Generalized Parton Distributions from Lattice QCD

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– QCDSF Collaboration –

LC2005

Cairns

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Special mention:

P. Hägler, D. Pleiter and J. Zanotti

Outline

Introduction

Lattice QCD

Generalized Parton Distributions

- Preliminaries
- Results
- Modelling

Summary and Outlook

Introduction

Much of our knowledge about QCD and the structure of hadrons has been derived from DIS experiments and measurements of elastic form factors

Perturbative QCD allowed us to extract quark and gluon distribution functions from the experimental data. A quantitative understanding of these distribution functions and, in particular, how quarks and gluons provide the mass (binding) and spin of the nucleon, is still missing

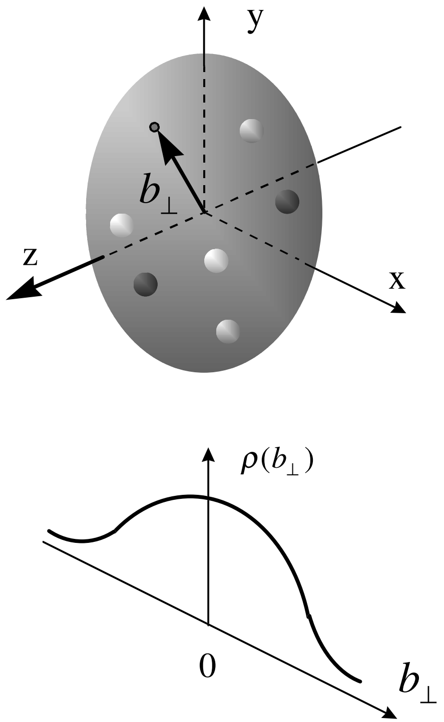
Continuing advances in computing power, and theoretical developments, such as

- $O(a)$ Improvement of the action and the operators, to reduce finite cut-off effects and to facilitate the extrapolation to the continuum limit,
- (Non)-perturbative renormalization and matching of the (bare) lattice operators,
- Chiral perturbation theory, to extrapolate reliably from the masses where the lattice calculations are performed to the physical pion mass,

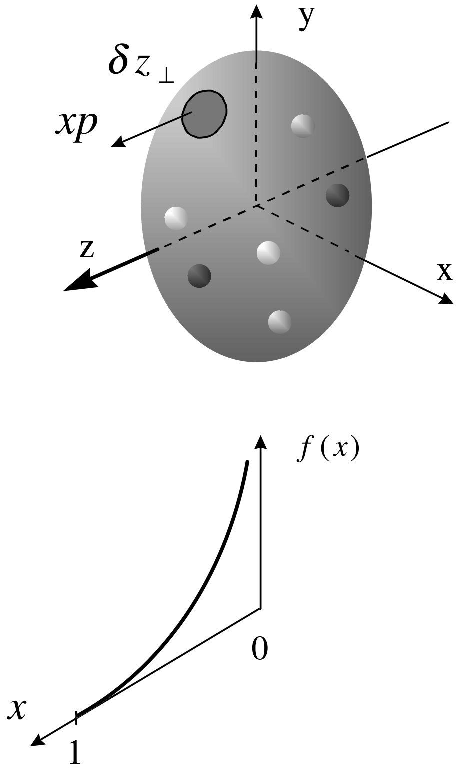
have now brought Lattice QCD to the point that definitive quantitative calculations of a host of hadron observables are becoming possible

Key quantities: Form factors, parton distributions, generalized parton distributions (GPD's)

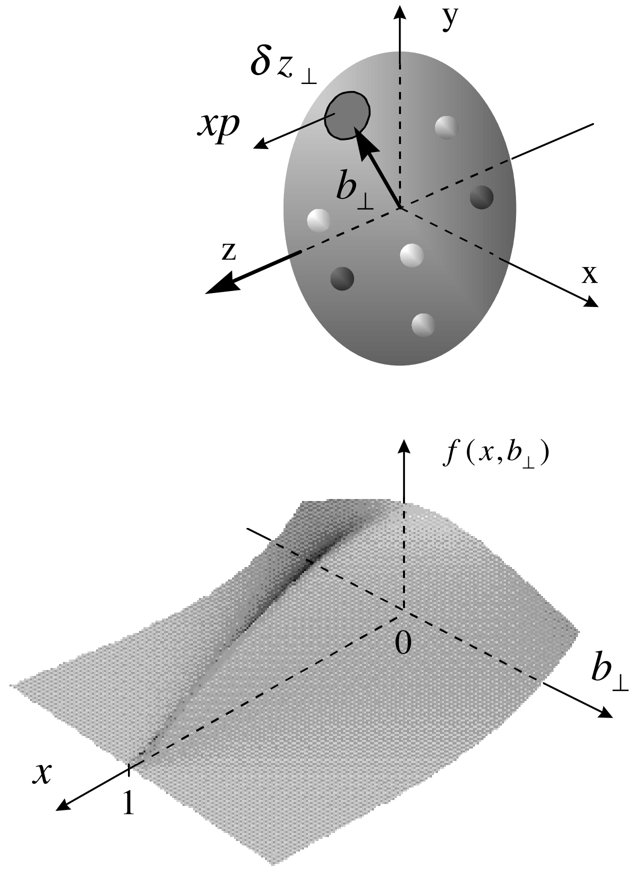
- Form factor



- Parton density



- Generalized parton distribution at η=0



Spatial resolution: $\delta z_\perp \sim 1/Q$

Lattice QCD

Action

$$S = S_G + S_F$$

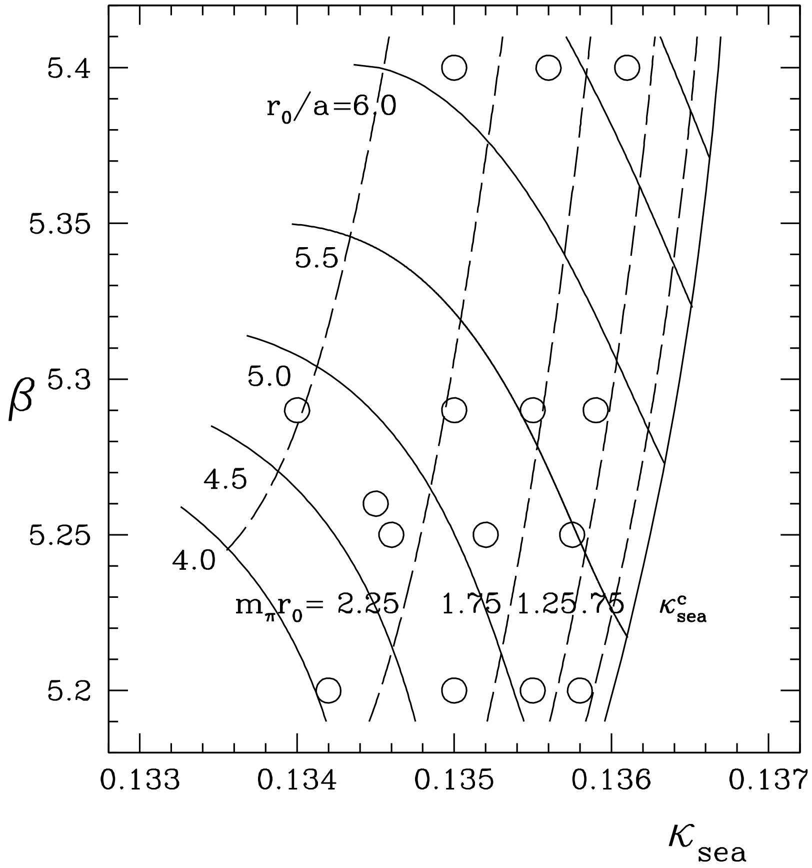
$$S_G = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{3} \mathrm{Re}\,\mathrm{Tr}\, U_{\mu\nu}(x)\right)$$

$$S_F = \sum_x \Big\{ \bar{\psi}(x)\psi(x) - \kappa\, \bar{\psi}(x) U_\mu^\dagger(x-\hat{\mu})[1+\gamma_\mu]\psi(x-\hat{\mu}) - \kappa\, \bar{\psi}(x) U_\mu(x)[1-\gamma_\mu]\psi(x+\hat{\mu}) - \frac{1}{2}\kappa\, c_{SW}\, g\, \bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x) \Big\}$$

Nonperturbatively $O(a)$ improved

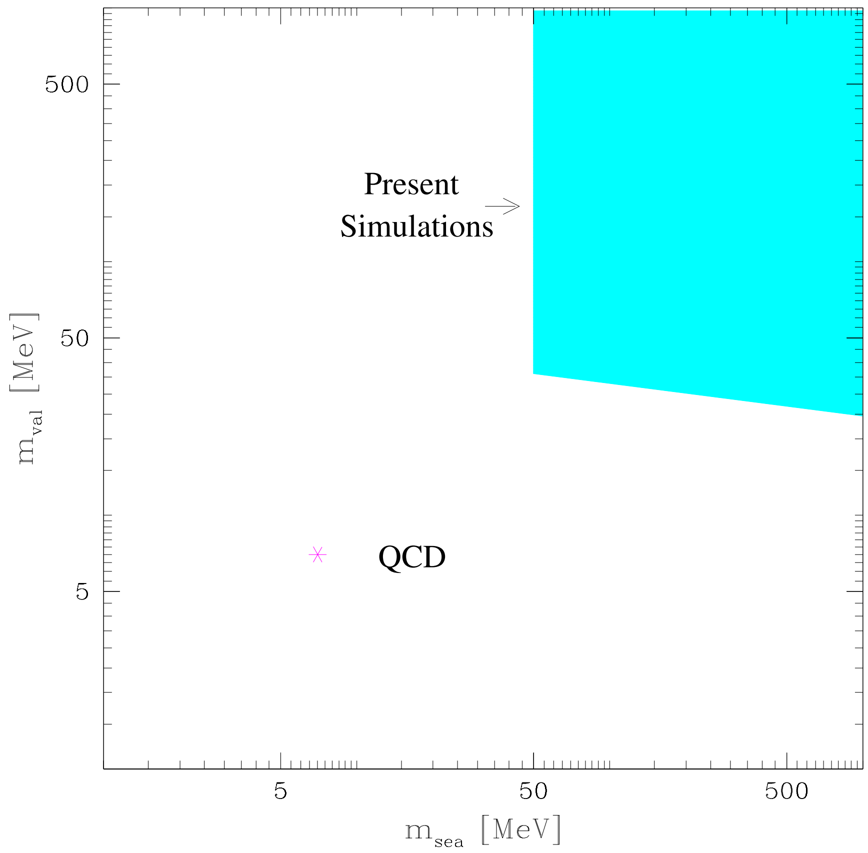
Sheikholeslami & Wohlert

Dynamical Wilson fermions $N_f = 2$



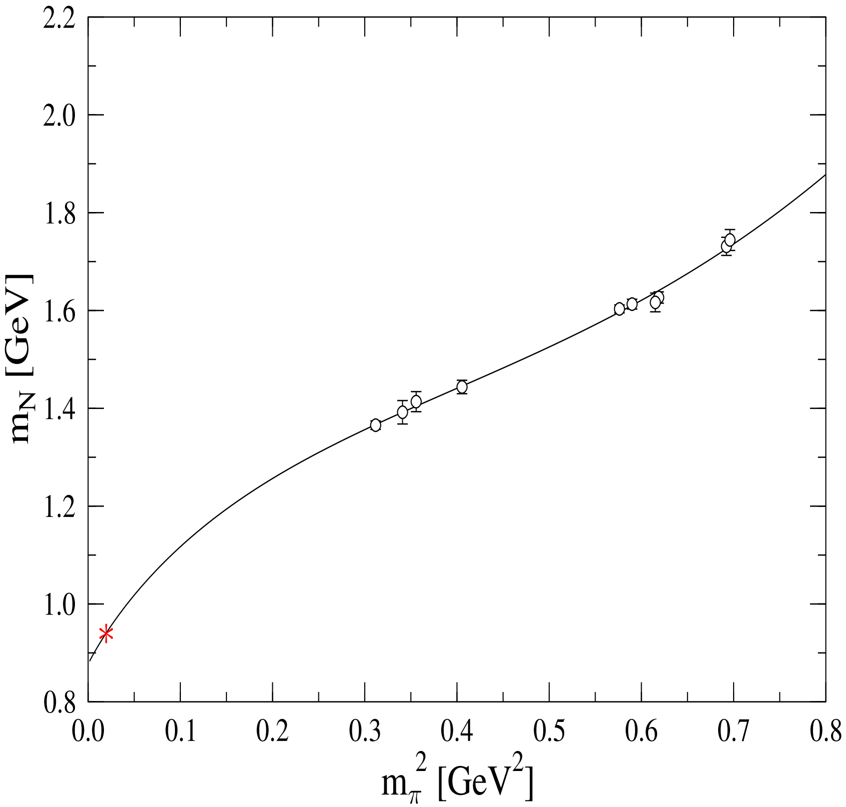
0.07 fm $\lesssim a \lesssim 0.12$ fm; 1 fm $\lesssim L \lesssim 2.2$ fm

Chiral extrapolation

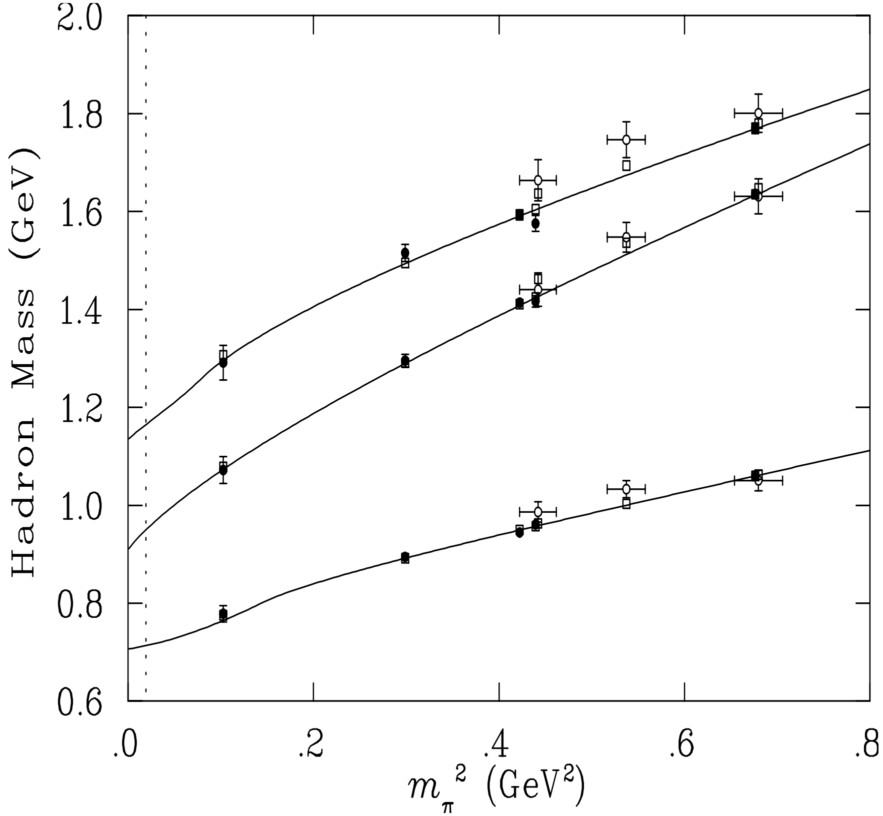


Lattice results have to be extrapolated to the physical quark masses ($*$). This needs theoretical guidance, which may be provided by chiral perturbation theory

Chiral limit



QCDSF

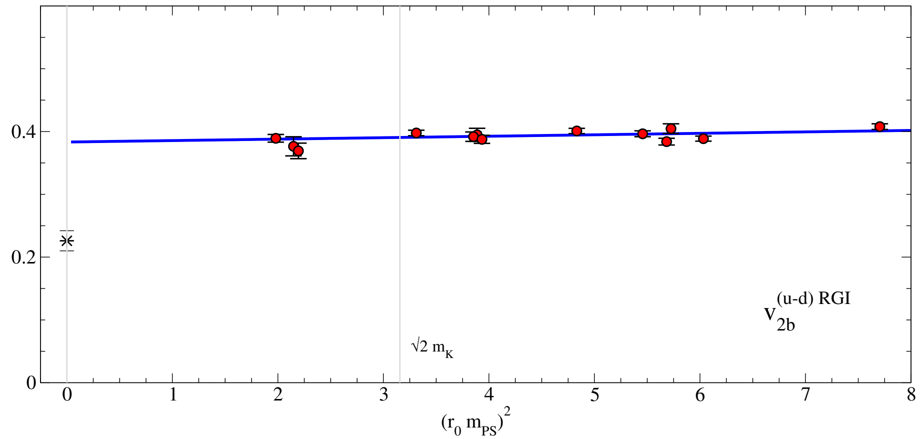


Leinweber, Thomas & Wright

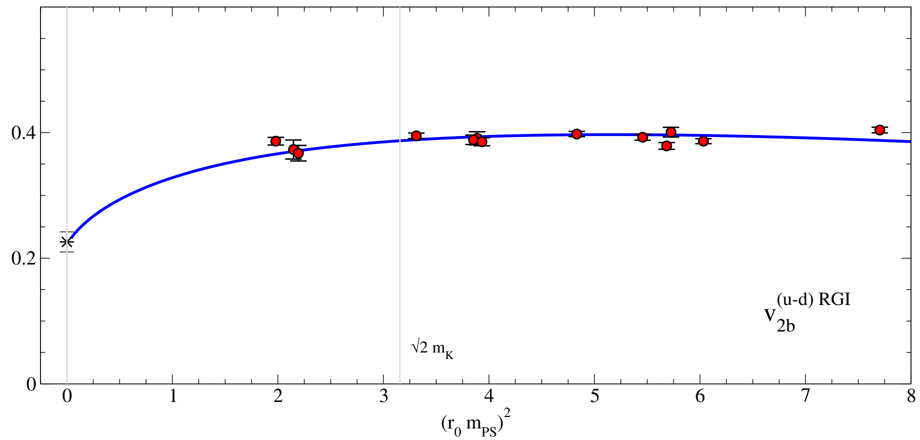
$r_0 = 0.467$ fm

Chiral limit *contd.*

$$\langle x \rangle = \int_0^1 dx\, x\, q(x, Q^2)$$



analytic



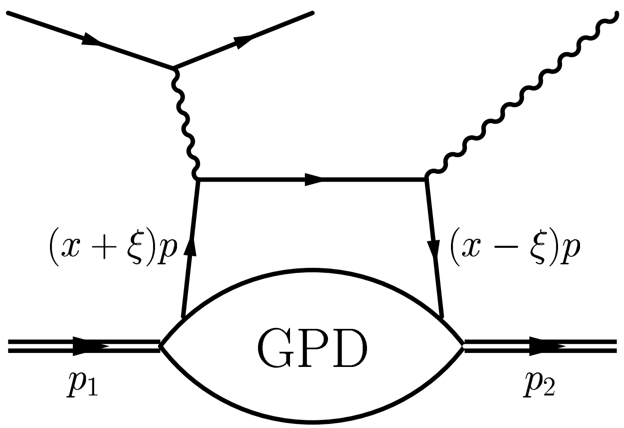
nonanalytic

Thomas, Melnitchouk & Steffen
Arndt & Savage
Chen & Ji

?

Generalized Parton Distributions

Preliminaries



$p = \frac{1}{2}(p_1+p_2)$, $\Delta = p_2 - p_1$, $Q = \frac{1}{2}(q_1+q_2)$

$\xi = 0$: Momentum transfer of the struck parton purely transverse, i.e. $\Delta = \mathbf{\Delta}_\perp$

$\Uparrow$

Of interest to us here only

Unpolarized	$H_q(x, \Delta^2, Q^2)$, $E_q(x, \Delta^2, Q^2)$, $\cdots$
Polarized	$\tilde{H}_q(x, \Delta^2, Q^2)$, $\tilde{E}_q(x, \Delta^2, Q^2)$, $\cdots$
Transversity	$H_q^T(x, \Delta^2, Q^2)$, $E_q^T(x, \Delta^2, Q^2)$, $\cdots$

Momentum transfer Δ^2 between initial and final hadron provides information on the transverse location of quarks (and gluons) in the fast moving hadron

At zero momentum transfer $(\Delta = 0)$ one recovers the ordinary unpolarized, polarized and transversity parton distributions

For example:

$$H_q\,(x, 0, Q^2) = q(x, Q^2)$$

$$\tilde{H}_q\,(x, 0, Q^2) = \Delta q(x, Q^2)$$

$$H_q^T(x, 0, Q^2) = \delta q(x, Q^2)$$

Müller et al., Ji, Radyushkin, Diehl et al., Burkardt

Theoretical basis: OPE

Twist-2 quark operators

$$\mathcal{O}^{q}_{\mu_1\cdots\mu_n} = \left(\frac{\mathrm{i}}{2}\right)^{n-1} \bar{q}\,\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n}\,q \;-\; \text{traces}$$

$$\mathcal{O}^{5q}_{\mu\mu_1\cdots\mu_n} = \left(\frac{\mathrm{i}}{2}\right)^{n} \bar{q}\,\gamma_{\mu}\gamma_5\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n}\,q \;-\; \text{traces}$$

$$\mathcal{O}^{Tq}_{\mu\nu\mu_1\cdots\mu_n} = \left(\frac{\mathrm{i}}{2}\right)^{n} \bar{q}\sigma_{\mu\nu}\,\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n}\,q \;-\; \text{traces}$$

We shall consider the leading twist contribution only

Classification according to representations of the Hypercubic Group $H(4)$

Off forward matrix elements

$$\langle p_1, s|\, \mathcal{O}^q_{\{\mu_1\cdots\mu_n\}}\, |p_2, s\rangle = \bar{u}(p_1, s)\big[A^q_n(\Delta^2)\gamma_{\{\mu_1} + \frac{\mathrm{i}\Delta^\alpha\sigma_{\alpha\{\mu_1}}{2m_N} B^q_n(\Delta^2)\big]$$

$$\times\, p_{\mu_2}\cdots p_{\mu_n\}}\, u(p_2, s) + \cdots$$

$$\langle p_1, s|\mathcal{O}^{5q}_{\{\mu\mu_1\cdots\mu_n\}}|p_2, s\rangle = \bar{u}(p_1, s)\big[\tilde{A}^q_{n+1}(\Delta^2)\gamma_{\{\mu}\gamma_5 + \frac{\mathrm{i}\Delta_{\{\mu}}{2m_N}\gamma_5\tilde{B}^q_{n+1}(\Delta^2)\big]$$

$$\times\, p_{\mu_1}\cdots p_{\mu_n\}}\, u(p_2, s) + \cdots$$

$$\langle p_1, s|\mathcal{O}^{Tq}_{\{\mu\nu\mu_1\cdots\mu_n\}}|p_2, s\rangle = \bar{u}(p_1, s)\big[A^{Tq}_{n+1}(\Delta^2)\ \sigma_{\mu\nu} + \frac{\mathrm{i}\gamma_{[\mu}\Delta_{\nu]}}{2m_N} B^{Tq}_{n+1}(\Delta^2)\big]$$

$$\times\, p_{\{\mu_1}\cdots p_{\mu_n\}}\big]\, u(p_2, s) + \cdots$$

Moments

$$\int_0^1 dx\, x^n\, H_q(x, \Delta^2, Q^2) = A^q_{n+1}(\Delta^2) \qquad\qquad A^q_{n+1}(0) = \langle x^n \rangle_q$$

$$\int_0^1 dx\, x^n\, E_q(x, \Delta^2, Q^2) = B^q_{n+1}(\Delta^2)$$

$$\int_0^1 dx\, x^n\, \tilde{H}_q(x, \Delta^2, Q^2) = \tilde{A}^q_{n+1}(\Delta^2) \qquad\qquad \tilde{A}^q_{n+1}(0) = \langle x^n \rangle_{\Delta q}$$

$$\int_0^1 dx\, x^n\, \tilde{E}_q(x, \Delta^2, Q^2) = \tilde{B}^q_{n+1}(\Delta^2)$$

$$\int_0^1 dx\, x^n\, H^T_q(x, \Delta^2, Q^2) = A^{Tq}_{n+1}(\Delta^2) \qquad\qquad A^{Tq}_{n+1}(0) = \langle x^n \rangle_{\delta q}$$

$$\int_0^1 dx\, x^n\, E^T_q(x, \Delta^2, Q^2) = B^{Tq}_{n+1}(\Delta^2)$$

$\uparrow$

Generalized form factors

In particular

Sachs form factors

$$A_1^q\ (\Delta^2) = F_1^q(\Delta^2)$$

$$B_1^q\ (\Delta^2) = F_2^q(\Delta^2)$$

$$G_e(\Delta^2) = F_1(\Delta^2) + \frac{\Delta^2}{4m_N^2} F_2(\Delta^2)$$

$$G_m(\Delta^2) = F_1(\Delta^2) + F_2(\Delta^2)$$

$$\tilde{A}_1^q\ (\Delta^2) = G_A^q(\Delta^2)$$

$$\tilde{B}_1^q\ (\Delta^2) = G_P^q(\Delta^2)$$

$$A_1^{Tq}(\Delta^2) = G_T^q(\Delta^2)$$

$$\frac{1}{2}\left(A_2^q(0) + B_2^q(0)\right) = J^q$$

$\uparrow$

Quark angular momentum

Ji

In impact parameter space:

$$H_q\,(x, \mathbf{b}_\perp^2, Q^2) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{2\pi)^2}\, \mathsf{e}^{\,\mathsf{i}\,\mathbf{b}_\perp \boldsymbol{\Delta}_\perp} H_q\,(x, \boldsymbol{\Delta}_\perp^2, Q^2)$$

$$\tilde{H}_q\,(x, \mathbf{b}_\perp^2, Q^2) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\, \mathsf{e}^{\,\mathsf{i}\,\mathbf{b}_\perp \boldsymbol{\Delta}_\perp} \tilde{H}_q\,(x, \boldsymbol{\Delta}_\perp^2, Q^2)$$

$$H_q^T(x, \mathbf{b}_\perp^2, Q^2) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\, \mathsf{e}^{\,\mathsf{i}\,\mathbf{b}_\perp \boldsymbol{\Delta}_\perp} H_q^T(x, \boldsymbol{\Delta}_\perp^2, Q^2)$$

Probabilistic interpretation

with

Burkardt

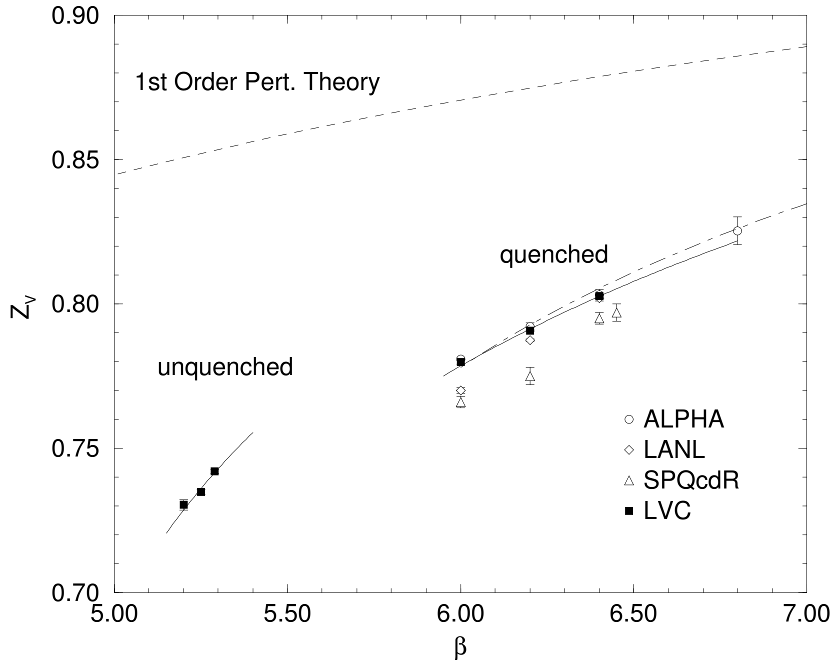
$$\int dx\, x^n d^2\mathbf{b}_\perp\, H_q\,(x, \mathbf{b}_\perp^2, Q^2) = \int dx\, x^n q(x, Q^2) = \langle x^n \rangle_q$$

$$\int dx\, x^n d^2\mathbf{b}_\perp\, \tilde{H}_q\,(x, \mathbf{b}_\perp^2, Q^2) = \int dx\, x^n \Delta q(x, Q^2) = \langle x^n \rangle_{\Delta q}$$

$$\int dx\, x^n d^2\mathbf{b}_\perp\, H_q^T(x, \mathbf{b}_\perp^2, Q^2) = \int dx\, x^n \delta q(x, Q^2) = \langle x^n \rangle_{\delta q}$$

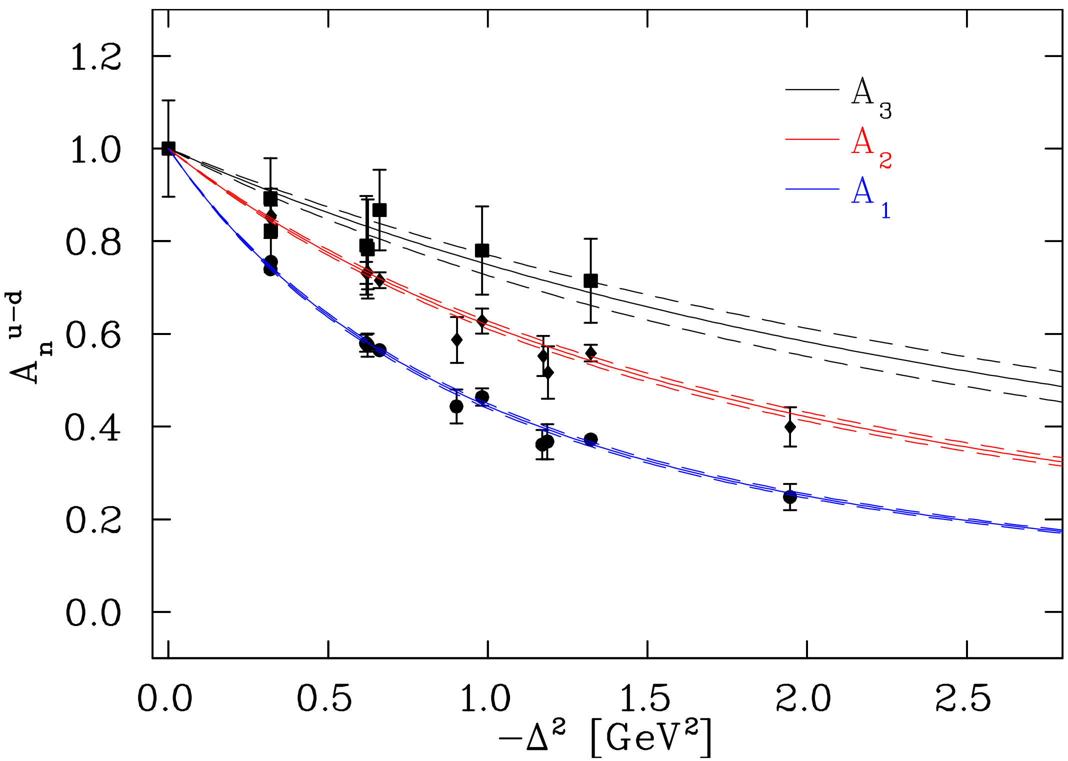
Results

Renormalization: $\mathcal{O}_{\mu_1\cdots\mu_n}(\mu) = Z_{\mathcal{O}}(a\mu)\,\mathcal{O}_{\mu_1\cdots\mu_n}(a)$



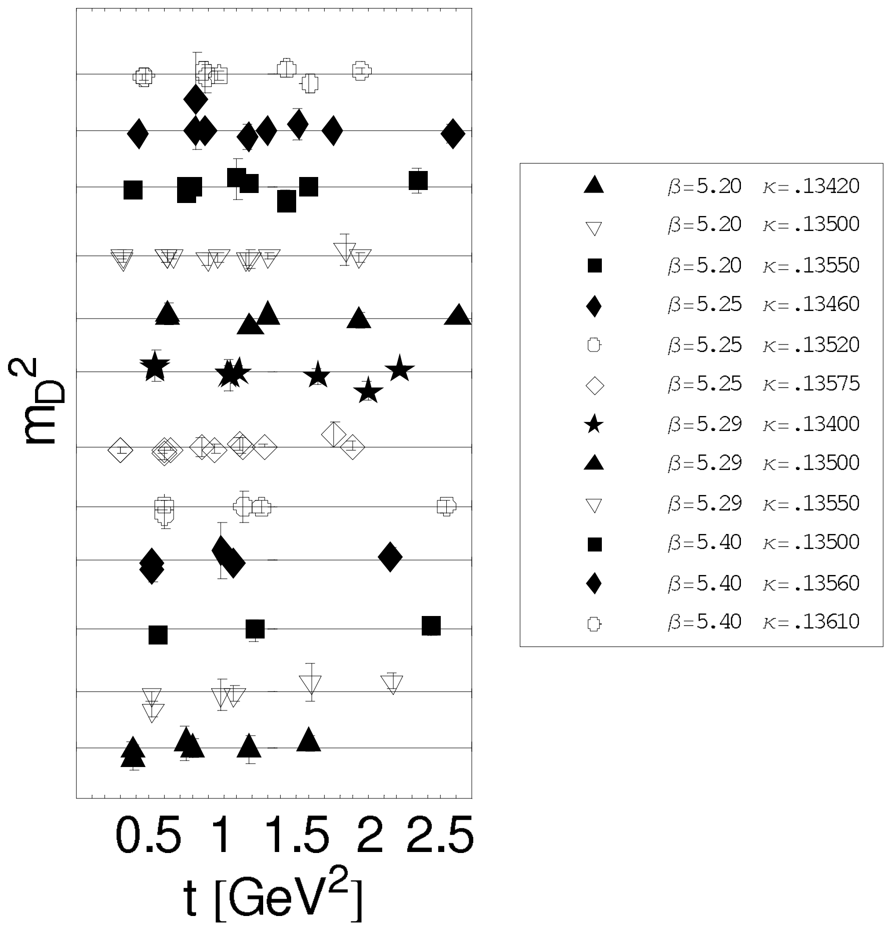
This is done nonperturbatively in the *RI-MOM* scheme

Unpolarized GFFs

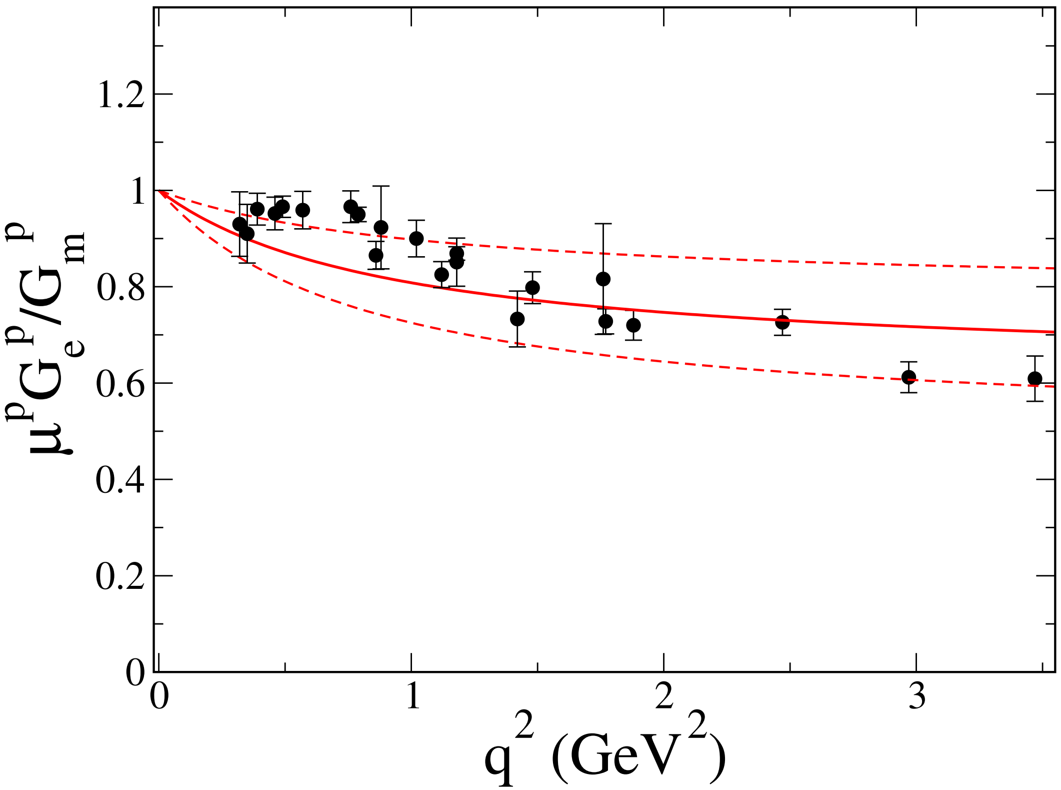


Dipole fit: $A_n(\Delta^2) = \dfrac{A_n(0)}{(1-\Delta^2/M_n^2)^2}$

Stability of the fit

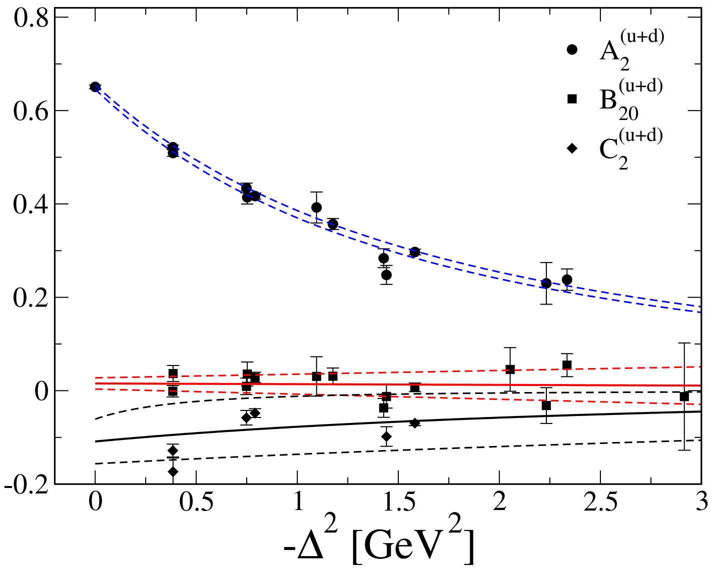


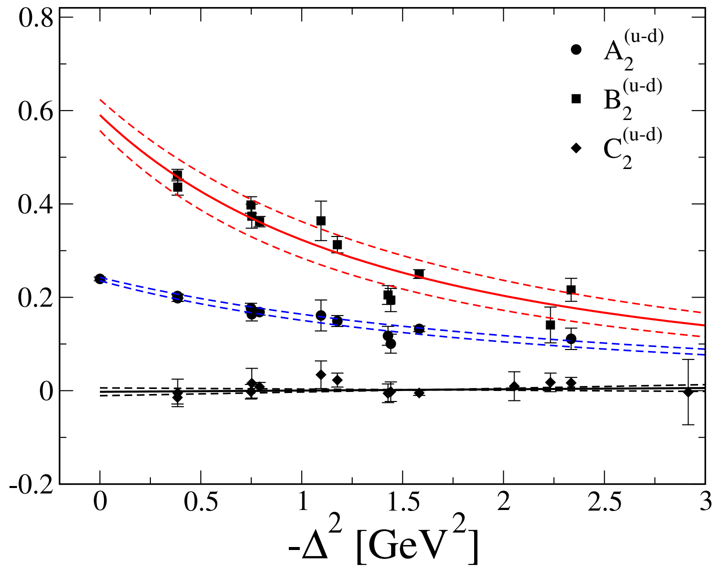
$$M^2_{1\,\mathrm{eff}} = \frac{t}{1 - \sqrt{A_1(0)/A_1(t)}} \qquad t = -\Delta^2$$



$\mu^p G_e^p / G_m^p$
q^2 (GeV^2)
0
0.2
0.4
0.6
0.8
1
1.2
0
1
2
3

(Orbital) angular momentum

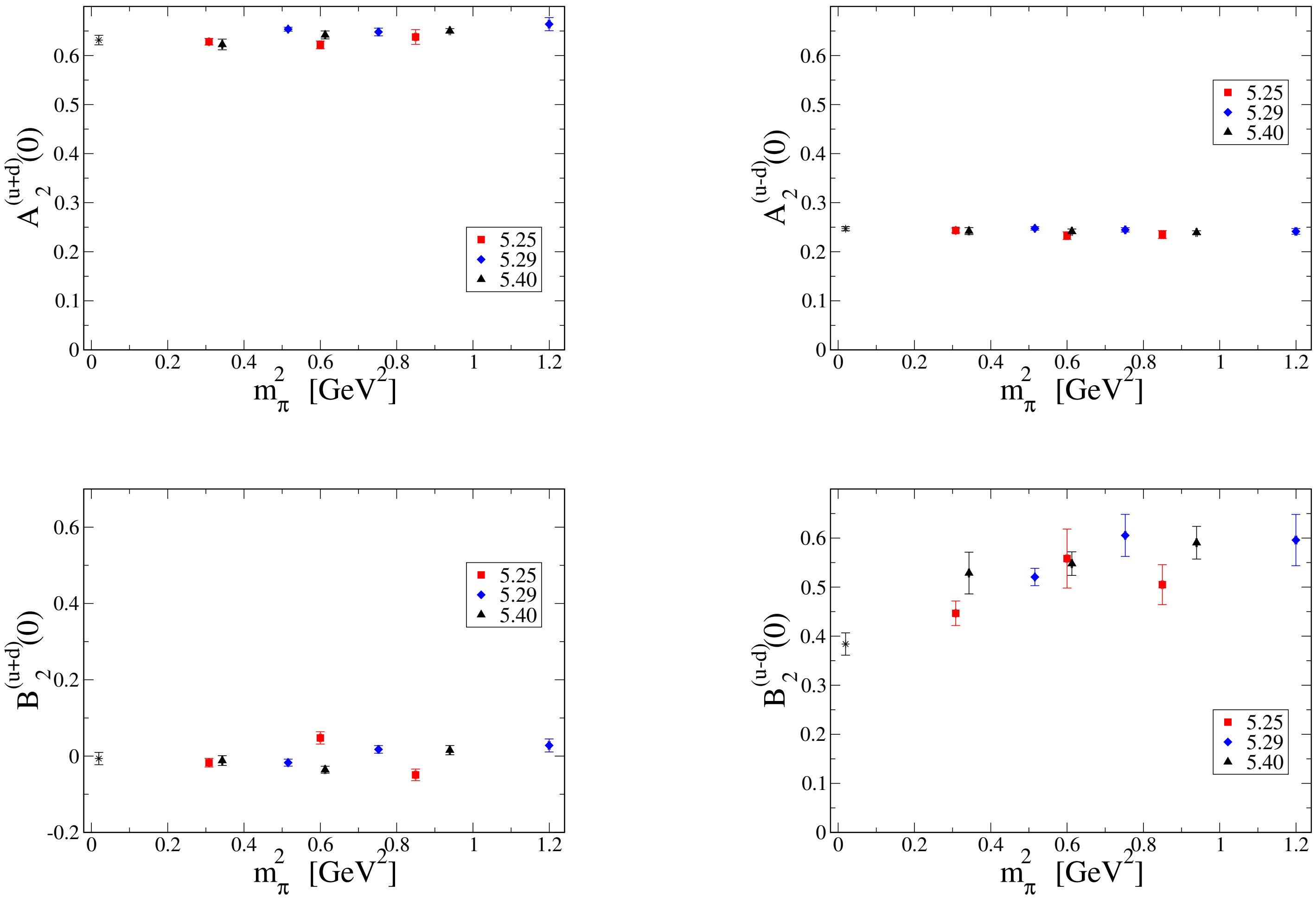




$$A_2(\Delta^2) = \frac{A_2(0)}{(1 - \Delta^2/M_2^2)^2}$$

$$B_2(\Delta^2) = \frac{B_2(0)}{(1 - \Delta^2/\hat{M}_2^2)^2}$$

Chiral extrapolation



$$J^{u+d} \approx \frac{1}{2} \langle x \rangle^{u+d} \qquad\qquad J^{u-d} \approx \frac{5}{4} \langle x \rangle^{u-d}$$

$$J^q = L^q + S^q, \quad S^q = \frac{1}{2}\Delta q$$

$$\boxed{L^{u+d} = 0.03(7)}$$

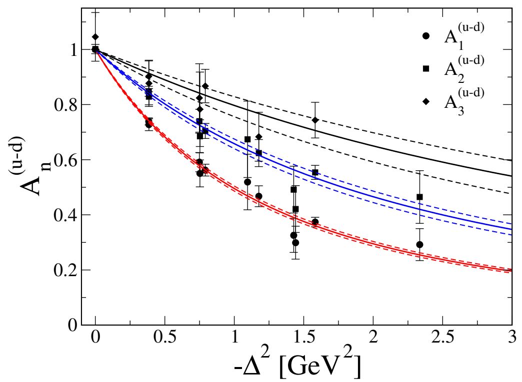
$\uparrow$

Valence quarks only

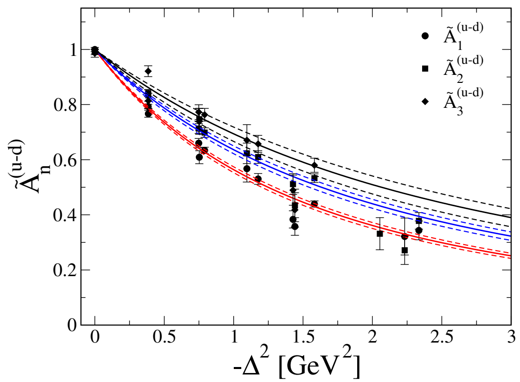
$$\boxed{L^{u-d} = -0.45(6)}$$

$\cdots$ but strong cancellations

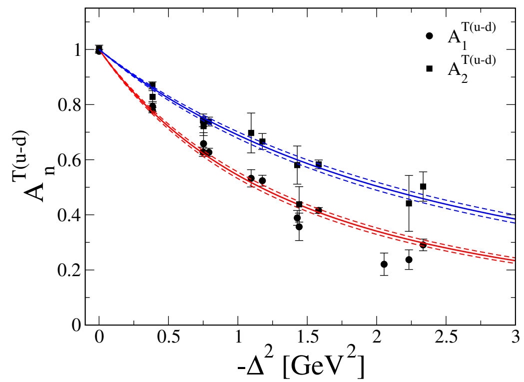
$\overline{MS}$, $Q^2 = 4$ GeV^2



Axial

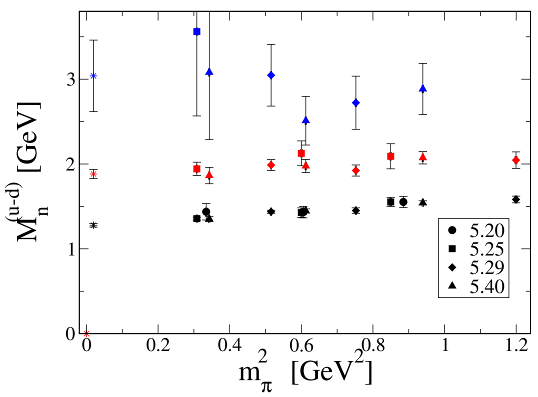


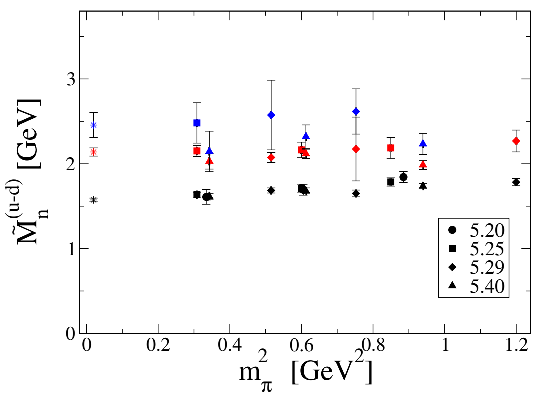
Tensor

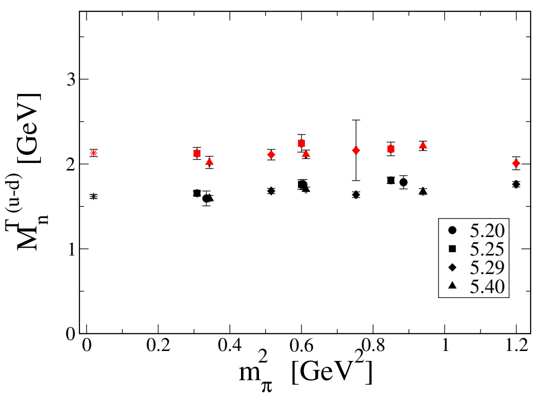


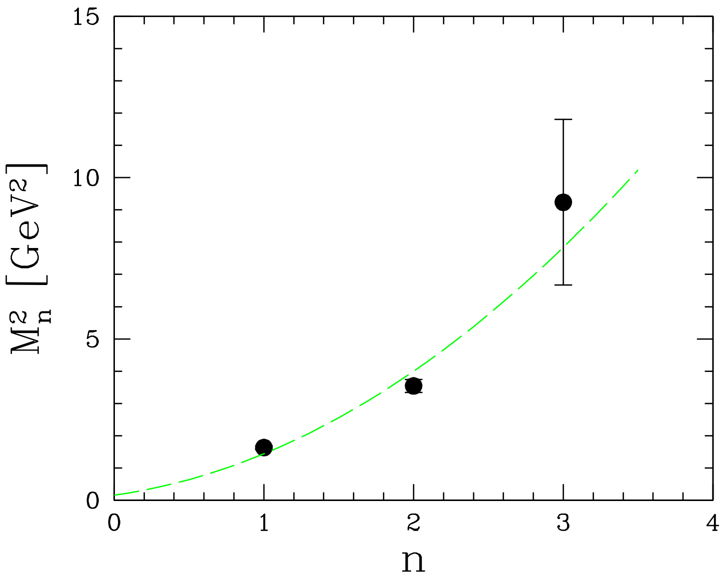
$$A_n(\Delta^2) = \frac{A_n(0)}{(1 - \Delta^2/M_n^2)^2}$$

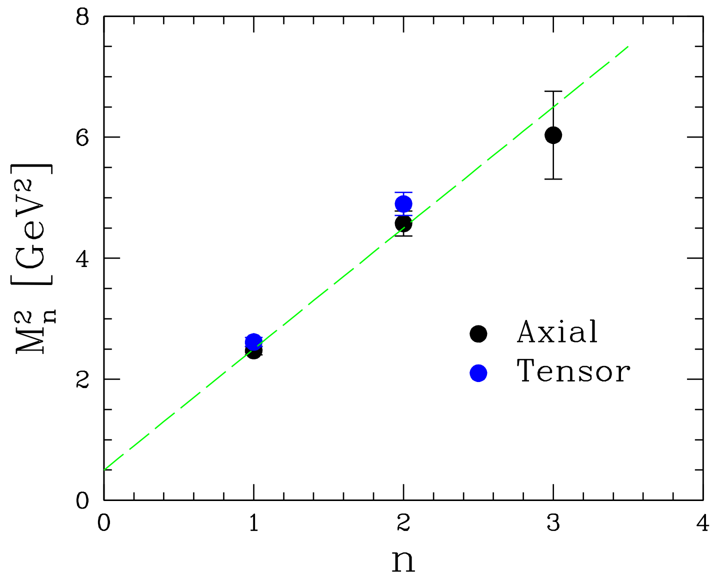
Chiral extrapolation









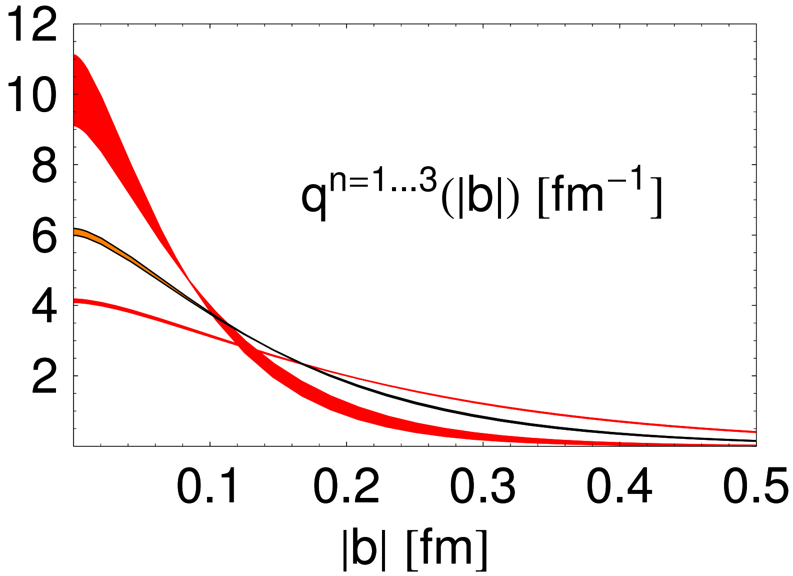


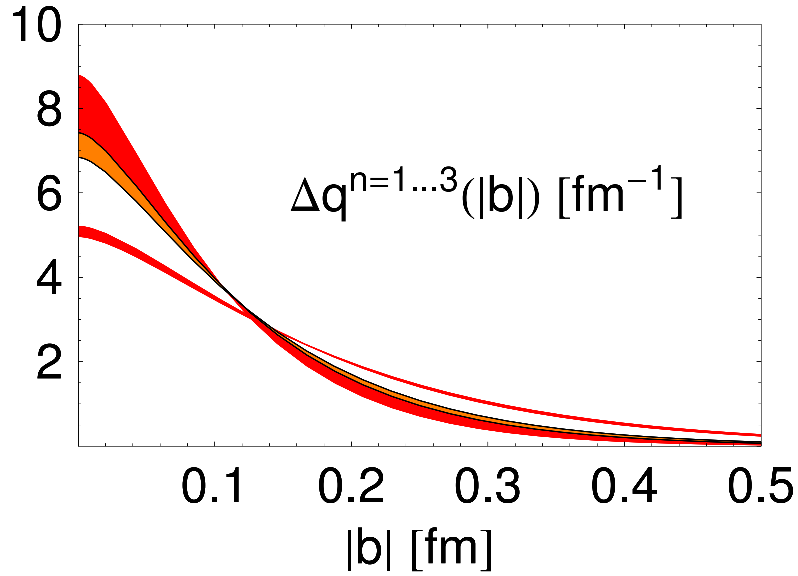
$$n = \alpha_0 + \alpha^\vee \sqrt{M_n^2}$$

square root

$$n = \alpha_0 + \alpha' \, M_n^2$$

linear trajectory





$$\int_0^1 dx\, x^n\, H_q(x, b^2, Q^2)$$

$$\int_0^1 dx\, x^n\, \tilde{H}_q(x, b^2, Q^2)$$

$$H_q(x, \Delta^2, Q^2) = \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}, \Delta^2\right) q(x, Q^2)$$

Similarly for $\tilde{H}_q$ and H_q^T

$$\int_0^1 dx\, x^n \underbrace{C(x, \Delta^2)}_{\text{By inverse Mellin transform}} = \frac{A_{n+1}(\Delta^2)}{A_{n+1}(0)} = \frac{1}{(1 - \Delta^2/M_n^2)^2}$$

$$H_q(x, b^2, Q^2) = \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}, b^2\right) q(x, Q^2)$$

$$C(x, b^2) = \int \frac{d^2\Delta}{(2\pi)^2} e^{ib\Delta} C(x, \Delta^2)$$

$b = \mathbf{b}_\perp$, $\Delta = \mathbf{\Delta}_\perp$

Modelling

$$\alpha(t) = \alpha_0 + \alpha^\vee \sqrt{t}$$

Unpolarized

$$\int_0^1 dx\, x^n\, C(x, \Delta^2) = \left[\frac{(n+1-\alpha_0)^2}{(n+1-\alpha_0)^2 - \alpha^{\vee\,2}\Delta^2} \right]^2$$

Solution:

$$\begin{aligned} C(x, \Delta^2) = \delta(x-1) &- \frac{3}{2}\alpha^\vee \sqrt{-\Delta^2}\, x^{-\alpha_0} \sin\left[\alpha^\vee \sqrt{-\Delta^2}\ln(1/x)\right] \\ &+ \frac{1}{2}\alpha^{\vee\,2}\Delta^2\, x^{-\alpha_0} \cos\left[\alpha^\vee \sqrt{-\Delta^2}\ln(1/x)\right] \ln(1/x) \end{aligned}$$

$$\langle b^2 \rangle = 7\,\alpha^{\vee\,2} x^{-\alpha_0} \left[\frac{1}{\alpha_0}\ln(1/x) - \frac{1}{\alpha_0^2}\left(1 - x^{\alpha_0}\right) \right] = \frac{7}{2}\alpha^{\vee\,2}\,(1-x)^2 + \mathcal{O}\left((1-x)^3\right)$$

Independent of parton distribution

The impact parameter b measures the distance of the struck quark from the center of momentum. A better measure of the transverse size of the hadron is the distance r between the struck quark and the spectator system:

$$\langle r^2 \rangle = \frac{\langle b^2 \rangle}{(1-x)^2}$$

Here

$$\langle r^2 \rangle = \frac{7}{2} \alpha^{\vee\,2} + \mathcal{O}\,(1-x)$$

$\Leftarrow$ Consistent with confinement

$\alpha(t) = \alpha_0 + \alpha' t$

Axial & Tensor

$$\int_0^1 dx\, x^n\, C(x, \Delta^2) = \left[\frac{n + 1 - \alpha_0\alpha'}{n + 1 - \alpha_0\alpha' - \alpha'\Delta^2}\right]^2$$

Solution:

$$C(x, \Delta^2) = \delta(x - 1) + \alpha'\Delta^2\,(2 - \alpha'\Delta^2 \ln x)\, x^{-\alpha'(\alpha_0 + \Delta^2)}$$

$$\langle b^2 \rangle = -\frac{4}{\alpha_0}\left(1 - x^{-\alpha_0\alpha'}\right) = 4\alpha'(1 - x) + \mathcal{O}\left((1 - x)^2\right)$$

Here

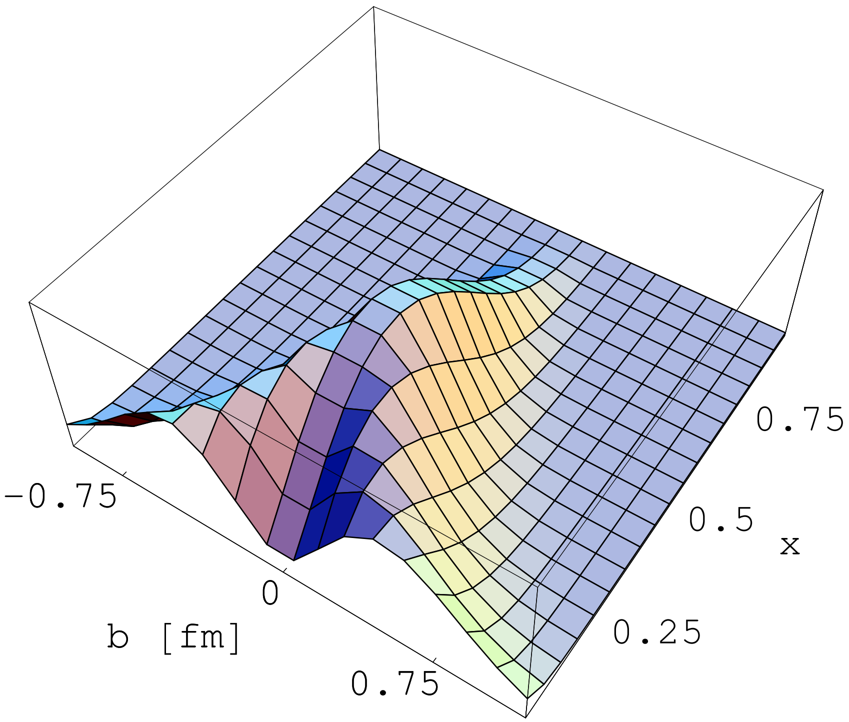
$$\langle r^2 \rangle \simeq \frac{4\alpha'}{1 - x}$$

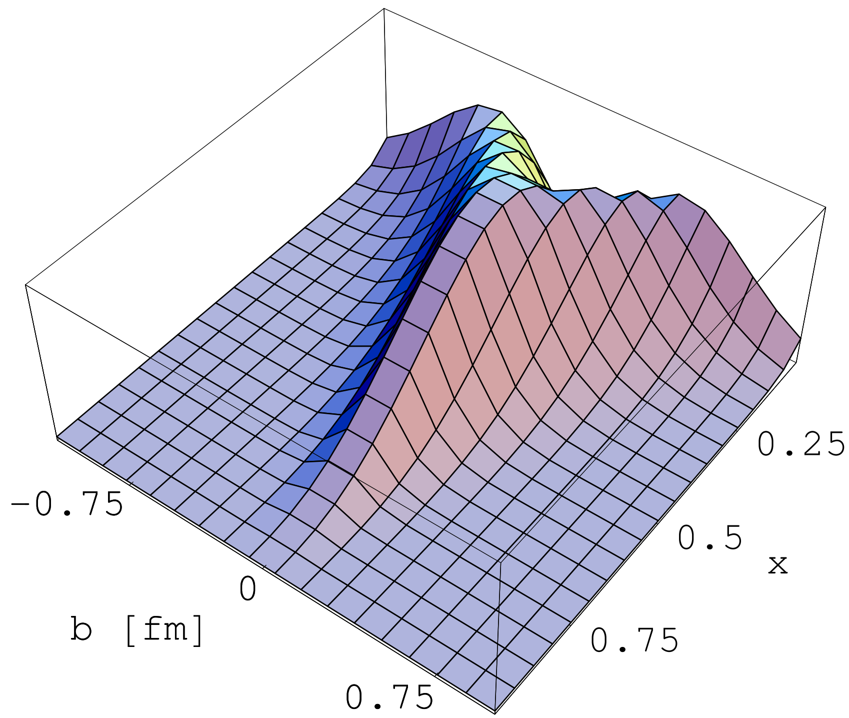
$\Leftarrow$ Spin radius divergent ?

$$H_q(x, b^2, Q^2) = \int_x^y \frac{dy}{y} C\left(\frac{x}{y}, b^2\right) q(x, Q^2)$$

$$\int_0^1 dx\, x^n C(x, \Delta^2) = \frac{A^q_{n+1}(\Delta^2)}{A^q_{n+1}(0)}$$

Valence





$Q^2 = 4$ GeV^2

Summary and Outlook

- Precision of numerical results is steadily improving
  - Computer power
  - Improved action
  - Improved operators
  - Renormalization
  - Chiral perturbation theory
- Modelling of the nucleon in terms of quarks (and gluons?) can be done on the lattice. First results on the charge and spin distribution look promising
- Ready to challenge Light-Cone QCD

  See also talk by James Zanotti
- To better constrain the (generalized) form factors need larger lattices
- To safely extrapolate to the chiral limit need to do simulations at $m_\pi \lesssim 300$ MeV

  Probably this can only be achieved with overlap fermions