QCD modeling of Hadron Physics: m_q Dependence and DSE Truncations

Peter Tandy
tandy@kent.edu
Center for Nuclear Research
Kent State University

KENT STATE
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Collaborators

- Pieter Maris, Univ of Pittsburgh
  - Much of this not possible without him
- Mandar Bhagwat, Kent State University
- Mike Pichowsky, Kent State University
- Craig Roberts, Argonne National Laboratory

Topics

- Context of QCD modeling of hadron observables via DSEs
  - Ladder-rainbow truncation and symmetry constraints
  - Ordering/organization of mechanisms
- Comparison with lattice-QCD: m_q dependence
- Limitations/failures of ladder-rainbow
- Beyond ladder-rainbow—some consequences from 3 gluon coupling
- m_q dependence as a diagnostic/guide beyond ladder-rainbow
- Summary

Organization chosen for DSE-based modeling

- EOMs: DSEs $\int \, D\bar{q}qG \; \frac{\delta}{\delta q(x)} \, e^{-\mathcal{S}[\bar{q},q,G]+(\bar{\eta},q)+(\bar{q},\eta)+(J,G)} = 0$
- Euclidean metric, p-space, covariant, no 3-space reduction
- Truncate to minimum 2-point, 3-point fns; IR phenomenology for ignorance
- Insist on preserving 1-loop QCD renorm group in UV
- Analtyic contin in external hadronic P^2 to mass shells
- Constraints for truncation: vector WTI, axial vector WTI
  E.g.
  $$-iP_\mu \Gamma_{5\mu}(k;P) = S^{-1}(k_+)\gamma_5 \frac{\tau}{2} + \gamma_5 \frac{\tau}{2} S^{-1}(k_-) \\ -2\, m_q(\mu)\, \Gamma_5(k;P)$$
- $\Rightarrow$ kernels of DSE$_q$ and $K_{\rm BSE}$ are related

Organization chosen for DSE-based modeling (2)

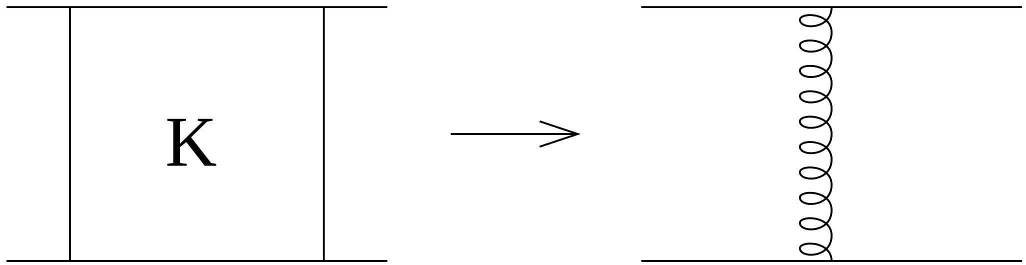
- Constraints for truncation: vector WTI, axial vector WTI
  - E.G. at 2,3-point fn level:
  - Rainbow DSE, ladder BSE, and IA for $F_\pi(Q^2)$ are symm-matched set
  - $\Rightarrow F_\pi(Q^2=0)=Q_\pi=1$, always
  - $\Rightarrow$ leading asymptotic $F_\pi(Q^2)$ phys content present
  - Hopefully the interpolation can't go too wrong
  - Present IR phenomenology: 1 param to fit $\langle\bar{q}q\rangle_\mu$
  - Goldstone nature of ps octet, and phys masses from explicit ch symm breaking, will always be correct—indep of model details
- A systematic symm-preserving correction scheme is available

Organization chosen for DSE-based modeling (3)

- DSE approach emphasizes p^2-depn of q-masses, connecting constituent to current masses
  e.g. DCSB$\Rightarrow$ π: $\Gamma^0_\pi(p^2) = i\gamma_5 \ \left[\frac{1}{4}\mathrm{tr}S_0^{-1}(p^2)\right] \ / f^0_\pi + \cdots$
- Can compare intermediate quantities with lattice-QCD for important guidance
- Present DSE organization emphasizes a certain ∞ sub-class of multiple gluon components
- Weakness: present choice of truncation may not be efficient for all processes
- Efficiency of description of observables is final guide

Ladder-Rainbow Model

- short-range part of interaction kernel fixed by pQCD —-one-gluon exchange with 1-loop renormalization group improvement



- $K_{\rm BSE} \rightarrow -\gamma_\mu \frac{\lambda^a}{2} 4\pi\alpha_{\rm eff}(q^2) D^{\rm free}_{\mu\nu}(q)\gamma_\nu \frac{\lambda^a}{2}$
- $\frac{Z_{1F}^2(\mu,\Lambda)}{Z_2^2(\mu,\Lambda)\, Z_3(\mu,\Lambda)} \rightarrow \left[\frac{\alpha_s(\Lambda^2)}{\alpha_s(\mu^2)}\right]$

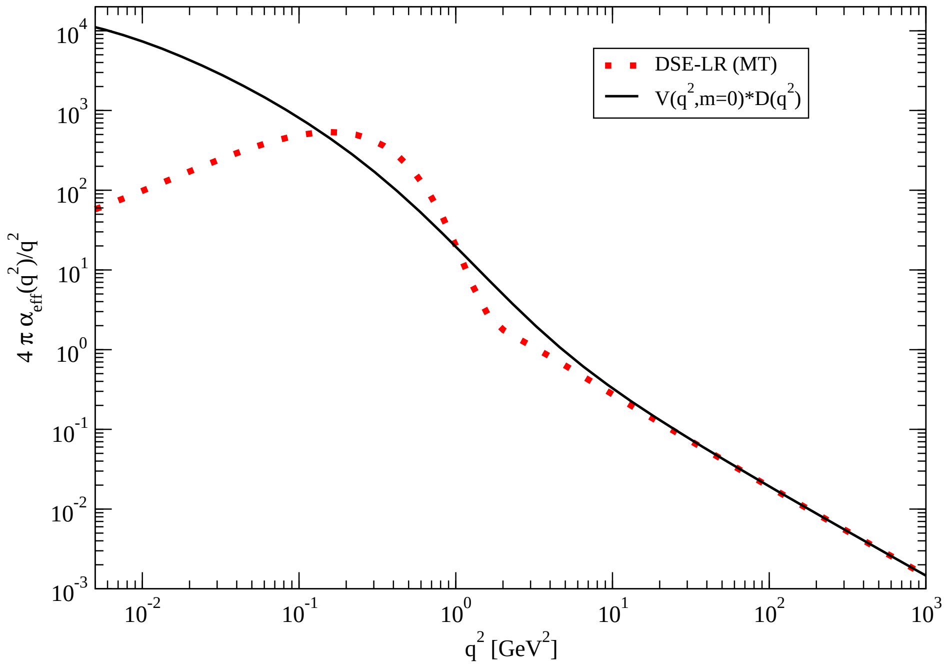
  $\alpha_{\rm eff}(q^2) \underset{UV}{\rightarrow} \alpha_s^{1-{\rm loop}}(q^2) = \frac{12\,\pi}{(11N_c - 2N_f)\ln(q^2/\Lambda^2)}$
- first term in a systematic expansion

Ladder-Rainbow Model

- short-range part of interaction kernel fixed by pQCD —-one-gluon exchange with 1-loop renormalization group improvement
- long-range part (IR, low-k^2) of interaction kernel fixed by $\langle \bar{q}q \rangle_{\mu=1\ \mathrm{GeV}} = -(240\mathrm{MeV})^3$
- single model parameter:

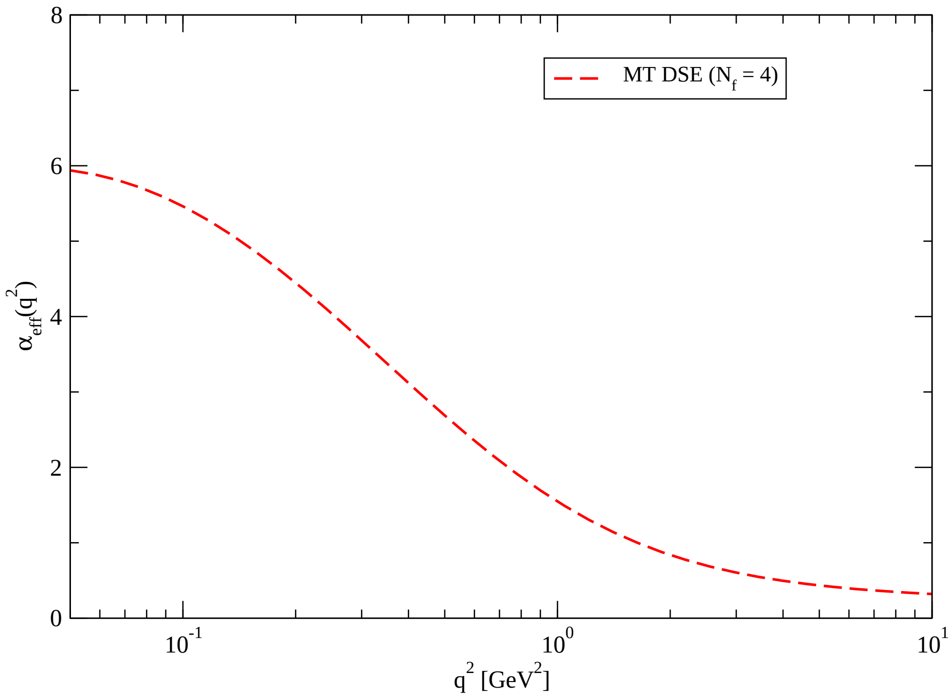
gluon mass scale $\sim$ 700 MeV

PM & P.C. Tandy, PRC60, 055214 (1999)



Ladder-Rainbow Model

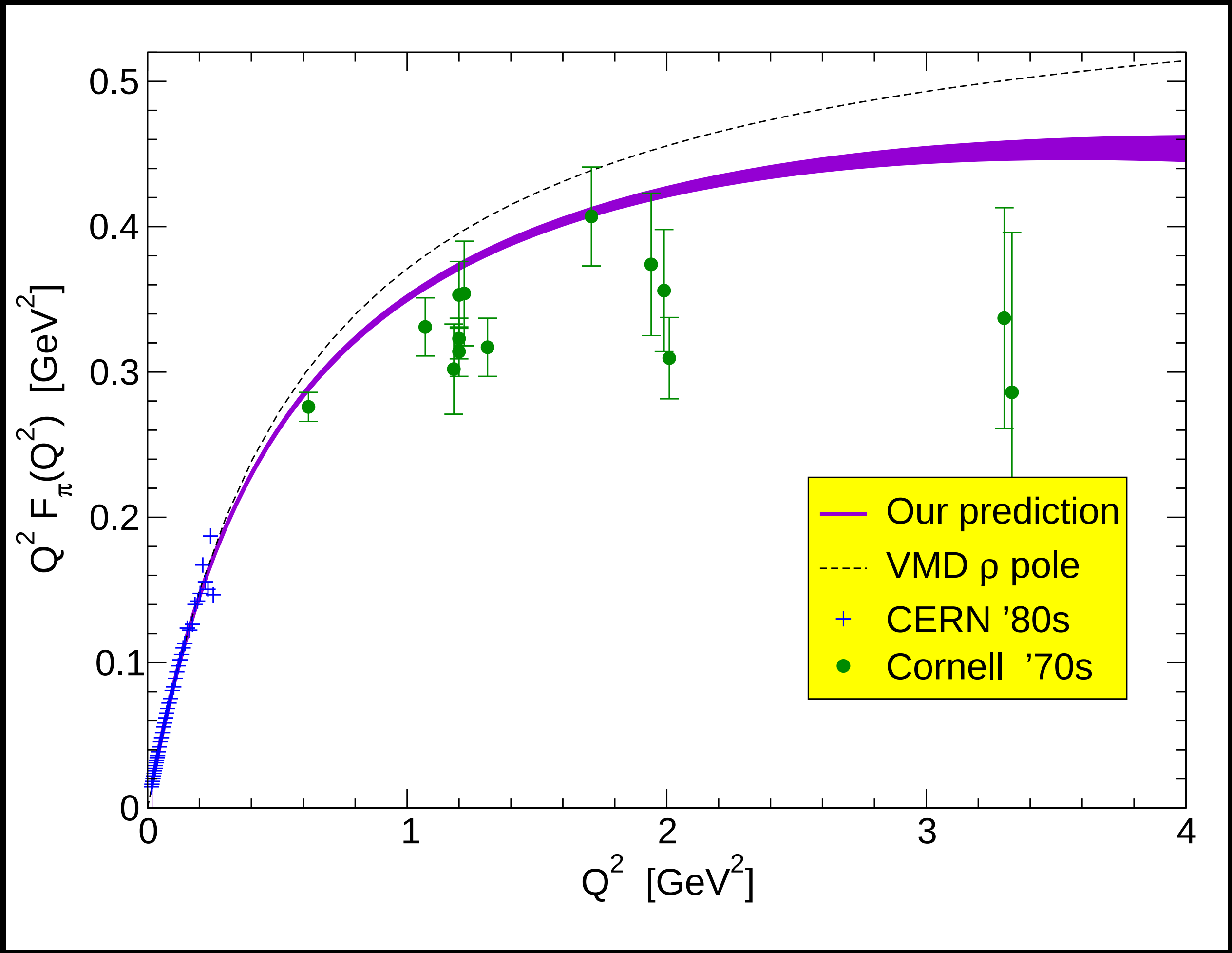
- short-range part of interaction kernel fixed by pQCD —-one-gluon exchange with 1-loop renormalization group improvement
- long-range part (IR, low-k^2) of interaction kernel fixed by $\langle \bar{q}q \rangle_{\mu=1\ \mathrm{GeV}} = -(240\mathrm{MeV})^3$
- Effective running coupling



Ladd-Rainb Model: Performance and Limitations

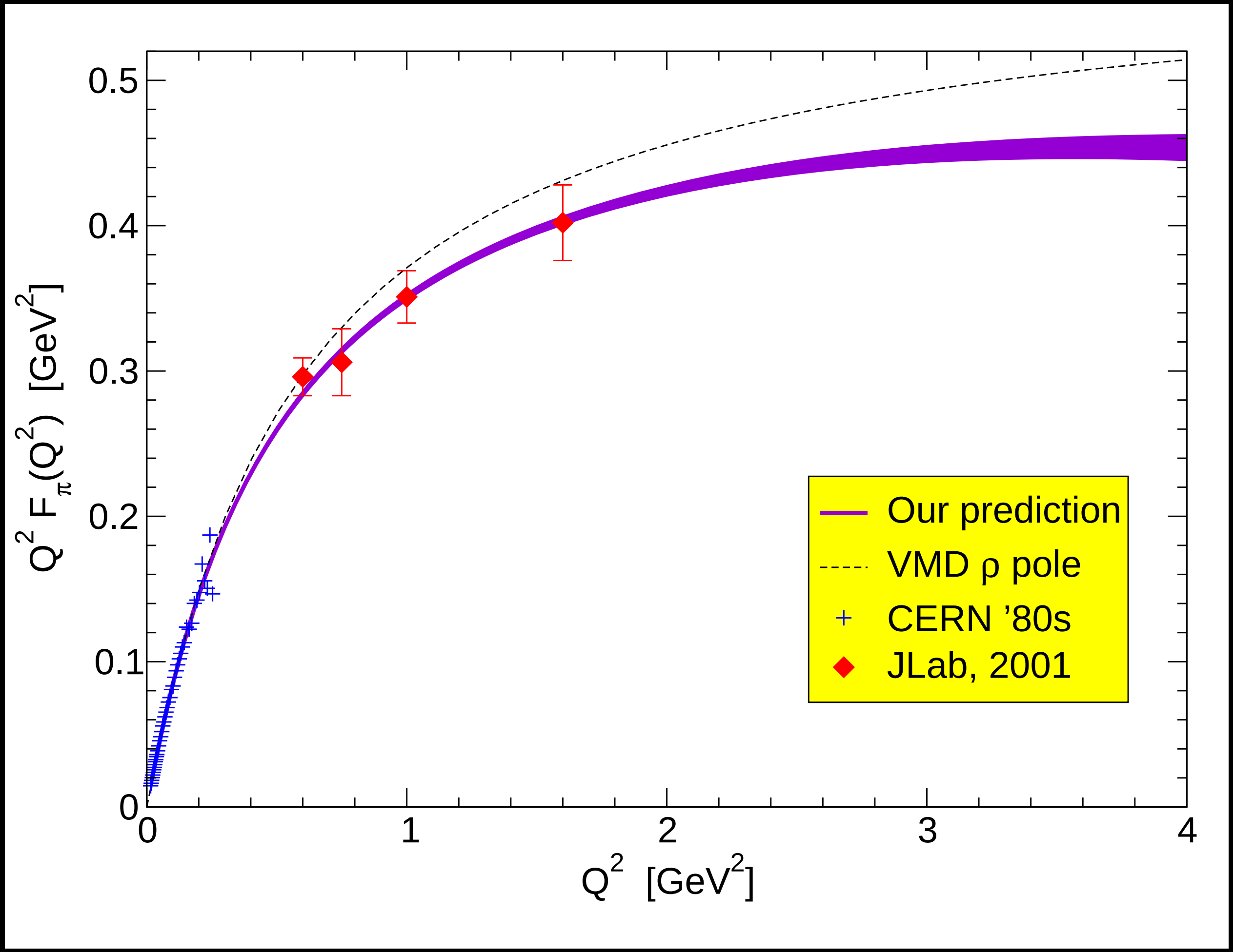
- Corrections are small in ps and vect meson channels
  Bender, et al. PLB380, 7 (96); Bender, Detmold, et al. PRC65, 065203 (02);
  Bhagwat, Höll, Krassnigg, Roberts & Tandy, PRC70, 035205 (2004)
- 1-parameter for $\langle \bar{q} q \rangle_\mu \Rightarrow$
- $M_\rho, M_\phi, M_{K^\star}$ to 5%; $f_\rho, f_\phi, f_{K^\star}$ to 10%
- Em form factors $Q^2 < 5$ GeV^2 good, but chiral loops not in
- Strong decays, em transition form factors satisfactory
- Limitations–corrections to ladder-rainbow needed:
  - Present LR model too attractive for axial vectors M_{a_1}, M_{b_1}
  - Chiral loops have to be added
  - Need extension to non-Abelian axial anomaly and $\eta - \eta'$
  - Etc

Pion electromagnetic form factor



PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]

Pion electromagnetic form factor



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JLab data from Volmer *et al*, PRL86, 1713 (2001) [nucl-ex/0010009]

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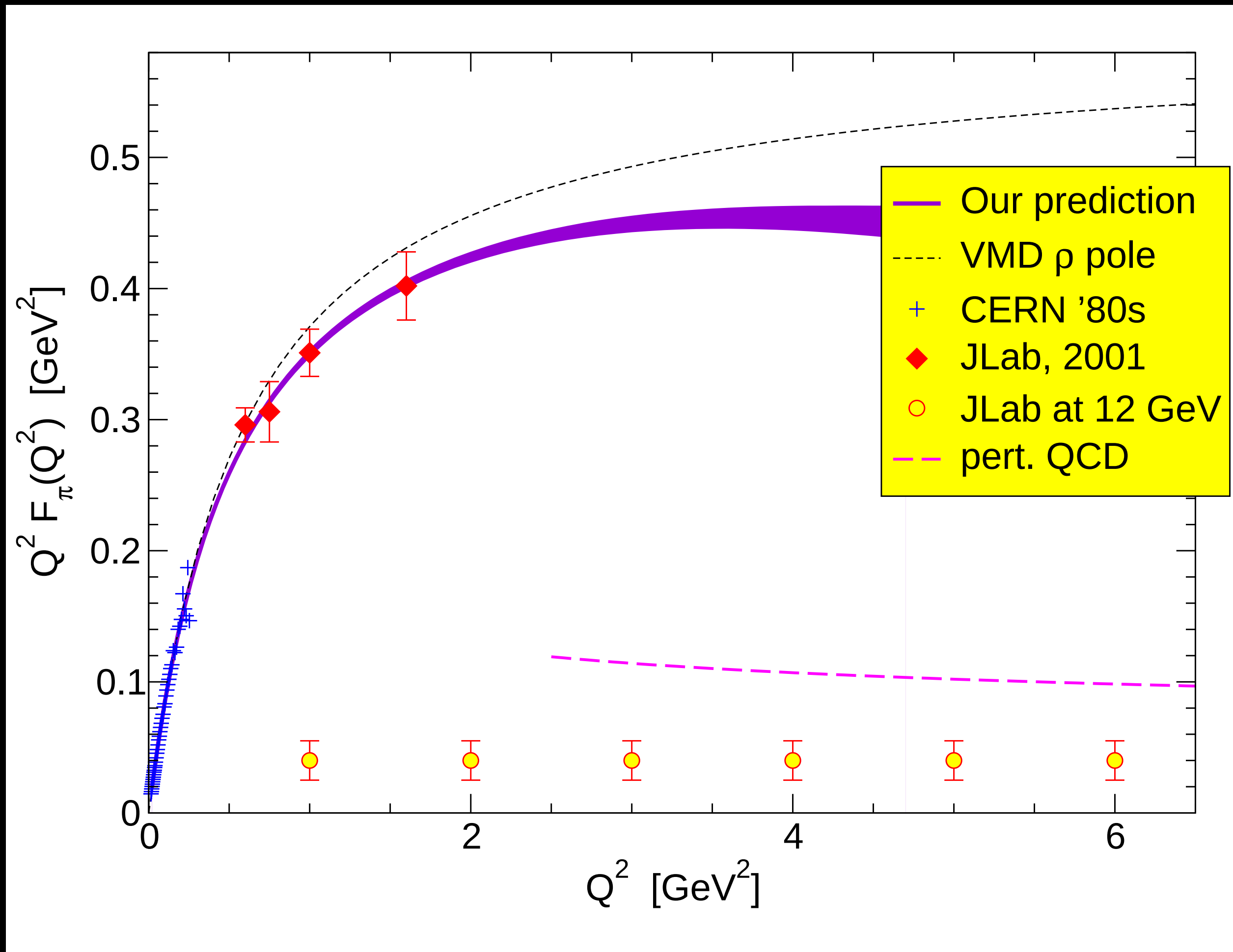

PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]

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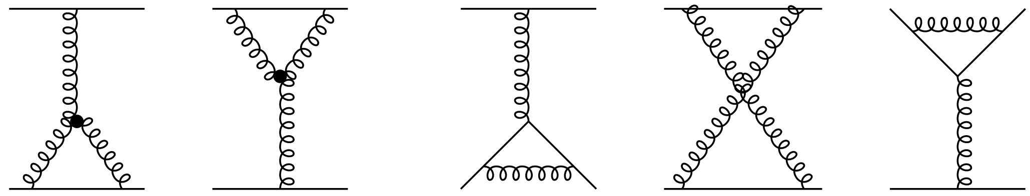
Beyond Ladder-Rainbow

- Preservation of AV-WTI: dressing of $\bar{q}\gamma q$ vertex in DSE $\Rightarrow$ corrections to ladd-rainb $K_{\rm BSE}$
- qq mesons: Feyn diagrammatic $\Sigma \Rightarrow K_{\rm BSE}$
- If Σ incl:

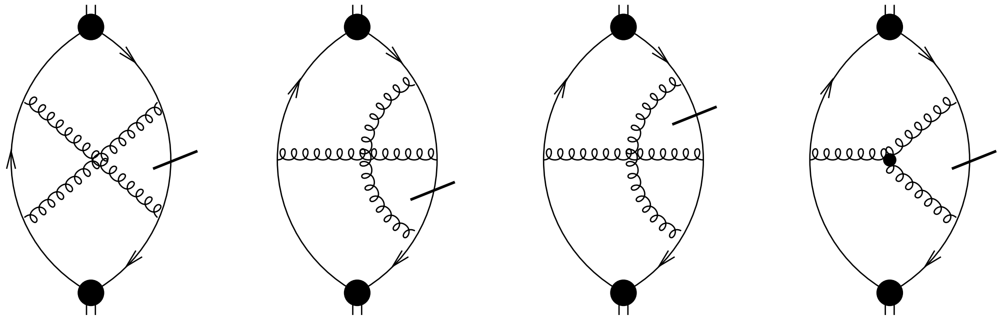
- $K_{\rm BSE} = -\frac{\delta\Sigma}{\delta S}$ incl:



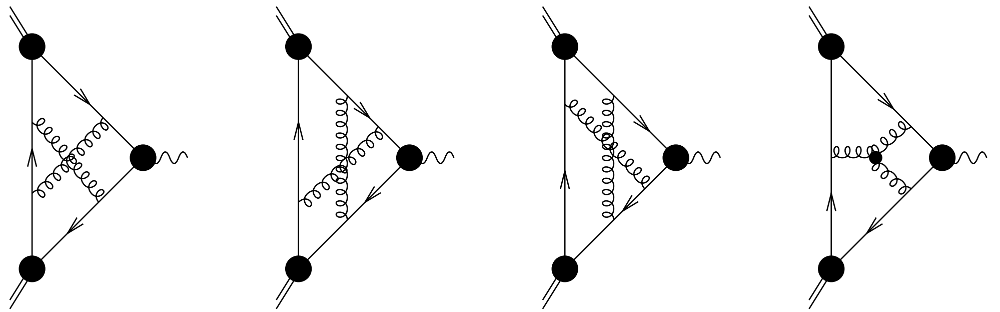
- $\frac{\delta}{\delta S}$ [closed q-loop] $\Rightarrow$ annihilation kernel for flavor singlets, e.g. $\eta - \eta'$

Beyond ladder-rainbow ⇒ beyond IA

Corrections to the ladd-rainb truncation ⇒ corresp corrections to the BSE norm condition



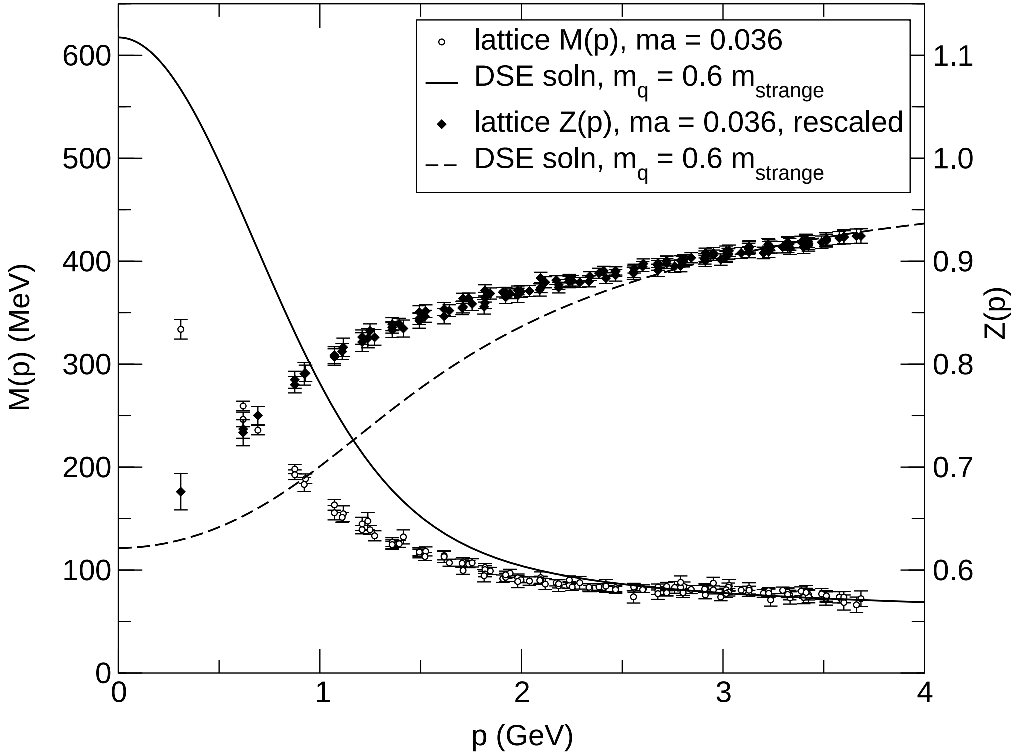
⇒ (via vector Ward Id) corresp corrections to IA for $F_\pi(Q^2 = 0)$:



and so on $\cdots$. Different organizations may absorb some into a wavefn.

Quark Propagator: DSE and Qu-Lattice

- Lattice: Bowman, Heller, Williams, PRD66, hep-lat/0203001
- Rainbow DSE: Jarecke, Maris, PCT, PRC67, nucl-th/0208019
- $S(p) = Z(p)\,[i\,\not{p} + M(p)]^{-1}$
- Ladd-rainb model too strong (attractive) in IR
- Beyond ladd-rainb needed
- Untangle vertex dressing from $D_{\rm gluon}(q)\cdots$



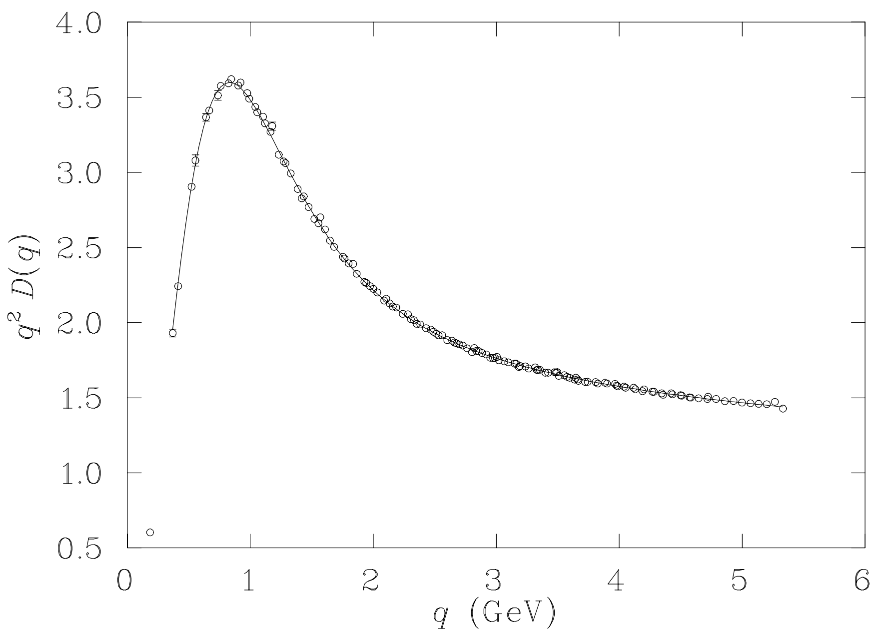
Untangle Vertex from Gluon Propagator

- Qu-lattice $D_{\rm gluon}(q)$

  Leinweber et al PRD60

  hep-lat/9811027

- Find $\Gamma^{\rm eff}_\nu(q,p)$ so DSE produces $S_{\rm latt}(p)$ from $D_{\rm latt}(q)$

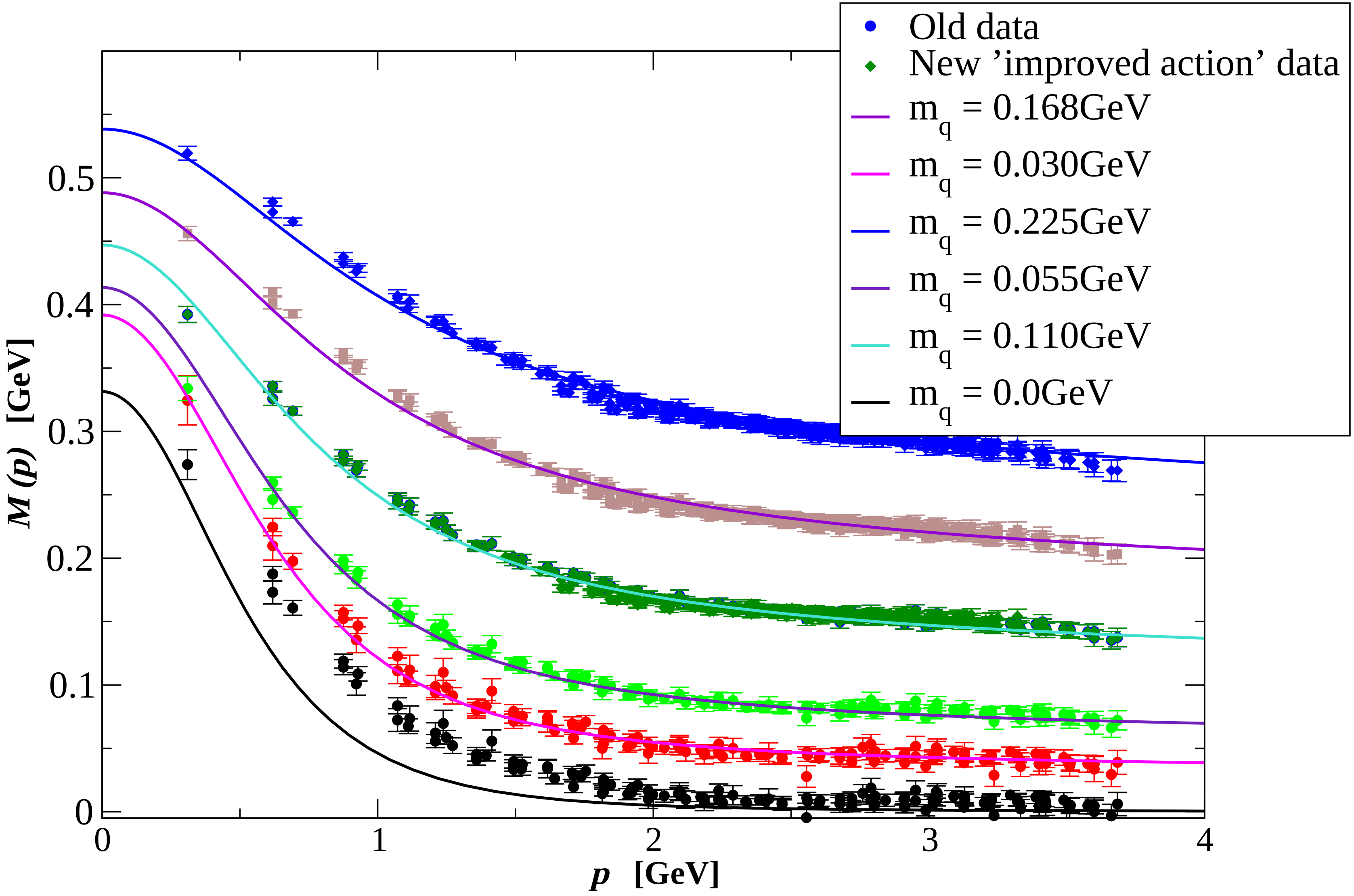


—Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (03)

$$g^2\gamma_\mu D(p-q)\, Z_{1F}(\mu,\Lambda)\, \Gamma_\nu(q,p) \to \gamma_\mu g^2 D(p-q)\gamma_\nu\, V(p-q)$$

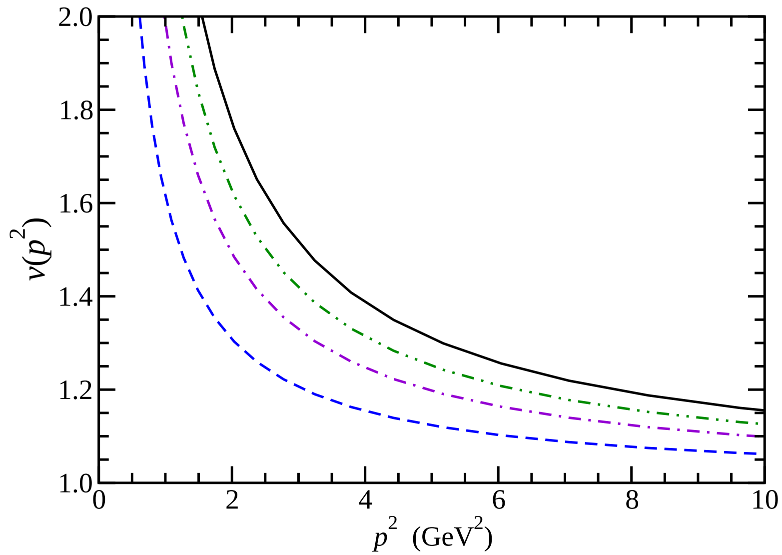
UV limit: $g^2\, D(k^2)\, V(k^2) \to \frac{4\pi\alpha_s^{1-\rm loop}(k^2)}{k^2}$

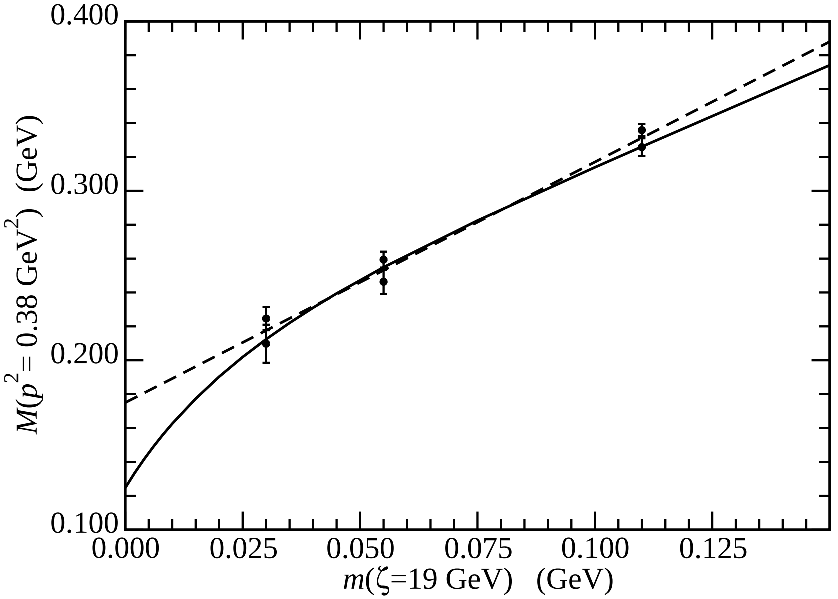
Qu-lattice $S(p), D(q)$ mapped to a DSE kernel



Lattice-assisted DSE Results

- Evident vertex enhancement
- Curvature in low m_q depn
- $M^{\mathrm{IR}}(p^2)$ 40% below linear
- Chiral Extrapolation
- $\langle \bar{q}q \rangle^{\mathrm{qu-lat}}_{\mu=1\ \mathrm{GeV}} = -(190\ \mathrm{MeV})^3$
- $\langle \bar{q}q \rangle^{\mathrm{qu-lat}} \approx \langle \bar{q}q \rangle^{\mathrm{expt}}/2$
- f_π 30% low

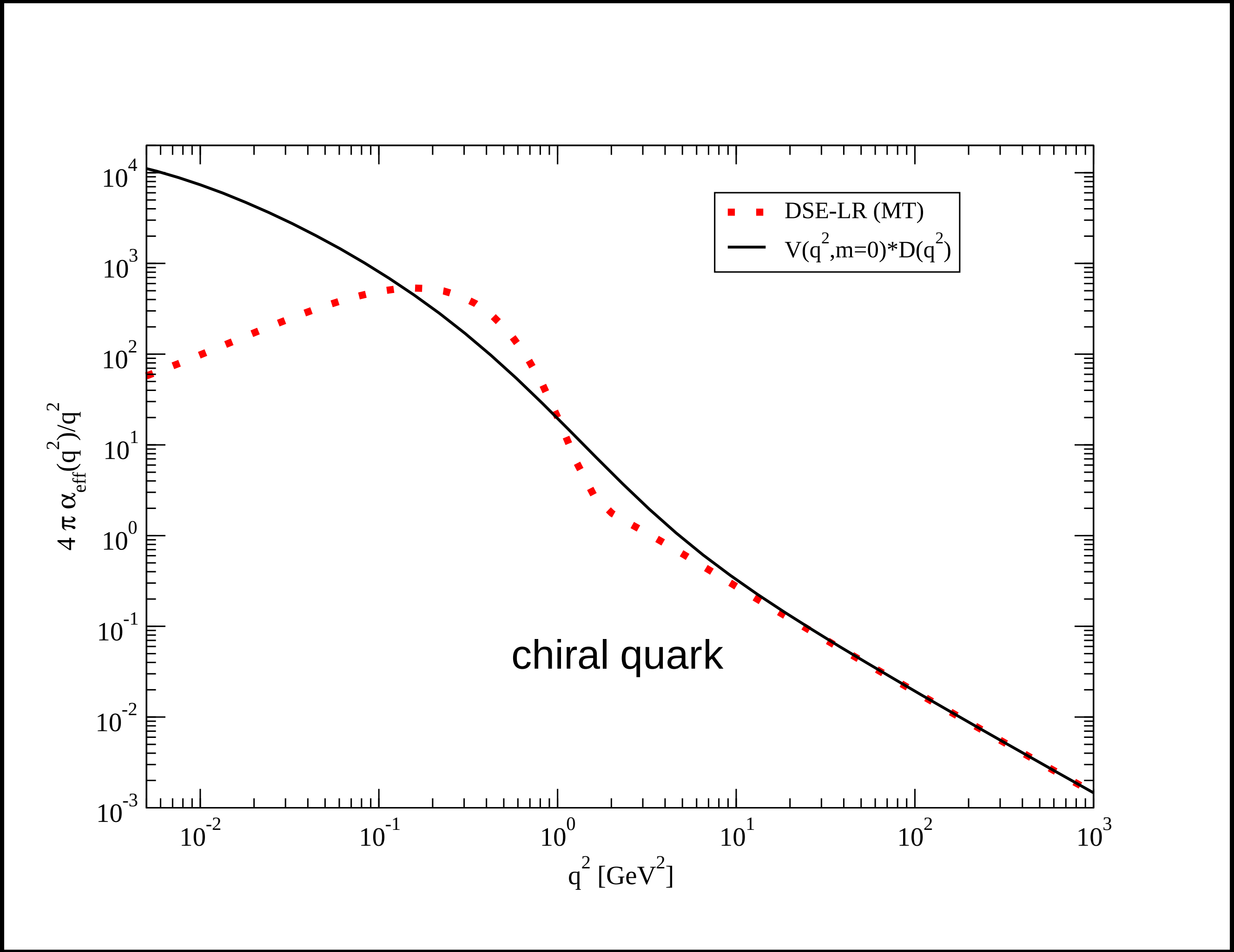




Lattice-assisted DSE Results

- Quenched lattice unable to provide a direct DSE model of observables
- But it indicates a strategy for further development $\cdots$

Quenched lattice $\Rightarrow m_q$ *Depn of DSE Kernel*



Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (2003)

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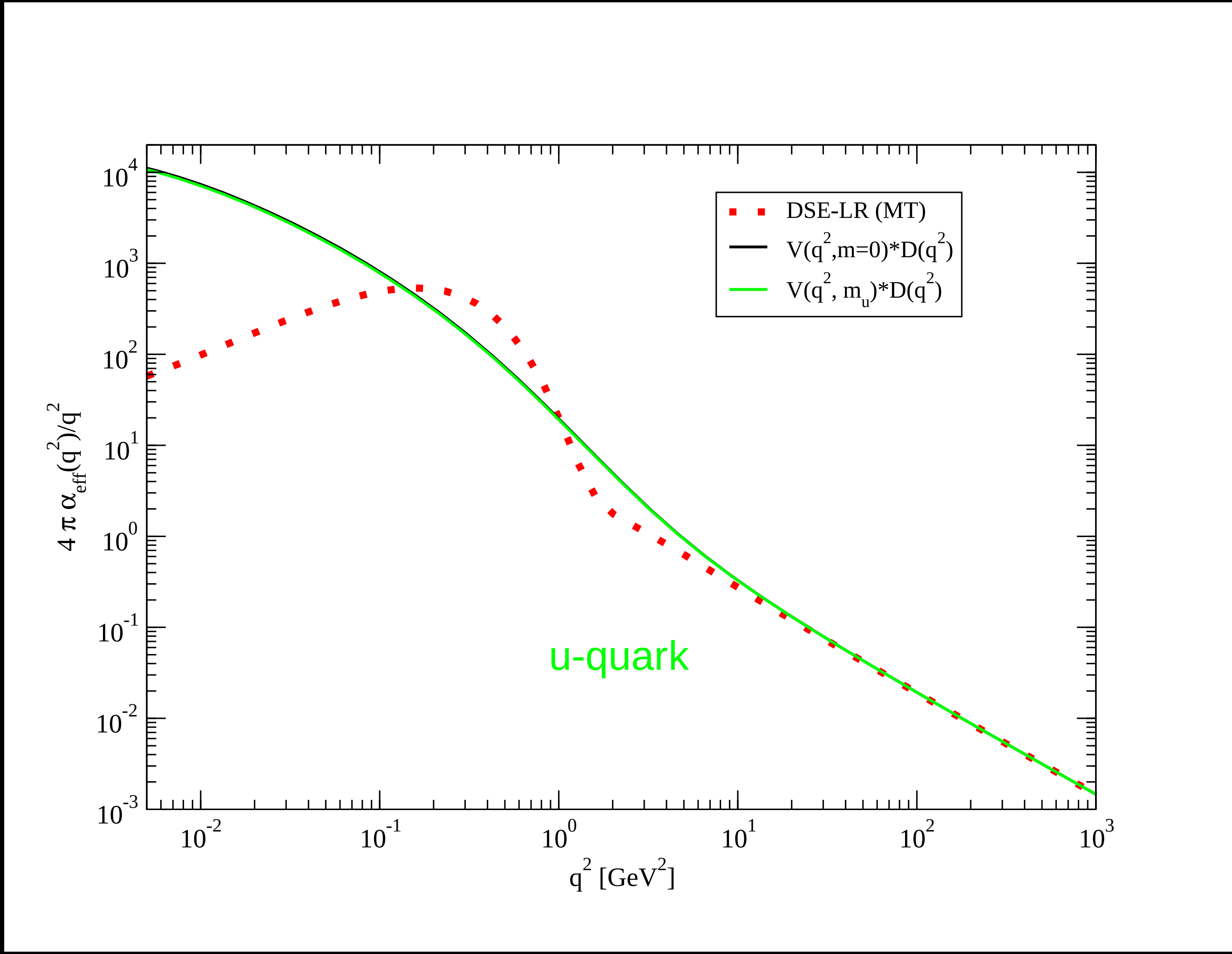

Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (2003)

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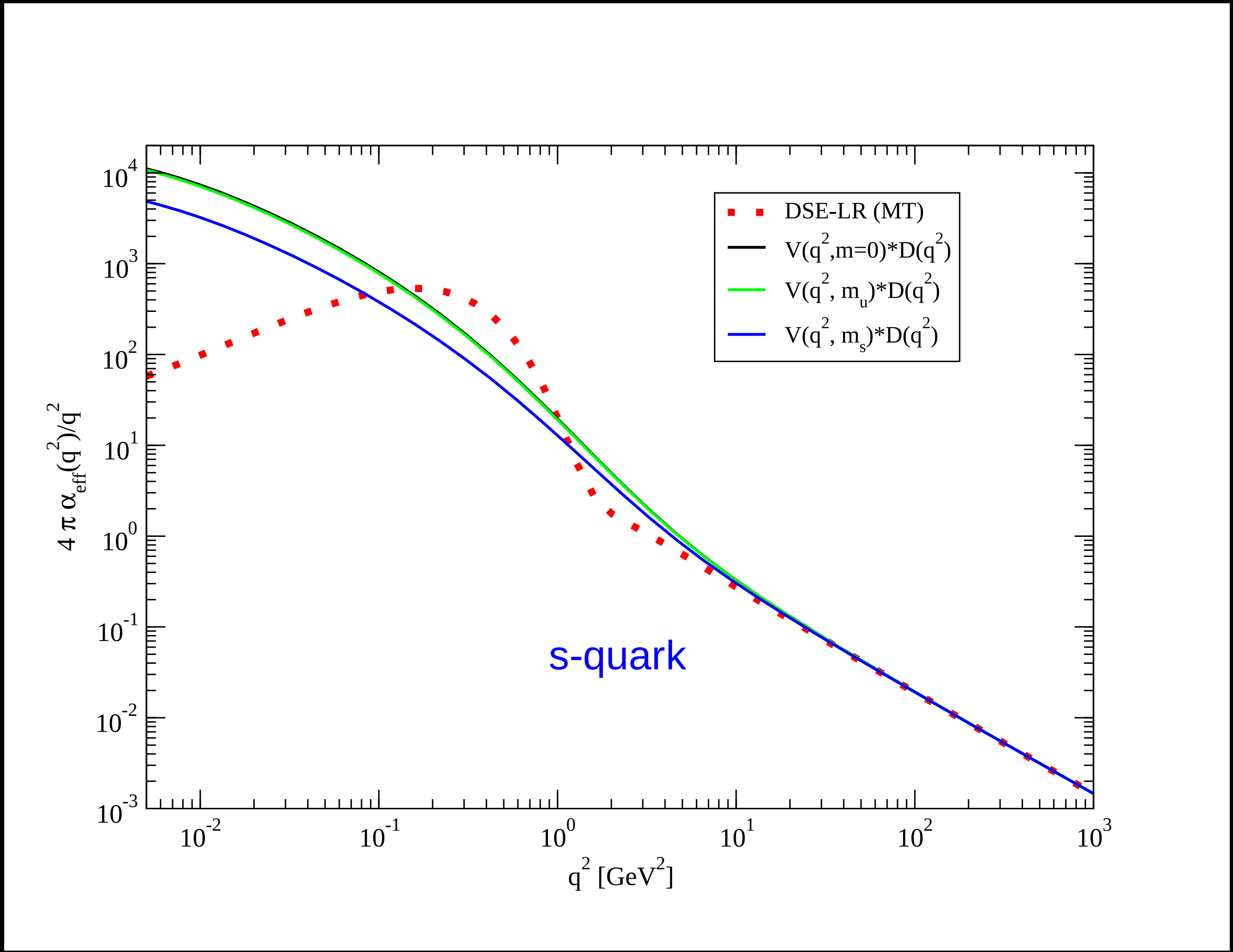
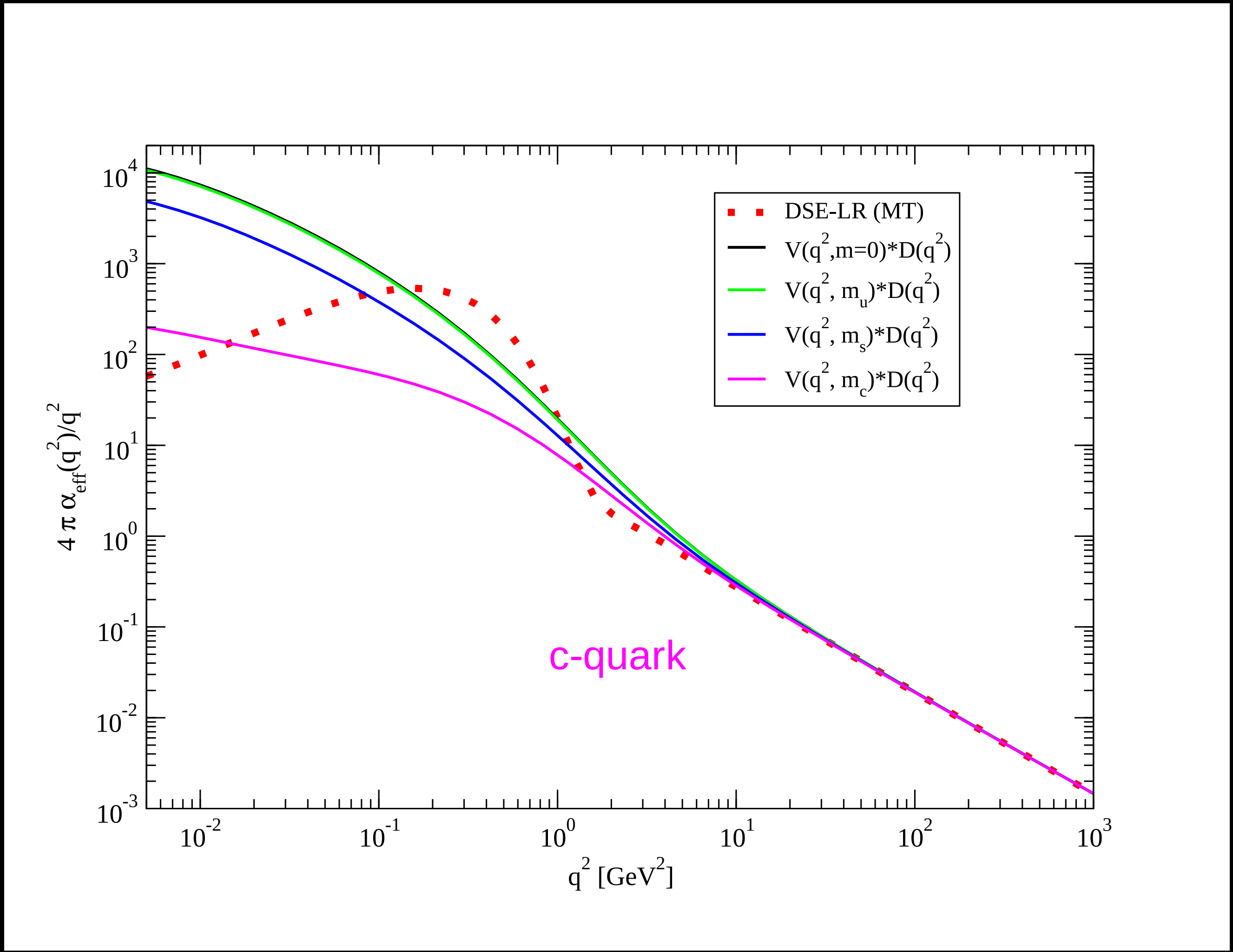


Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (2003)

Quenched lattice ⇒ m_q Depn of DSE Kernel



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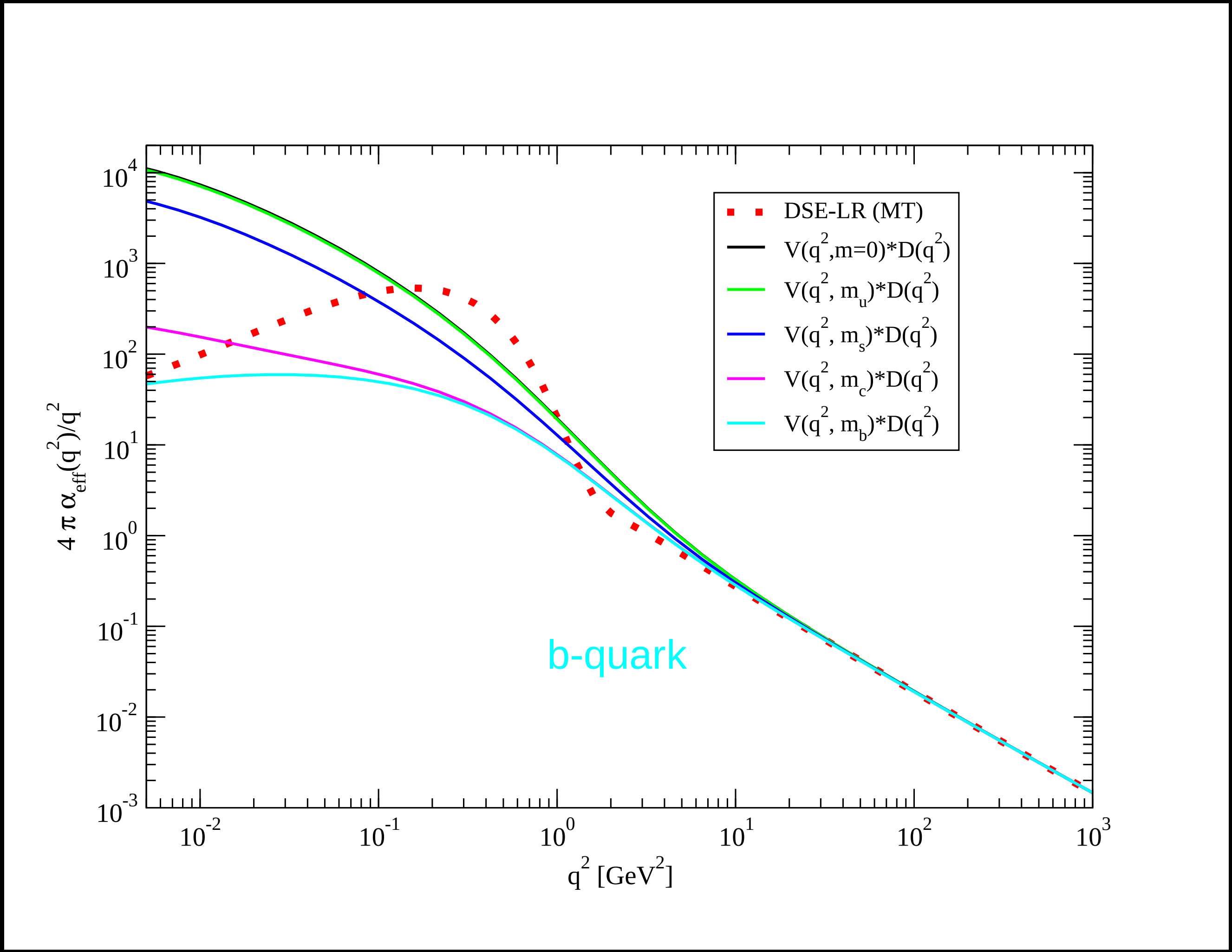
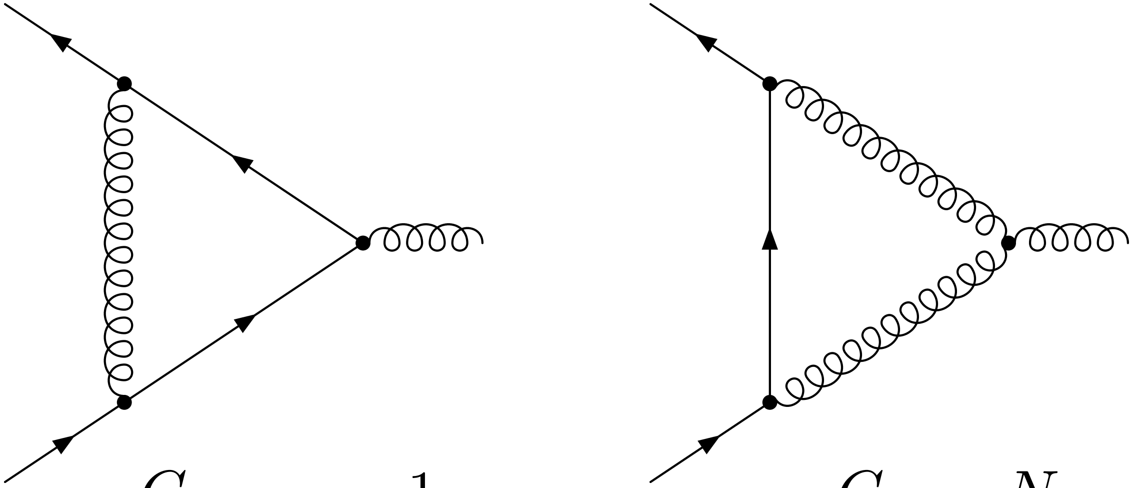
Quenched lattice $\Rightarrow m_q$ *Depn of DSE Kernel*



Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (2003)

Dressed gluon-quark vertex: 1-loop pQCD



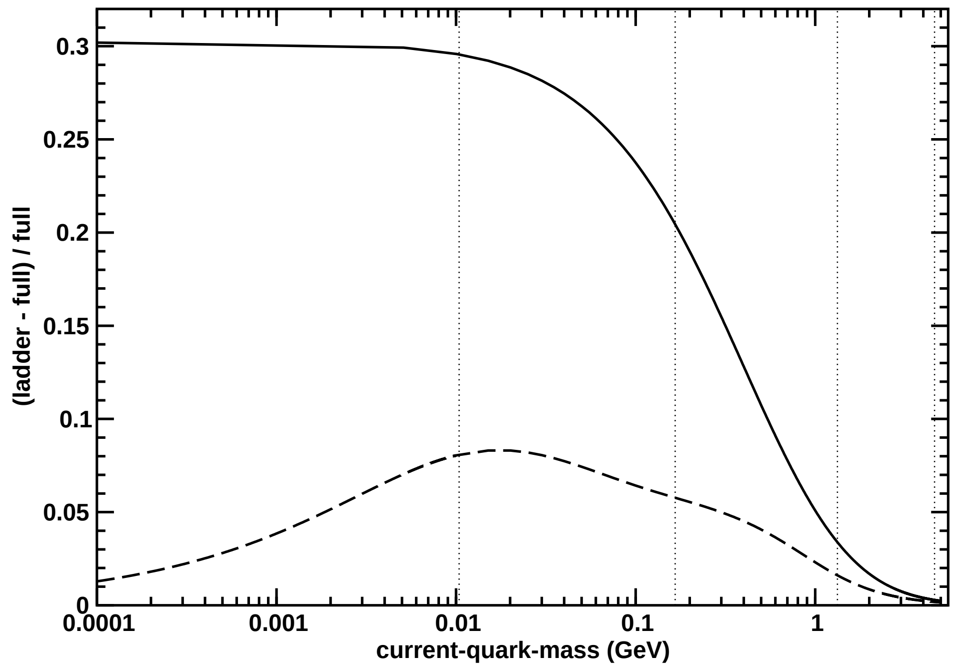
$(C_F - \frac{C_A}{2}) = -\frac{1}{2N_c}$ (weak repulsive)

$\frac{C_A}{2} = \frac{N_c}{2}$ (v. strong attractive)

- Satisfies Slavnov-Taylor Id to $\mathcal{O}(g^3)$
  $ik_\nu\Gamma_\nu = G(k^2)[(1+B)\,S^{-1}(p_+) - S^{-1}(p_-)\,(1+\tilde{B})]$
- (Abelian, QED) color singlet channel: $C_F = (N_c^2-1)/2N_c$ (strong attractive)

Estimate Effect of 3-Gluon Vertex on Mesons

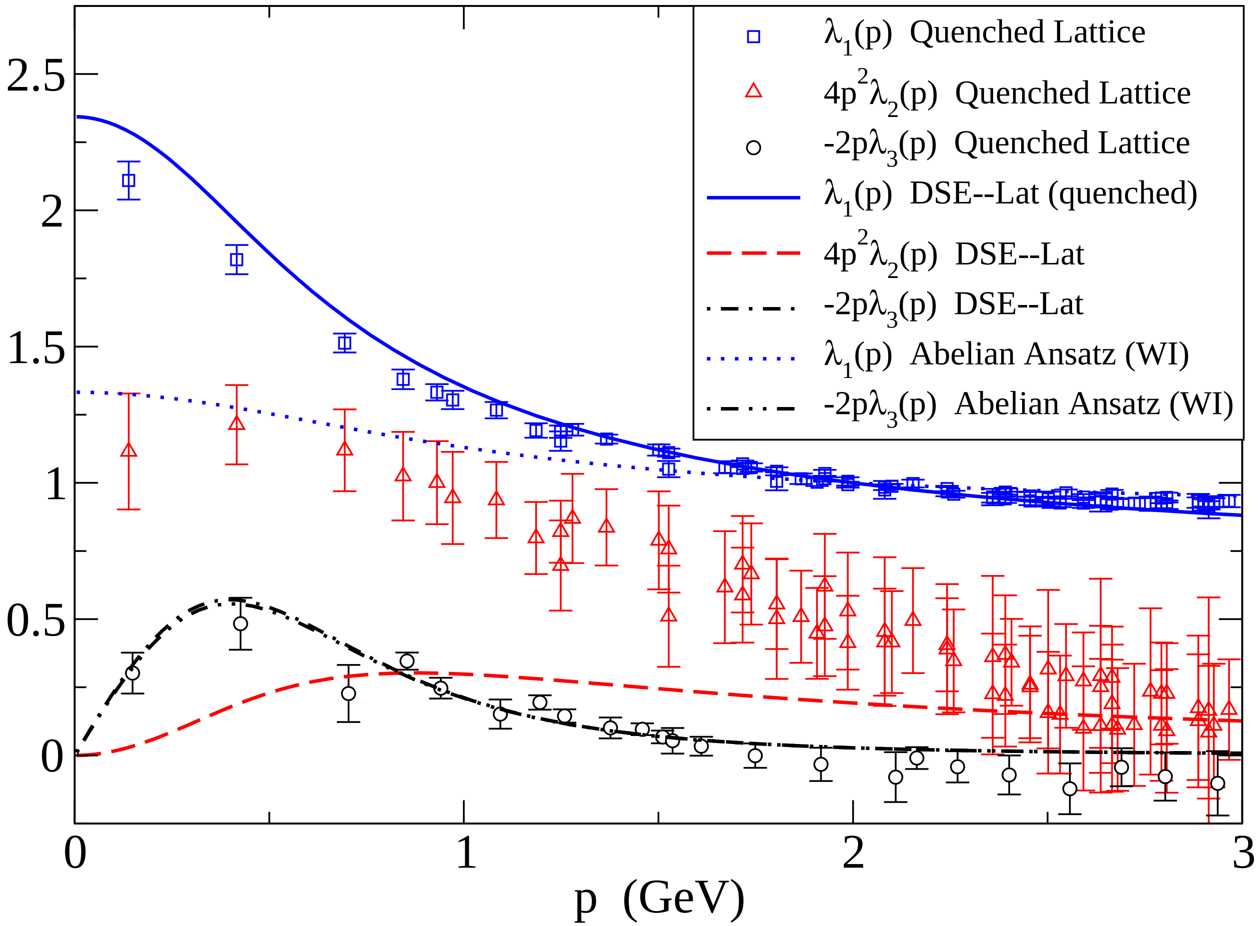
- Enters quark-gluon vertex and K_{BSE}, preserves chiral symmetry
- Implemented in DSE$_q$ and meson BSE via (algebraic) MN model
- nucl-th/0403012, Bhagwat, Höll, Krassnigg, Roberts, PCT
- cf Ladder-rainbow:
  30% reduction in M_{V}
  minor change in M_{PS}



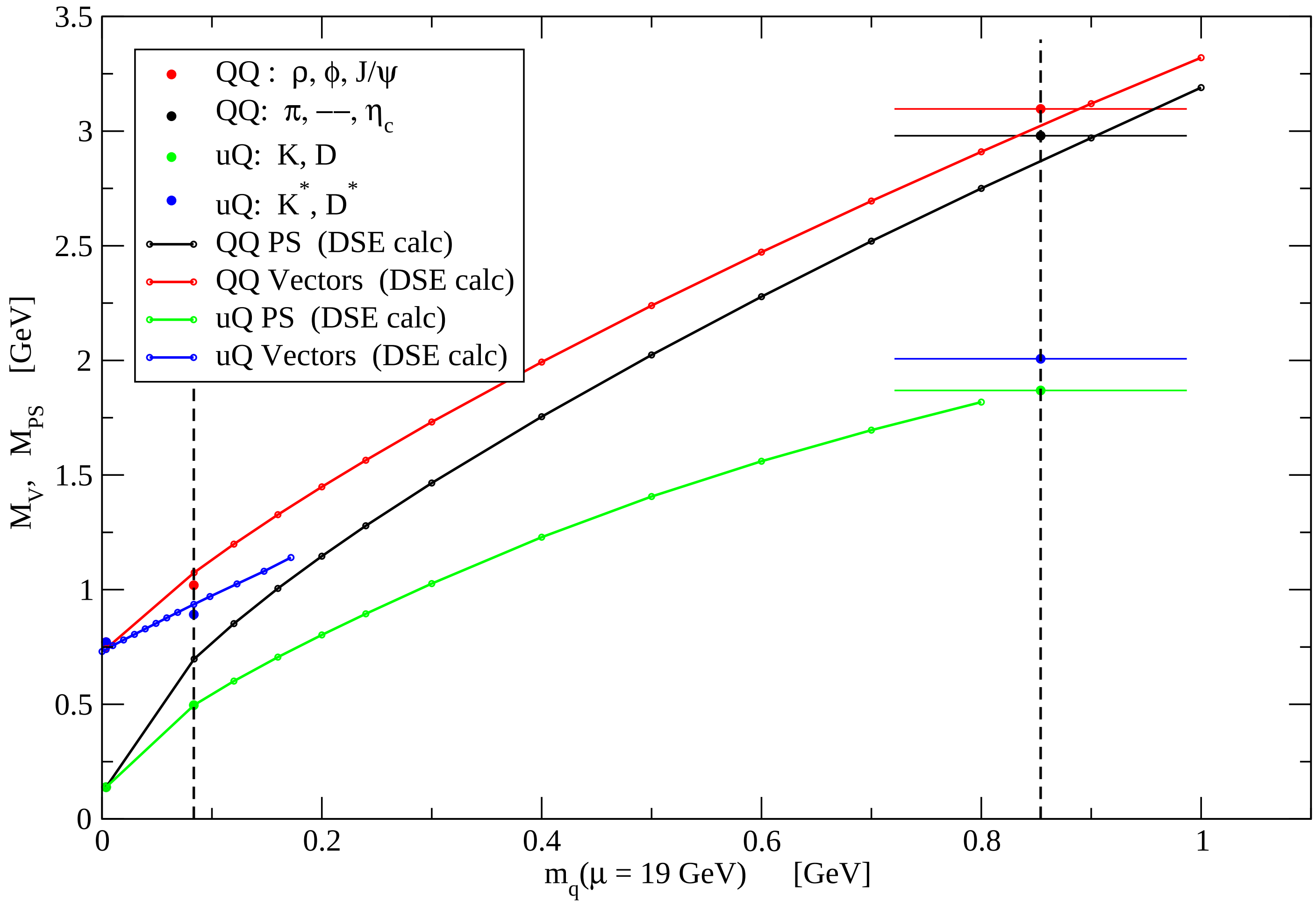
DSE Model for $k = 0$ gluon-quark vertex

$$\Gamma_\nu(p;0) = \gamma_\nu \, \lambda_1(p) - 4p_\nu \, \not{p} \lambda_2(p) - 2ip_\nu \lambda_3(p)$$

WI: $\lambda_1 \sim A, \quad \lambda_2 \sim -A'/2, \quad \lambda_3 \sim B'$



MT Model too attractive at large m_q ?



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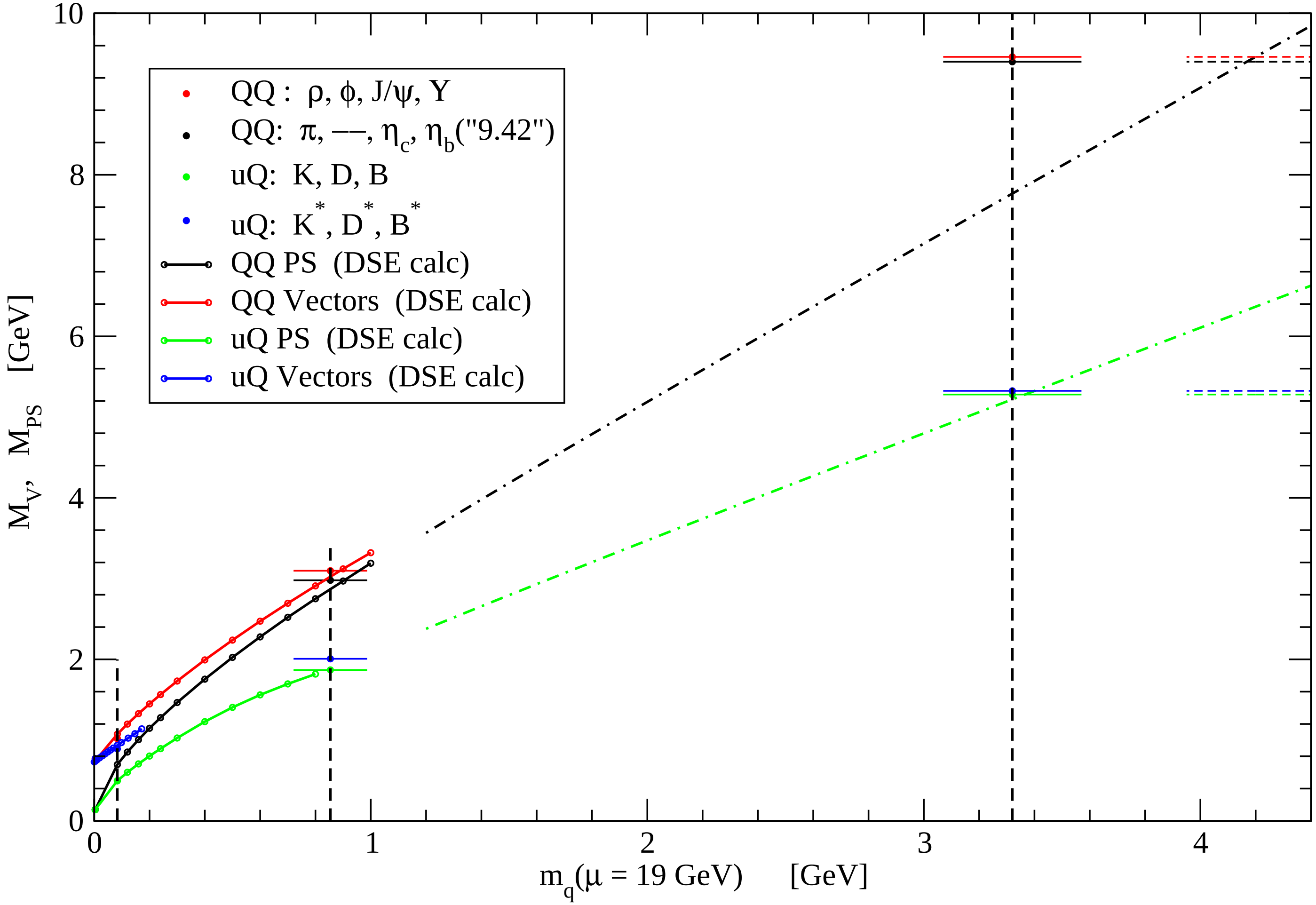
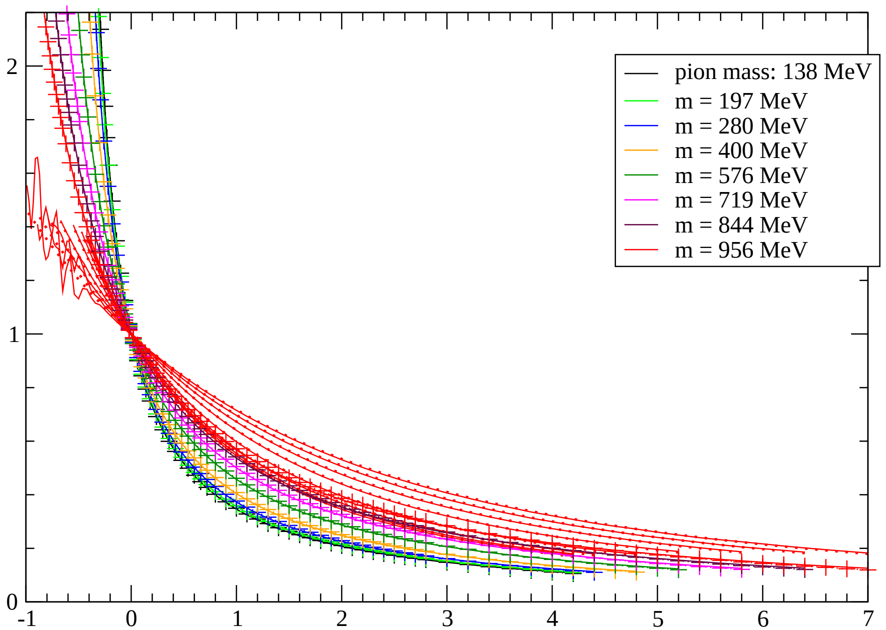
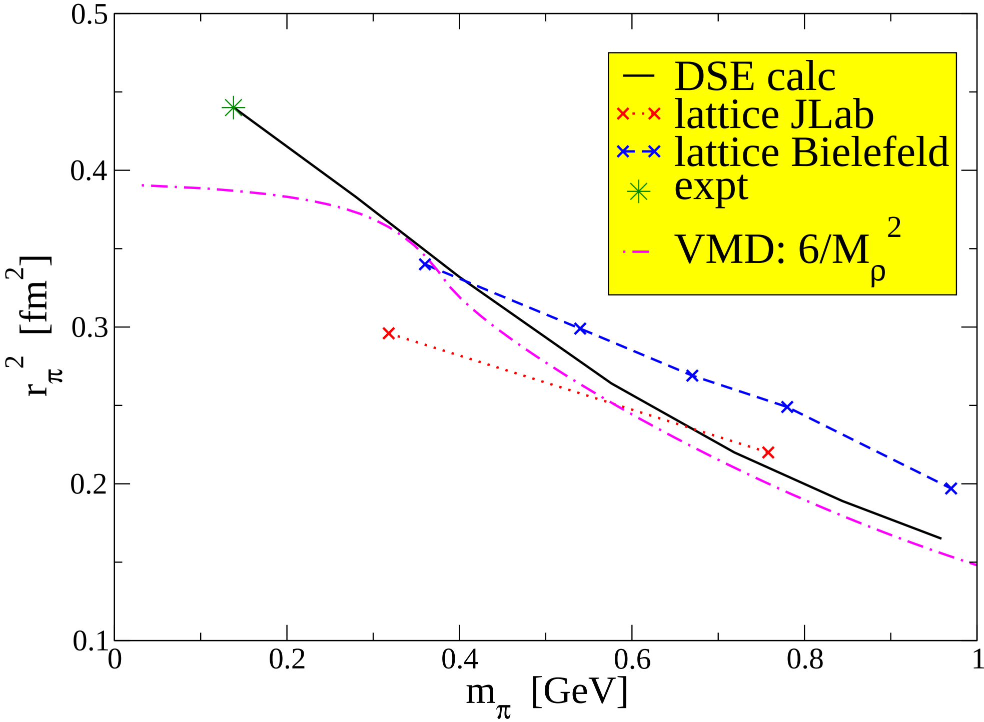


m_q dependence of pion EM form factor and radius

- Work in progress with Pieter Maris
- Ladder-rainbow—no chiral loops, yet

F(Q^2): colored: best calc. ; red: using BC Ansatz





- Also have $M_{\rm ps}(m_q)$, $f_{\rm ps}(m_q)$ for qq, uq, sq mesons.

Summary

- Hadron observables and dynamics modeled from QCD -DSEs—covariant, quark confining, DχSB
- Summary of ordering/organization
- Preserve 1-loop QCD renorm, vector, axial vector WTIds
- $\langle \bar{q} q \rangle_\mu \Rightarrow$ 1 IR parameter
- Propagators compare to lattice-QCD $S(p)$
- Successes/limitations/failures of ladder-rainbow truncation
- Beyond LR truncation:
- 3-gluon coupling, dressed $\bar{q}\gamma q$ vertex $\Rightarrow$ 30% attraction to ground state vector mesons
- m_q-dependence as a diagnostic beyond LR

Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle^0_\mu$	(0.236 GeV)^3	(0.241†)^3
m_π	0.1385 GeV	0.138†
f_π	0.0924 GeV	0.093†
m_K	0.496 GeV	0.497†
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	7.6 $\cdot 10^6$ s^{-1}	7.38
$\Gamma(K_{\mu 3})$	5.2 $\cdot 10^6$ s^{-1}	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
$m_{K^\star}$	0.892 GeV	0.936
$f_{K^\star}$	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^\star K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^\star K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^\star K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

BSE Norm Condition

$$2P_\mu \quad = \quad N_c \frac{\partial}{\partial P_\mu} \mathrm{Tr} \int_q^\Lambda \Big\{ \bar{\Gamma}_\pi(q;Q) S_+(P)\, \Gamma_\pi(q;Q)\, S_-(P)$$
$$+ \int_k^\Lambda \bar{\chi}_\pi(k;Q)\, K(k,q;P)\, \chi_\pi(q;Q) \Big\}_{Q\to P}$$

Beyond ladder BSE kernel $\Rightarrow \frac{\partial}{\partial P_\mu} K(k,q;P) \neq 0$

Light meson sector is well understood