Numerical Evidence for the Maldacena Conjecture in Two-Dimensional $\mathcal{N} = (8, 8)$ Super Yang–Mills Theory

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Introduction

- Maldacena conjecture: $\exists$ correspondences between certain string theories and super Yang–Mills (SYM) theories [hep-th/9711200, hep-th/9802042].
- can test by computing a correlator of the stress-energy tensor in both cases, from a weak-coupling supergravity approximation to the string theory and for the corresponding strong coupling in SYM theory.
- for SYM theory, need nonperturbative technique.
- will use a Hamiltonian approach in a Fock basis
- consistent expansion and simple vacuum $\longrightarrow$ light-cone coordinates.
- diagonalize a discrete matrix representation.
- compute correlator by inserting sum over eigenstates.

Acknowledgments

- work done in collaboration with S. Pinsky, N. Salwen, and U. Trittmann [hep-th/0506225].
- see also
  - F. Antonuccio, A. Hashimoto, O. Lunin, and S. Pinsky, JHEP **9907**, 029 (1999, [hep-th/9906087].
  - J. R. Hiller, O. Lunin, S. Pinsky, and U. Trittmann, PLB **482**, 409 (2000), [hep-th/0003249].
  - M. Harada, J. R. Hiller, S. Pinsky, and N. Salwen, PRD **70**, 045015 (2004), [hep-th/0404123].
- supported in part by
  - US Department of Energy.
  - Minnesota Supercomputing Institute.

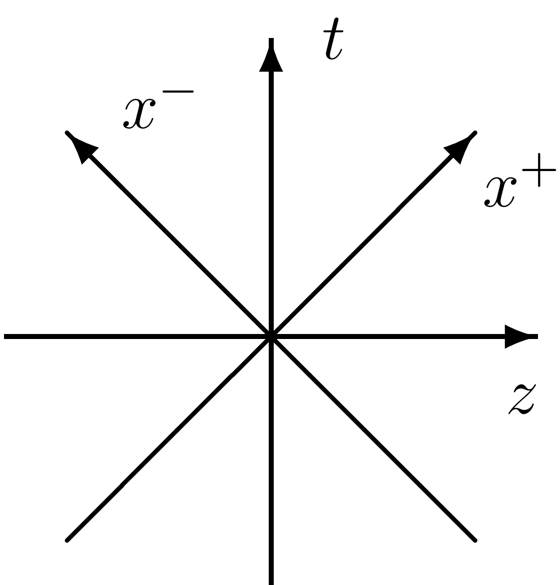
Outline

- light-cone coordinates.
- (supersymmetric) discrete light-cone quantization.
- SYM theory in two dimensions.
- stress-energy correlator.
- results.
- conclusions.

Light-cone coordinates

Dirac, RMP **21**, 392 (1949).

- time: $x^{+} = (t+z)/\sqrt{2}$.
- space: $x^{-} \equiv (t-z)/\sqrt{2}$.
- energy: $p^{-} = (E-p_z)/\sqrt{2}$.
- momentum: $p^{+} \equiv (E+p_z)/\sqrt{2}$.
- mass-shell condition: $p^2 = m^2 \;\; \Rightarrow \;\; p^{-} = \frac{m^2}{2p^{+}}$



Discrete light-cone quantization (DLCQ)

Pauli and Brodsky, PRD **32**, 1993 (1985); 2001 (1985).
Brodsky, Pauli, and Pinsky, Phys. Rep. **301**, 299 (1997).

- impose a light-cone box $-L < x^- < L$ and periodic BC $\rightarrow$ discrete grid: $p_i^+ \rightarrow \frac{\pi}{L} n_i$.
- the limit $L \rightarrow \infty$ is exchanged for a limit in terms of the integer *resolution* $K \equiv \frac{L}{\pi} P^+$ for fixed total momentum P^+.
- $n_i > 0 \;\Rightarrow\;$ # of particles $\leq K$.
- integrals are replaced by discrete sums $\int dp^+ f(p^+) \simeq \frac{\pi}{L} \sum_n f(n\pi/L)$.
- Dirac delta functions become Kronecker deltas $\delta(p^+ - p^{'+}) \rightarrow \frac{L}{\pi} \delta_{nn'}$.

Supersymmetric DLCQ (SDLCQ)

Matsumura, Sakai, and Sakai, PRD **52**, 2446 (1995).
Lunin and Pinsky, AIP Conf. Proc. **494**, 140 (1999).

- supersymmetry algebra: $\{Q^+, Q^+\} = 2\sqrt{2}P^+$, $\{Q^-, Q^-\} = 2\sqrt{2}P^-$, $\{Q^+, Q^-\} = -4P_\perp$.
- discretize supercharge Q^- and compute $P^-_{\mathrm{SDLCQ}} = \frac{1}{2\sqrt{2}}\{Q^-, Q^-\} \neq P^-_{\mathrm{DLCQ}}$.
- preserves supersymmetry.
- for ordinary DLCQ, recover supersymmetry only in infinite resolution limit.
- will use the large-N_c approximation.

$\mathcal{N}$=(8,8) SYM theory

- reduce $\mathcal{N}$=1 SYM theory from ten to two dimensions.
- the action in light-cone gauge ($A_- = 0$) is

$$S^{LC}_{1+1} = \int dx^+ dx^- \mathrm{tr}\Big[\partial_+ X_I \partial_- X_I + i\theta^T_R \partial^+ \theta_R + i\theta^T_L \partial^- \theta_L$$

$$+\tfrac{1}{2}(\partial_- A_+)^2 + gA_+ J^+ + \sqrt{2} g \theta^T_L \beta_I [X_I, \theta_R] + \tfrac{g^2}{4}[X_I, X_J]^2\Big].$$

- the X_I with $I = 1, \ldots, 8$ are the scalar remnants of the transverse components of the 10-D gauge field A_μ.
- the two-component spinor fields θ_R and θ_L are remnants of the right-moving and left-moving projections of the sixteen-component spinor.
- $J^+ = i[X_I, \partial_- X_I] + 2\theta^T_R \theta_R$.
- $\beta_1 \equiv \sigma_1$, $\beta_2 \equiv \sigma_3$.

Supercharges

- dimensionally reduce the 10-D supercurrent, to obtain

$$Q^-_\alpha = g\int dx^- \mathrm{tr}\left(-2^{3/4}J^+\frac{1}{\partial_-}u_\alpha + 2^{-1/4}i[X_I, X_J](\beta_I\beta_J)_{\alpha\eta}u_\eta\right)$$

where $\alpha, \eta = 1, \ldots, 8$
and the u_α are the components of θ_R.

- the mode expansions of the dynamical fields are

$$X_{Ipq}(x^-) = \frac{1}{\sqrt{2\pi}}\int_0^\infty \frac{dk^+}{\sqrt{2k^+}}[A_{Ipq}(k^+)e^{-ik^+x^-} + A^\dagger_{Iqp}(k^+)e^{ik^+x^-}]$$

$$u_{\alpha pq}(x^-) = \frac{1}{\sqrt{2\pi}}\int_0^\infty \frac{dk^+}{\sqrt{2}}[B_{\alpha pq}(k^+)e^{-ik^+x^-} + B^\dagger_{\alpha qp}(k^+)e^{ik^+x^-}],$$

where $p, q = 1, 2, \ldots, N_c$.

Creation/annihilation operators

The operators A and B satisfy

$$[A_{Ipq}(k^+), A^\dagger_{Jrs}(k^{'+})] = \delta_{IJ}\delta_{pr}\delta_{qs}\delta(k^+ - k^{'+}),$$

$$\{B_{\alpha pq}(k^+), B^\dagger_{\beta rs}(k^{'+})\} = \delta_{\alpha\beta}\delta_{pr}\delta_{qs}\delta(k^+ - k^{'+}).$$

In the discrete approximation, we rescale the annihilation operators $\sqrt{\frac{L}{\pi}}a(k) = A(k^+ = \frac{\pi k}{L}),\ \ \sqrt{\frac{L}{\pi}}b(k) = B(k^+ = \frac{\pi k}{L})$,

so that

$$[a_{Ipq}(k), a^\dagger_{Jrs}(k')] = \delta_{IJ}\delta_{pr}\delta_{qs}\delta_{kk'},$$

$$\{b_{\alpha pq}(k), b^\dagger_{\beta rs}(k')\} = \delta_{\alpha\beta}\delta_{pr}\delta_{qs}\delta_{kk'}$$

and

$$X_{Ipq}(x^-) = \frac{1}{\sqrt{2\pi}}\sum_{k=1}^{\infty}\frac{1}{\sqrt{2k}}[a_{Ipq}(k)e^{-i\frac{\pi}{L}kx^-} + a^\dagger_{Iqp}(k^+)e^{i\frac{\pi}{L}kx^-}],$$

$$u_{\alpha pq}(x^-) = \frac{1}{\sqrt{2L}}\sum_{k=1}^{\infty}\frac{1}{\sqrt{2}}[b_{\alpha pq}(k)e^{-i\frac{\pi}{L}kx} + b^\dagger_{\alpha qp}(k)e^{i\frac{\pi}{L}kx^-}].$$

Symmetries

- diagonalize any one of the $P_\alpha^- = \{Q_\alpha^-, Q_\alpha^-\}/2\sqrt{2}$.
- various symmetries can be used to block diagonalize.
- transformations that leave Q_8^- unchanged are generated by

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	b_1	b_2	b_3	b_4	b_5	b_6	b_7
1	a_1	a_8	$-a_5$	$-a_4$	$-a_3$	a_6	$-a_7$	a_2	b_1	b_4	$-b_3$	b_2	b_7	$-b_6$	b_5
2	a_2	a_1	$-a_5$	$-a_6$	$-a_3$	$-a_4$	$-a_8$	$-a_7$	b_4	b_3	b_2	b_1	b_5	$-b_6$	$-b_7$
3	a_2	a_1	$-a_6$	a_8	a_7	$-a_3$	a_5	a_4	b_1	$-b_2$	b_6	b_5	b_4	b_3	$-b_7$
4	$-a_1$	$-a_2$	$-a_3$	$-a_4$	$-a_5$	$-a_6$	$-a_7$	$-a_8$	b_1	b_2	b_3	b_4	b_5	b_6	b_7
5	a_1	a_2	a_3	$-a_4$	$-a_5$	a_6	$-a_7$	$-a_8$	b_1	b_2	$-b_3$	$-b_4$	$-b_5$	$-b_6$	b_7
6	$-a_1$	a_2	a_3	$-a_4$	a_5	$-a_6$	$-a_7$	a_8	b_1	$-b_2$	b_3	$-b_4$	$-b_5$	b_6	$-b_7$
7	a_1	$-a_2$	a_3	a_4	$-a_5$	$-a_6$	$-a_7$	a_8	$-b_1$	b_2	b_3	$-b_4$	b_5	$-b_6$	$-b_7$

$\mathcal{N}$=(2,2) SYM theory

- reduce $\mathcal{N}$=1 SYM theory from four to two dimensions.
- the action is the same as for $\mathcal{N}$=(8,8), except that the indices run from 1 to 2 instead of 1 to 8.
- there are also fewer symmetries.
- the smaller number of fields allows calculations at higher resolution; we have reached $K = 14$ for the (2,2) theory vs $K = 11$ for the (8,8) theory.
- however, there is no conjecture of correspondence or any separate estimate of the correlator's behavior.

Stress-energy correlator

- the stress-energy correlation function is

$$F(x^-, x^+) \equiv \langle T^{++}(x) T^{++}(0) \rangle.$$

- for the string theory corresponding to $\mathcal{N}$=(8,8) SYM theory, F can be calculated on the string-theory side in a weak-coupling super-gravity approximation

$$F(x^-, x^+) = \frac{N_c^{\frac{3}{2}}}{g_{\mathrm{YM}} r^5}, \text{ with } r \equiv \sqrt{x^+ x^-}.$$

- will compute this in strongly-coupled SYM theory and compare, considering both $\mathcal{N}$=(8,8) and (2,2).

Fourier transform

- fix the total momentum $P^+ = P_-$.
- compute the Fourier transform and express the transform in a spectral decomposed form

$$
\begin{aligned}
\tilde{F}(P_-, x^+) &= \frac{1}{2L}\langle T^{++}(P_-, x^+)T^{++}(-P_-, 0)\rangle \\
&= \sum_i \frac{1}{2L}\langle 0|T^{++}(P_-, 0)|i\rangle \mathrm{e}^{-iP_+^i x^+}\langle i|T^{++}(-P_-, 0)|0\rangle .
\end{aligned}
$$

- the position-space form is recovered by Fourier transforming with respect to $P_- = K\pi/L$.
- continue to Euclidean space by taking $r = \sqrt{2x^+x^-}$ to be real.

Final form

This yields

$$F(x^-,x^+) = \sum_i \left|\frac{L}{\pi}\langle 0|T^{++}(K)|i\rangle\right|^2 \left(\frac{x^+}{x^-}\right)^2 \frac{M_i^4 K_4(M_i\sqrt{2x^+x^-})}{8\pi^2 K^3}.$$

The stress-energy operator is given by

$$T^{++}(x^-,x^+) = \mathrm{tr}\left[(\partial_- X^I)^2 + \frac{1}{2}(iu^\alpha\partial_- u^\alpha - i(\partial_- u^\alpha)u^\alpha)\right].$$

In terms of the discretized creation operators, we find

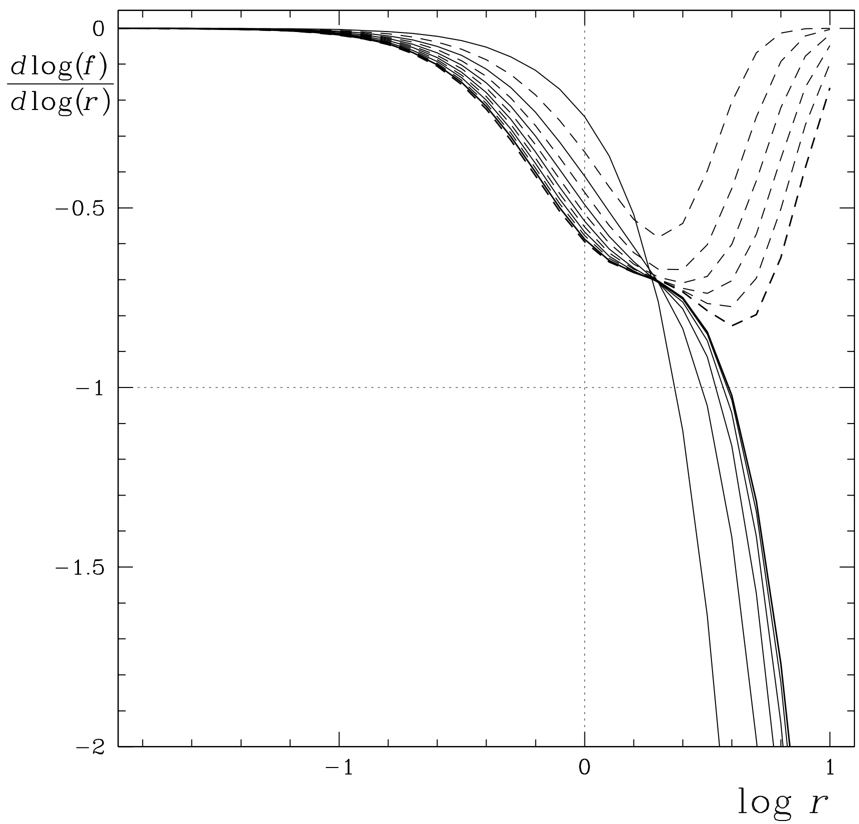
$$T^{++}(-K)|0\rangle = \frac{\pi}{2L}\sum_{k=1}^{K-1}\left[-\sqrt{k(K-k)}a^\dagger_{Iij}(K-k)a^\dagger_{Iji}(k)\right.$$
$$\left. + \left(\frac{K}{2}-k\right) b^\dagger_{\alpha ij}(K-k)b^\dagger_{\alpha ji}(k)\right]|0\rangle.$$

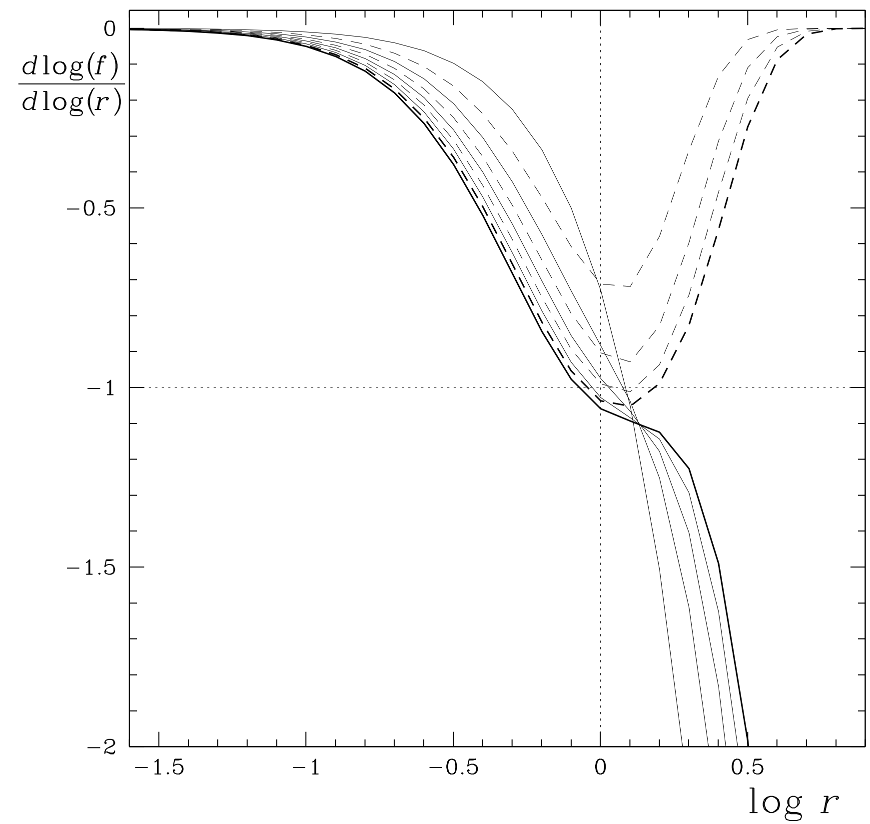
Thus $(L/\pi)\langle 0|T^{++}(K)|i\rangle$ is independent of L.
Also, only one symmetry sector contributes.

Computation

- the correlator behaves like $1/r^4$ at small r:
  $\left(\frac{x^-}{x^+}\right)^2 F(x^-, x^+) = \frac{N_c^2(2n_b+n_f)}{4\pi^2 r^4}(1 - 1/K)$.
- rescale by defining
  $f \equiv \langle T^{++}(x) T^{++}(0) \rangle \left(\frac{x^-}{x^+}\right)^2 \frac{4\pi^2 r^4}{N_c^2(2n_b+n_f)}$.
- then $f \to (1 - 1/K)$ for small r.
- compute f numerically by
  - computing entire spectrum for "small" matrices.
  - using Lanczos iterations for large matrices.

Plot of (2,2) & (8,8) log derivative



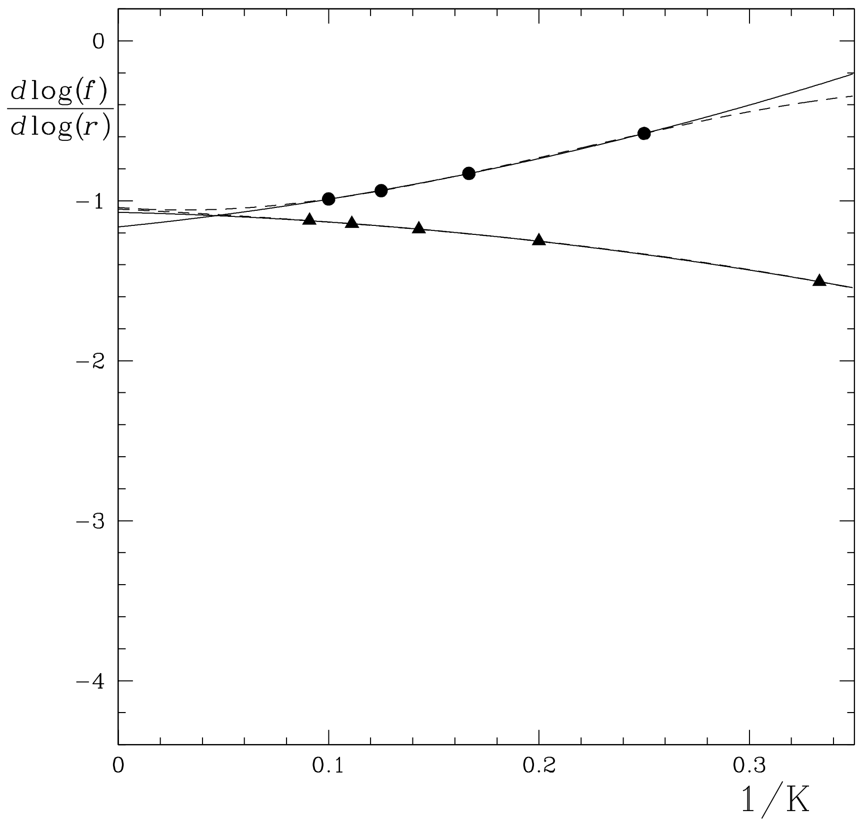


Comments

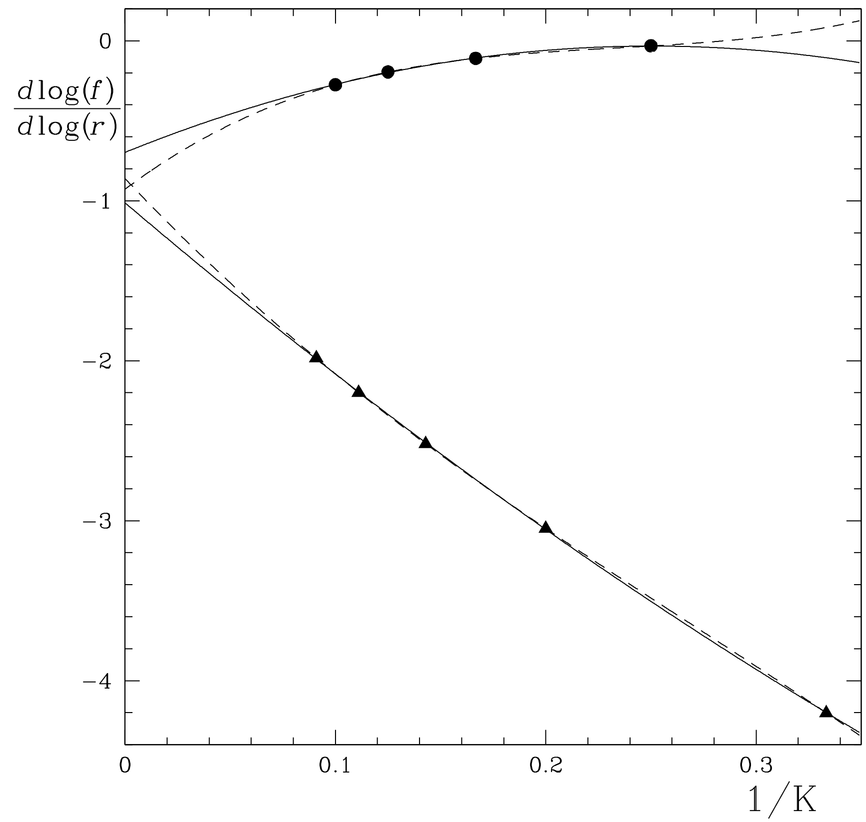
- at small r, the graphs for different K match the expected $(1 - 1/K)$ behavior.
- at large r the behavior is different between odd and even K; the difference gets smaller as K gets bigger.
- for even K there is exactly one massless state that contributes to the correlator, while there is no massless state for odd K.
- the lowest massive state dominates for odd K at large r; however, this state becomes massless as $K \to \infty$.
- for most of the range, can extrapolate to infinite K.

Sample (8,8) extrapolations

Fit quadratic and cubic to K odd and even separately:

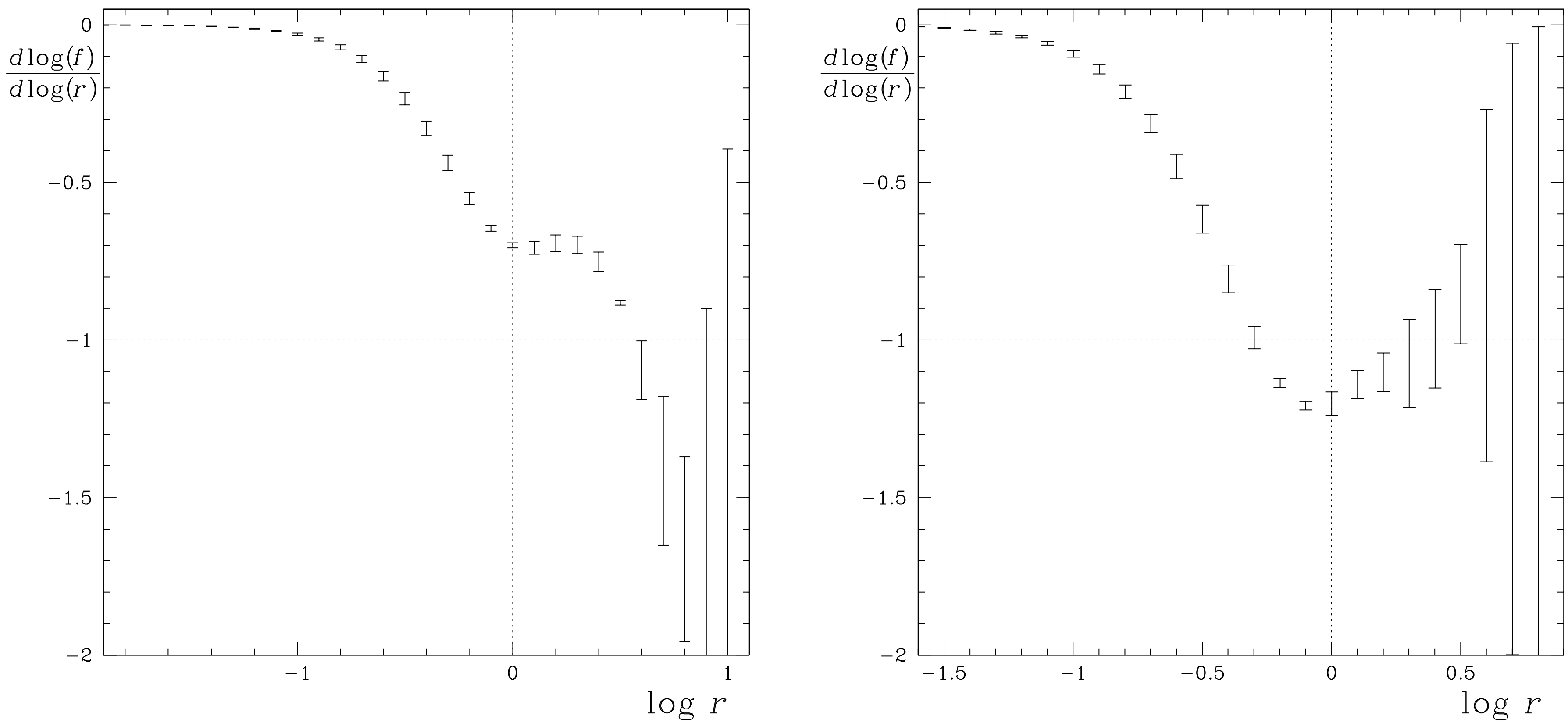


$\log(r) = 0.2$



$\log(r) = 0.5$

Plot of (2,2) & (8,8) extrapolations



Conclusions

- the (2,2) and (8,8) results differ in their behavior at intermediate r.
- the calculations do distinguish between theories that differ in the amount of extended supersymmetry.
- the result for the (8,8) theory is consistent with the Maldacena conjecture.
- additional calculations at higher resolution are needed to reduce the extrapolation errors and explicitly confirm the conjecture.