Non perturbative renormalization in LFD with Fock state truncation

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Plan

- General framework
- Definition of counterterms and bare coupling constants
- Cancelation of divergences with Fock state truncation
- Example : electromagnetic form factor of the electron for N=2

General framework

- spin 1/2 fermion coupled to a boson (scalar or gauge)

- explicitly covariant formulation of LFD

  light front position characterized by a four-vector ω

  $\omega\cdot x = 0$ with $\omega^2 = 0$

- Fock state truncation to N particles

  ex: two body truncation for a scalar boson

  $$|\Phi(p)\rangle = |1\rangle + |2\rangle$$

  with the general structure for the two body component

$$\bar{u}(k_1)\Gamma_2 u(p) = b_1 \bar{u}(k_1) u(p) + b_2 \frac{m}{\omega\cdot p} \bar{u}(k_1) \not{\omega}\, u(p)$$

Definition of counterterms and bare coupling constants

- Mass counterterm $\delta m\, \bar{\Psi}\Psi$ and physical mass m
- Bare coupling constants $g_0\, \bar{\Psi}\, \phi\, \Psi$ for scalar boson
- ω - dependent counterterms

ex.: $$Z_\omega \frac{m}{i\omega\cdot\partial} \bar{\Psi}\, \not{\omega}\, \Psi$$

- insure the 2PGF is ω - independent at $s = m^2$
- should be zero in an exact calculation

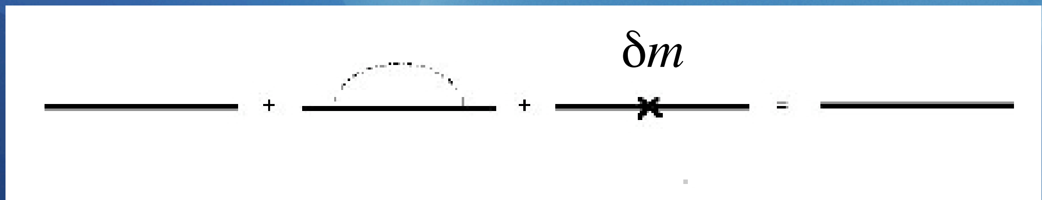
➡ explicit check of the approximation

- invariant mass regularization

Cancelation of divergences with Fock state truncation

- **Mass counterterm**

  - **Simple example for N=2**



at $\not{p} = m$

couples two different Fock components

- $\delta m \longrightarrow \delta m^{(2)}$ or more generally $\delta m^{(n)}$

n: total number of particles in which the fermion can fluctuate

- in the equation of motion $\hat{P}^2 \, \phi(p) \;=\; M^2 \, \phi(p)$

  for the N-body truncation

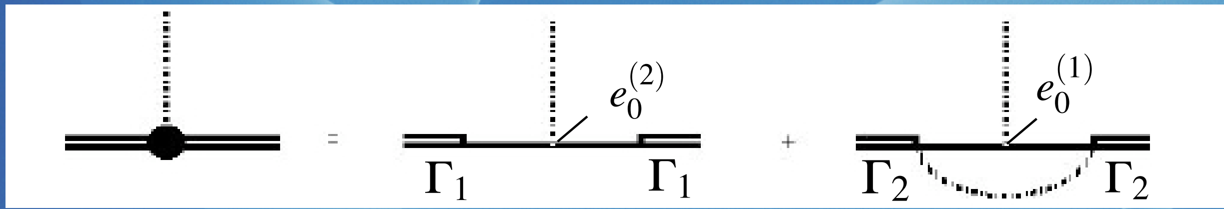
$$\delta m^{(1)} = 0 \; , \; \delta m^{(2)} = -g^2 \Sigma(p^2 = m^2) \; , \; \delta m^{(3)} \; , \; \ldots \; , \delta m^{(n)}$$

- systematic determination of $\delta m^{(n)}$

  by doing successive calculations for N = 2, 3, ...n

❑ Bare coupling constants

➢ **Determination from the electromagnetic form factor at** $Q^2 = 0$

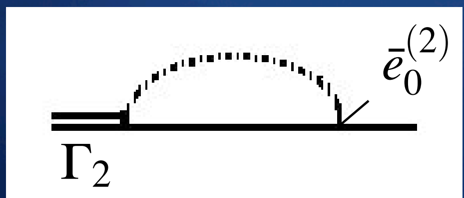


+ contact terms

⇨ **determination of** $e_0^{(n)}$ **similarly to** $\delta m^{(n)}$

➢ **The electromagnetic probe is here considered as an instantaneous external field which does not enter into the counting rule for the maximum number of particles in the w.f.**

➢ **This is NOT the case for the calculation of the N-body component of the wave function**



the left side of the vertex cannot fluctuate anymore in a 2 body state while the right side can

⇨ $\bar{e}_0^{(n)} \neq e_0^{(n)}$

- ➢ **determination of** $\bar{e}_0^{(n)}$

  diagramatically (for N=2) or from the non-perturbative condition

$$\bar{u}(k_1)\Gamma_2(s=m^2)u(p) = g_{phys}\,\bar{u}(k_1)u(p)$$

  after extraction, and cancelation, of the ω **- dependent piece**

- ➢ **should be done for any regularization procedure**
  and even for finite renormalization contributions

Example: electromagnetic form factor of the electron for N=2

➢ **Calculation of the state vector**

$$\bar{u}(k_1)\,\Gamma_2\,u(p) = \bar{e}_0^{(2)}\sqrt{Z_2}\,\bar{u}(k_1)\ \ u(p)$$

with the normalization constant Z_2 **determined from the normalization of the state vector**

$$\langle p'|p\rangle = 2\,p_0\,\delta^3(p'-p)$$

we get $$Z_2 = \frac{1}{1+\left[\bar{e}_0^{(2)}\right]^2 I(L)}$$

where I(L) is logarithmically divergent

➢ **Calculation of $\bar{e}_0^{(2)}$**

$$\bar{e}_0^{(2)} \sqrt{Z_2} \equiv e \qquad\qquad \left[\bar{e}_0^{(2)}\right]^2 = \frac{e^2}{1 - e^2\, I(L)}$$

with that one has $\quad Z_2 = 1 \; - \; e^2\, I(L)$

➢ **Calculation of $e_0^{(2)}$**

$$e \equiv \sqrt{Z_2}\, e_0^{(2)} \sqrt{Z_2} + \sqrt{Z_2}\, e_0^{(1)} \left[\bar{e}_0^{(2)}\right]^2 I(L)\, \sqrt{Z_2}$$

so that $\quad e_0^{(2)} \equiv e \qquad$ **Ward-Identity satisfied exactly**

➢ **Only possible because**

- **extraction of the ω-dependent piece of the two-body wave function**
- **different physical content for $e_0^{(2)}$ (elm f.f.) and $\bar{e}_0^{(2)}$ (w.f.)**

Conclusions

- ➢ Systematic strategy to calculate physical masses and coupling constants

- ➢ Analytic N=2 results

  recover perturbative calculations including W.I.

- ➢ Fully non-perturbative calculation for N=3 under way for scalar bosons

❑ Equation of motion

$$\hat{P}^2\,\phi(p) = M^2\,\phi(p) \quad \textbf{with} \quad \hat{P}_\mu = \hat{P}^0_\mu + \hat{P}^{int}_\mu$$

$$\hat{P}^{int}_\mu = \omega_\mu \int H^{int}(x)\delta(\omega\cdot x)\, d^4x = \omega_\mu \int_{-\infty}^{+\infty} \tilde{H}^{int}(\omega\tau)\frac{d\tau}{2\pi}$$

$$\frac{1}{2\pi}\int \tilde{H}^{int}(\omega\tau)\frac{d\tau}{\tau}\mathcal{G}(p) = -\mathcal{G}(p) \equiv -\lambda(M^2)\mathcal{G}(p)$$

with $\mathcal{G}(p) = 2(\omega\cdot p)\hat{\tau}\phi(p)$

- **Equation of motion for a fermion coupled to a scalar boson**