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CERN/EP

Experimental Constraints on the CKM Matrix

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Workshop on CP Violation
Adelaide, Australia July 3-8, 1998.

Introduction

The Cabibbo-Kobayashi-Maskawa matrix describes the mixing of the quark mass eigenstates with the weak interaction ones.

$$\mathrm{V_{CKM}} = \begin{pmatrix} \mathrm{V_{ud}} & \mathrm{V_{us}} & \mathrm{V_{ub}} \\ \mathrm{V_{cd}} & \mathrm{V_{cs}} & \mathrm{V_{cb}} \\ \mathrm{V_{td}} & \mathrm{V_{ts}} & \mathrm{V_{tb}} \end{pmatrix} \simeq$$

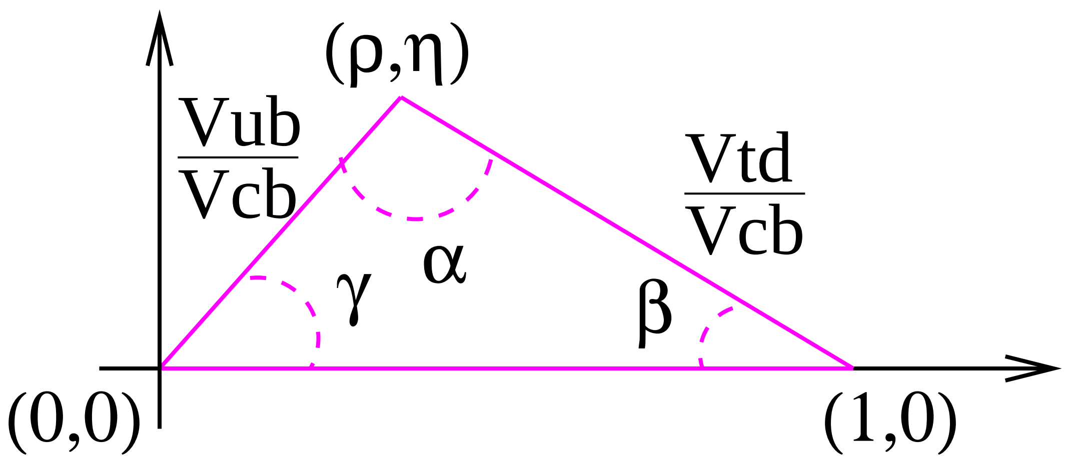
$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

(L. Wolfenstein)

A, ρ and $\eta \simeq \mathcal{O}(1)$ $\quad \lambda = \sin\theta_C$

$\frac{\delta A}{A} = 4\%$ e $\frac{\delta\lambda}{\lambda} = 1\%$

No direct measurements of ρ and η exist to date.

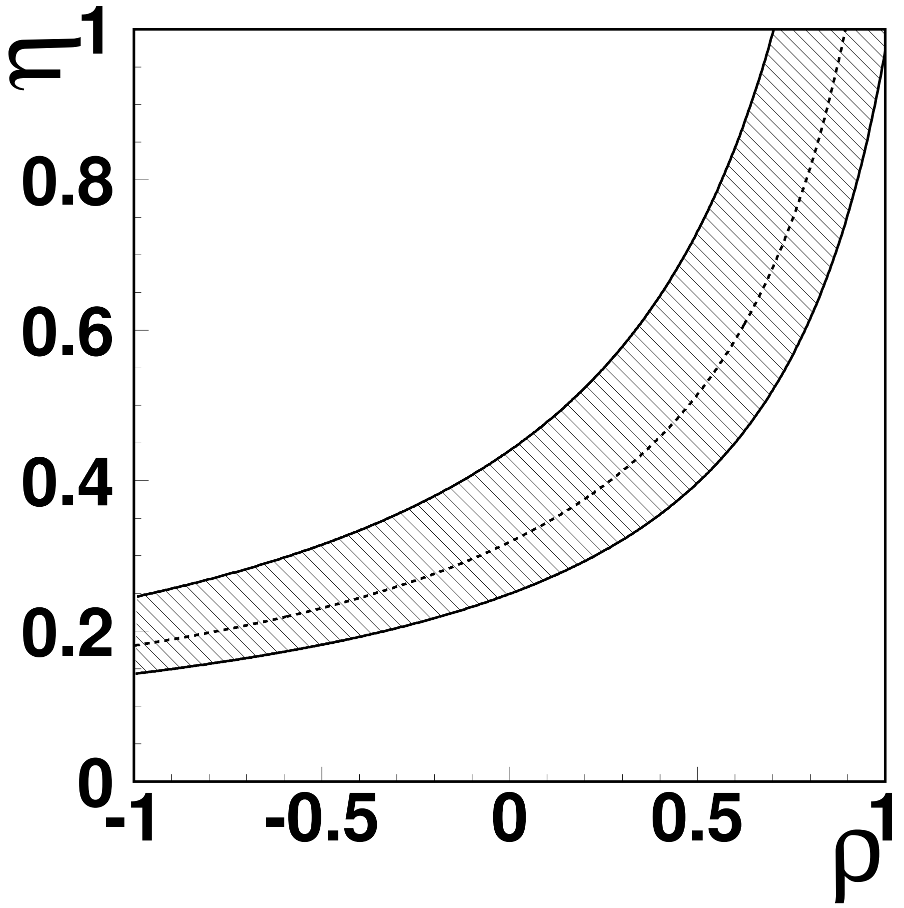


Constraints

$$|K_S\rangle = p|K^0\rangle + q|\bar{K^0}\rangle \qquad |K_L\rangle = p|K^0\rangle - q|\bar{K^0}\rangle$$

$$\frac{p}{q} = \frac{1+\epsilon_K}{1-\epsilon_K}$$

Information from precise measurements of $K_S \to \pi^+\pi^-$ and $K_L \to \pi^+\pi^-$ decays.

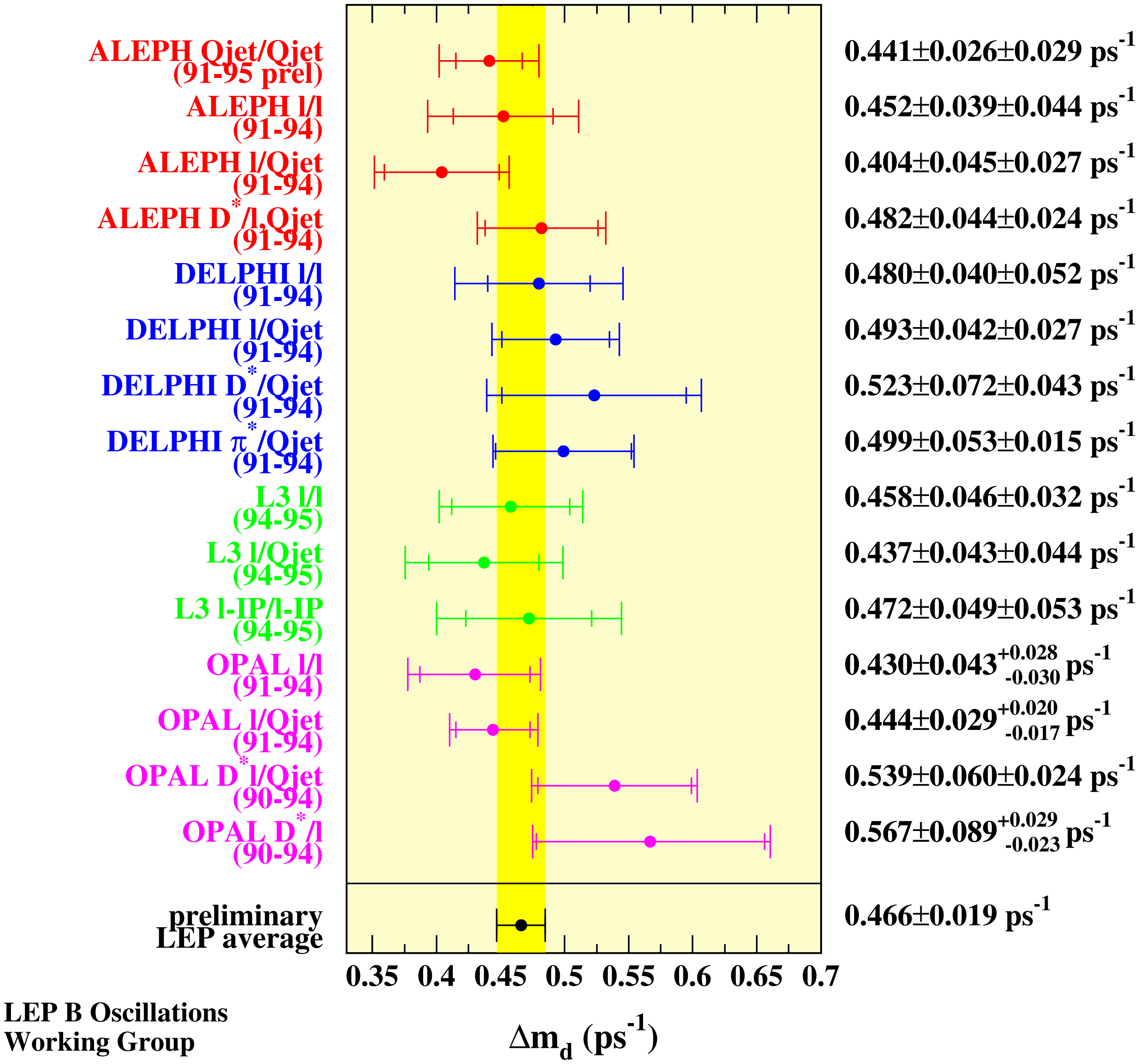


$$|\epsilon_K| = \frac{G_F^2 f_K^2 m_K m_W^2}{6\sqrt{2}\pi^2 \Delta m_K} B_K \left(A^2\lambda^6\eta\right) \left[y_c \left(\eta_{ct} f_3(y_c, y_t)\eta_{cc}\right)\right.$$
$$\left. + \eta_{tt} y_t f_2(y_t) A^2\lambda^4 (1-\rho) \right]$$

Constraints

$$|B_L\rangle = p|B_d^0\rangle + q|\bar{B}_d^0\rangle \qquad |B_H\rangle = p|B_d^0\rangle - q|\bar{B}_d^0\rangle$$

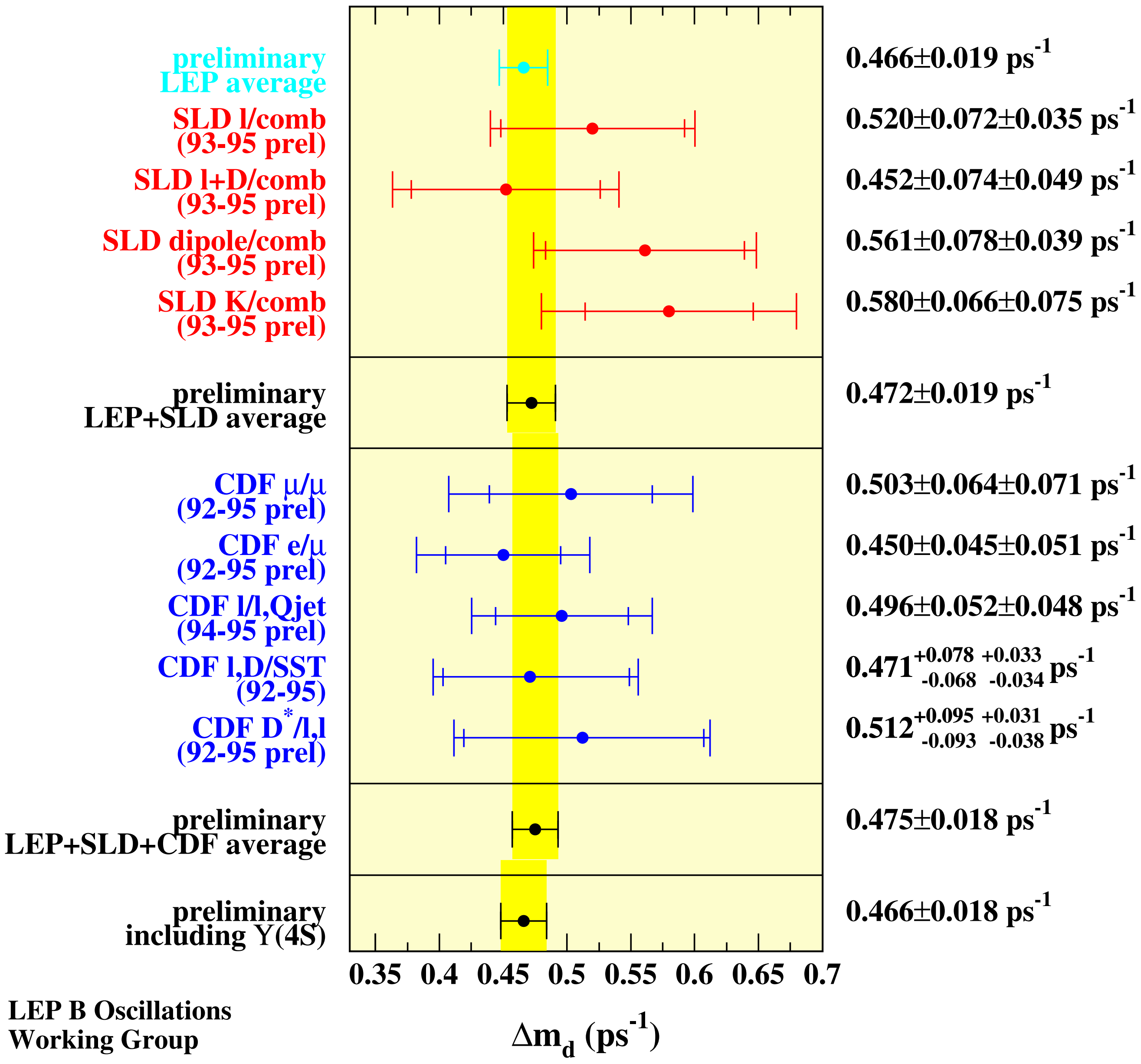
$$\Delta m_d = m_{B_H} - m_{B_L}$$



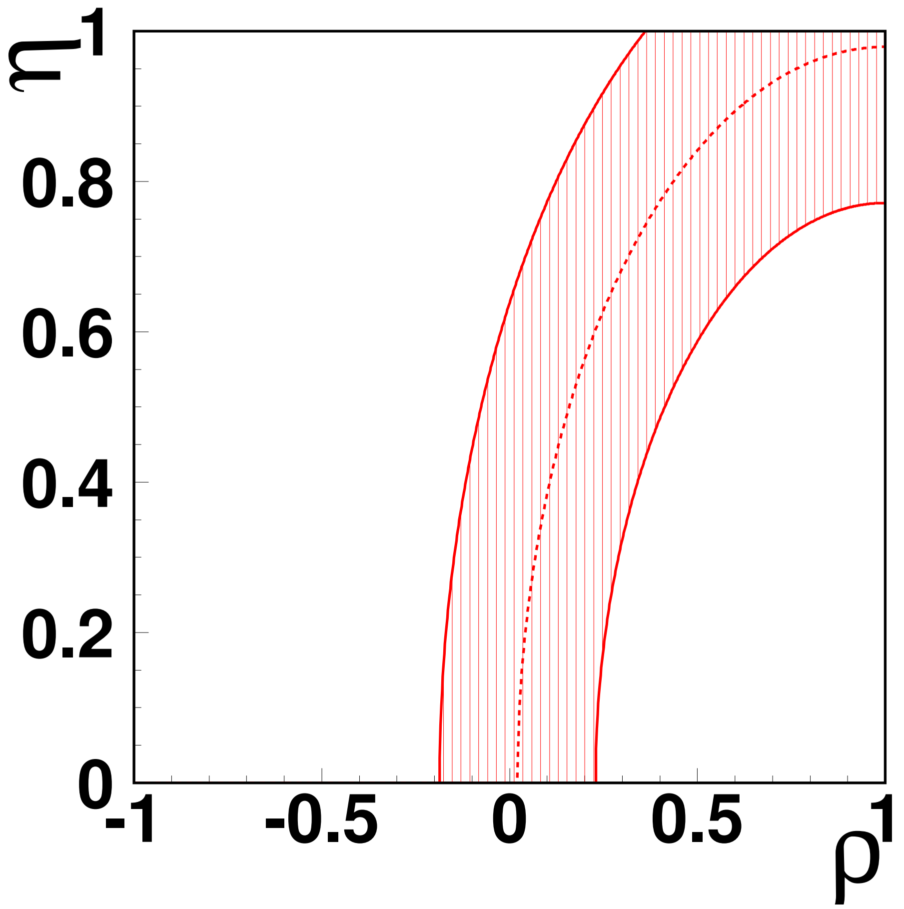
Constraints

$$|B_L\rangle = p|B_d^0\rangle + q|\bar{B}_d^0\rangle \qquad |B_H\rangle = p|B_d^0\rangle - q|\bar{B}_d^0\rangle$$

$$\Delta m_d = m_{B_H} - m_{B_L}$$



Constraints



$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_W^2 m_B^2 \left(f_{B_d}\sqrt{B_{B_d}}\right)^2 \eta_B y_t f_2(y_t) A\lambda^6 \left[(1-\rho)^2 + \eta^2\right]$$

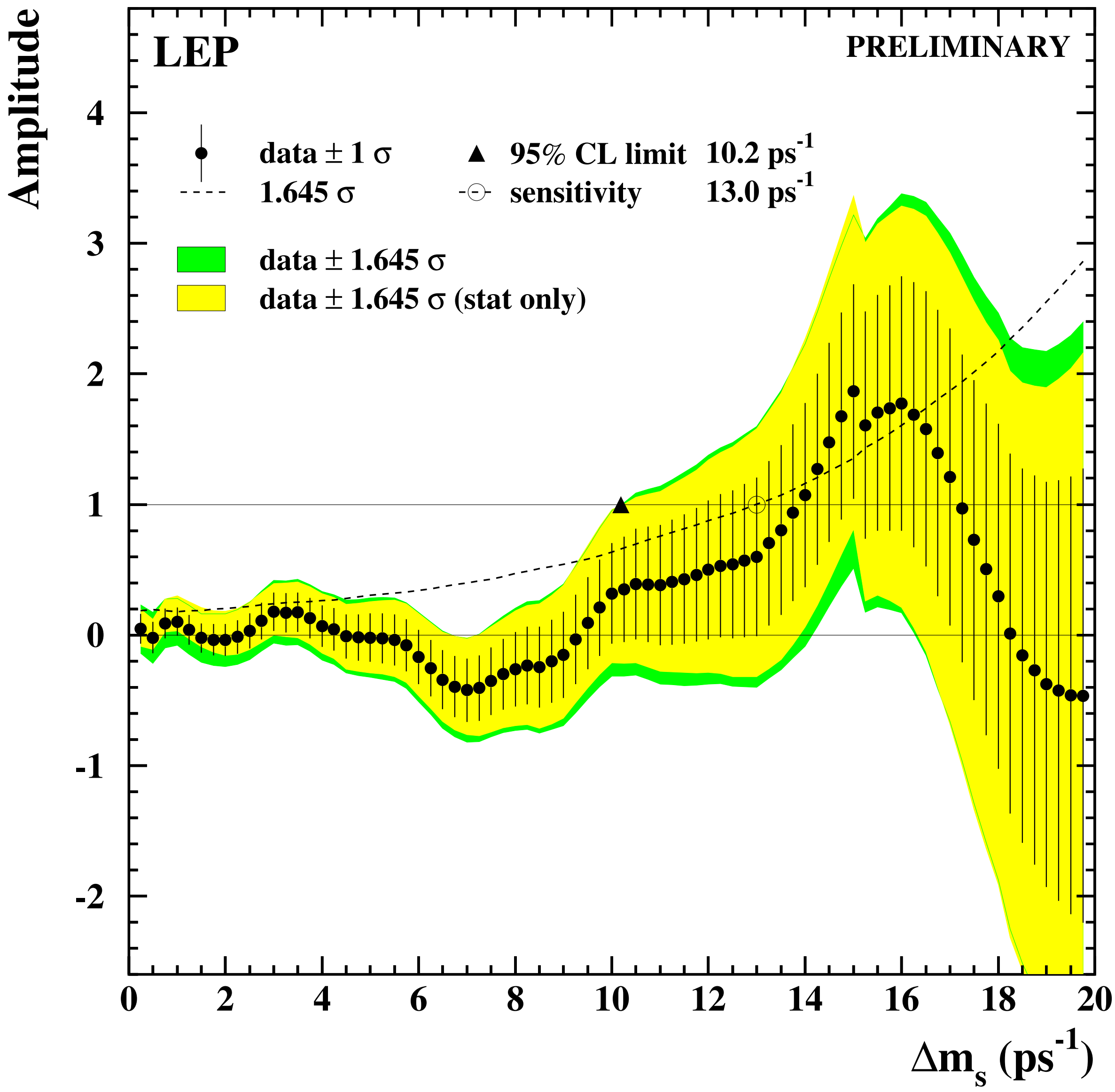
Constraints

$$|B_{sL}\rangle = p|B_s^0\rangle + q|\bar{B}_s^0\rangle \qquad |B_{sH}\rangle = p|B_s^0\rangle - q|\bar{B}_s^0\rangle$$

$$\Delta m_s = m_{B_{sH}} - m_{B_{sL}}$$

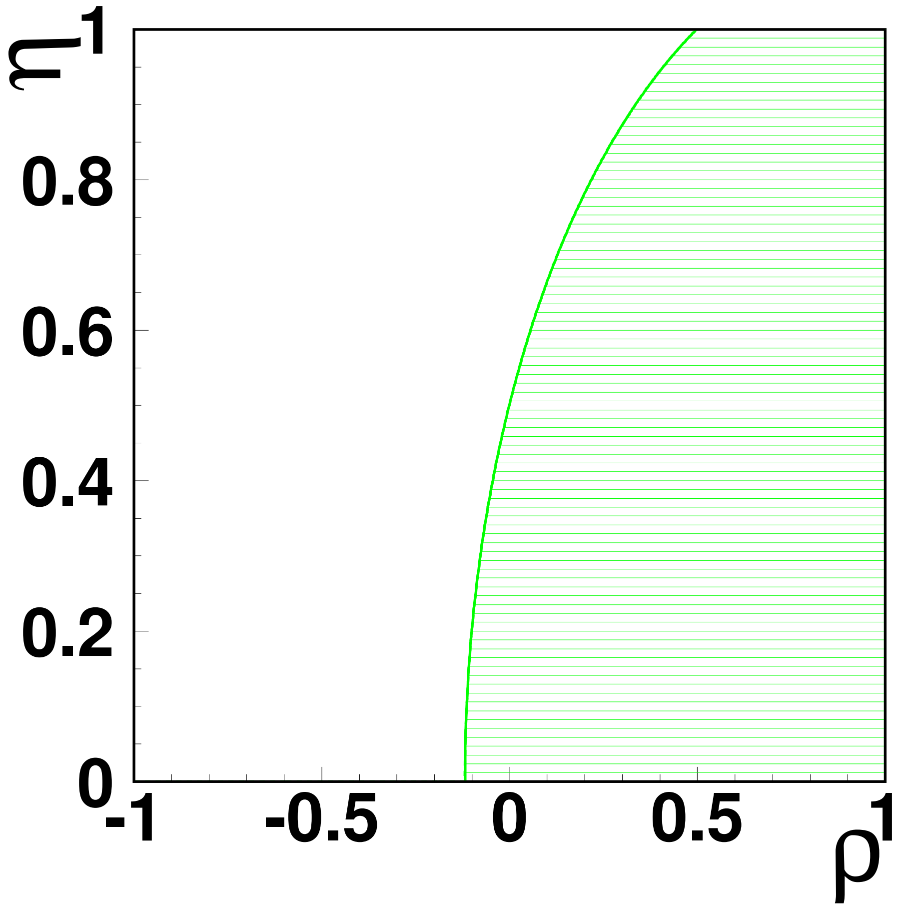
Faster oscillation, not observed yet.

$$P\left[B_s^0 \to (B_s^0, \bar{B}_s^0)\right] = \frac{1}{2\tau_s} e^{-t/\tau_s} \left(1 \pm \mathcal{A} \cos \Delta m_s\right)$$



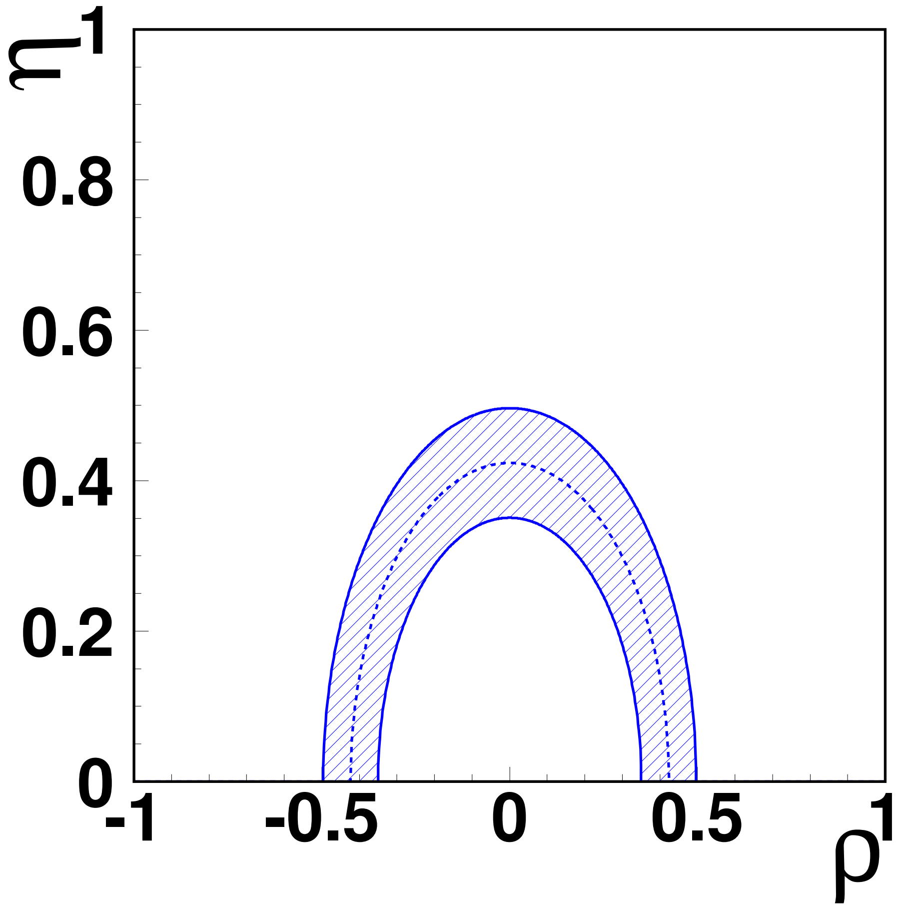
$$\Delta m_s < 10.2\,\mathrm{ps}^{-1}\ 95\%\,\mathrm{C.L.}$$

Constraints



$$\Delta m_s = \Delta m_d \frac{1}{\lambda^2} \frac{m_{B_s}}{m_{B_d}} \xi^2 \frac{1}{(1-\rho)^2 + \eta^2}$$

Constraints



$$|V_{ub}|/|V_{cb}| = \lambda\sqrt{\rho^2+\eta^2}$$

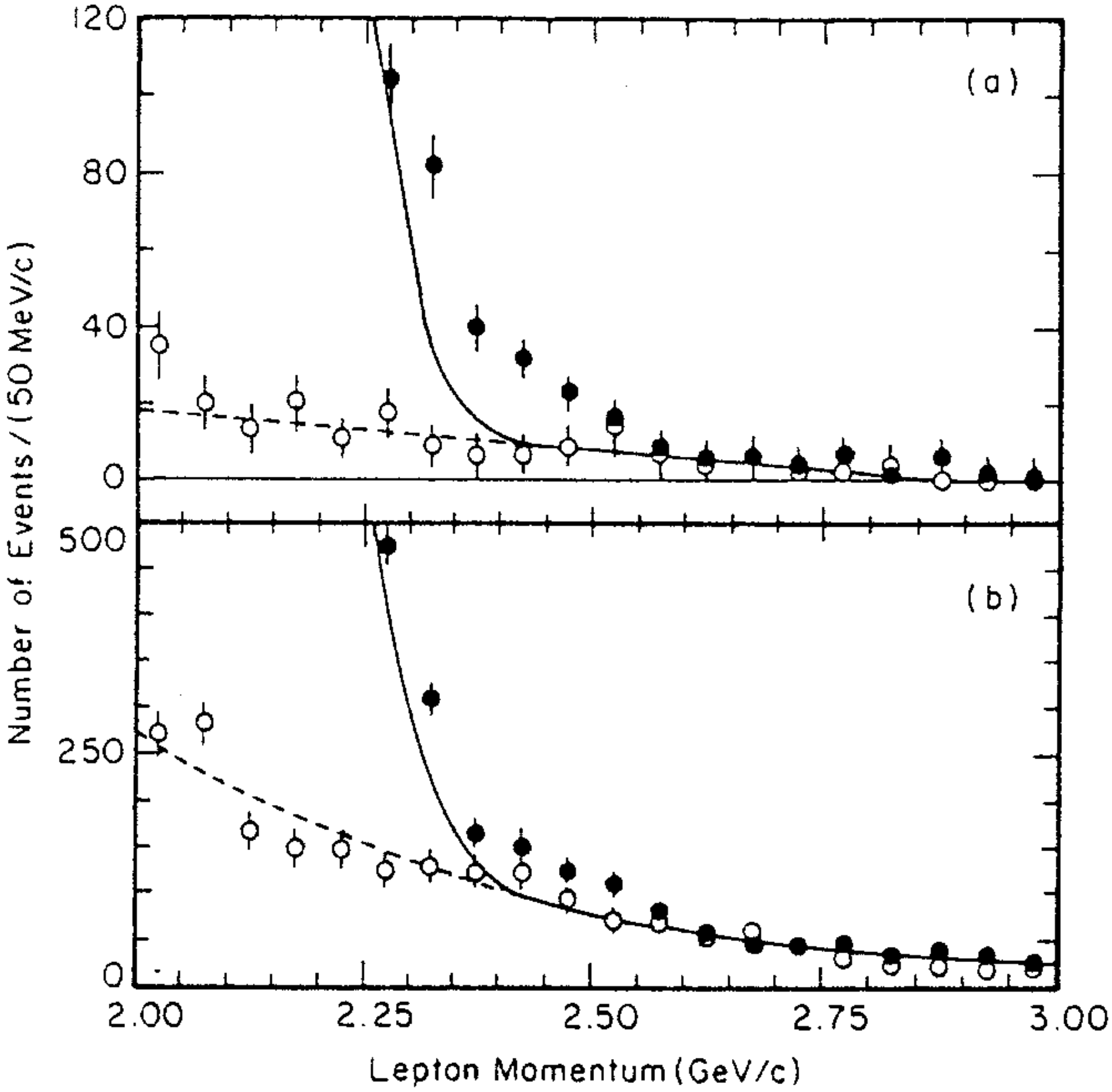
Each previous constraints has a factor with sensible theoretical uncertainty or controversial calculations:

$$\begin{aligned} |\epsilon_K| &\Longrightarrow B_K \\ \Delta m_d &\Longrightarrow f_{B_d}\sqrt{B_{B_d}} \\ \Delta m_s &\Longrightarrow \xi \end{aligned}$$

That from $|V_{ub}|$ does not need any further theoretical factors.

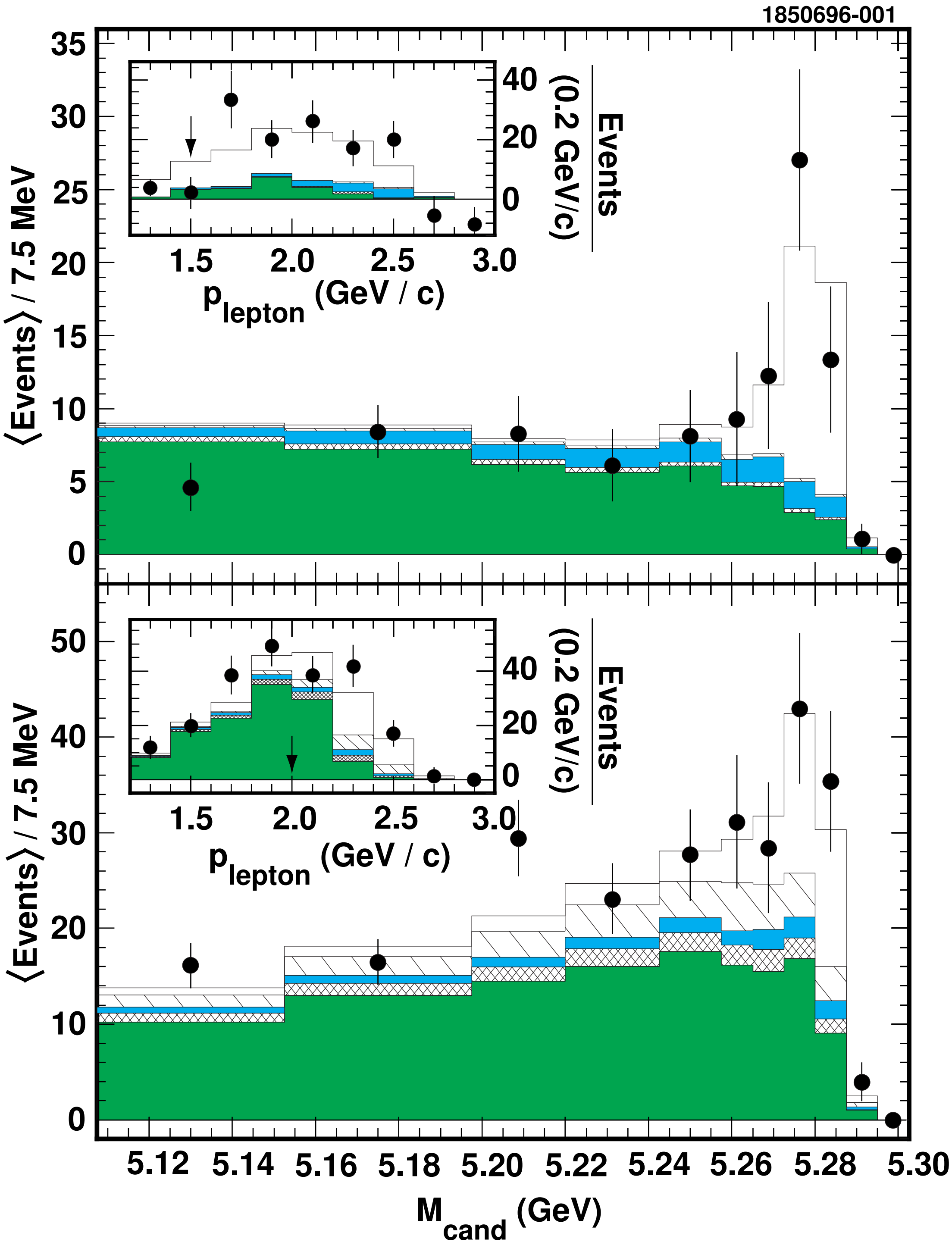
V_{ub} at CLEO

$$m_{X_c} > m_{X_u} \Longrightarrow p_\ell^c < p_\ell^u$$



- Inclusive measurement only for $2.3\,\mathrm{GeV} < p_\ell < 2.6\,\mathrm{GeV}$
- Theoretical problems for the extrapolation to the full spectrum
- $V_{ub} = (3.2 \pm 1.2) \times 10^{-3} \left(\text{from } \frac{|V_{ub}|}{|V_{cb}|} = 0.08 \pm 0.02\right)$

V_{ub} at CLEO



- Exclusive measurement: $B \to \pi \ell \nu$ ($\uparrow$) & $B \to (\rho, \omega) \ell \nu$ ($\downarrow$)
- Strong theory dependence for the extraction of V_{ub}
- $V_{ub} = (3.3 \pm 0.2^{-0.4}_{+0.3} \pm 0.7) \times 10^{-3}$

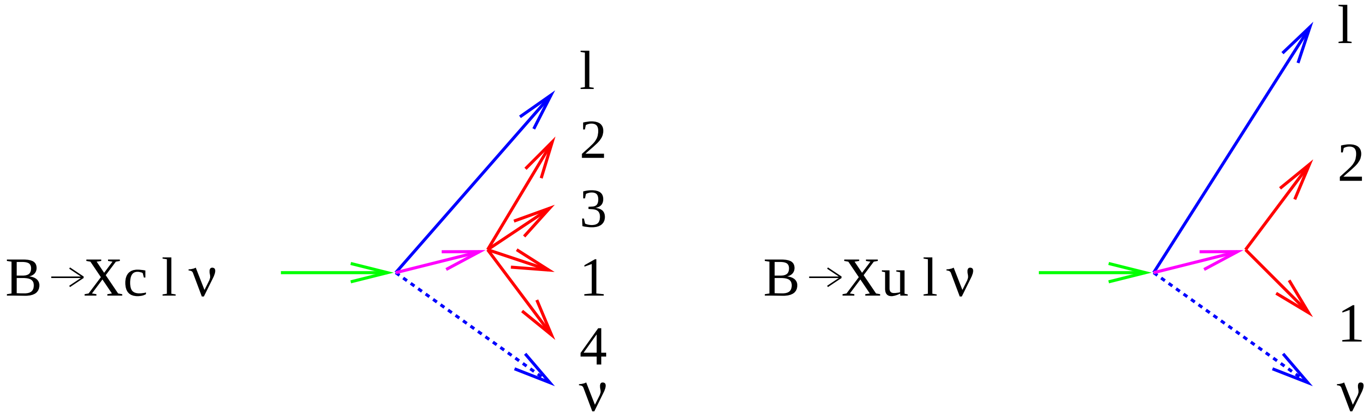
$\mathrm{V_{ub}}$ at LEP

$\mathrm{V_{ub}}$ can be extracted from the inclusive measurement of $\mathrm{Br(B \to X_u \ell \nu)}$:

$$|\mathrm{V_{ub}}| = 0.00458 \times \sqrt{\frac{\mathrm{Br(B \to X_u \ell \nu)}}{0.002}} \times \sqrt{\frac{1.6\,\mathrm{ps}}{\tau_{\mathrm{B}}}} \pm 4\%_{\mathrm{theory}}$$

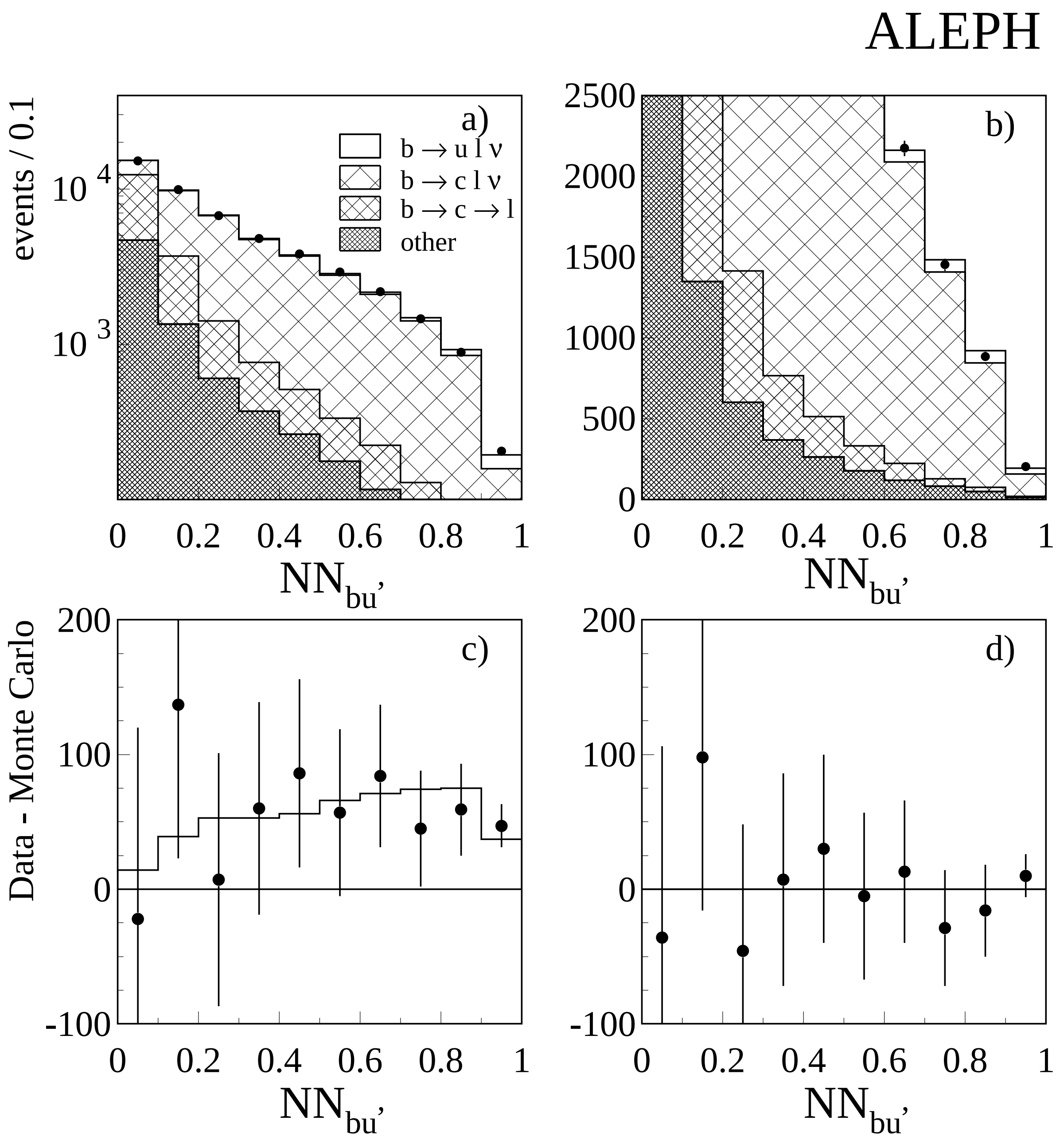
(N. Uraltsev Int. J. Mod. Phys. **A 11** (1996) 515)

The B boost makes the measurement feasible.



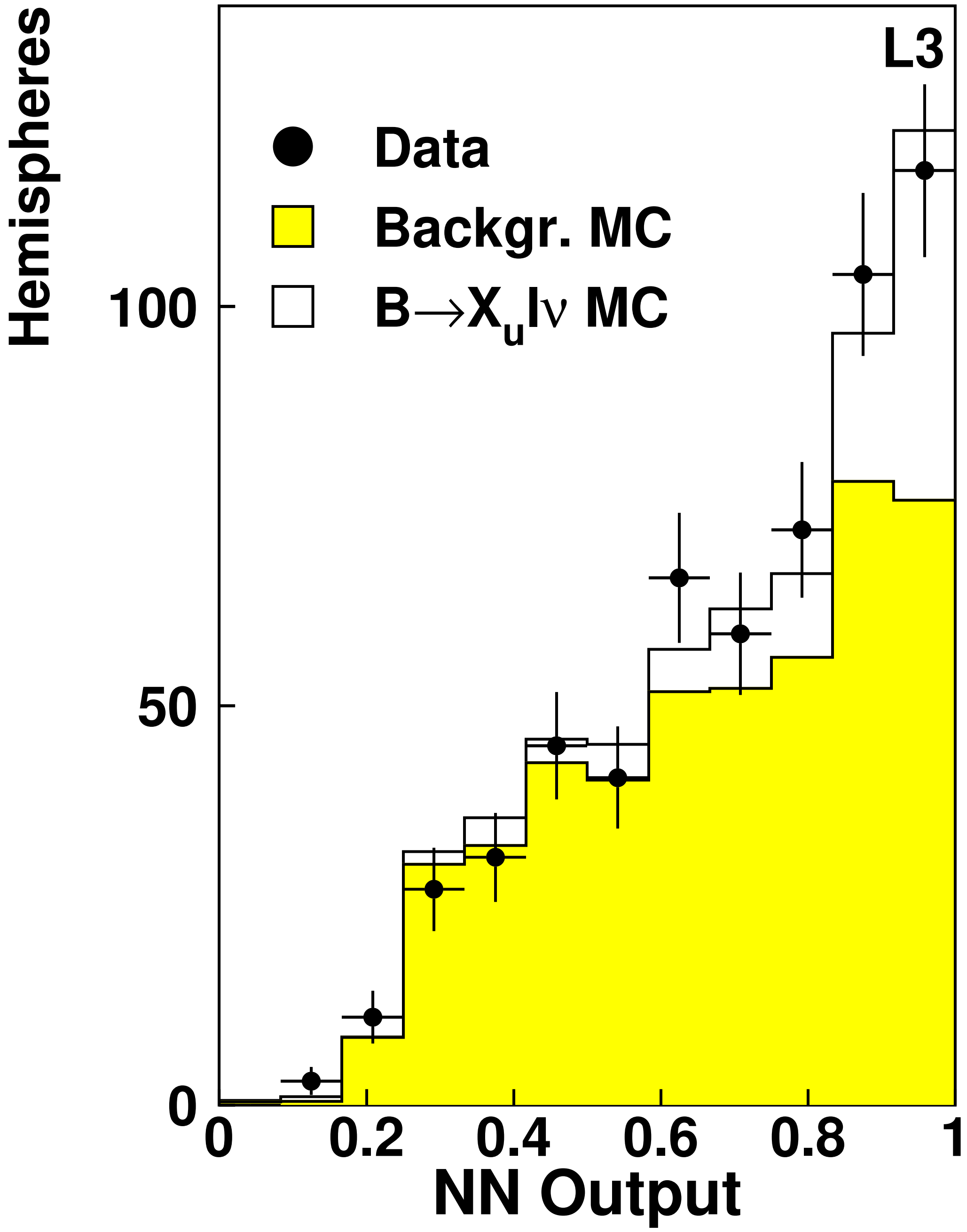
$$\boxed{\begin{array}{rcl} p_\ell, p_t|_{\mathrm{u}} & > & p_\ell, p_t|_{\mathrm{c}} \\ \mathrm{multiplicity}\ |_{\mathrm{u}} & < & \mathrm{multiplicity}\ |_{\mathrm{c}} \\ m_{1\ell}|_{\mathrm{u}} & > & m_{1\ell}|_{\mathrm{c}} \\ p_{1\ell}|_{\mathrm{u}} & > & p_{1\ell}|_{\mathrm{c}} \\ p_1|_{\mathrm{u}} & > & p_1|_{\mathrm{c}} \\ p_2|_{\mathrm{u}} & < & p_2|_{\mathrm{c}} \end{array}}$$

With a neural network approach:



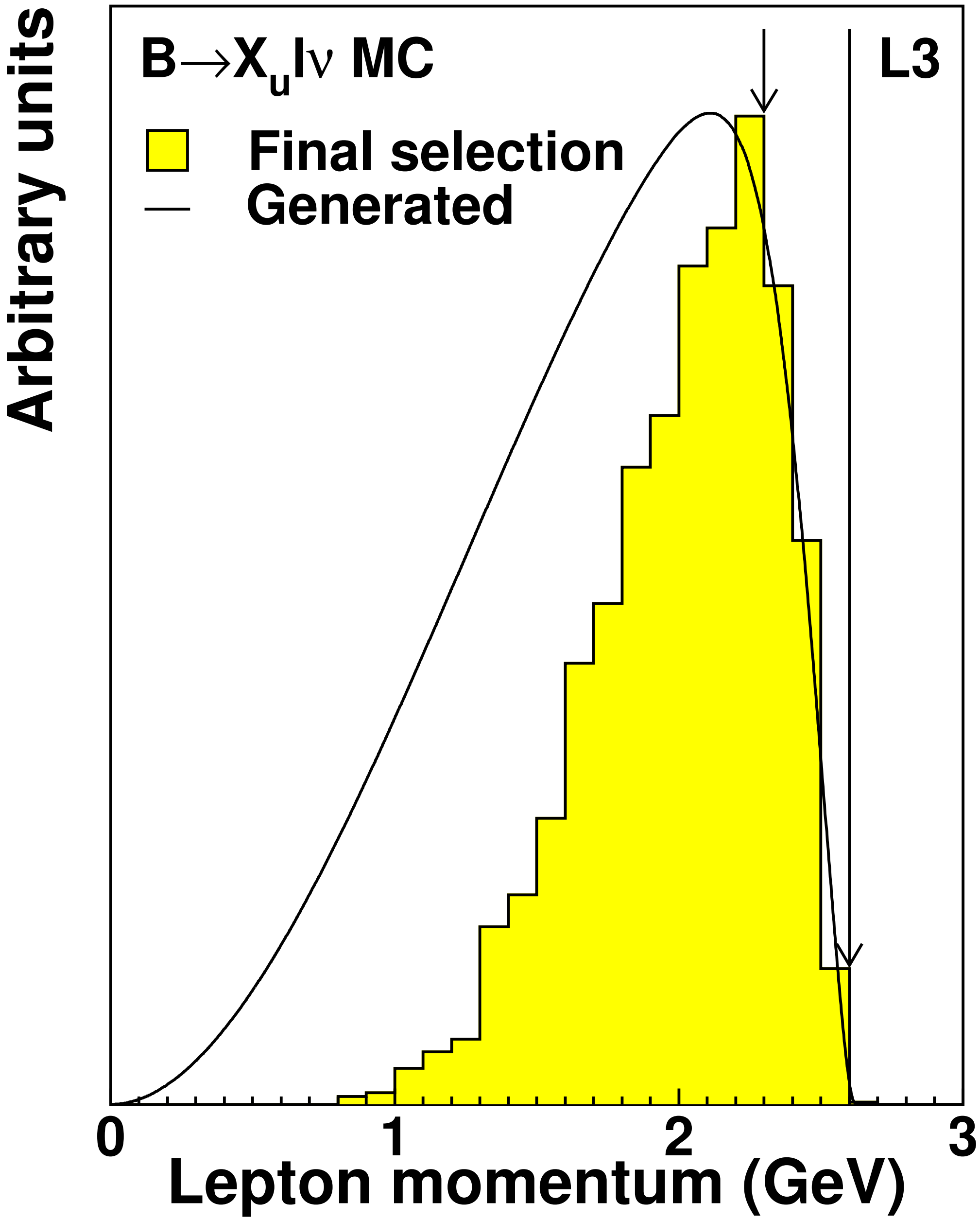
$$\mathrm{ALEPH} : \mathrm{Br}(b \to X_u \ell \nu) = 1.73 \pm 0.55 \pm 0.55 \times 10^{-3}$$

With a counting method and a neural network approach:



$$\mathrm{L3} : \mathrm{Br}(\mathrm{b} \to \mathrm{X_u}\ell\nu) = 3.0 \pm 1.0 \pm 1.7 \times 10^{-3}$$

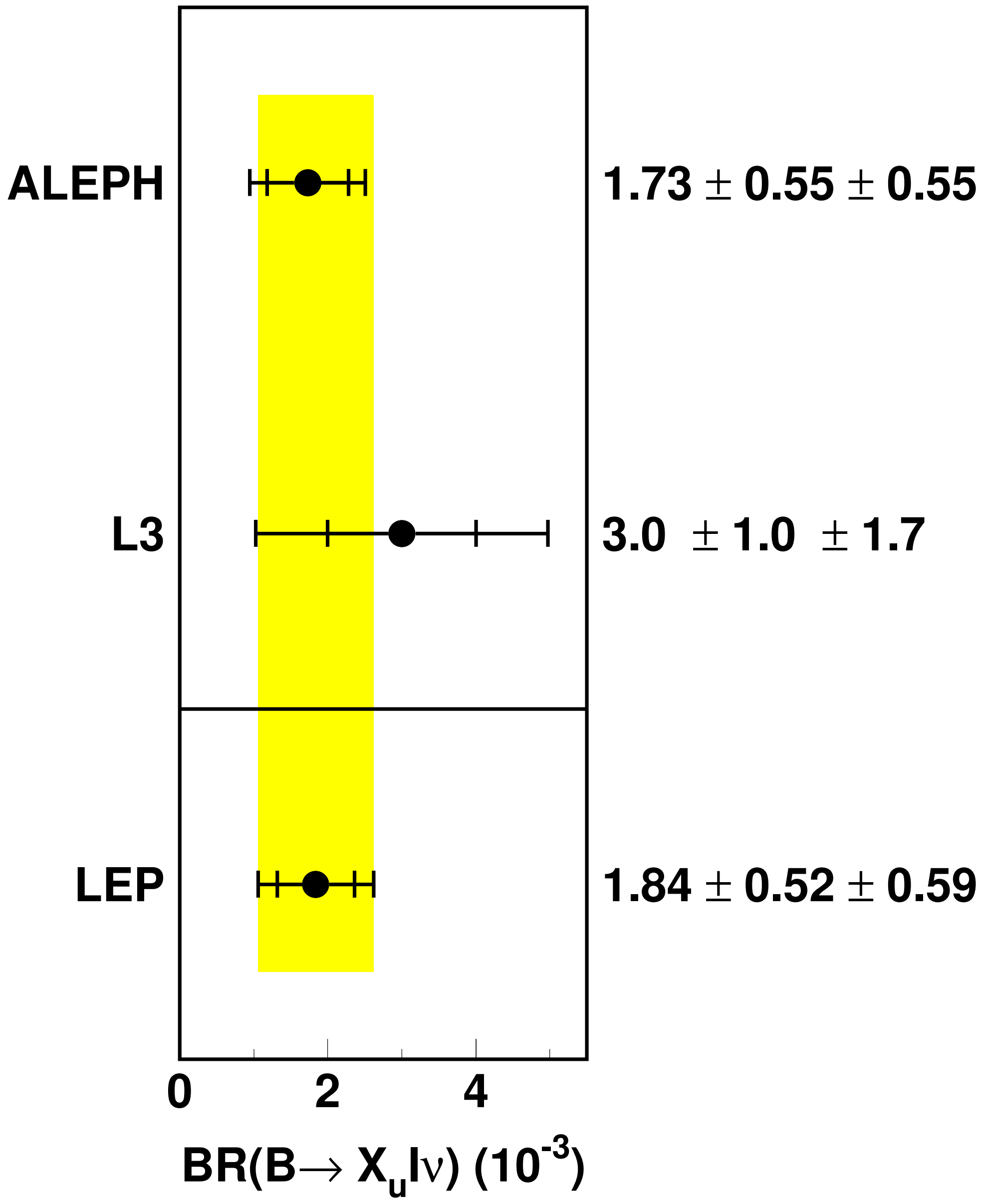
Cross Checks



Large fraction of the spectrum with respect to $\Upsilon(4S)$.

Combinations

Combining ALEPH and L3 measurements of $\mathrm{Br}(\mathrm{B} \to \mathrm{X_u}\ell\nu)$:

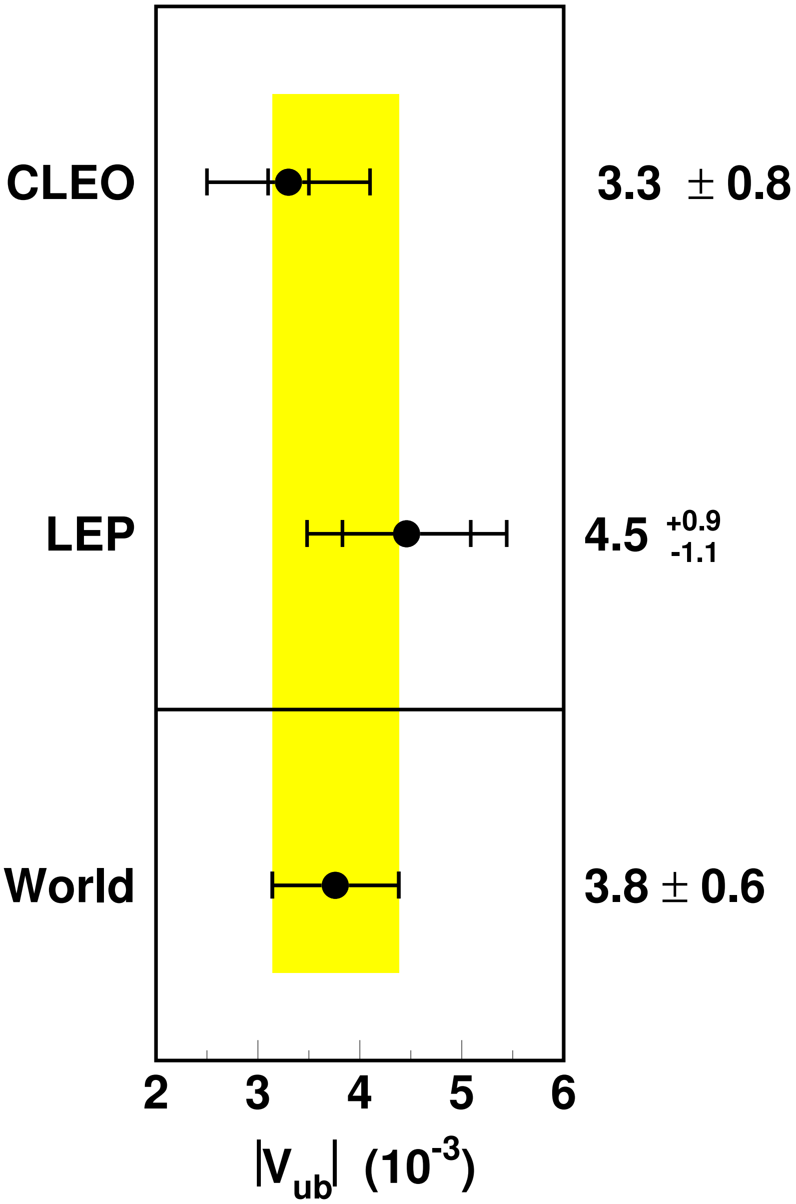


Giving the magnitude of $\mathrm{V_{ub}}$ at LEP:

$$|\mathrm{V_{ub}}| = (4.5^{+0.6}_{-0.7}(\mathrm{stat.})^{+0.7}_{-0.8}(\mathrm{syst.}) \pm 0.17(\mathrm{theo.})) \times 10^{-3}$$

Combinations

From LEP inclusive and CLEO exclusive measurements:

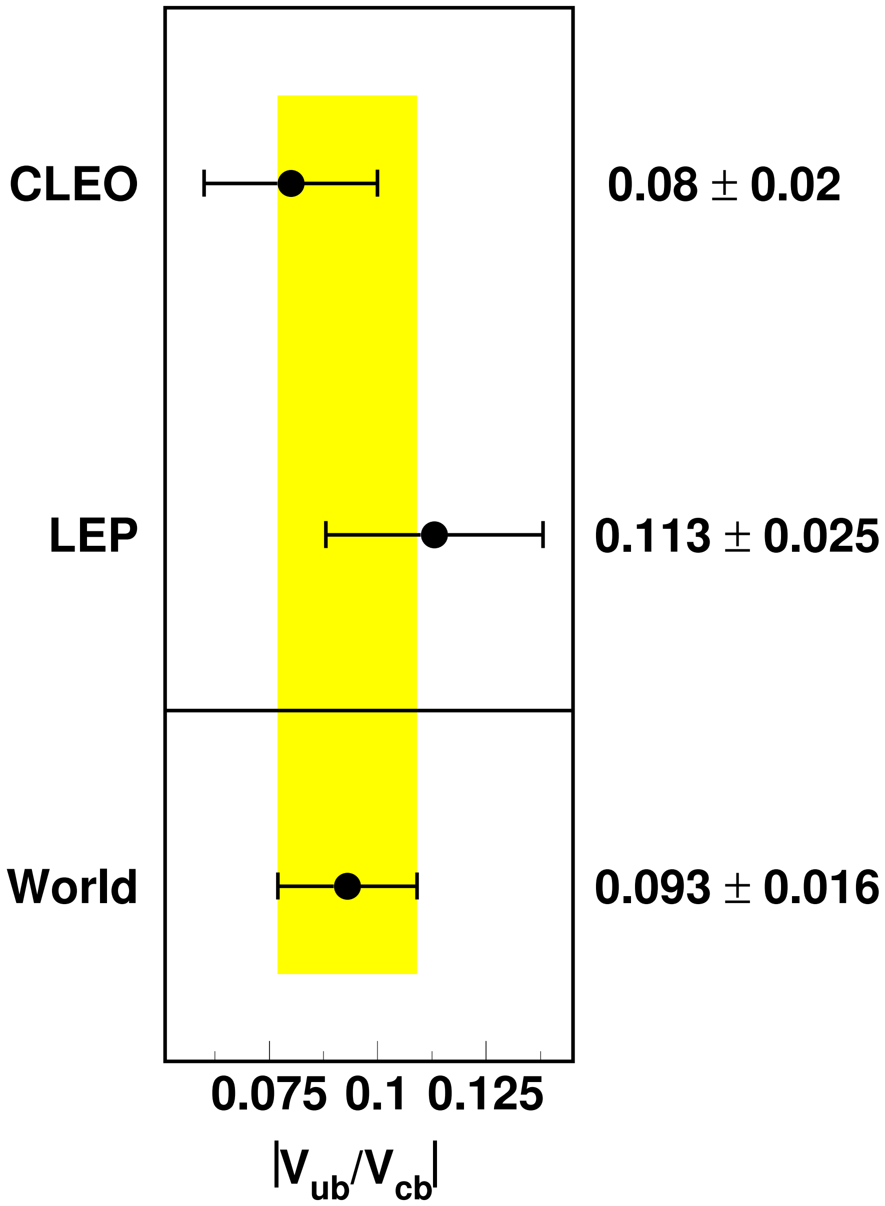


World average of $|V_{ub}|$:

$$|V_{ub}| = (3.8 \pm 0.6) \times 10^{-3}$$

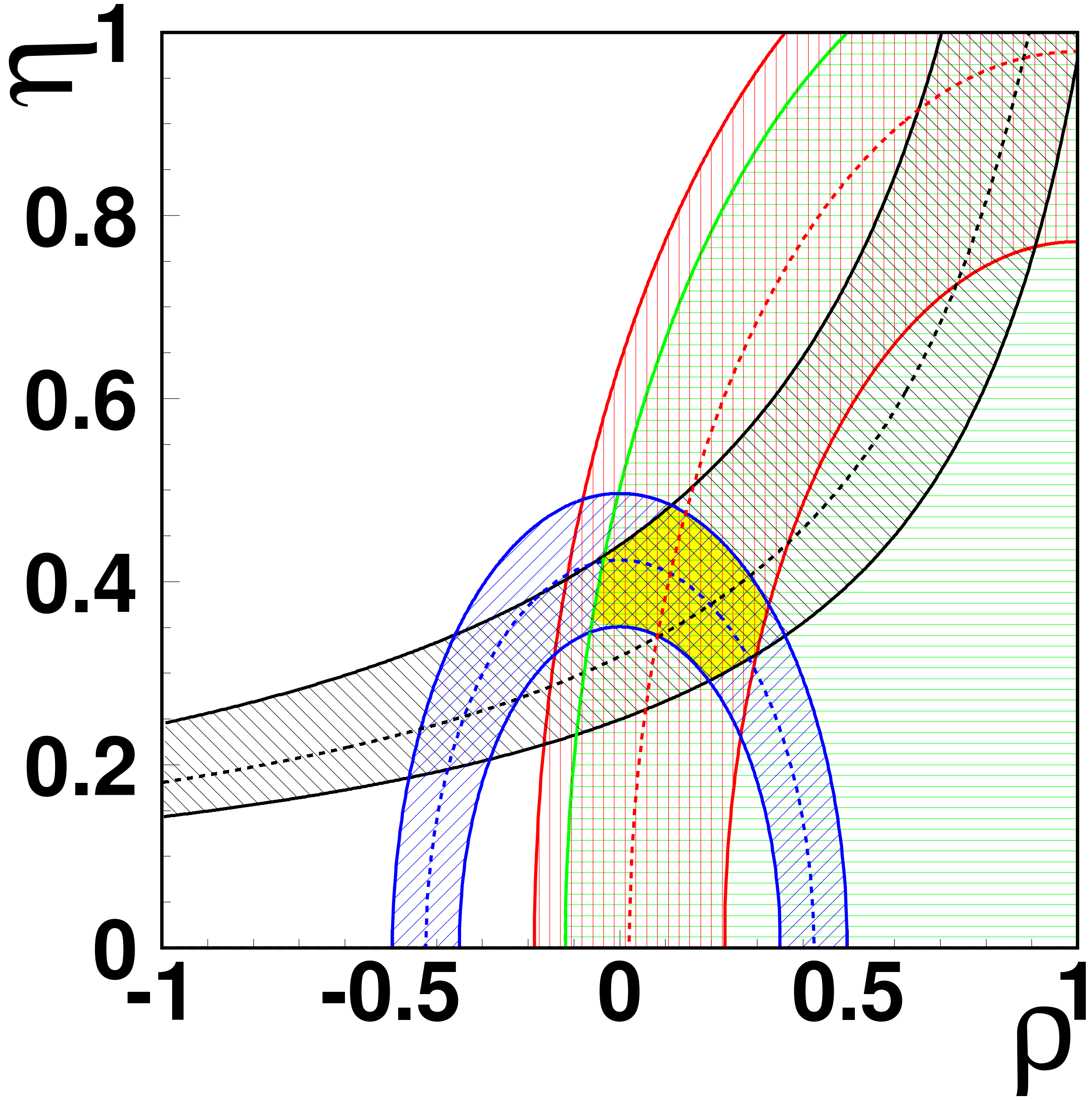
Combinations

The combined $|V_{ub}|/|V_{cb}|$ constraint can be extracted from the LEP $|V_{ub}|$ with $|V_{cb}| = (39.5 \pm 1.7) \times 10^{-3}$ and from the CLEO inclusive measurement.



Fit

The current constraints at $\pm 1\sigma$ are:



Fit

$$\chi^2 = \frac{(\widehat{m_c} - m_c)^2}{\sigma^2_{m_c}} + \frac{(\widehat{m_t} - m_t)^2}{\sigma^2_{m_t}} + \frac{\left(\widehat{B_K} - B_K\right)^2}{\sigma^2_{B_K}} +$$

$$+ \frac{\left(\widehat{A} - A\right)^2}{\sigma^2_A} + \frac{\left(\widehat{f_{B_d}\sqrt{B_{B_d}}} - f_{B_d}\sqrt{B_{B_d}}\right)^2}{\sigma^2_{f_{B_d}\sqrt{B_{B_d}}}} + \frac{(\hat{\xi} - \xi)^2}{\sigma^2_\xi} +$$

$$+ \frac{(\widehat{\eta_{cc}} - \eta_{cc})^2}{\sigma^2_{\eta_{cc}}} + \frac{(\widehat{\eta_{ct}} - \eta_{ct})^2}{\sigma^2_{\eta_{ct}}} + \frac{\left(\widehat{\frac{|\mathrm{V_{ub}}|}{|\mathrm{V_{cb}}|}} - \frac{|\mathrm{V_{ub}}|}{|\mathrm{V_{cb}}|}\right)^2}{\sigma^2_{\frac{|\mathrm{V_{ub}}|}{|\mathrm{V_{cb}}|}}} +$$

$$+ \frac{\left(\widehat{|\epsilon_K|} - |\epsilon_K|\right)^2}{\sigma^2_{|\epsilon_K|}} + \frac{\left(\widehat{\Delta m_d} - \Delta m_d\right)^2}{\sigma^2_{\Delta m_d}} +$$

$$\frac{+\left(1 - \mathcal{A}\left(\Delta m_s\right)\right)^2}{\sigma^2_{\mathcal{A}(\Delta m_s)}}$$

Fit parameters: ρ, η, m_c, m_t, B_K, A, $f_{B_d}\sqrt{B_{B_d}}$, ξ, η_{cc}, η_{ct}.

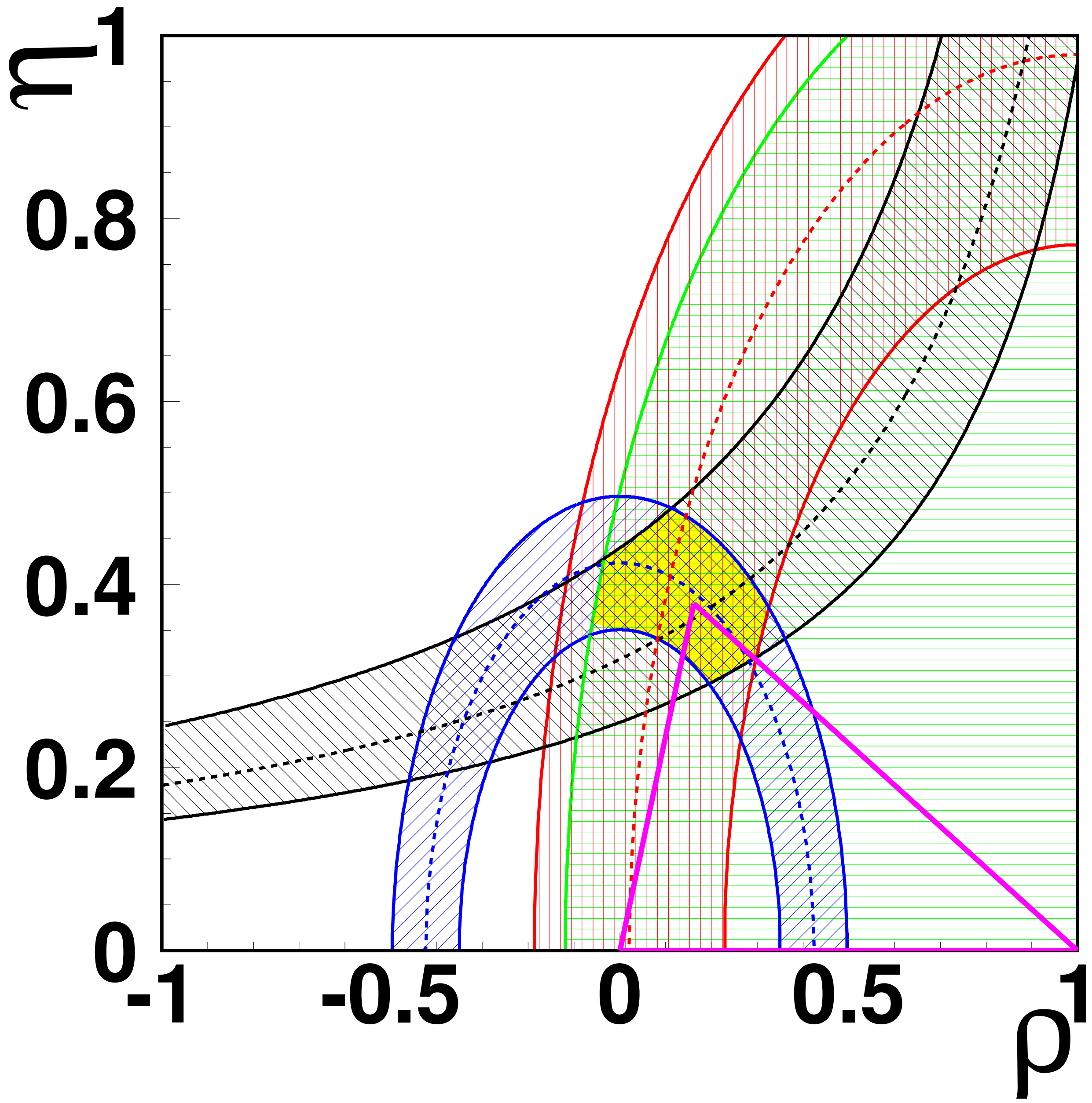
$\frac{|\mathrm{V_{ub}}|}{|\mathrm{V_{cb}}|}$, $|\epsilon_K|$, Δm_d and $\Delta \mathrm{m_s}$ known functions of these parameters.

(Δm_s inclusion idea in P. Paganini *et al.* `hep-ph/9711261`)

Numerical values used in the fit:

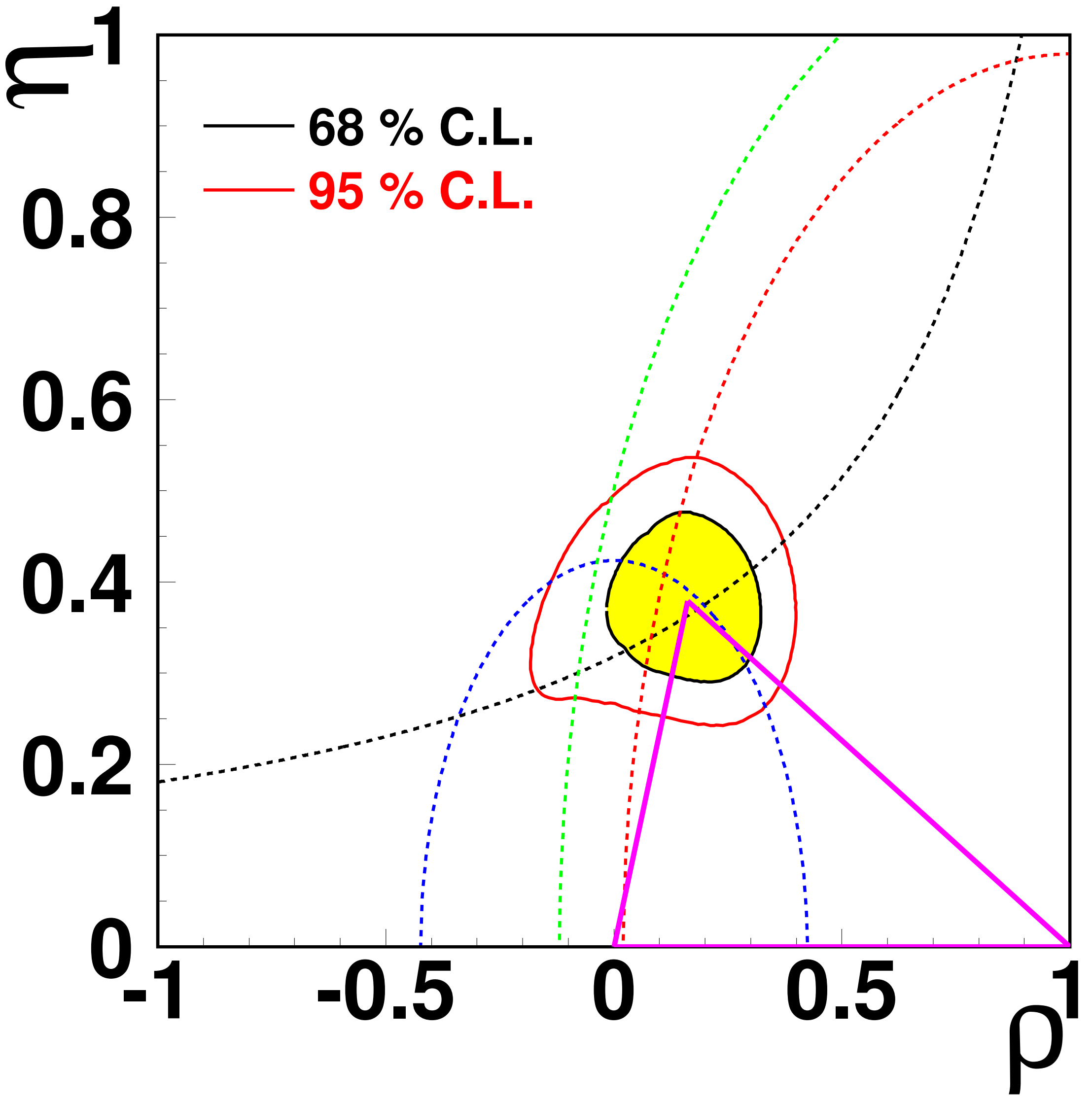
Fixed in the fit		
λ	$=$	0.2196 ± 0.0023
G_F	$=$	$(1.16639 \pm 0.00001) \times 10^{-5}\,\mathrm{GeV}^{-2}$
f_K	$=$	$0.1598 \pm 0.0015\,\mathrm{GeV}$
Δm_K	$=$	$(0.5304 \pm 0.0014) \times 10^{-2}\,\mathrm{ps}^{-1}$
m_K	$=$	$0.497672 \pm 0.000031\,\mathrm{GeV}$
m_W	$=$	$80.375 \pm 0.064\,\mathrm{GeV}$
m_{B_d}	$=$	$5.2792 \pm 0.0018\,\mathrm{GeV}$
m_{B_s}	$=$	$5.3692 \pm 0.0020\,\mathrm{GeV}$
m_B	$=$	$5.290 \pm 0.002\,\mathrm{GeV}$
η_B	$=$	0.55 ± 0.01
η_{tt}	$=$	0.574 ± 0.004
Varied in the fit		
A	$=$	0.819 ± 0.035
η_{ct}	$=$	0.47 ± 0.04
η_{cc}	$=$	1.38 ± 0.53
$\bar{m}_c(m_c)$	$=$	$(1.25 \pm 0.15)\,\mathrm{GeV}$
$\bar{m}_t(m_t)$	$=$	$(166.8 \pm 5.3)\,\mathrm{GeV}$
$\lvert\epsilon_K\rvert$	$=$	$(2.280 \pm 0.019) \times 10^{-3}$
Δm_d	$=$	$(0.466 \pm 0.019)\,\mathrm{ps}^{-1}$
$f_{B_d}\sqrt{B_{B_d}}$	$=$	$(0.201 \pm 0.042)\,\mathrm{GeV}$
B_K	$=$	0.87 ± 0.14
ξ	$=$	1.14 ± 0.08

The fit unitarity triangle:



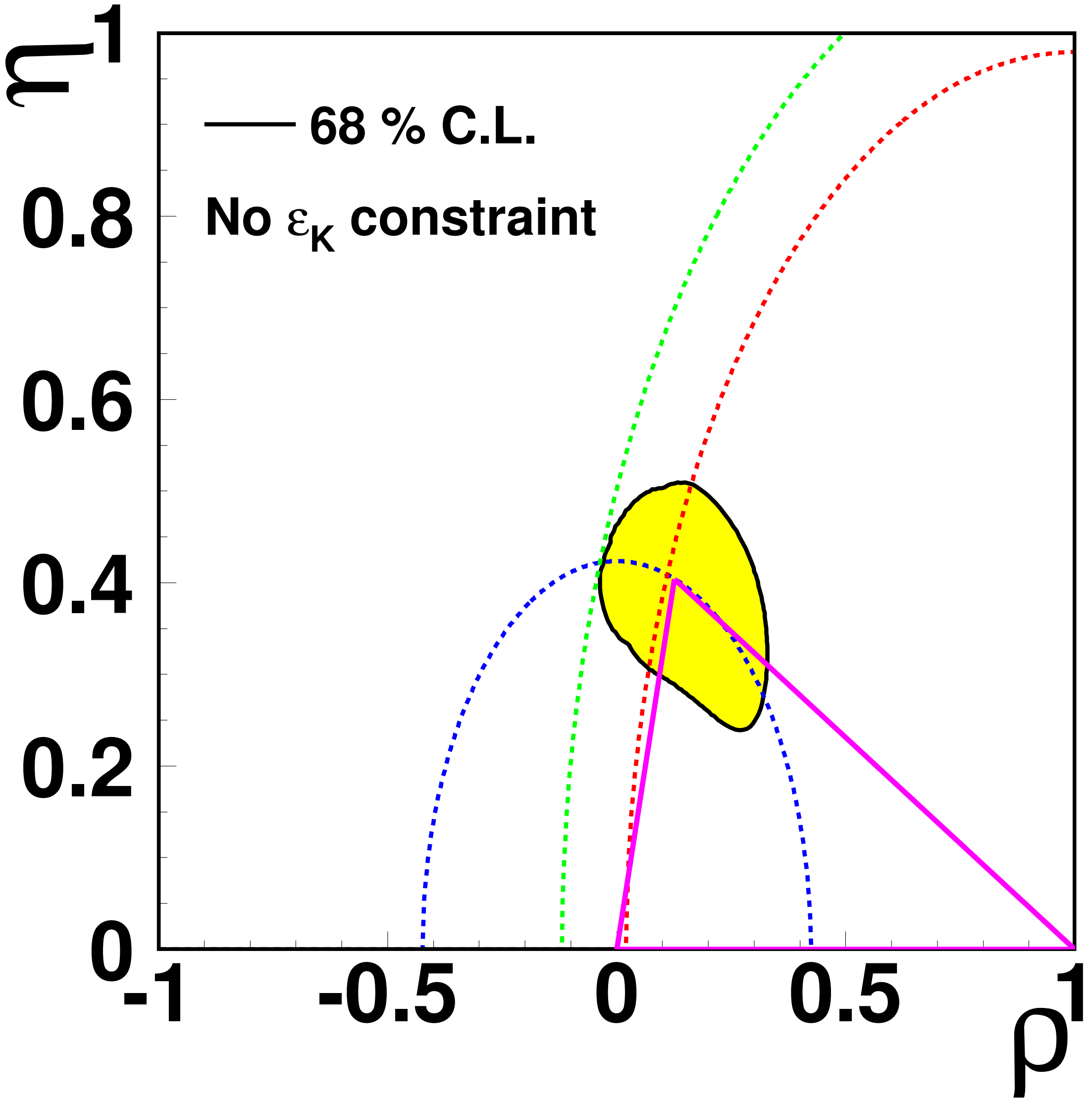
Fit

Confidence regions:



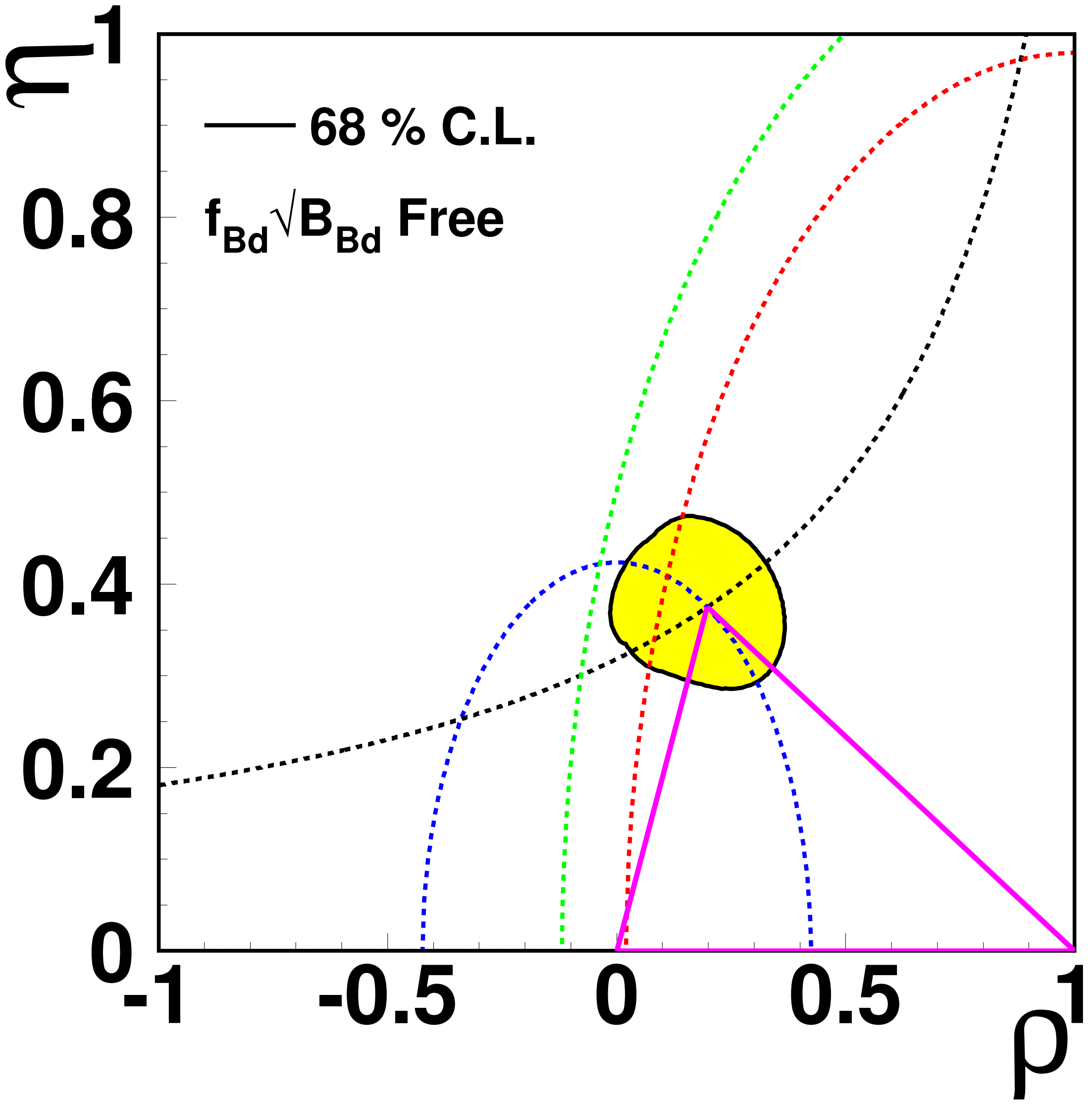
$$\rho = 0.162^{-0.110}_{+0.112}$$
$$\eta = 0.379^{-0.060}_{+0.063}$$

To date the measurement of ϵ_K is the only direct hint of a non vanishing η: remove that experimental constraint from the fit.



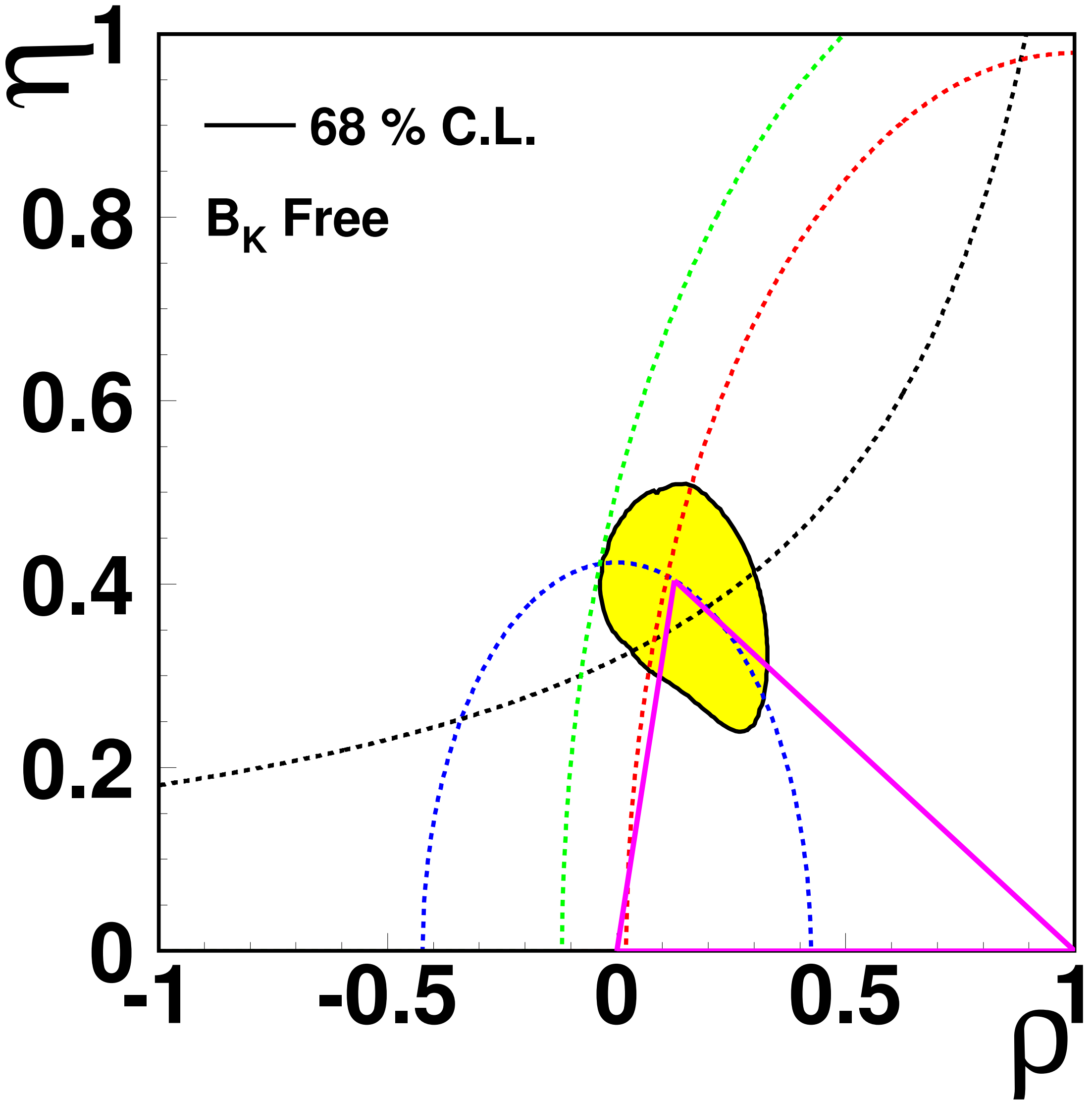
	Free fit	Original fit
ρ	$0.126^{-0.080}_{+0.145}$	$0.162^{-0.110}_{+0.112}$
η	$0.404^{-0.098}_{+0.070}$	$0.379^{-0.060}_{+0.063}$

$f_{B_d}\sqrt{B_{B_d}}$ has a large theoretical error : leave it free in the fit and derive experimental favoured values.



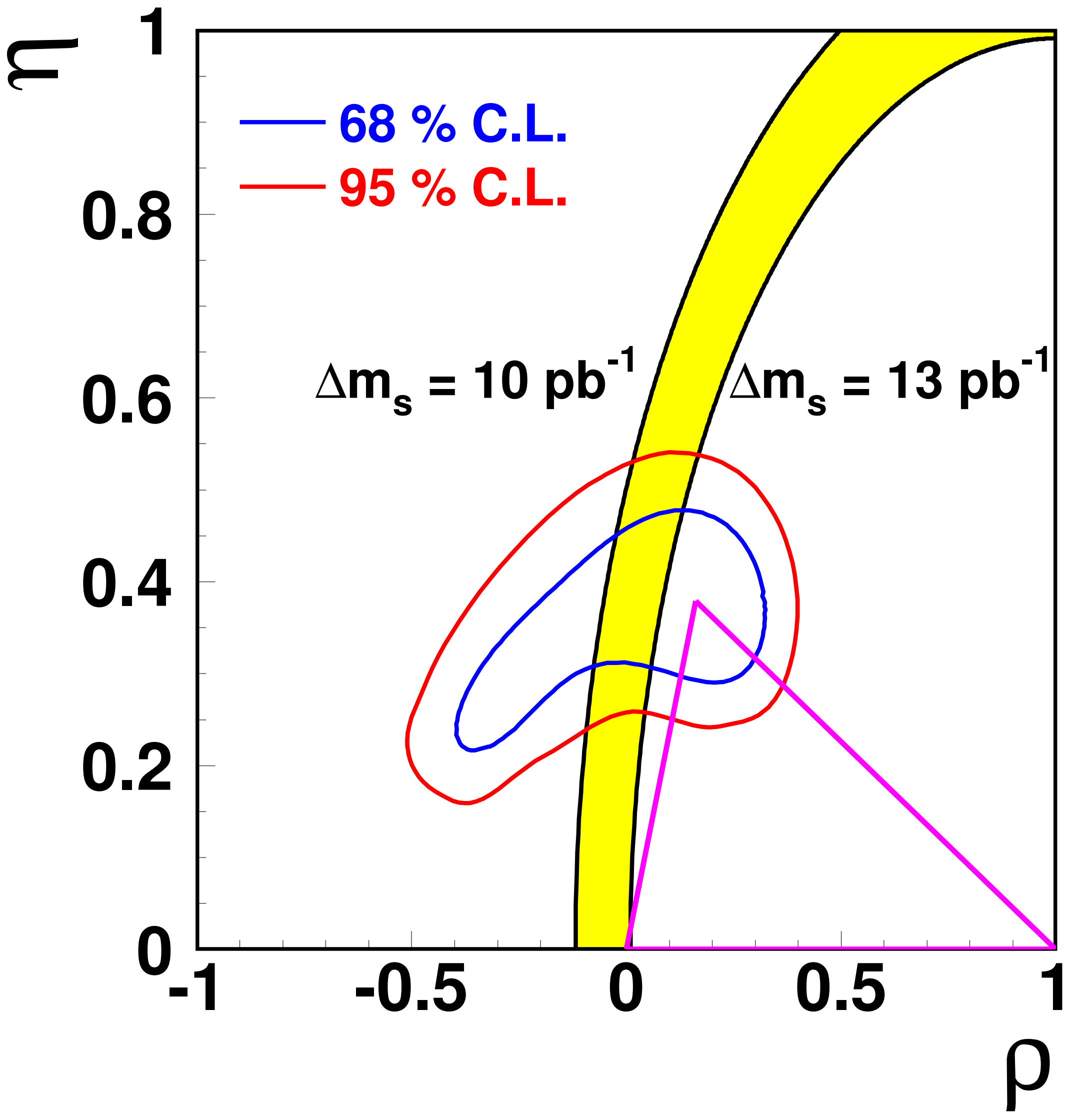
	Free fit	Original fit
ρ	$0.197^{-0.137}_{+0.118}$	$0.162^{-0.110}_{+0.112}$
η	$0.375^{-0.060}_{+0.065}$	$0.379^{-0.060}_{+0.063}$
$f_{B_d}\sqrt{B_{B_d}}$	$0.222^{-0.027}_{+0.030}$ GeV	0.201 ± 0.042 GeV

B_K has a large theoretical error: leave it free in the fit and derive experimental favoured values.



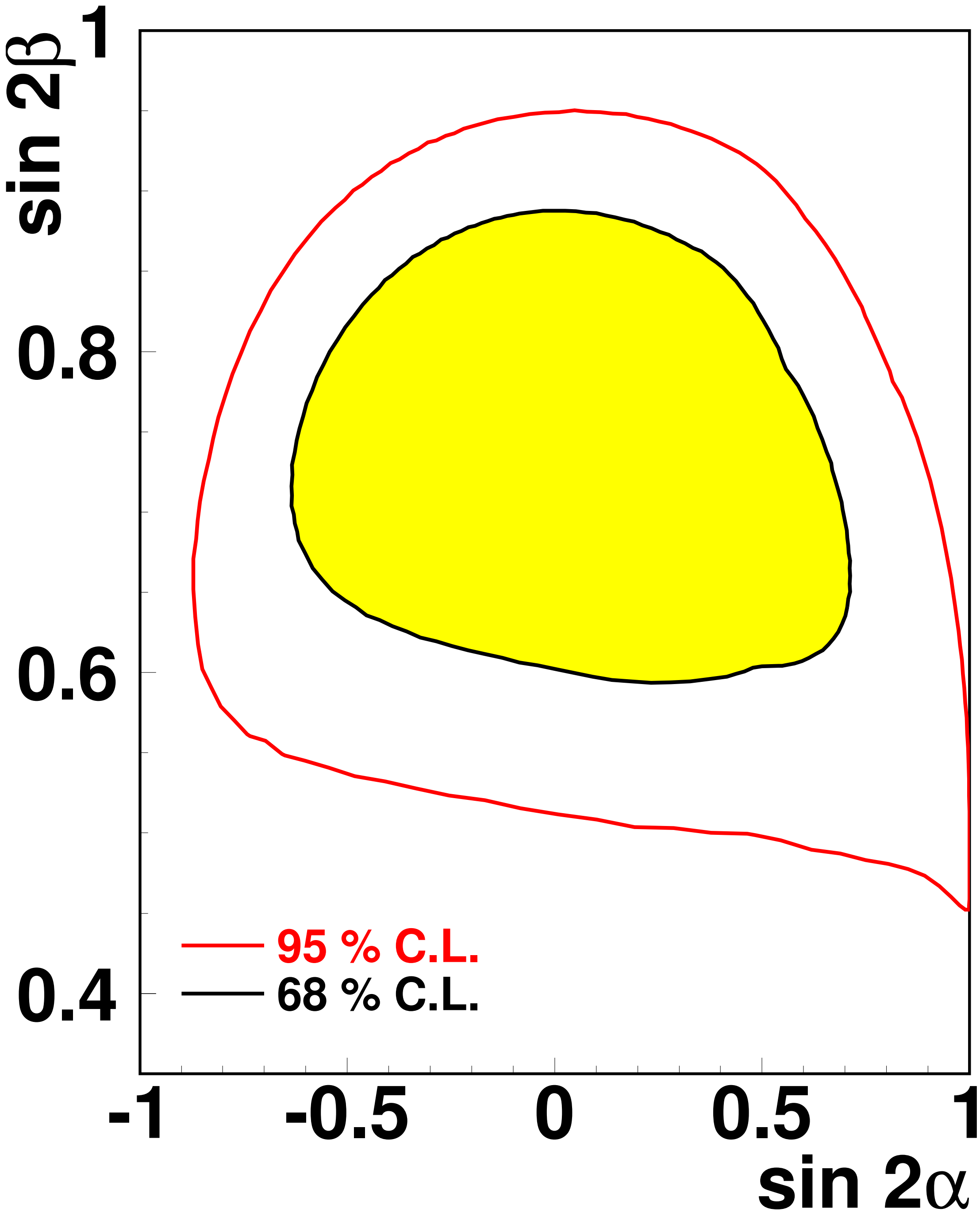
	Free fit	Original fit
ρ	$0.126^{-0.084}_{+0.145}$	$0.162^{-0.110}_{+0.112}$
η	$0.404^{-0.098}_{+0.069}$	$0.379^{-0.060}_{+0.063}$
B_K	$0.75^{-0.16}_{+0.34}$	0.87 ± 0.14

The Δm_s constraint has a big impact on ρ negative error.



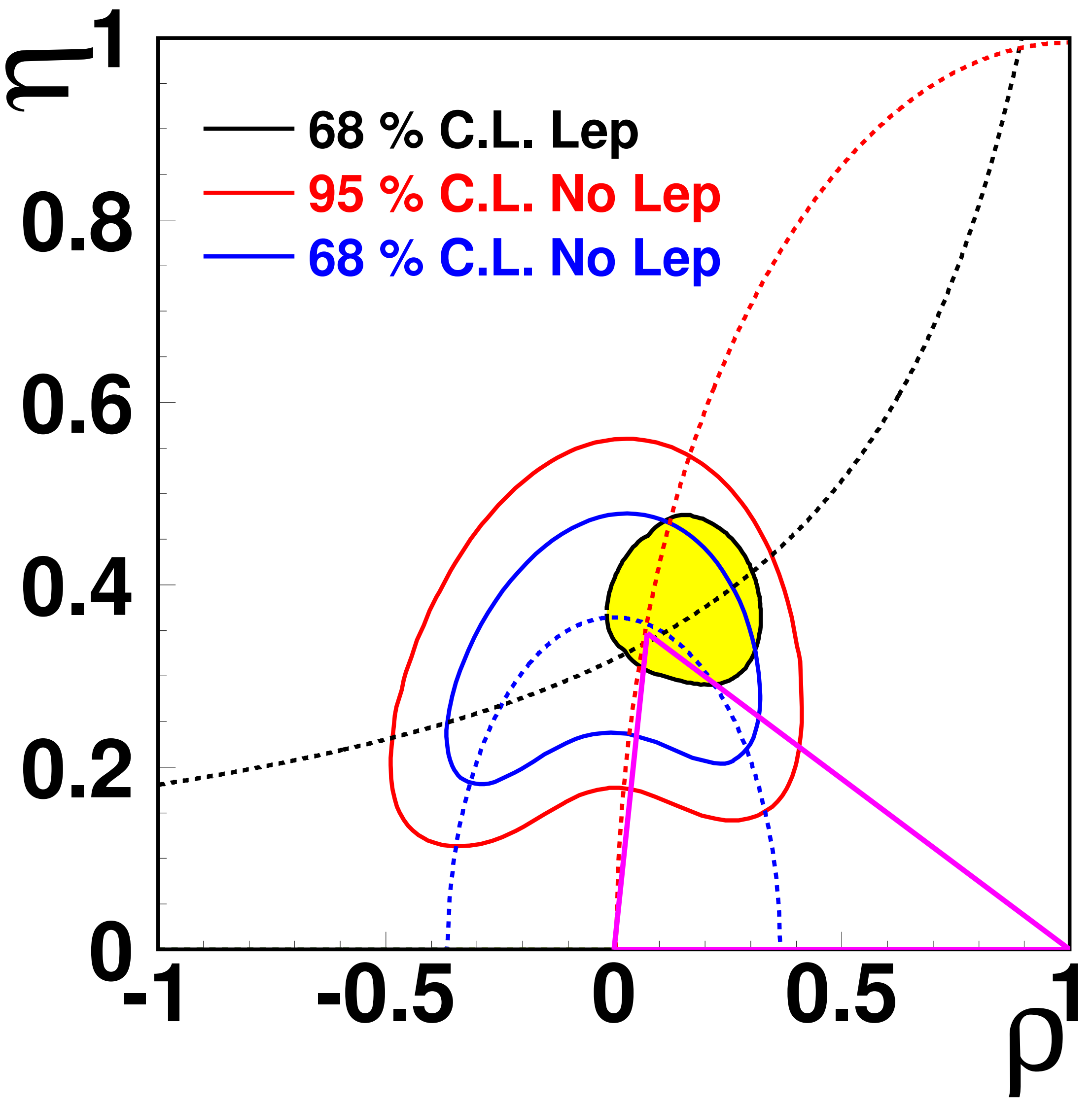
	Free fit	Original fit
ρ	$0.162^{-0.191}_{+0.112}$	$0.162^{-0.110}_{+0.112}$
η	$0.379^{-0.060}_{+0.064}$	$0.379^{-0.060}_{+0.063}$

$\mathrm{B_s}$ oscillations can still be in the LEP reach ($\Delta m_s < 13.0\,\mathrm{ps}^{-1}$)



$$\begin{aligned} \sin 2\alpha &= 0.04^{-0.48}_{+0.45} \\ \sin 2\beta &= 0.71^{-0.10}_{+0.10} \\ \gamma &= (67^{-14}_{+15})^\circ \end{aligned}$$

Removing all the LEP measurements:



	Lep excluded	Original fit
ρ	$0.073^{-0.340}_{+0.184}$	$0.162^{-0.110}_{+0.112}$
η	$0.346^{-0.098}_{+0.086}$	$0.379^{-0.060}_{+0.063}$
$\sin 2\alpha$	$0.29^{-1.02}_{+0.71}$	$0.04^{-0.48}_{+0.45}$
$\sin 2\beta$	$0.66^{-0.12}_{+0.27}$	$0.71^{-0.10}_{+0.10}$
γ	$(78^{-30}_{+57})^\circ$	$(67^{-14}_{+15})^\circ$

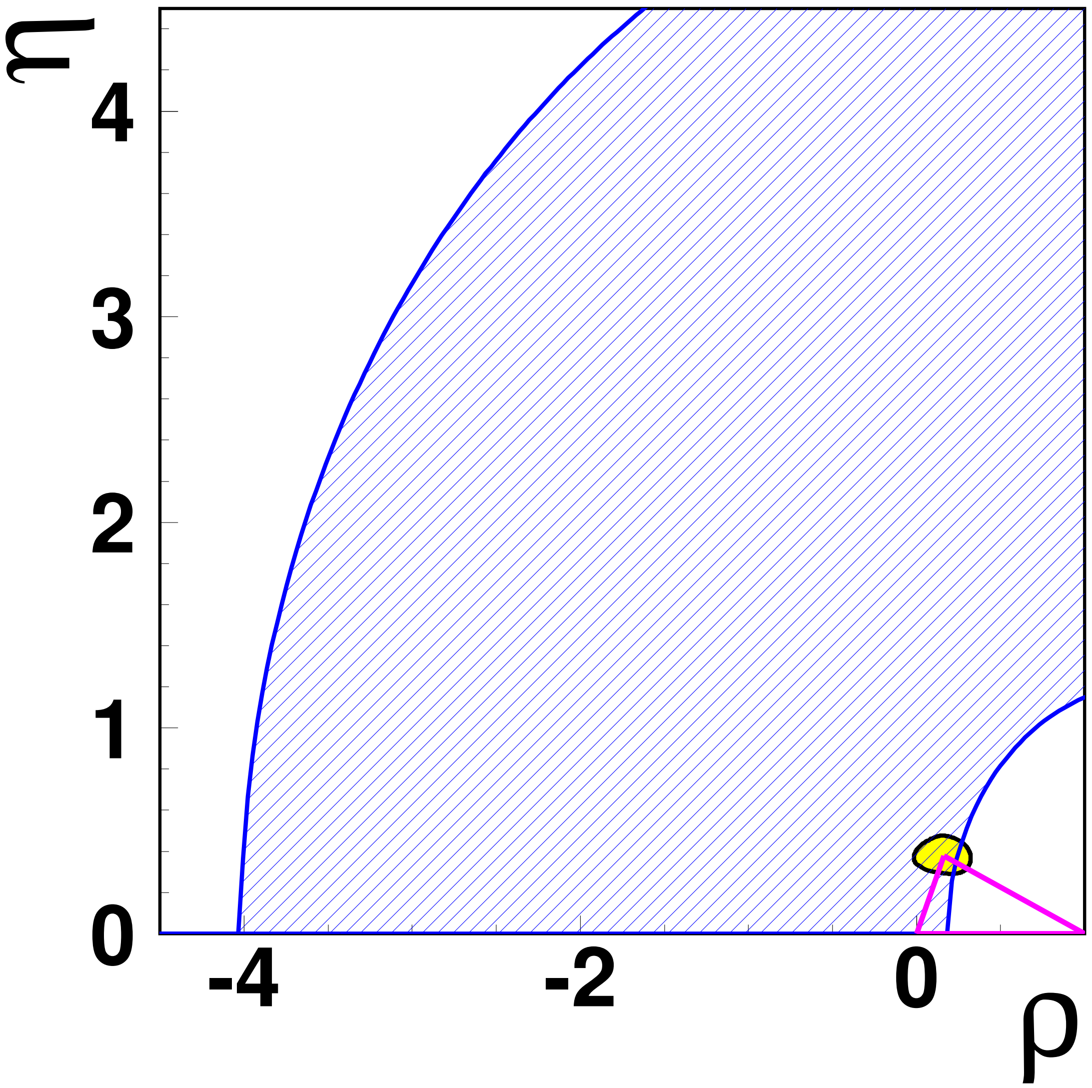
Near future prospects

Rare kaon decays can give further constraints:

$$\mathrm{Br}(\mathrm{K}^{\pm} \to \pi^{\pm}\nu\bar{\nu}) \approx 1.0 \times 10^{-10}\mathrm{A}^2\left[\eta^2 + (1.4-\rho)^2\right]$$

With one candidate event BNL-787 quotes:

$$\mathrm{Br}(\mathrm{K}^{\pm} \to \pi^{\pm}\nu\bar{\nu}) = (4.2^{+9.7}_{-3.5}) \times 10^{-10}$$



Results with much more data will be available soon

Conclusions

- Recent LEP measurements of Δm_d and limits on Δm_s help to constrain the CKM matrix
- New combined LEP measurement:
  $|\mathrm{V_{ub}}| = (4.5^{+0.6}_{-0.7}(\mathrm{stat.})^{+0.7}_{-0.8}(\mathrm{syst.}) \pm 0.17(\mathrm{theo.})) \times 10^{-3}$
- From a simultaneous fit one gets:

$$\begin{aligned} \rho &= 0.162^{-0.110}_{+0.112} \\ \eta &= 0.379^{-0.060}_{+0.063} \end{aligned}$$

- Together with indications on the theory parameters:

$$\begin{aligned} f_{B_d}\sqrt{B_{B_d}} &= 0.222^{-0.027}_{+0.030}\,\mathrm{GeV} \\ B_K &= 0.75^{-0.16}_{+0.34} \end{aligned}$$

- Indirect measurement of the angles of the unitarity triangle:

$$\begin{aligned} \sin 2\alpha &= 0.04^{-0.48}_{+0.45} \\ \sin 2\beta &= 0.71^{-0.10}_{+0.10} \\ \gamma &= (67^{-14}_{+15})^\circ \end{aligned}$$

- The precision on $\sin 2\beta$ is comparable with the expected one from direct measurement at future B factories
- More input to the fits in the near future from rare kaon decays