Second Moment of the Pion's Distribution Amplitude from Lattice QCD

James Zanotti

NIC/DESY Zeuthen

July 11, 2005

Acknowledgments

QCDSF: M. Göckeler, R. Horsley, D. Pleiter, P. Rakow, G. Schierholz, W. Schroers

Outline

Introduction

- Amplitude for converting a pion into $q\bar{q}$ pair

$$\langle 0|\bar{d}(0)\gamma_\mu\gamma_5 u(x)|\pi^+(p)\rangle = if_\pi p_\mu \int_0^1 du\ e^{-iupx}\Phi_\pi(u,x^2) + \ldots$$

- Factorise into hard and soft parts

$$\Phi_\pi(u,x^2) = c(u,x^2,\mu^2) \otimes \phi_\pi(u,\mu^2)$$

Introduction

- Amplitude for converting a pion into $q\bar{q}$ pair

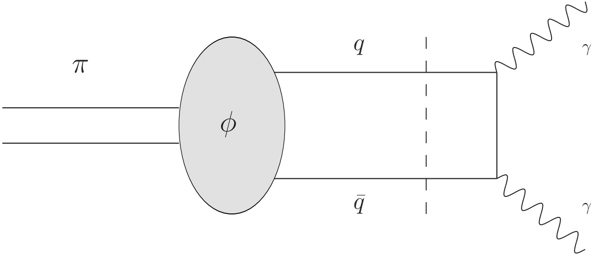
$$\langle 0|\bar{d}(0)\gamma_\mu\gamma_5 u(x)|\pi^+(p)\rangle = i f_\pi p_\mu \int_0^1 du\ e^{-iupx}\Phi_\pi(u,x^2) + \dots$$

- Factorise into hard and soft parts

$$\Phi_\pi(u,x^2) = c(u,x^2,\mu^2) \otimes \phi_\pi(u,\mu^2)$$

- Coefficient function (perturbative)
- Distribution amplitude (non-perturbative)

Distribution Amplitudes



- Measured in exclusive processes
- On the Light Cone

$$\langle 0|\bar{d}(0)\gamma_\mu\gamma_5 u(z)|\pi^+(p)\rangle = i f_\pi p_\mu \int_0^1 du\ e^{-iupz}\phi_\pi(u,\mu)$$

where $z^2 = 0$

- Describes how the pion's longitudinal momentum is shared between its quark and anti-quark constituents

Moments of Distribution Amplitudes

- nth moment of the pion's distribution amplitude

$$\langle \xi^n \rangle \equiv \int \mathrm{d}\xi\, \xi^n \, \phi(\xi, Q^2), \quad \xi = x_q - x_{\bar{q}}$$

- extracted from matrix elements of twist-2 operators

$$\langle 0 | \mathcal{O}_{\{\mu_0 \ldots \mu_n\}}(0) | \pi(p) \rangle = f_\pi p_{\mu_0} \ldots p_{\mu_n} \langle \xi^n \rangle + \cdots$$

$$\mathcal{O}_{\mu_0 \ldots \mu_n}(0) = (-i)^n \overline{\psi} \gamma_{\mu_0} \gamma_5 \overleftrightarrow{D}_{\mu_1} \ldots \overleftrightarrow{D}_{\mu_n} \psi$$

- normalisation $\rightarrow \langle \xi^0 \rangle = 1$
- $\langle \xi^1 \rangle = 0$
- First non-trivial moment $n = 2$

Moments of Distribution Amplitudes

- $H(4)$-representation $\implies$ use operators
  - $\vec{p} = (1,1,0)$:

$$\mathcal{O}^a_{\{412\}} = \frac{1}{6}\left(\mathcal{O}_{412} + \mathcal{O}_{421} + \mathcal{O}_{124} + \mathcal{O}_{142} + \mathcal{O}_{214} + \mathcal{O}_{241}\right)$$

  - $\vec{p} = (1,0,0)$:

$$\mathcal{O}^b_{\{411\}} = \left(\mathcal{O}_{\{411\}} - \frac{\mathcal{O}_{\{422\}} + \mathcal{O}_{\{433\}}}{2}\right)$$

- Mixings?

Moments of Distribution Amplitudes

- Forward matrix elements:
  - $\mathcal{O}^b$ suffers from operator mixing
  - $\mathcal{O}^a$ does not
- Non-forward matrix elements: hep-lat/0410009
  - Mix with operators containing external ordinary derivatives

$$\mathcal{O}^{a\,\prime}_{\{412\}} = -\frac{1}{4}\partial_{\{4}\partial_1\left(\bar{\psi}\gamma_{2\}}\gamma_5\psi\right)$$

  - Mixing coefficient expected to be small
  - WIP $\Longrightarrow$ use only $\mathcal{O}^a_{\{412\}}$

Lattice Two-Point Functions

$$C^{\mathcal{O}}(t,\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \left\langle \mathcal{O}(\vec{x},t) J(\vec{0},0)^\dagger \right\rangle$$
$$\rightarrow \frac{Z}{2E} \langle 0|\mathcal{O}(0)|\pi(p)\rangle e^{-Et}, \qquad t \gg 0$$

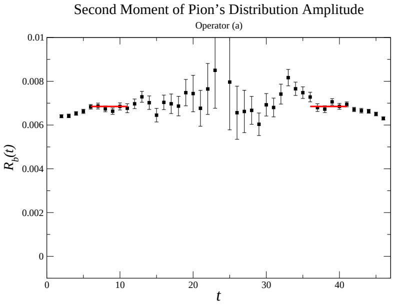
where

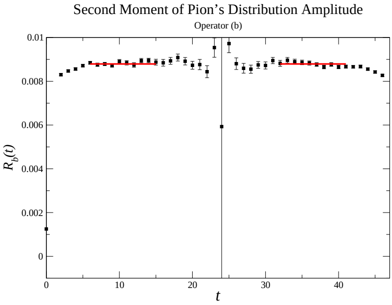
$$Z = \langle \pi(p)|J(0)^\dagger|0\rangle$$
$$J(x) \equiv \pi(x) = \overline{\psi}(x)\gamma_5\psi(x), \quad J(x) \equiv A_4(x) = \overline{\psi}(x)\gamma_5\gamma_4\psi(x)$$

- Construct ratio

$$R^a = \frac{C^{\mathcal{O}^a_{\mu\nu\rho}}(t)}{C^{\mathcal{O}_4}(t)} = p_\nu p_\rho \, \langle \xi^2 \rangle^a$$
$$R^b = \frac{C^{\mathcal{O}^b_{\mu\nu\nu}}(t)}{C^{\mathcal{O}_4}(t)} = p_\nu^2 \, \langle \xi^2 \rangle^b$$

Ratios


Second Moment of Pion's Distribution Amplitude
Operator (a)
$R_b(t)$
0.01
0.008
0.006
0.004
0.002
0
0
10
20
30
40
t


Second Moment of Pion's Distribution Amplitude
Operator (b)
$R_b(t)$
0.01
0.008
0.006
0.004
0.002
0
0
10
20
30
40
t

Renormalisation

- Renormalise bare lattice operators in scheme, $\mathcal{S}$ and at scale, M

$$\mathcal{O}^{\mathcal{S}}(M) = Z^{\mathcal{S}}_{\mathcal{O}}(M)\mathcal{O}_{bare}$$

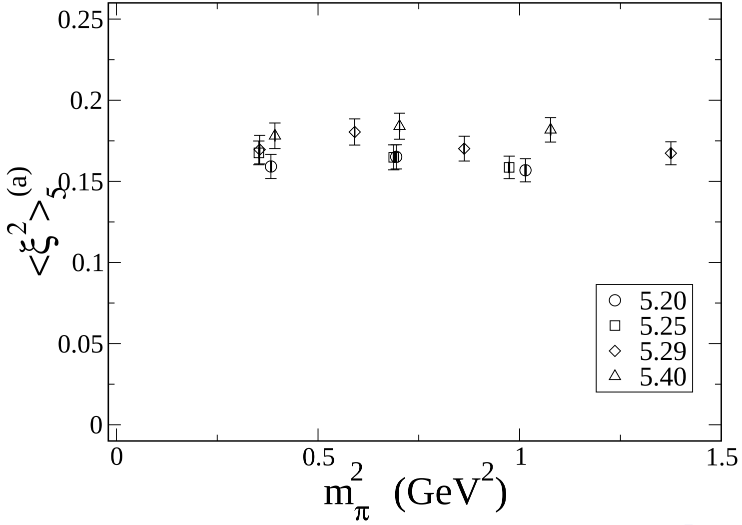
$$\langle\xi^2\rangle^{\mathcal{S}}(M) = \frac{Z^{\mathcal{S}}_{\mathcal{O}}(M)}{Z^{\mathcal{S}}_{\mathcal{O}_4}(M)}\langle\xi^2\rangle_{bare}$$

- We use $\mathcal{S} = \overline{\mathrm{MS}}$ at $M^2 = \mu^2 = 5\,(\mathrm{GeV})^2$

Lattice Parameters

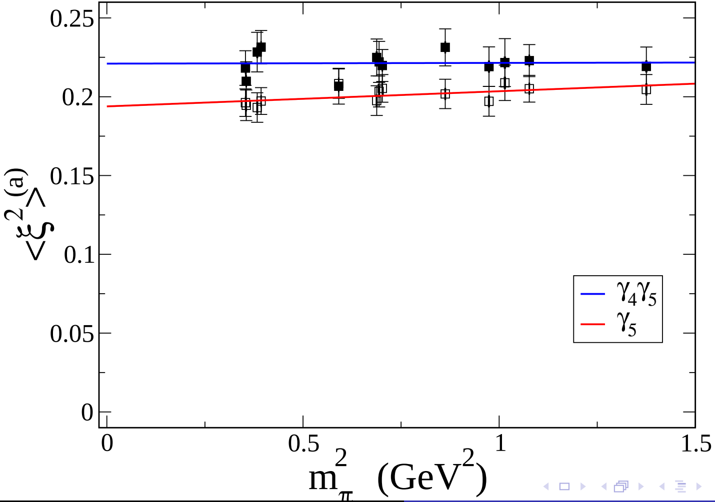
β	$\kappa_{\rm sea}$	Volume	$N_{\rm traj}$	a (fm)	m_π (GeV)
5.20	0.13420	$16^3 \times 32$	O(5000)	0.1226	0.9407(19)
5.20	0.13500	$16^3 \times 32$	O(8000)	0.1052	0.7780(24)
5.20	0.13550	$16^3 \times 32$	O(8000)	0.0992	0.5782(30)
5.25	0.13460	$16^3 \times 32$	O(5800)	0.1056	0.9217(20)
5.25	0.13520	$16^3 \times 32$	O(8000)	0.0973	0.7746(25)
5.25	0.13575	$24^3 \times 48$	O(5900)	0.0904	0.5552(14)
5.29	0.13400	$16^3 \times 32$	O(4000)	0.1039	1.0952(18)
5.29	0.13500	$16^3 \times 32$	O(5600)	0.0957	0.8674(17)
5.29	0.13550	$24^3 \times 48$	O(2000)	0.0898	0.7180(13)
5.40	0.13500	$24^3 \times 48$	O(3700)	0.0821	0.9692(14)
5.40	0.13560	$24^3 \times 48$	O(3500)	0.0784	0.7826(17)
5.40	0.13610	$24^3 \times 48$	O(3500)	0.0745	0.5856(22)

Quark Mass Dependence



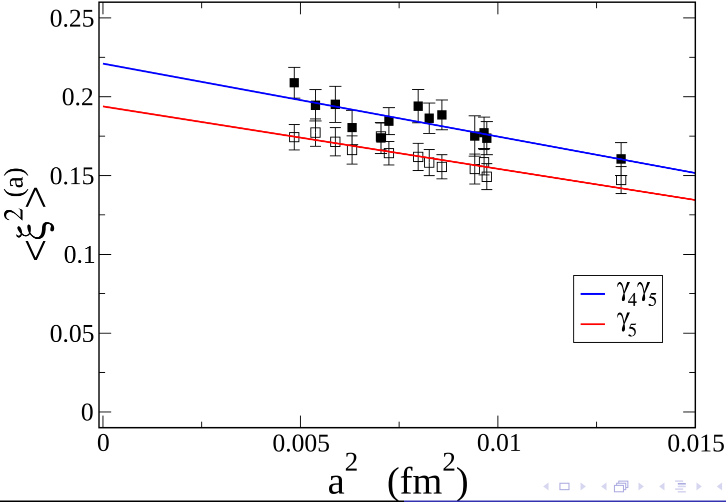
Quark Mass Dependence

$\langle \xi^2 \rangle = m_0 + c_1 m_\pi^2 + c_2 a^2 \quad \Longrightarrow \quad \chi^2/\mathrm{dof} < 1$



Quark Mass Dependence

$\langle \xi^2 \rangle = m_0 + c_1 m_\pi^2 + c_2 a^2 \quad \Longrightarrow \quad \chi^2/\mathrm{dof} < 1$



Result for Second Moment

- $J(x) \equiv \pi(x) = \overline{\psi}(x)\gamma_5\psi(x)$

$$\langle \xi^2 \rangle = 0.221(8)$$

- $J(x) \equiv A_4(x) = \overline{\psi}(x)\gamma_5\gamma_4\psi(x)$

$$\langle \xi^2 \rangle = 0.194(5)$$

- Average

$$\langle \xi^2 \rangle = 0.208(7)$$

- Compare with
  - hep-lat/0211037 (quenched) $\rightarrow \langle \xi^2 \rangle = 0.28(5)$

Gegenbauer Moments

- Expansion in terms of Gegenbauer polynomials $C_n^{\frac{3}{2}}$

$$\phi(u,\mu^2) = 6u(1-u)\sum_{n=0}^{\infty} a_n(\mu^2)C_n^{\frac{3}{2}}(2u-1),\ n \text{ even}$$

- a_n small for $n > 4$

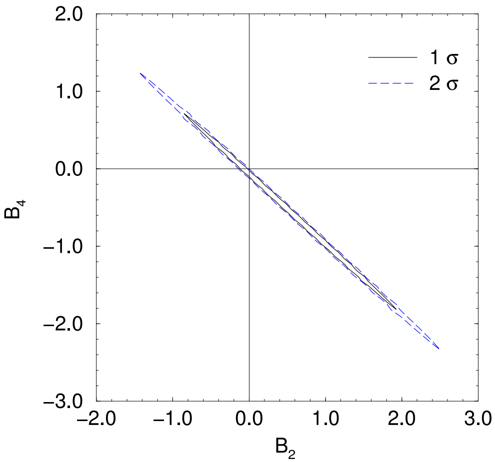
$$\phi(u,\mu^2) = 6u(1-u)\left\{1 + a_2(\mu^2)C_2^{\frac{3}{2}}(2u-1) + a_4(\mu^2)C_4^{\frac{3}{2}}(2u-1)\right\}$$

- Asymptotic limit, $\mu^2 \to \infty$

$$\phi_{\rm as}(u) = 6u(1-u) \implies \langle\xi^2\rangle_{\rm as} = 0.2$$

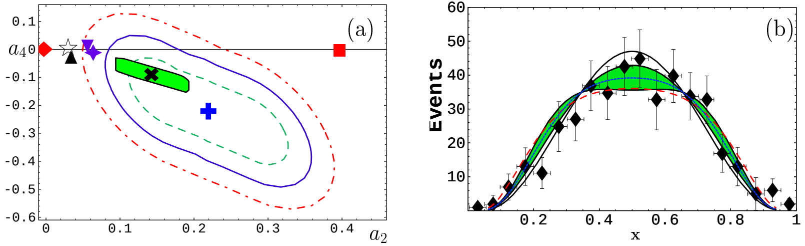
Gegenbauer Moments

- M.Diehl *et al.* hep-ph/0108220, CLEO data for $F_{\pi\gamma}$:



Gegenbauer Moments

- P.Bakulev *et al.* hep-ph/0212250, CLEO data for $F_{\pi\gamma}$, including twist-4:



Gegenbauer Moments

- $0 \leq a_2(1\mathrm{Gev}) \leq 0.3,\ -0.15 \leq a_4(1\mathrm{Gev}) \leq 0.15$
- S.Dalley *et al.* hep-ph/0212086:
  $a_2(0.5\mathrm{Gev}) = 0.15(2),\ \ a_4(0.5\mathrm{Gev}) = 0.04(1)$

$$\rightarrow a_2(2.4\mathrm{Gev}) = 0.07(1),\ \ a_4(2.4\mathrm{Gev}) = 0.01(1)$$

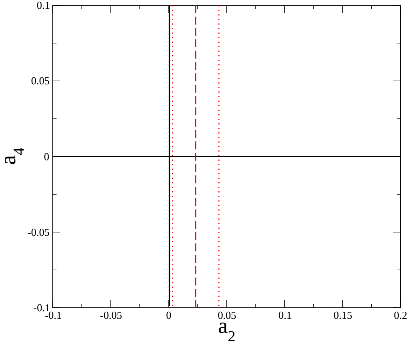
- P.Ball *et al.* hep-ph/0507076:

$$a_2(1\mathrm{Gev}) = 0.19(19),\ \ a_4(1\mathrm{Gev}) \geq -0.07$$

Gegenbauer Moments

$$\langle \xi^2 \rangle = \int_0^1 d\xi \, \xi^2 \phi(\xi, 5\mathrm{GeV}^2) = \frac{1}{35}(7 + 12 a_2) = 0.208$$

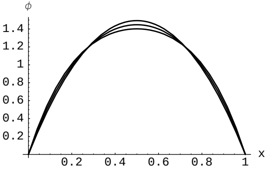
$$\Longrightarrow \; a_2 = 0.02333$$



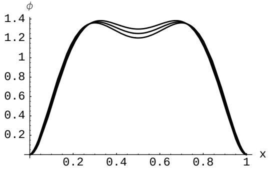
Gegenbauer Moments

$a_2 = 0.023(20)$

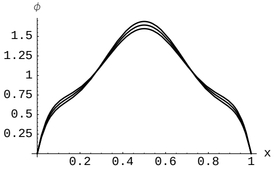
$a_4 = 0.0$



$a_4 = -0.07$



$a_4 = 0.07$



Summary and Future Work

- Lattice calculation of $\langle \xi^2 \rangle \rightarrow a_2$
- Chiral extrapolation
- Operator mixings
- Finite volume effects
- Pion form factors and GPDs
- Nucleon distribution amplitudes
- Nucleon form factors and GPDs → see talk by G.Schierholz
- K meson $\implies \langle \xi^1 \rangle \neq 0$
- Higher twist