Generalized parton distributions from form factors

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1. Impact parameter parton distributions

2. Proton images from form factor data

3. Spin and the Pauli form factors

4. Conclusions

How are hadrons made from quarks and gluons?
Hadron structure as seen in short-distance processes

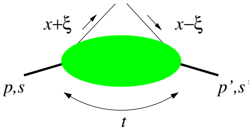
considerable information in parton densities:

- longitudinal momentum and helicity distribution of quarks/gluons in a fast-moving hadron

but: no information on transverse structure/dynamics
$\rightsquigarrow$ try to obtain a three-dimensional picture

- transverse momentum dependent parton densities
- impact parameter dependent parton densities
  $\rightsquigarrow$ this talk

Generalized parton distributions

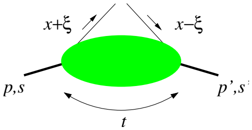


$$\int dz^- \, e^{ixP^+z^-} \langle p', s' | \, \bar{q}(-\tfrac{1}{2}z) \, \gamma^+ q(\tfrac{1}{2}z) \, | p, s \rangle_{z^+=0,\, \boldsymbol{z}=\mathbf{0}}$$

$$\propto \quad H^q \, \bar{u}(p', s')\gamma^+ u(p, s) + E^q \, \bar{u}(p', s') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2m} u(p, s)$$

- light-cone coordinates: $v^{\pm} = (v^0 \pm v^3)/\sqrt{2}, \;\; \boldsymbol{v} = (v^1, v^2)$
- H^q and E^q depend on
  x and ξ: plus-momentum fractions w.r.t. $P = \frac{1}{2}(p + p')$
  invariant momentum transfer $t = (p - p')^2$

Generalized parton distributions



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- proton spin structure:
  $H^q \leftrightarrow s = s'$ for $p = p'$ get usual parton density:
  $$H^q(x, \xi = 0, t = 0) = q(x)$$
  $E^q \leftrightarrow s \neq s'$ decouples for $p = p'$
- similar definitions for polarized quarks and for gluons

Impact parameter

- states with definite light-cone momentum p^+ and transverse position (impact parameter):

$$|p^+, \boldsymbol{b}\rangle = \int d^2\boldsymbol{p}\, e^{-i\boldsymbol{b}\cdot\boldsymbol{p}}\, |p^+, \boldsymbol{p}\rangle$$

formal: eigenstates of 2 dim. position operator

- can exactly localize proton in 2 dimensions (in 3 dim only within Compton wavelength) and stay in frame where proton moves fast $\rightsquigarrow$ parton interpretation

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- proton is extended object: what exactly means position $\boldsymbol{b}$?

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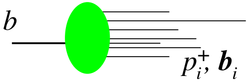
formal: eigenstates of 2 dim. position operator

- proton is extended object: what exactly means position $\boldsymbol{b}$?
- consequence of Lorentz invariance (transverse boosts):



$$\boldsymbol{b} = \frac{\sum_i p_i^+ \boldsymbol{b}_i}{\sum_i p_i^+} \qquad (i = q, \bar{q}, g)$$

is center of momentum of the partons in proton D. Soper '77
nonrelativistic analog: Galilei invariance ⇒ center of mass

Impact parameter GPDs

in following specialize to $\xi = 0$

- matrix element

$$\int dz^- \, e^{ixP^+z^-} \langle p^+, \boldsymbol{b} | \, \bar{q}(-\tfrac{1}{2}z) \, \gamma^+ q(\tfrac{1}{2}z) \, | p^+, \boldsymbol{b} \rangle_{z^+=0,\, \boldsymbol{z}=\boldsymbol{0}}$$

- $\rightsquigarrow$ impact parameter distribution

$$q(x, b^2) = (2\pi)^{-2} \int d^2\boldsymbol{\Delta} \, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}} \, H^q(x, \xi = 0, t = -\boldsymbol{\Delta}^2)$$

- $q(x, b^2)$ gives distribution of quarks with
  - longitudinal momentum fraction x
  - transverse distance b from proton center

M. Burkardt '00

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- $q(x, b^2)$ gives distribution of quarks with
  - longitudinal momentum fraction x
  - transverse distance b from proton center — M. Burkardt '00
- average impact parameter

$$\langle b^2 \rangle_x = \frac{\int d^2b \, b^2 \, q(x, b^2)}{\int d^2b \, q(x, b^2)} = 4 \, \frac{\partial}{\partial t} \log H^q(x, \xi, t) \Big|_{t=0}$$

Evolution

- $q(x, b^2)$ fulfills usual DGLAP evolution equation
  for non-singlet (e.g. $q_{\mathsf{NS}} = q - \bar{q}$ or $q_{\mathsf{NS}} = u - d$):

$$\mu^2 \frac{d}{d\mu^2}\, q_{\mathsf{NS}}(x, b^2, \mu^2) = \int_x^1 \frac{dz}{z} \left[P\Big(\frac{x}{z}\Big) \right]_+ q_{\mathsf{NS}}(z, b^2, \mu^2)$$

  evolution local in b (let $1/\mu \ll b$ to be safe)

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- average

$$\langle b^2 \rangle_x = \frac{\int d^2 b \, b^2 \, q_{\mathrm{NS}}(x, b^2)}{\int d^2 b \, q_{\mathrm{NS}}(x, b^2)}$$

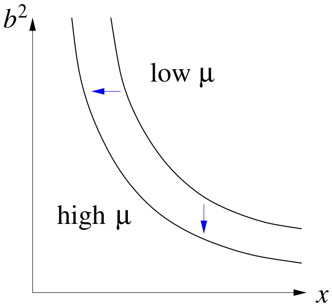
  evolves as

$$\mu^2 \frac{d}{d\mu^2} \langle b^2 \rangle_x = -\frac{1}{q_{\mathrm{NS}}(x)} \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}\right) q_{\mathrm{NS}}(z) \Big[\langle b^2 \rangle_x - \langle b^2 \rangle_z \Big]$$

Evolution

$$\mu^2 \frac{d}{d\mu^2} \langle b^2 \rangle_x = -\frac{1}{q_{\mathsf{NS}}(x)} \int_x^1 \frac{dz}{z} P\Big(\frac{x}{z}\Big) q_{\mathsf{NS}}(z) \Big[\langle b^2 \rangle_x - \langle b^2 \rangle_z \Big]$$

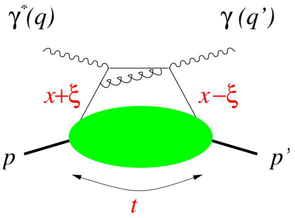
- if $\langle b^2 \rangle_x$ decreases with x $\Rightarrow$ $\langle b^2 \rangle_x$ decreases with μ^2

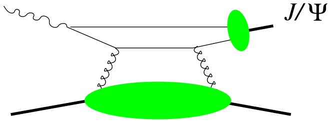


Relation to observables

- generalized parton distributions $H^q(x,\xi,t)$, $E^q(x,\xi,t)$, $H^g(x,\xi,t)$, ... appear in exclusive processes
- factorization theorems similar to inclusive processes

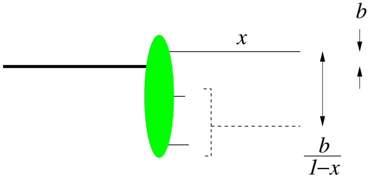
J. Collins et al. '96





- measured t dependence $\rightsquigarrow$ information on b distribution

Large x



- for $x \to 1$ get $b \to 0$
  nonrel. analog:
  center of mass of atom
- $\Leftrightarrow$ t dependence becomes flat

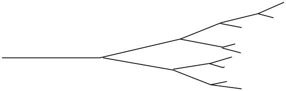
- $d = b/(1-x)$
  $=$ distance of selected parton from spectator system
  gives lower bound on overall size of proton
- finite size of configurations with $x \to 1$ implies

$$\langle b^2 \rangle \sim (1-x)^2$$

M. Burkardt, '02, '04

Small-x behavior

- partons with smaller $x \to$ broader in b
- Gribov diffusion: parton branching as random walk in b space



$$\to \langle b^2 \rangle \propto \alpha' \log(1/x)$$

- Regge phenomenology: simplest ansatz

$$H(x, \xi = 0, t) \sim \left(\frac{1}{x}\right)^{\alpha_0 + \alpha' t} = x^{-\alpha_0}\, e^{t\alpha' \log(1/x)}$$

use as effective power-law in limited range of x and t

- works well for forward parton distributions

Small-x behavior

$$H(x, \xi = 0, t) \sim \left(\frac{1}{x}\right)^{\alpha_0 + \alpha' t} = x^{-\alpha_0} \, e^{\, t\alpha' \log(1/x)}$$

- exclusive J/Ψ production $\rightarrow$ gluon distribution
  - photoproduction — H1 preliminary (DIS 05)

    $\alpha_0 = 1.224 \pm 0.010 \pm 0.012$

    $\alpha' = 0.164 \pm 0.028 \pm 0.030 \;\; \text{GeV}^{-2}$
  - similar in electroproduction
- values very different in soft processes $\gamma p \rightarrow \rho p$, $pp \rightarrow pp$, ...

  for α_0 well-known from inclusive $\gamma^* p \rightarrow X$ vs. $\gamma p \rightarrow X$

Small-x behavior

$$H(x, \xi = 0, t) \sim \left(\frac{1}{x}\right)^{\alpha_0 + \alpha' t} = x^{-\alpha_0} \, e^{t\alpha' \log(1/x)}$$

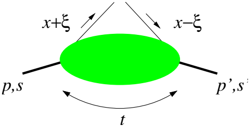
- in nonsinglet sector (quarks only, no gluons) from CTEQ6M distributions including error estimates: for $\mu = 2$ GeV and $10^{-5} < x < 10^{-2}$ have

  $u_v + d_v \sim x^{-0.38 \text{ to } -0.495}$

  $(u_v - d_v)/(u_v + d_v) \approx x^{-0.165 \text{ to } +0.13}$

- similar to $\alpha_0 = 0.4 \ldots 0.5$ from soft processes
- information on α' in partonic regime?
  . . . wait a few slides

Moments



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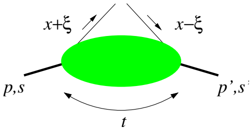
- Mellin moments: $\int dx\, x^n$ $\rightarrow$ local operator $\rightarrow$ form factors
- $\int dx$ $\rightarrow$ vector current

  $\sum_q e_q \int dx\, H^q(x, \xi, t) = F_1(t)$ Dirac f.f.

  $\sum_q e_q \int dx\, E^q(x, \xi, t) = F_2(t)$ Pauli f.f.

  Lorentz invariance $\Rightarrow$ lowest moments independent of ξ

Moments



$$\int dz^- \, e^{ixP^+z^-} \langle p', {\color{red}s'} | \, \bar{q}(-\tfrac{1}{2}z) \, \gamma^+ q(\tfrac{1}{2}z) \, | p, {\color{red}s} \rangle_{z^+=0,\, \boldsymbol{z}=\mathbf{0}}$$

$$\propto \quad {\color{blue}H^q} \, \bar{u}(p', s')\gamma^+ u(p, s) + {\color{blue}E^q} \, \bar{u}(p', {\color{red}s'}) \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2m} u(p, s)$$

- Mellin moments: $\int dx \, x^n$ $\rightarrow$ local operator $\rightarrow$ form factors
- $\int dx \, x$ $\rightarrow$ energy-momentum tensor

  sum rule $\frac{1}{2}\int dx \, x(H^q + E^q) = J^q(t)$ X. Ji '96

  $J^q(0) =$ total angular momentum carried by quark flavor q (helicity and orbital part)

Lattice

→ G. Schierholz' talk

Mellin moments of GPDs can be calculated in lattice QCD

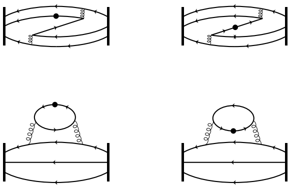


figure:

J. Negele, hep-lat/0211022

systematic uncertainties from

- omission of "disconnected" diagrams
  but: cancel in difference of u and d
- extrapolation to physical pion mass

Proton images from form factor data

M.D., Th. Feldmann, R. Jakob, P. Kroll, hep-ph/0408173

- idea: use data on electromagnetic nucleon form factors
  to constrain interplay of x and b dependence
- sum rule for Dirac form factor:

$$F_1^p(t) = \int_0^1 dx \left[\tfrac{2}{3} H_v^u(x,t) - \tfrac{1}{3} H_v^d(x,t)\right]$$

$$F_1^n(t) = \int_0^1 dx \left[\tfrac{2}{3} H_v^d(x,t) - \tfrac{1}{3} H_v^u(x,t)\right]$$

where $H_v^q(x,t) = H^q(x,\xi=0,t) - H^{\bar{q}}(x,\xi=0,t)$
$=$ valence distribution of flavor q in proton

e.m. current sensitive to $q - \bar{q}$

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- develop simple, physically motivated parametrization
  and fit to form factor data

exponential ansatz for t dependence

$$H_v^q(x,t) = q_v(x)\,\mathrm{exp}[\,t f_q(x)\,]$$

- for $q_v(x)$ take CTEQ6M parameterization at scale $\mu = 2$ GeV
  results stable within CTEQ error estimates of q_v
- gives impact parameter distribution of valence quarks

$$q_v(x,b) = \frac{q_v(x)}{4\pi f_q(x)}\,\mathrm{exp}\left[-\frac{b^2}{4f_q}\right]$$

and average squared impact parameter $\langle b^2\rangle_x^q = 4f_q(x)$

ansatz for $f_q(x)$

- for $x \to 0$ simple Regge phenomenology suggests

$$H_v(x,t) \sim x^{-\alpha(0)} \exp[\, t\alpha' \log(1/x) \,]$$

with $\alpha_0 \approx 0.5$ and $\alpha' \approx 0.9 \ \text{GeV}^{-2}$ K. Goeke et al. '01

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- for $x \to 1$ demand finite distance quark to spectators:

$$\langle b^2 \rangle_x^q = 4 f_q(x) \sim (1-x)^2$$

M. Burkardt '02, M. Guidal et al. '04

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- interpolating ansatz:

$$f_q(x) = \alpha'(1-x)^3 \log(1/x) + B_q(1-x)^3 + A_q(1-x)^2$$

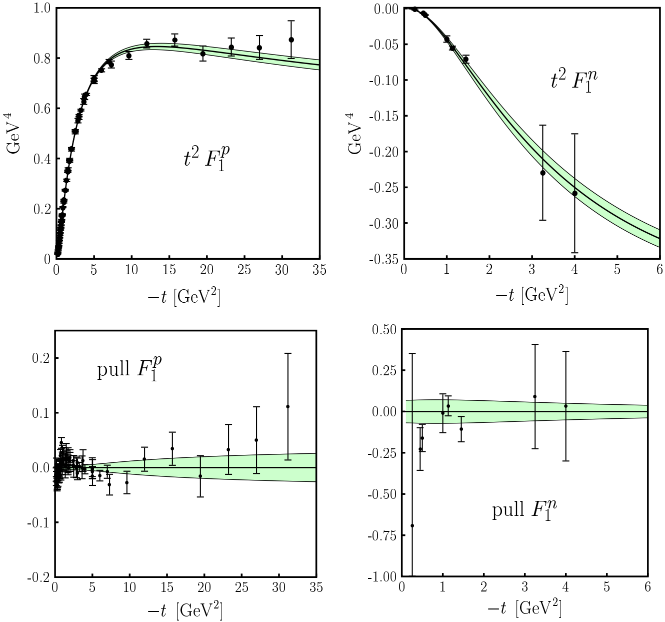
multiply $\log(1/x)$ with $(1-x)^3$ so that α' controls $f_q(x)$ at small x but not at large x

fit

$$H_v^q(x,t) = q_v(x) \exp[\, t f_q(x) \,]$$
$$f_q(x) = \alpha'(1-x)^3 \log(1/x) + B_q(1-x)^3 + A_q(1-x)^2$$

to data of $F_1^p(t)$ and $F_1^n(t)$

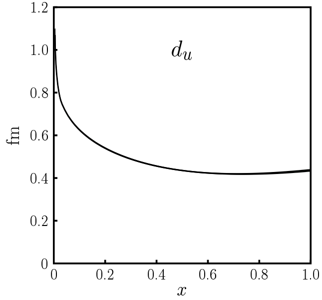
- set $\alpha' = 0.9$ GeV^2
  leaving it free get $\alpha' = 0.97 \pm 0.04$ GeV^{-2}
- fit parameters $B_u = B_d$ and A_u and A_d
  leaving B_u and B_d independent $\rightarrow$ only slight improvement
  setting $A_u = A_d$ $\rightarrow$ fit clearly worse

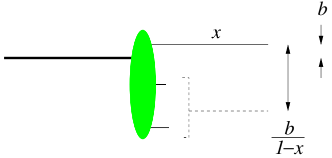


χ^2/d.o.f.
$= 1.93$

pull =
data/fit -1

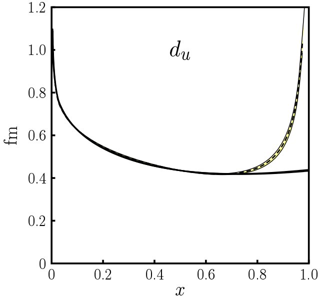
Lessons from the fit

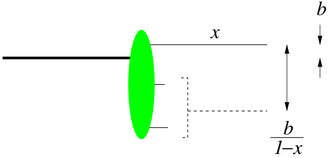




- clear drop with x of average distance $d = b/(1-x)$

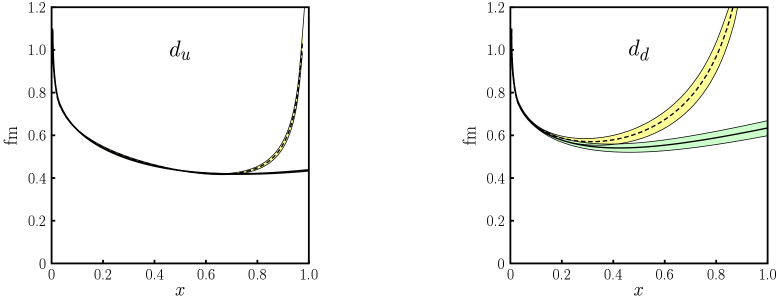
Lessons from the fit





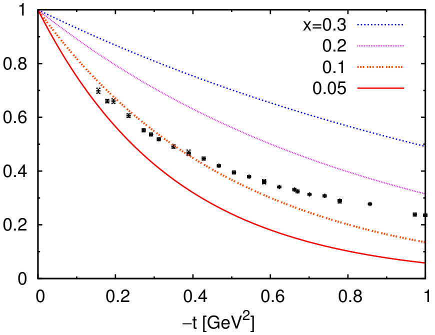
- $f_q(x) = \alpha'(1-x)^2 \log(1/x) + B_q(1-x)^2 + A_q(1-x)$
  (upper curve) gives similarly good fit to data
- region $x \gtrsim 0.8$ contributes less than 5% to form factors
  $\to$ data cannot fix asymptotic behavior of $d_q(x)$ for $x \to 1$

Lessons from the fit



- d quark distribution less well determined
  (contributes little to F_1^p, where data are best)
- to describe both F_1^p and F_1^n well
  fit wants $d_d(x) > d_u(x)$ for moderate to large x
  $\leftrightarrow$ d quarks more "spread out" than u quarks

- strong correlation of x and t dependence



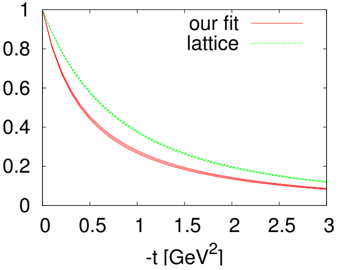
no factorization of type

$$H(x,t) = q(x)\, F_1(t)$$

- lines: $\frac{2}{3} H_v^u(x,t) - \frac{1}{3} H_v^d(x,t)$ normalized to 1 at $t = 0$
- data points: $F_1^p(t) = \int_0^1 dx \left[\frac{2}{3} H_v^u(x,t) - \frac{1}{3} H_v^d(x,t) \right]$
- effect important even far away from $x = 1$

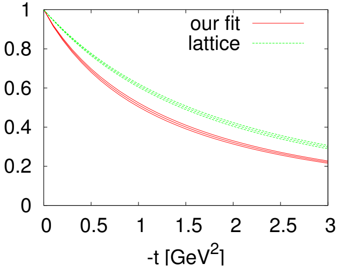
Comparison with lattice results

$$\int dx\,(H^u - H^d) = F_1^p(t) - F_1^n(t)$$



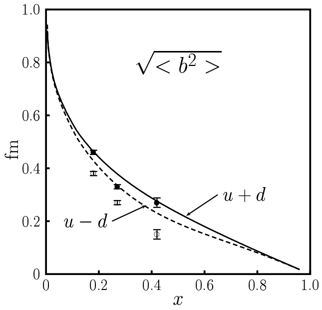
$$\int dx\,x\,(H^u - H^d)$$

normalized to 1 at $t = 0$



- lattice calculation QCDSF Collab., hep-lat/0410023 extrapolated to physical pion mass (from $m_\pi = 553 \ldots 1090$ MeV)
- our results for $\int dx\,x\,H$: only valence quark contribution lattice: both valence and sea

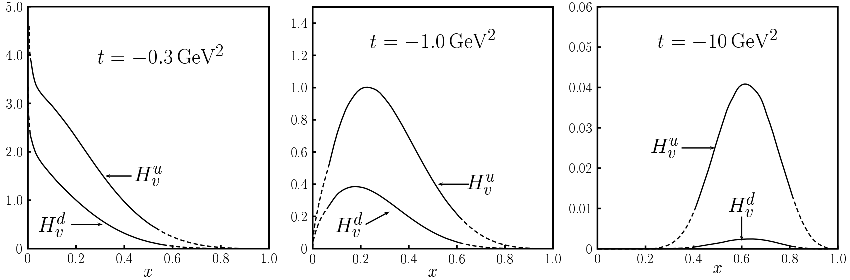
Comparison with lattice results (more)



J. Negele et al., hep-lat/0404005

- Wilson fermions
- $m_\pi = 870$ MeV
- typical x in $\int dx\, x^n q(x,b)$ estimated as

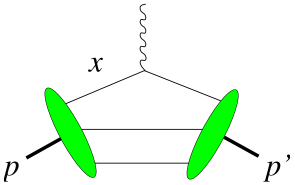
$$\langle x \rangle = \frac{\int dx\, x^{n+1} q(x)}{\int dx\, x^n q(x)}$$



as $-t$ increases

- distribution shifts towards large x
- much less d than u

Large t and the Feynman mechanism



- if impose that spectators have virtualities $\sim \Lambda^2$ then

$$1 - x \sim \Lambda / \sqrt{-t}$$

- large-t asymptotics with our ansatz:

  $\langle 1 - x \rangle_t \sim 1/\sqrt{-t}$ for $q(x) \sim (1-x)^{\beta_q}$ at large x

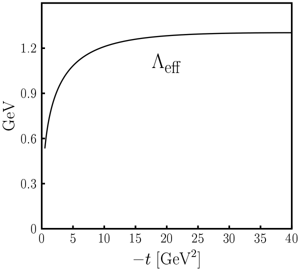
  (saddle point approx.)

- $\rightsquigarrow$ Drell-Yan relation: $F_1^q(t) \sim |t|^{-\frac{1+\beta_q}{2}}$

  CTEQ6M distributions at $\mu = 2$ GeV:

  $\beta_u \sim 3.4$ and $\beta_d \sim 5.0$ (for $0.5 < x < 0.9$)

Large t and the Feynman mechanism



- if impose that spectators have virtualities $\sim \Lambda^2$ then

$$1 - x \sim \Lambda / \sqrt{-t}$$

- numerically seen for $-t \gtrsim 5\ \text{GeV}^2$

$$\Lambda_{\text{eff}} = \frac{\langle 1 - x \rangle_t}{\sqrt{-t}} \qquad \text{with} \qquad \langle x \rangle_t = \frac{\int dx\, x H_v(x,t)}{\int dx\, H_v(x,t)}$$

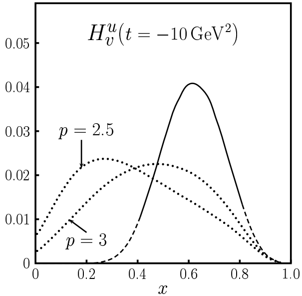
- does not prove that Feynman mechanism dominates but provides natural picture if assume this dominance

Large t and the Feynman mechanism

- with same form of $f_q(x)$ as before get good fit of data with

$$H_v^q(x,t) = q_v(x)\left(1 - \frac{t f_q(x)}{p}\right)^{-p}$$

when $p \gtrsim 2.5$ (for $p = \infty$ recover exponential form)



- still get large-t asymptotics $\langle 1-x\rangle_t \sim 1/\sqrt{-t}$
- but for small p only realized at very large t (else saddle point approx. bad)
- taking exponential form presently is theory driven

Spin and the Pauli form factors

- sum rule

$$F_2^p(t) = \int_0^1 dx \left[\tfrac{2}{3} E_v^u(x,t) - \tfrac{1}{3} E_v^d(x,t)\right]$$
$$F_2^n(t) = \int_0^1 dx \left[\tfrac{2}{3} E_v^d(x,t) - \tfrac{1}{3} E_v^u(x,t)\right]$$

  for valence distributions

$$E_v^q(x,t) = E^q(x,\xi=0,t) - E^{\bar{q}}(x,\xi=0,t)$$

- $E \leftrightarrow$ nucleon helicity flip $\langle \downarrow | \mathcal{O} | \uparrow \rangle$

  polarization along $\pm x$ axis $|X\pm\rangle = (\,|\uparrow\rangle \pm |\downarrow\rangle\,)/\sqrt{2}$

$$\langle X+ |\mathcal{O}| X+\rangle - \langle X- |\mathcal{O}| X-\rangle = \langle \uparrow |\mathcal{O}| \downarrow\rangle + \langle \downarrow |\mathcal{O}| \uparrow\rangle$$

- quark density in proton state $|X+\rangle$

$$q_v^X(x, \mathbf{b}) = q_v(x, b) - \frac{b^y}{m} \frac{\partial}{\partial b^2} \, e_v^q(x, b)$$

shifted in y direction
where $e_v^q(x, b)$ is Fourier transform of $E_v^q(x, t)$

- related with spin asymmetries in momentum space (Sivers effect) M. Burkardt '02

(similar effects for transverse quark polarization)
M.D. and P. Hägler '05, M. Burkardt '05

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- positivity bound M. Burkardt '03

$$\Big[E^q(x,t=0)\Big]^2 \;\le\; m^2\Big[q(x)+\Delta q(x)\Big]\Big[q(x)-\Delta q(x)\Big] \times 4\,\frac{\partial}{\partial t}\ln\Big[H^q(x,t)\pm\tilde{H}^q(x,t)\Big]_{t=0}$$

$\Rightarrow$ E^q must fall faster than H^q at large x
orbital angular momentum carried by partons with smaller x

- ansatz for valence distributions (as for H_v^q)

$$E_v^q(x,t) = e_v(x) \exp[\, t\, g_q(x)\,]$$
$$g_q(x) = \alpha'(1-x)^3 \log(1/x) + D_q(1-x)^3 + C_q(1-x)^2$$

- shape of forward limit $e_v^q(x)$ not known $\rightarrow$ ansatz

$$e_v^q = \mathcal{N}_q\, x^{-\alpha}(1-x)^{\beta_q}$$

$\mathcal{N}_q$ determined by p and n magnetic moments

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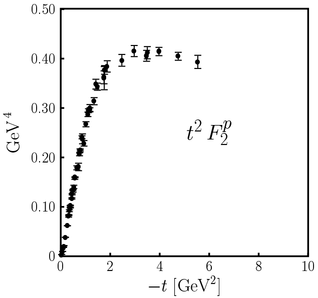
$$e_v^q = \mathcal{N}_q\, x^{-\alpha}(1-x)^{\beta_q}$$

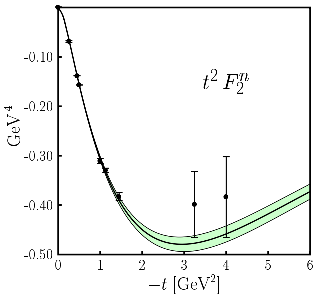
  $\mathcal{N}_q$ determined by p and n magnetic moments

- obtain good fit of $F_2^p(t)$ and $F_2^n(t)$
  - $\alpha' = 0.9$ GeV^{-2} and $\alpha = 0.55$

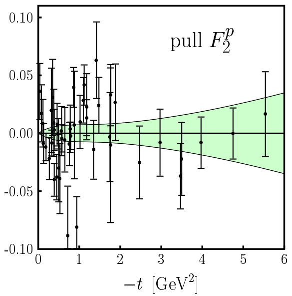
    ok with Regge phenomenology

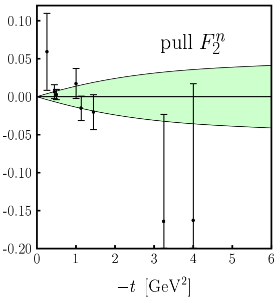
- wide regions of C_q, D_q and β_q allowed
  but positivity constraints seriously limit parameter space
  in particular $\beta_d \geq 5$ and $\beta_u \geq 3.5$



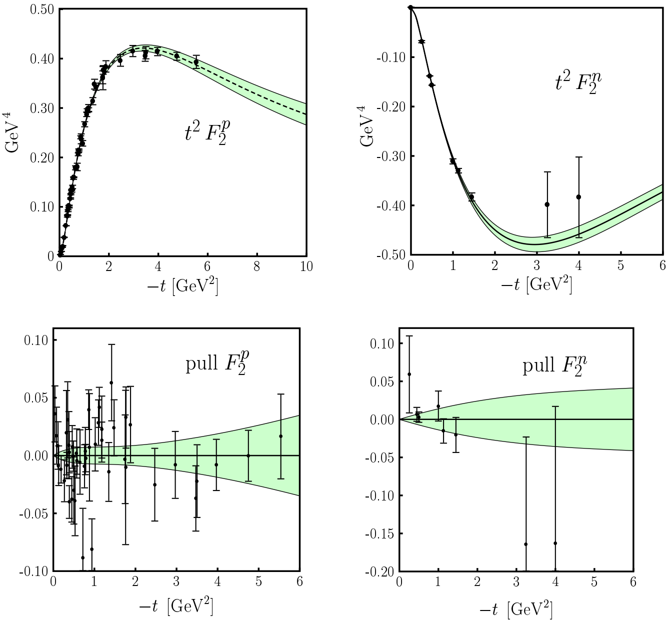


χ^2/d.o.f. $= 1.31$





pull = data/fit -1



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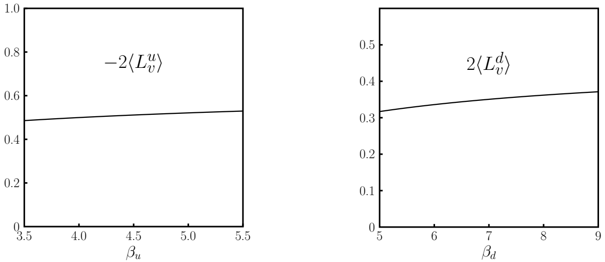
pull $=$ data/fit -1

- orbital angular momentum carried by valence quarks

$$\langle L_v^q \rangle = \frac{1}{2} \int dx \left[x e_v^q(x) + x q_v(x) - \Delta q_v(x) \right]$$



- individual u and d rather well determined
- $2\langle J_v^u \rangle = 2\langle L_v^u \rangle + 0.93$ and $2\langle J_v^d \rangle = 2\langle L_v^d \rangle - 0.34$
- strong cancellations in $\langle L_v^u + L_v^d \rangle$

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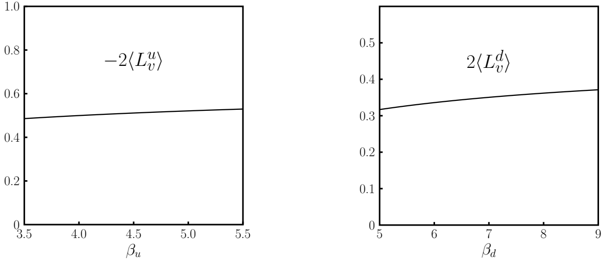


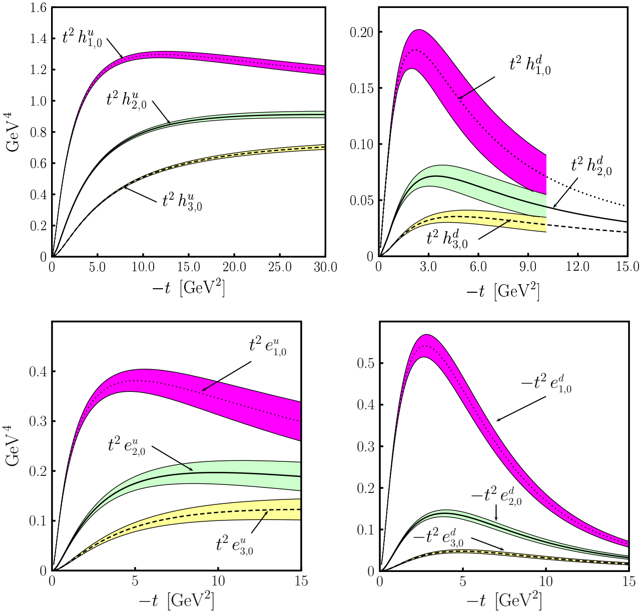
- $2\langle L_v^u - L_v^d \rangle = -(0.81 \ldots 0.90)$ at $\mu = 2$ GeV

  lattice:

  QCDSF: $2\langle L_v^u - L_v^d \rangle = -0.9 \pm 0.12$ G. Schierholz, this meeting

  LHPC: $2\langle L_v^u - L_v^d \rangle = -0.25 \pm 0.05$ for $m_\pi = 897$ MeV

  from hep-lat/0410017



moments of GPDs

$$h^q_{n,0}(t) = \int_0^1 dx\, x^{n-1}\, H^q_v(x,t)$$

$$e^q_{n,0}(t) = \int_0^1 dx\, x^{n-1}\, E^q_v(x,t)$$

Drell-Yan relation at work

Conclusions

impact parameter dependent parton distributions

- longitudinal momentum
  and transverse position of partons $\rightsquigarrow$ 3D information
  fully consistent with relativity and parton picture
- operator definition in QCD
  $\longrightarrow$ evolution equations, lattice calculations
- observables:
  - GPDs from hard exclusive processes
    simultaneous measurement of t and x_B
  - elastic form factors
    no momentum fraction measured, but access to large t

Conclusions

theory input $\oplus$ nucleon form factor data
$\rightsquigarrow$ quantitative information on $q - \bar{q}$ distributions

- strong dependence of impact parameter profile on x
  hint at different impact parameter profile for u and d quarks
- at small x consistent with hadronic Regge phenomenology
- at large x natural implementation of Feynman mechanism
  for $|t| \gtrsim 5$ GeV^2
  care needed with asymptotic expansions/power laws

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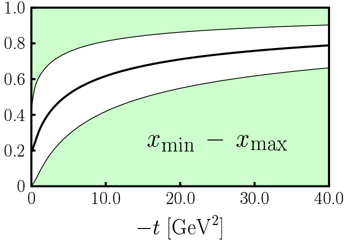
- strong dependence of impact parameter profile on x
  hint at different impact parameter profile for u and d quarks
- at small x consistent with hadronic Regge phenomenology
- at large x natural implementation of Feynman mechanism for $|t| \gtrsim 5$ GeV^2
  care needed with asymptotic expansions/power laws
- Drell-Yan relation test in experiment or lattice
- despite large uncertainties on helicity-flip distribution E^q
  rather good determination of $\langle L_v^u - L_v^d \rangle$ within our ansatz
  find $J^d \approx 0$

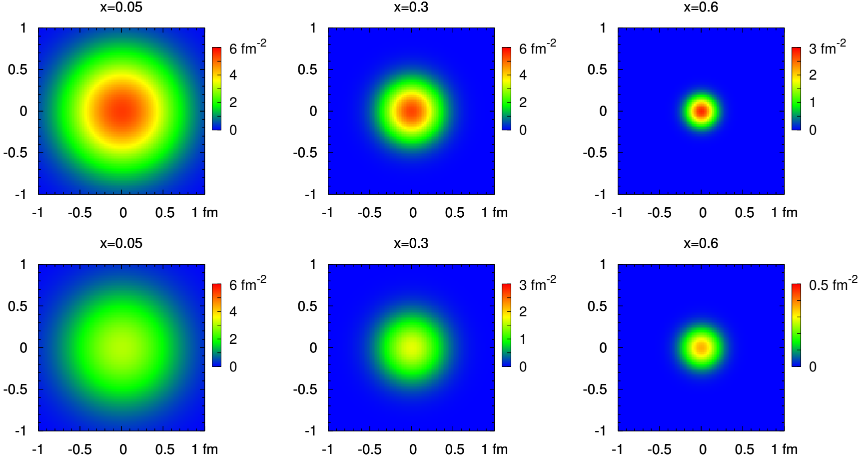
Relevant region of x



white region contributes 90% to integral

$$F_1^p(t) = \int_0^1 dx\, [\tfrac{2}{3} H_v^u(x,t) - \tfrac{1}{3} H_v^d(x,t)]$$

quark density in transverse plane
top: $u_v(x,b)$ bottom: $d_v(x,b)$



valence d quark density in transverse plane
top: unpolarized bottom: proton polarized along x-axis