Phase Transitions in High Density QCD

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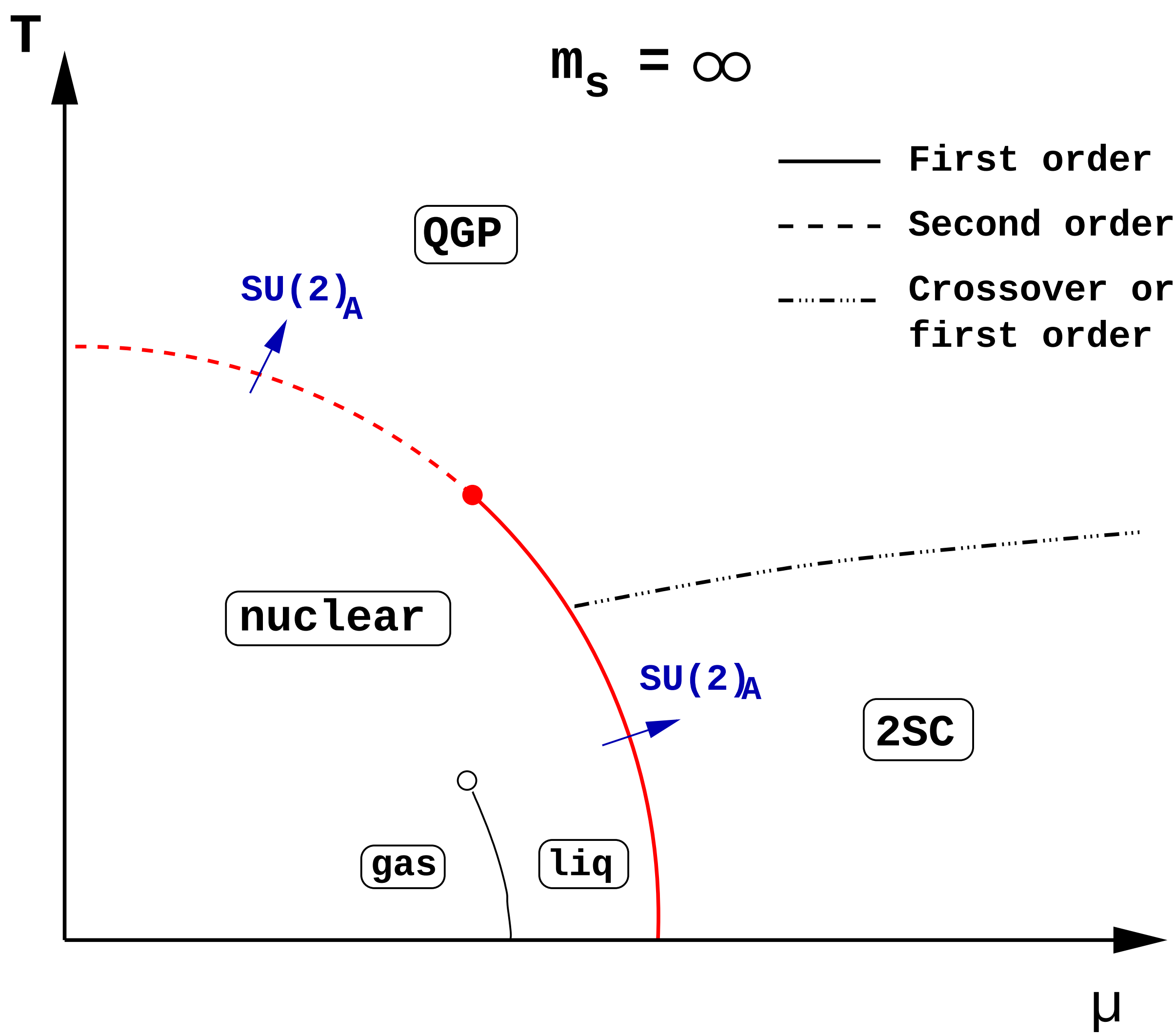
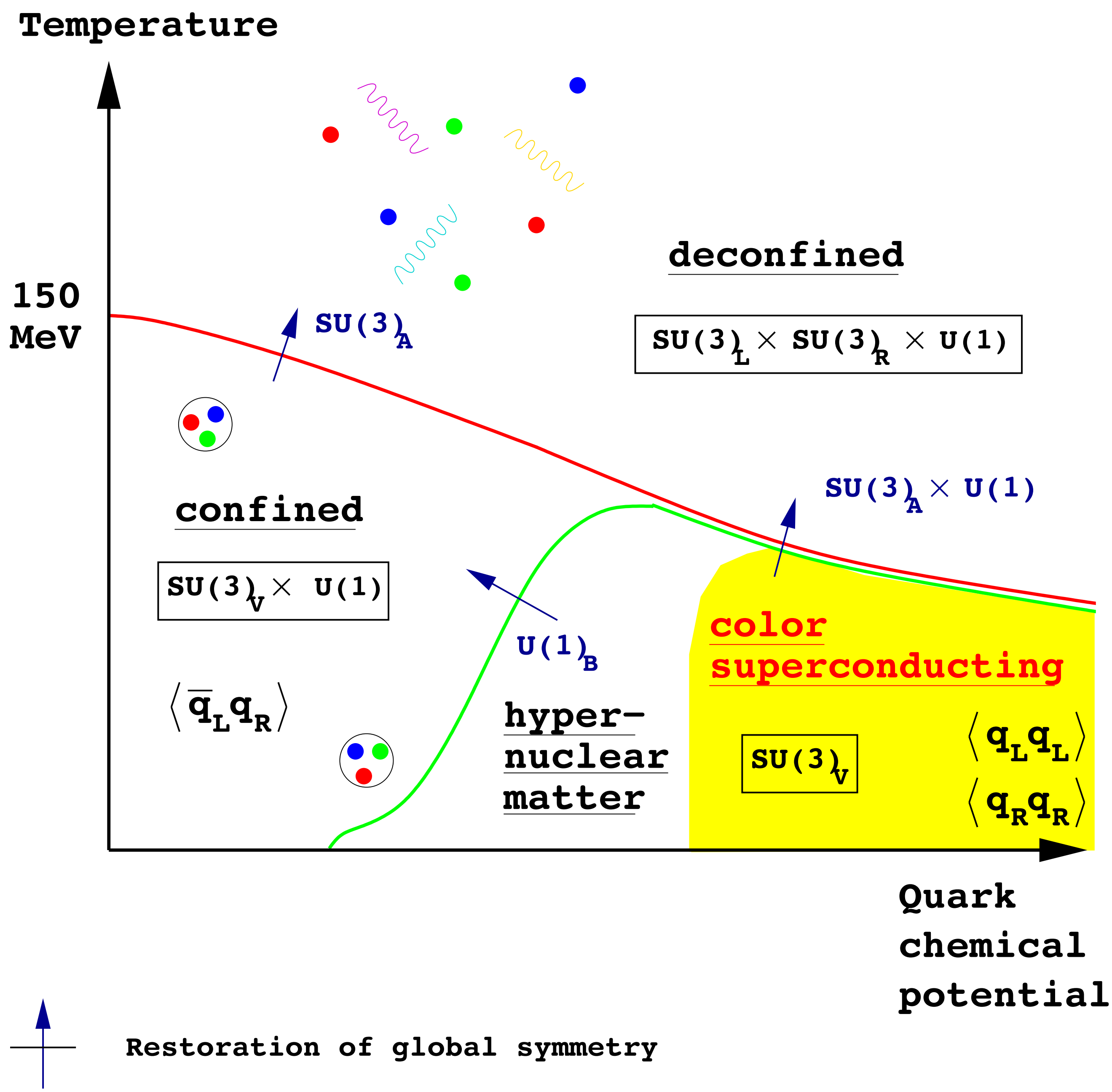
(Based on works with D. Toublan)

I. Introduction

1. The QCD phase diagram at nonzero T, μ is quite complicated

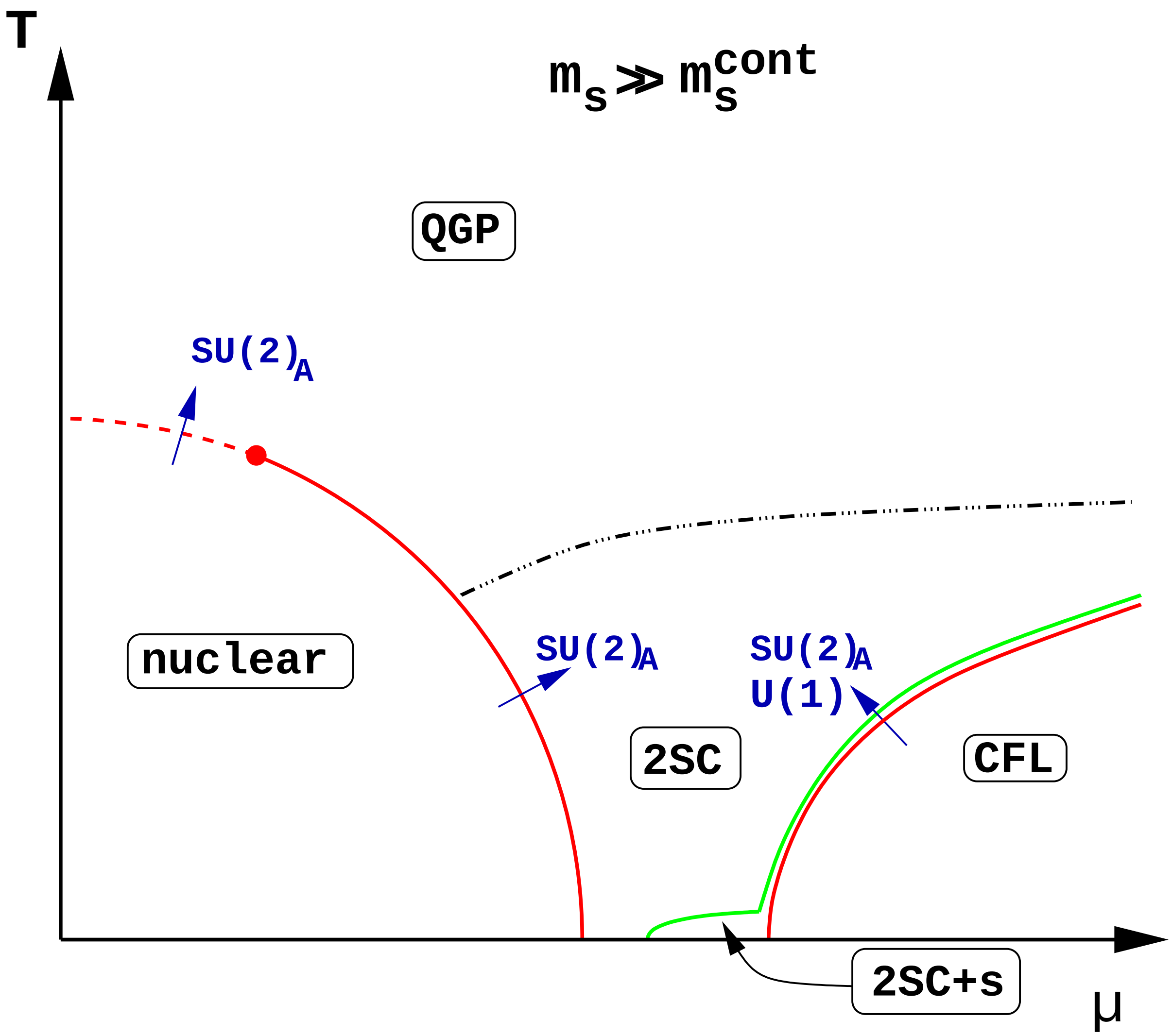


Conjectured phase diagram for QCD, $N_f = 2, m_q = 0$. (From Mark Alford's review paper).



Conjectured phase diagram for QCD, $N_f = 3, m_q = 0$.
(From Mark Alford's review paper).

When $m_s \neq 0$ phase diagram becomes really very complicated at intermediate μ... with possibilities of K condensation, η condensation, LOFF phase, crystalline phase, to name just a few...



Conjectured phase diagram for QCD, $N_f = 3, m_s \neq 0$. (From Mark Alford's review paper).

2. Few Remarks:

a) On the lattice one can study the phase diagram at $T \neq 0, \mu \simeq 0$. One can not analyze the large μ behavior due to the sign problem;

b) We are interested in behavior when μ varies at small $T \simeq 0$. It might be relevant for the physics of neutron stars. At large $\mu \gg \Lambda_{QCD}$ the system in the deconfinement phase; at small $\mu \simeq 0$ the system in the hadronic phase. Something should occur on the way from $(\mu \simeq 0) \Rightarrow (\mu \gg \Lambda_{QCD})$;

c) If one knows the most important vacuum configurations at $\mu = 0$ (hint: **instantons?**) one can answer many questions about the phase transitions.

3. On the phenomenological side: The development of the **instanton liquid model** (Shuryak and Co.) has encountered successes: chiral symmetry breaking, resolution of the $U(1)$ problem, spectrum etc and failures: a) confinement can not be described by well separated lumps with integer topological charges; b) lattice calculations suggest that T_c for confinement and chiral phase transitions are very close to each other (hint: both phenomena originated from the same vacuum configurations?); c)....

II. Main Goals and Results

1. We argue that the instantons is the driving force for confinement-deconfinement phase transition at nonzero μ (they are not necessary small well-localized lumps, see below).

2. We argue that the low-energy effective chiral Lagrangian corresponds to a statistical system of interacting pseudo-particles with fractional $1/N_c$ charges. (dual representation)

3. We shall identify these objects with **instanton quarks** suspected long ago (demonstration of a link between confinement and instantons in 2d): V.Fateev et al, B.Berg and M.Luscher, A. Belavin et al, (1979).

4. We make some very specific predictions which can be tested with traditional Monte Carlo techniques, by studying QCD at nonzero isospin chemical potential μ_I where there is no sign problem. In particular we predict that the confinement-deconfinement transition and the topological charge density distribution must experience sharp changes exactly at the same critical value $\mu_c(T)$. We estimate $\mu_c(T)$ for different N_c, N_f.

III. Main Logic of the Presentation

1. I use a TRICK which allows me to represent the low energy effective lagrangian in terms of dual variables (a statistical system of some interacting pseudo-particles).

2. I test this trick in the weak coupling regime in QCD (large chemical potential, Color Superconductor) where all calculations are under complete theoretical control.

3. I observe that the instanton-instanton(II) and instanton -anti-instanton ($\bar{I}I$) interactions at large distances are very different from the naive semiclassical calculations.

4. I apply the same TRICK to QCD at zero chemical potential and $T = 0$. I advocate the picture that in the strongly coupled theories the instantons and anti-instantons lose their individual properties (instantons will "melt") their sizes become very large and they overlap.

5. The description in terms of the instantons and anti-instantons is not appropriate any more, and alternative degrees of freedom should be used to describe the physics. The relevant description is that of instanton-quarks with fractional topological charges $1/N_c$.

6. I approach the phase transition region from the high density side where instanton calculations under complete theoretical control.

7. I argue that the phase transition has an universal nature for μ and isotopical chemical potential μ_I for different N_c and N_f. For $\mu_I \neq 0$ our predictions can be tested with traditional Monte Carlo techniques, by studying QCD at nonzero isospin chemical potential.

• The key observation here is there existence of the free parameter θ (it plays the crucial role in mapping of one problem to another). The θ plays the role of the messenger between colorless (chiral effective Lagrangian fields) and colorful (instanton quarks) objects.

IV. Color Superconductivity for Pedestrians

($\mu\bar{\Psi}\gamma_0\Psi$ - term, $\mu \gg \Lambda_{QCD}$)

1. If there is a channel in which the quark-quark interaction is attractive, than the true ground state of the system will be a complicated coherent state of Cooper pairs like in **BCS** theory (ordinary superconductor).

2. Diquark condensates break color symmetry (CFL phase, $N_c = N_f = 3$):

$$\langle\psi^{ia}_{L\alpha}\psi^{jb}_{L\beta}\rangle^* \sim \epsilon_{\alpha\beta\gamma}\epsilon^{ij}\epsilon^{abc}X^{\gamma}_{c},$$
$$\langle\psi^{ia}_{R\alpha}\psi^{jb}_{R\beta}\rangle^* \sim \epsilon_{\alpha\beta\gamma}\epsilon^{ij}\epsilon^{abc}Y^{\gamma}_{c}$$

3. $SU(3)_c \times U(1)_{EM} \times SU(3)_L \times SU(3)_R \times U(1)_B$

$$\Downarrow$$

$$SU(3)_{c+L+R} \times U(1)^*_{EM}$$

a) Color gauge group is completely broken;
b) $U(1)_B$ is spontaneously broken;
c) $U(1)_{EM}$ is not broken;
d) $U(1)_A$ is broken spontaneously and explicitly (by instantons)

4. Goldstone fields are the phases of the condensate

$$\Sigma_\gamma^\beta = \sum_c X_c^\beta Y_\gamma^{c*} \sim e^{i\lambda^a \pi^a} e^{i\varphi_A}. \qquad (1)$$

5. $U(1)_A$ is spontaneously broken. The symmetry is broken also explicitly by the instantons. Effective lagrangian is

$$L_A \sim f_A^2[(\partial_0 \varphi_A)^2 - u^2(\partial_i \varphi_A)^2] + a\mu^2\Delta^2 \cos(\varphi_A - \theta)$$

Coefficient a can be explicitly calculated from the t'Hooft formula *(Son, Stephanov, AZ, 2001)*.

6. To compute a we start from the instanton induced effective four-fermion interaction,

$$\begin{aligned} L_{\text{inst}} &= e^{-i\theta}\int d\rho\, n_0(\rho)\left(\frac{4}{3}\pi^2\rho^3\right)^2\Big\{(\bar{u}_R u_L)(\bar{d}_R d_L) + \\ &+ \frac{3}{32}\Big[(\bar{u}_R\lambda^a u_L)(\bar{d}_R\lambda^a d_L) \\ &- \frac{3}{4}(\bar{u}_R\sigma_{\mu\nu}\lambda^a u_L)(\bar{d}_R\sigma_{\mu\nu}\lambda^a d_L)\Big]\Big\} + \text{H.c.} \end{aligned} \quad (2)$$

where $n_0(\rho)$ in the presence of $\mu \neq 0$ is given by

$$n_0(\rho) = C_N\left(\frac{8\pi^2}{g^2}\right)^{2N_c}\rho^{-5}\exp\left(-\frac{8\pi^2}{g^2(\rho)}\right)e^{-N_f\mu^2\rho^2} \quad (3)$$

with

$$C_N = \frac{0.466e^{-1.679N_c}1.34^{N_f}}{(N_c-1)!(N_c-2)!}\,, \quad (4)$$

7. Averaging Eq. (2) in the superconducting background, we find

$$V_{\text{inst}}(\varphi) = -\int d\rho\, n_0(\rho) \left(\frac{4}{3}\pi^2\rho^3\right)^2 12|X|^2 \cos(\varphi_A - \theta).$$

where

$$|X| = \frac{3}{2\sqrt{2}\pi}\frac{\mu^2\Delta}{g}.$$

Using formula for $n_0(\rho)$ we get the final result

$$a(\mu \gg \Lambda_{QCD}) = 5\times 10^4 \left(\ln\frac{\mu}{\Lambda_{QCD}}\right)^7 \left(\frac{\Lambda_{QCD}}{\mu}\right)^{29/3} \ll 1$$

8. The η' is light: $m^2_{\eta'} \sim \frac{\mu^2\Delta^2}{f^2} \cdot a \sim (\frac{\Lambda_{QCD}}{\mu})^b \to 0.$

9. Weak coupling regime: dilute gas approximation leads exactly to the combination $\left(e^{i(\varphi_A-\theta)} + e^{-i(\varphi_A-\theta)}\right)$ which is expected from the very beginning.

V. Instanton interactions in dense QCD

1. Partition function for η'

$$Z = \int \mathcal{D}\varphi_A \; e^{-f^2 u \int d^4x (\partial\varphi_A)^2} \, e^{a' \int d^4x \cos(\varphi_A(x)-\theta)} \, ,$$

2. Different representation for η'

$$\mathrm{e}^{a' \int d^4x \cos(\varphi_A(x)-\theta)} \equiv$$

$$\sum_{M=0}^{\infty} \frac{(a'/2)^M}{M!} \left(\int d^4x \sum_{Q=\pm 1} e^{iQ(\varphi(x)-\theta)} \right)^M$$

$$= \sum_{M_\pm=0}^{\infty} \frac{(a'/2)^M}{M_+! M_-!} \int d^4x_1 \ldots \int d^4x_M \; e^{i \sum_{a=0}^{M} Q_a (\varphi_A(x_a)-\theta)} \; .$$

The last line is aclassical partition function of the gas of $M = M_+ + M_-$ identical particles of charges $+1$ or -1 placed in an external potential given by $i(\theta - \varphi(x))$.

3. For each term the path integral is Gaussian and can be easily taken:

$$\int \mathcal{D}\varphi \, e^{-f^2 u \int d^4x (\partial\varphi)^2} \, e^{i\sum_{a=0}^{M} Q_a(\varphi(x_a)-\theta)} =$$

$$e^{-i\theta \sum_{a=0}^{M} Q_a} \, e^{-\frac{1}{2f^2 u}\sum_{a>b=0}^{M} Q_a Q_b G(x_a - x_b)} \, .$$

4. Thus we obtain the dual CG representation for the partition function

$$Z = \sum_{M_\pm=0}^{\infty} \frac{(a/2)^M}{M_+! M_-!} \int d^4x_1 \ldots \int d^4x_M \; e^{-i\theta \sum_{a=0}^{M} Q_a} \; \cdot$$

$$e^{-\frac{1}{2f^2 u}\sum_{a>b=0}^{M} Q_a Q_b G(x_a - x_b)} \, , \quad G(x_a - x_b) = \frac{1}{4\pi^2 (x_a - x_b)^2} \, .$$

The two representations of the partition function are equivalent.

5. Physical Interpretation.

a) Since $Q_{\rm net} \equiv \sum_a Q_a$ is the total charge and it appears in the action multiplied be the parameter θ, one concludes that $Q_{\rm net}$ *is* the total topological charge of a given configuration.

b) Each charge Q_a in a given configuration should be identified with an integer topological charge well localized at the point x_a. This, by definition, corresponds to a small instanton positioned at x_a.

c) Further support for the identification: every particle with charge Q_a brings along a factor of fugacity $\sim a'$ which contains the classical one-instanton suppression factor $\exp(-8\pi^2/g^2(\rho))$ in the density of instantons.

6. The following hierarchy of scales exists: The typical size of the instantons $\bar{\rho} \sim \mu^{-1}$ is much smaller than the short-distance cutoff of our effective low-energy theory, Δ^{-1},

$$\begin{array}{ccccccc} \text{(size)} & \ll & \text{(cutoff)} & \ll & (II \text{ distance}) & \ll & \text{(Debye)} \\ \mu^{-1} & \ll & \Delta^{-1} & \ll & (\sqrt{a}\mu\Delta)^{-1/2} & \ll & (\sqrt{a}\Delta)^{-1} \end{array}$$

Due to this hierarchy, ensured by large $\mu/\Lambda_{\rm QCD}$, we acquire analytical control.

7. The starting low-energy effective Lagrangian contains only a colorless field φ_A, we have ended up with a representation of the partition function in which objects carrying color (the instantons) can be studied.

8. In particular, II and $I\bar{I}$ interactions (at very large distances) are exactly the same up to a sign, order g^0, and are Coulomb-like. This is in **contrast with semiclassical expressions** when II interaction is zero and $I\bar{I}$ interaction is order $1/g^2$.

9. Very complicated picture of the **bare** II and $I\bar{I}$ interactions becomes very simple for **dressed** instantons/anti-instantons when all integrations over all possible sizes, color orientations and interactions with background fields are properly accounted for!

10. As expected, the ensemble of small $\rho \sim 1/\mu$ instantons can not produce confinement. This is in accordance with the fact that CS phase is not in confining phase.

VI.Chiral Lagrangian ($\mu = 0 \quad \theta \neq 0$)

1. We keep only the diagonal elements of the chiral matrix $U = \exp\{i \mathrm{diag}(\phi_1, \ldots, \phi_{N_f})\}$ which are relevant in the description of the ground state. Singlet combination is defined as $\phi = \mathrm{Tr}\, U$.

2. Effective lagrangian for the ϕ is

$$L_{\eta'} = f^2(\partial_\mu \phi)^2 + E \cos\left(\frac{\phi - \theta}{N_c}\right) + \sum_{a=1}^{N_f} m_a \cos \phi_a \quad (6)$$

3. A Sine-Gordon structure for the singlet combination corresponds to the following behavior of the $(2k)^{\mathrm{th}}$ derivative of the vacuum energy in pure gluodynamics (Veneziano, 1979)

$$\left.\frac{\partial^{2k} E_{vac}(\theta)}{\partial\, \theta^{2k}}\right|_{\theta=0} \sim \int \prod_{i=1}^{2k} dx_i \langle Q(x_1)...Q(x_{2k})\rangle \sim (\frac{i}{N_c})^{2k},$$

where $Q = \frac{g^2}{32\pi^2} G_{\mu\nu} \widetilde{G}_{\mu\nu}$ is topological density. *Veneziano originally thought that this relation implies the periodicity to be $2\pi N_c$ rather than 2π.*

VII. Dual Representation for the Chiral Lagrangian ($\mu = 0 \quad \theta \neq 0$)

1. One can represent the Sine Gordon effective field theory in terms of the classical statistical model (Coulomb Gas representation)

$$
\begin{aligned}
Z &= \sum_{Q_a^{(0)} = \pm \frac{1}{N_c}} \frac{(\frac{E}{2})^{M_0}}{M_0!} \int (dx_1^{(0)} \dots dx_{M_0}^{(0)})\; e^{-S_{CG}} \\
S_{CG} &= i\theta Q_T^{(0)} + \frac{1}{2f^2} \left\{ \sum_{b,c=1}^{M_0} Q_b^{(0)}\; G(x_b^{(0)} - x_c^{(0)})\; Q_c^{(0)} \right\}. \\
Q_T^{(0)} &= \sum_{b=1}^{M_0} Q_b^{(0)} - \text{total charge for the configuration}
\end{aligned}
$$

2. One can identify $Q_T^{(0)}$ as the total topological charge of the given configuration. Indeed, the θ parameter appears in the original Lagrangian only in the combination $i\theta \frac{G_{\mu\nu}\widetilde{G}_{\mu\nu}}{32\pi^2} d^4x$.

3. The fundamental difference in comparison with the previous case: while the total charge in integer, the individual charges are fractional $\pm 1/N_c$. The fact that species $Q_i^{(0)}$ has charges $\sim 1/N_c$ is a direct consequence of the θ/N_c dependence in the underlying QCD with frozen (non-dynamical) quarks.

4. Due to the 2π periodicity of the theory, only configurations which contain an integer topological number contribute to the partition function. Therefore, the number of particles for each given configuration $Q_i^{(0)}$ with charges $\sim 1/N_c$ must be proportional to N_c.

5. The number of integrations over $d^4 x_i^{(0)}$ exactly equals $4N_c k$, where k is integer. This number, $4N_c k$, exactly corresponds to the number of zero modes in the k-instanton background, and we *conjecture* that at low energies (large distances) the fractionally charged species-$Q_i^{(0)}$ pseudo-particles are the **instanton-quarks** suspected long ago.

VIII. Interpretation. Speculations.

1. There is an interesting connection between the CG statistical ensemble and the $2d\ CP^{N_c}$ models. An exact accounting and resummation of the n-instanton solutions maps the original problem to a $2d$-CG with fractional charges (dubbed in 1979 as the instanton-quarks). These pseudo-particles do not exist separately as individual objects; rather, they appear in the system all together as a set of $\sim N_c$ instanton-quarks so that the total topological charge of each configuration is always integer.

2. One immediate objection: it has long been known that instantons can explain most low energy QCD phenomenology (chiral symmetry breaking, resolution of the $U(1)$ problem, spectrum, etc) with the exception confinement; and we claim that confinement can arise in this picture: how can this be consistent?

3.In dilute gas approximation quark confinement can not be described. However, in strongly coupled theories the instantons and anti-instantons lose their individual properties their sizes become very large, they overlap. Confinement is the possibility.

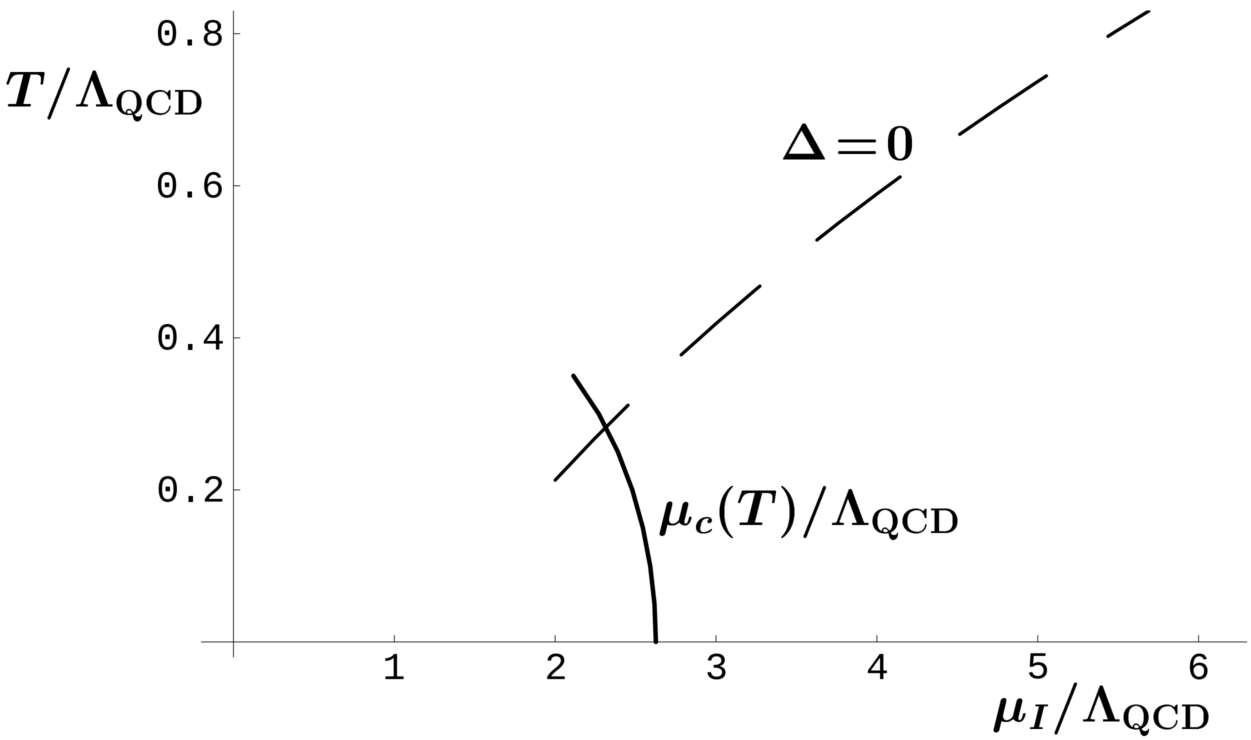
IX. Conjecture and Results

1. We thus **conjecture** that the confinement-deconfinement phase transition takes place at precisely the value where the dilute instanton calculation breaks down: At low μ color is confined (because of the instanton-quarks), whereas at large μ color is not confined (because of dilute instantons).

2. We have determined the critical chemical potential in different cases at nonzero baryon or isospin chemical potential assuming $m_s = 150\ MeV$ for $N_f = 3$ case,

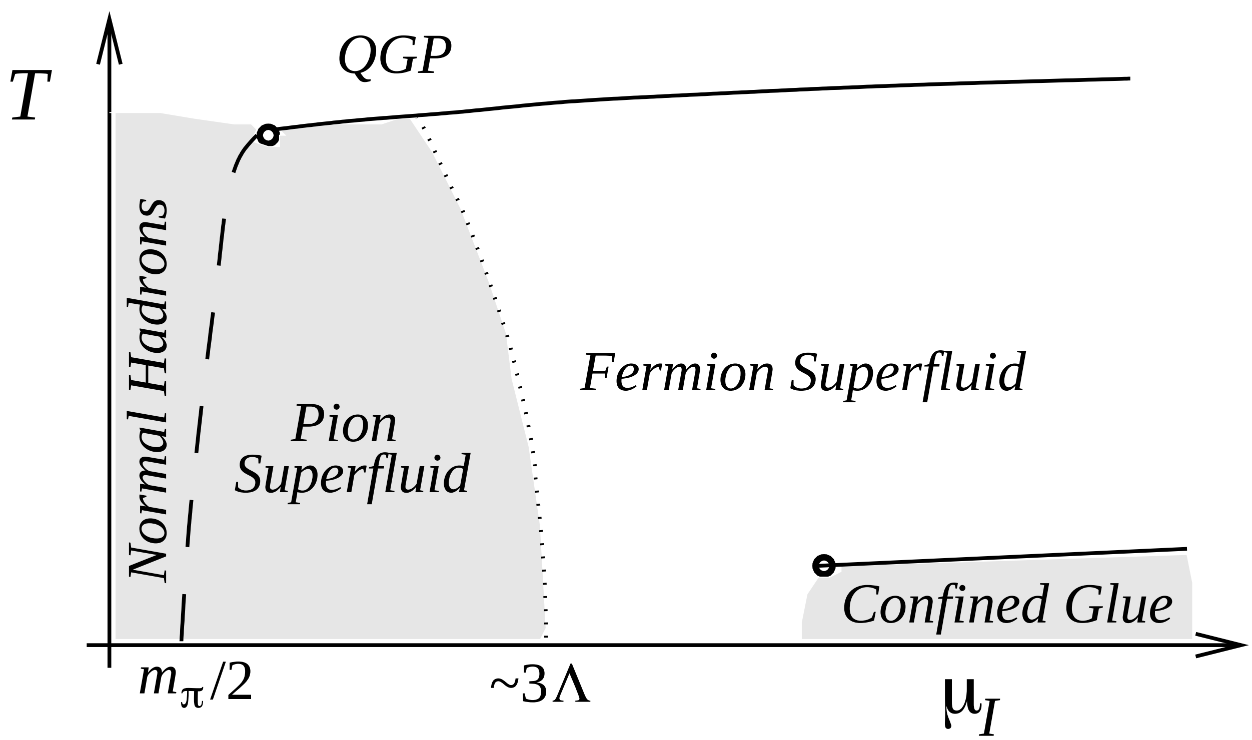
	$N_c\!=\!3, N_f\!=\!2$	$N_c\!=\!N_f\!=\!3$	$N_c\!=\!N_f\!=\!2$
μ_{Bc}/Λ	2.3	1.4	3.5
μ_{Ic}/Λ	2.6	1.5	3.5

3. One can generalize the calculations for $T \neq 0$ when gap is still large and the instanton calculations are still justified.



Critical isospin chemical potential $\mu_I(T)$ for the confinement-deconfinement phase transition as a function of temperature (solid curve) at $N_c = 3, N_f = 2$ where direct lattice calculation are possible.

4. The phase diagram of QCD at nonzero temperature and isospin chemical potential should look like....



Phase diagram of QCD at nonzero temperature and isospin chemical potential. First and second order phase transitions are depicted by solid and dashed curves, respectively. The confined phases are shaded.

X. Future Directions

1. We claim that the topological charge density distribution measured as a function of μ_I will experience sharp changes at the same critical value $\mu_I = \mu_c(T)$ where the phase transition occurs.
a).There are well- established lattice methods which allow to measure the topological density distribution.
b). Independently, there are well- established lattice method which allow to introduce μ_I into the system.
Combine these two measurements!
2. There is a close relation between instanton quarks and the "periodic instantons" which have the internal structure resembling the instanton-quarks (van Baal).
3. One can hope to understand the relation between old picture advocated by 't Hooft and Mandelstam and confinement due to the instanton-quarks. The key point of the 't Hooft - Mandelstam approach is the assumption that dynamical monopoles exist and Bose condense. One can argue (semiclassical analysis) that the instanton-quarks carry the magnetic charges. In this case both pictures could be the two sides of the same coin.