Lambda^2_{QCD}$ and/or $x_i \to 1$

- Use PQCD (minimally connected tree graphs)

- AdS/CFT (duality between string theory and conformal field theory)

In practice: QCD: Approximately Conformal

$$\left|\Psi_h(P^+,\vec{P}_\perp)\right\rangle = \sum_{n,\lambda_i}\int \left[dx_i\ d^2\vec{k}_{\perp i}\right]\psi_{n/h}(x_i,\vec{k}_{\perp i},\lambda_i)\left|n : x_iP^+, x_i\vec{P}_\perp + \vec{k}_{\perp i},\lambda_i\right\rangle$$

Conformal Behavior:

$$\psi_{n/h}(\vec{k}_\perp) \to (k_\perp)^\ell \left[\frac{1}{\vec{k}_\perp^2}\right]^{n+\delta_n+\ell-1}.$$

PQCD: Ji, Ma, Yuan

AdS/CFT : de Teramond, Sjb

Model Form

$$\psi_{n/h}(x_i,\vec{k}_{\perp i},\lambda_i,l_{zi}) \sim \frac{(g_s\ N_C)^{\frac{1}{2}(n-1)}}{\sqrt{N_C}}\prod_{i=1}^{n-1}(k_{i\perp}^{\pm})^{|l_{zi}|}\left[\frac{\Lambda_o}{\mathcal{M}^2-\sum_i\frac{\vec{k}_{\perp i}^2+m_i^2}{x_i}+\Lambda_o^2}\right]^{n+|l_z|-1}$$

$$\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i, l_{zi}) \sim \frac{(g_s \ N_C)^{\frac{1}{2}(n-1)}}{\sqrt{\mathcal{N}_C}}$$

$$\times \prod_{i=1}^{n-1} (k_{i\perp}^{\pm})^{|l_{zi}|} \left[\frac{\Lambda_{QCD}}{\mathcal{M}^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} + \Lambda_{QCD}^2} \right]^{n+|l_z|-1}$$

de Teramond, SJB

The form is compatible with the scaling properties predicted by the AdS/CFT correspondence including orbital angular momentum.

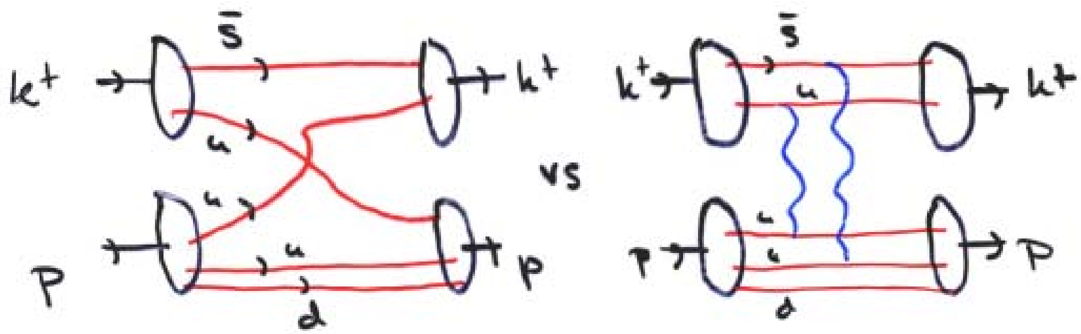
New Perspectives on QCD from AdS/CFT

- LFWFs: Fundamental description of hadrons at amplitude level
- QCD is Nearly Conformal
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra
- Quark-interchange and scattering amplitudes

Angular Distribution $-t/s = \frac{1}{2}(1-\cos\theta_{cm})$

$$\frac{d\sigma}{dt} = \frac{1}{s^{n_{TOT}-2}} F(t/s)$$

determined by scattering mechanism



Quark Interchange   gluon exchange

analogous to spin exchange in atom - atom scattering   Van der Waals

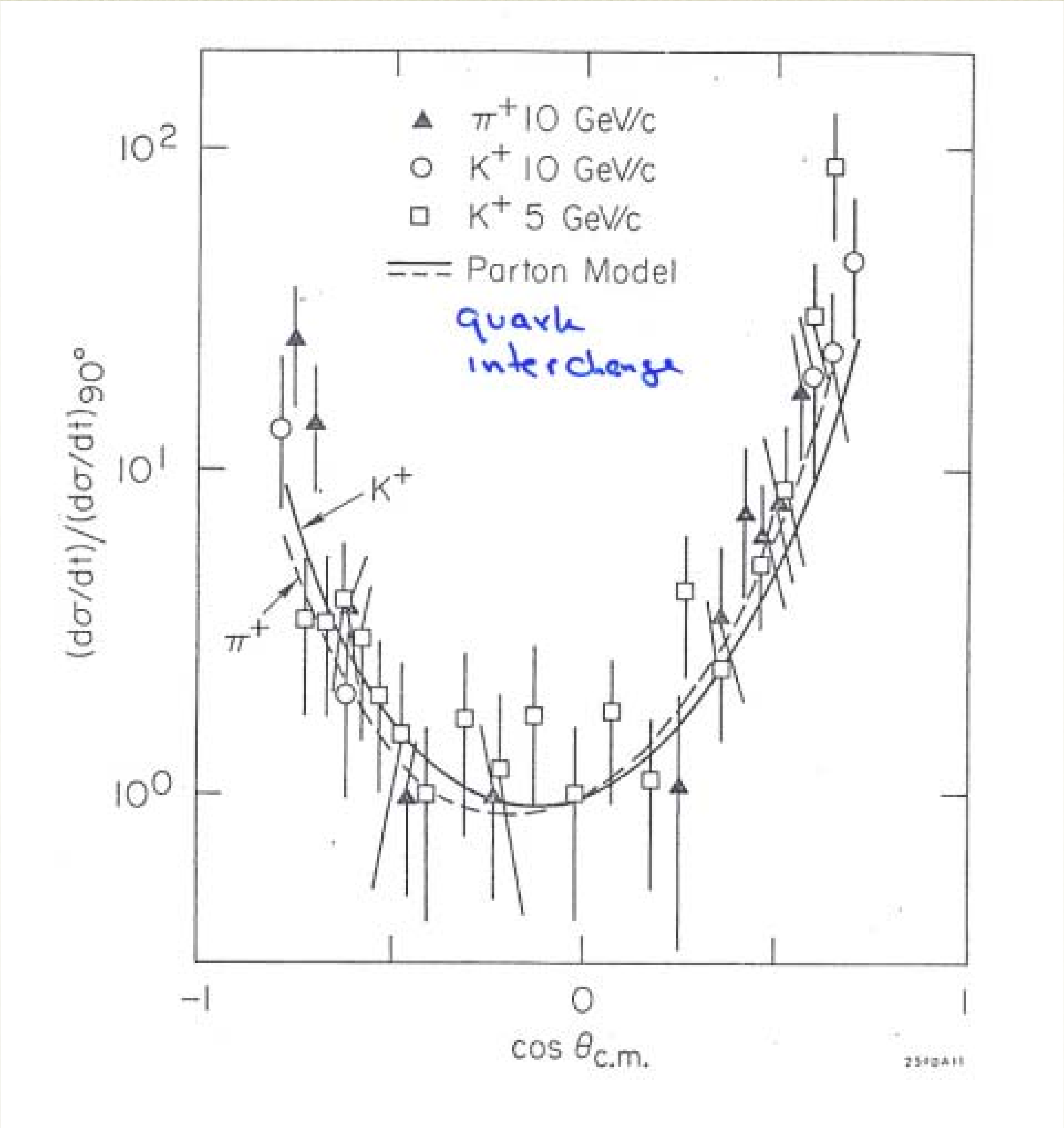
Blankenbecler, Gunion, sjb

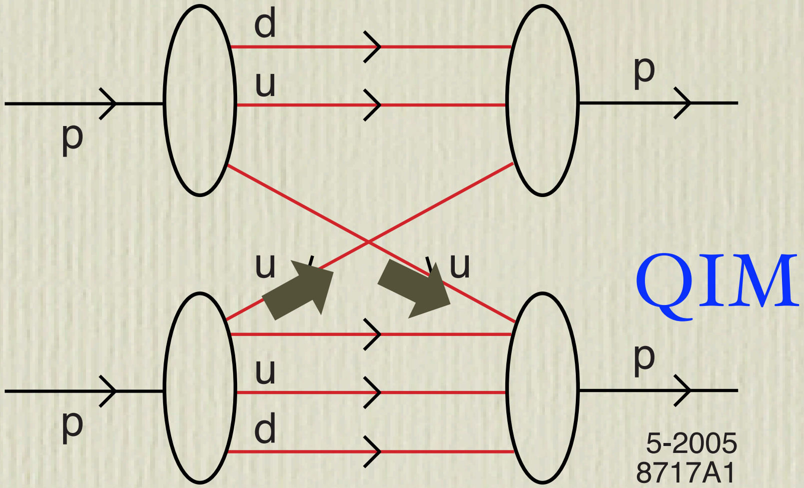
MIT Bag Model predicts dominance of quark interchange: deTar

Large N_c: Quark Interchange Dominant

$$\mathcal{M} \sim \frac{1}{u}\frac{1}{t^2}$$

't Hooft limit, AdS/CFT


π+ 10 GeV/c
K+ 10 GeV/c
K+ 5 GeV/c
Parton Model
quark interchange
10²
10¹
10⁰
(dσ/dt)/(dσ/dt)90°
K+
π+
-1
0
1
cos θc.m.



$$\frac{d\sigma}{dt}(pp \to pp) = C\,\frac{F_p^{\,2}(t)\,F_p^{\,2}(u)}{s^2}$$

$$\frac{d\sigma}{dt} = \frac{1}{s^{10}} f(\theta_{\mathrm{c.m.}})\,, \quad f(\theta_{\mathrm{c.m.}}) \sim \left(\frac{1}{1-\cos^2\theta}\right)^4 .$$

The biggest failure of the interchange mechanism is in the spin correlation. For all angles we predict from Table I

$$A_{nn} = \frac{1}{3}\,\frac{1-(\frac{3}{31})^2\chi^2}{1+\frac{1}{3}(\frac{3}{31})^2\chi^2}\,, \tag{3.11}$$

where

$$\chi = \frac{f(\theta) - f(\pi-\theta)}{f(\theta)+f(\pi-\theta)}\,.$$

Thus A_{nn} is predicted to be within 2% of $\frac{1}{3}$ even when $\chi = 1$ [$\chi = 0$ for the form in Eq. (3.6)]. The data clearly indicate that A_{nn} is not a constant near $\frac{1}{3}$.

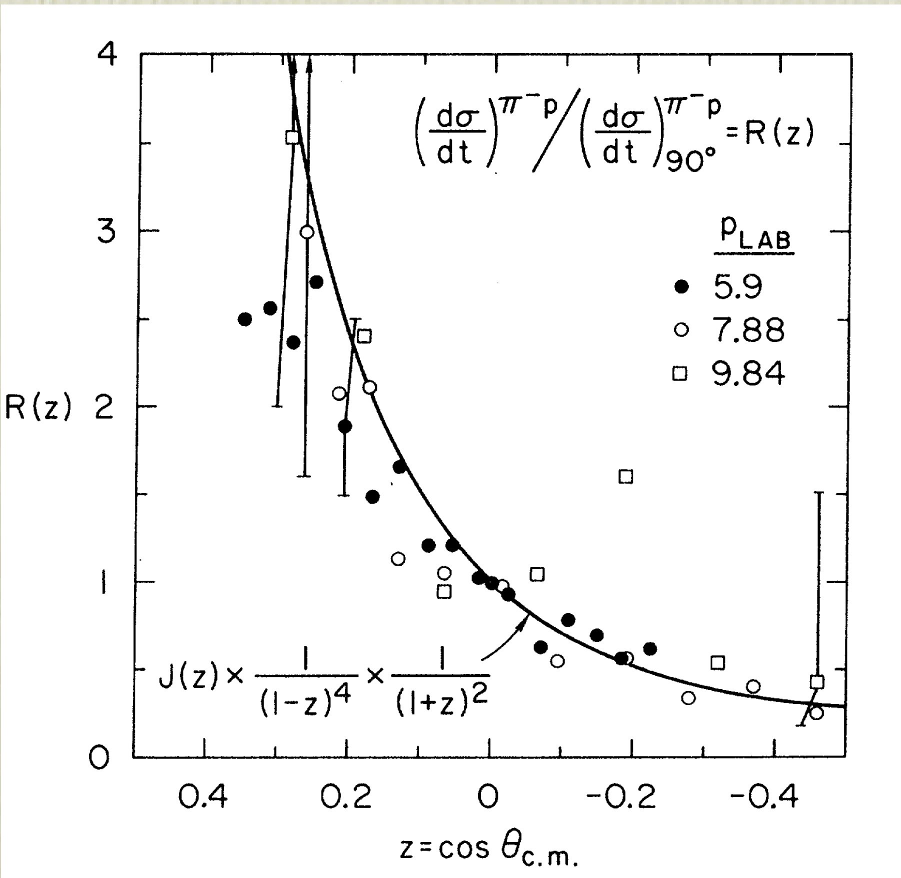
Our expectation, then, is that there is an additional amplitude which strongly interferes with the quark-interchange contributions at Argonne energies; most plausibly, the quark-interchange contribution is dominant at asymptotic t and u, and the interfering amplitude is most important at low t and u. As we shall discuss below, the behavior of A_{ll} and A_{ss} in the interference region can play an important role in sorting out the possible subasymptotic contributions.

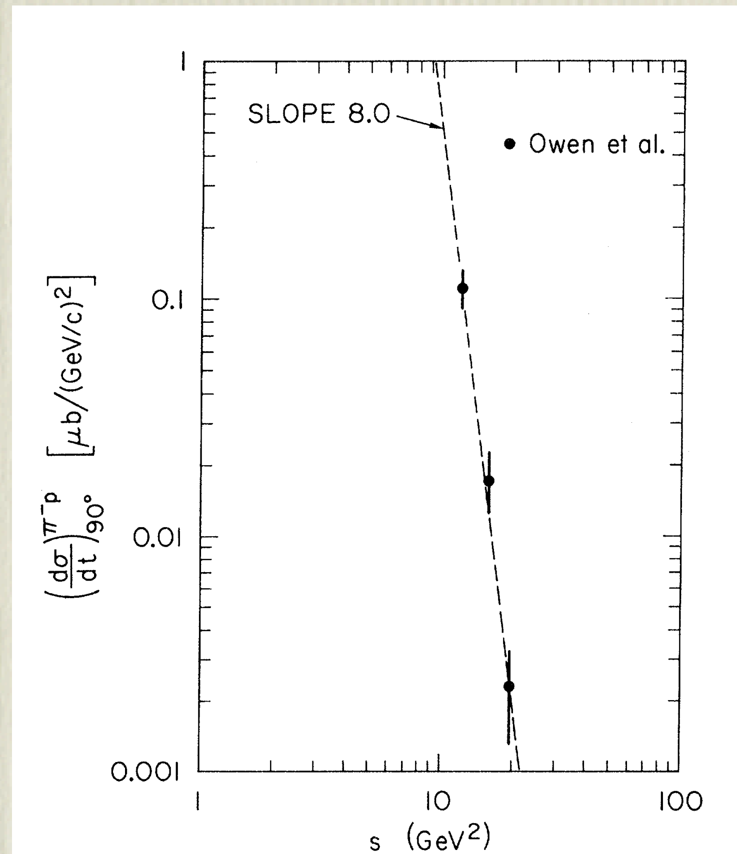
These results for the quark-interchange model have also been obtained by Farrar, Gottlieb, Sivers, and Thomas,[12] who also consider the possibility that nonperturbative effects (quark-quark scattering via instantons) can explain the data.

$$M_{FI} = \langle \psi_F | E - K | \psi_I \rangle$$
$$\equiv \langle \psi_F | \Delta | \psi_I \rangle$$
$$= \frac{1}{2(2\pi)^3} \int d^2k \int_0^1 \frac{dx}{x^2(1-x)^2} \, \Delta \psi_C(\vec{k}_\perp - x\vec{r}_\perp, x) \psi_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) \psi_A(\vec{k}_\perp - x\vec{r}_\perp + (1-x)\vec{q}_\perp, x) \psi_B(\vec{k}_\perp, x),$$

where

$$\Delta = s - M_A{}^2 - M_B{}^2 - K_a - K_b - K_c - K_d$$
$$= M_A{}^2 + M_B{}^2 - S_A(\vec{k}_\perp + (1-x)\vec{q}_\perp - x\vec{r}_\perp, x) - S_B(\vec{k}_\perp, x)$$
$$= M_C{}^2 + M_D{}^2 - S_C(\vec{k}_\perp - x\vec{r}_\perp, x) - S_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) \,.$$





Why is quark-interchange dominant over gluon exchange?

Example: $M(K^+p \to K^+p) \propto \frac{1}{ut^2}$

Exchange of common u quark

$M_{QIM} = \int d^2k_\perp dx \; \psi_C^\dagger \psi_D^\dagger \Delta \psi_A \psi_B$

Holographic model (Classical level):

Hadrons enter 5th dimension of AdS_5

Quarks travel freely within cavity as long as separation $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.

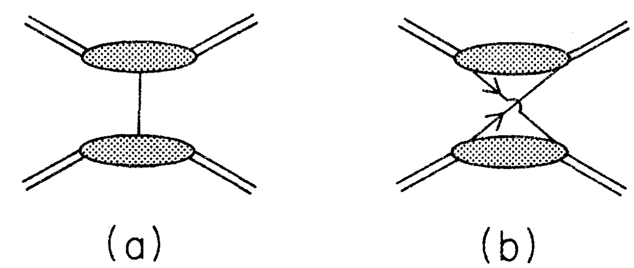


FIG. 1. The two basic types of interactions between hadrons: (a) gluon interchange and (b) constituent interchange.

AdS/CFT and QCD

- Non-Perturbative Derivation of Dimensional Counting Rules (Strassler and Polchinski)
- Light-Front Wavefunctions: Confinement at Long Distances and Conformal Behavior at short distances (de Teramond and Sjb)
- Power-law fall-off at large transverse momentum, $x \rightarrow 1$
- Hadron Spectra, Regge Trajectories

New Perspectives on QCD from AdS/CFT

- LFWFs: Fundamental description of hadrons at amplitude level
- QCD is Nearly Conformal
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra
- Quark-interchange dominates scattering amplitudes

Outlook

- Only one scale Λ_{QCD} determines hadronic spectrum (slightly different for mesons and baryons).
- Ratio of Nucleon to Delta trajectories determined by zeroes of Bessel functions.
- String modes dual to baryons extrapolate to three fermion fields at zero separation in the AdS boundary.
- Only dimension $3, \frac{9}{2}$ and 4 states $\overline{q}q$, qqq, and gg appear in the duality at the classical level!
- Non-zero orbital angular momentum and higher Fock-states require introduction of quantum fluctuations.
- Simple description of space and time-like structure of hadronic form factors.
- Dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model. Modified by gluonic quantum fluctuations.
- Covariant version of the bag model with confinement and conformal symmetry.

Light-Front QCD Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme

Light-Front QCD Phenomenology

- Final-state corrections to DIS: Leading-twist Diffractive DIS, Sivers Effect, Nuclear Shadowing
- Non-universality of Nuclear Anti-shadowing
- Exclusive QCD Processes
- Exclusive heavy hadron decays
- Angular Momentum Sum Rules: only n-1 L_z

New Directions for obtaining LFWFs

- Use AdS/CFT motivated LFWFS as an initial condition & expansion functions for hadron wfs in QCD; variational methods; coordinate space Harinandrath, Vary, sjb

- DLCQ for φ^4 theory -- reproduce classical results without zero modes Harinandrath, Vary

- Nonpertubative renormalization methods; truncated LF Fock space; optimization; regularization
  Glazek Mathiot, Karmanov Hiller, McCartor, sjb
- Adaptive perturbation theory +DLCQ Weinstein

- Bethe Salpeter Eq. in Minkowski space; lcg

Light-Front Thermodynamics

- Ideal for relativisitic systems
- Boost-invariant formalism
- Construct covariant partition function from DLCQ solutions
- Recent papers: Das; Raufeisen & sjb

GSI Anti-Proton Facility

- Comprehensive QCD program with intense, continuous, polarized anti-proton beams & variety of targets targets

- p_{lab} = 15 GeV/c: Ideal energy domain for study of valence regime

- Anti-proton exclusive channels, spin asymmetries, transversity

- Timelike DVCS, meson electroproduction, charm at threshold , color transparency, hadronization, hidden color, exotics, higher twist, ...

Jlab 12 GeV Upgrade

- Comprehensive QCD DIS program with intense, continuous, polarized beams & targets + extensive facilities
- Ideal energy domain for study of valence regime
- Parity violation, nuclear targets, exclusive channels, spin asymmetries, correlations
- DVCS, meson electroproduction, charm at threshold , color transparency, hadronization, hidden color, exotics, higher twist, ...
- Complimentary to GSI antiproton program