Cairns, July 7, 2005

Light-front Hamiltonians for heavy quarks and gluons

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What is the mechanism of binding of quarks and gluons in QCD? The binding occurs typically above threshold, which means that the sum of masses of constituents is smaller than the mass of the bound state. How can this happen in a relativistic quantum theory? A boost-invariant Hamiltonian formulation of quantum choromodynamics is used to develop a heuristic picture for the binding mechanism of heavy quarks and gluons.

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Light-Cone QCD and Nonperturbative Hadron Physics, Cairns, July 7-15, 2005.

Plan:

1. Introduce Hamiltonians for effective particles in QFT: light-front.

2. Describe an emerging heuristic picture of binding in gauge theories.

3. Describe what comes out of the picture for gluonium and heavy quarkonia.

Acknowledgements:
Tomasz Masłowski - Poincaré algebra and gluonium
Jarosław Młynik - Renormalization group for Hamiltonians and quarkonium
Jakub Narębski - Models of hybrids made of heavy gluons and quarks
Lech Stawikowski - Schrödinger equation with Coulomb + oscillator potential
Marek Więckowski - Theory of bound states of constituents and scattering
Kenneth G. Wilson - Initial RG studies.

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KEY POINTS:

1. In the region of binding, the copuling constant increases to ~ 1.

$$\begin{aligned} \alpha &= \frac{g^2}{4\pi}\,, \\ g &= g(\lambda)\,, \\ H &= H_\lambda\,, \\ H_\lambda &= ? \end{aligned} \tag{1}$$

2. One can use RGPEP equations for evaluating H_λ in perturbation theory.

$$q_\lambda = U_\lambda\, q_{can}\, U_\lambda^\dagger\,, \tag{2}$$

$$H_\lambda(q_\lambda) = [H_{can} + H_{CT}]_{reg}\,(q_{can})\,. \tag{3}$$

Regularization, renormalization (both not Lagrangian) ab initio Hamiltonian, en block, in the Fock space.

Can it be relativistic? Yes.

3. RGPEP extends to the Poincaré algebra in QFT (T. Masłowski):

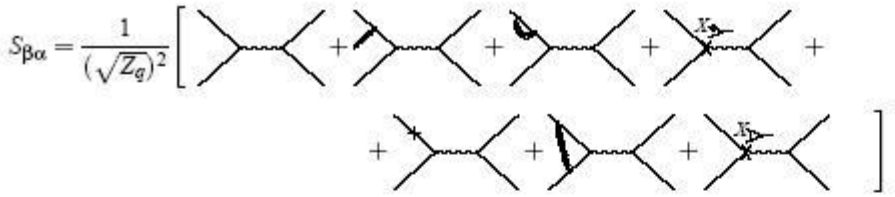
$$\begin{aligned}
[P^\mu, P^\nu] &= 0\,, \\
[P^\mu, M^{\nu\rho}] &= i(g^{\mu\nu}P^\rho - g^{\mu\rho}P^\nu)\,, \\
[M^{\mu\nu}, M^{\rho\sigma}] &= i(g^{\mu\rho}M^{\sigma\nu} - g^{\mu\sigma}M^{\rho\nu} + g^{\nu\rho}M^{\mu\sigma} - g^{\nu\sigma}M^{\mu\rho})\,.
\end{aligned}$$

Example of ϕ^3:

$$\begin{aligned}
J^1_\lambda &= 1/2(M^{-2}_\lambda - M^{+2}_\lambda)\,, \\
J^2_\lambda &= -1/2(M^{-1}_\lambda - M^{+1}_\lambda)\,, \\
J^3_\lambda &= M^{12}_\lambda
\end{aligned}$$

$$[J^i_\lambda, J^j_\lambda] = i\epsilon_{ijk}J^k_\lambda + o(g^3)\,.$$

4. Więckowski's Theorem (checked expl. 1 loop ϕ^3 in 5+1 dimensions):

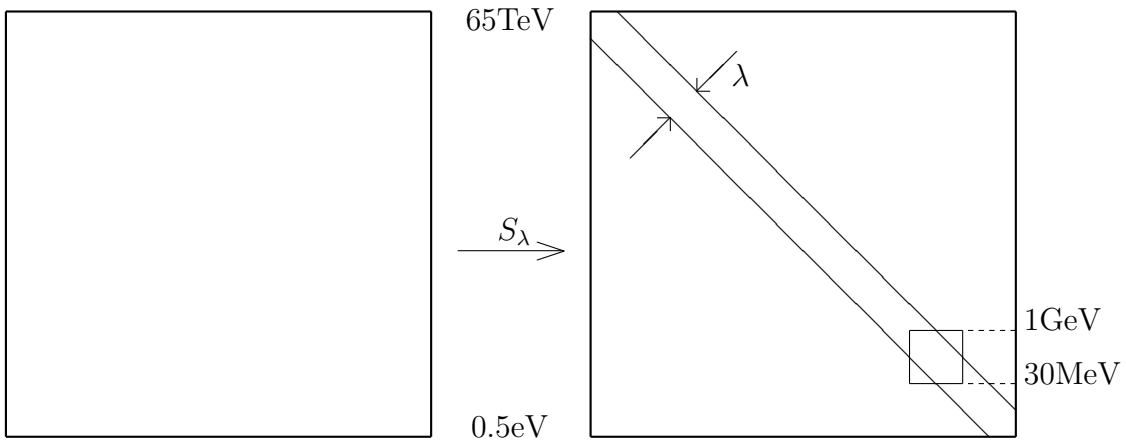


Structure of Hamiltonian CTs is calculated from RGPEP, and there exist finite parts in these counterterms that produce covariant amplitude of the type $e^+e^- \to hadrons$ in one loop. Marek.Wieckowski@fuw.edu.pl

The same S matrix for scattering of physical particles can be obtained using:

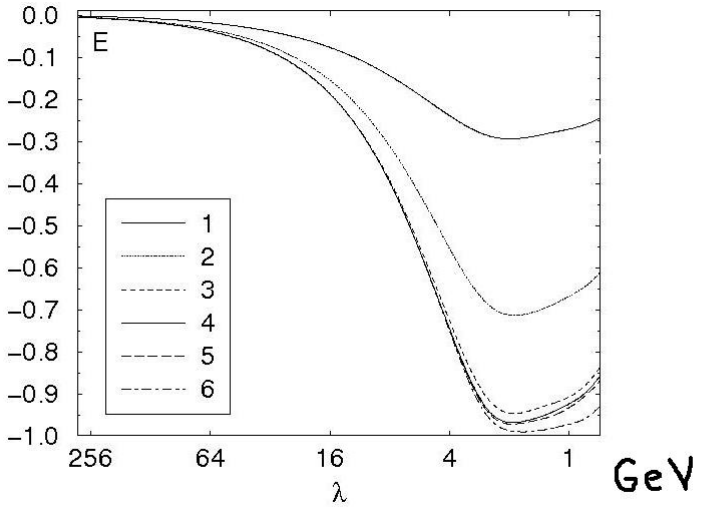
1. a bare Hamiltonian $[H_{can} + H_{CT}]_{reg}$, and representing the in and out particles with creation and annihilation operators q_{can} for bare-particles,
2. an effective Hamiltonian H_λ and effective operators q_λ.

In each order of perturbation theory, the result for the S matrix is the same, provided that the connection between q_{can} and q_λ, which also means between H_{reg} and H_λ, is calculated up to this order. H_λ contains formfactors in interaction terms.



5. One can calculate bound states using Window Hamiltonians: Like one can calculate non-perturbative atomic physics using the Coulomb potential, which is only of formal order e^2. RGPEP produces $H_\lambda(g_\lambda)$ without small denominators in perturbation theory.

Results of non-perturbative diagonalization of W_λ obtained from first 6 orders. Asymptotically free model for



bound-state calculation, with **J. Młynik.**.

6. Convergence in the Fock space in the effective-particle basis

Gluonium:

$$\Psi\rangle \;=\; |g_\lambda g_\lambda\rangle + |g_\lambda g_\lambda g_\lambda\rangle + ...$$

Quarkonium:

$$\Psi\rangle \;=\; |Q_\lambda \bar{Q}_\lambda\rangle + |Q_\lambda \bar{Q}_\lambda g_\lambda\rangle + ...$$

Growth of g_λ compensated by the narrowing formfactor f_λ.

Akin to nuclear physics with practically fixed number of nucleons.

7. Assume a mass gap between states $|g_\lambda g_\lambda g_\lambda\rangle$ and $|g_\lambda g_\lambda\rangle$ due to non-abelian potentials.

$$\begin{aligned} |g_\lambda g_\lambda g_\lambda\rangle &= \mathcal{R}|g_\lambda g_\lambda\rangle\,, \\ H_{gg}|g_\lambda g_\lambda g_\lambda\rangle &= E|g_\lambda g_\lambda\rangle\,, \\ H_{gg} &= \frac{1}{\sqrt{\mathcal{P}+\mathcal{R}^\dagger\mathcal{R}}}(\mathcal{P}+\mathcal{R}^\dagger)H_\lambda(\mathcal{P}+\mathcal{R})\frac{1}{\sqrt{\mathcal{P}+\mathcal{R}^\dagger\mathcal{R}}}\,. \end{aligned}$$

$$\begin{aligned} \langle k|H_{gg}|k'\rangle &= \langle k|T_\lambda + T^{\delta m^2}_{\lambda\epsilon} + f_\lambda W_{gg,gg} + \\ &+ \frac{1}{2}Y_{gg,ggg}\left(\frac{1}{E_k - T_\lambda - T^{\mu^2}} + \frac{1}{E_{k'} - T_\lambda - T^{\mu^2}}\right)Y_{ggg,gg}|k'\rangle\,, \\ T_\lambda|k\rangle &= E_k|k\rangle\,. \end{aligned}$$

The gap ansatz depends on the relative momenta of 3 gluons:

$$T^{\mu^2} = \frac{1}{3!}\sum_{123}\int[123]\sum_{i=1}^{3}\frac{\mu_i^2(123)}{p_i^+}|123\rangle\langle 123|\,,$$

$\mu_i^2(123)$, $i = 1, 2, 3$.

8. Let us make a gap ansatz **correctable** order by order:

$H_\lambda = T + V$, acting in the gluon Fock space.

$\alpha_\lambda = \frac{g_\lambda^2}{4\pi}$.

Take $\lambda = \lambda_0$ near the binding scale.

Physical value: $\alpha_{\lambda_0} = \alpha_{physical} \sim 1/3$ or $3/4$.

One is free to add

$$\left[1 - \left(\frac{\alpha_{\lambda_0}}{\alpha_{physical}}\right)^2\right] T_\mu \,,$$

where T_μ is a mass term (function of x and $\kappa^\perp$) for gluons.

For the right $\alpha_{\lambda_0} = \alpha_{physical}$ the added term is zero.

$$H_{\lambda_0} \;=\; T_{\lambda_0} + T_\mu + \left[V_{\lambda_0} - \left(\frac{\alpha_{\lambda_0}}{\alpha_{physical}}\right)^2 T_\mu\right] .$$

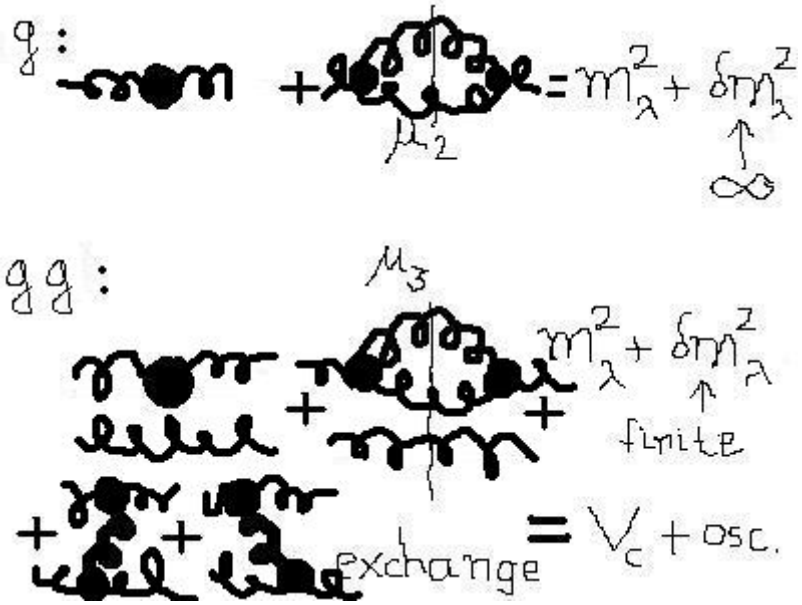


FIG. 1: How binding works.

Transition from the Lepage-Brodsky colinear region to the binding region.

$$
\begin{aligned}
v_{fin}(12,34) \;=\; & -\frac{1}{x_1x_2}\varepsilon_1^{\perp *}\alpha^\perp\,\varepsilon_2^{\perp *}\beta^\perp\,\varepsilon_3^\perp\varepsilon_4^\perp + \frac{1}{x_1x_4}\varepsilon_1^{\perp *}\alpha^\perp\,\varepsilon_4^\perp\beta^\perp\,\varepsilon_2^{\perp *}\varepsilon_3^\perp \\
& + \frac{1}{x_2x_3}\varepsilon_3^\perp\alpha^\perp\,\varepsilon_2^{\perp *}\beta^\perp\,\varepsilon_1^{\perp *}\varepsilon_4^\perp - \frac{1}{x_3x_4}\varepsilon_3^\perp\alpha^\perp\,\varepsilon_4^\perp\beta^\perp\,\varepsilon_1^{\perp *}\varepsilon_2^{\perp *} \\
& - \frac{1}{x_1x_5}\varepsilon_1^{\perp *}\alpha^\perp\,\varepsilon_3^\perp\beta^\perp\,\varepsilon_2^{\perp *}\varepsilon_4^\perp + \frac{1}{x_2x_5}\varepsilon_4^\perp\alpha^\perp\,\varepsilon_2^{\perp *}\beta^\perp\,\varepsilon_1^{\perp *}\varepsilon_3^\perp \\
& + \frac{1}{x_3x_5}\varepsilon_3^\perp\alpha^\perp\,\varepsilon_1^{\perp *}\beta^\perp\,\varepsilon_2^{\perp *}\varepsilon_4^\perp - \frac{1}{x_4x_5}\varepsilon_2^{\perp *}\alpha^\perp\,\varepsilon_4^\perp\beta^\perp\,\varepsilon_1^{\perp *}\varepsilon_3^\perp\,,
\end{aligned}
$$

$$
\begin{aligned}
\alpha^\perp &= x_1\kappa_{34}^\perp - x_3\kappa_{12}^\perp\,, \\
\beta^\perp &= x_2\kappa_{34}^\perp - x_4\kappa_{12}^\perp\,, \\
f_\lambda &= \exp[-(\mathcal{M}_a - \mathcal{M}_b)^2/\lambda^2]\,, \\
\widetilde{\mathcal{F}}_2 &= x_2x_3\frac{1+x_1-x_3}{x_2\alpha^{\perp 2} + x_3\beta^{\perp 2}}\,, \\
ff &= \exp\left[-\frac{x_2\alpha^{\perp 2} + x_3\beta^{\perp 2}}{x_2x_3(x_1-x_3)\lambda^2}\right]\,.
\end{aligned}
$$

9. How binding works

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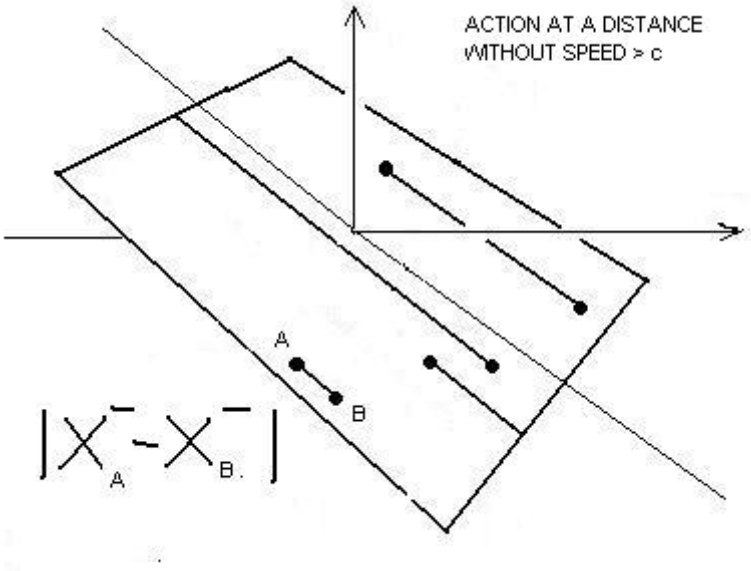


FIG. 2: Interaction at a distance (potential) without tachyons.

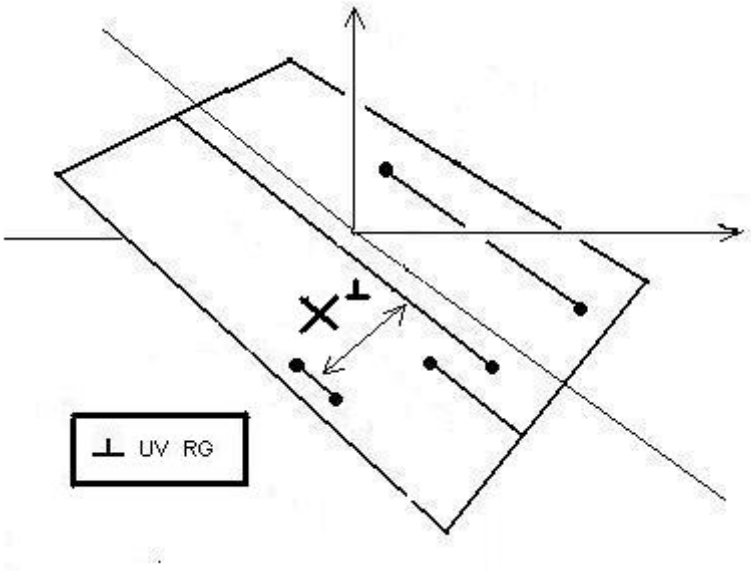
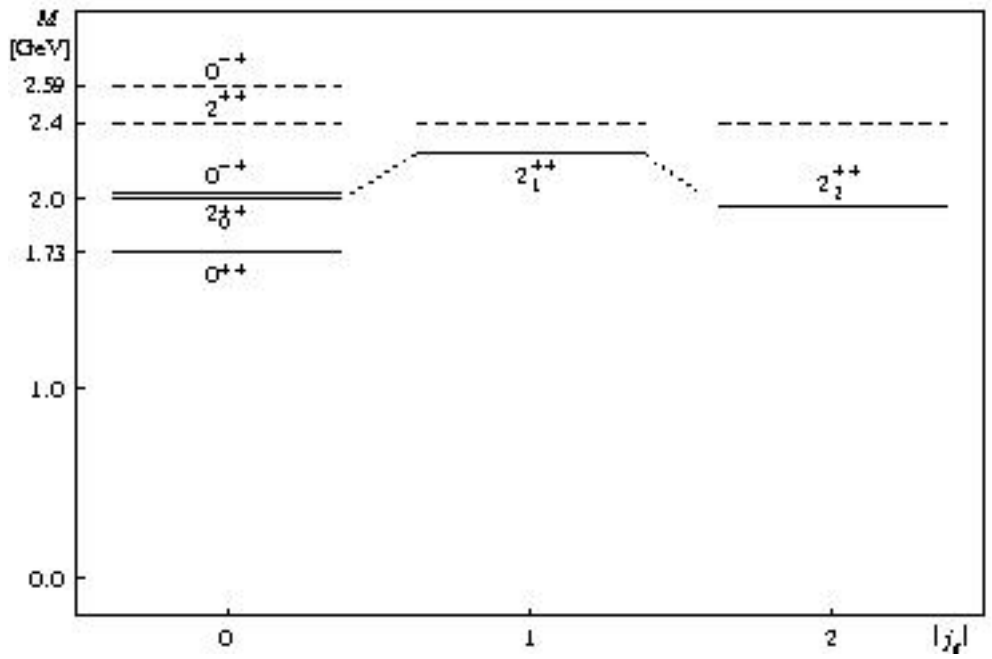


FIG. 3: $UV^{\perp}$ renormalization group (Hamiltonians) for effective particles.



$M(j_z = 0)$ [GeV]	$M(\|j_z\| = 1)$ [GeV]	$M(\|j_z\| = 2)$ [GeV]
1.73	2.25	1.97
2.01	2.63	2.54
2.06	2.64	2.6
2.41	2.66	2.67
2.41	2.85	2.83

Gluonium masses (not energies). Dashed = lattice. Bold from H_λ. 0^{++} matched to lattice 1.73 GeV: $\lambda = 1.92$ GeV, $b = 2\lambda$, $\delta = 0.2$, $\alpha_s = 0.44$. Also, $n_{max} = 9$, $l_{max} = 15$. Results perhaps similar in size to lattice 0^{++}, 2^{++}, 0^{-+}, 0^{*++}, 2^{-+} but only lowest $j_z \neq 0$ are well separated; denser spectrum than on lattice.

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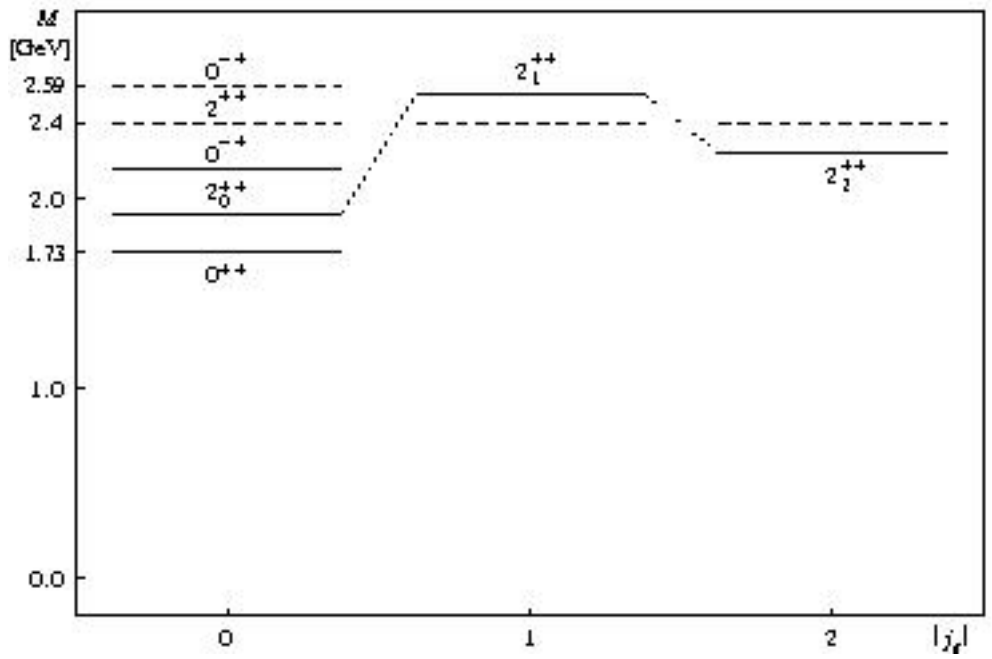
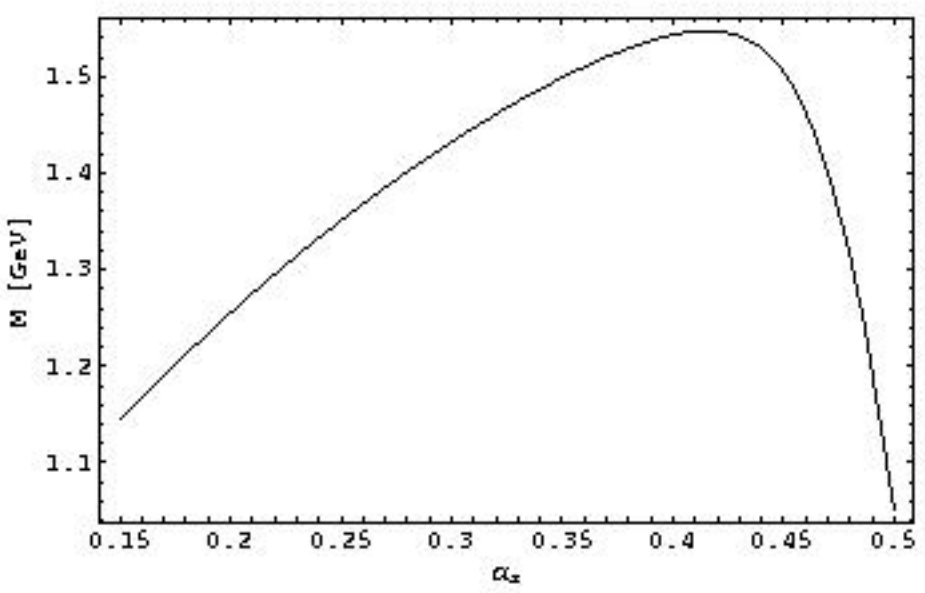


$M(j_z = 0)$ [GeV]	$M(\|j_z\| = 1)$ [GeV]	$M(\|j_z\| = 2)$ [GeV]
1.73	2.56	2.27
1.92	3.01	2.9
2.19	3.03	2.97
2.71	3.05	3.07
2.75	3.27	3.25

The same as before, but changed λ and α_s keeping 0^{++} at 1.73 GeV. Degeneracy for $j = 2$ is not appearing but 2_1^{++} and 2_2^{++} can vary. Here, $\lambda = 2.14$ GeV, $b = 2\lambda$, $\delta = 0.2$, $\alpha_s = 0.47$, $n_{max} = 9$, $l_{max} = 15$. Tomasz.Maslowski@fuw.edu.pl



Typical dependence of the lightest gluonium masses on α.

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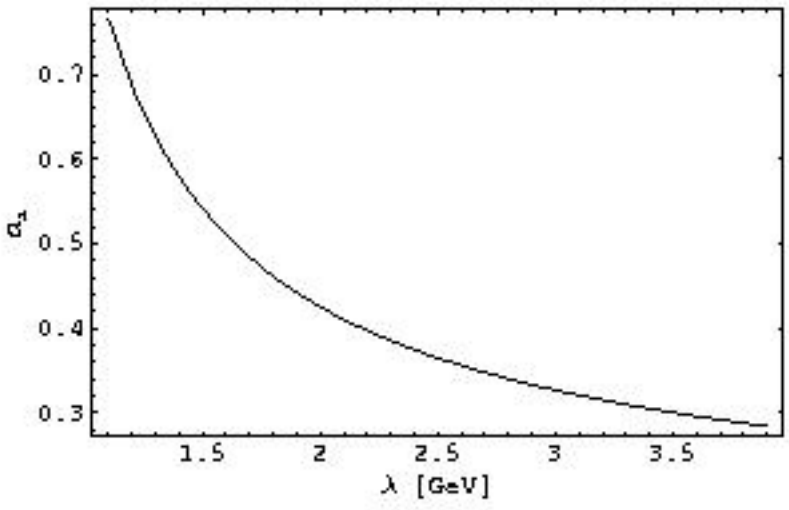

α_s as a perturbative function of λ for $\Lambda_{QCD} = 522$ MeV, no quarks.

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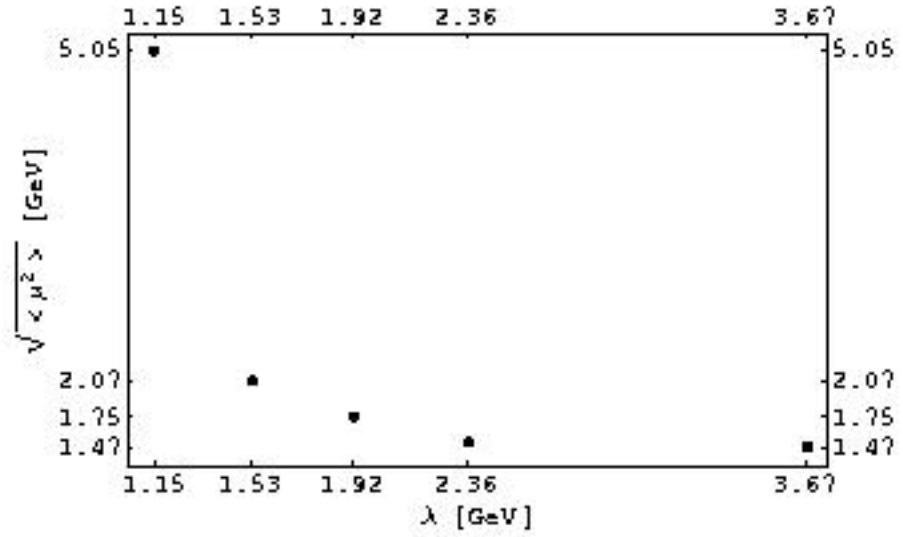
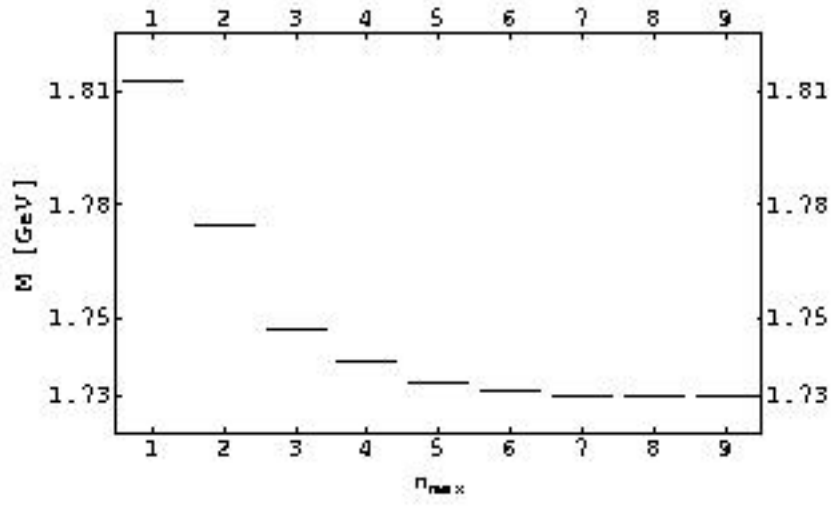


Variation of the expectation value of the effective gluon mass with parameters, assuming that $\alpha_s(\lambda)$ is perturbative, parameters b and δ are constrained so that $M(0^{++}) = 1.73$ Gev. 10 hrs per point per calculation, and several calculations per point to identify central values in the stability region. Tomasz.Maslowski@fuw.edu.pl

$M(0^{++})$	b/λ	δ	$\sqrt{\langle\mu^2\rangle}$	$M(2_0^{++})/M(0^{++})$	$M(2_1^{++})$	$M(2_2^{++})$
1.72	1.0	0.085	1.12	1.19	2.16	1.91
1.72	1.5	0.15	1.46	1.17	2.23	1.96
1.73	2.0	0.2	1.75	1.16	2.25	1.97
1.77	3.0	0.27	2.27	1.16	2.33	2.04



Convergence with radial number, $l_{max} = 15$, and n_{max} varies from 1 to 9. $\lambda = 1.92$ GeV, $b = 2\lambda$, $\delta = 0.2$, $\alpha_s = 0.44$.

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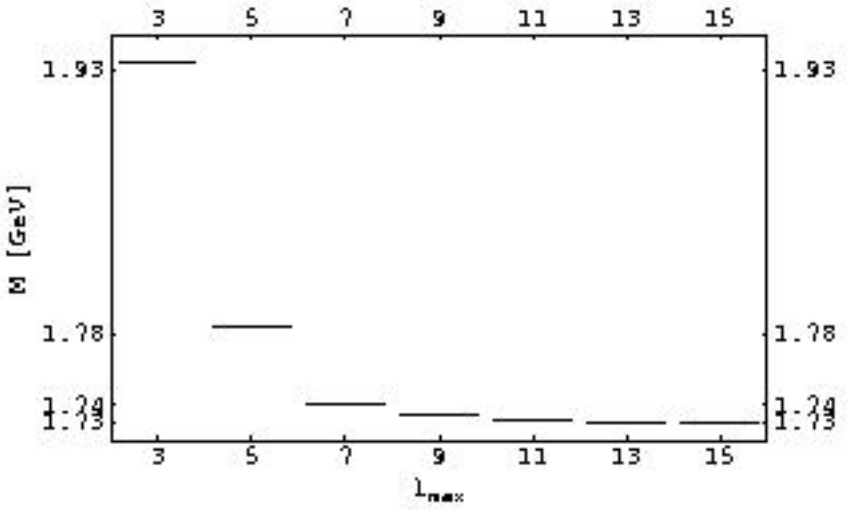

Convergence with angular momentum, $n_{max} = 9$, and l_{max} varies from 3 do 15. $\lambda = 1.92$ GeV, $b = 2\lambda$, $\delta = 0.2$, $\alpha_s = 0.44$.

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QUARKONIUM

All details of the mass ansatz in $|Q_\lambda \bar{Q}_\lambda g_\lambda\rangle$ disappear.

The leading interaction in the resulting equation is spherically symmetric

The boost-invariant wavefunction in the sector $|Q_\lambda \bar{Q}_\lambda\rangle$, satisfies the Shrödinger equation of the form

$$\left[\frac{p^2}{m_q} - \frac{\alpha}{r} + \frac{1}{2}kr^2\right]\,\psi \;=\; \frac{M^2 - 4m_q^2}{4m_q}\,\psi\,.$$

This is how strings may arise.

Are we in the ball park? Answer from **L. Stawikowski**: Yes.

PDG:
$m_c = 1.15 - 1.35$ GeV
$m_b = 4.1 - 4.4$ GeV

$$\left[\frac{p^2}{m_q} - \frac{\alpha}{r} + \frac{1}{2}kr^2\right]\psi = \frac{M^2 - 4m_q^2}{4m_q}\psi\,.$$

Υ data:
3S 9460.3 + 563.0 + 331.9
2S 9460.3 + 563.0
1S 9460.3

The Coulomb and oscillator mix.

For 560 and 330, one needs one negative eigenvalue x.
$m_b = 5017.2$
$\alpha = 0.84130$
$k = 0.03991$ ($\omega = 0.1261$)

m_b determines the scale of M_{1S}, M_{2S}, M_{3S}
α determines $M_{1S} - M_{2S}$
k determines $M_{2S} - M_{3S}$

THEN:
$M_{1P} = 9.9172$ (exp 9.8884)
$M_{2P} = 10.2521$ (exp 10.2519)

THEN:
$m_b \to m_c = 1.624$ to fit $M_{1S} = 3.0969$ for J/Ψ
$M_{2S} = 3.6828$ (exp 3.6861)
$M_{1P} = 3.5113$ (exp 3.4940).

Three aspects:

1. One needs higher order analysis to tackle 10% effects.
2. Numbers were calculated by a freshman.
3. His code in Mathematica runs several hours on his ancient home PC (36 basis states).

CONCLUSION

1. We are in the ball park
(symmetry and numbers)

2. We have an image to hold on to
(self-interaction- mechanism for binding above threshold).

3. We have a theory to refine our image
(full formalism for effective particles in QFT, open for development).

4. We have puzzles to solve
(4th order, fermion seagul terms, condensate terms, chiral symmetry).

5. So far, no vacuum was involved.

6. Contact interactions (δ^3-functions) are under theoretical control.

7. We can expect surprises
(mass and polarization of gluons, values of quark masses, CMS and IMF, structure of hybrids).

(**8.** Infrared limit cycle in the chiral extrapolation $\rightarrow$ baryons.)