Electromagnetic form factors of hadrons

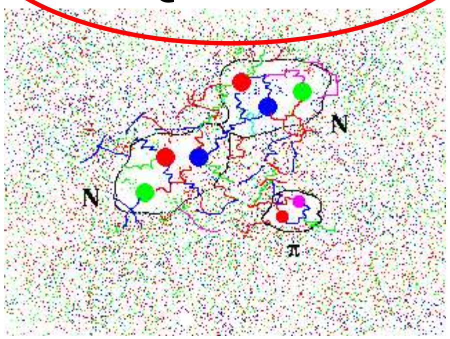
Pieter Maris
pim6@pitt.edu

University of Pittsburgh
and
Kent State University

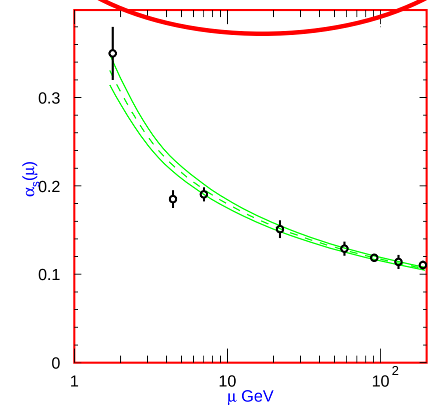
see review article by PM and C.D. Roberts
Dyson–Schwinger equations: a tool for hadron physics
Int. J. Mod. Phys. E12, 297 (2003) [nucl-th/0301049]

Hadron physics

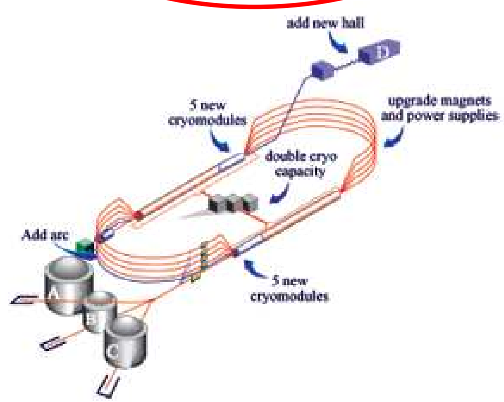
nonperturbative QCD



perturbative QCD

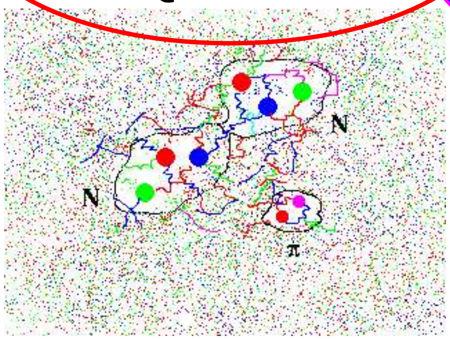


Hadron experiments

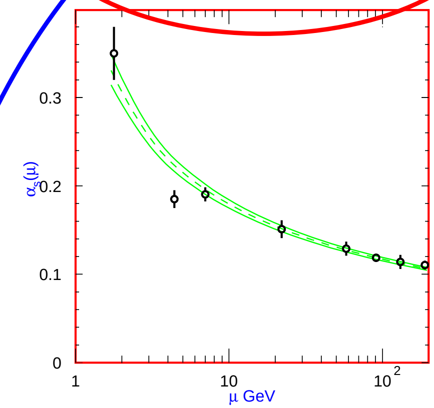


Hadron physics

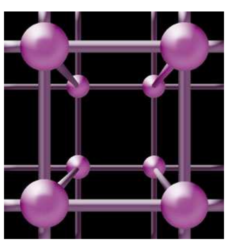
nonperturbative QCD



perturbative QCD

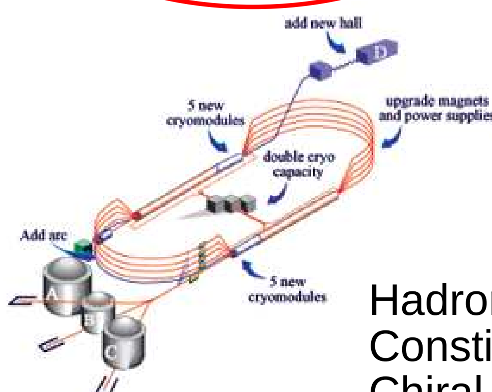


Hadron experiments



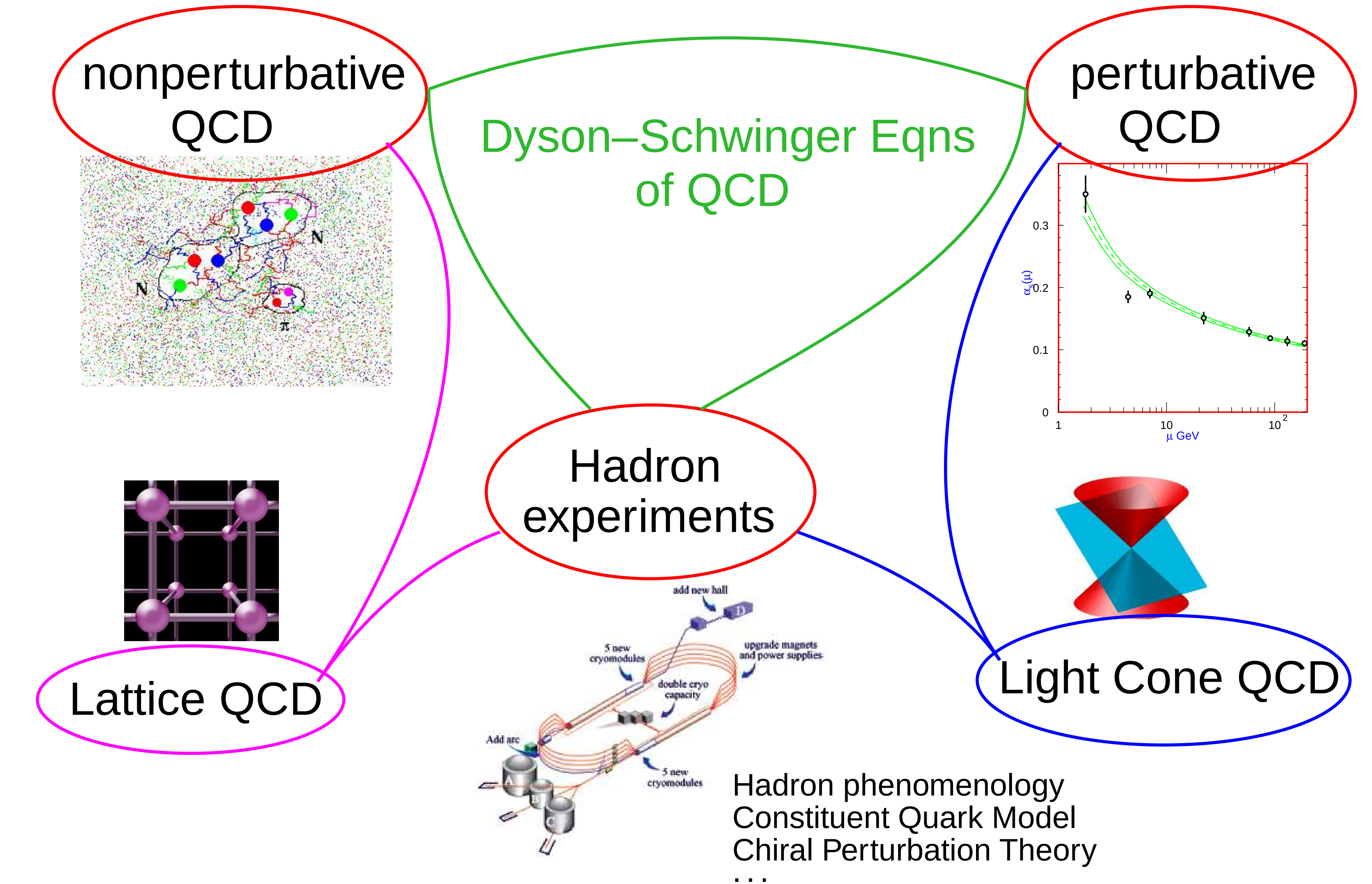
Lattice QCD

Light Cone QCD

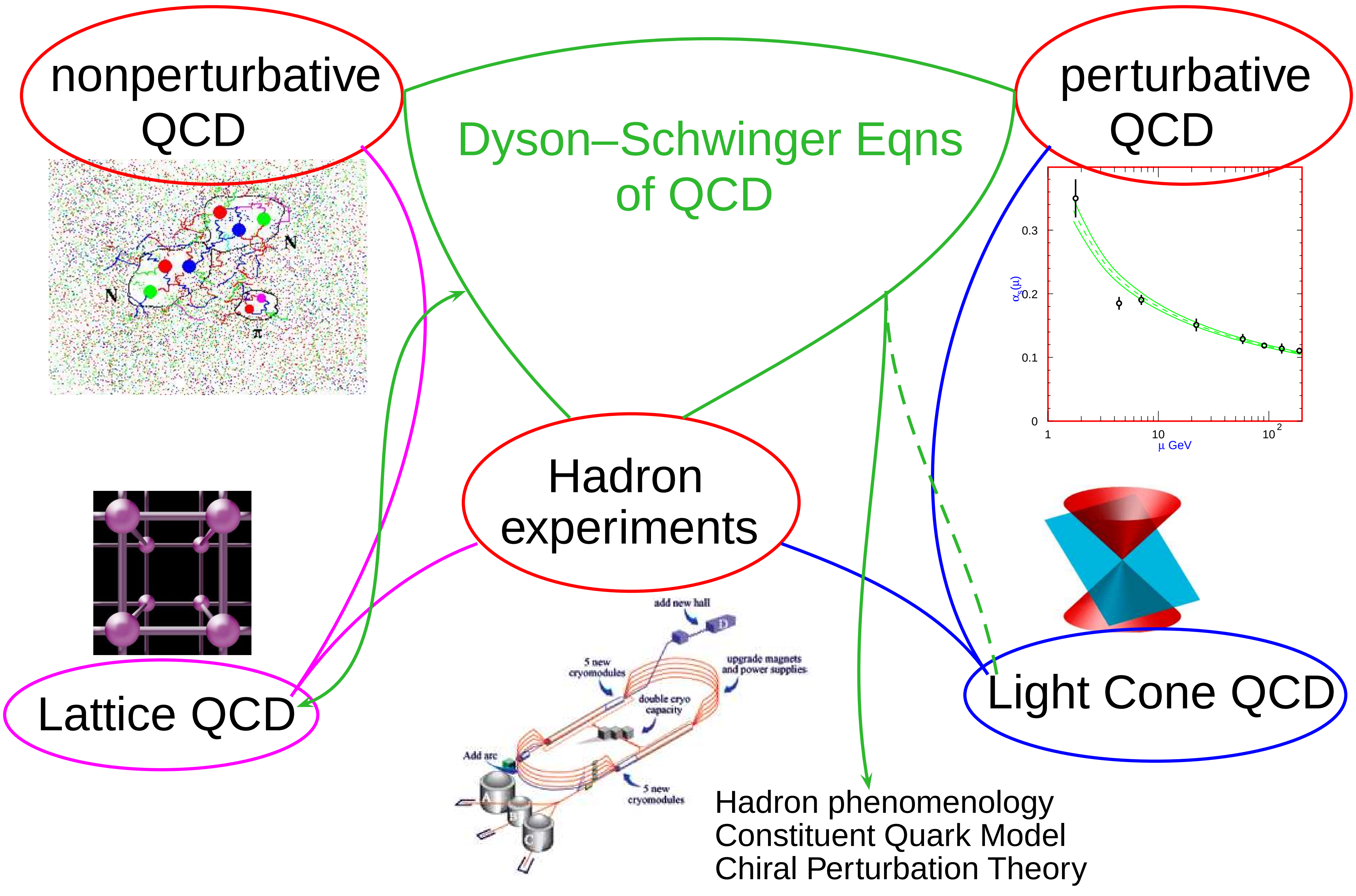


Hadron phenomenology
Constituent Quark Model
Chiral Perturbation Theory
. . .

Hadron physics

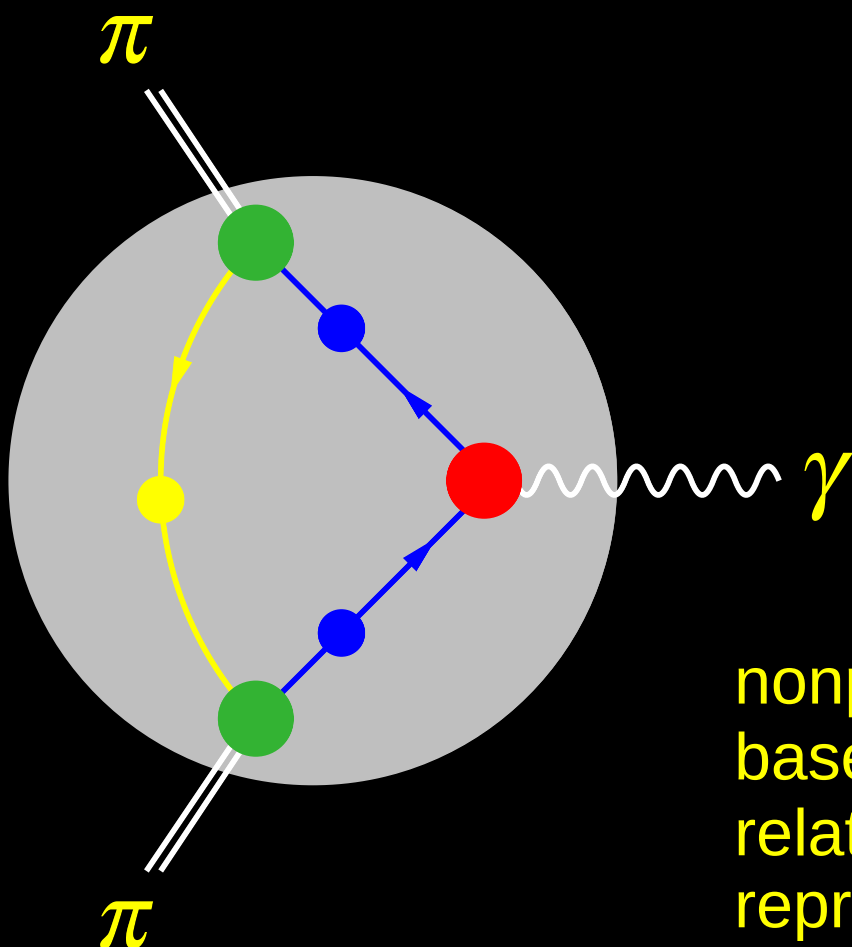


Hadron physics



Pion electromagnetic form factor

Pion-photon coupling: set of Dyson–Schwinger equations for



quark propagator
pion wave function
quark-photon coupling

nonperturbative QFT approach
based on QCD dynamics
relativistic, Poincaré invariant
reproduces pQCD results
in perturbative regime

Quark propagator

Quantum fluctuations modify the quark propagator S_0

$$S_0(p) = \frac{1}{i\not{p} + m_q} \quad \longrightarrow \quad S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

- Perturbation theory, renormalisation group equation: running quark mass

$$M(p^2) \;=\; \frac{\hat{m}_q}{\left(\frac{1}{2}\ln(p^2/\Lambda^2_{QCD})\right)^{\gamma_m}}$$

  with anomalous dimension $\gamma_m = 12/(33 - 2N_f)$

- To any order in perturbation theory: $M(p^2) \propto \hat{m}_q$
- Perturbatively, $M(p^2)$ diverges as $p^2 \downarrow \Lambda^2_{QCD}$

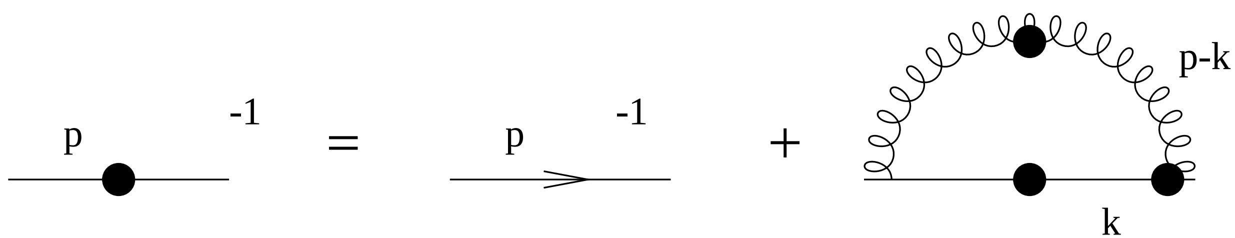
Quark propagator

Quantum fluctuations modify the quark propagator S_0

$$S_0(p) = \frac{1}{i \not p + m_q} \quad \longrightarrow \quad S(p) = \frac{Z(p^2)}{i \not p + M(p^2)}$$

- Nonperturbatively: QCD gap equation

$$S(p)^{-1} \;=\; i \not p \, Z_2 + m_q(\mu)\, Z_4 + \int^\Lambda \frac{d^4k}{(2\pi)^4}\, g^2 D_{\mu\nu}(q)\gamma_\mu \tfrac{\lambda^a}{2} S(k)\Gamma_\nu(k,p)\tfrac{\lambda^a}{2}$$



- Nonlinear integral equation for the quark propagator
- Allows for nontrivial solution $M(p^2) \neq 0$ even if $m_q(\mu) = 0$

Nonperturbative quark propagator

- QCD gap eqn has indeed a solution $M(p^2) \neq 0$ for $m_q = 0$

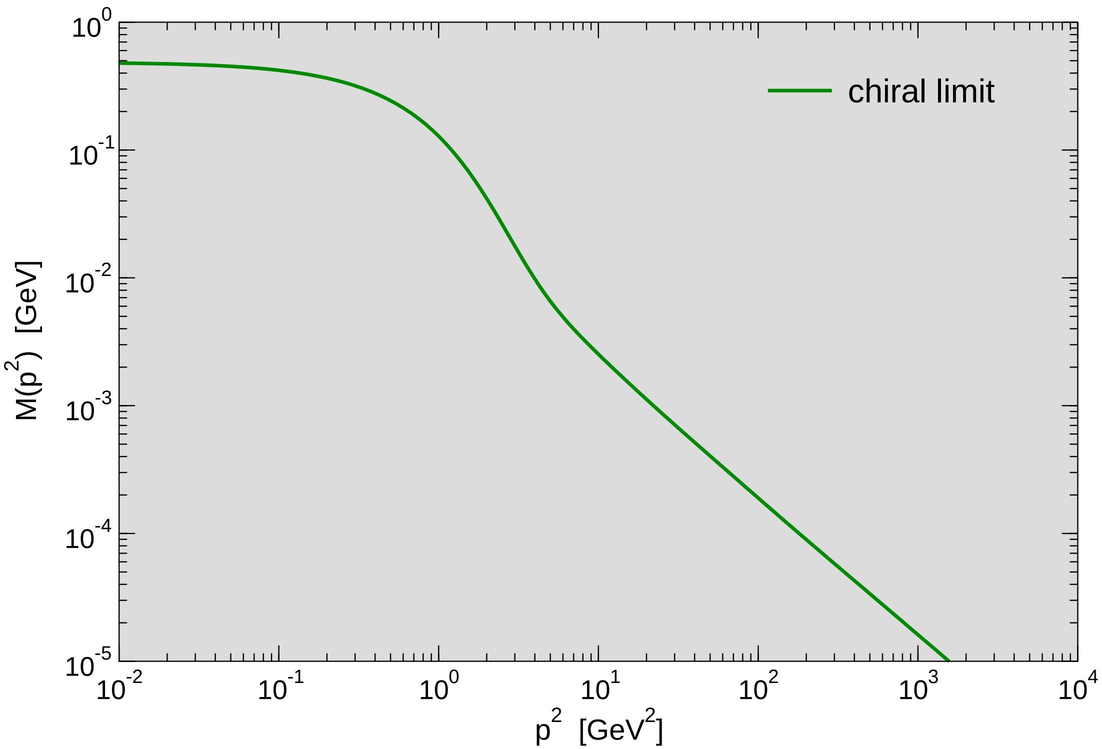


Fig. adapted from PM & C.D. Roberts, PRC56, 3369 (1997) [nucl-th/9708029]

$$M(p^2) \xrightarrow{\text{large } p^2} \frac{2\pi^2\gamma_m}{3} \frac{-\langle \bar{q}q \rangle^0}{p^2 \left(\frac{1}{2} \ln(p^2/\Lambda^2_{QCD}) \right)^{1-\gamma_m}}$$

Nonperturbative quark propagator

- QCD gap eqn has indeed a solution $M(p^2) \neq 0$ for $m_q = 0$

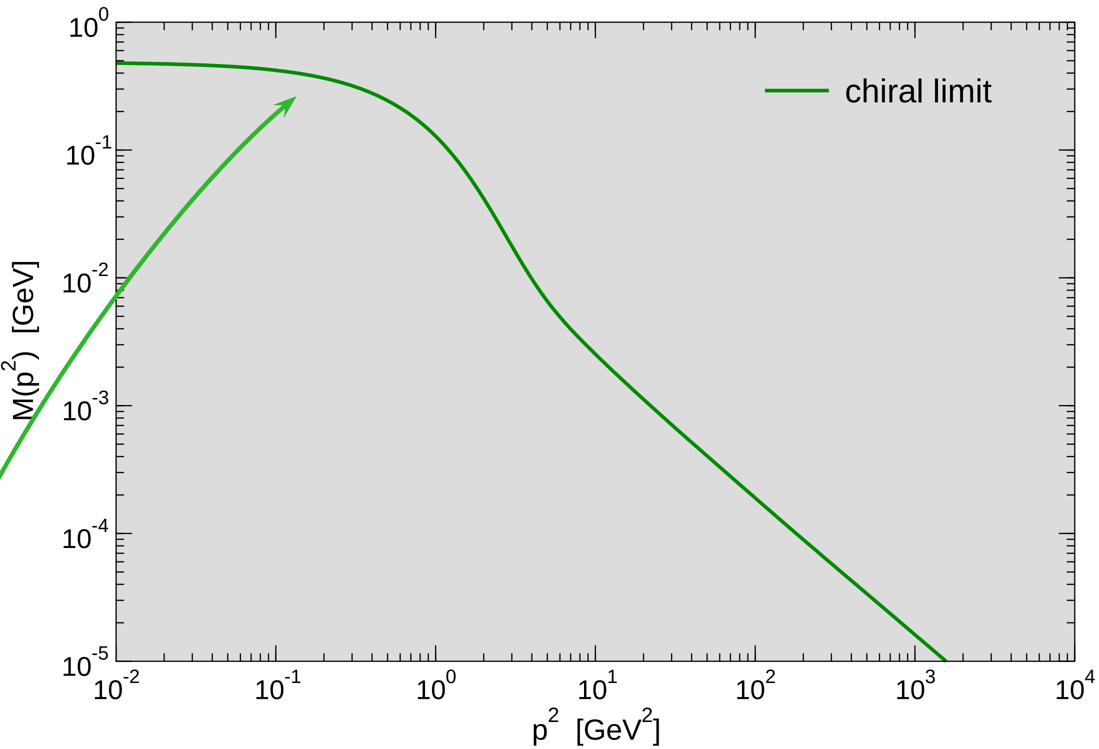


Fig. adapted from PM & C.D. Roberts, PRC56, 3369 (1997) [nucl-th/9708029]

- Details at low momenta depend on details of the low-momentum behavior of QCD running coupling $\alpha_s(Q^2)$

Nonperturbative quark propagator

- QCD gap eqn has indeed a solution $M(p^2) \neq 0$ for $m_q = 0$

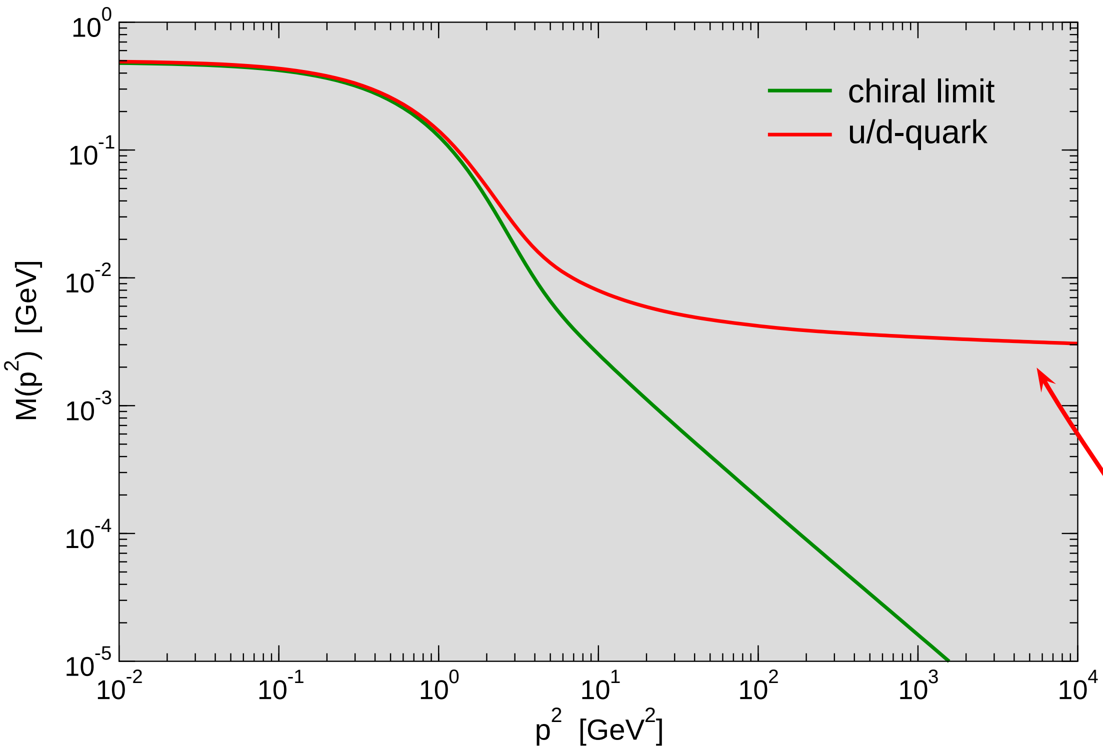


Fig. adapted from PM & C.D. Roberts, PRC56, 3369 (1997) [nucl-th/9708029]

- If $m_q \neq 0$, $M(p^2)$ describes the evolution from a current quark mass at high energies

Nonperturbative quark propagator

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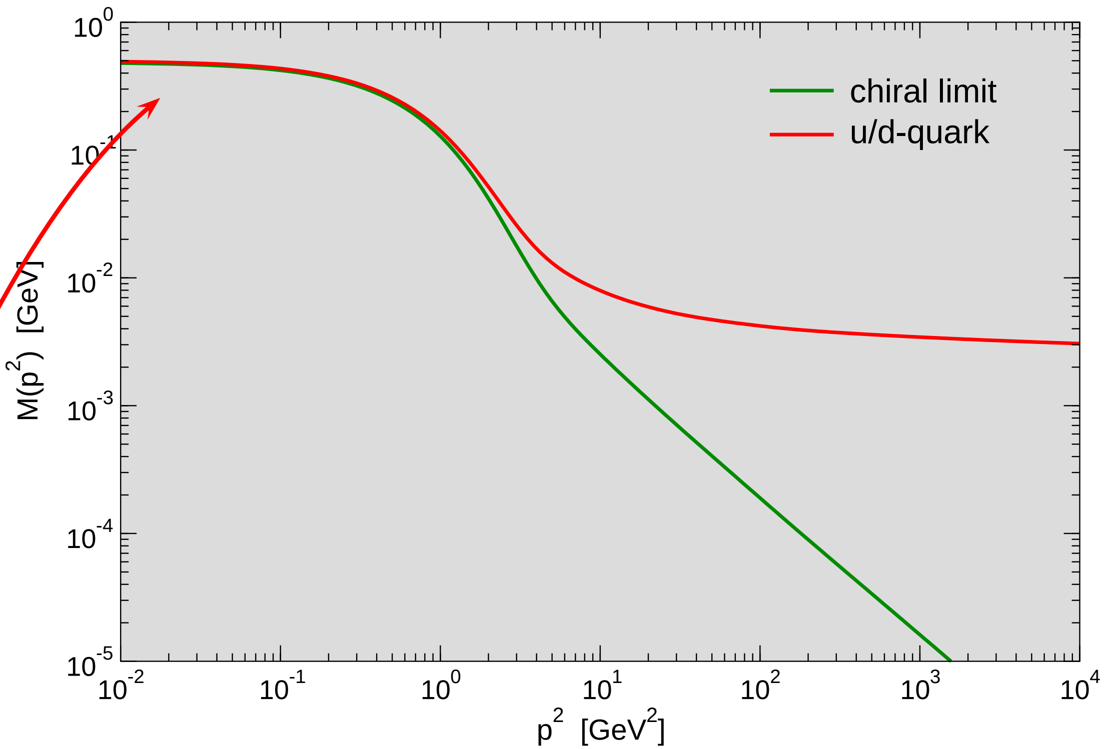


Fig. adapted from PM & C.D. Roberts, PRC56, 3369 (1997) [nucl-th/9708029]

- If $m_q \neq 0$, $M(p^2)$ describes the evolution from a current quark mass at high energies to a constituent-like quark mass at low energies

Nonperturbative quark propagator

- QCD gap eqn has indeed a solution $M(p^2) \neq 0$ for $m_q = 0$

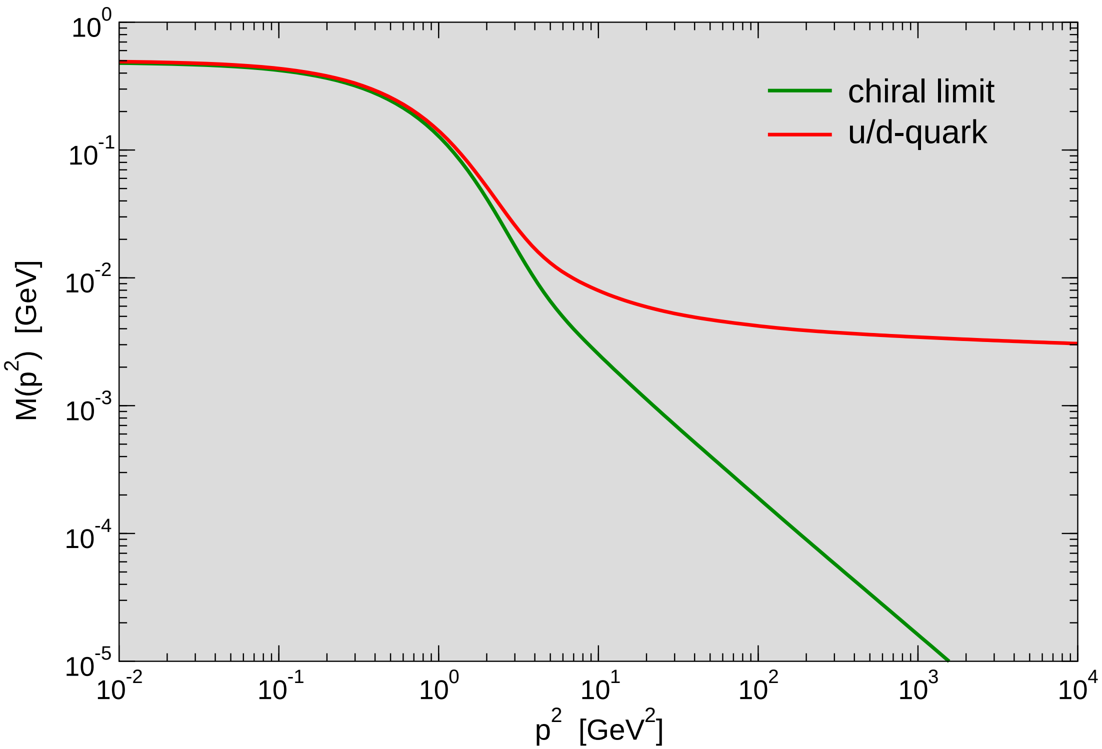
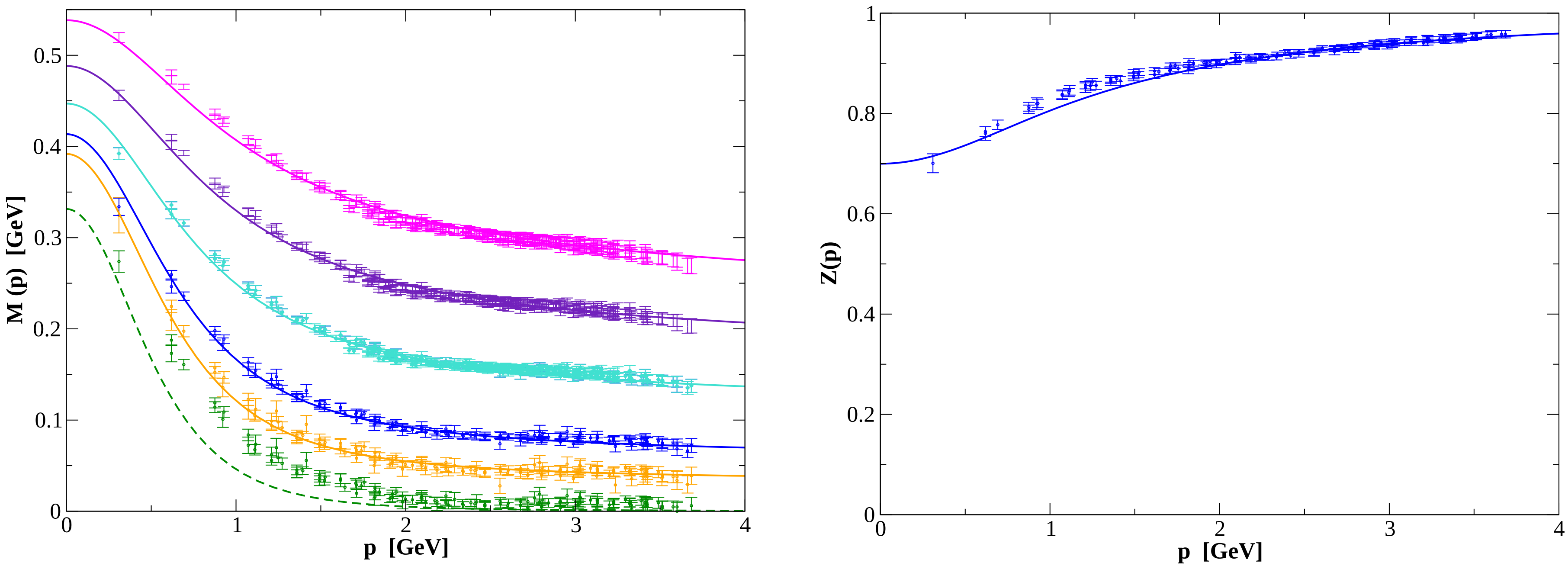


Fig. adapted from PM & C.D. Roberts, PRC56, 3369 (1997) [nucl-th/9708029]

$M(p^2)$ connects hadronic physics with perturbative QCD

Nonperturbative quark propagator

Longstanding predictions from solution of the QCD gap eqn have been confirmed by lattice simulations of QCD

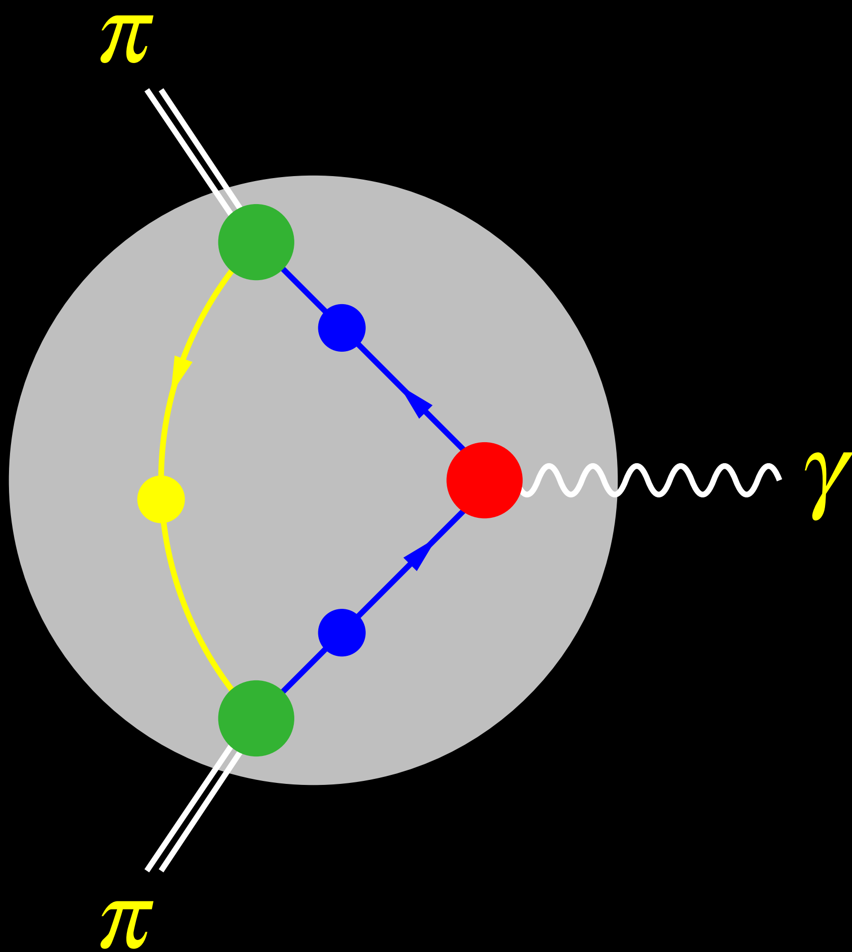


DSE soln from Bhagwat, Pichowsky, Roberts, Tandy,
Phys.Rev.C68, 015203 (2003) [nucl-th/0304003]

Lattice data from Bowman, Heller, Leinweber, Williams,
Nucl.Phys.Proc.Suppl.119, 323 (2003) [hep-lat/0209129]

Pion electromagnetic form factor

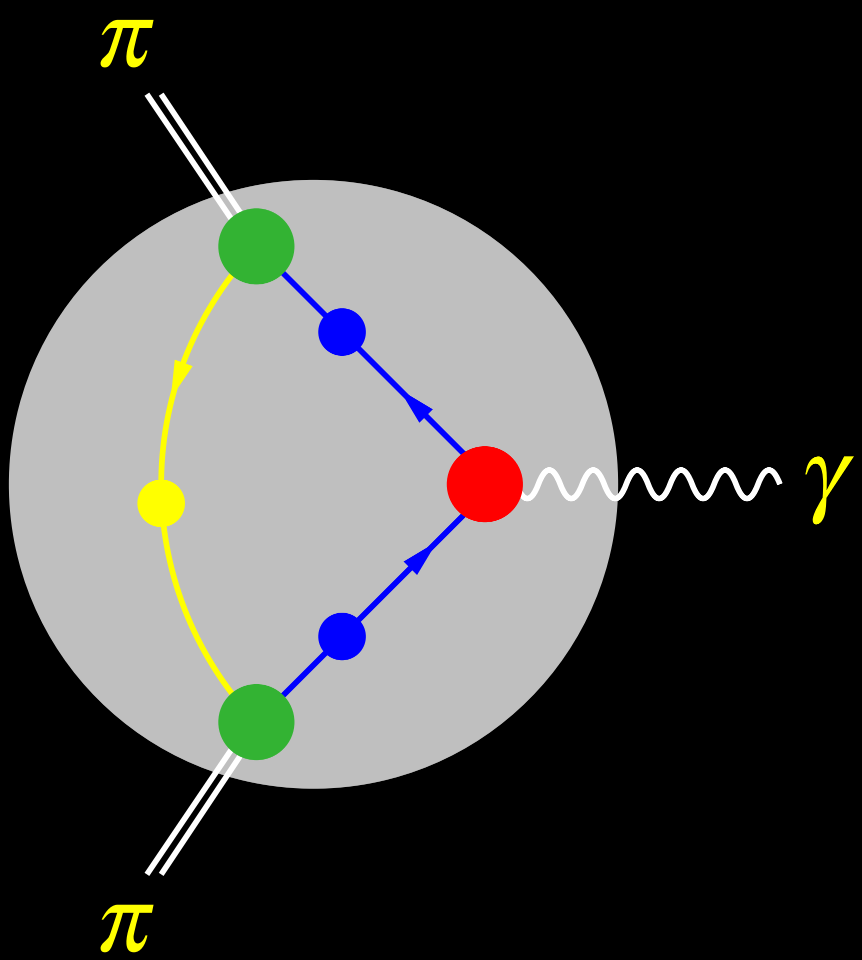
Pion-photon coupling



quark propagator

Pion electromagnetic form factor

Pion-photon coupling

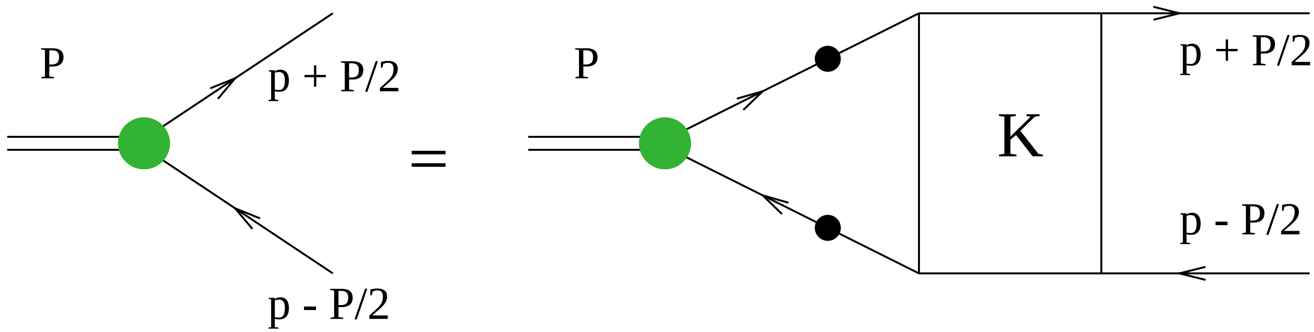


pion wave function

Mesons

Quark-antiquark bound states satisfy homogeneous
Bethe–Salpeter equation at mass pole $P^2 = -m^2_{\text{meson}}$

$$\Gamma_H(p;P) \;=\; \int \frac{d^4k}{(2\pi)^4}\, K(p,k;P)\; S(k_+)\,\Gamma_H(k;P)\,S(k_-)$$



- Kernel $K(p,k;P)$: 1PI quark-antiquark scattering kernel
- Quark propagators nonperturbatively dressed
- Euclidean formulation: integration variable Euclidean quark propagator arguments $k_\pm = k \pm P/2$ complex

Pion

- Quark-antiquark bound state
- Goldstone bosons associated with dynamical chiral symmetry breaking
- Axial-vector Ward–Takahashi identity in the chiral limit relates pseudoscalar BSA

$$\Gamma_\pi(k;P) = \gamma_5\big[iE_\pi(k;P) + \not{P} F_\pi(k;P) + \not{k}\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\big]$$

to inverse quark propagator $S^{-1}(k) = i\not{k} A(k^2) + B(k^2)$

$$f_\pi\, E_\pi(k;0) = B(k^2)$$

$\Longrightarrow$ existence of a massless, pseudoscalar bound state

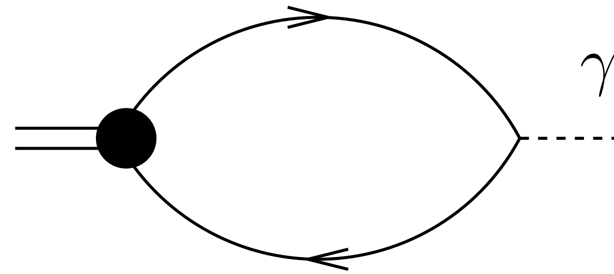
Pions are (pseudo-)Goldstone bosons of QCD

Exact pseudoscalar mass relation

- In general, AV-WTI relates different spin projections of the BS wavefunction $\chi_H(k;P) = S(k_+)\Gamma_H(k;P)S(k_-)$

$$f_{PV} = Z_2 \int_k \mathrm{Tr}[\chi_H(k;P)\gamma_5\gamma_\mu] \frac{P_\mu}{m_H^2}$$

$$r_{PS}(\mu) = Z_4 \int_k \mathrm{Tr}[\chi_H(k;P)\gamma_5]$$



$$f_{PV}\; m_H^2 \;\; = \;\; 2\, m_q(\mu)\; r_{PS}(\mu)$$

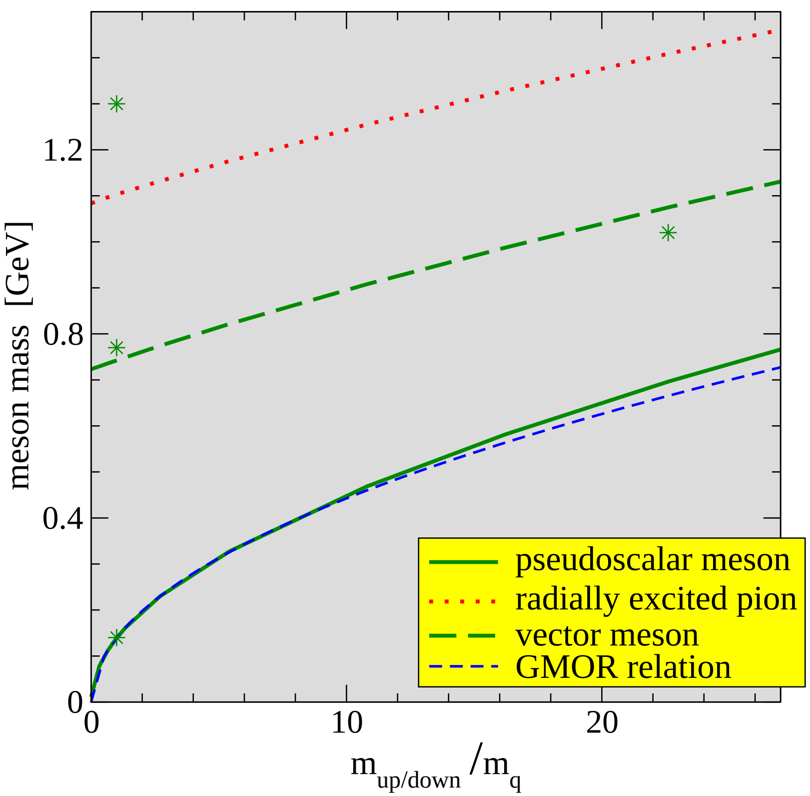
PM, Roberts, Tandy, PLB420, 267 (1998) [nucl-th/9707003]

- Exact statement, with observable consequences for both light mesons and heavy mesons

Ivanov, Kalinovsky, Roberts, PRD60, 034018 (1999) [nucl-th/9812063]

Consequences of pseudoscalar mass relation

$$f_{PV}\, m_H^2 \;=\; 2\, m_q(\mu)\, r_{PS}(\mu)$$



In chiral limit

$$r_{PS}(\mu) \longrightarrow \frac{Z_4}{f_\pi} \int_k \mathrm{Tr}[S_{m_q=0}(k)]$$

and thus

$$f_\pi^2\, m_\pi^2 = 2\, m_q(\mu)\, \langle \bar{q} q \rangle_\mu$$

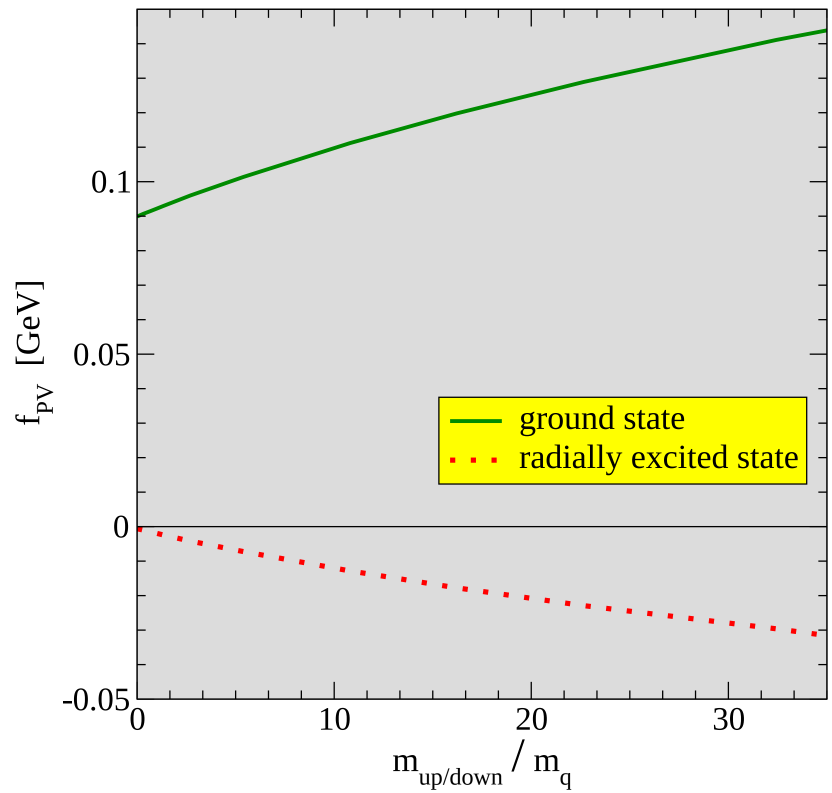
Gell-Mann–Oakes–Renner relation
Phys. Rev. **175**, 2195 (1968)

Consequences of pseudoscalar mass relation

$$f_{PV}\, m_H^2 \;=\; 2\, m_q(\mu)\, r_{PS}(\mu)$$

Ground state pion:
mass vanishes in chiral limit

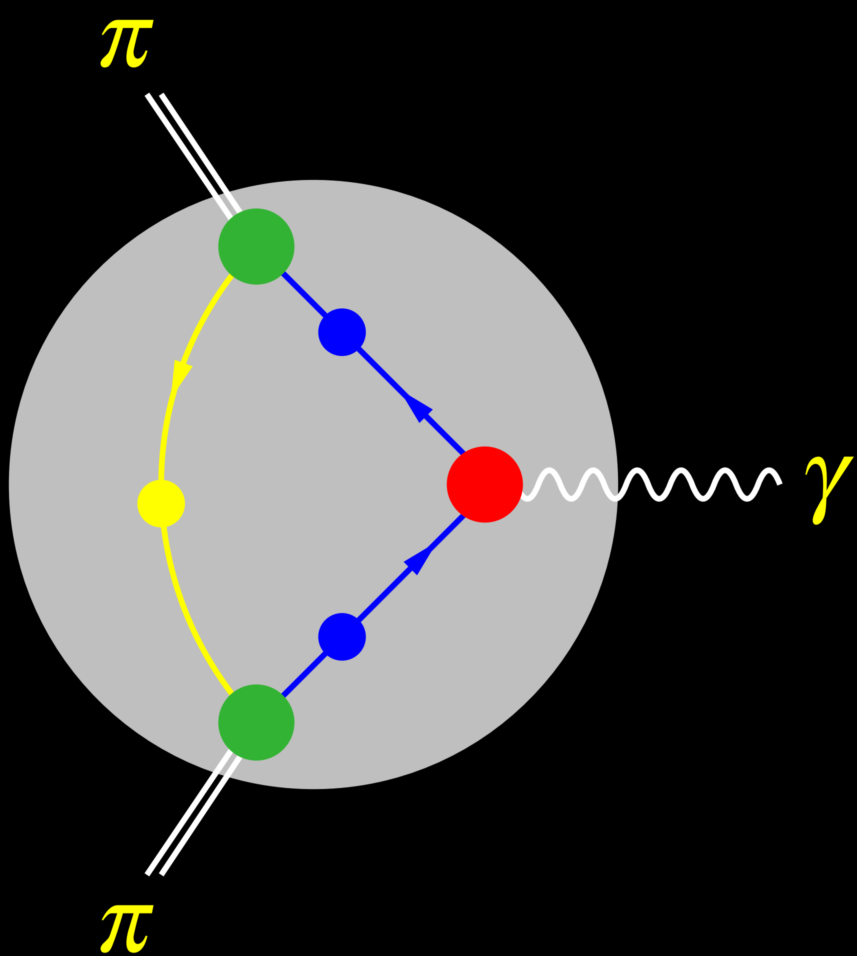
Excited pion:
mass nonzero in chiral limit
but decay constant vanishes



Höll, Krassnigg, Roberts, PRC70, 042203 (2004) [nucl-th/0406030]

Pion electromagnetic form factor

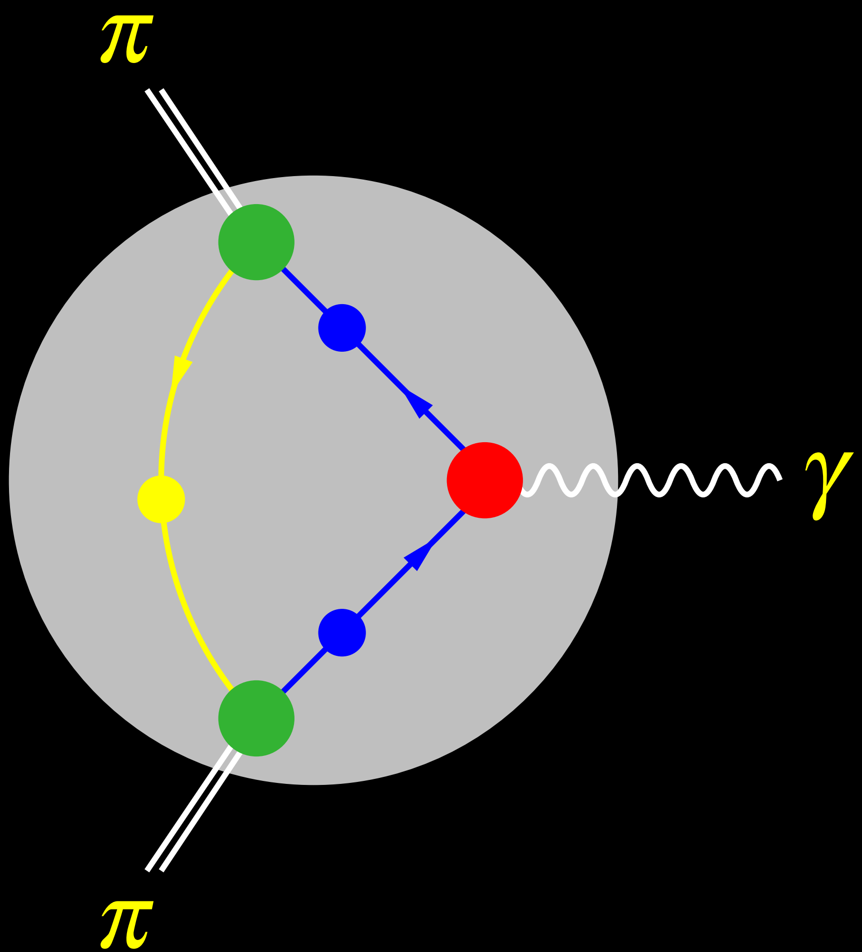
Pion-photon coupling



pion wave function

Pion electromagnetic form factor

Pion-photon coupling



quark-photon coupling

Quark-photon coupling

- Electromagnetic current conservation

$$\partial_\mu J^\mu = 0$$

- Vector Ward–Takahashi identity

$$i\,P_\mu\,\Gamma_\mu(q_+,q_-;P) = S^{-1}(q+P/2) - S^{-1}(q-P/2)$$

- Differential Ward–Takahashi identity

$$i\,\Gamma_\mu(q,q;0) = \frac{\partial}{\partial q_\mu} S^{-1}(q)$$

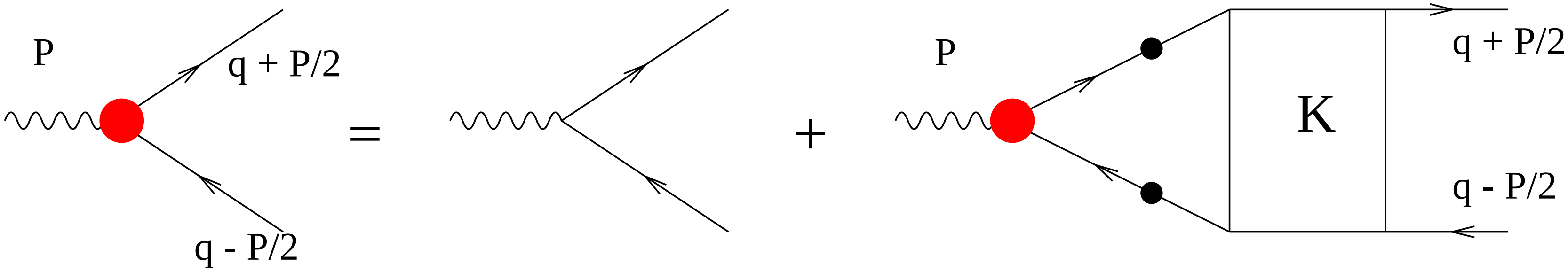
$\Longrightarrow$ longitudinal components uniquely determined
$\Longrightarrow$ constraints on transverse components at $P=0$ only

Quark-photon coupling

- Electromagnetic current conservation

$$\partial_\mu J^\mu \;=\; 0$$

- Inhomogeneous Bethe–Salpeter equation for the quark-photon vertex



guarantees current conservation

- Same kernel K as meson bound state eqn
- Solve for $$\Gamma^T_\mu(k_+, k_-; P) \;=\; \sum_{i=0}^{8} T^i_\mu(k, P) F_i(k^2, k \cdot P; P^2)$$

Quark-photon coupling

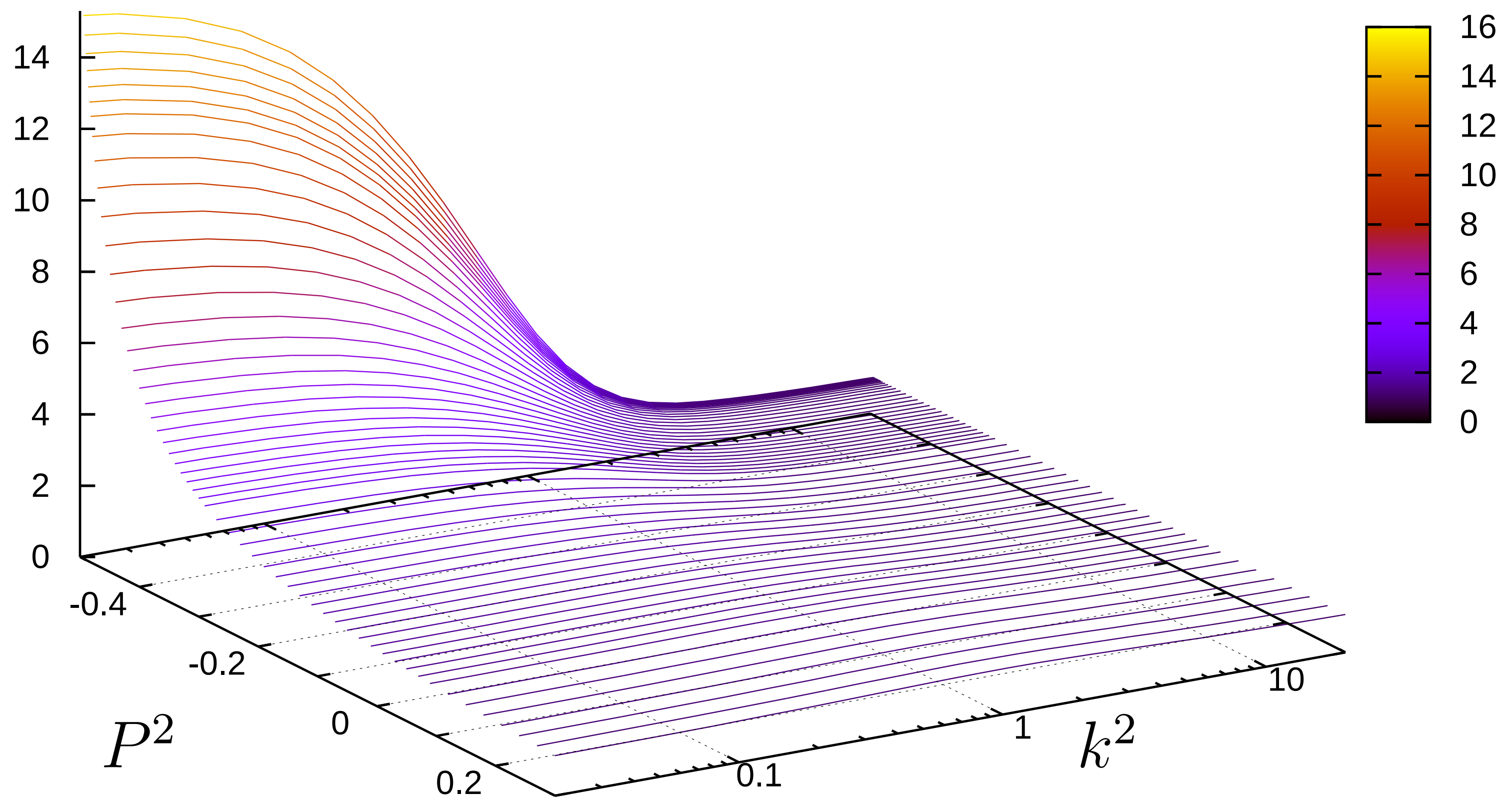
Vector mesons appear as poles in quark-photon vertex

- Homogeneous vector BSE has solution at $P^2 = -m_{\rho,\omega,\phi,\ldots}$
- Inhomogeneous quark-photon vertex BSE has poles at corresponding values of P^2

$$\Gamma_\mu(k;P) \;\; \simeq \;\; \Gamma_\mu^{\text{Reg}}(k;P) + \frac{f_\rho \, m_\rho}{P^2 + m_\rho^2} \, \Gamma_\mu^\rho(k;P)$$

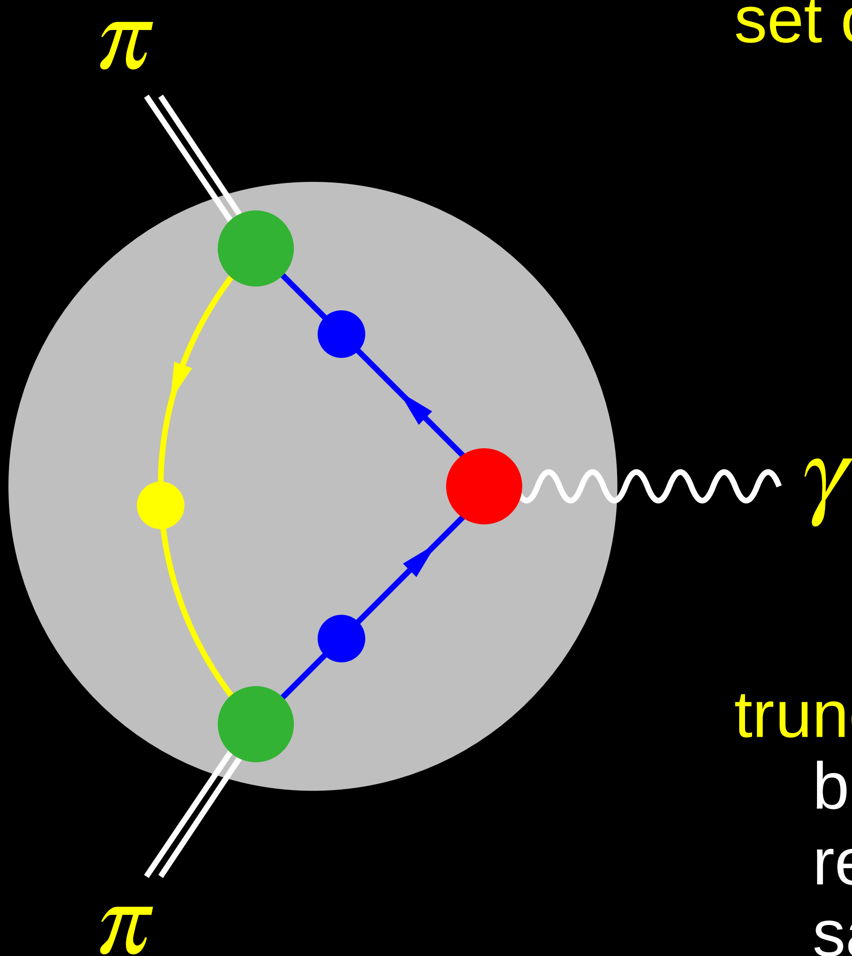
Quark-photon coupling

Vector mesons appear as poles in quark-photon vertex



$F_1^0(k^2; P^2)$ zeroth Cheb. mom. of canonical Dirac structure γ_μ^T

Pion electromagnetic form factor



set of Dyson–Schwinger equations

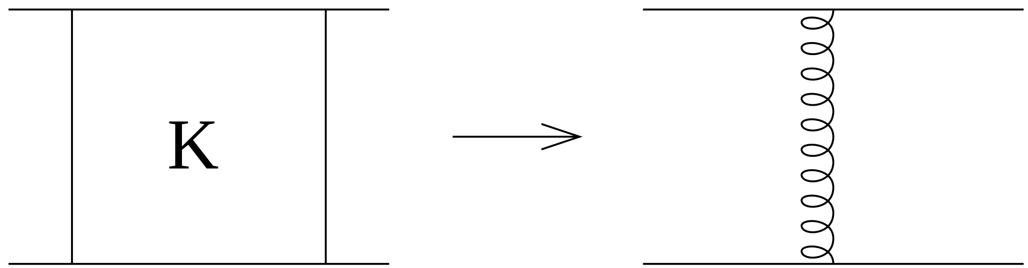
quark propagator
pion wave function
quark-photon coupling

truncation of DSEs

based on QCD dynamics
relativistic, Poincaré invariant
satisfies relevant Ward identities
reduces to pQCD
in perturbative regime

Truncation

- Short-range part of the interaction (i.e. high-Q^2 behavior) is fixed by asymptotic freedom: one-gluon exchange

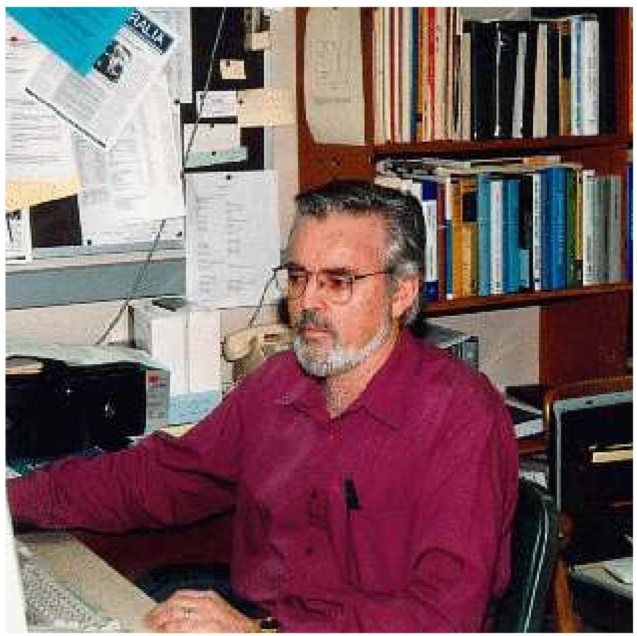


  - first term in a systematic expansion that respects relevant Ward–Takahashi identities
  - corrections are small in the pseudoscalar and vector channels

Bender, Roberts, & Smekal, PLB380, 7 (1996) [nucl-th/9602012];
Bender, Detmold, Roberts & Thomas, PRC65, 065203 (2002) [nucl-th/0202082];
Bhagwat, Höll, Krassnigg, Roberts & Tandy, PRC70, 035205 (2004) [nucl-th/0403012]

Truncation

- Short-range part of the interaction (i.e. high-Q^2 behavior) is fixed by asymptotic freedom: one-gluon exchange
- Model for the long-range part (low-Q^2) of the interaction



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Truncation

- Short-range part of the interaction (i.e. high-Q^2 behavior) is fixed by asymptotic freedom: one-gluon exchange
- Model for the long-range part (low-Q^2) of the interaction
- Single model parameter
  - fixed to give vacuum condensate (energy gap)

$$\langle \bar{q}q \rangle \;=\; -(240\text{MeV})^3$$

PM & P.C. Tandy, PRC60, 055214 (1999) [nucl-th/9905056]

  - IR: regulated $1/q^4$ behavior related to linearly rising potential between static quarks

Truncation

- Short-range part of the interaction (i.e. high-Q^2 behavior) is fixed by asymptotic freedom: one-gluon exchange
- Model for the long-range part (low-Q^2) of the interaction
- Single model parameter
- Apply to a range of meson observables and compare with experiments

Intermezzo: Frame (in)dependence

- In principle,
  all frames are equivalent
- In practice,
  some frames are easier than others . . .
- Form factor

$$\Lambda_\mu(P,Q) \;=\; 2\,P_\mu\,F_\pi(Q^2) \quad = \quad N_c \int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\big[\bar{\Gamma}^\pi\,S\,i\Gamma_\mu\,S\,\Gamma^\pi\,S\big]$$

- Most convenient frame
  - incoming photon $Q = (0, q, 0, 0)$
  - pions $P \pm Q/2 = (iE, \pm q/2, 0, 0)$
  - both pions are moving
- No matter what frame is used, at least one pion is moving

Intermezzo: Frame (in)dependence

- In principle,
  all frames are equivalent
- In practice,
  some frames are easier than others ...
- Form factor

$$\Lambda_\mu(P,Q) \;=\; 2\,P_\mu\;F_\pi(Q^2) \quad = \quad N_c \int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\big[\bar{\Gamma}^\pi\,S\,i\Gamma_\mu\,S\,\Gamma^\pi\,S\big]$$

- BSA of moving pion
  - interpolation and extrapolation of rest frame BSA
    - solve BSE only once
    - need for interpolation and extrapolation of solution

Intermezzo: Frame (in)dependence

- In principle,
  all frames are equivalent
- In practice,
  some frames are easier than others ...
- Form factor

$$\Lambda_\mu(P,Q) \;=\; 2\,P_\mu\,F_\pi(Q^2) \quad = \quad N_c \int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\big[\bar{\Gamma}^\pi\,S\,i\Gamma_\mu\,S\,\Gamma^\pi\,S\big]$$

- BSA of moving pion
  - solve BSE in exactly the same frame
    as used in form factor calculation
    - no need for interpolation nor extrapolation
    - solve BSE for every value of Q^2

Intermezzo: Frame (in)dependence

- In principle,
  all frames are equivalent
- In practice,
  some frames are easier than others . . .

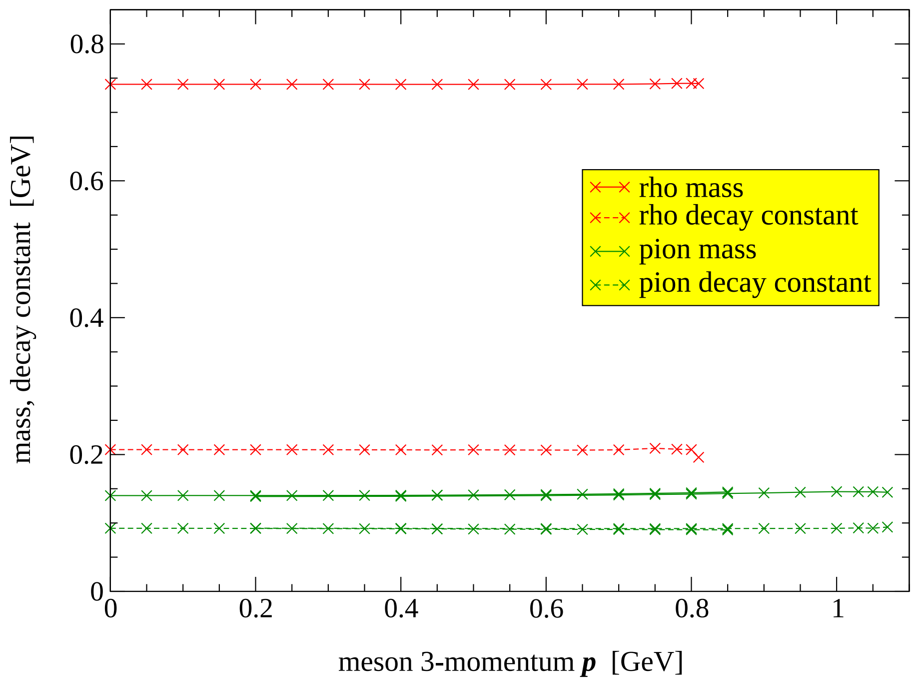
- Meson BSE
  discrete solutions at $P^2 = -m^2$

$$\Gamma_M(p;P) \ = \ \frac{-4}{3} \int \frac{d^4k}{(2\pi)^4} \, \alpha((p-k)^2) D_{\mu\nu}(p-k) \, \gamma_\mu \, \chi_M(k;P) \, \gamma_\nu$$

where $q = p - k$
- rest frame: $P = (im, 0, 0, 0)$
- moving meson: $P = (iE, q, 0, 0)$ where $E^2 = m^2 + q^2$

Intermezzo: Frame (in)dependence

- In principle,
  all frames are equivalent
- In practice,
  some frames are easier than others . . .

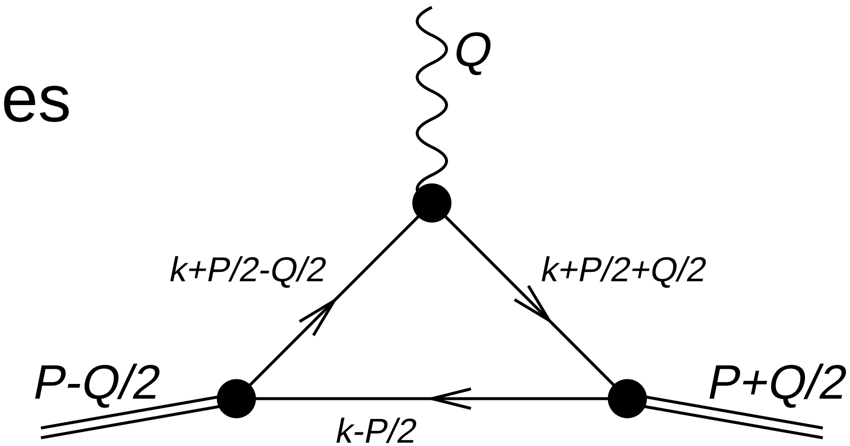


Numerically expensive:
BSA functions of
3 independent variables

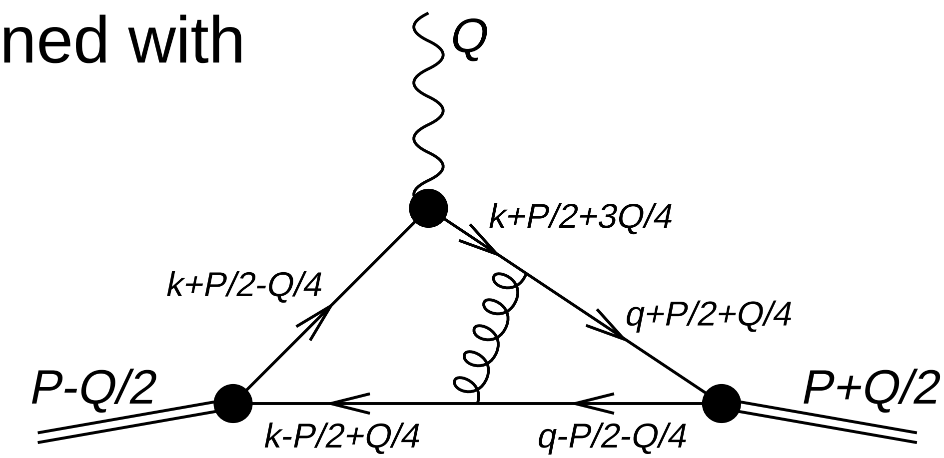
Currently limited
by singularities
in quark propagator

Triangle diagram integration

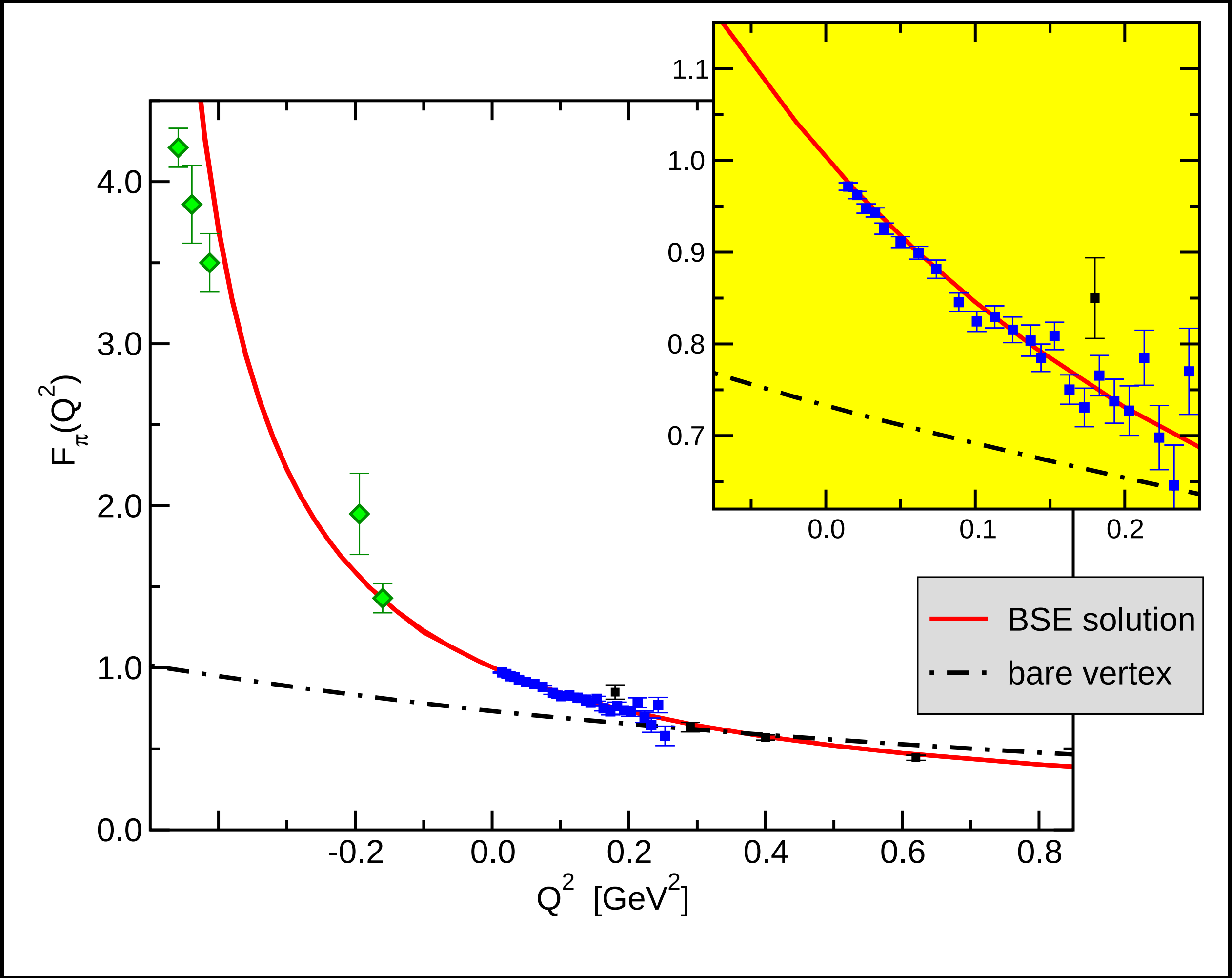
- Within numerical accuracy, results independent of
  - choice integration variables
  - form factor frame



- Best results obtained with

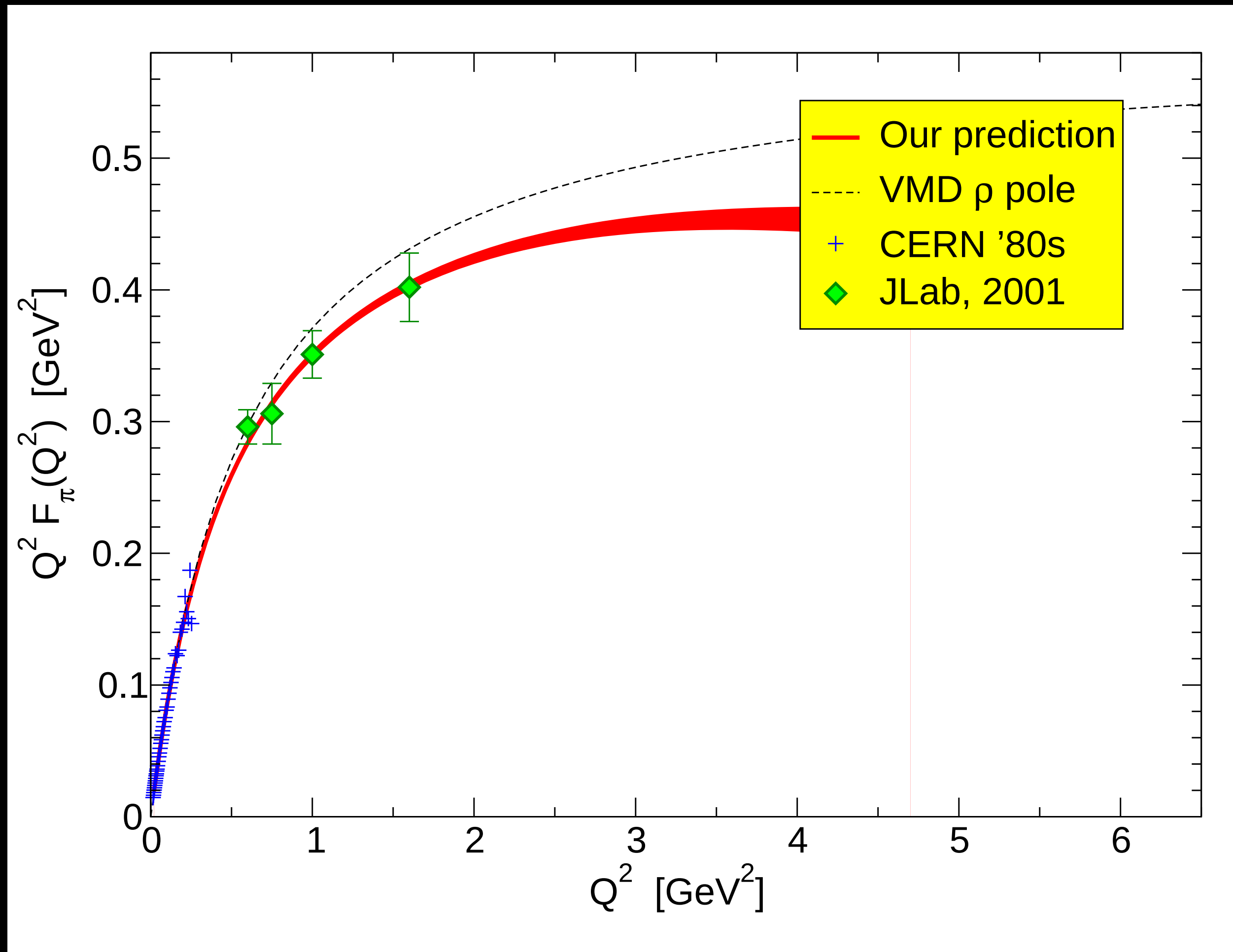


Pion electromagnetic form factor



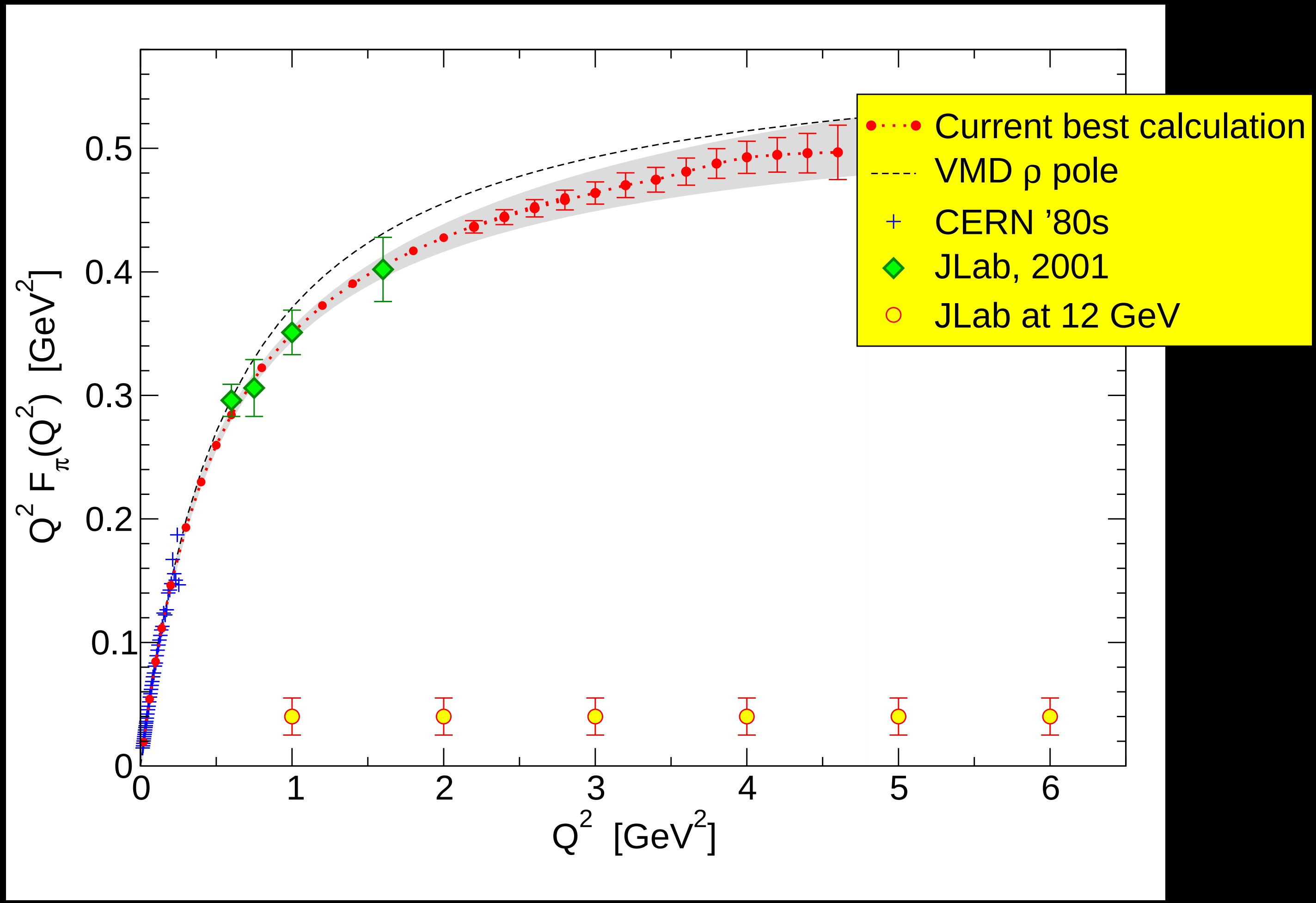
PM and Tandy, PRC61,045202 (2000) [nucl-th/9910033]

Pion electromagnetic form factor



PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]
JLab data from Volmer *et al*, PRL86, 1713 (2001) [nucl-ex/0010009]

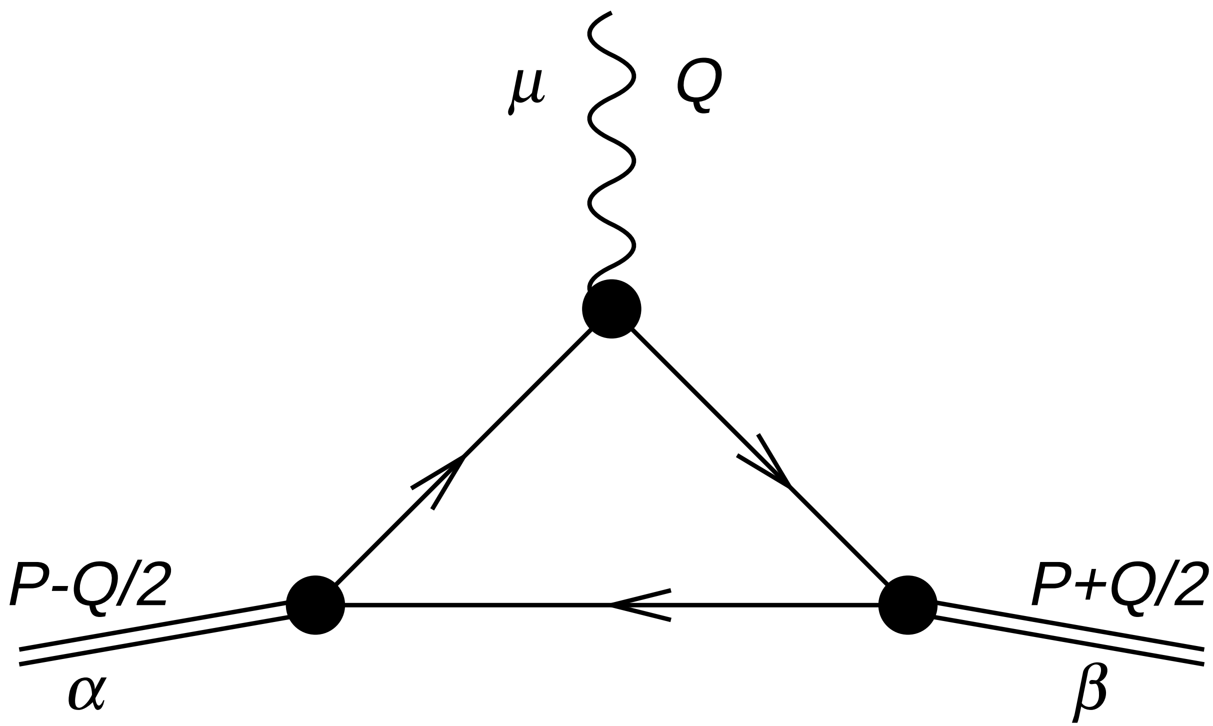
Pion electromagnetic form factor



Rho electromagnetic form factor

- General structure of vector form factors

$$\Lambda_{\mu\alpha\beta}(P,Q) \;=\; N_c \int_k \mathrm{Tr}\big[\bar{\Gamma}^{\rho}_{\beta}\, S\, i\Gamma_{\mu}\, S\, \Gamma^{\rho}_{\alpha}\, S\big] \;=\; \sum T^{[j]}_{\mu\alpha\beta}(P,Q)\; F_j(Q^2)$$



Rho electromagnetic form factor

- General structure of vector form factors

$$\Lambda_{\mu\alpha\beta}(P,Q) \;=\; N_c \int_k \mathrm{Tr}\left[\bar{\Gamma}^{\rho}_{\beta}\, S\, i\Gamma_{\mu}\, S\, \Gamma^{\rho}_{\alpha}\, S\right] \;=\; \sum T^{[j]}_{\mu\alpha\beta}(P,Q)\; F_j(Q^2)$$

- Commonly used covariant form factors

$$\begin{aligned}
T^{[1]}_{\mu\alpha\beta}(P,Q) &= 2\, P_\mu\, \mathcal{P}^T_{\alpha\gamma}(P^-)\mathcal{P}^T_{\gamma\beta}(P^+) \\
T^{[2]}_{\mu\alpha\beta}(P,Q) &= \left(Q_\alpha - P^-_\alpha \frac{Q^2}{2\,m^2}\right)\, \mathcal{P}^T_{\mu\beta}(P^+) \\
&\quad - \left(Q_\beta + P^+_\beta \frac{Q^2}{2\,m^2}\right)\, \mathcal{P}^T_{\mu\alpha}(P^-) \\
T^{[3]}_{\mu\alpha\beta}(P,Q) &= \frac{P_\mu}{m^2}\, \left(Q_\alpha - P^-_\alpha \frac{Q^2}{2\,m^2}\right)\, \left(Q_\beta + P^+_\beta \frac{Q^2}{2\,m^2}\right)
\end{aligned}$$

with $P_\pm = P \pm Q/2$

Rho electromagnetic form factor

- General structure of vector form factors

$$\Lambda_{\mu\alpha\beta}(P,Q) \;=\; N_c \int_k \mathrm{Tr}\left[\bar{\Gamma}^{\rho}_{\beta}\, S\, i\Gamma_{\mu}\, S\, \Gamma^{\rho}_{\alpha}\, S\right] \;=\; \sum T^{[j]}_{\mu\alpha\beta}(P,Q)\; F_j(Q^2)$$

- Electric monopole, magnetic dipole and quadrupole form factors

$$\begin{aligned} G_{\mathrm{E}}(Q^2) &= F_1(Q^2) + \frac{2}{3}\,\frac{Q^2}{4\,m^2} G_{\mathrm{Q}}(Q^2) \\ G_{\mathrm{M}}(Q^2) &= -F_2(Q^2) \\ G_{\mathrm{Q}}(Q^2) &= F_1(Q^2) + F_2(Q^2) + \left(1 + \frac{Q^2}{4\,m^2}\right) F_3(Q^2) \end{aligned}$$

  - Magnetic dipole moment $G_{\mathrm{M}}(Q^2 = 0) = \mu$
  - Quadrupole moment $G_{\mathrm{Q}}(Q^2 = 0) = \mathcal{Q}$

Rho electromagnetic form factor

Same procedure as for pion form factor

- Solve quark DSE
- Solve ρ-meson BSE
- Solve quark-photon vertex
- Calculate form factor

Lemieux at Pittsburgh
Supercomputing Center

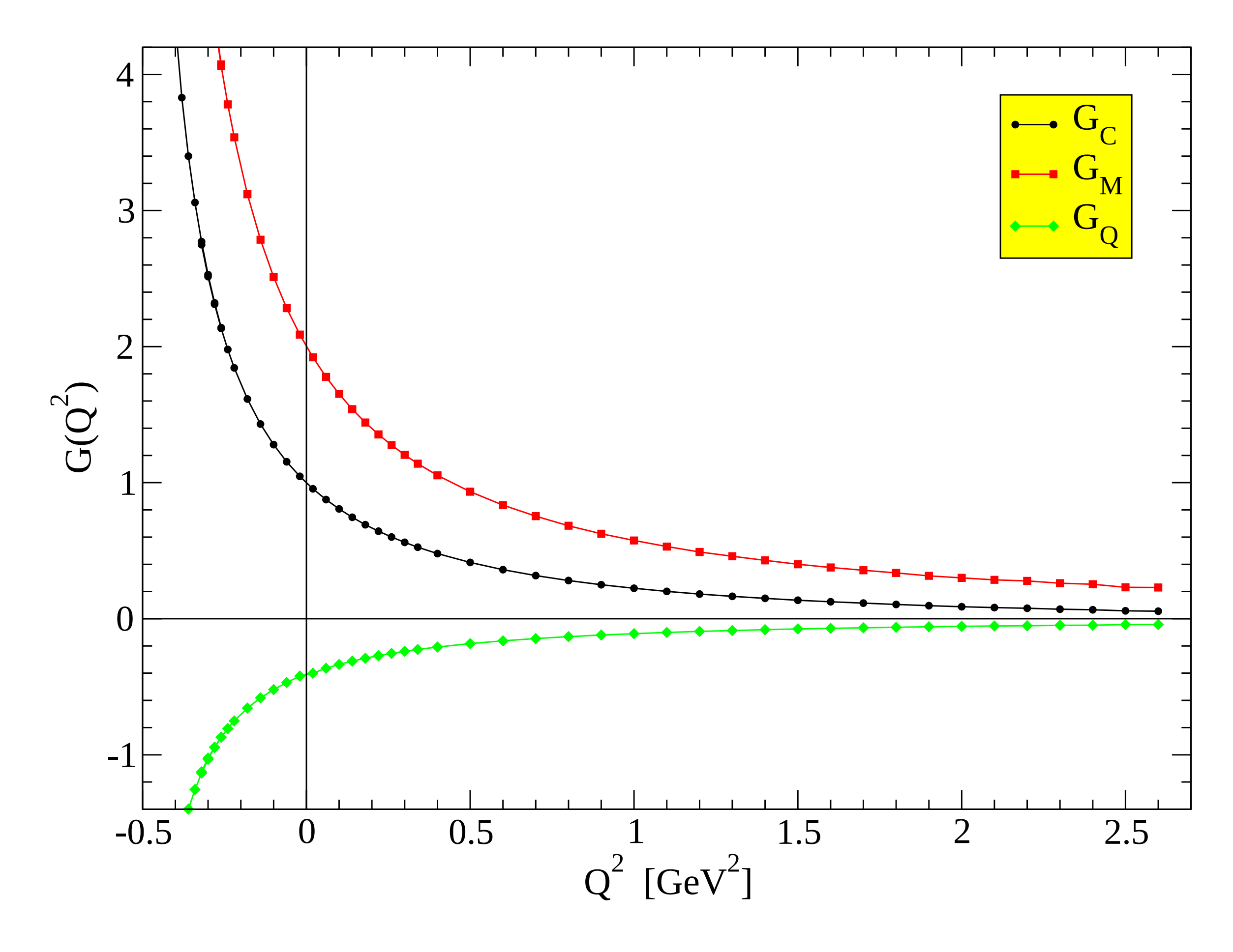
Rho electromagnetic form factor

Same procedure as for pion form factor

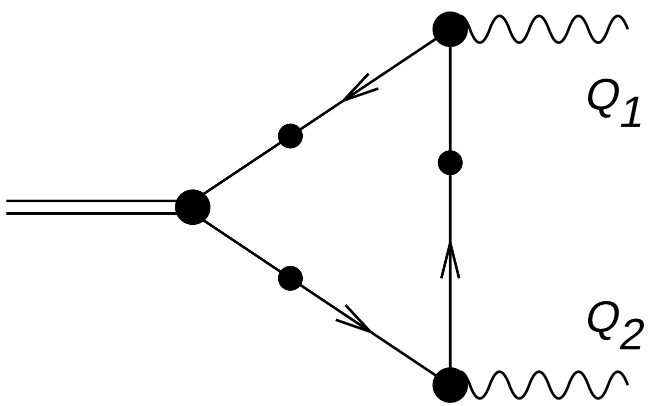
- Solve quark DSE
- Solve ρ-meson BSE
- Solve quark-photon vertex
- Calculate form factor

	μ	$\mathcal{Q}$
present calculation	2.00	0.41
Choi & Ji, PRD70, 053015 (2004)	1.92	0.43
Jaus, PRD67, 094010 (2003)	1.83	0.33
de Melo & Frederico, PRC55, 2043 (1997)	2.17	0.78
Hawes & Pichowsky, PRC59, 1743 (1999)	2.69	0.84
Aliev, Kanik, & Savci, PRC68, 056002 (2003)	2.3(5)	—

Rho electromagnetic form factor



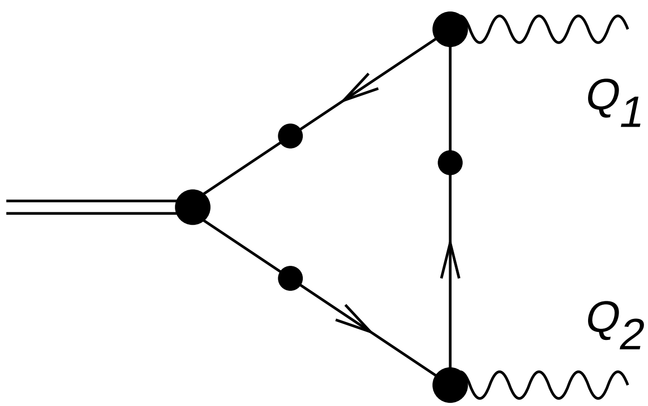
Transition form factors: $\pi^0\,\gamma\gamma$



$$\Lambda_{\mu\nu}(Q_1, Q_2) = \frac{2i\,\alpha\,g_{\pi\gamma\gamma}}{\pi\ f_\pi}\epsilon_{\mu\nu\alpha\beta}\,Q_{1,\alpha}\,Q_{2,\beta}\,F(Q_1^2, Q_2^2)$$

- Value at $Q^2 = 0$ governed by axial anomaly related to the divergence of the AVV diagram
- Axial anomaly follows automatically if WTI is satisfied
  - DSE calculation physical pion: $g_{\pi\gamma\gamma} \approx 0.50$
  - experimental $\pi^0 \to \gamma\gamma$ coupling: $g_{\pi\gamma\gamma} \approx 0.50$

Transition form factors: $\pi^0\,\gamma\gamma$



$$\Lambda_{\mu\nu}(Q_1, Q_2) =$$
$$\frac{2i\,\alpha\,g_{\pi\gamma\gamma}}{\pi\;f_\pi}\epsilon_{\mu\nu\alpha\beta}\,Q_{1,\alpha}\,Q_{2,\beta}\,F(Q_1^2, Q_2^2)$$

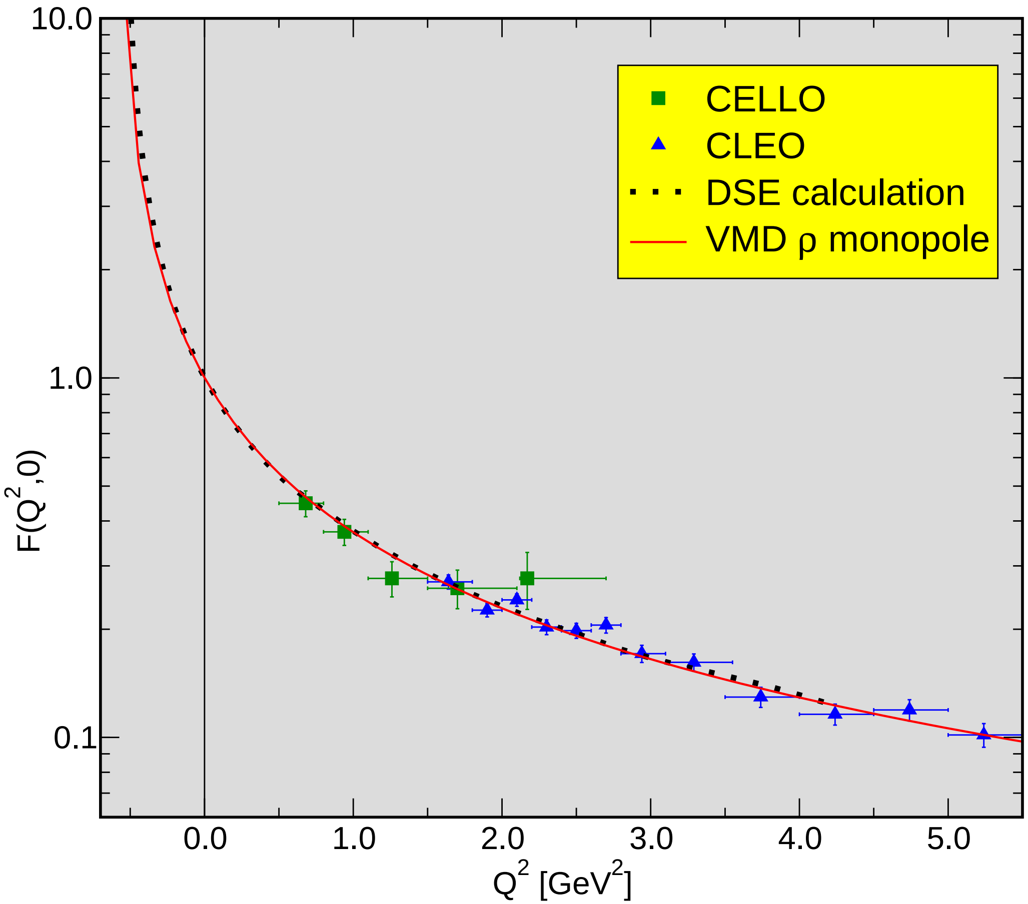
- Asymptotic behavior related to pion D.A. $\phi_\pi(x)$

$$F(Q_1^2, Q_2^2) \quad\rightarrow\quad 4\pi^2 f_\pi^2 \left\{ \frac{J(\omega)}{Q_1^2 + Q_2^2} + \mathcal{O}\left(\frac{\alpha_s}{\pi}, \frac{1}{(Q_1^2 + Q_2^2)^2}\right) \right\}$$
$$J(\omega) \quad = \quad \frac{4}{3}\int_0^1 \frac{dx}{1 - \omega^2(2x - 1)}\phi_\pi(x)$$

where $\omega = \frac{Q_1^2 - Q_2^2}{Q_1^2 + Q_2^2}$ characterizes photon asymmetry

Asymmetric $\pi\ \gamma^{\star}\gamma$ *form factor*

One photon on-shell: $Q_1^2 = Q^2$, $Q_2^2 = 0$, $\omega = 1$

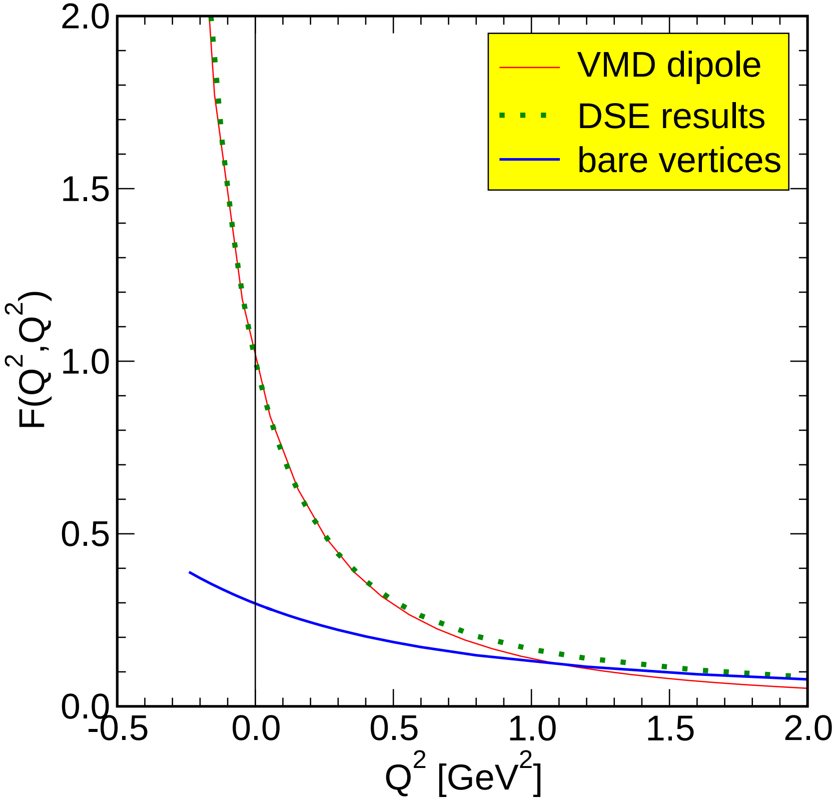


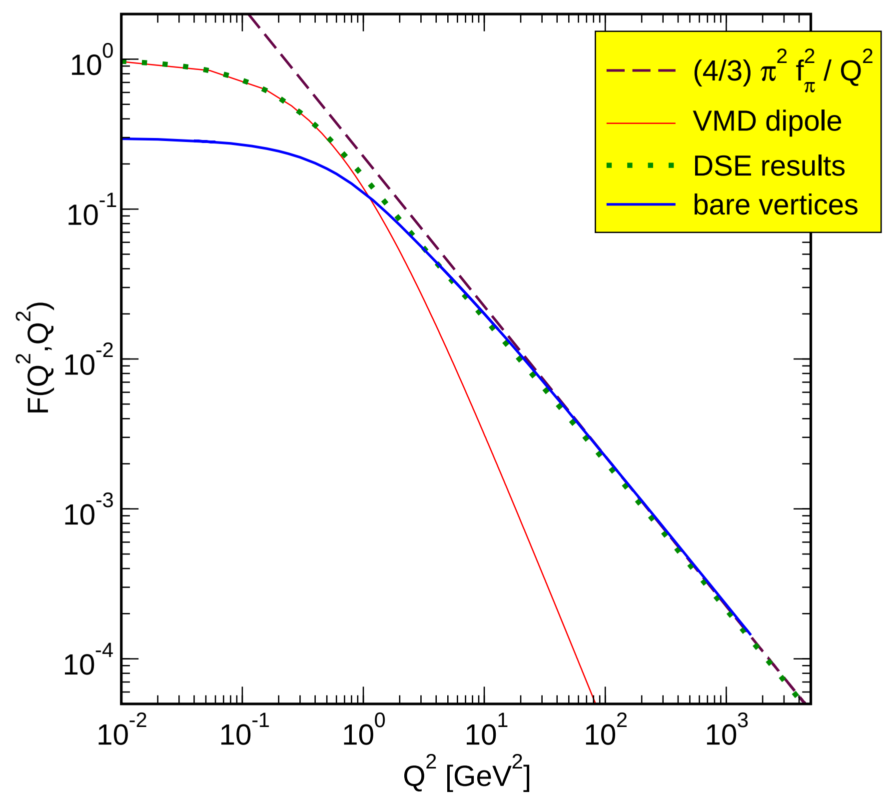
Transition radius:
DSE calc.: $r^2 = 0.39$ fm^2
expt.: $r^2 = 0.42 \pm 0.04$ fm^2

Asymptotic behavior appears to be somewhat below pQCD

Symmetric $\pi\ \gamma^{\star}\gamma^{\star}$ form factor

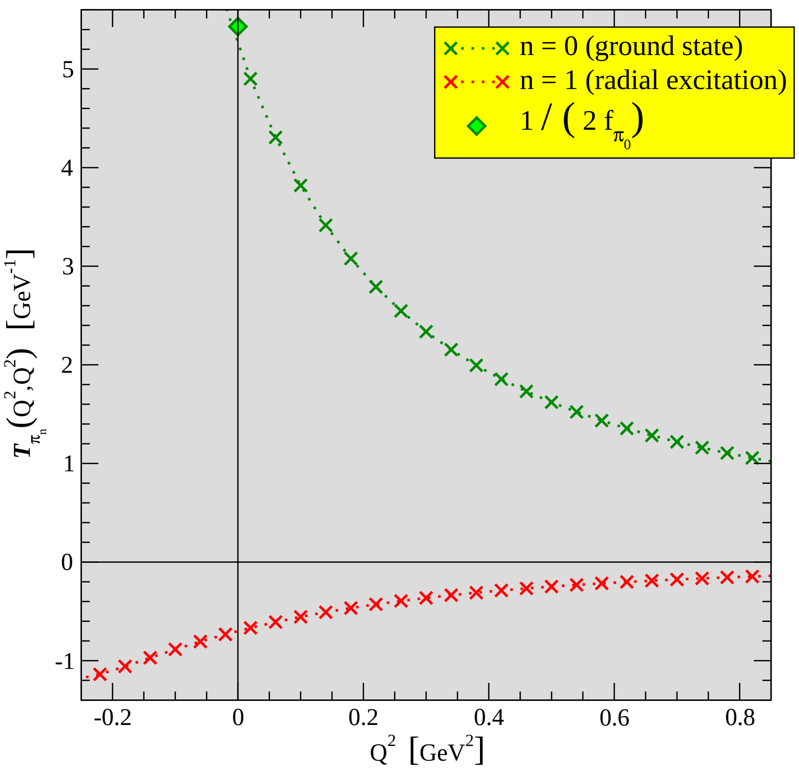
Both photons equally vitual: $Q_1^2 = Q_2^2$, $\omega = 0$





- Dipole in time-like region, in agreement with VMD
- Asymptotic behavior in perfect agreement with pQCD

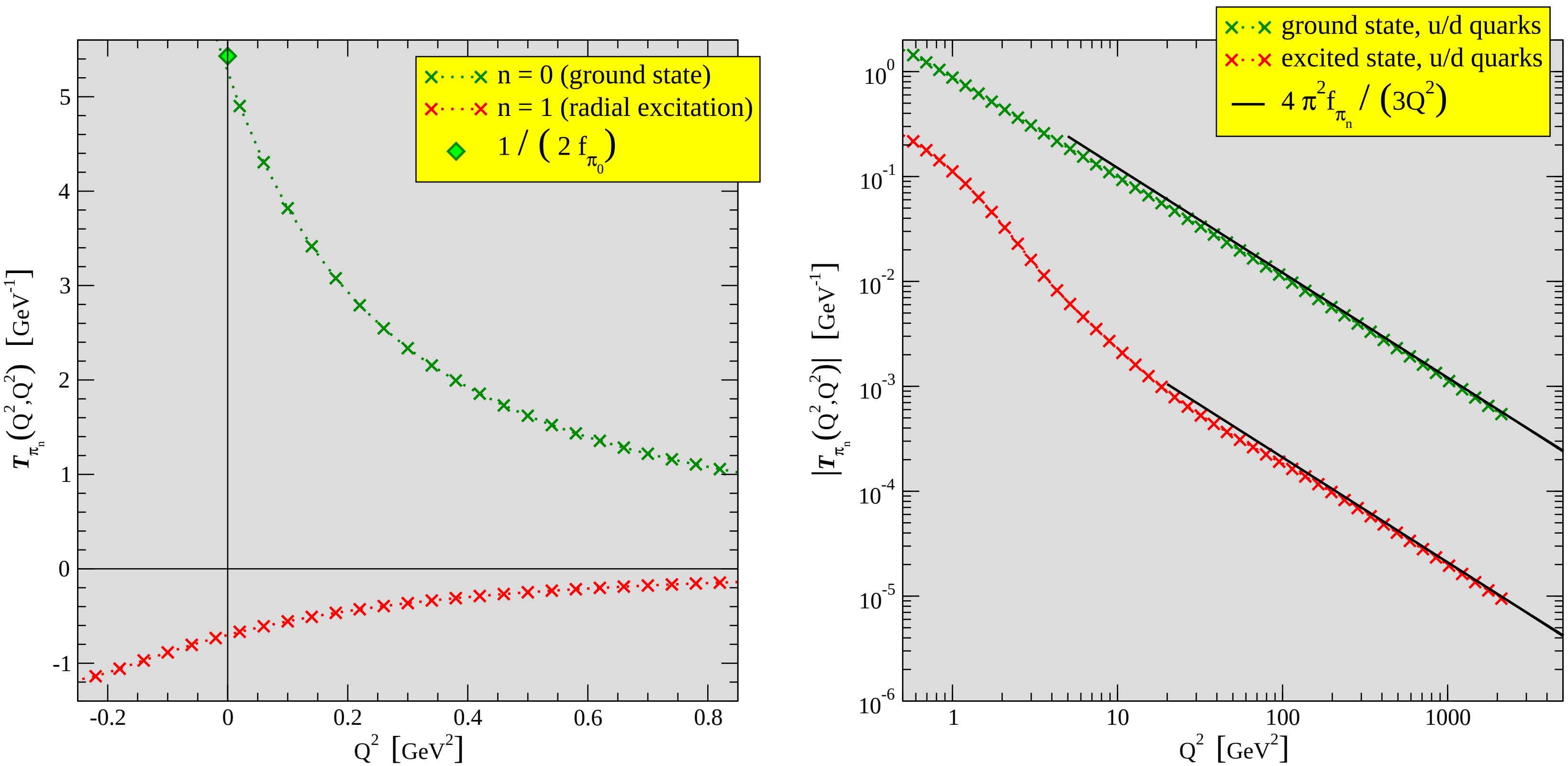
Excited $\pi\ \gamma^\star\gamma^\star$ *form factor*



- ground state pion
  $T_{\pi_0}(0,0) = \frac{1}{2f_{\pi_0}} \approx 5.4$
- radially excited state
  $T_{\pi_1}(0,0) \neq \frac{1}{2f_{\pi_1}}$
- excited pseudoscalar meson decouples from AVV anomaly

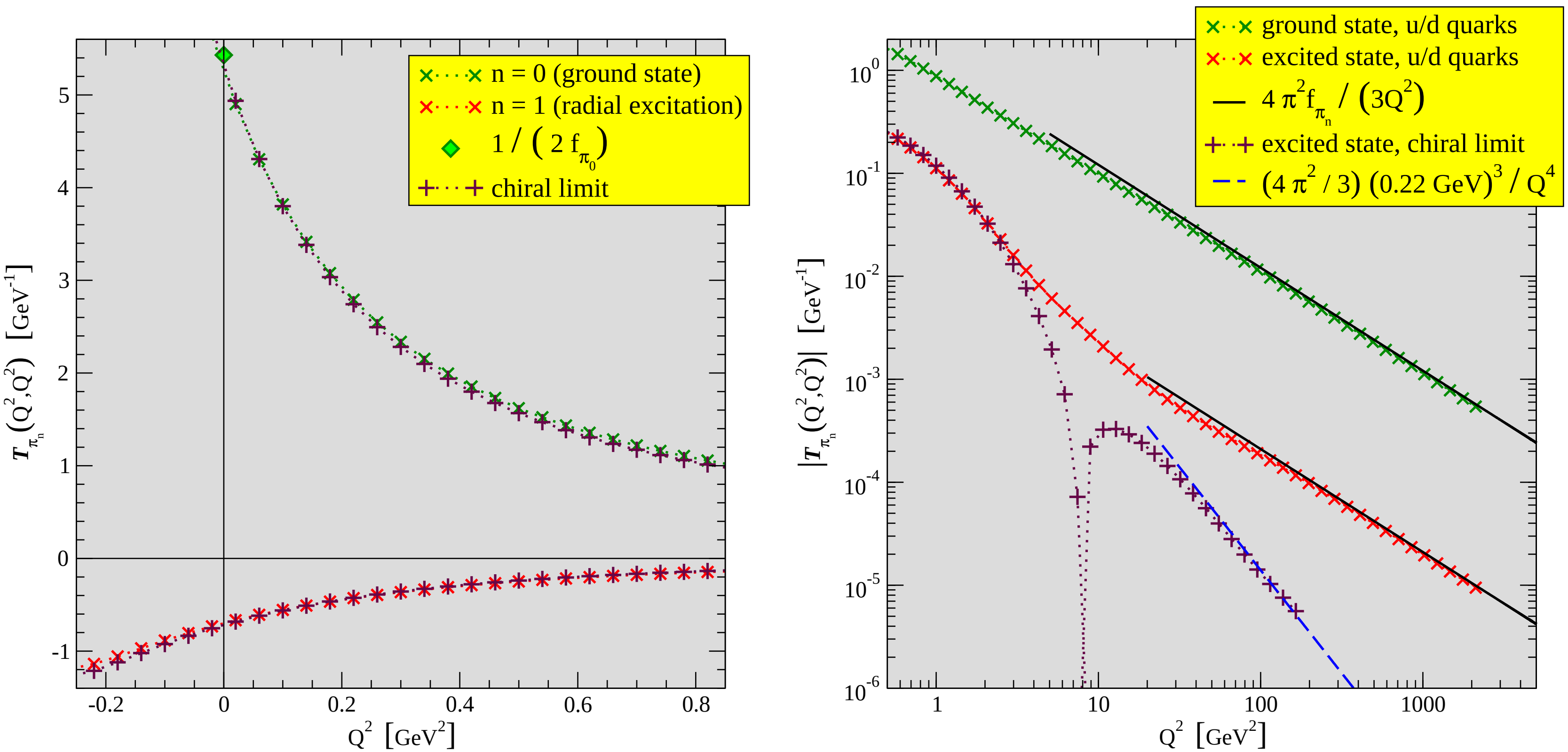
$$\Lambda_{\mu\nu}(Q_1,Q_2) \quad = \quad \frac{2i\,\alpha}{\pi}\epsilon_{\mu\nu\alpha\beta}\,Q_{1,\alpha}\,Q_{2,\beta}\,T_{\pi_n}(Q_1^2,Q_2^2)$$

Excited $\pi\ \gamma^\star\gamma^\star$ form factor



Asymptotically, $T_{\pi_n}(Q^2,Q^2) \longrightarrow \frac{4\pi^2 f_{\pi_n}}{3\,Q^2}$
both for ground state and for excited state

Excited $\pi\ \gamma^\star\gamma^\star$ *form factor*



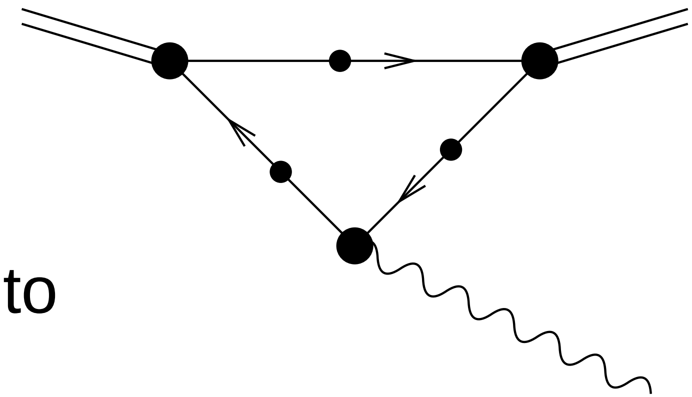
Chiral limit excited state

$f_{\pi_1} \to 0$, but T_{π_1} does not diverge, nor does it vanish

Asymptotically, $T_{\pi_n}(Q^2,Q^2) \propto \frac{1}{Q^4}$

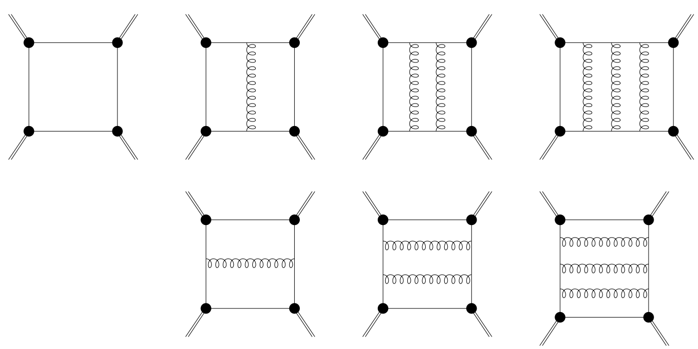
Other meson form factors

- $K^{\pm}$, K^0 electromagnetic form factors
- $\rho\,\pi\,\gamma$ transition form factor
- $K^{\star}\,K\,\gamma$ transition form factors
- weak K_{l3} decay
- ...



Same method can also be applied to

- Strong decays
- Scattering processes
  iff the kernel is also iterated inside the box



Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle\bar{q}q\rangle^0_\mu$	$(0.236\ \text{GeV})^3$	$(0.241^\dagger)^3$
m_π	0.1385 GeV	$0.138^\dagger$
f_π	0.0924 GeV	$0.093^\dagger$
m_K	0.496 GeV	$0.497^\dagger$
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	$7.6 \cdot 10^6\ \text{s}^{-1}$	7.38
$\Gamma(K_{\mu 3})$	$5.2 \cdot 10^6\ \text{s}^{-1}$	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
$m_{K^\star}$	0.892 GeV	0.936
$f_{K^\star}$	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^\star K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

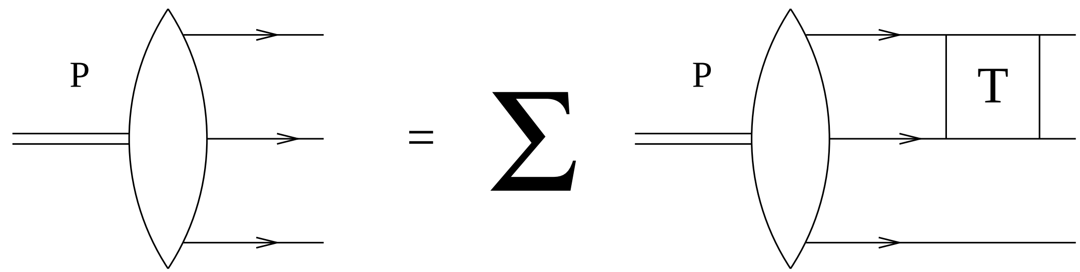
$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^\star K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^\star K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

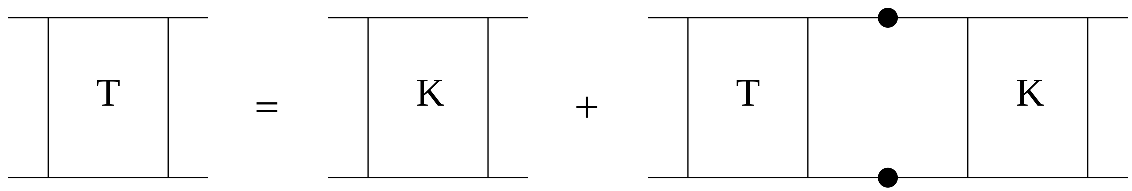
Baryons – bound states of three quarks

- If one discards intrinsic three-body interactions, three-body bound state eqn reduces to relativistic Faddeev eqn



- Involves the quark-quark scattering matrix T, which satisfies a Bethe–Salpeter equation

$$T_{ij} \quad = \quad K_{ij} + \int K_{ij}\, S_i S_j\, T_{ij}$$



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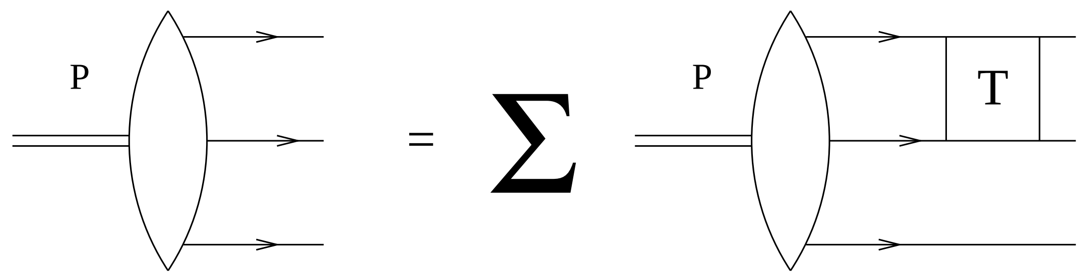

- Involves the quark-quark scattering matrix T, which satisfies a Bethe–Salpeter equation

$$T_{ij} \;=\; K_{ij} + \int K_{ij}\, S_i S_j\, T_{ij}$$

- One-gluon exchange is attractive in color anti-triplet diquark channel with effective coupling half that of that in meson channel

Baryons – bound states of three quarks

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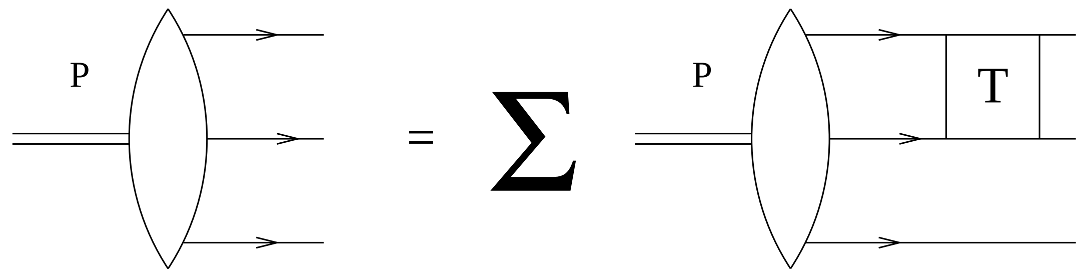


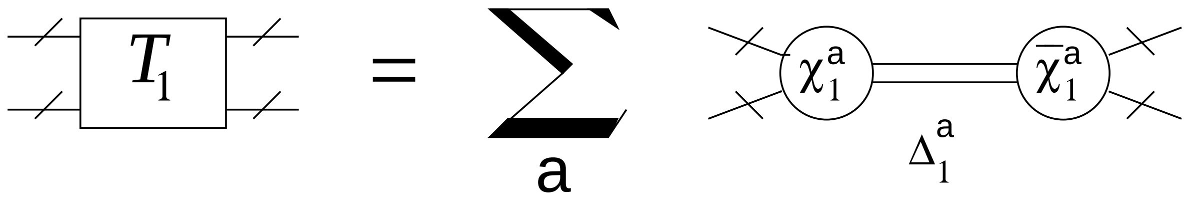
- Involves the quark-quark scattering matrix T, which satisfies a Bethe–Salpeter equation

$$T_{ij} \;\; = \;\; K_{ij} + \int K_{ij} \, S_i S_j \, T_{ij}$$

- Color anti-triplet diquark can couple with a quark to form a color-singlet baryon

Baryons – quark-diquark bound states

- Approximate T matrix by diquarks



$$T_{ij}(k_i,k_j;p_i,p_j) \;\approx\; \sum \bar{\chi}^{J^P}_{ij}(k_i,k_j;K)\,\Delta^{J^P}(K)\,\chi^{J^P}_{ij}(p_i,p_j;K)$$

where $K = k_1 + k_2 = p_1 + p_2$ is the diquark momentum

- Bethe–Salpeter equation for diquarks
  - Typical mass scales lightest diquarks
    - scalar (0^+) diquark $m \approx 0.7 \sim 0.8$ GeV
    - axialvector (1^+) diquark $m \approx 0.9 \sim 1.0$ GeV

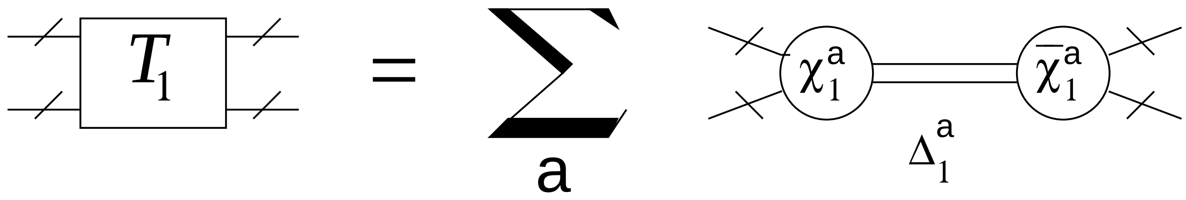
PM, FBS32, 41 (2002) [nucl-th/0204020]

  - radii about 10% larger than corresponding mesons

PM, FBS35, 117 (2004) [nucl-th/0409008]

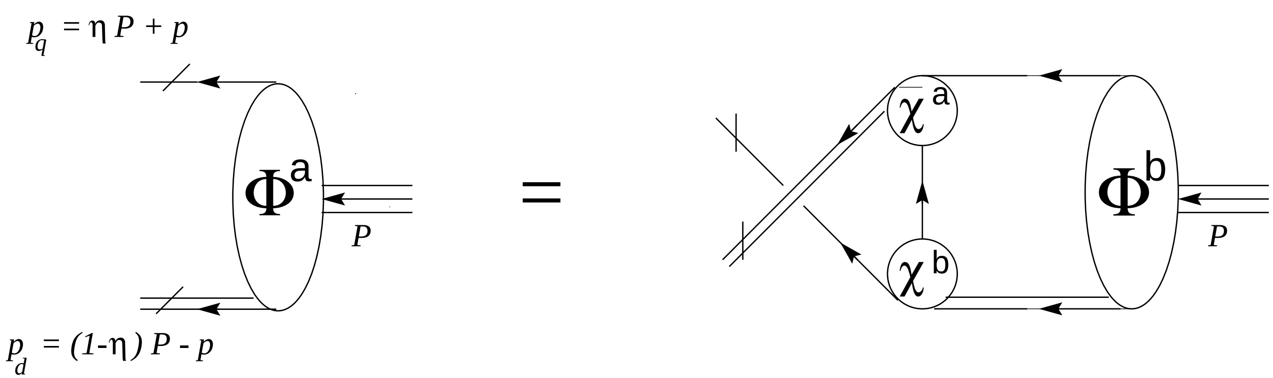
Baryons – quark-diquark bound states

- Approximate T matrix by diquarks



- Incorporate both scalar (0^+) and axialvector (1^+) diquarks
- Solve Faddeev equation for quark-diquark bound state

Cahilll, Roberts, Praschifka, Austr. J. Phys. 42, 129 (1989)



see e.g. Oettel, Von Smekal and Alkofer, CPC144, 63 (2002) [hep-ph/0109285];

program available from CPC Program Library at http://cpc.cs.qub.ac.uk/summaries/ADPT

Baryons – quark-diquark bound states

- Approximate T matrix by diquarks
- Incorporate both scalar (0^+) and axialvector (1^+) diquarks
- Solve Faddeev equation for quark-diquark bound state
- Can fit baryon octet and decouplet masses

e.g. Oettel, Hellstern, Alkofer and Reinhardt, PRC58, 2459 (1998) [nucl-th/9805054]

	M_N	M_Δ	M_Λ	M_Σ	M_Ξ	M_Σ	M_Ξ	M_Ω
calc.	0.939	1.232	1.123	1.134	1.307	1.373	1.545	1.692
expt.	0.939	1.232	1.116	1.193	1.315	1.384	1.530	1.672

Baryons – quark-diquark bound states

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- Can fit baryon octet and decouplet masses
- However, nucleon mass receives significant contributions of about -200 MeV due to pion loop corrections

Better to fit “core” masses
$M_N^{\text{core}} \approx 1.18$ GeV
$M_\Delta^{\text{core}} \approx 1.33$ GeV

Hecht, Oettel, Roberts, Schmidt, Tandy, Thomas, PRC65, 055204 (2002) [nucl-th/0201084]

Ishii, PLB 431, 1 (1998)

Pearce, Afnan, PRC34, 9991 (1986)

Baryons – quark-diquark bound states

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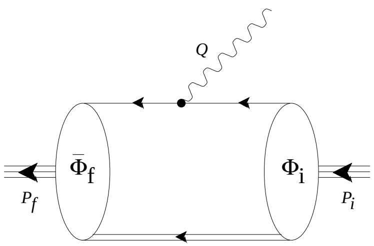
  Better to fit "core" masses
  $M_N^{\text{core}} \approx 1.18$ GeV
  $M_\Delta^{\text{core}} \approx 1.33$ GeV
- Decomposition of Faddeev amplitudes in partial waves shows presence of p-waves, in addition to s- and d-waves

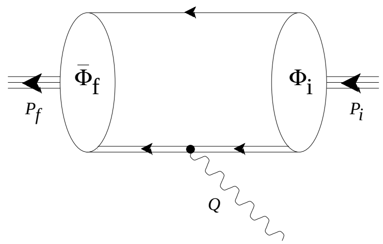
See e.g. PhD thesis of Martin Oettel, arXiv:nucl-th/0012067

$\Longrightarrow$ intrinsic quark orbital angular momentum

Nucleon electromagnetic form factors

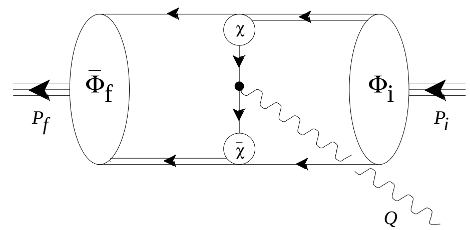
- Impulse approximation



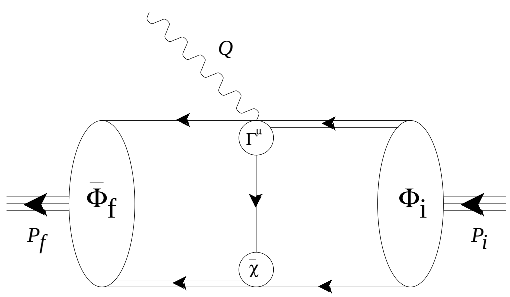


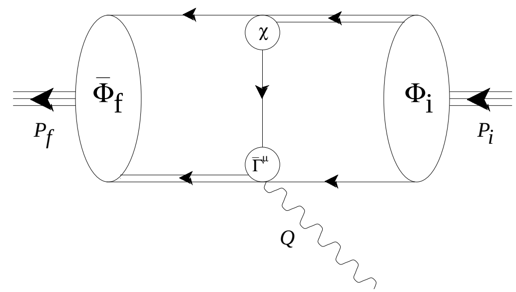
- quark-photon vertex
- scalar diquark-photon vertex
- a.v. diquark-photon vertex
- scalar-a.v.diquark transition

- Exchange current



- Seagull terms





Oettel, Pichowksy, von Smekal, Eur.Phys.J.A8, 251 (2000)

Model calculations

Höll, Alkofer, Kloker, Krassnigg, Roberts, Wright, nucl-th/0501033

- Quark propagators nonperturbatively dressed
  Use confined parametrisation of DSE solution
  Parameters fixed in light meson sector

  Burden, Roberts and Thomson, PLB371, 163 (1996) [nucl-th/9511012]

- Diquarks also confined
  Use confined parametrisation of diquark propagator
- Fit diquark masses to "core" masses,
  leaving room for pion cloud

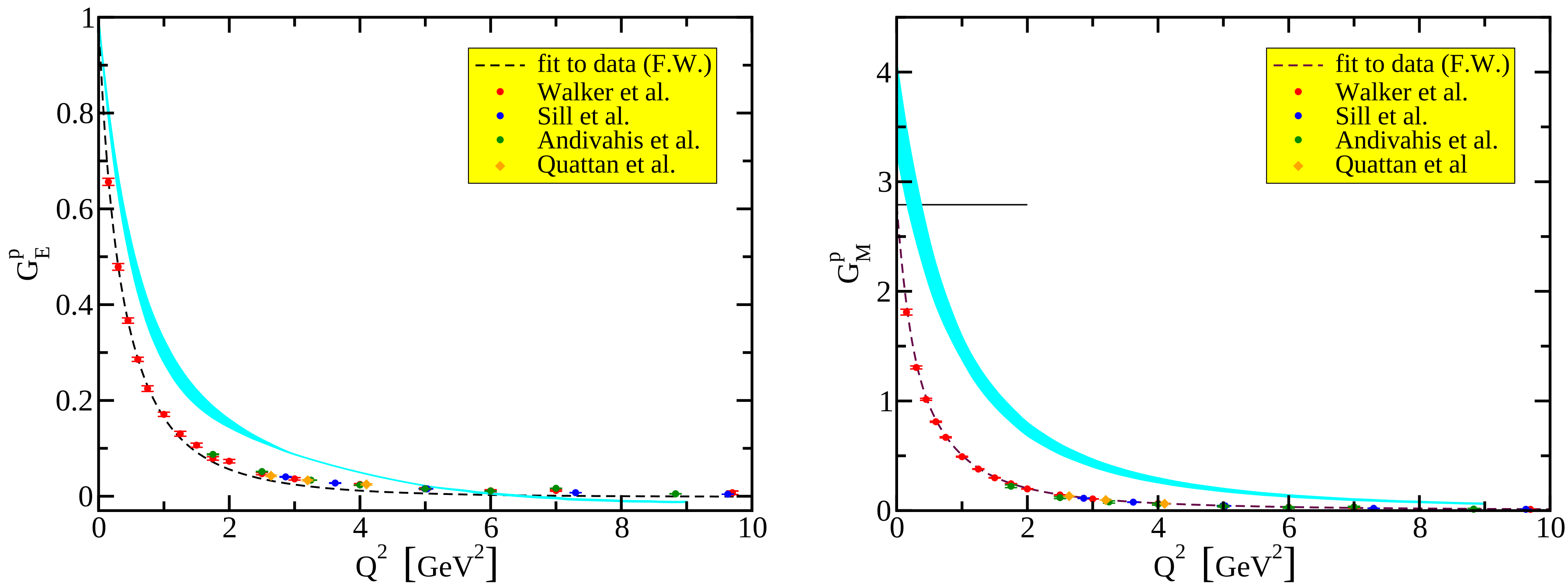
M_N	M_Δ	M_{0^+}	M_{1^+}
1.18	1.33	0.79	0.89

- Photon vertices constrained by current conservation

Model calculations – results

Alkofer, Höll, Kloker, Krassnigg, Roberts, nucl-th/0412046,nucl-th/0501033

- Qualitative agreement with experimental data



Most recent: Qattan *et al.*, PRL94, 142301 (2005) [nucl-ex/0410010]

Fit to data: Friedrich, Walcher, Eur.Phys.J.A17, 607 (2003) [hep-ph/0303054]

Sill *et al.*, PRD48, 29 ('93); Andivahis *et al.*, PRD50, 5491 ('94); Walker *et al.*, PRD49, 5671 ('94)

Model calculations – results

Alkofer, Höll, Kloker, Krassnigg, Roberts, nucl-th/0412046,nucl-th/0501033

- Qualitative agreement with experimental data
- Significant underestimation of the charge radii
  $\Longrightarrow$ room for pion cloud

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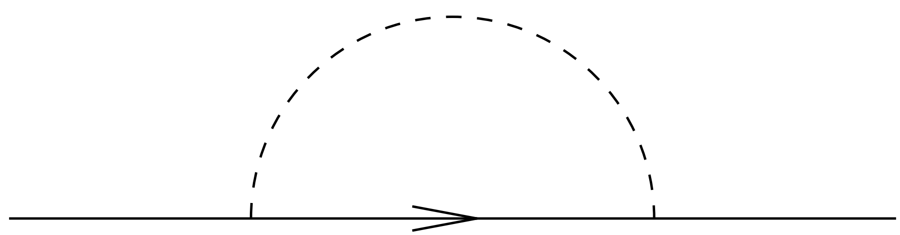
- Significant underestimation of the charge radii
  Expect increase of proton radius due to pion loops

Ashley, Leinweber, Thomas, Young, Eur.Phys.J.A19, 9 (2004)

	r_p	μ_p	r_p^μ
q-(qq) core	0.595	3.63	0.449
$+\ \pi$-loop corr.	0.762	3.05	0.761
expt.	0.847	2.79	0.836



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- WTIs constrain photon-quark and photon-diquark coupling, but do not uniquely determine photon vertices
- Role of transverse VMD-like contributions to photon vertices, which would increase the proton charge radius

Latest results for $\mu G_E^P(Q^2)/G_M^P(Q^2)$

Rosenbluth separation

$$\frac{d\sigma}{d\Omega}\frac{\epsilon(1+\tau)}{\sigma_{\text{Mott}}} =$$

$$\tau\left(G_M^p(Q^2)\right)^2 + \epsilon\left(G_E^p(Q^2)\right)^2$$

Polarization transfer

$$\frac{G_E^p}{G_M^p} =$$

$$-\frac{P_t}{P_l}\frac{(E_e+E'_e)\tan(\frac{1}{2}\theta_e)}{2\,m_p}$$

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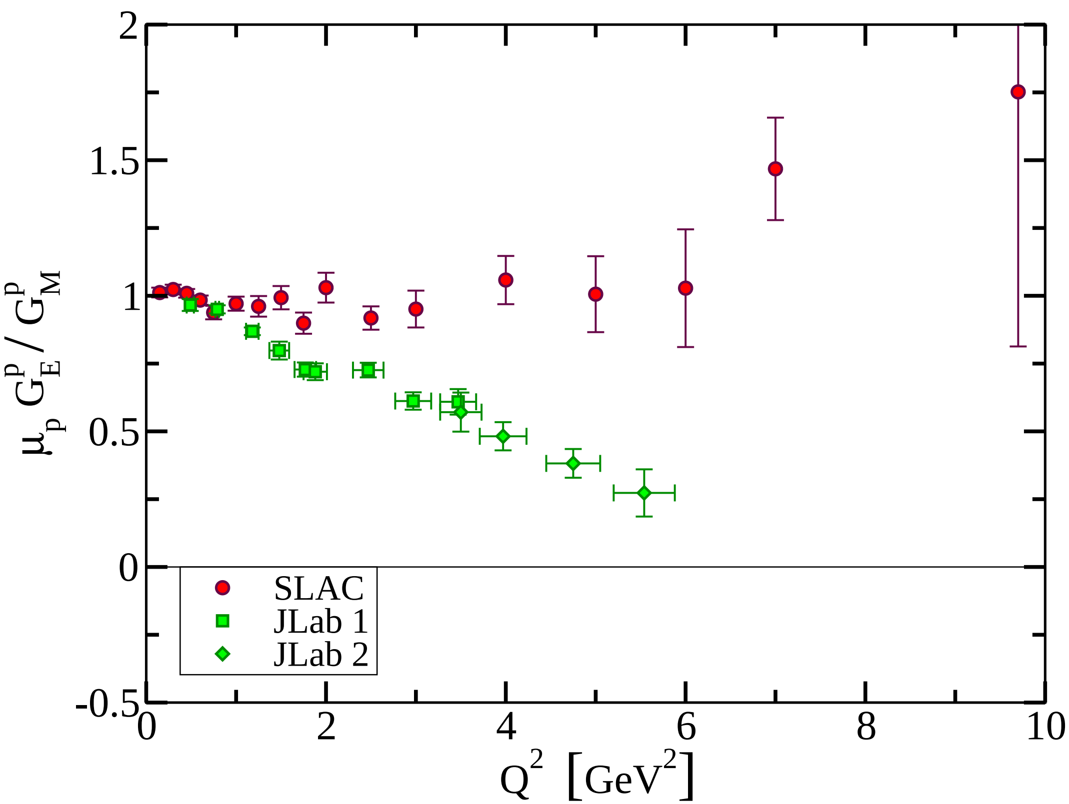
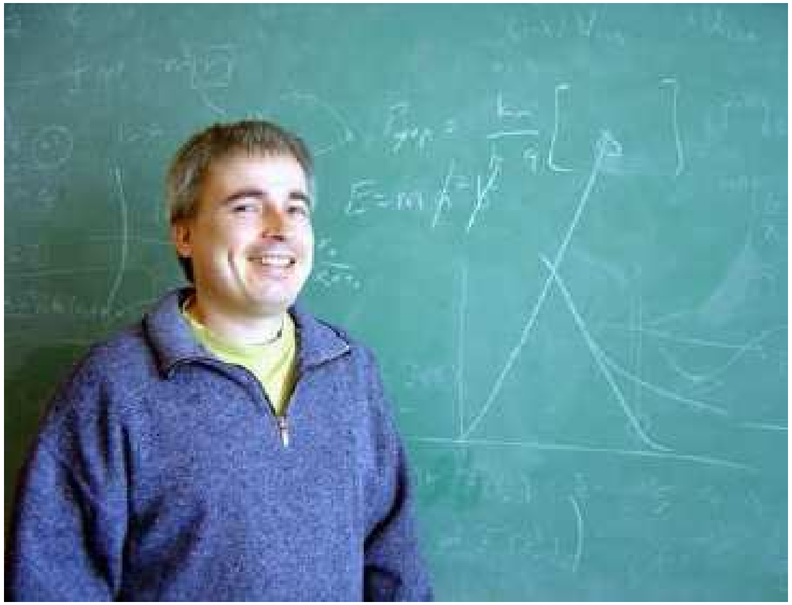


SLAC: R.C. Walker *et al.*, PRD 49, 5671 (1994)

Jlab 2: M.K. Jones *et al.*, PRL 84, 1398 (2000)

Jlab 2: O.Gayou *et al.*, PRL 88, 092301 (2002)

Latest results for $\mu G_E^P(Q^2)/G_M^P(Q^2)$



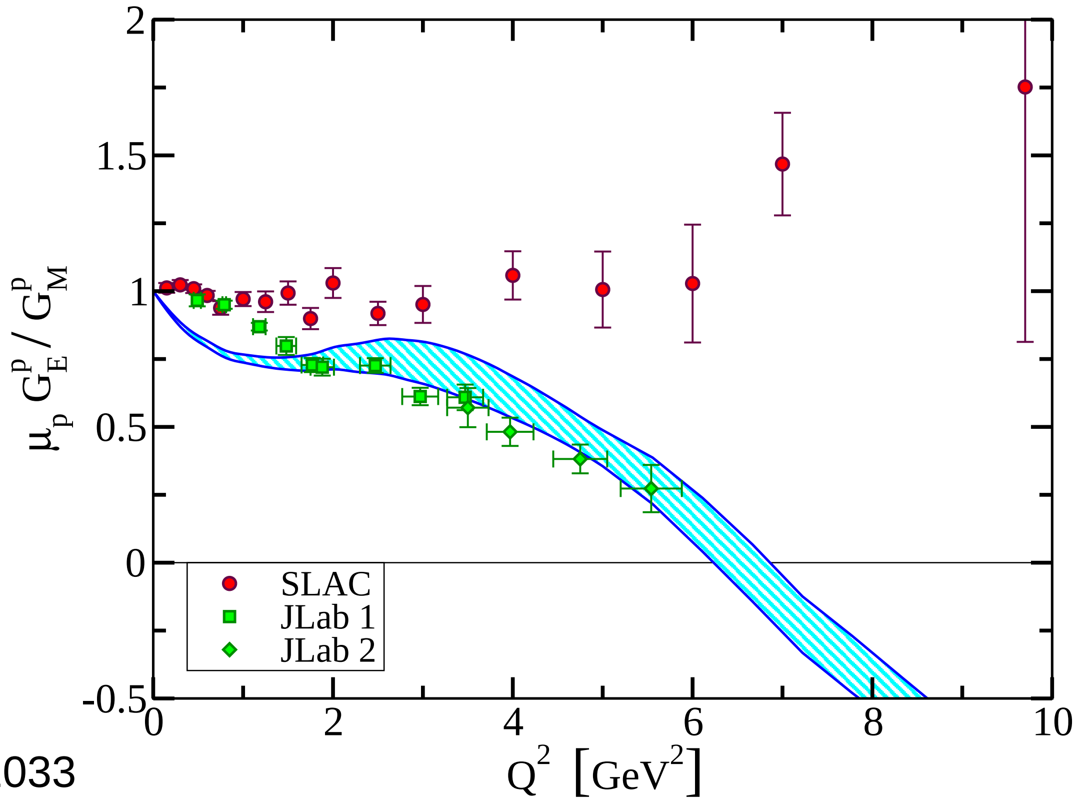
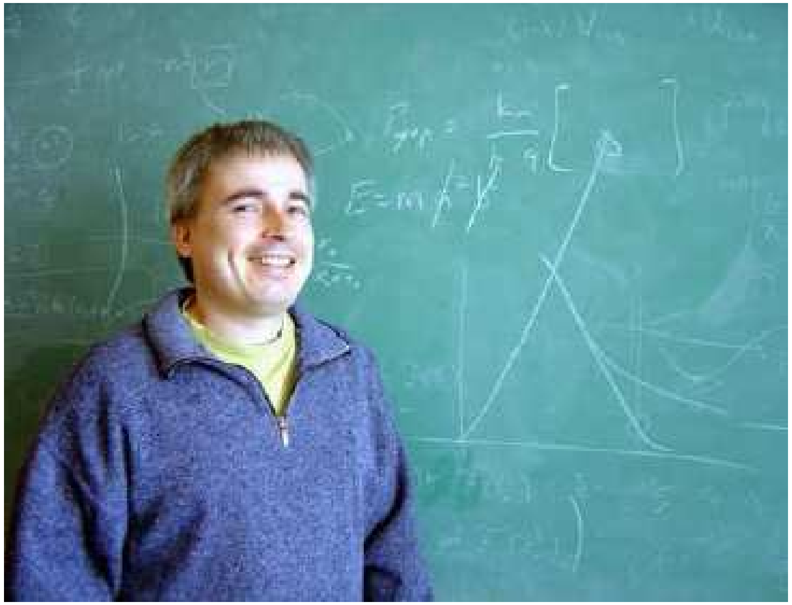


Fig. adapted from Höll, Alkofer, Kloker, Krassnigg, Roberts, Wright, nucl-th/0501033

- Calculations support polarization transfer data
- Pion cloud effects important at small Q^2

Latest results for $\mu G_E^P(Q^2)/G_M^P(Q^2)$



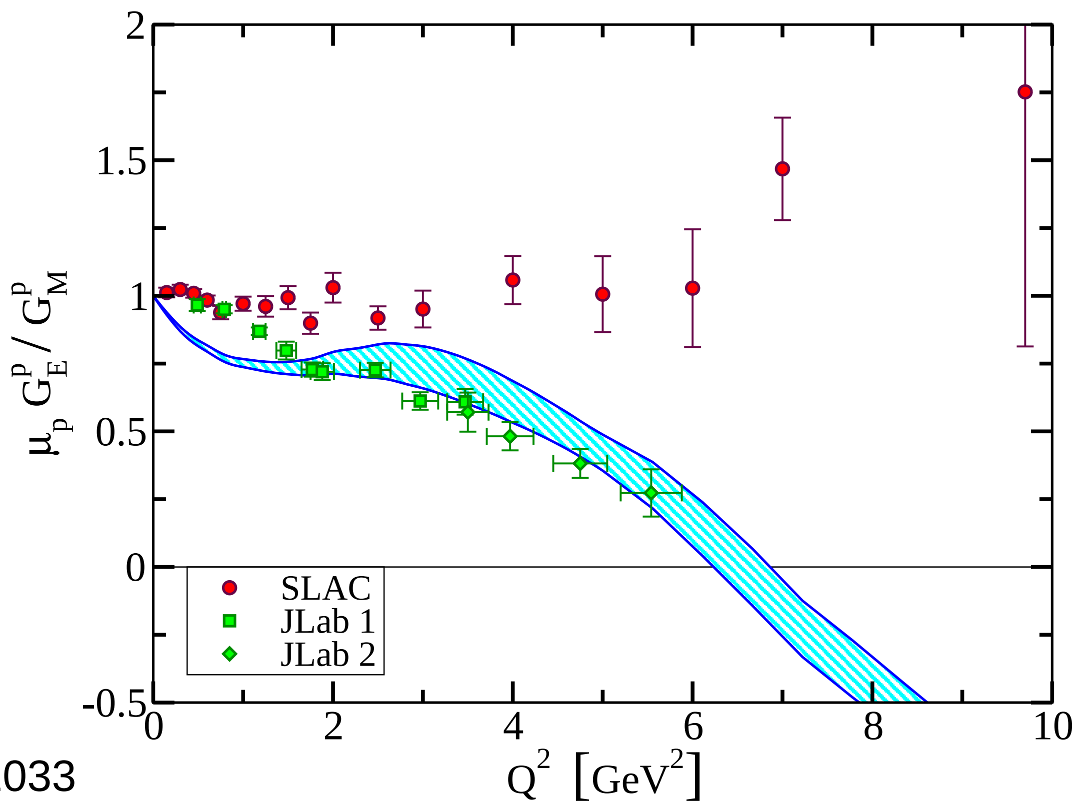
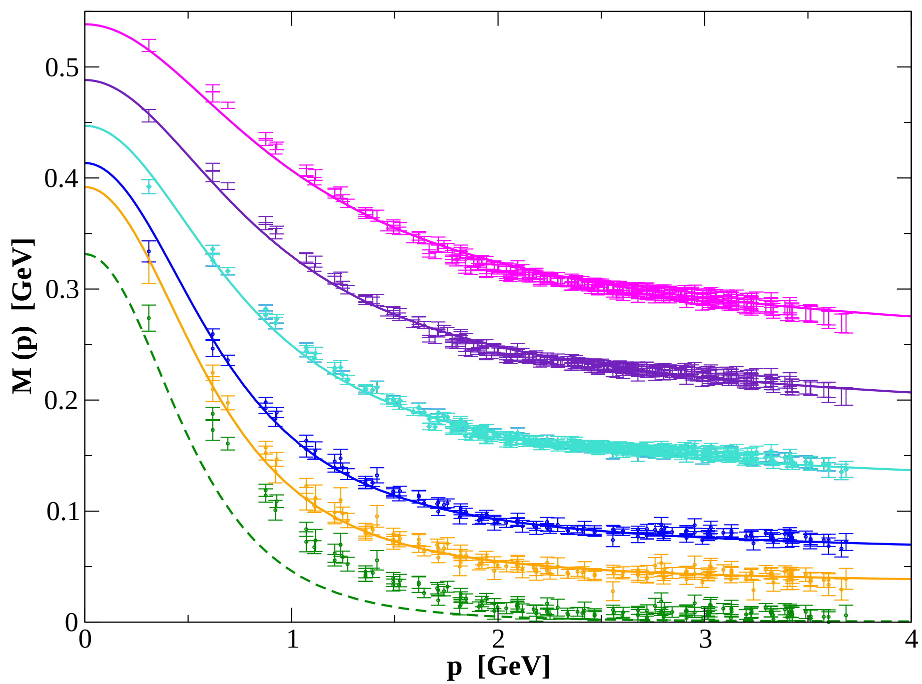


Fig. adapted from Höll, Alkofer, Kloker, Krassnigg, Roberts, Wright, nucl-th/0501033

- Expect zero crossing around $Q^2 \approx 6.5\ \mathrm{GeV}^2$
- Compare quenched lattice: $Q^2_{(0)} \approx 5.8 \ldots 6.5\ \mathrm{GeV}^2$
  Matevosyan, Miller, Thomas, nucl-th/0501044

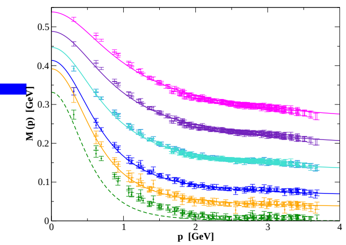
Conclusions

- Nonperturbative quark propagator understood
  - quark mass function $M(p^2)$
    evolves from current mass in the ultraviolet region
    to a constituent-like mass in the infrared region

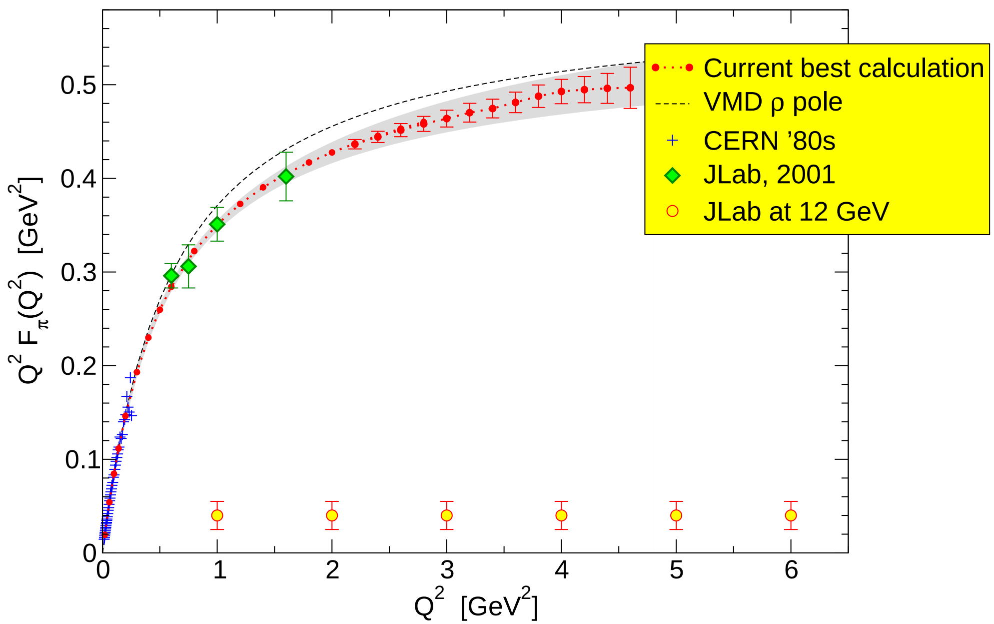


  - connects perturbative QCD with hadronic physics

Conclusions

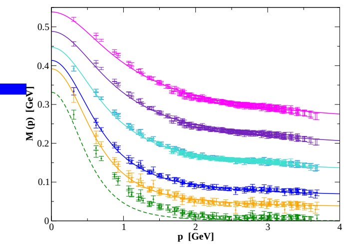


- Nonperturbative quark propagator understood
- Covariant Dyson–Schwinger, Bethe–Salpeter approach successful in describing the light meson sector
  - masses, electroweak form factors



  - strong and electroweak decays, scattering processes

Conclusions



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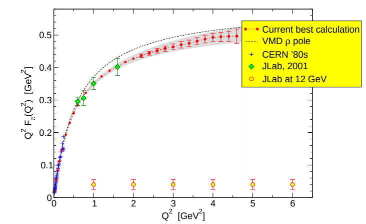
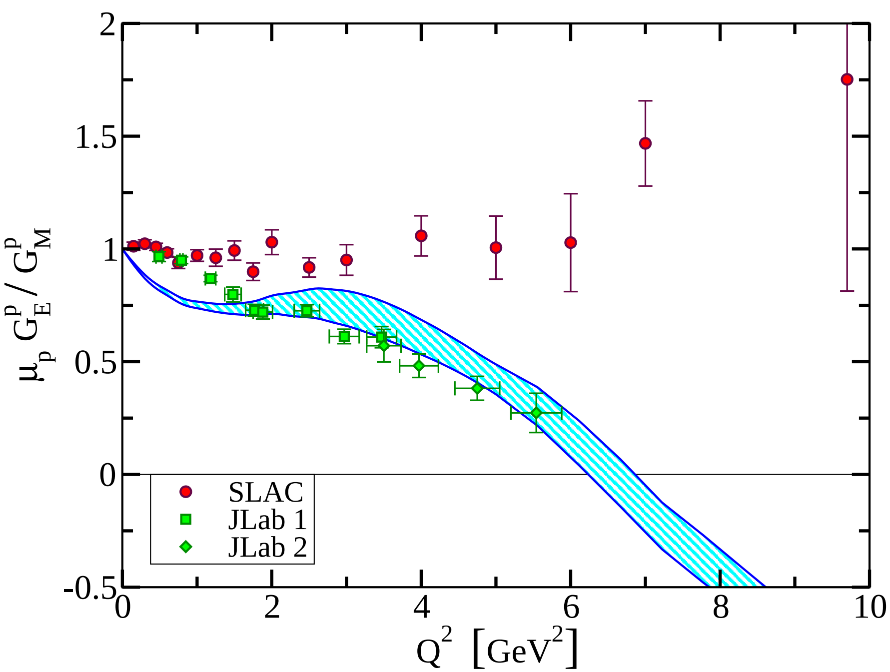
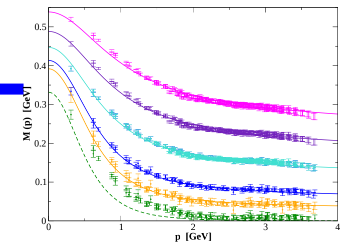


- Poincaré covariant quark-diquark model can describe octet and decouplet baryons



- both 0^+ and 1^+ diquarks
- support pol. transfer data for G_E/G_M
- pion loops important at low Q^2

Conclusions



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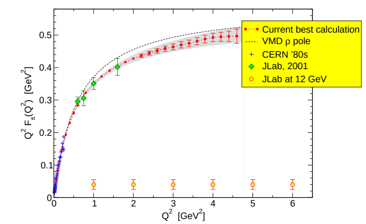

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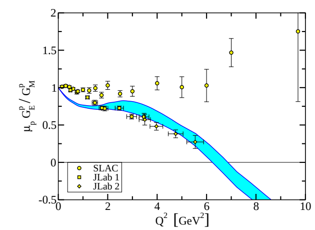


- Enjoy Cairns

Thanks to all of the collaborators

Reinhard Alkofer
Mandar Bhagwat
Jaques Bloch
Christian Fischer
Arne Höll
Markus Kloker
Andreas Krassnigg
Felipe Llanes–Estrada
Martin Oettel
Mike Pichowsky
Craig Roberts
Lorenz von Smekal
Peter Tandy – Monday
Stewart Wright – tomorrow
...

Chiral limit axial-vector Ward–Takahashi identity

$$-iP_\mu\,\Gamma_{5\mu}(k;P) = S^{-1}(k_+)\,\gamma_5 + \gamma_5\,S^{-1}(k_-)$$

Axial-vector vertex

$$\begin{aligned}
\Gamma_{5\mu}(k;P) &= \Gamma^{\text{Reg}}_{5\mu}(k;P) + \frac{P_\mu}{P^2+m_\pi^2}\,f_\pi\,\Gamma_\pi(k;P) + \mathcal{O}(P) \\
\Gamma^{\text{Reg}}_{5\mu}(k;P) &= \gamma_\mu F_R(k;P) + \gamma\cdot k k_\mu G_R(k;P) - \sigma_{\mu\nu}\,k_\nu\,H_R(k;P)
\end{aligned}$$

Limit $P \to 0$ gives

$$\begin{aligned}
f_\pi E_\pi(k;P=0) &= B(k^2) \\
F_R(k;0) + 2\,f_\pi F_\pi(k;0) &= A(k^2) \\
G_R(k;0) + 2\,f_\pi G_\pi(k;0) &= 2A'(k^2) \\
H_R(k;0) + 2\,f_\pi H_\pi(k;0) &= 0
\end{aligned}$$

Axial-vector Ward–Takahashi identity

$$-iP_\mu \,\Gamma_{5\mu}(k;P) = S^{-1}(k_+)\,\gamma_5 + \gamma_5\, S^{-1}(k_-) - 2\,m_q(\mu)\,\Gamma_5(k;P)$$

- Pseudoscalar mesons show up as poles in the axial-vector and pseudoscalar vertices, $\Gamma_{5\mu}(k;P)$ and $\Gamma_5(k;P)$
- Residues of these poles

$$\begin{aligned} f_{PV}\, P_\mu &= Z_2 \int_q \mathrm{Tr}\big[S(q_+)\,\Gamma_H(q;P)\,S(q_-)\,\gamma_5\gamma_\mu\big] \\ r_{PS}(\mu) &= Z_4 \int_q \mathrm{Tr}\big[S(q_+)\,\Gamma_H(q;P)\,S(q_-)\,\gamma_5\big] \end{aligned}$$

are related via AV-WTI: $\quad f_{PV}\, m_H^2 \;=\; 2\,r_{PS}(\mu)\,m_q(\mu)$

- Combination $r_{PS}(\mu)m_q(\mu)$ is renormalisation-point independent and gauge independent

PM, Roberts, Tandy, PLB420, 267 (1998) [nucl-th/9707003]

Current Conservation

- Impulse approximation

$$\Lambda_\nu(P,Q) = 2\,P_\nu\, F_\pi(Q^2)$$
$$= N_c \int \mathrm{Tr}\Big[S(q)\,\Gamma_\pi(k_+;P_-)S(q_+)\,i\Gamma_\nu(q;Q)\,S(q_-)\,\bar{\Gamma}_\pi(k_-;P_+)\Big]$$

- Insertion of vector Ward–Takahashi identity

$$i\,Q_\mu\,\Gamma_\mu(q_+,q_-;Q) = S^{-1}(q+Q/2) - S^{-1}(q-Q/2)$$

gives $Q_\mu \Lambda_\nu(P,Q) = 0$

provided that a translationally-invariant regularisation of divergent integrals is used

Current Conservation

- Impulse approximation

$$\Lambda_\nu(P,Q) = 2\,P_\nu\;F_\pi(Q^2)$$
$$= N_c \int \mathrm{Tr}\Big[S(q)\,\Gamma_\pi(k_+;P_-)S(q_+)\,i\Gamma_\nu(q;Q)\,S(q_-)\,\bar\Gamma_\pi(k_-;P_+)\Big]$$

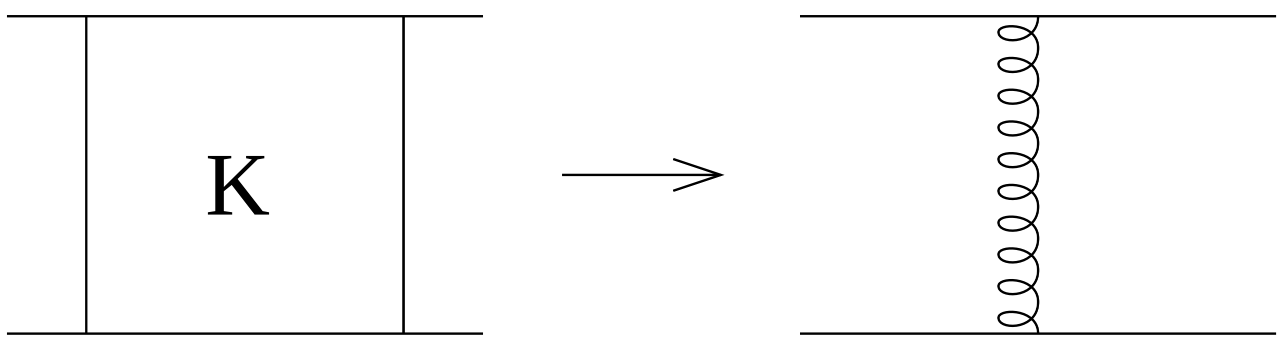
- Canonical normalisation condition for mesons

$$2\,P_\mu = N_c\,\frac{\partial}{\partial P_\mu}\int^\Lambda \frac{d^4q}{(2\pi)^4}\left\{\mathrm{Tr}\Big[\bar\Gamma_H(q;Q)S(q+\tfrac{1}{2}P)\,\Gamma_H(q;Q)\,S(q-\tfrac{1}{2}P)\Big] + \int^\Lambda \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\Big[\bar\chi_H(k;Q)\,K(k,q;P)\,\chi_H(q;Q)\Big]\right\}$$

- Insertion of differential WTI identity gives $F_\pi(0) = 1$ provided that $K(q,p;P)$ is independent of P

Current Conservation – continued

- If $K(q,p;P)$ is independent of P
  and quark-photon vertex Γ_μ satisfies WTI
  then e.m. current conserved in impulse approximation
- Kernel $K(q,p;P)$ is independent of P
  in rainbow/ladder approximation



$$\mathrm{K}(p,k;P) \;\rightarrow\; -\alpha\big((p-k)^2\big)\, D^{\mathrm{free}}_{\mu\nu}(p-k)\;\gamma_\mu \frac{\lambda^a}{2} \otimes \gamma_\nu \frac{\lambda^a}{2}$$

Current Conservation – continued

- If $K(q,p;P)$ is independent of P
  and quark-photon vertex Γ_μ satisfies WTI
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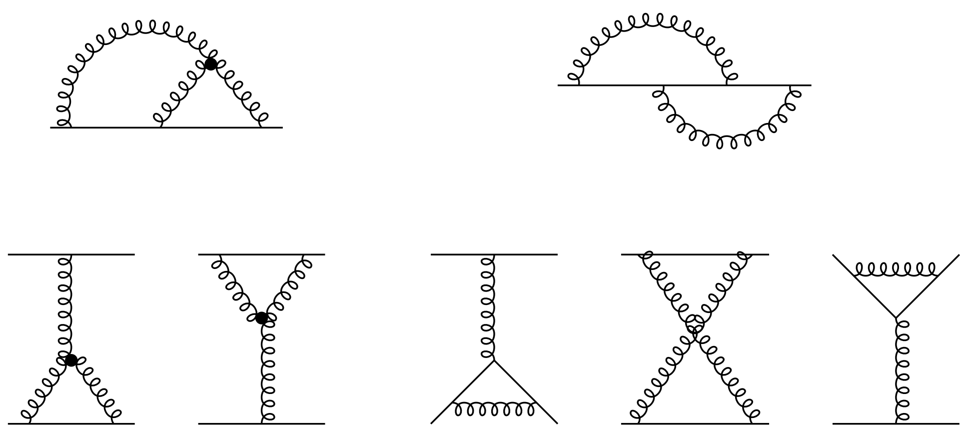
- Kernel $K(q,p;P)$ is independent of P
  in rainbow/ladder approximation
- Beyond rainbow/ladder/impulse approximation

  corrections to rainbow DSE and ladder BSE



Current Conservation – continued

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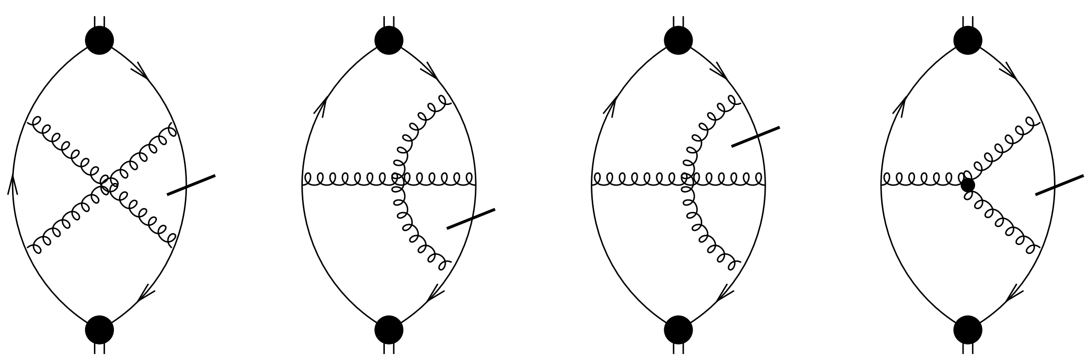
  in rainbow/ladder approximation
- Beyond rainbow/ladder/impulse approximation

  corrections to the BS normalisation condition



Current Conservation – continued

- If $K(q, p; P)$ is independent of P
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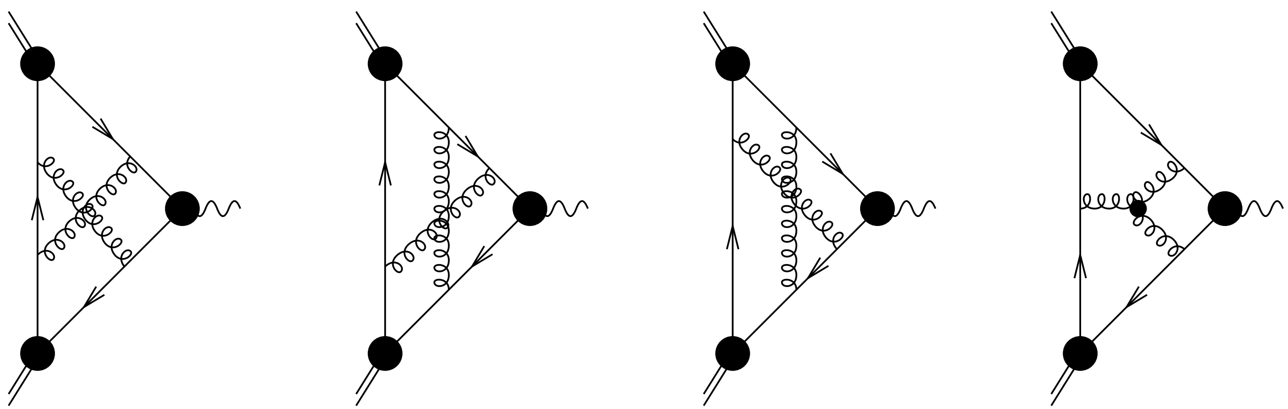
- Beyond rainbow/ladder/impulse approximation

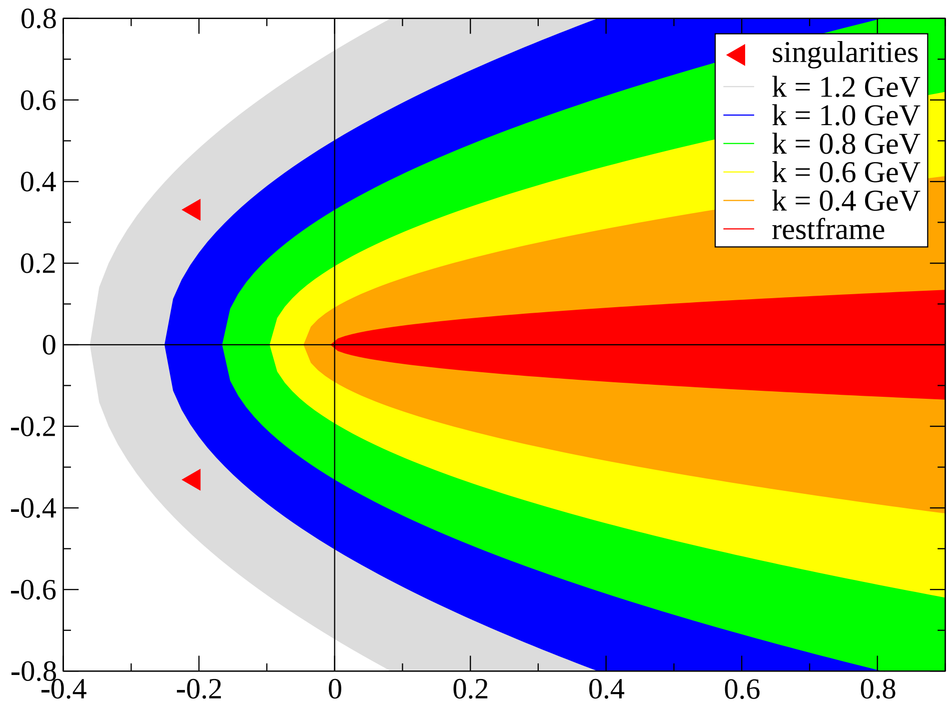
  corrections to impulse approximation



Quark propagator in complex plane

Range covered by argument of quark propagator in the BSE increases with increasing 3-momentum of the meson

$$k_{\pm} = k \pm P/2 = k^2 + iEk\cos\alpha + kp\sin\alpha\cos\beta - m^2$$



Limited by pair of c.c. singularities at $k^2 = -0.207 \pm 0.331 \mathrm{GeV}^2$

Analytic structure of QCD propagators

Alkofer, Detmold, Fischer, PM, PRD70, 014014 (2004) [hep-ph/0309077] and references therein

Gluon propagator: singularity at origin

Lerche, von Smekal, PRD65, 125006 (2002)

Zwanziger, PRD 65, 094039 (2002)

Fischer, Alkofer, and Reinhardt, Phys. Rev. D 65, 094008 (2002)

Alkofer, Fischer, von Smekal, Acta Phys. Slov. 52, 191 (2002)

Fischer, Alkofer, PRD 67, 094020 (2003)

Pawlowski, Litim, Nedelko, von Smekal, PRL93 (2004) 152002

. . .

Analytic structure of QCD propagators

Alkofer, Detmold, Fischer, PM, PRD70, 014014 (2004) [hep-ph/0309077] and references therein

Gluon propagator: singularity at origin

Quark propagator:

- Real mass singularity on time-like axis
  - simple pole
  - start of brach-cut
- Pair of c.c. mass-like singularities
  - simple pole
  - start of brach-cut
- Entire function
  essential singularity at infinity,
  serious problems for Wick rotation
- Non-analytic function, then we’re in trouble . . .

Asymptotic behavior $\pi\gamma\gamma$ form factor

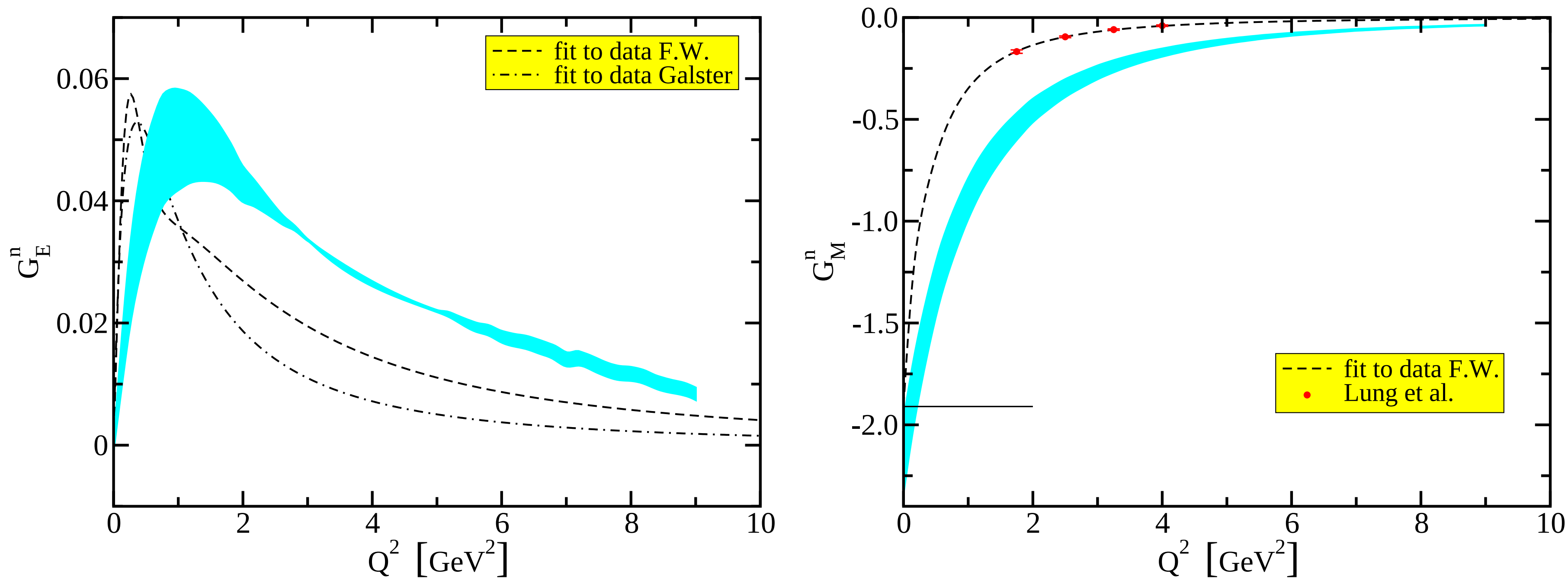
Asymmetric: $Q_1^2 = Q^2$, $Q_2^2 = 0$, $\omega = 1$

	$J(1)$
lightcone pQCD (BL, 1980)	2
Anikin *et al.*	1.8
current DSE model numerical estimate	1.7
experimental estimate	1.6

Symmetric: $Q_1^2 = Q_2^2$, $\omega = 0$

	$J(1)$
Anikin *et al.*, PLB475, 361 (2000)	4/3
DSE analysis (model independent)	4/3
current DSE model numerical result	1.3

Model calculations – Neutron form factors



Fit to data: Friedrich, Walcher, Eur.Phys.J.A17, 607 (2003) [hep-ph/0303054]

Fit to data: Galster *et al*, NPB32, 221 (1971) ; Lung *et al*, PRL70, 718 (1993)

Chiral loop corrections

Corrections estimated using

$$\langle r_N^2 \rangle^{\text{1-loop}} = \mp \frac{1+5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2+\lambda^2}\right)$$

$$\langle (r_N\mu)^2 \rangle^{\text{1-loop}} = -\frac{1+5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2+\lambda^2}\right) + \frac{g_A^2\, m_N}{16\pi^2 f_\pi^2 \mu_N} \frac{2}{m_\pi} \arctan\left(\frac{\lambda}{m_\pi}\right)$$

$$(\mu_N^2)^{\text{1-loop}} = \mp \frac{g_A^2\, m_N}{4\pi^2 f_\pi^2} \frac{2m_\pi}{\pi} \arctan\left(\frac{\lambda^3}{m_\pi^3}\right)$$

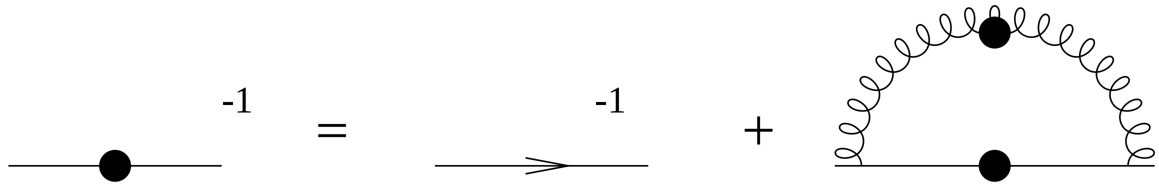
for regularisation-dependent pion loops

Ashley, Leinweber, Thomas, Young, Eur.Phys.J.A19, 9 (2004)

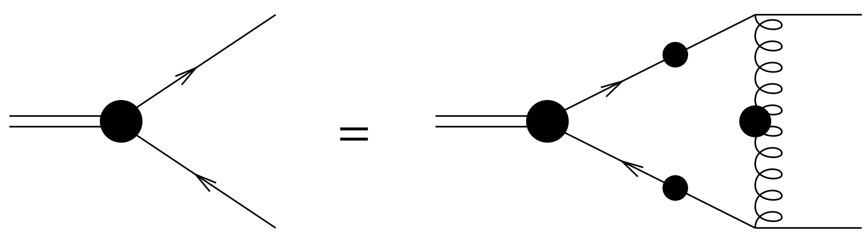
effective range: $\lambda \approx 0.3\ \text{GeV}$ so $R \approx 0.7\ \text{fm}$

Rainbow–Ladder truncation

- Rainbow approximation for quark DSE
  $Z_1^g\, g^2 D_{\mu\nu}(q)\, \Gamma_\nu(k,p) \;\rightarrow\; Z_2^2\, \mathcal{G}(q^2)\, D_{\mu\nu}^{\rm free}(q)\, \gamma_\nu$



- Ladder approximation for meson and vertex
  $\mathrm{K}(p,k;P) \;\rightarrow\; -Z_2^2\, \mathcal{G}(q^2)\, D_{\mu\nu}^{\rm free}(q)\; \gamma_\mu \frac{\lambda^a}{2} \otimes \gamma_\nu \frac{\lambda^a}{2}$
  with $q = p - k$



- Respects vector and axialvector Ward identities