Further nonperturbative calculation of the electron's magnetic moment

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Light-Cone QCD and Nonperturbative Hadron Physics
Cairns, 2005

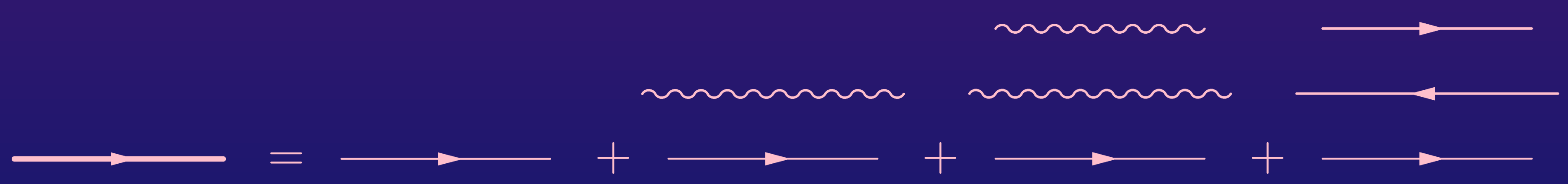
DEVELOP NONPERTURBATIVE METHOD ($\longrightarrow$ QCD)

- ✠ *Regularize and renormalize*
- ✠ *Find induced operators*
- ✠ *Preserve symmetries*
  - ✠✠ *Include Pauli-Villars fields*
  - ✠✠ *Add counterterms*

Further nonperturbative calculation of the electron's magnetic moment

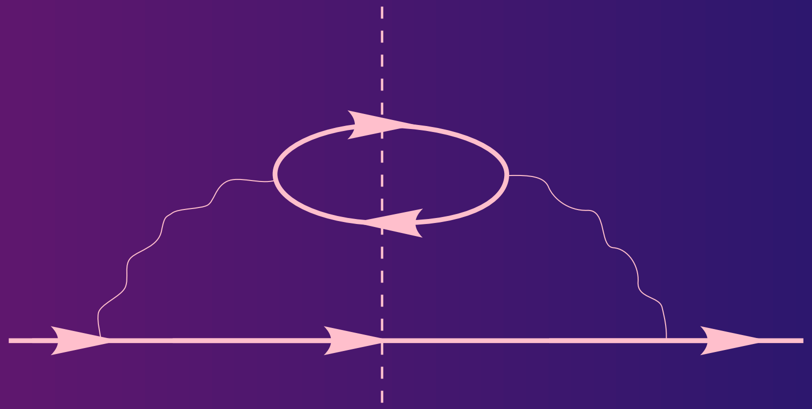
"A nonperturbative calculation of the electron's magnetic moment"
S.J.Brodsky, V.A.Franke, J.R.Hiller, G.McCartor, S.A.Paston, E.V.Prokhvatilov

- *Extend the nonperturbative calculation of the electron's magnetic moment (light-cone quantization in Feynman gauge, (3+1)-dimensions, Pauli-Villars regularization)*
  - *Include states:*
    One electron; one electron — one photon; one electron — two photons; two electrons and one positron states



WHY?

- Testing the improvement in answer
  - Expect to receive the next order contribution to the anomalous electron's magnetic moment — Sommerfield-Petermann term (α^2 term)
- Expansion methodology (include electron-positron loop)



- Further test of singularity prescription
- Develop numerical procedures of solving coupled integral equations

$$\sum_{i=0}^{1}\left(-\frac{1}{4}(-1)^{i}F_{i}^{\mu\nu}\,F_{i,\mu\nu}+(-1)^{i}\overline{\psi}_{i}(i\gamma^{\mu}\partial_{\mu}-m_{i})\psi_{i}+B_{i}\partial_{\mu}A_{i}^{\mu}+\frac{1}{2}B_{i}B_{i}\right)-e\,\overline{\psi}\,\gamma^{\mu}\,\psi\,A_{\mu},$$

where $$A^{\mu}=\sum_{i=0}^{1}A_{i}^{\mu},\qquad \psi=\sum_{i=0}^{1}\psi_{i},\qquad F_{i}^{\mu\nu}=\partial^{\mu}A_{i}^{\nu}-\partial_{\nu}A_{i}^{\mu}$$

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$$\textit{where}\qquad A^\mu=\sum_{i=0}^{1}A_i^\mu,\qquad \psi=\sum_{i=0}^{1}\psi_i,\qquad F_i^{\mu\nu}=\partial^\mu A_i^\nu-\partial_\nu A_i^\mu$$

$$P^-=\sum_{i,s}\int d\underline{p}\frac{m_i^2+p_\perp^2}{p^+}(-1)^i\,b_{i,s}^\dagger(\underline{p})\,b_{i,s}(\underline{p})+\sum_{l,\mu}\int d\underline{k}\frac{m_l^2+k_\perp^2}{k^+}(-1)^l\epsilon^\mu a_l^{\mu\dagger}(\underline{k})\,a_l^\mu(\underline{k})+$$

$$+\sum_{i,j,l,s,\mu}\int d\underline{p}\,d\underline{q}\left\{b_{i,s}^\dagger(\underline{p})\left[b_{j,s}(\underline{q})Q_{ij,2s}^\mu(\underline{p},\underline{q})+b_{j,-s}(\underline{q})R_{ij,-2s}^\mu(\underline{p},\underline{q})\right]a_{l\mu}^\dagger(\underline{q}-\underline{p})+h.c.\right\}$$

$$\sum_{i=0}^{1}\left(-\frac{1}{4}(-1)^i F_i^{\mu\nu}\,F_{i,\mu\nu} + (-1)^i\overline{\psi}_i(i\gamma^\mu\partial_\mu - m_i)\psi_i + B_i\partial_\mu A_i^\mu + \frac{1}{2}B_iB_i\right) - e\,\overline{\psi}\,\gamma^\mu\,\psi\,A_\mu,$$

$$\textit{where} \qquad A^\mu = \sum_{i=0}^{1} A_i^\mu, \qquad \psi = \sum_{i=0}^{1}\psi_i, \qquad F_i^{\mu\nu} = \partial^\mu A_i^\nu - \partial_\nu A_i^\mu$$

$$P^- = \sum_{i,s}\int d\underline{p}\frac{m_i^2+p_\perp^2}{p^+}(-1)^i\,b_{i,s}^\dagger(\underline{p})\,b_{i,s}(\underline{p}) + \sum_{l,\mu}\int d\underline{k}\frac{m_l^2+k_\perp^2}{k^+}(-1)^l\epsilon^\mu a_l^{\mu\dagger}(\underline{k})\,a_l^\mu(\underline{k}) +$$

$$+\sum_{i,j,l,s,\mu}\int d\underline{p}\,d\underline{q}\left\{b_{i,s}^\dagger(\underline{p})\left[b_{j,s}(\underline{q})Q_{ij,2s}^\mu(\underline{p},\underline{q}) + b_{j,-s}(\underline{q})R_{ij,-2s}^\mu(\underline{p},\underline{q})\right]a_{l\mu}^\dagger(\underline{q}-\underline{p}) + h.c.\right\}$$

$$\Phi_+(\underline{P}) = \sum_i z_i b_{i+}^\dagger(\underline{P})|0\rangle + \sum_{ijs}\int d\underline{q}\,f_{ijs}(\underline{q})\,b_{is}^\dagger(\underline{P}-\underline{q})\,a_j^\dagger(\underline{q})|0\rangle +$$

$$+\sum_{ijks}\int d\underline{q_1}d\underline{q_2}\,f_{ijks}(\underline{q_1},\underline{q_2})\frac{1}{\sqrt{1+\delta_{jk}}}\,b_{is}^\dagger(\underline{P}-\underline{q_1}-\underline{q_2})\,a_j^\dagger(\underline{q_1})\,a_k^\dagger(\underline{q_2})|0\rangle + \ldots$$

$$V^0_{ij\pm}(\underline{p},\underline{q}) = \frac{e}{\sqrt{16\pi^3}}\frac{\vec{p}_\perp\cdot\vec{q}_\perp \pm i\vec{p}_\perp\times\vec{q}_\perp + m_i m_j + p^+q^+}{p^+q^+\sqrt{q^+-p^+}}$$

$$V^3_{ij\pm}(\underline{p},\underline{q}) = \frac{-e}{\sqrt{16\pi^3}}\frac{\vec{p}_\perp\cdot\vec{q}_\perp \pm i\vec{p}_\perp\times\vec{q}_\perp + m_i m_j - p^+q^+}{p^+q^+\sqrt{q^+-p^+}}$$

$$V^1_{ij\pm}(\underline{p},\underline{q}) = \frac{e}{\sqrt{16\pi^3}}\frac{p^+\left(q^1 \pm iq^2\right) + q^+\left(p^1 \mp ip^2\right)}{p^+q^+\sqrt{q^+-p^+}}$$

$$V^2_{ij\pm}(\underline{p},\underline{q}) = \frac{e}{\sqrt{16\pi^3}}\frac{p^+\left(q^2 \mp iq^1\right) + q^+\left(p^2 \pm ip^1\right)}{p^+q^+\sqrt{q^+-p^+}}$$

$$U^0_{ij\pm}(\underline{p},\underline{q}) = \frac{\mp e}{\sqrt{16\pi^3}}\frac{m_j\left(p^1 \pm ip^2\right) - m_i\left(q^1 \pm iq^2\right)}{p^+q^+\sqrt{q^+-p^+}}$$

$$U^3_{ij\pm}(\underline{p},\underline{q}) = \frac{\pm e}{\sqrt{16\pi^3}}\frac{m_j\left(p^1 \pm ip^2\right) - m_i\left(q^1 \pm iq^2\right)}{p^+q^+\sqrt{q^+-p^+}}$$

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$$P^+P^-|\Phi_+\rangle = M^2|\Phi_+\rangle$$

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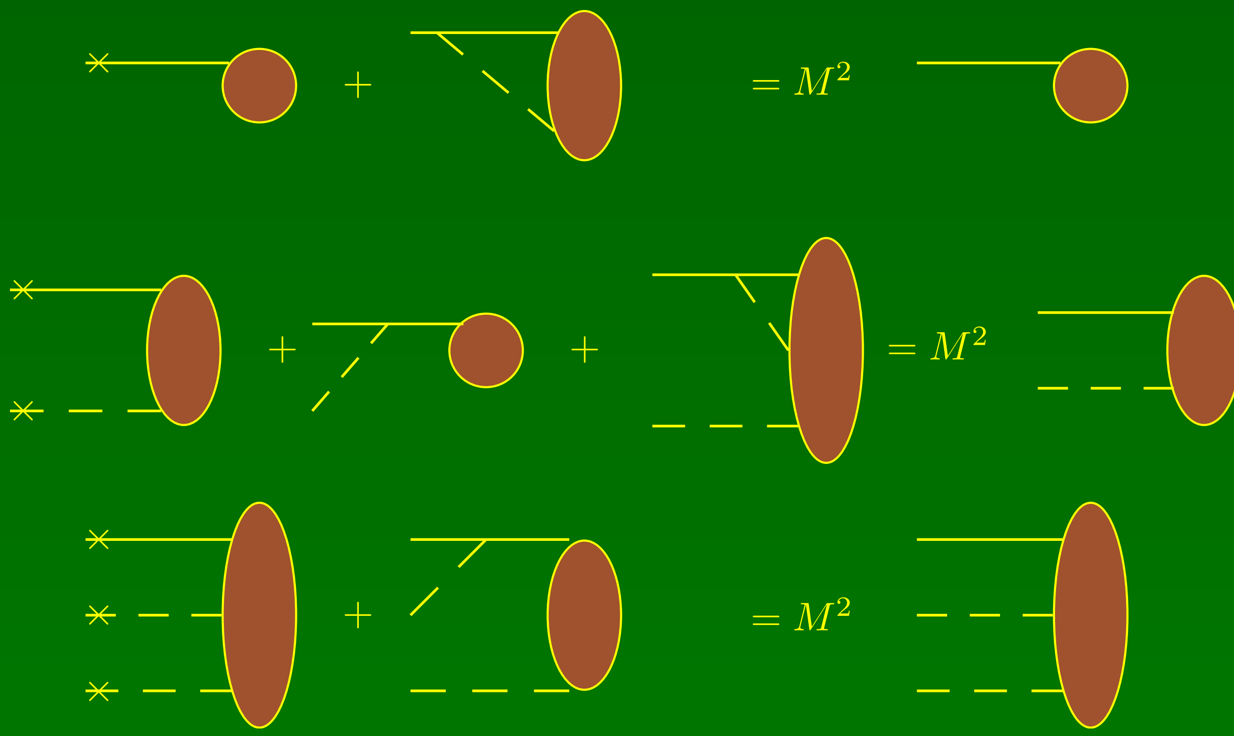

THE EQUATION FOR TWO-PARTICLE AMPLITUDES ONLY:

$$\left[M^2 - \frac{m_i^2 + q_\perp^2}{1-y} - \frac{\mu_j^2 + q_\perp^2}{y}\right] G^\lambda_{ijs}(y, q_\perp) =$$

$$\frac{e^2}{8\pi^3} \sum_l I_{ijl}(y, q_\perp)\, G^\lambda_{ljs}(y, q_\perp) +$$

$$+ \frac{e^2}{4\pi^2} \sum_{n,k,s',\lambda'} \int\limits_0^1 dy' \int\limits_0^{+\infty} q'_\perp dq'_\perp\, J^{(0)\lambda\lambda'}_{ijs,nks'}(y, q_\perp; y', q'_\perp)\, G^{\lambda'}_{nks'}(y', q'_\perp) +$$

$$+ \frac{e^2}{8\pi^3} \sum_{n,k,s',\lambda'} \int\limits_0^{1-y} dy' \int\limits_0^{+\infty} q'_\perp dq'_\perp\, J^{(2)\lambda\lambda'}_{ijs,nks'}(y, q_\perp; y', q'_\perp)\, G^{\lambda'}_{nks'}(y', q'_\perp)$$

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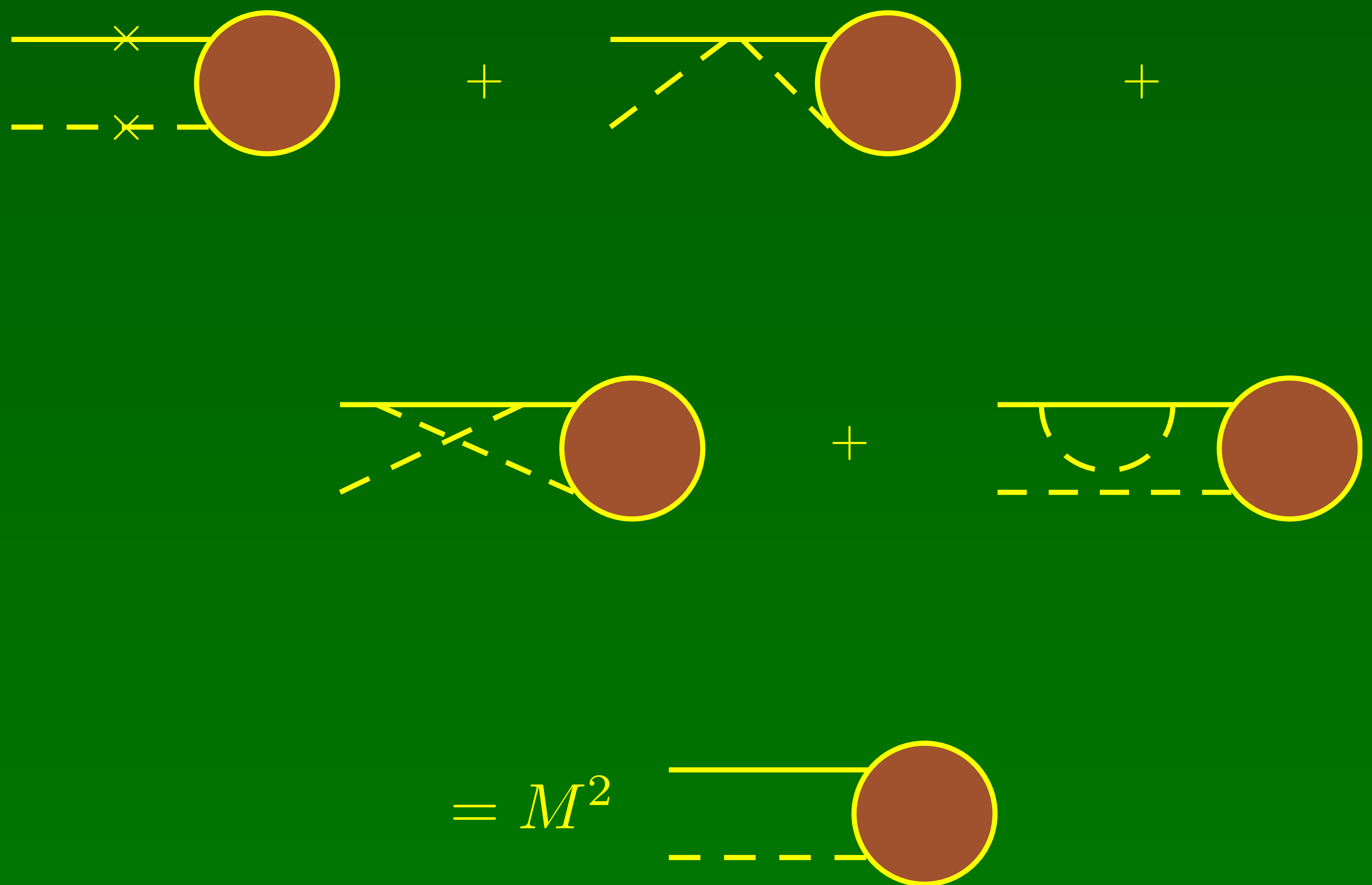

$$J^{(0)(-)[+]}_{ij-\frac{1}{2},nk-\frac{1}{2}}(y,q_\perp;y',q'_\perp) = (-1)^{n+k}\frac{1}{\sqrt{y\,y'}}\sum_l \frac{(-1)^l}{M^2-m_l^2}\cdot\frac{m_l\,q'_\perp\,(m_i - m_l\,(1-y))}{(1-y)(1-y')}$$

$$J^{(2)\,[-]\,(+)}_{ij+\frac{1}{2},nk-\frac{1}{2}}(y,q_\perp;y',q'_\perp) = \sum_l \frac{(-1)^{l+n+k}}{\sqrt{y\,y'}}\int\limits_0^{2\pi} d\varphi'\cdot$$

$$\cdot\frac{-1}{\left[\frac{m_l^2+q_\perp^2+{q'_\perp}^2}{1-y-y'}+\frac{\mu_j^2+q_\perp^2}{y}+\frac{\mu_k^2+{q'_\perp}^2}{y'}-M^2\right]+\frac{2\,q_\perp\,q'_\perp}{1-y-y'}\cos(\varphi-\varphi')}\cdot$$

$$\cdot\left(-\frac{q_\perp e^{-2i\varphi+i\varphi'}\Big(-q_\perp m_n e^{i\varphi'}+q'_\perp e^{i\varphi}(m_l-m_n)\Big)}{(1-y)(1-y')(1-y-y')}\right)$$

$$J^{(2)\,[-]\,(-)}_{ij+\frac{1}{2},nk-\frac{1}{2}}(y,q_\perp;y',q'_\perp) = \sum_l \frac{(-1)^{l+n+k}}{\sqrt{y\,y'}} \int\limits_0^{2\pi} d\varphi' \cdot$$

$$\cdot \frac{-1}{\left[\frac{m_l^2+q_\perp^2+{q'_\perp}^2}{1-y-y'}+\frac{\mu_j^2+q_\perp^2}{y}+\frac{\mu_k^2+{q'_\perp}^2}{y'}-M^2\right]+\frac{2\,q_\perp\,q'_\perp}{1-y-y'}\,\cos(\varphi-\varphi')} \cdot$$

$$\cdot \frac{1}{(1-y')(1-y-y')^2} \cdot$$

$$\cdot \left(e^{-i(\varphi+\varphi')}\Big((q_\perp e^{i\varphi}+q'_\perp e^{i\varphi'})(q_\perp m_n e^{i\varphi'}+q'_\perp e^{i\varphi}(m_n-m_l))\Big) + \right.$$

$$\left. + \frac{\Big(m_l(1-y)-m_i(1-y-y')\Big)\Big(q_\perp q'_\perp + e^{i(\varphi+\varphi')}(m_l m_n + {q'_\perp}^2)\Big)}{1-y} \right)$$

CURRENTLY

- ✠ Numerical solution (first without electron-positron loop)
  - ✠✠ Discretization via Gauss-Legendre quadrature
  - ✠✠ Numerical integration of the singular kernel
  - ✠✠ Solving matrix eigenvalue problem by applying the Lanczos diagonalization method

NEXT STEP

- ✠ Adding electron positron loop — better a nonperturbative approximation to QED
  - ✠✠ Further extension of the method

EXTENSION OF THE METHOD

- ✠ *Currently*
  - ✠✠ 1 *PV electron*, 1 *PV photon*
  - ✠✠ *PV electron mass* $\longrightarrow \infty$
  - ✠✠ *Answer is regularized by PV photon mass*
- ✠ *PV electron breaks gauge invariance*
- ✠ *PV electron mass* $\longrightarrow \infty$ *restores gauge invariance*
- ✠ *Only PV electron regulates loop*
- ✠ *Need new methodology*
  - ✠✠ *Counterterms ?*