Lattice Investigations of Nucleon Structure at Light Quark Masses

James Zanotti
University of Edinburgh
[QCDSF Collaboration]

Tony's 60th B'day Party, Adelaide, February 15-19, 2010

Topics to Cover

- Form Factors
  - Provide information on size, shape and internal (charge) densities
  - eg. Neutron has charge zero, but charge density? +/-?
  - Good place to search for chiral non-analytic behaviour
- Nucleon Axial Charge, g_A
  - Neutron beta decay, chiral symmetry breaking
  - Study finite size effects
- Transversity (Tensor charge, g_T)
  - Not yet measured experimentally (lattice prediction)
- $\langle x \rangle_q, \langle x \rangle_{\Delta q}$:Look for curvature at light quark masses

Challenges from Tony for the Lattice

- Magnetic moments

  PRD60:034014 (1999)

  PRD71:014001 (2005)

- Radii

  PRD79:094001 (2009)

- g_A

  PRD66:054501 (2002)

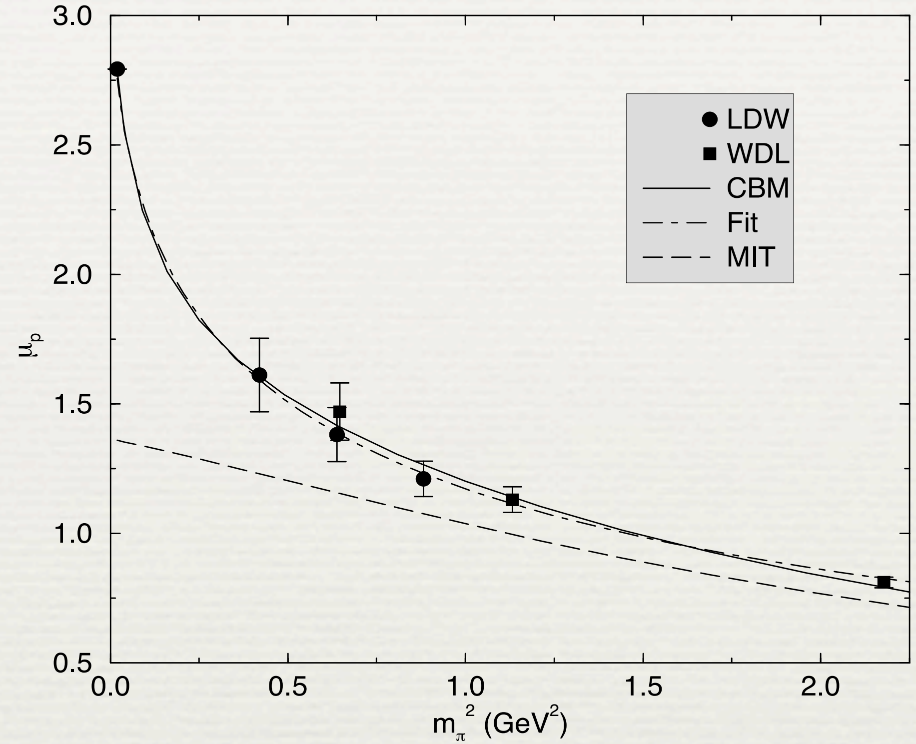
  hep-lat/0502002

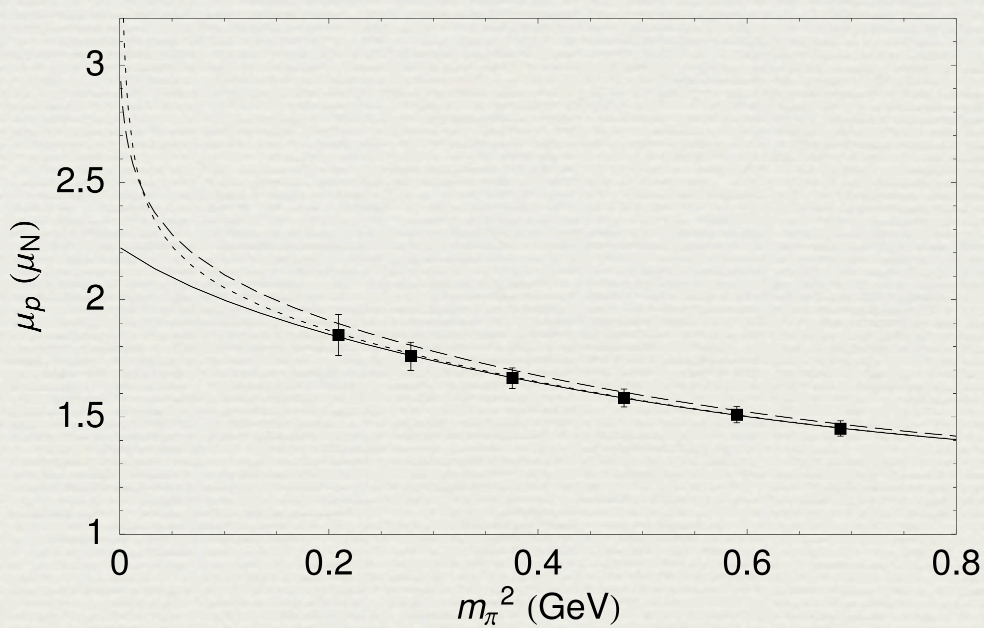
- $\langle x \rangle_q, \ \langle x \rangle_{\Delta q}$

  PRD66:054501 (2002)

Challenges from Tony for the Lattice

- Magnetic moments
  - PRD60:034014 (1999)
  - PRD71:014001 (2005)
- Radii
  - PRD79:094001 (2009)
- g_A
  - PRD66:054501 (2002)
  - hep-lat/0502002
- $\langle x \rangle_q, \ \langle x \rangle_{\Delta q}$
  - PRD66:054501 (2002)





Challenges from Tony for the Lattice

- Magnetic moments

  PRD60:034014 (1999)

  PRD71:014001 (2005)

- Radii

  PRD79:094001 (2009)

- g_A

  PRD66:054501 (2002)

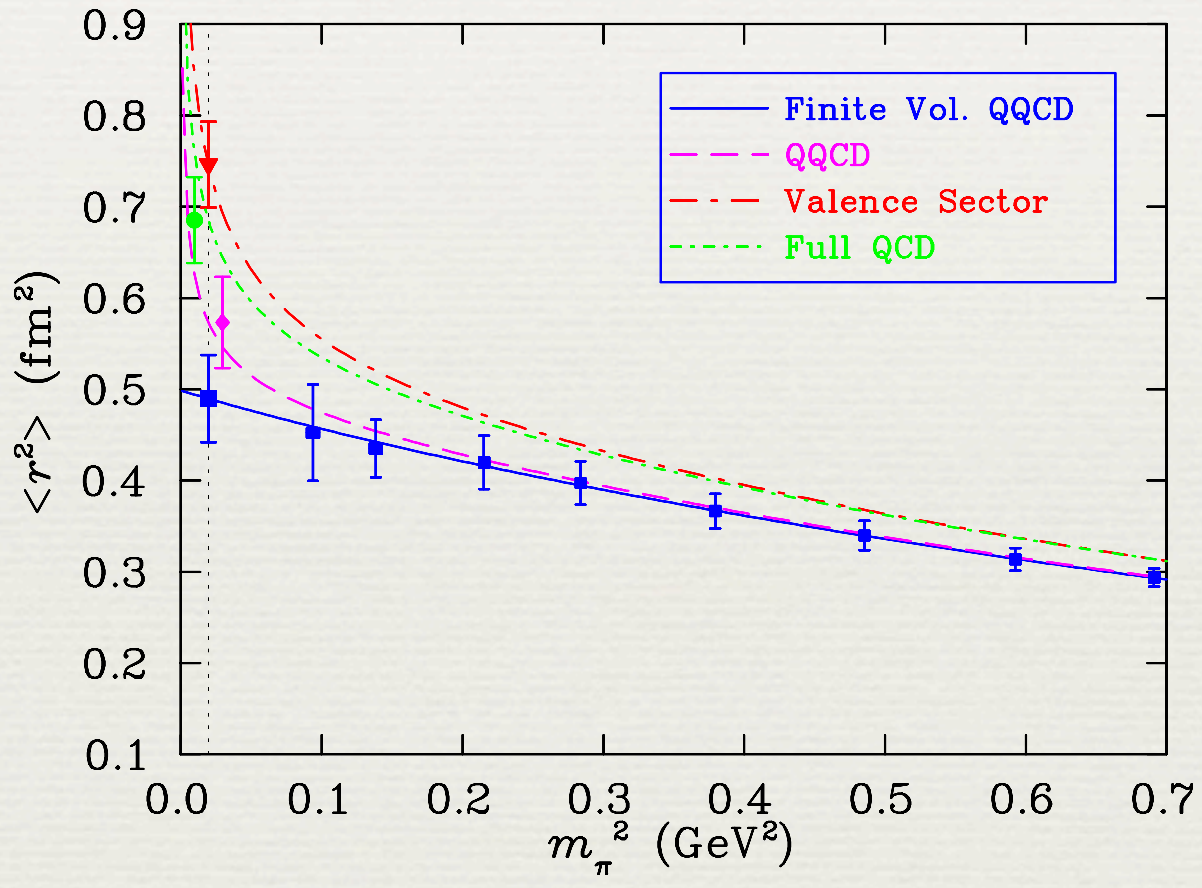
  hep-lat/0502002

- $\langle x \rangle_q, \ \langle x \rangle_{\Delta q}$

  PRD66:054501 (2002)

Challenges from Tony for the Lattice

- Magnetic moments
  - PRD60:034014 (1999)
  - PRD71:014001 (2005)
- Radii
  - PRD79:094001 (2009)
- g_A
  - PRD66:054501 (2002)
  - hep-lat/0502002
- $\langle x \rangle_q, \ \langle x \rangle_{\Delta q}$
  - PRD66:054501 (2002)



Challenges from Tony for the Lattice

- Magnetic moments

  PRD60:034014 (1999)

  PRD71:014001 (2005)

- Radii

  PRD79:094001 (2009)

- g_A

  PRD66:054501 (2002)

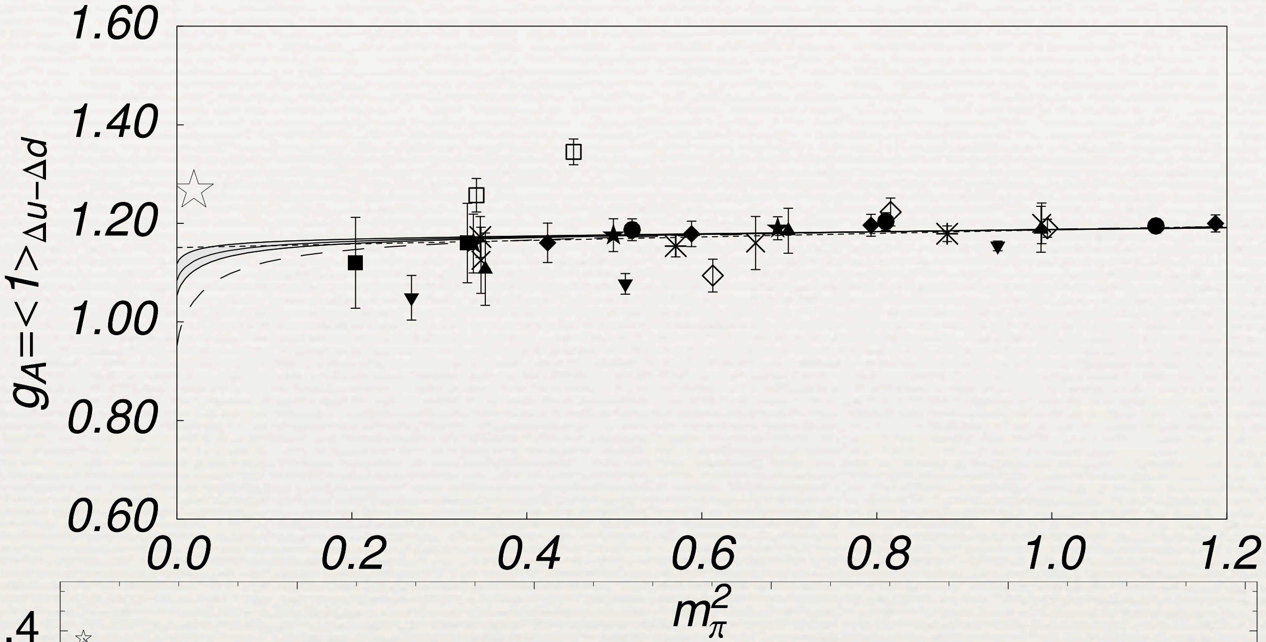
  hep-lat/0502002

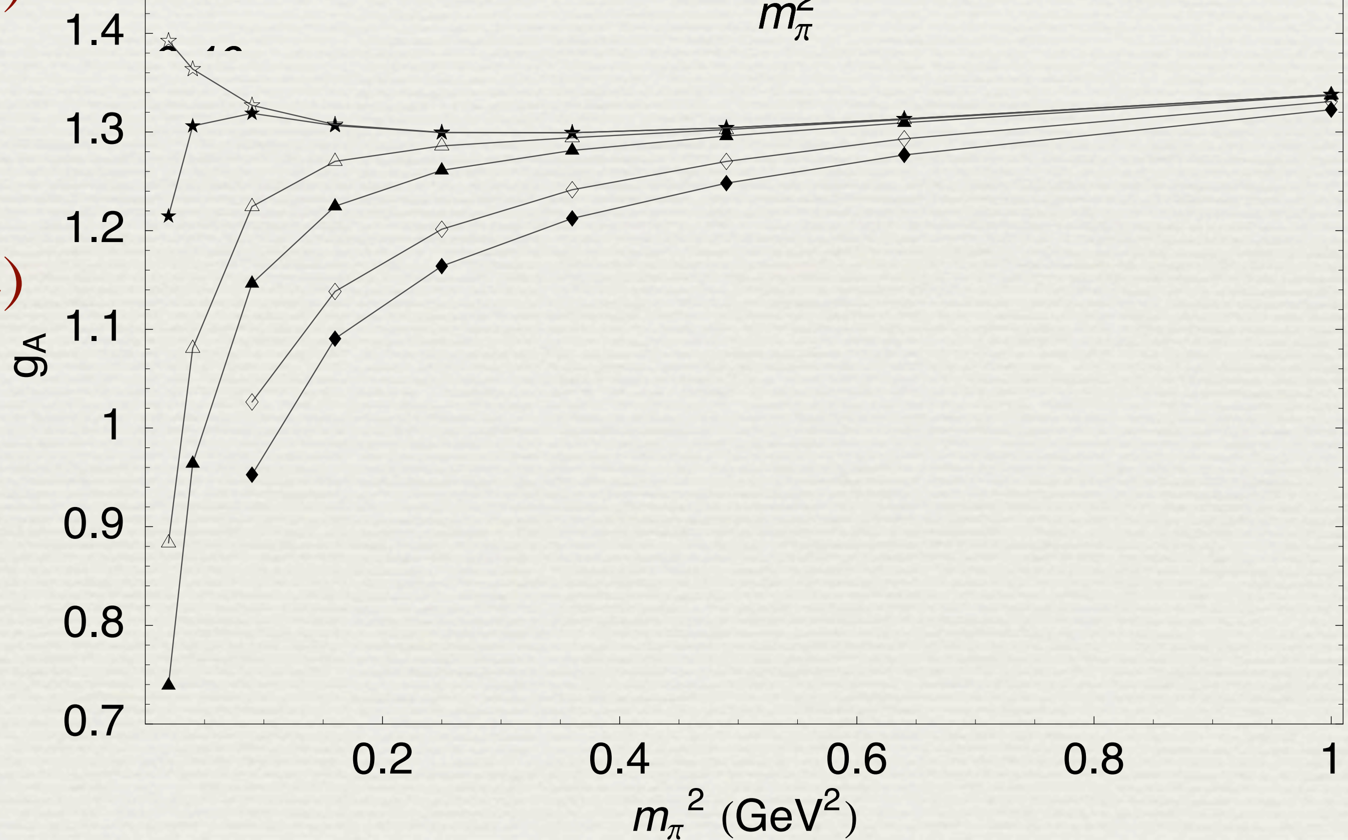
- $\langle x \rangle_q, \ \langle x \rangle_{\Delta q}$

  PRD66:054501 (2002)

Challenges from Tony for the Lattice

- Magnetic moments
  - PRD60:034014 (1999)
  - PRD71:014001 (2005)
- Radii
  - PRD79:094001 (2009)
- g_A
  - PRD66:054501 (2002)
  - hep-lat/0502002
- $\langle x \rangle_q, \ \langle x \rangle_{\Delta q}$
  - PRD66:054501 (2002)





Challenges from Tony for the Lattice

- Magnetic moments

  PRD60:034014 (1999)

  PRD71:014001 (2005)

- Radii

  PRD79:094001 (2009)

- g_A

  PRD66:054501 (2002)

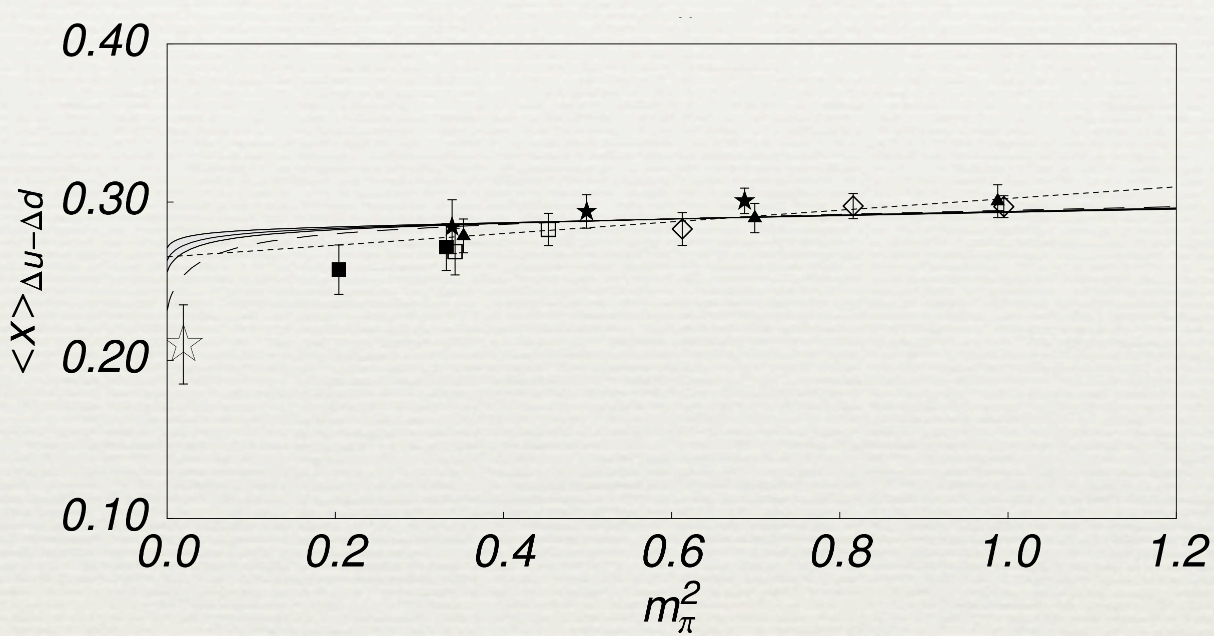
  hep-lat/0502002

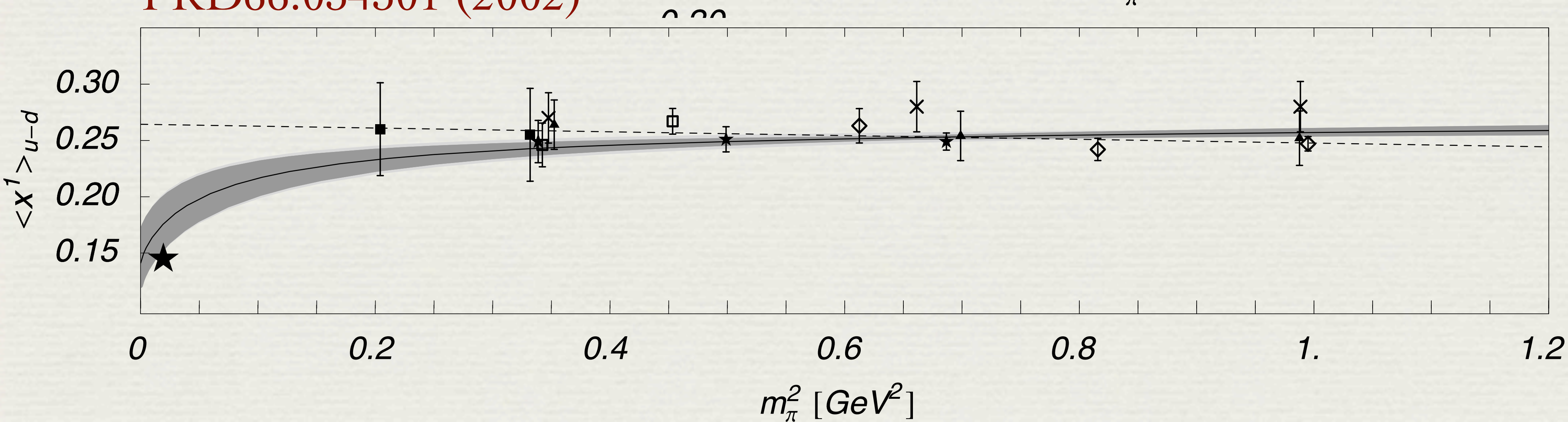
- $\langle x \rangle_q, \ \langle x \rangle_{\Delta q}$

  PRD66:054501 (2002)

Challenges from Tony for the Lattice

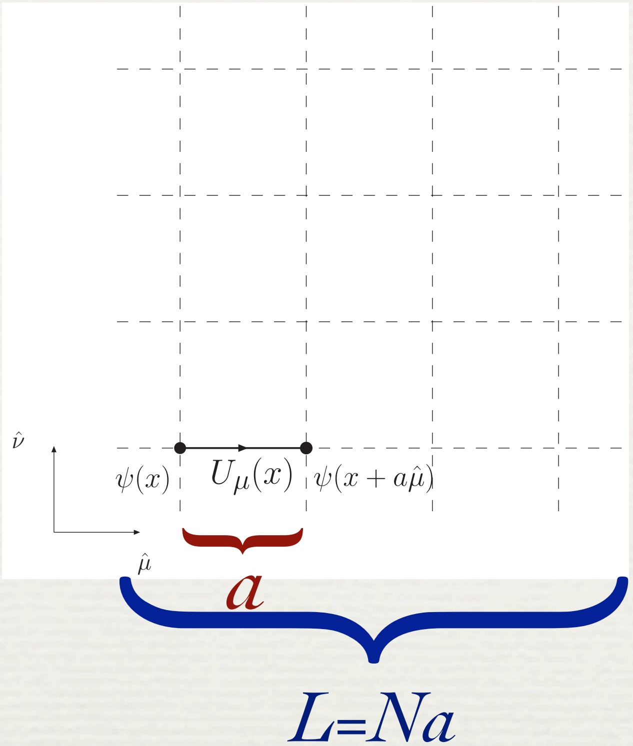
- Magnetic moments
  - PRD60:034014 (1999)
  - PRD71:014001 (2005)
- Radii
  - PRD79:094001 (2009)
- g_A
  - PRD66:054501 (2002)
  - hep-lat/0502002
- $\langle x \rangle_q, \ \langle x \rangle_{\Delta q}$
  - PRD66:054501 (2002)





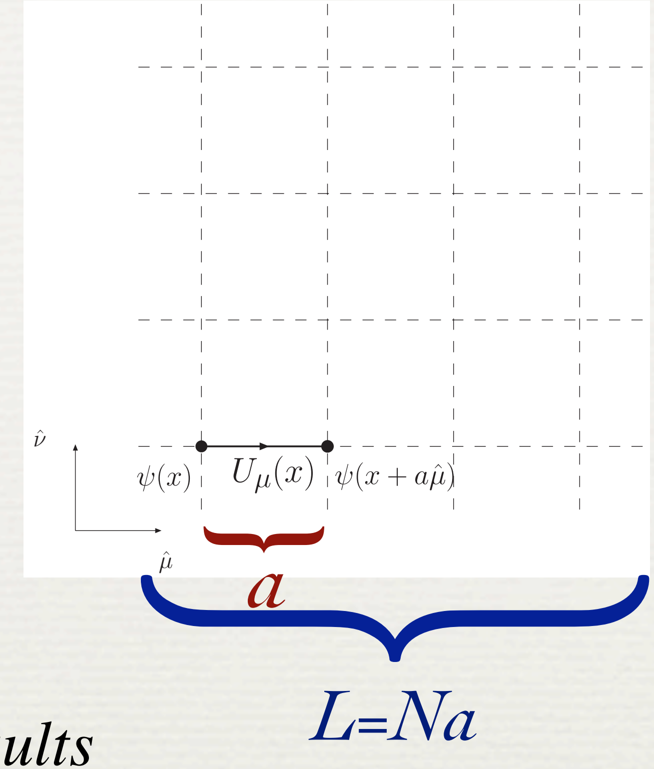
The Lattice

- Discretise space-time with lattice spacing ***a*** volume $L^3 \times T$
- Quark fields reside on sites
- Gauge fields on the links



The Lattice

- Discretise space-time with lattice spacing ***a*** volume $L^3 \times T$
- Quark fields reside on sites
- Gauge fields on the links



- *Lattice simulations for QCD give first principle results*
- *but need to have control of [ideally in this order]:* — *'Goal'*
  - *Statistical errors,* $N_{conf} \sim O(1000)$ — $N_{conf} \to \infty$
  - *Volume:* $L \sim 1.5\,\text{fm} \to 3\,\text{fm}$ — $L \to \infty$
  - *Continuum limit:* $a \sim 0.1\,\text{fm} \to 0.04\,\text{fm}$ — $a \to 0$
  - *Chiral extrapolation:* $m_\pi \sim 500\,\text{MeV} \to 200\,\text{MeV}$ — $m_\pi \to 140\,\text{MeV}$
- *difficult, need Tflop++ machines to approach the theoretical goal*

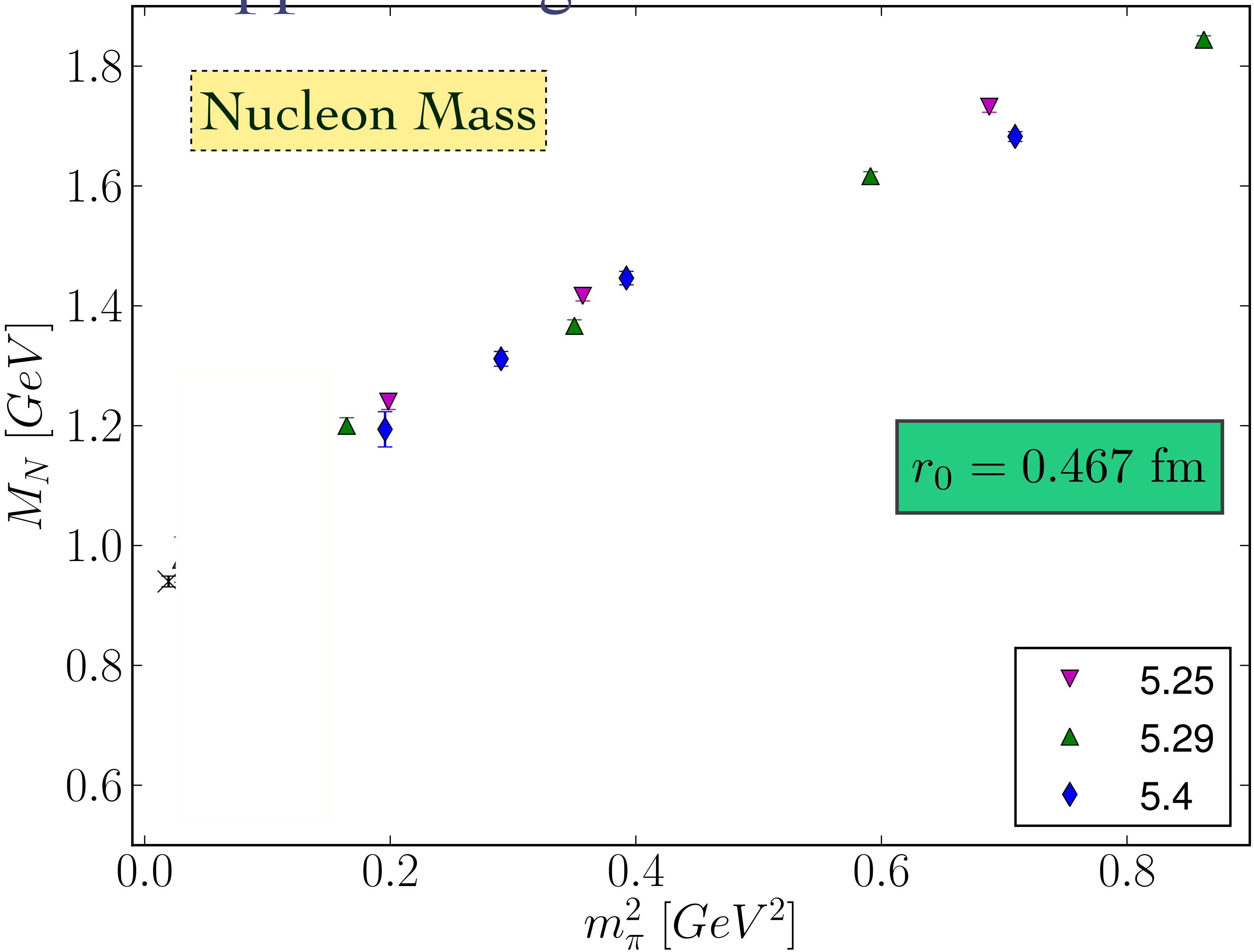
Lattice Techniques - QCDSF

- *O(a)-improved Wilson (Clover) fermions*
- *Wilson gauge action*
- $N_f = 2$ *dynamical configurations*
- *4* β *values* ➡ $0.07\,\mathrm{fm} < a < 0.12\,\mathrm{fm}$

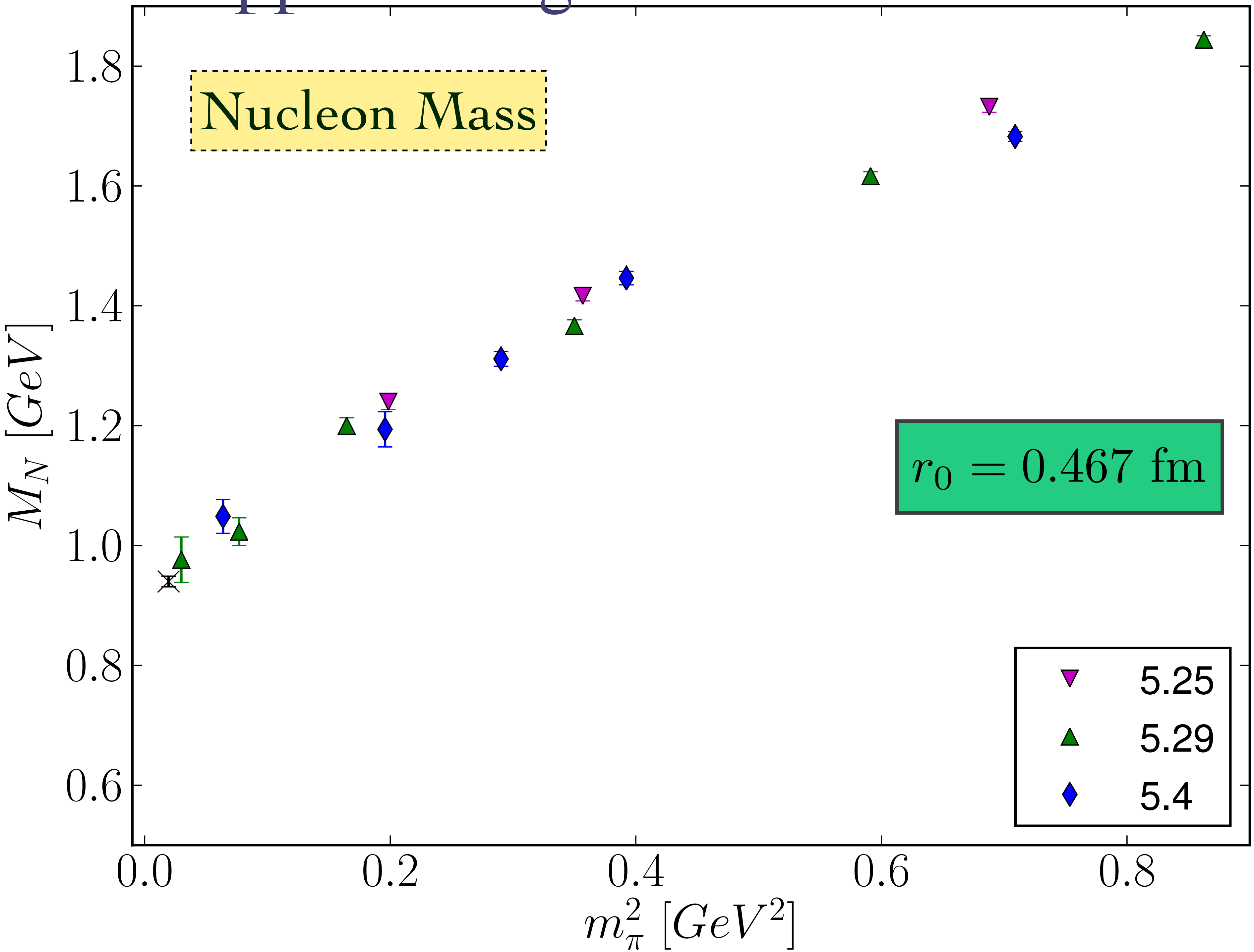
$$150\ \mathrm{MeV} < m_\pi < 1.2\ \mathrm{GeV}$$

$$1.1\ \mathrm{fm} < L < 3.2\ \mathrm{fm}$$

Approaching The Chiral Limit



Approaching The Chiral Limit



Nucleon Form Factor

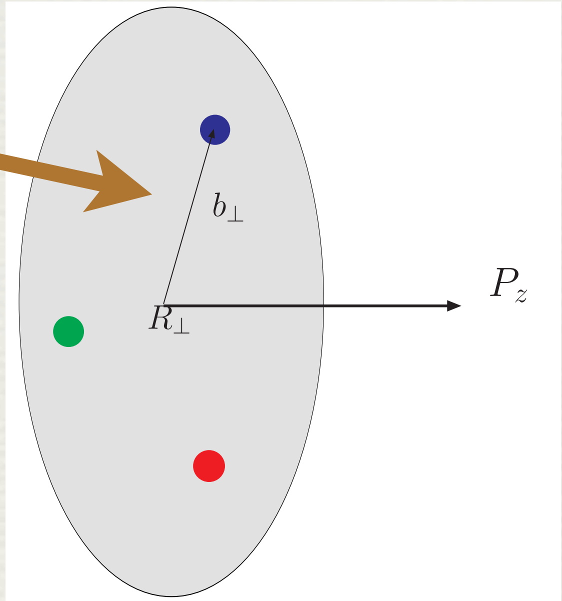
Electromagnetic Form Factors

$$\langle p',\, s'|J^\mu(\vec{q})|p,\, s\rangle \;= \bar{u}(p',\, s')\left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_2(q^2)\right]u(p,\, s)$$

Quark (charge) distribution in transverse plane

$$q(b_\perp^2) = \int d^2q_\perp\, \mathrm{e}^{-i\vec{b}_\perp\cdot q_\perp} F_1(q^2)$$

Distance of (active) quark to the centre of momentum in a fast moving nucleon



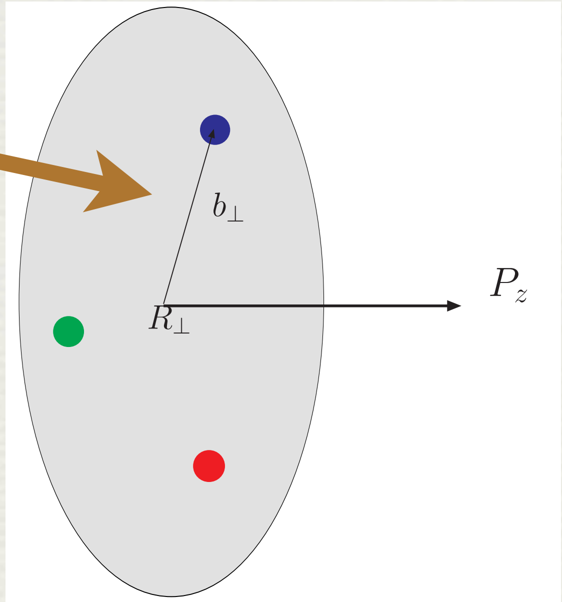
Electromagnetic Form Factors

$$\langle p',\, s'|J^{\mu}(\vec{q})|p,\, s\rangle \;= \bar{u}(p',\, s')\left[\gamma^{\mu}F_1(q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_2(q^2)\right]u(p,\, s)$$

Quark (charge) distribution in transverse plane

$$q(b_\perp^2) = \int d^2q_\perp\, \mathrm{e}^{-i\vec{b}_\perp\cdot q_\perp} F_1(q^2)$$

Distance of (active) quark to the centre of momentum in a fast moving nucleon



Provide information on the size and internal charge densities

Scaling of Form Factors

From dimensional counting *[Brodsky & Farrar, 1973]*

$$\frac{F(0)}{(1+Q^2/M^2)^p}$$

$$F_1 \propto \frac{1}{Q^4} \quad \text{(dipole?)}$$

$$F_2 \propto \frac{1}{Q^6} \quad \text{(tripole?)}$$

for $Q^2 > \zeta_{\mathrm{pQCD}}$

$$Q^2 \frac{F_2}{F_1} \propto \mathrm{const}$$

$$\frac{G_E}{G_M} \propto \mathrm{const}$$

Scaling of Form Factors

From dimensional counting *[Brodsky & Farrar, 1973]*

$$F_1 \propto \frac{1}{Q^4} \qquad \text{(dipole?)}$$

$$\frac{F(0)}{(1+Q^2/M^2)^p}$$

$$F_2 \propto \frac{1}{Q^6} \qquad \text{(tripole?)}$$

for $Q^2 > \zeta_{\mathrm{pQCD}}$

$$Q^2 \frac{F_2}{F_1} \propto \text{const} \qquad \frac{G_E}{G_M} \propto \text{const}$$

JLab

$$Q \frac{F_2}{F_1} \propto \qquad \frac{G_E}{G_M} \propto$$

Scaling of Form Factors

From dimensional counting *[Brodsky & Farrar, 1973]*

$$\frac{F(0)}{(1+Q^2/M^2)^p}$$

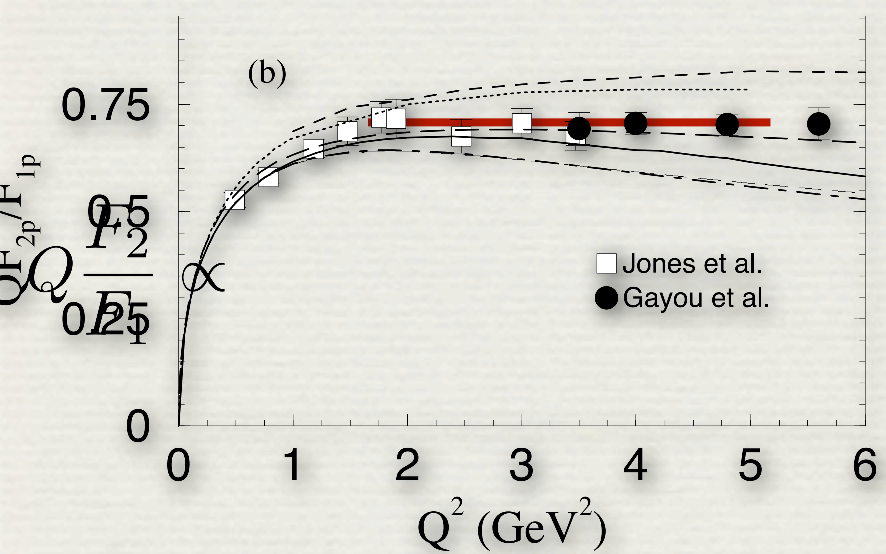
$$F_1 \propto \frac{1}{Q^4} \quad \text{(dipole?)}$$

$$F_2 \propto \frac{1}{Q^6} \quad \text{(tripole?)}$$

for $Q^2 > \zeta_{\mathrm{pQCD}}$

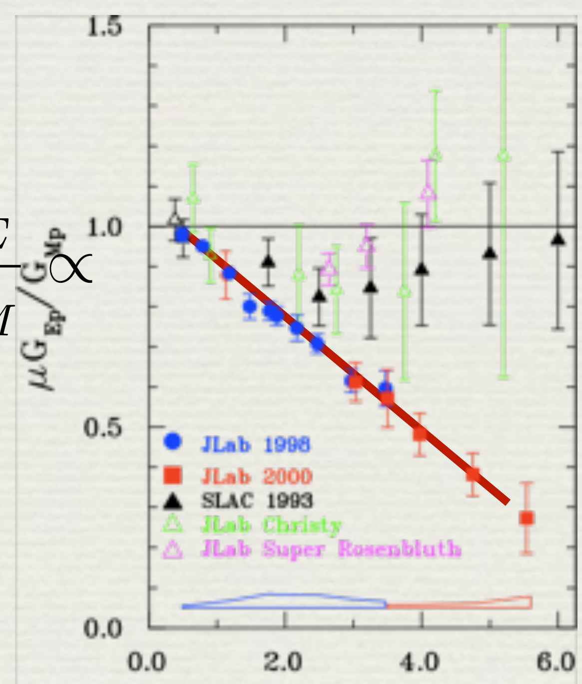
$$Q^2 \frac{F_2}{F_1} \propto \text{const}$$

$$\frac{G_E}{G_M} \propto \text{const}$$



JLab

$$\frac{G_E}{G_M} \propto$$



Form Factor Radii & Magnetic Moments

Search for non-analytic behaviour predicted by Chiral Perturbation Theory

- *Form factor radii:*

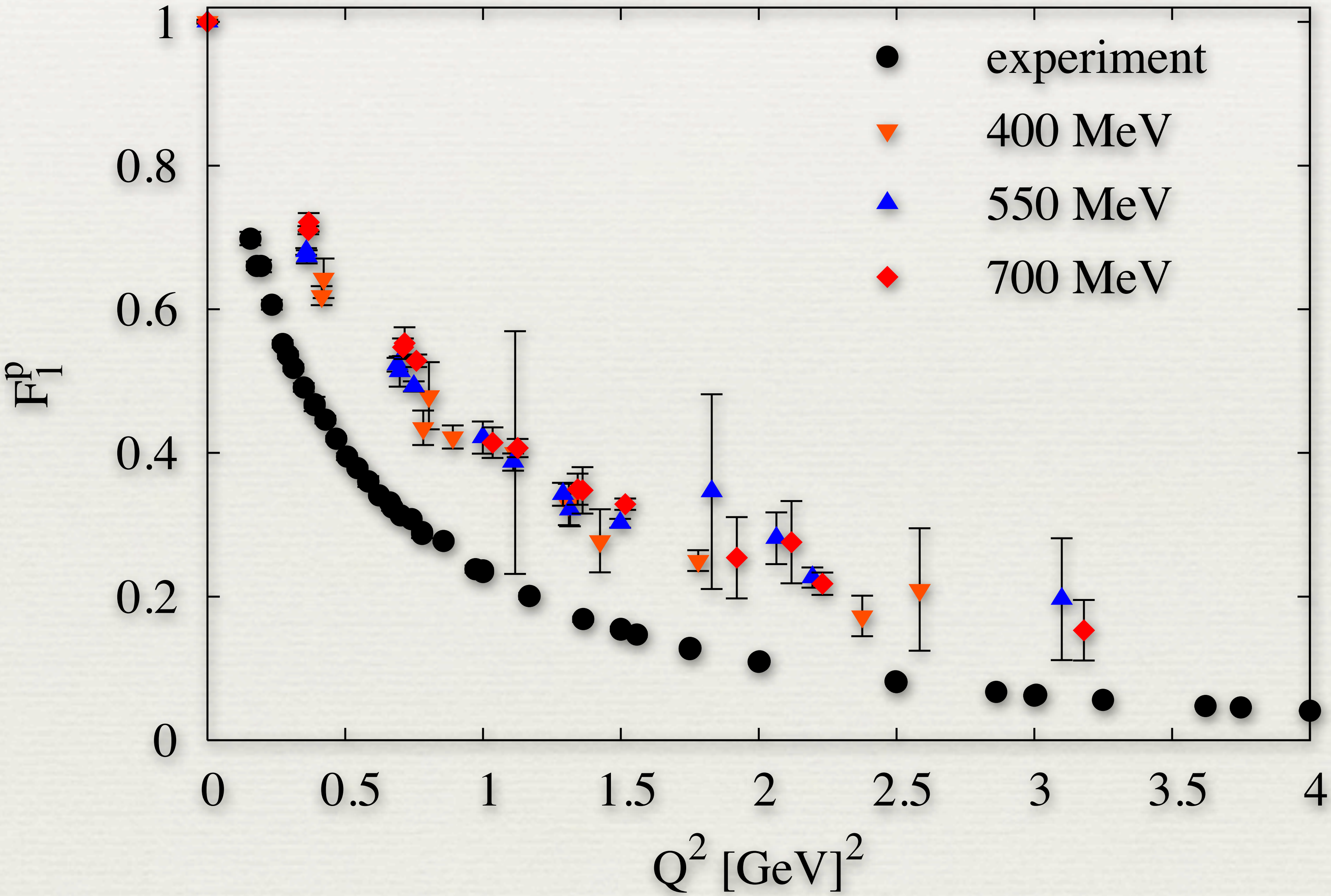
$$r_i^2 = -6 \frac{dF_i(q^2)}{dq^2}\Big|_{q^2=0}$$

- *Magnetic moment* μ */anomalous magnetic moment* κ

$$\mu = 1 + \kappa = G_m(0)$$

Form Factors: $F_1^{(p)}$

Comparison with experiment

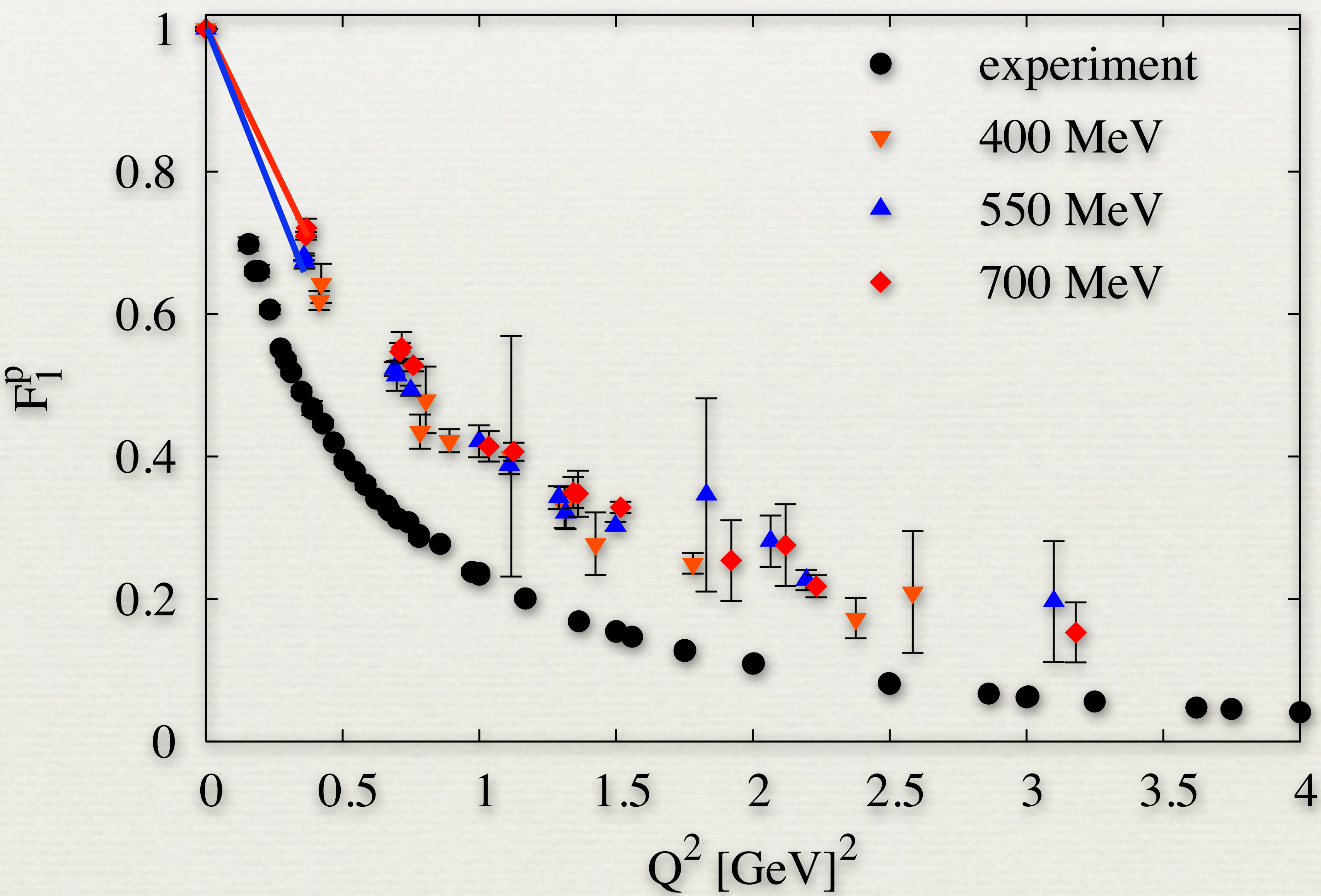


Form Factors: $F_1^{(p)}$

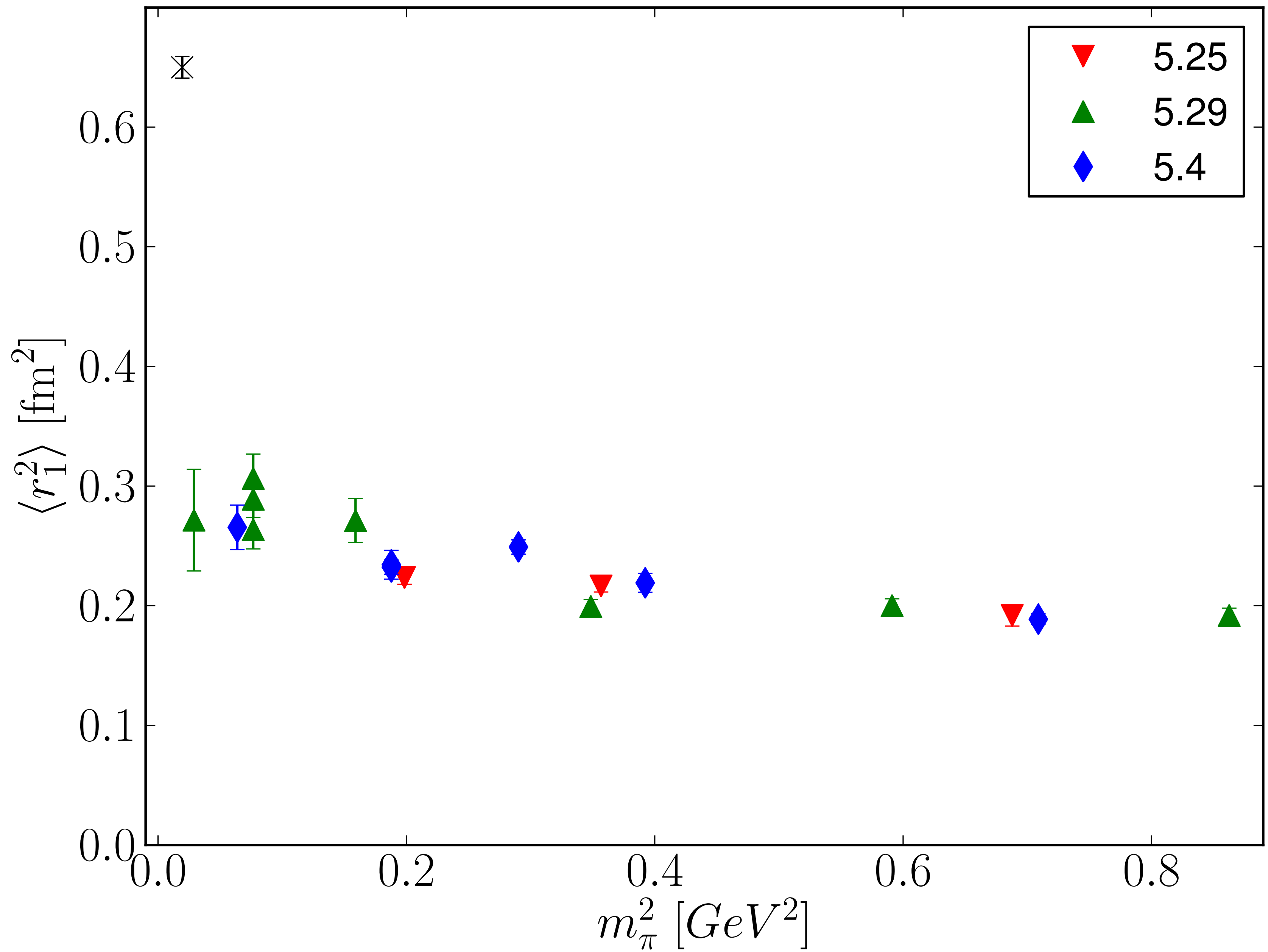
Comparison with experiment

charge radius

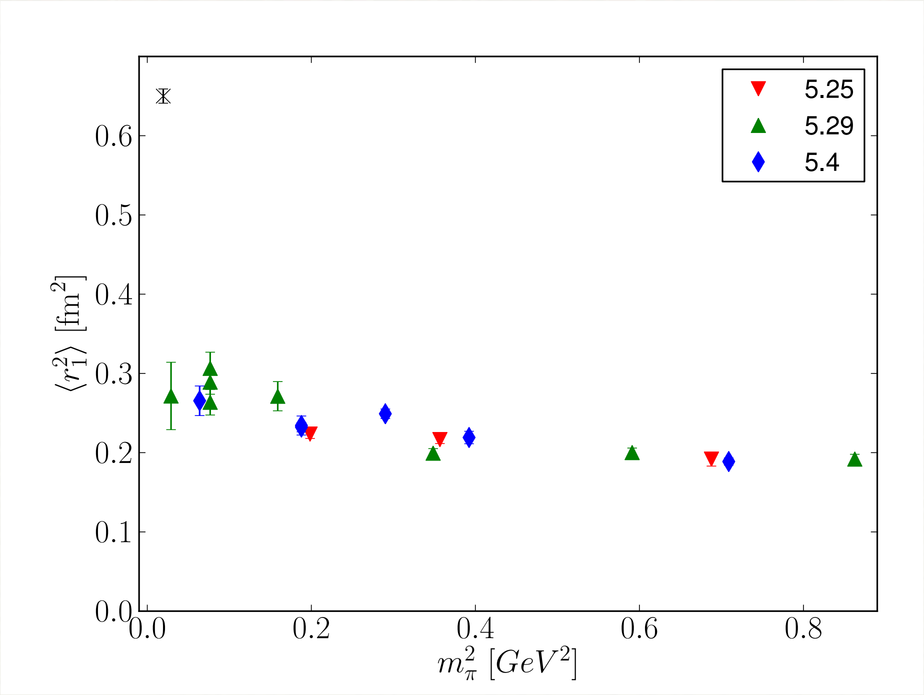
$$r_i^2 = -6 \frac{dF_i(q^2)}{dq^2}\bigg|_{q^2=0}$$



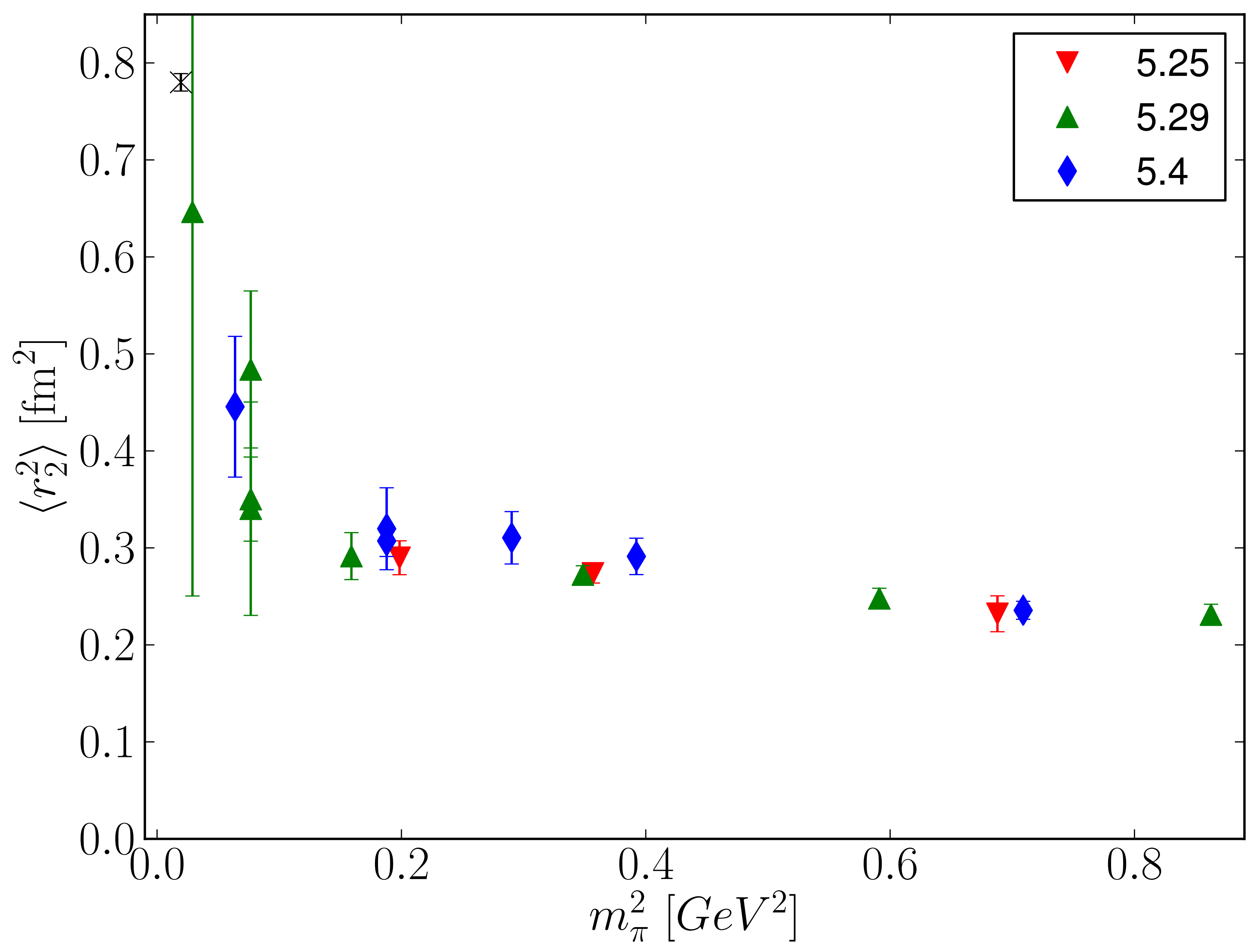
Form Factor Radii



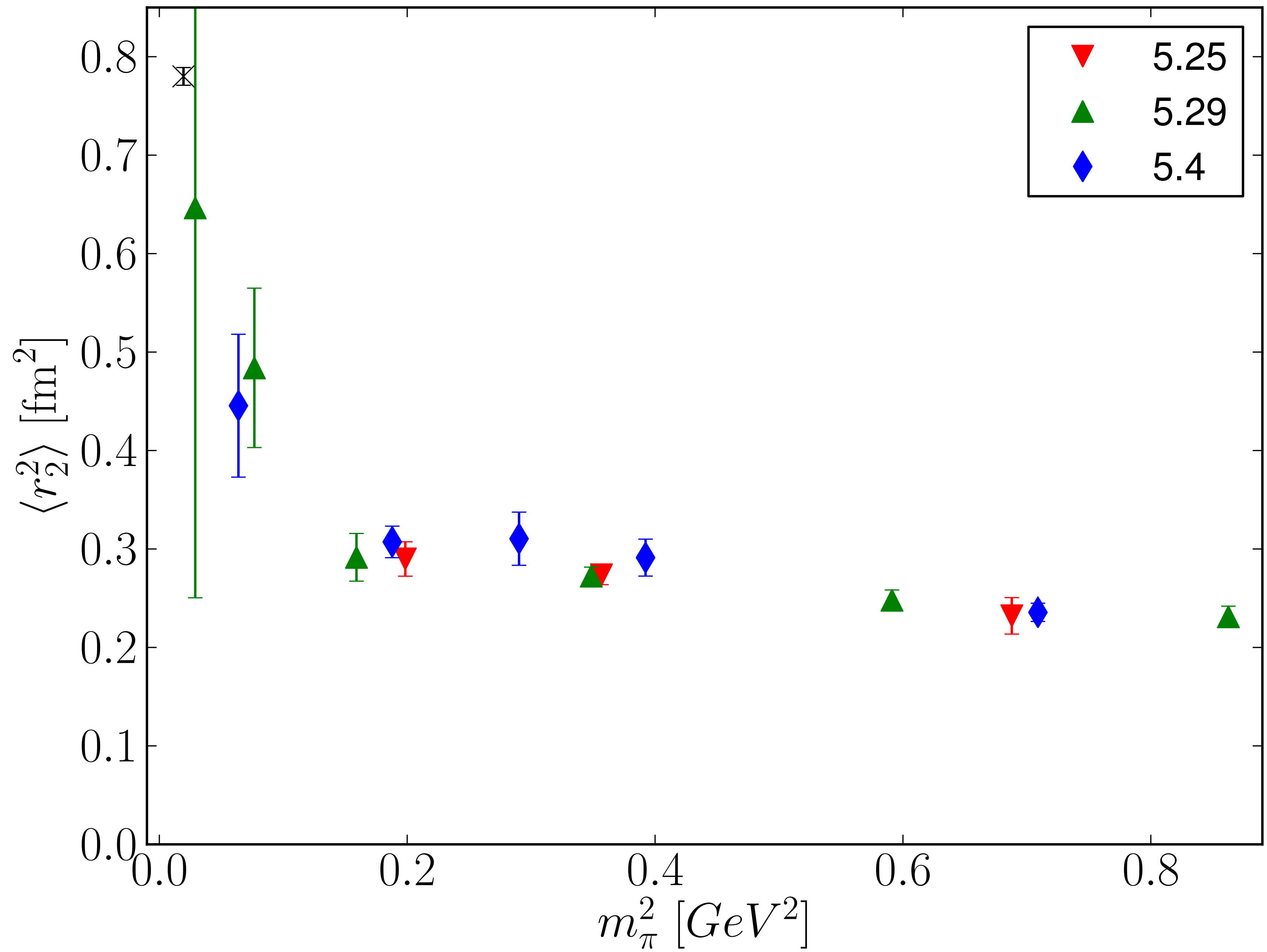
Form Factor Radii



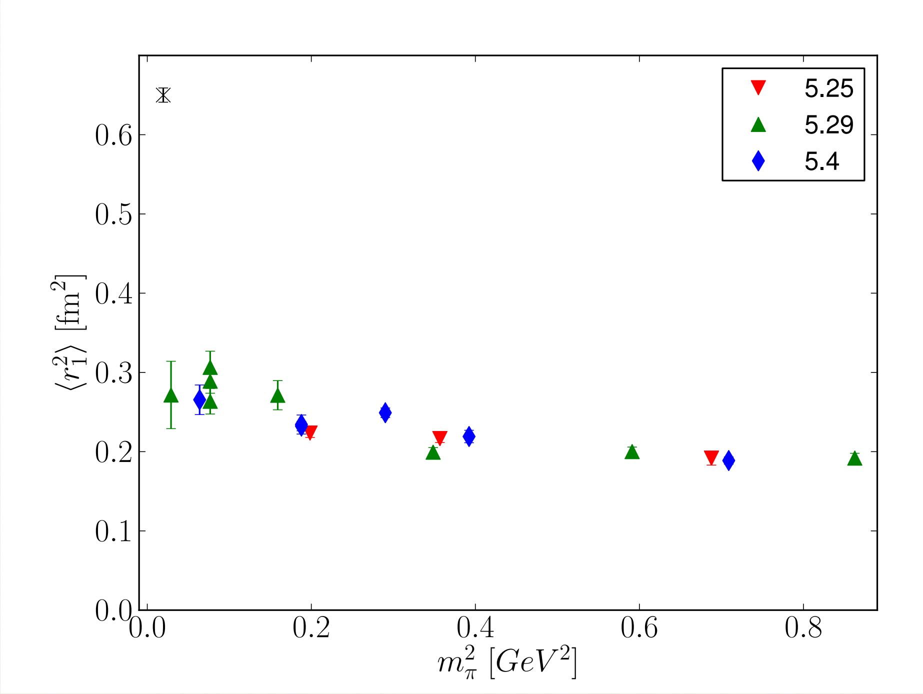
Form Factor Radii

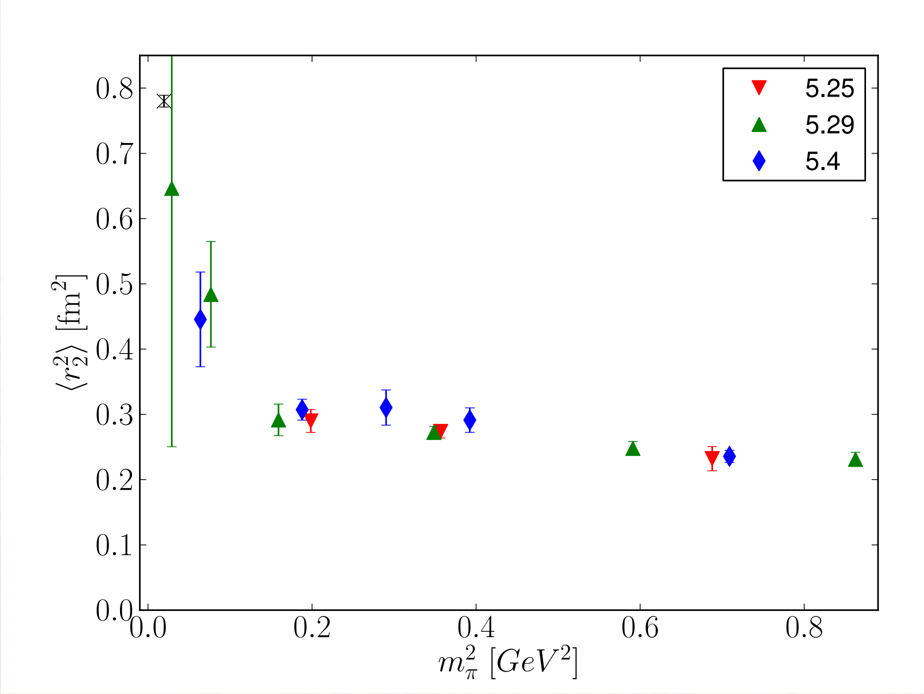


Form Factor Radii



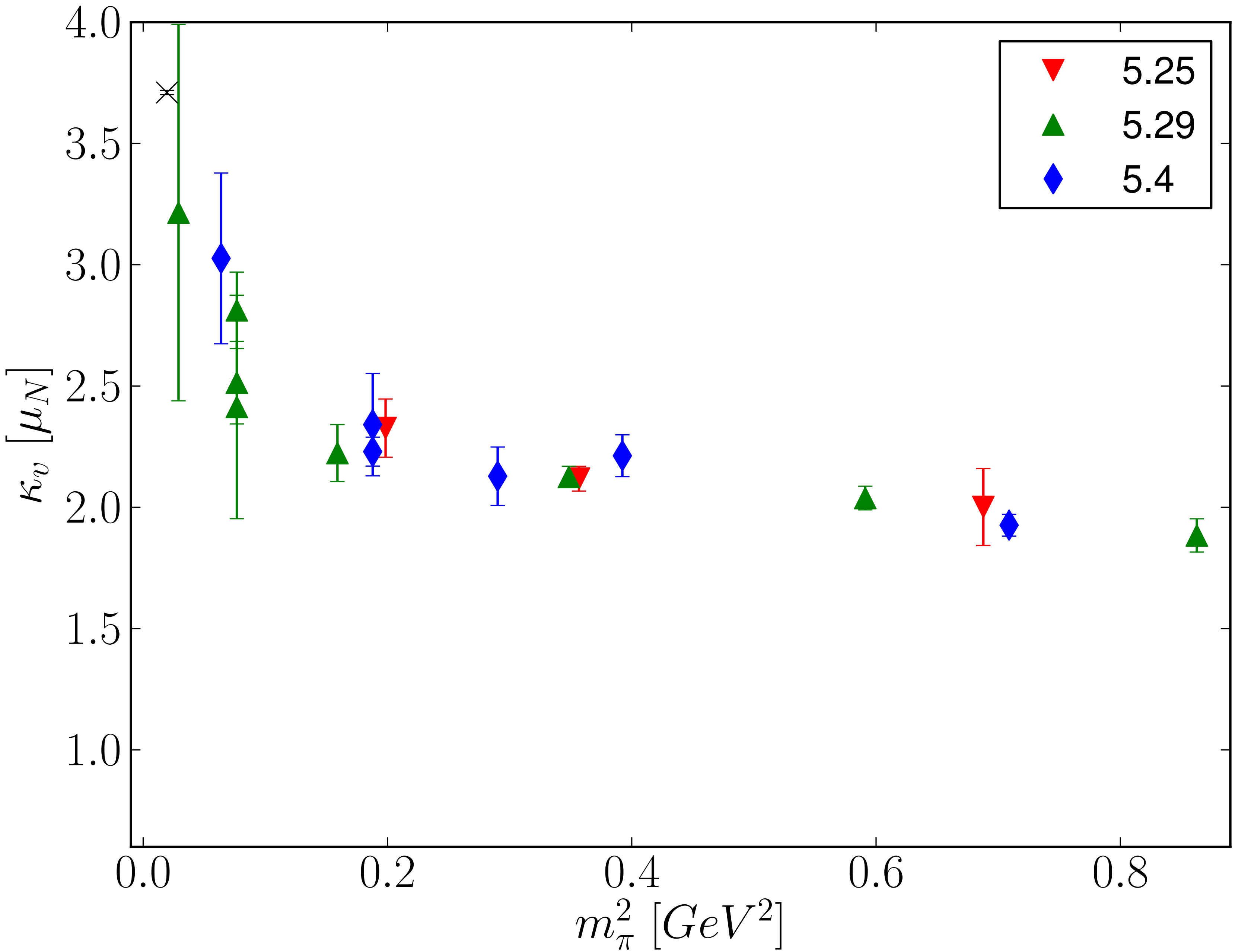
Form Factor Radii



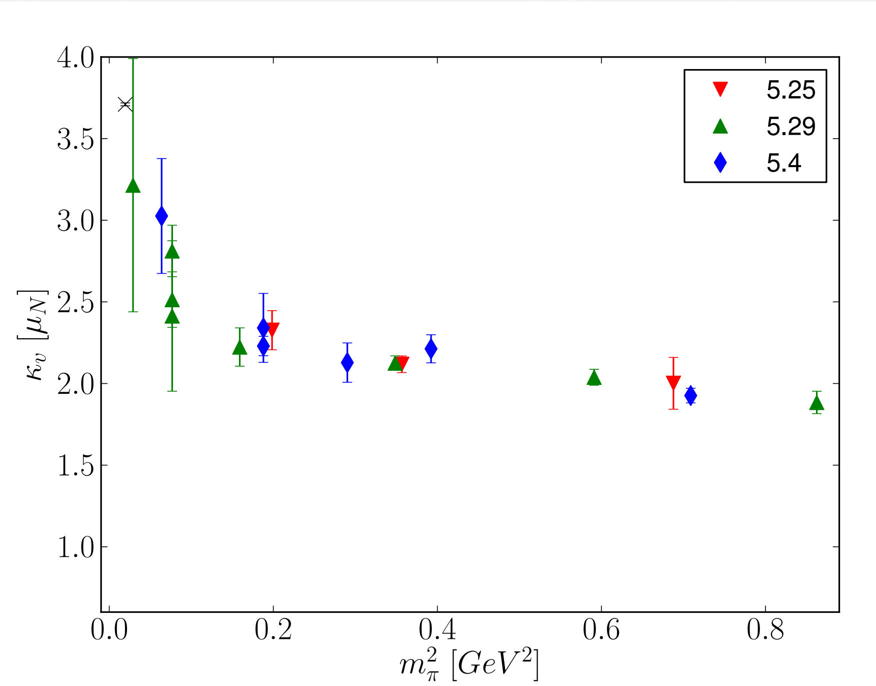


- *Chiral perturbation theory: Dramatic non-analytic predicted*
  Need $m_\pi < 300$ MeV
- *Evidence for divergence in r_2 but not r_1* $r_2 \propto \frac{1}{m_\pi}$
- *Radius measures slope at Q^2=0, but smallest Q^2>0.25GeV^2*
  - *Twisted boundary conditions*
- *Finite volume effects?*

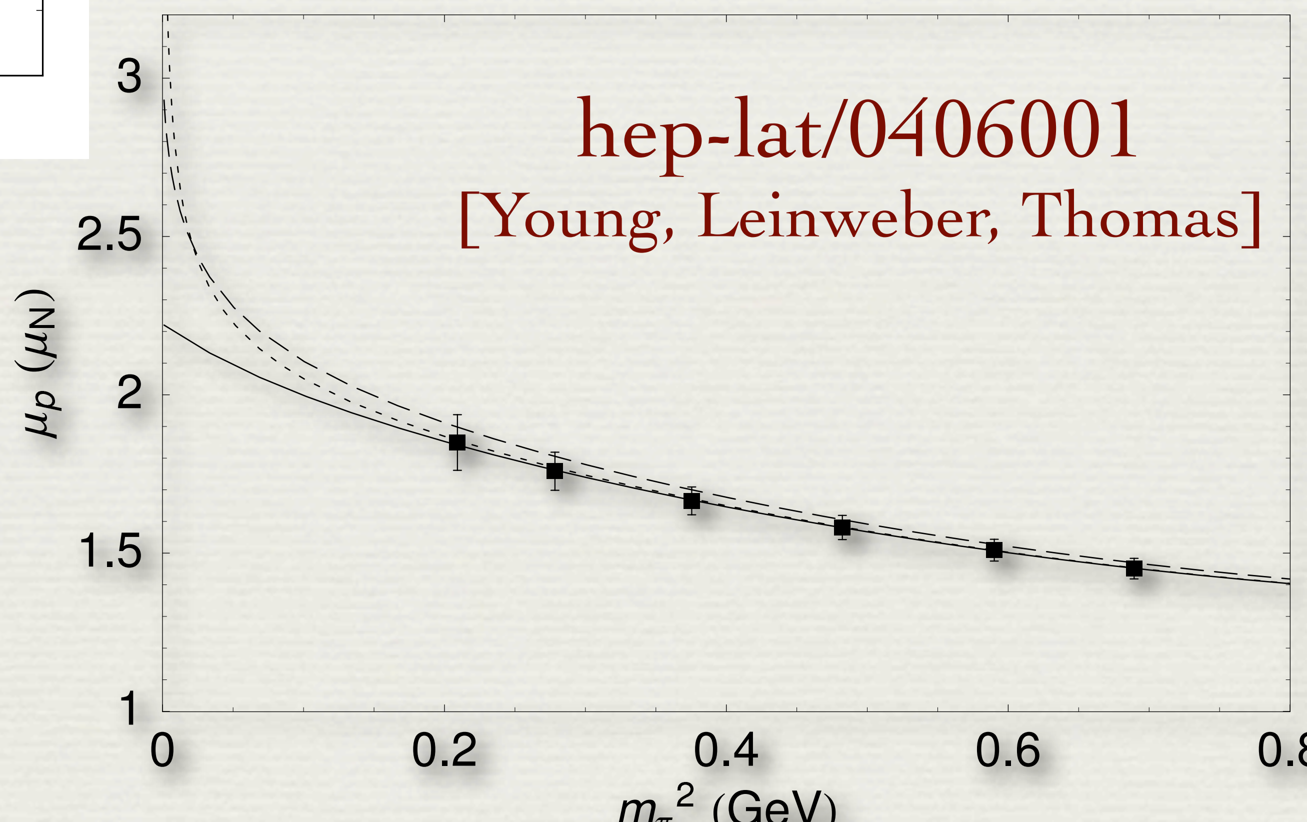
Anomalous Magnetic Moment



Anomalous Magnetic Moment

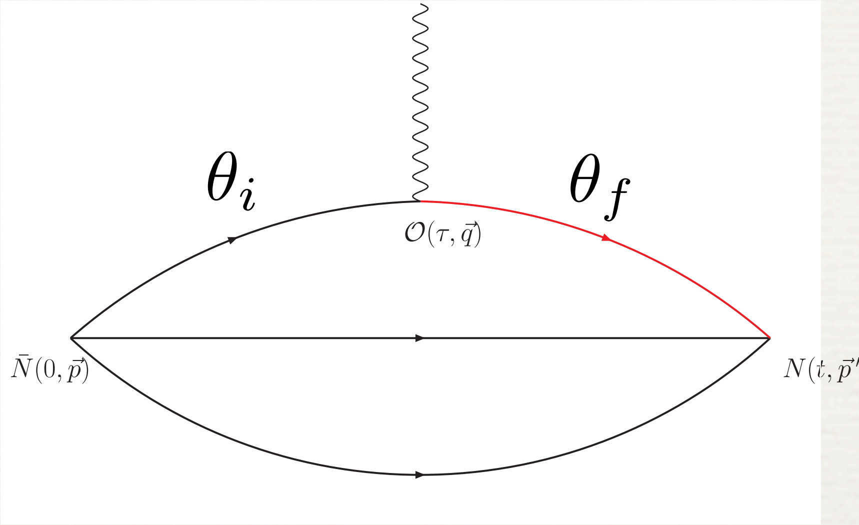


- *Chiral perturbation theory: Dramatic non-analytic predicted in the infinite volume*
- *Finite volume effects should suppress magnetic moment*



Accessing Small Q^2: Partially Twisted Boundary Conditions

hep-lat/0411033, hep-lat/0703005



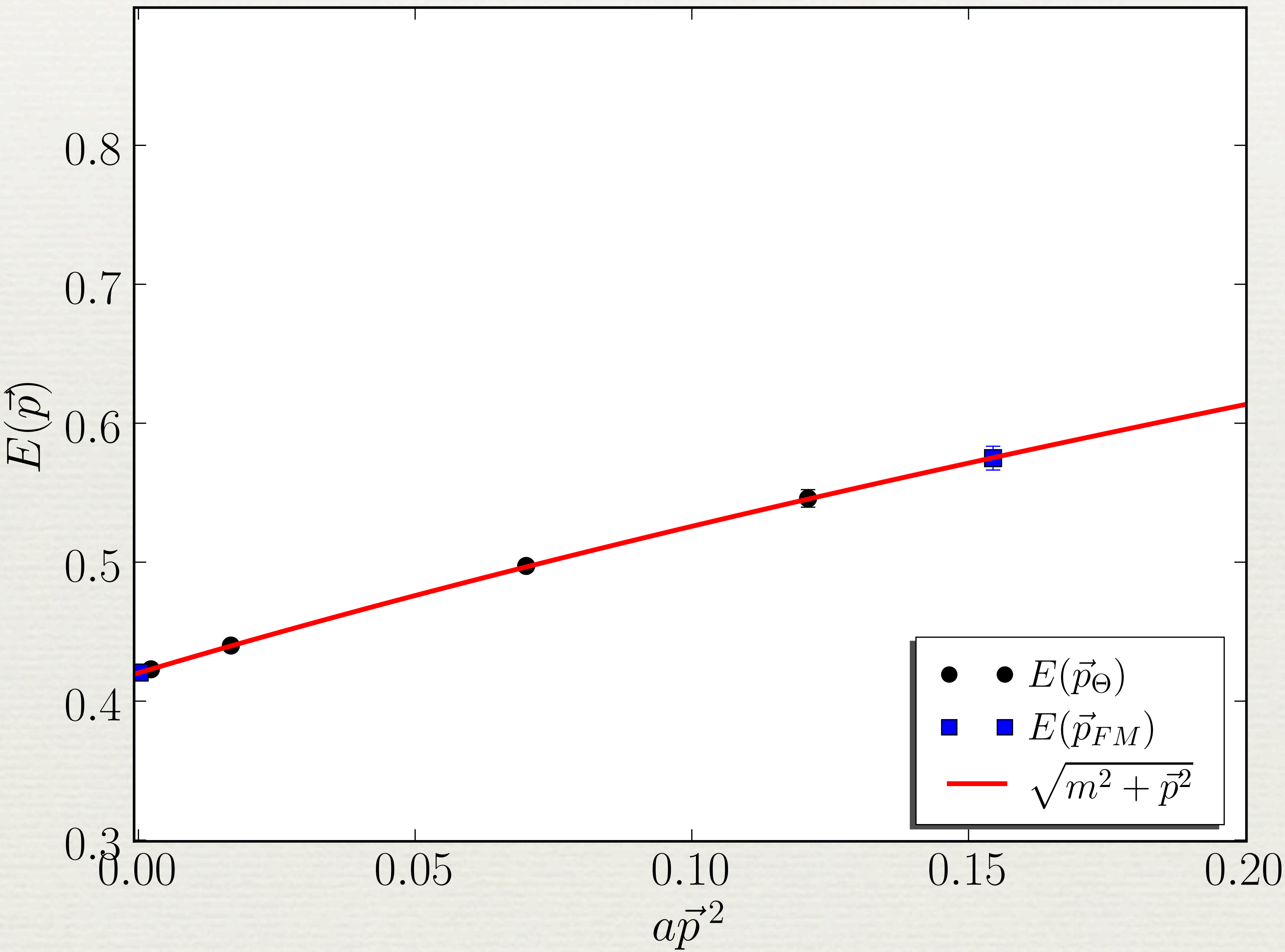
- On a periodic lattice with spatial volume L^3, momenta are discretised in units of $2\pi/L$
- Modify boundary conditions on the valence quarks

$$\psi(x_k + L) = e^{i\theta_k}\psi(x_k), \qquad (k = 1,2,3)$$

- allows to tune the momenta continuously $\quad \vec{p}_{\rm FT} + \vec{\theta}/L$
- Introduces additional finite volume effect $\sim e^{-m_\pi L}$

$$q^2 = (p_f - p_i)^2 = \left\{ [E_f(\vec{p}_f) - E_i(\vec{p}_i)]^2 - \left[(\vec{p}_{{\rm FT},f} + \vec{\theta}_f/L) - (\vec{p}_{{\rm FT},i} + \vec{\theta}_i/L) \right]^2 \right\}$$

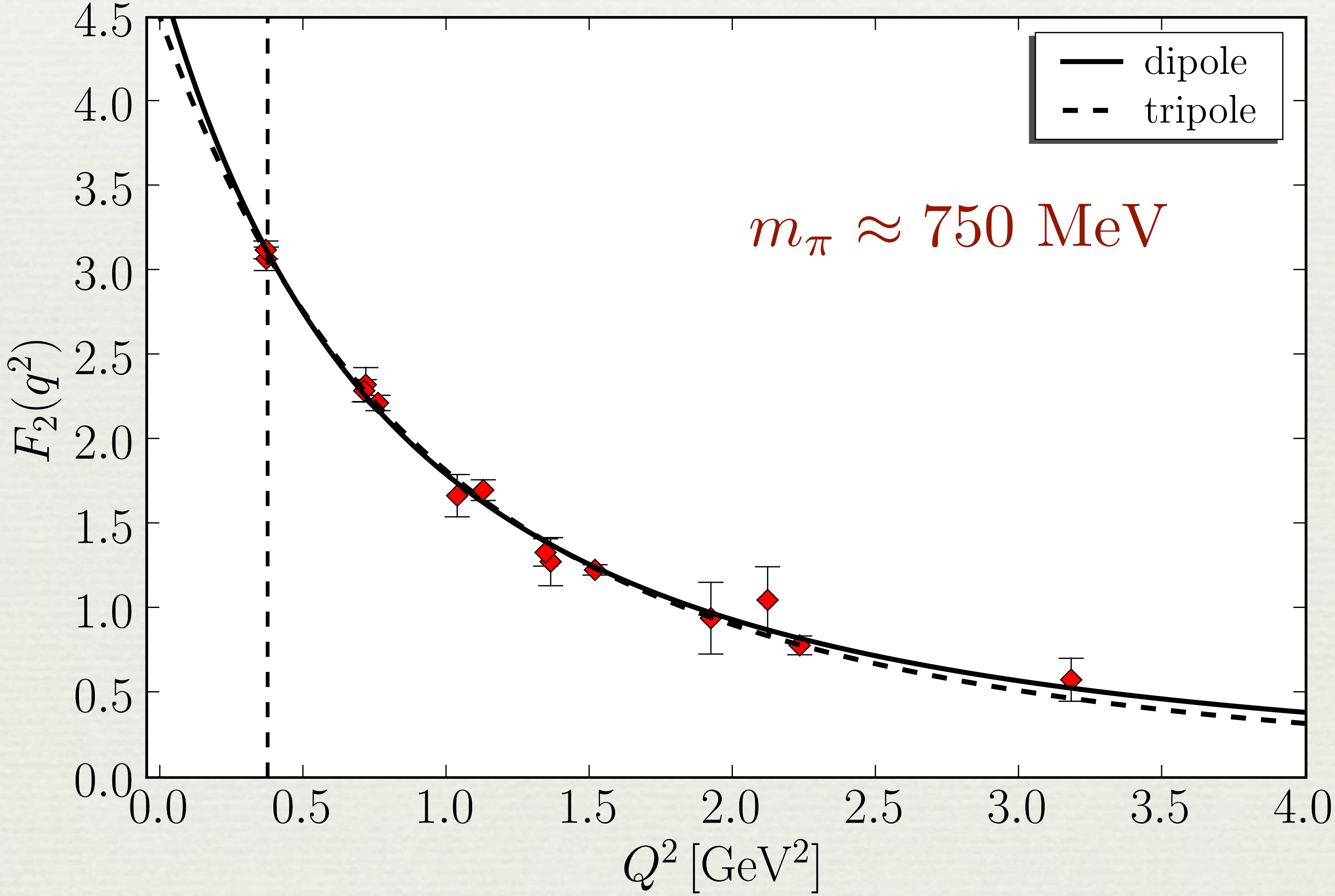
Pion Dispersion Relation



Accessing Small Q^2: Partially Twisted Boundary Conditions

QCDSF: N_f=2 Clover

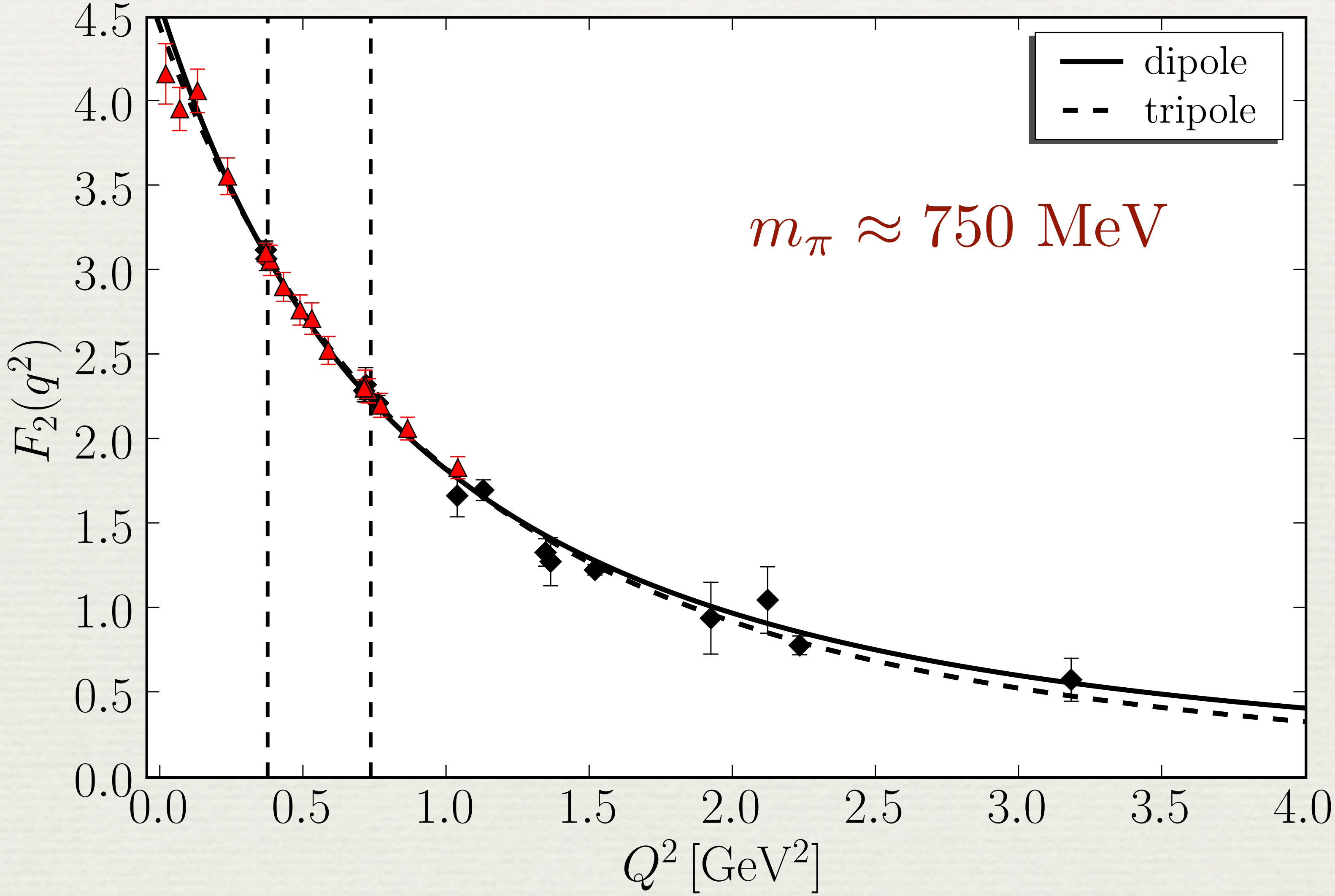
We need to extrapolate $F_2(q^2)$ to q^2=0 → **Model dependence**



Accessing Small Q^2: Partially Twisted Boundary Conditions

QCDSF: N_f=2 Clover

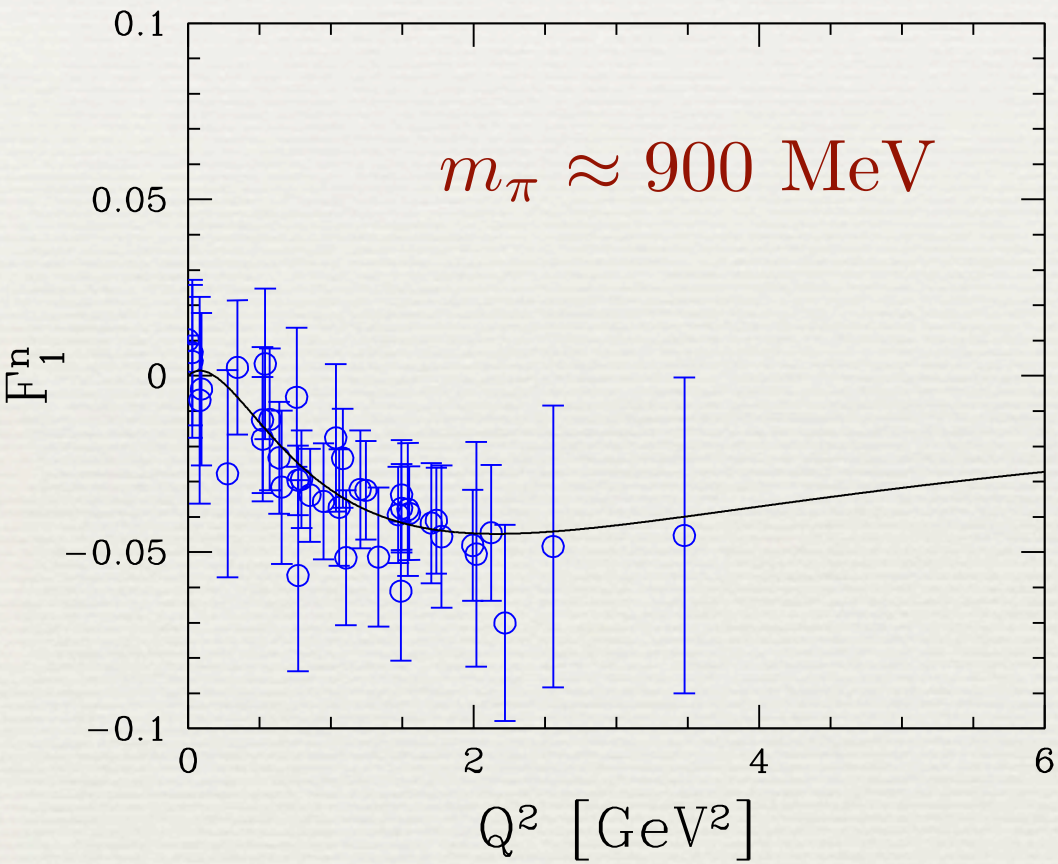
We need to extrapolate $F_2(q^2)$ to q^2=0 → **Model dependence**

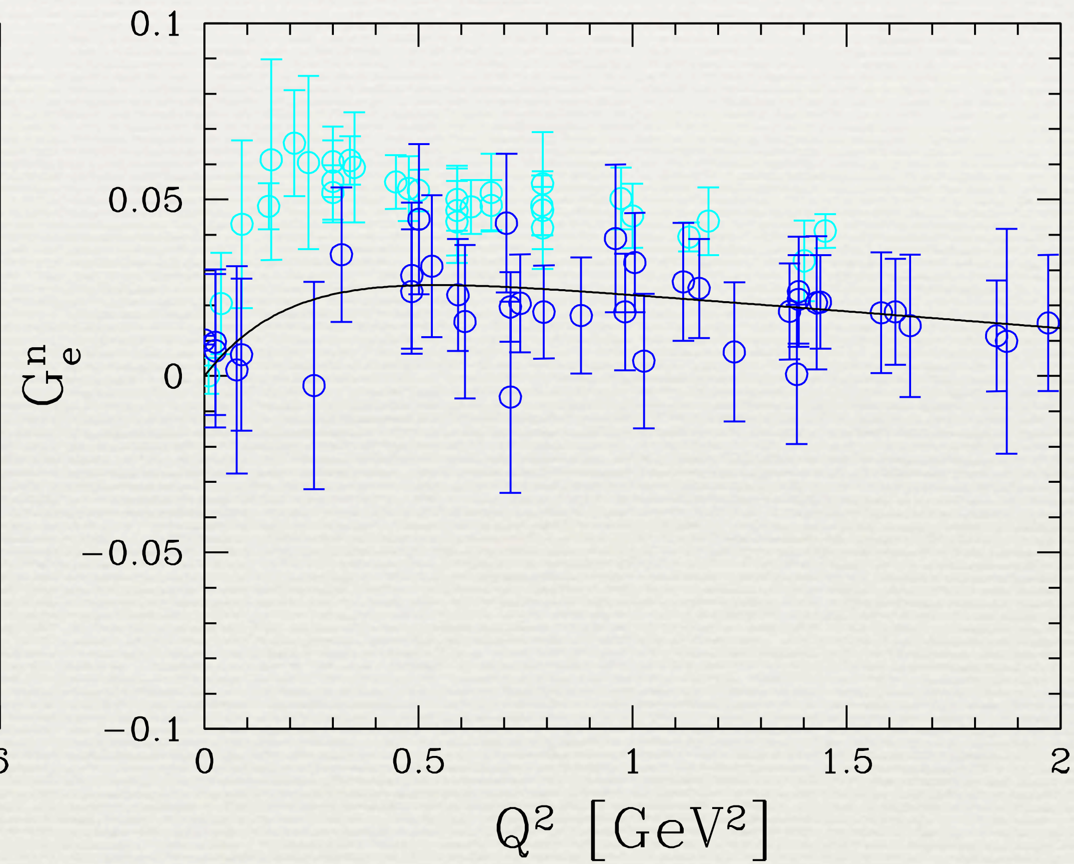


Neutron Form Factors

$$F^n = -\frac{1}{3}F^u - \frac{2}{3}F^d$$

$$\mathcal{G}_E(q^2) = F_1(q^2) - \frac{q^2}{(2m)^2}F_2(q^2)$$





F_1 neutron negative at small Q^2

How does "hump" change with quark mass?

$$g_A$$

Axial charge, g_A

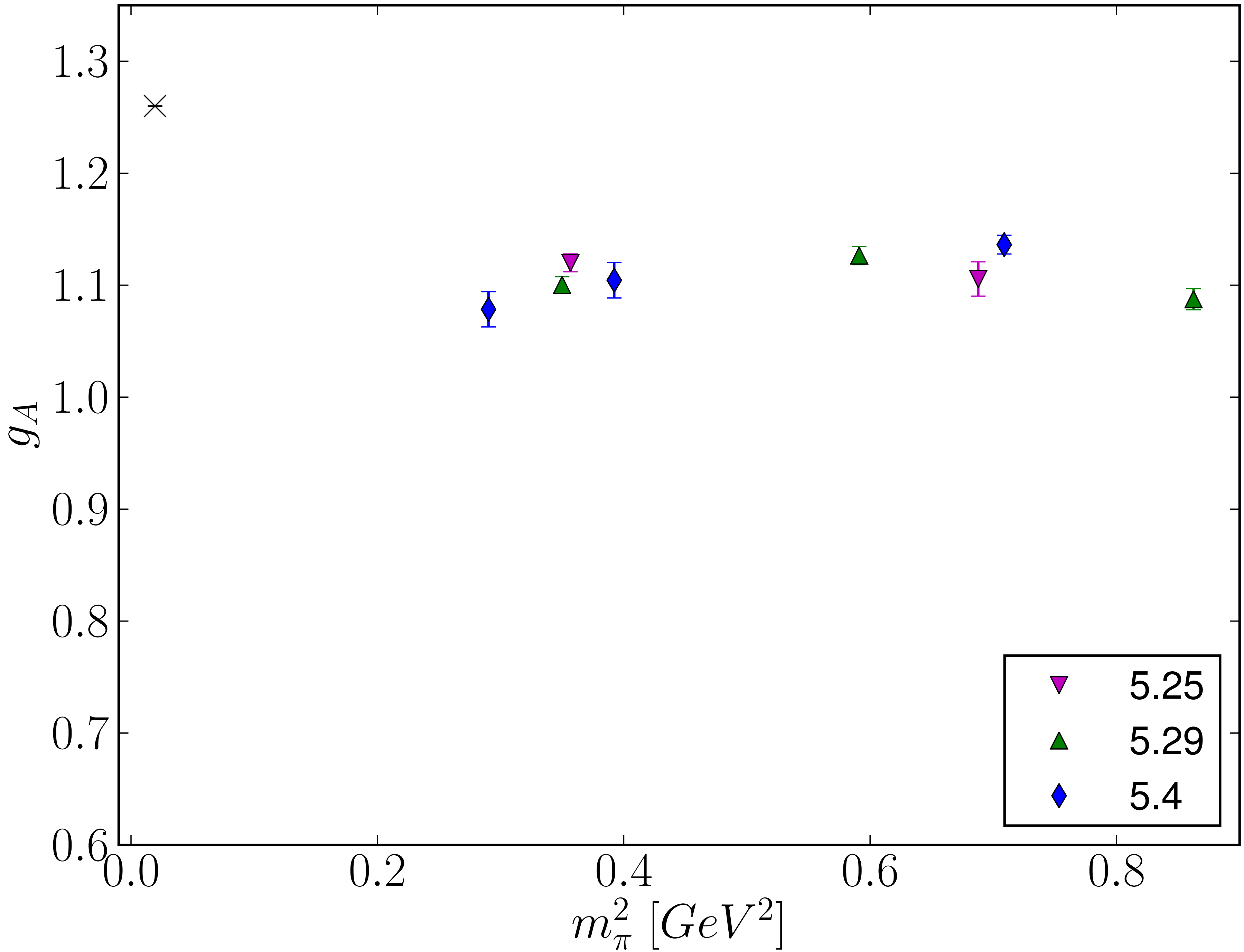
- *Governs neutron* β *decay*
- *Given by the forward nucleon matrix elements*

$$\langle p, s|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|p, s\rangle = 2 g_A s_\mu$$

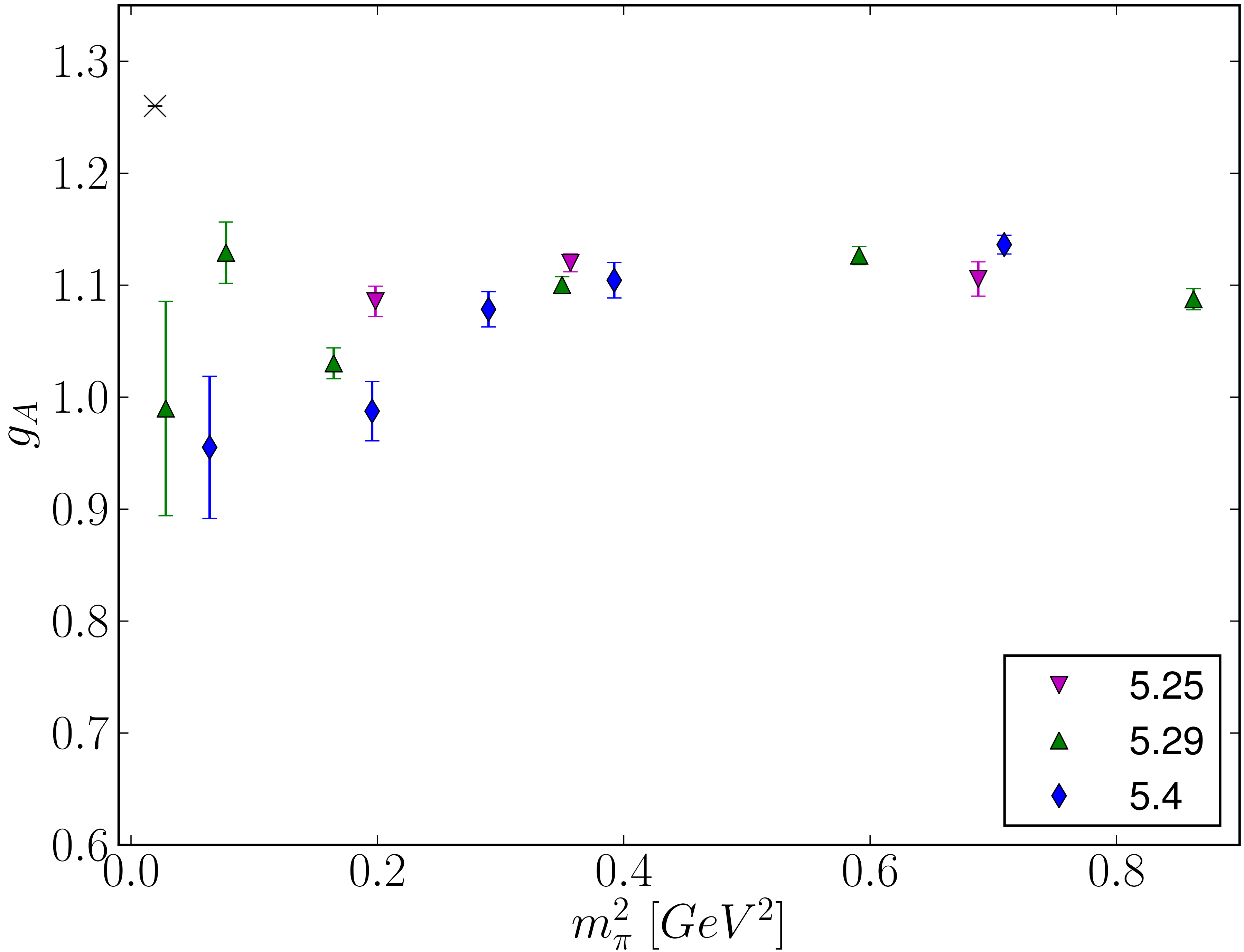
  - p *- nucleon momentum*
  - s *- spin vector,* $s^2 = -m_N^2$
  - $g_A = \Delta u - \Delta d$
- *Renormalised improved axial vector current*

$$A_\mu(x) = Z_A(1 + b_A a m_q)\left(\bar{q}(x)\gamma_\mu\gamma_5 q(x) + a c_A \partial_\mu \bar{q}(x)\gamma_5 q(x)\right)$$

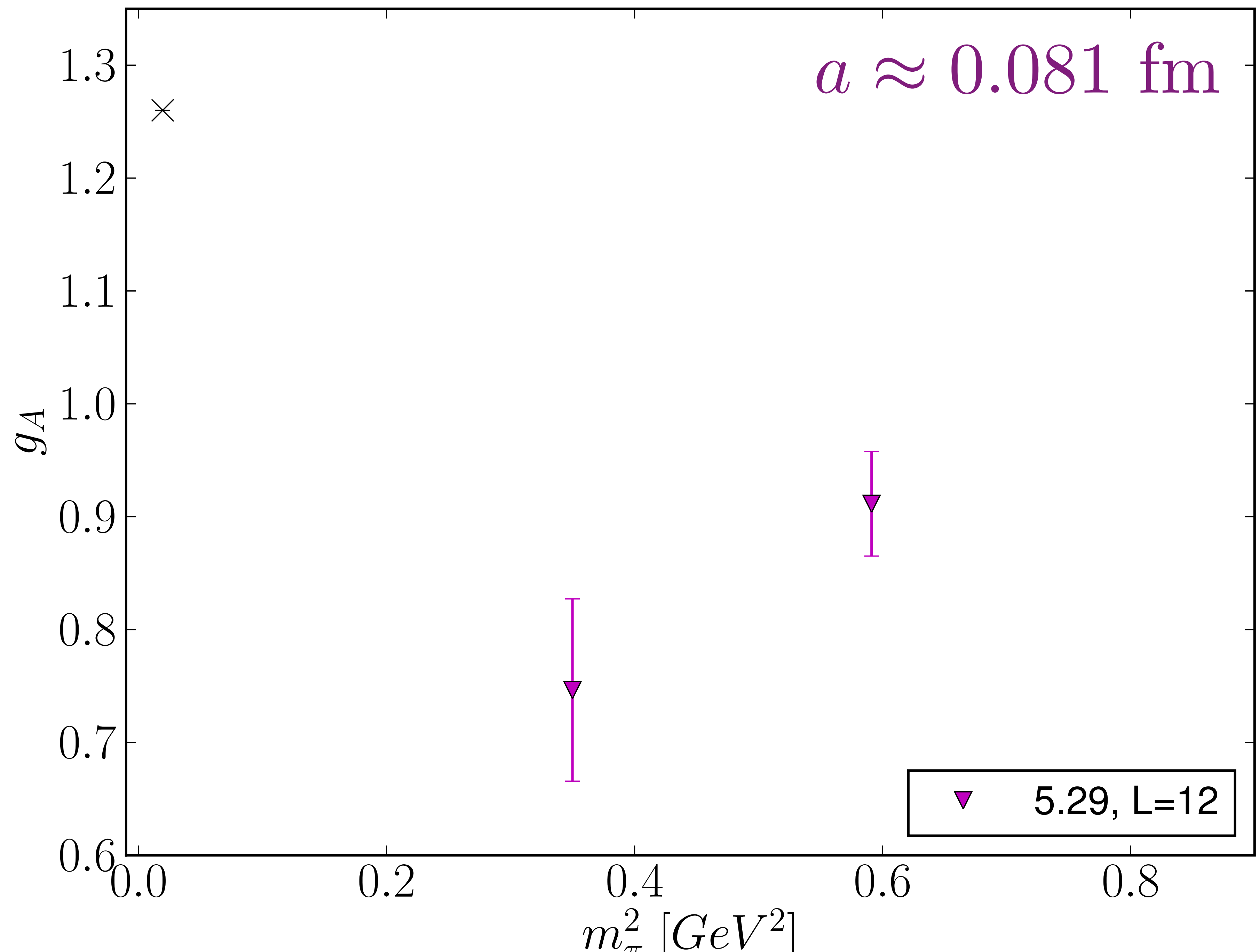
  - $m = (1/\kappa - 1/\kappa_c)/(2a)$ *is the bare quark mass*
  - *derivative operator vanishes for forward matrix elements*
  - b_A *is only known perturbatively*



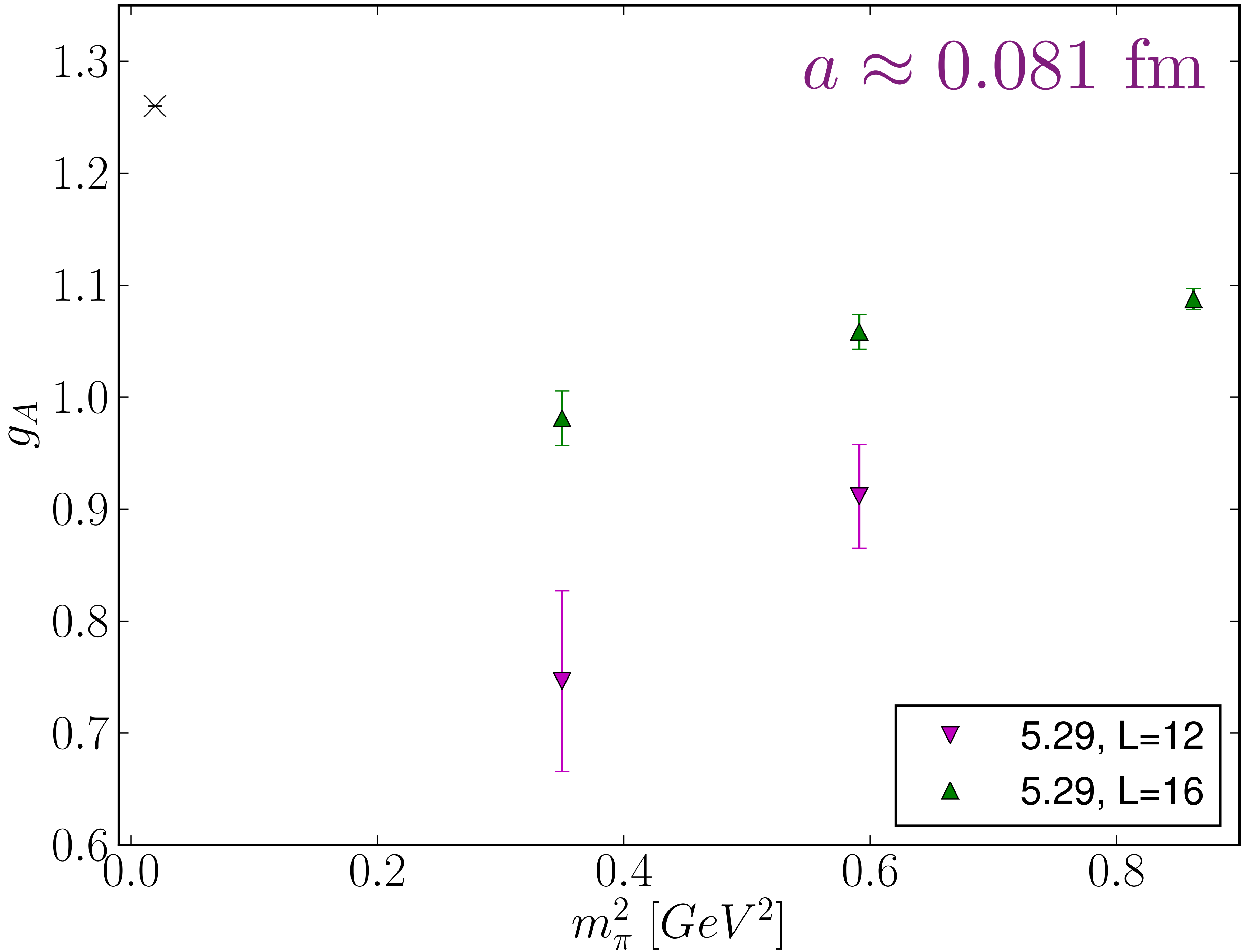
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
g_A
0.0
0.2
0.4
0.6
0.8
m_π^2 [GeV^2]
5.25
5.29
5.4



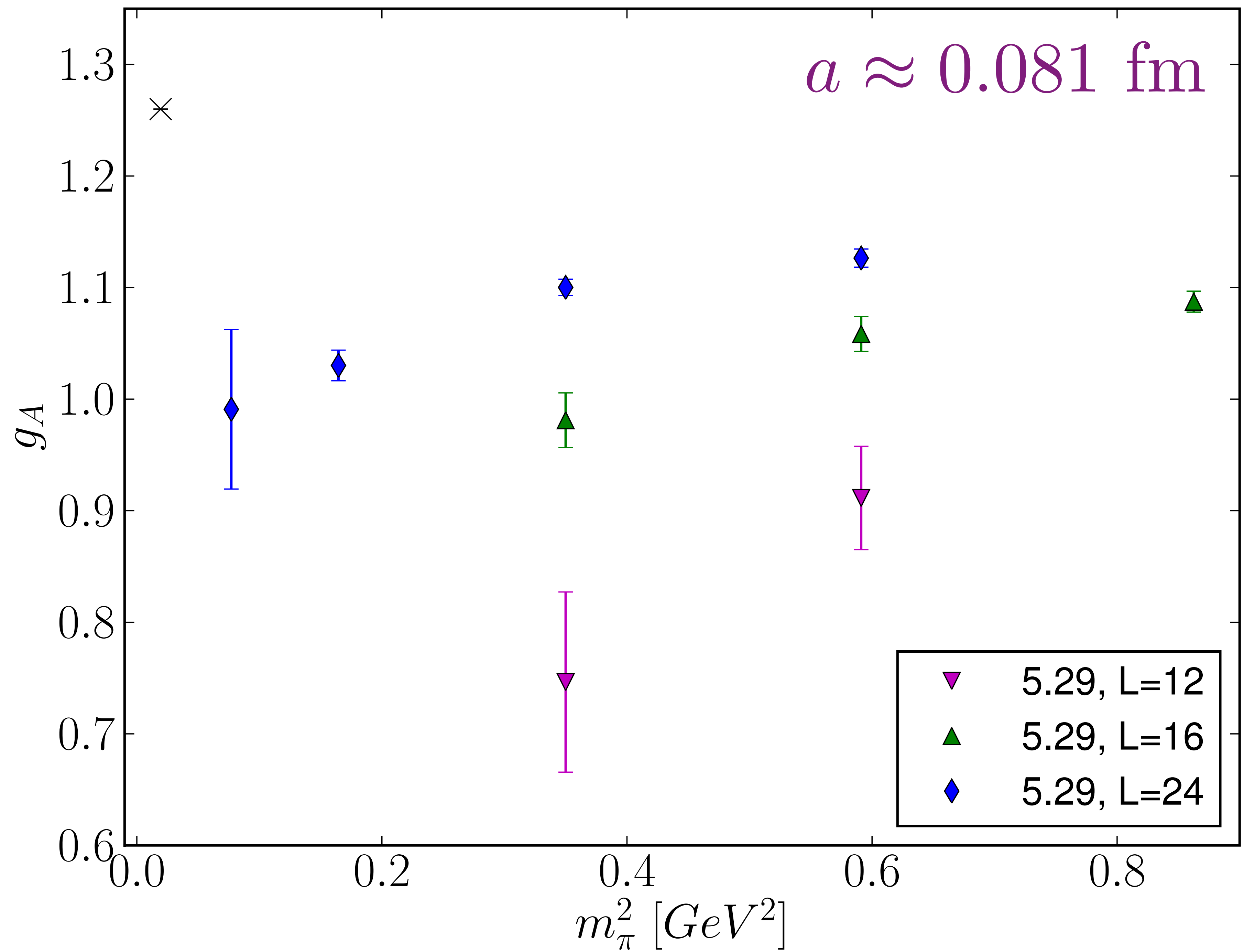
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
g_A
0.0
0.2
0.4
0.6
0.8
m_π^2 $[GeV^2]$
5.25
5.29
5.4



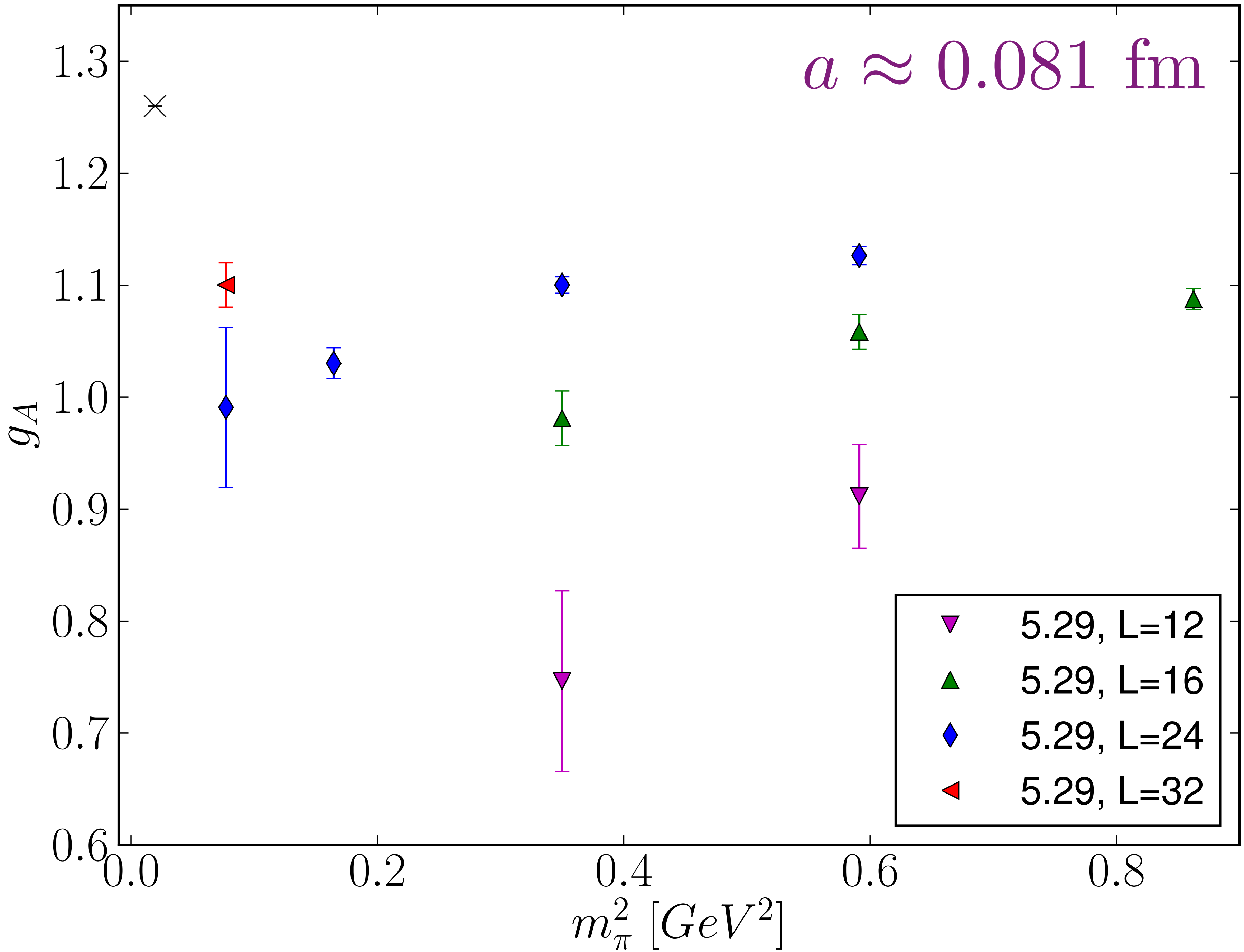
$a \approx 0.081$ fm
g_A
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.0
0.2
0.4
0.6
0.8
m_π^2 [GeV^2]
5.29, L=12



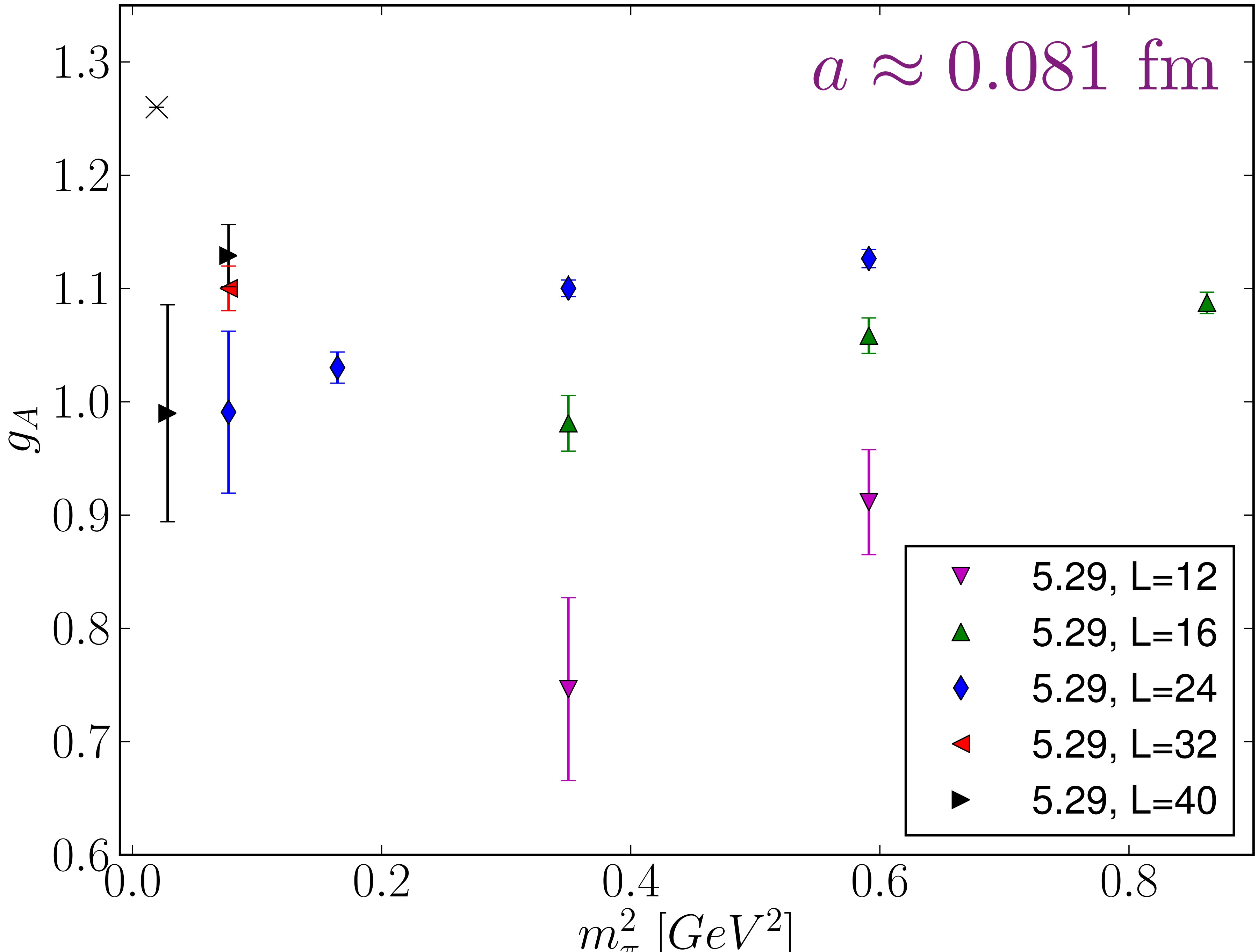
$a \approx 0.081$ fm
g_A
m_π^2 $[GeV^2]$
5.29, L=12
5.29, L=16
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.0
0.2
0.4
0.6
0.8



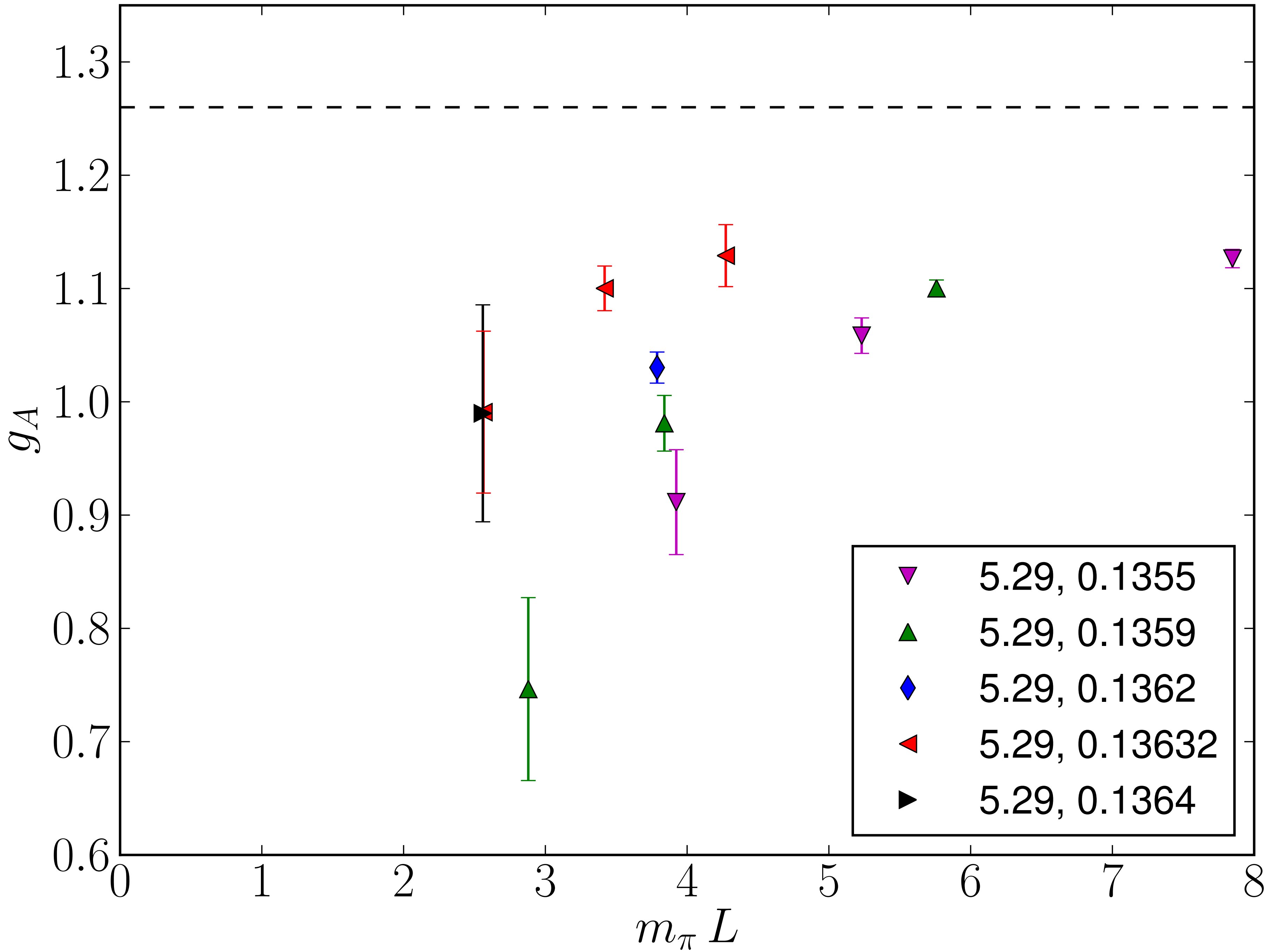
$a \approx 0.081$ fm
g_A
m_π^2 [GeV^2]
5.29, L=12
5.29, L=16
5.29, L=24
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.0
0.2
0.4
0.6
0.8



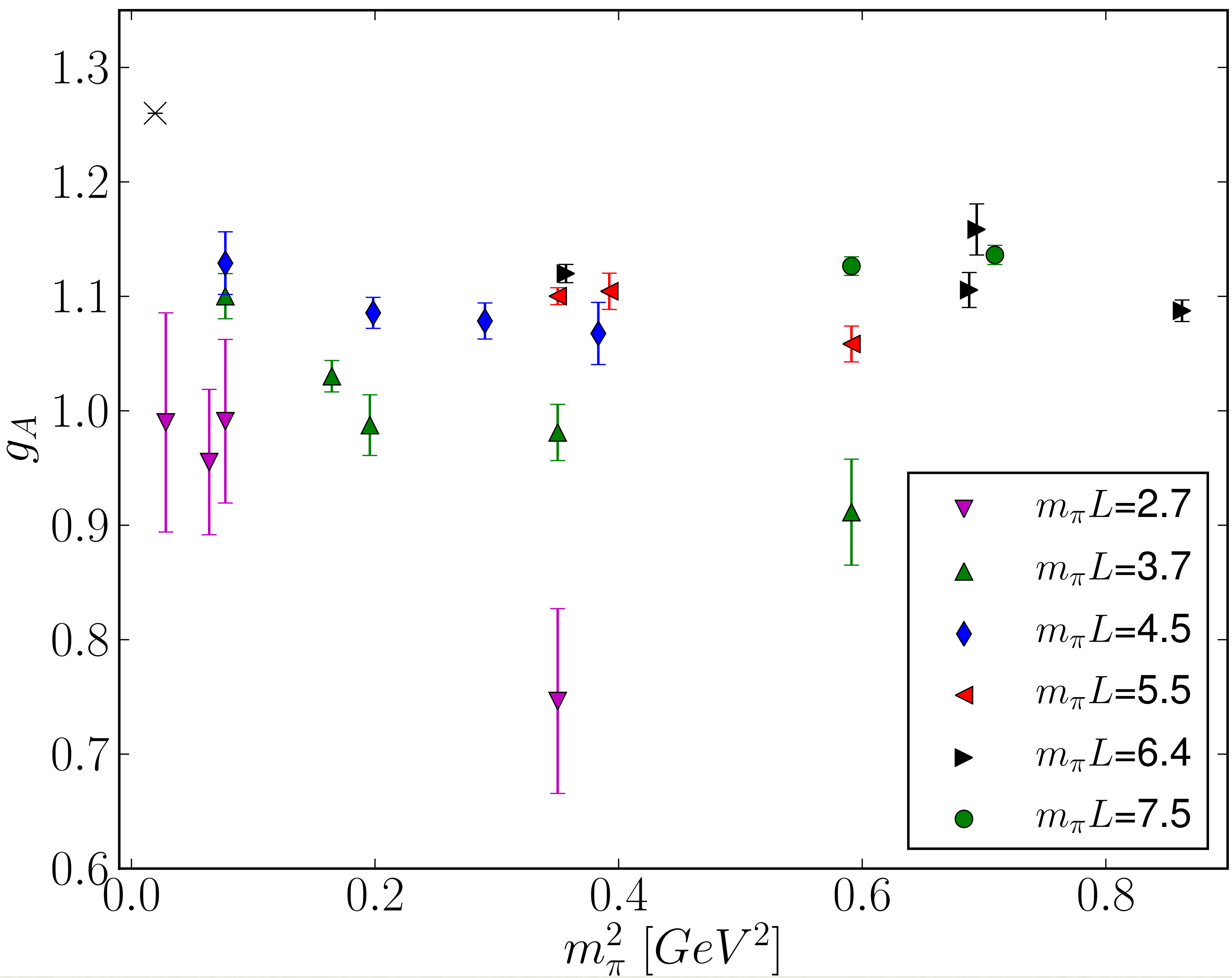
$a \approx 0.081$ fm
g_A
m_π^2 $[GeV^2]$
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.0
0.2
0.4
0.6
0.8
5.29, L=12
5.29, L=16
5.29, L=24
5.29, L=32



$a \approx 0.081$ fm
g_A
m_π^2 $[GeV^2]$
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.0
0.2
0.4
0.6
0.8
5.29, L=12
5.29, L=16
5.29, L=24
5.29, L=32
5.29, L=40

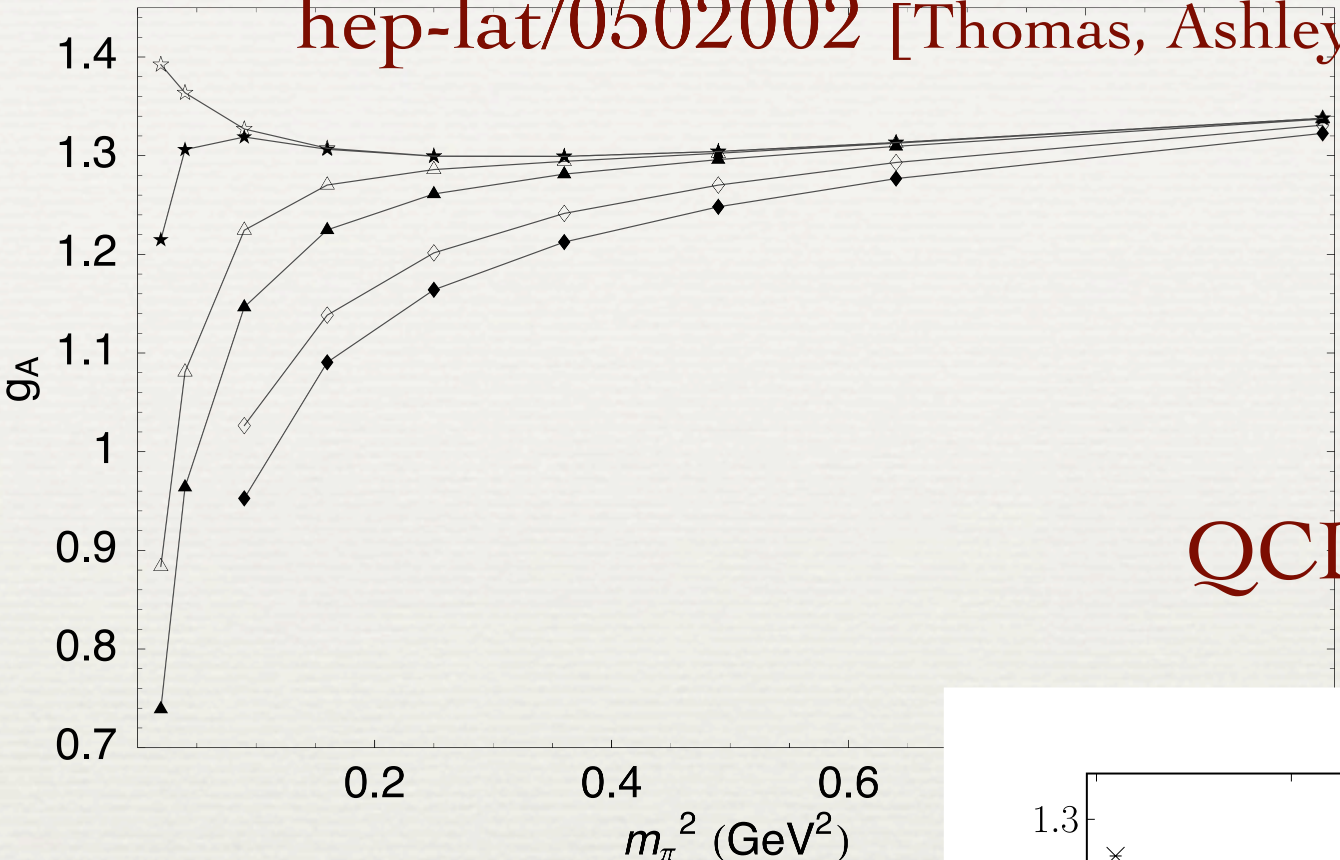


g_A
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0
1
2
3
4
5
6
7
8
$m_\pi L$
5.29, 0.1355
5.29, 0.1359
5.29, 0.1362
5.29, 0.13632
5.29, 0.1364

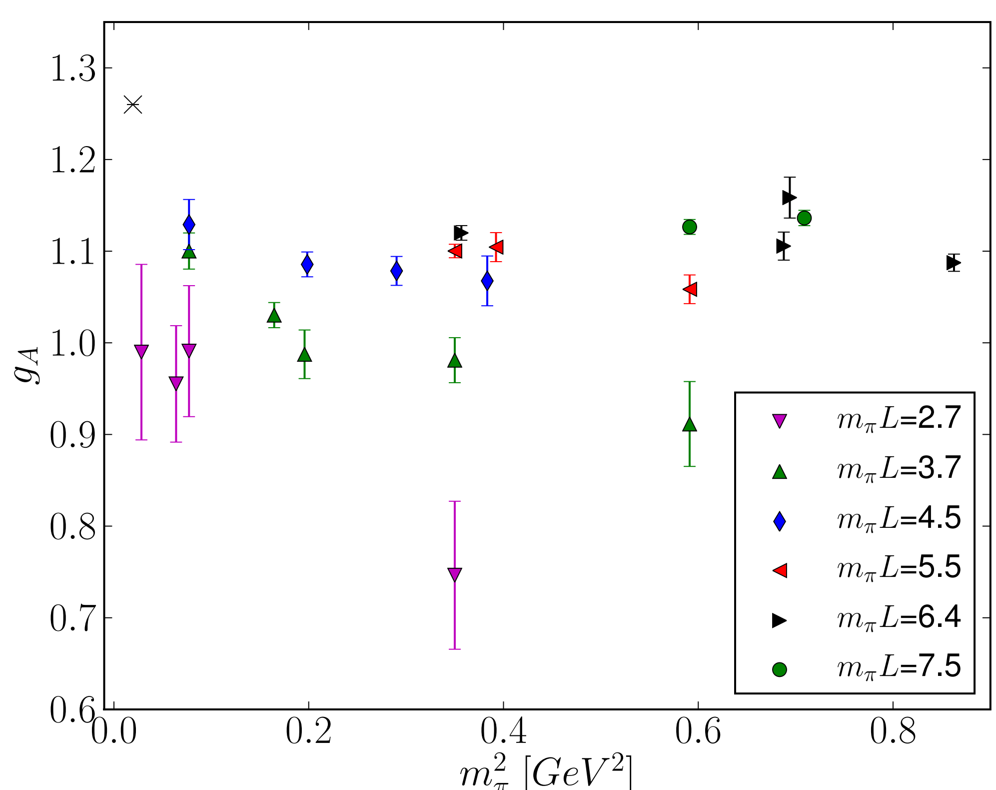


1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
g_A
0.0
0.2
0.4
0.6
0.8
$m_\pi^2\ [GeV^2]$
$m_\pi L$=2.7
$m_\pi L$=3.7
$m_\pi L$=4.5
$m_\pi L$=5.5
$m_\pi L$=6.4
$m_\pi L$=7.5

hep-lat/0502002 [Thomas, Ashley, Leinweber, Young]

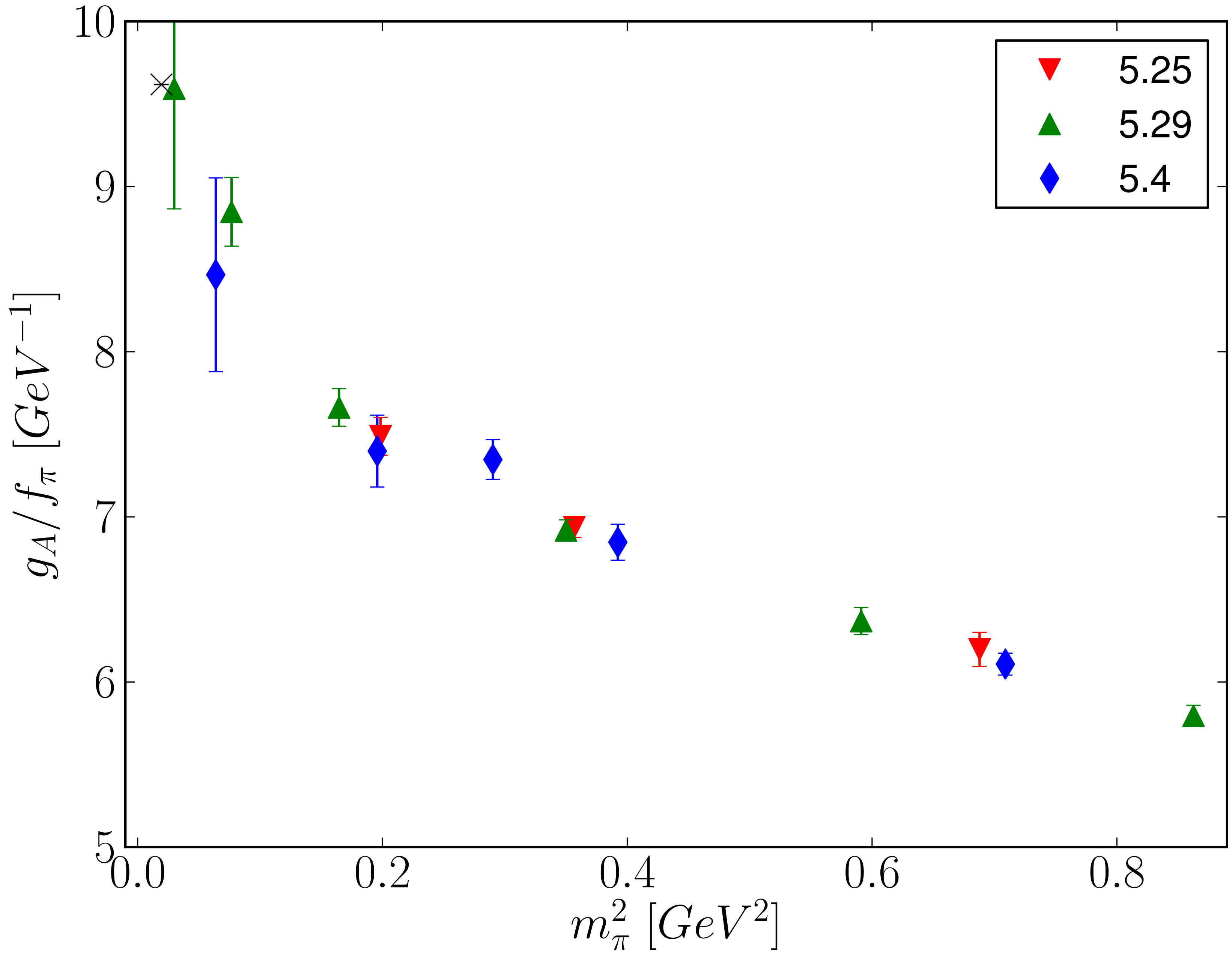


QCDSF (preliminary)



- For $m_\pi < 300$ MeV require $L > 3$ fm
- $m_\pi < 200$ MeV $L > 3.5$ fm ?
- Overall all trend still low.
    - Renormalisation?
    - Chiral physics?

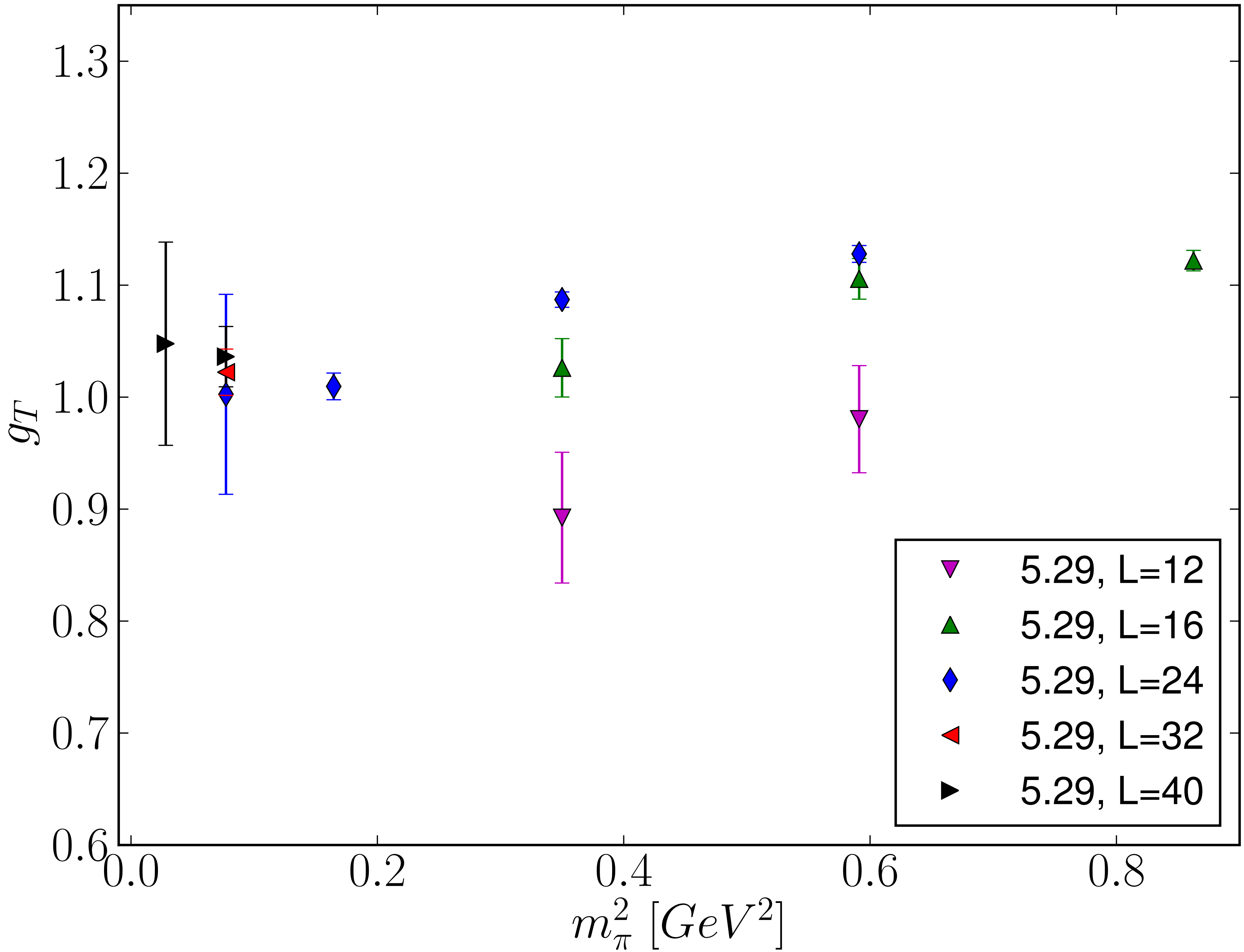
g_A/f_π [GeV^{-1}]



$$g_T$$

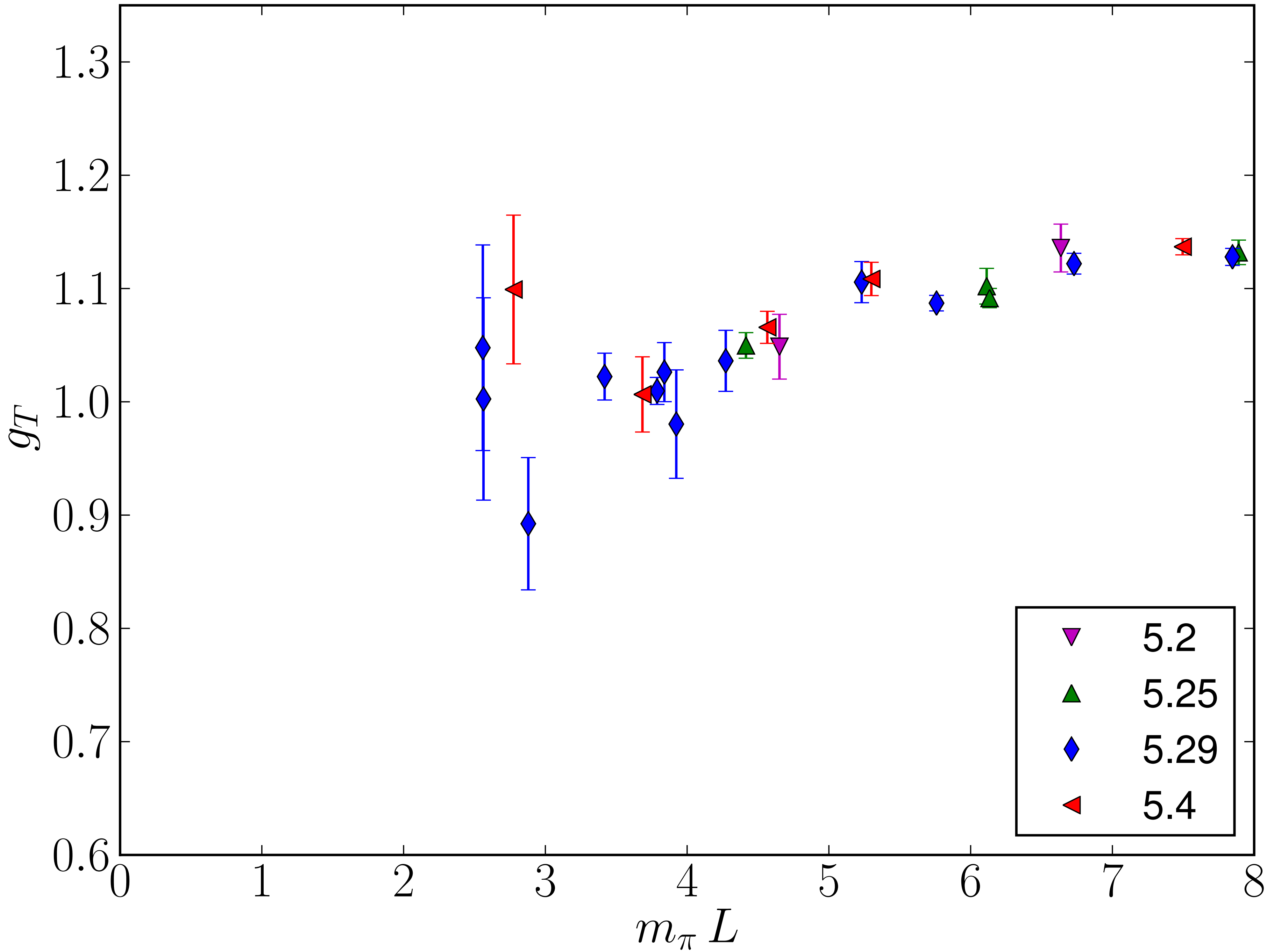
$$\mathcal{O}^{\sigma}_{i4} = \bar{q}\gamma_5\sigma_{i4}q$$

g_T

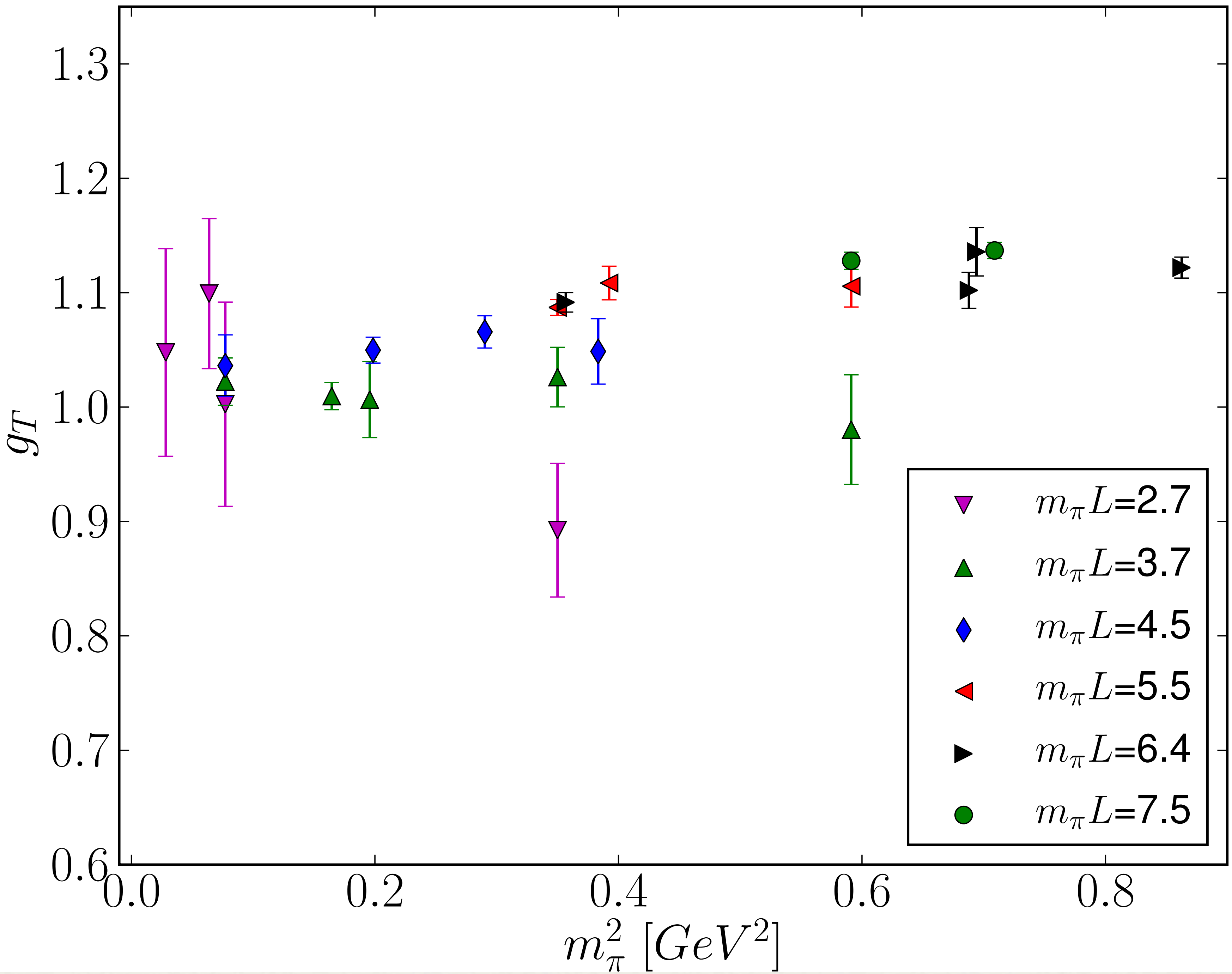


g_T
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.0
0.2
0.4
0.6
0.8
$m_\pi^2\ [GeV^2]$
5.29, L=12
5.29, L=16
5.29, L=24
5.29, L=32
5.29, L=40

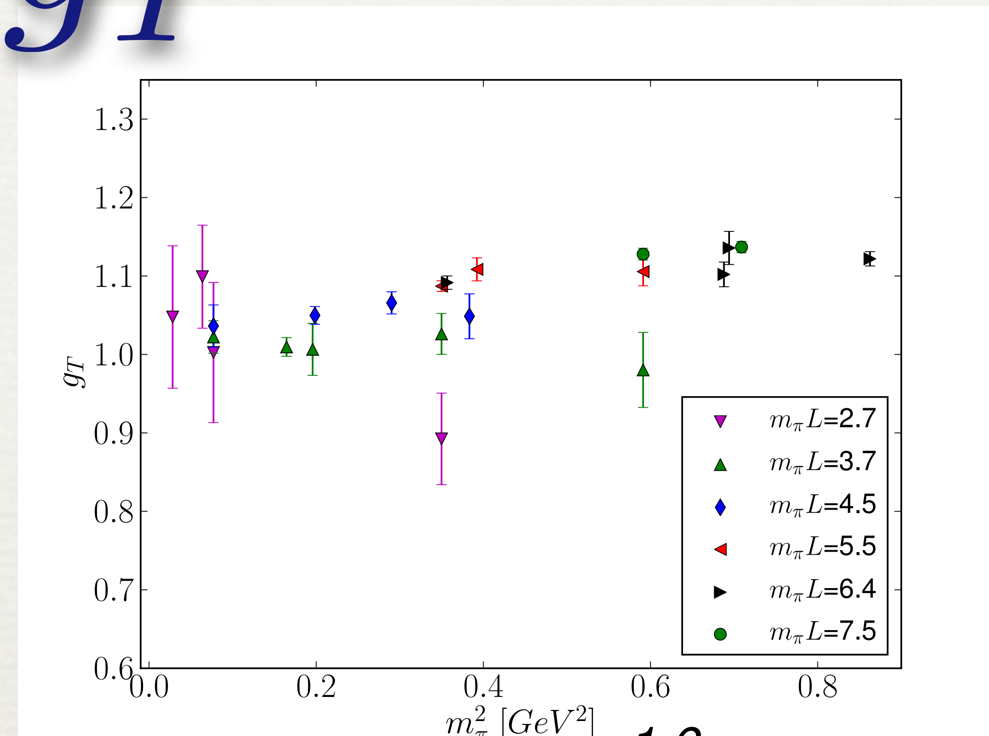
g_T



g_T

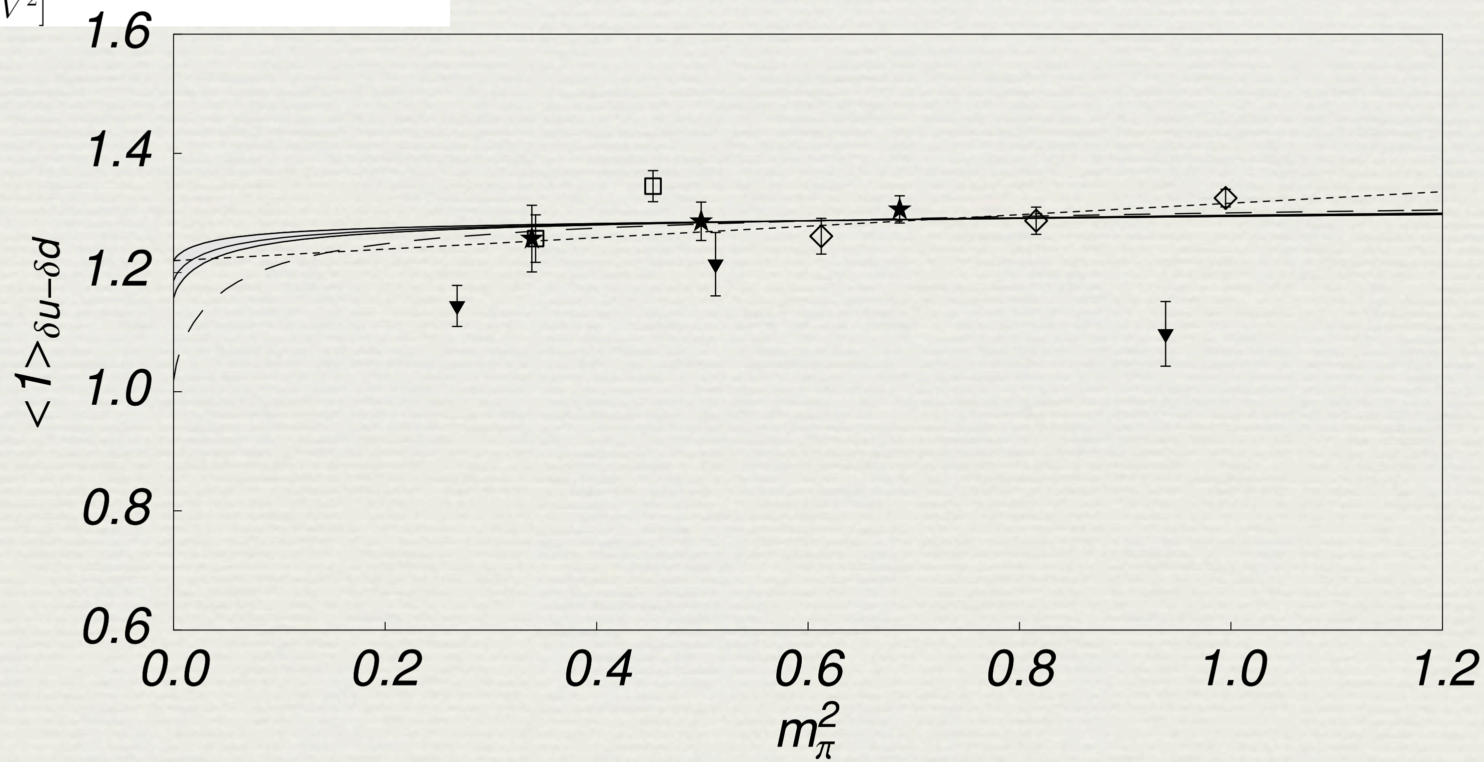

1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.0
0.2
0.4
0.6
0.8
g_T
m_π^2 $[GeV^2]$
$m_\pi L$=2.7
$m_\pi L$=3.7
$m_\pi L$=4.5
$m_\pi L$=5.5
$m_\pi L$=6.4
$m_\pi L$=7.5

g_T



PRD66:054501 (2002)

[Detmold, Melnitchouk, Thomas]



$$\langle x \rangle$$

$\langle x \rangle$

- *Forward MEs with no momentum transfer provide moments of quark distributions (or structure functions)*

$$\langle N(\vec{p})| \left[\mathcal{O}_q^{\{\mu_1\cdots\mu_n\}} - \mathrm{Tr}\right] |N(\vec{p})\rangle^{\mathcal{S}} = 2v_n^{(q)\mathcal{S}} \left[p^{\mu_1}\cdots p^{\mu_n} - \mathrm{Tr}\right]$$

$$\int_0^1 dx\, x^{n-1} \left[q(x) + (-1)^n \bar{q}(x)\right] = v_n^{(q)\mathcal{S}}$$

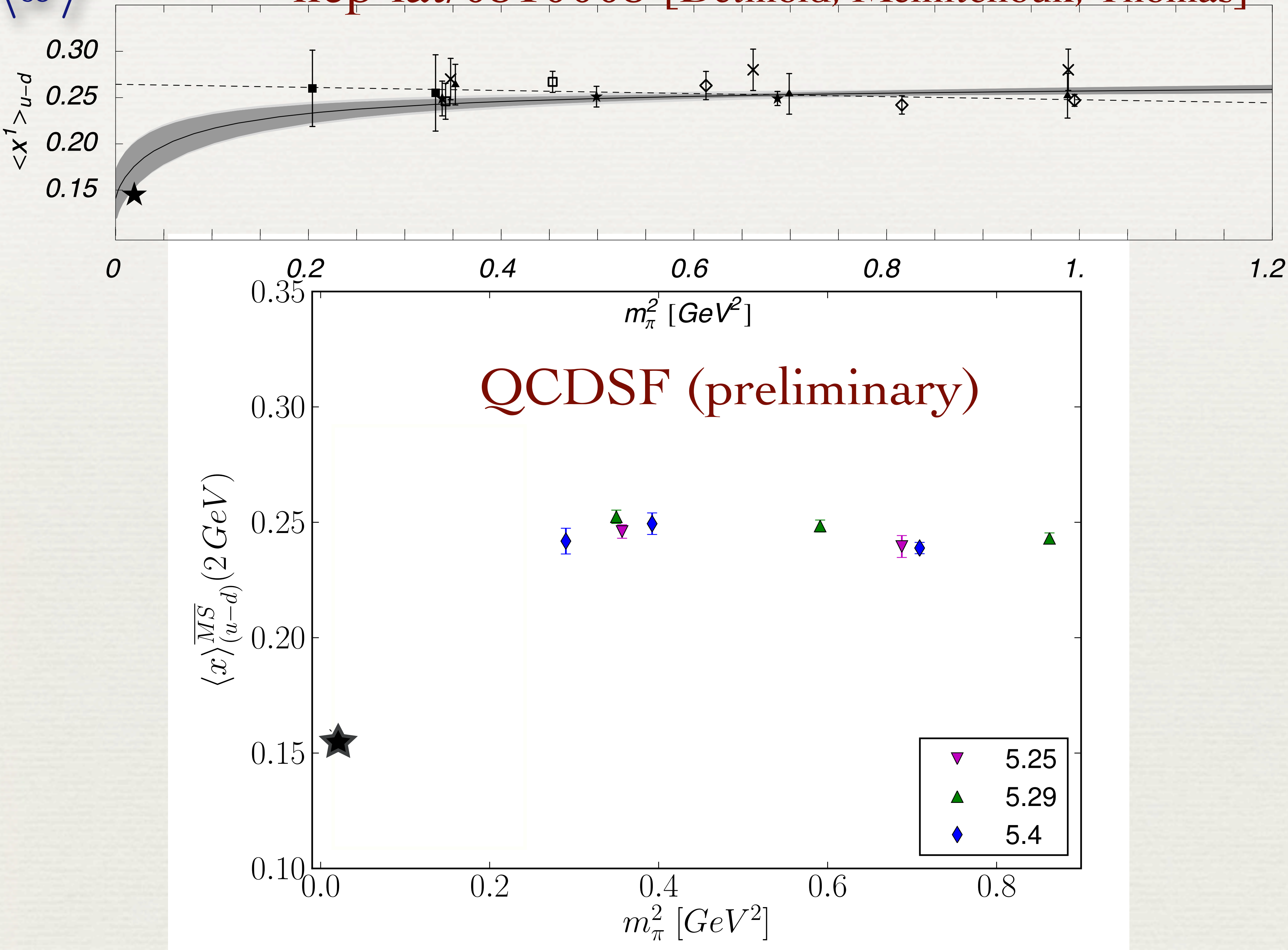
- *Consider u-d*

- *Nonperturbative renormalisation using Rome-Southampton method (RI'-MOM), then convert to $\overline{MS}$ at 2 GeV*

$$\mathcal{O}_q^{\mu_1\cdots\mu_n} = \bar{q}\,\gamma^{\mu_1}\,\overleftrightarrow{D}^{\mu_2}\ldots\overleftrightarrow{D}^{\mu_n}\,q \qquad \overleftrightarrow{D} = \frac{1}{2}(\overrightarrow{D} - \overleftarrow{D})$$

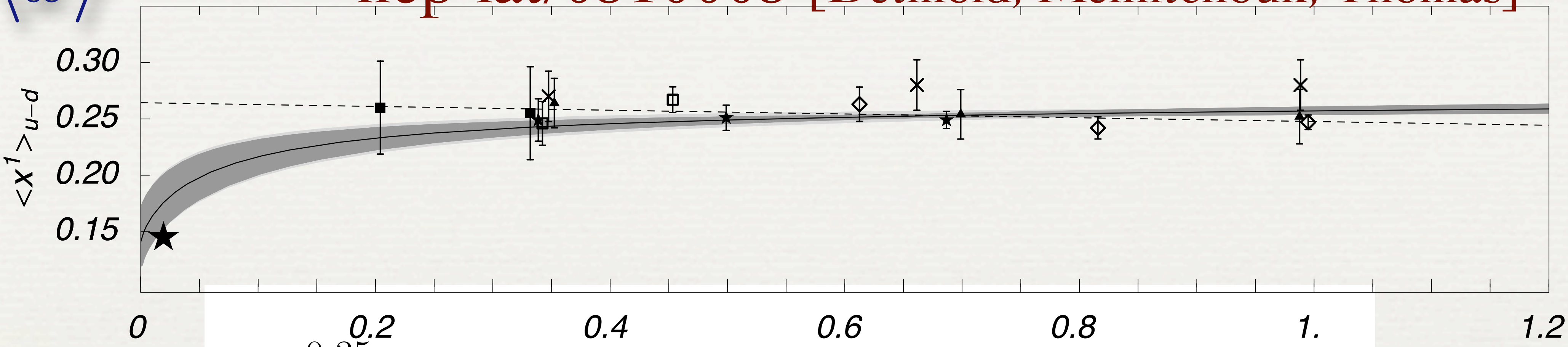
$\langle x \rangle$

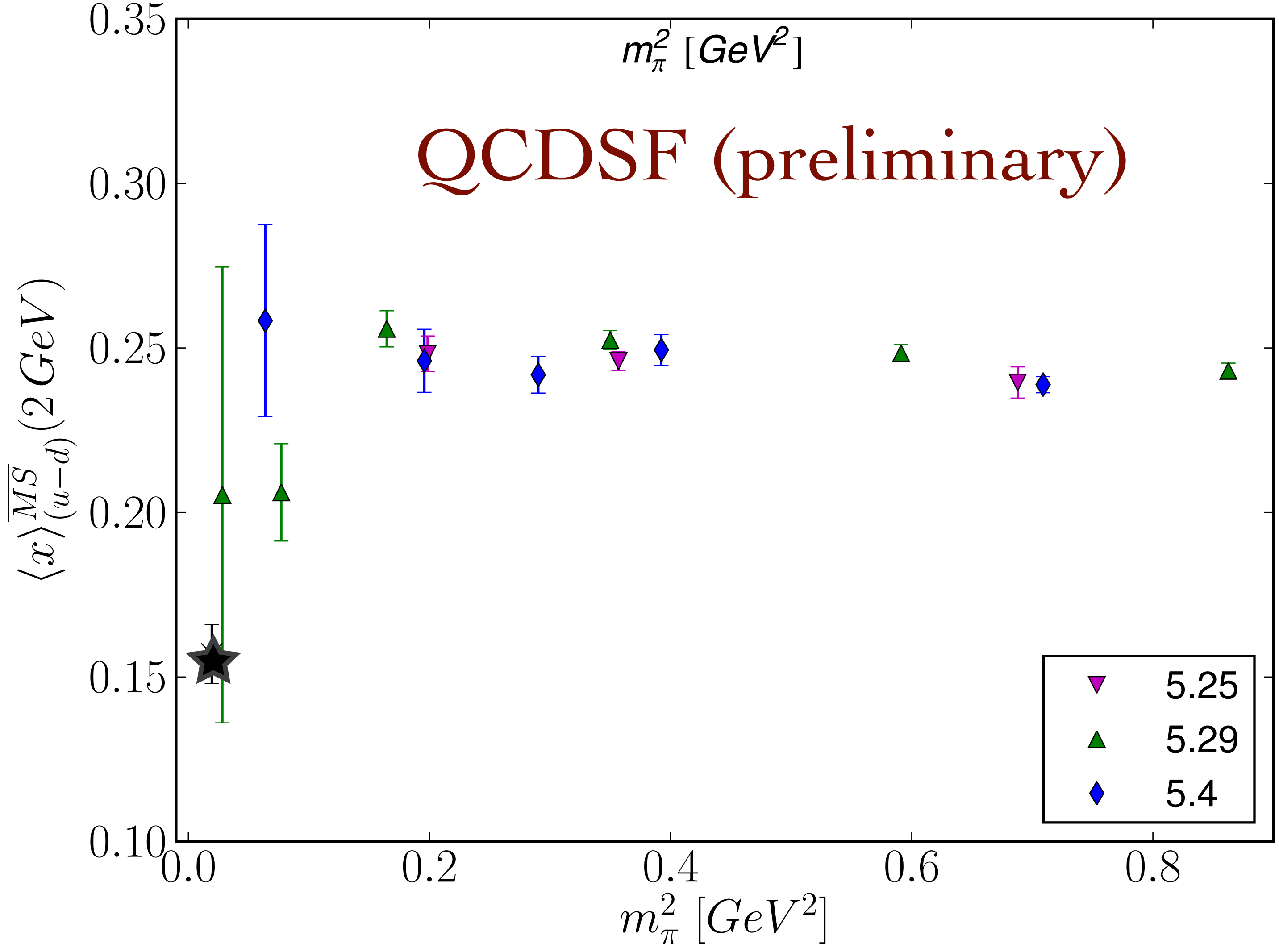
hep-lat/0310003 [Detmold, Melnitchouk, Thomas]



$\langle x \rangle$

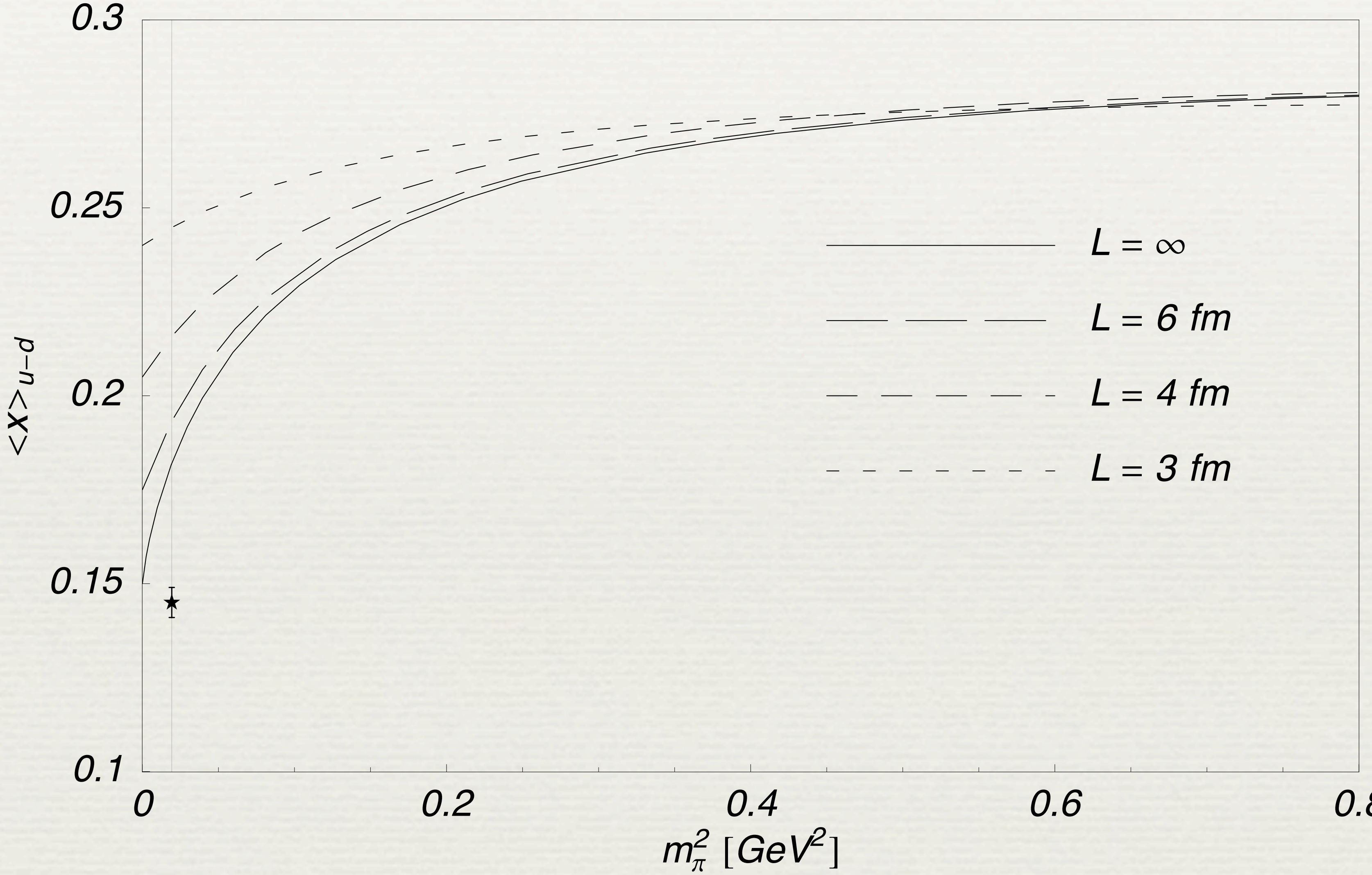
hep-lat/0310003 [Detmold, Melnitchouk, Thomas]





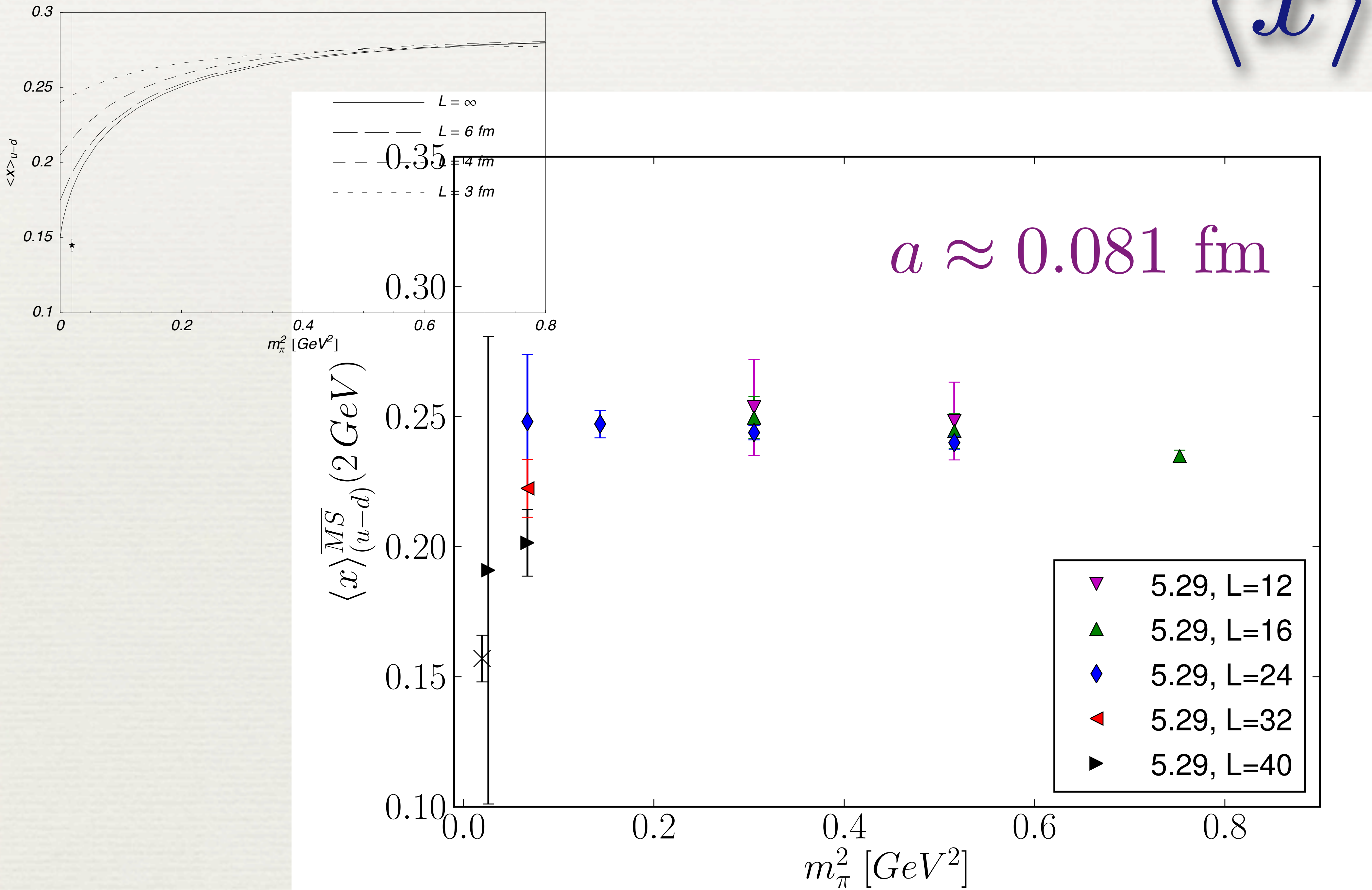
hep-lat/0310003 [Detmold, Melnitchouk, Thomas]

$\langle x \rangle$

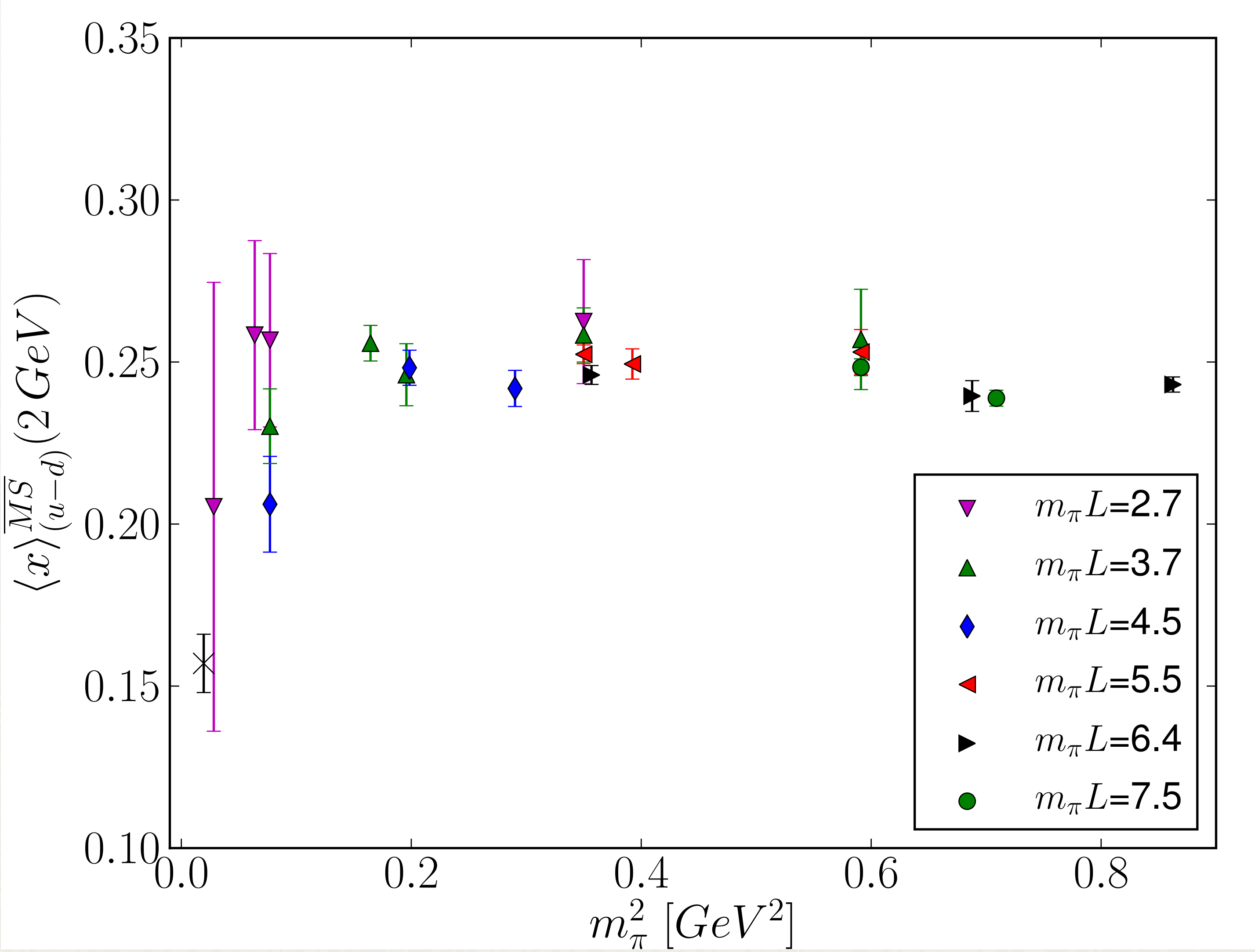


QCDSF (preliminary)

$\langle x \rangle$



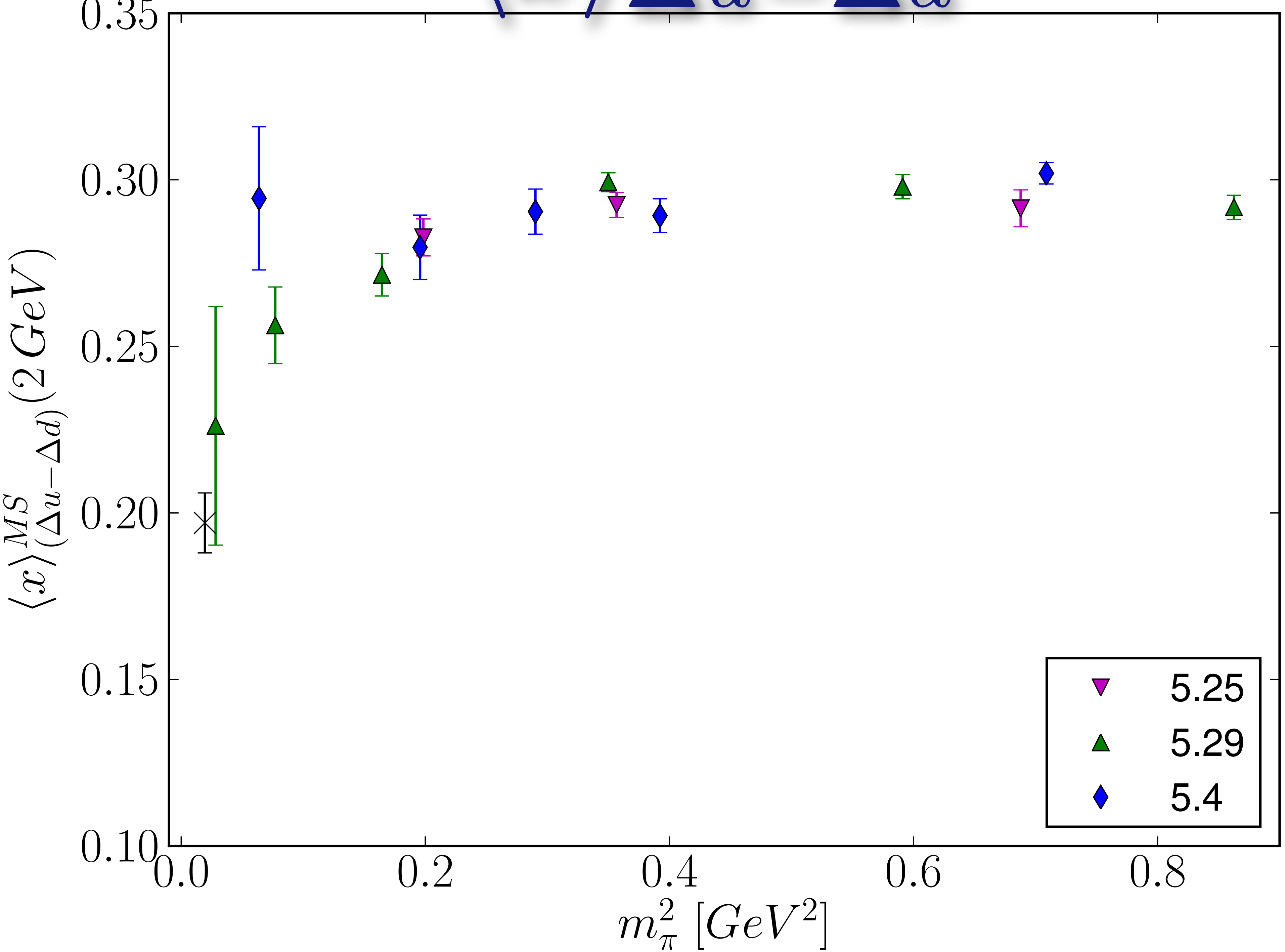
$\langle x \rangle$



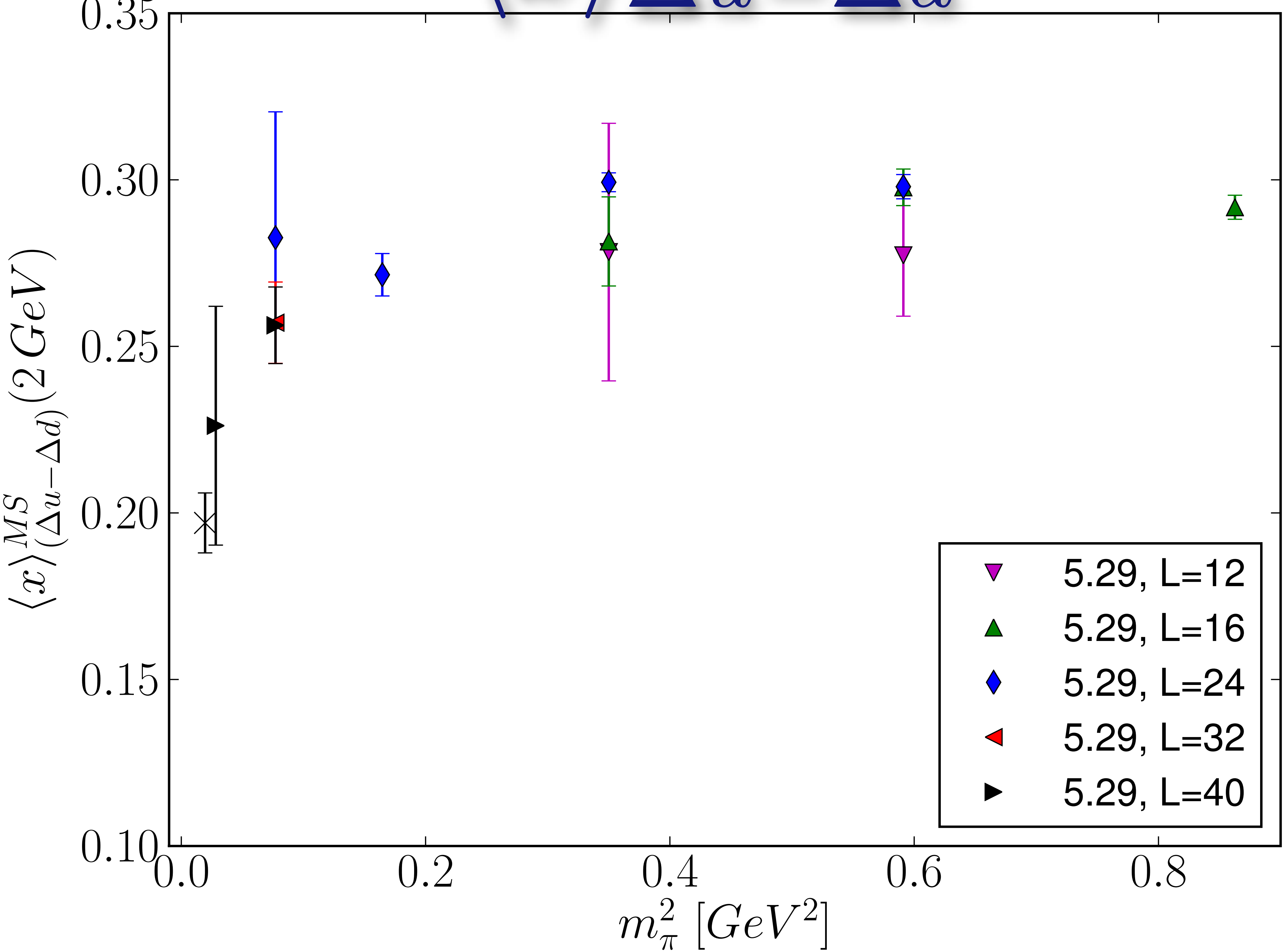
- *Results confirm predictions that "flat" behaviour will persist on small volumes*
- *Evidence for curvature with large volume results*

$$\langle x \rangle_{\Delta u - \Delta d}$$

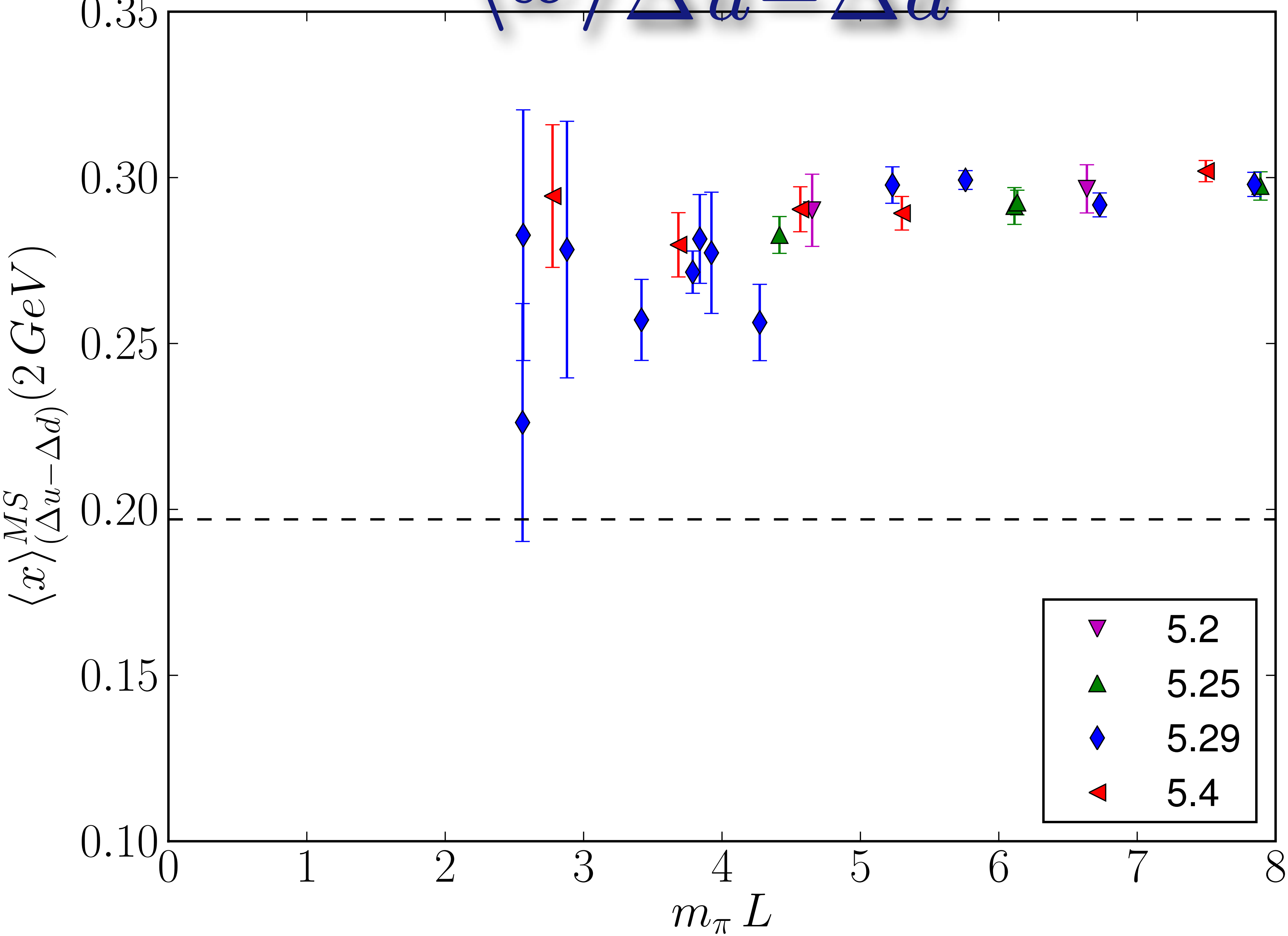
$$\langle x \rangle_{\Delta u - \Delta d}$$



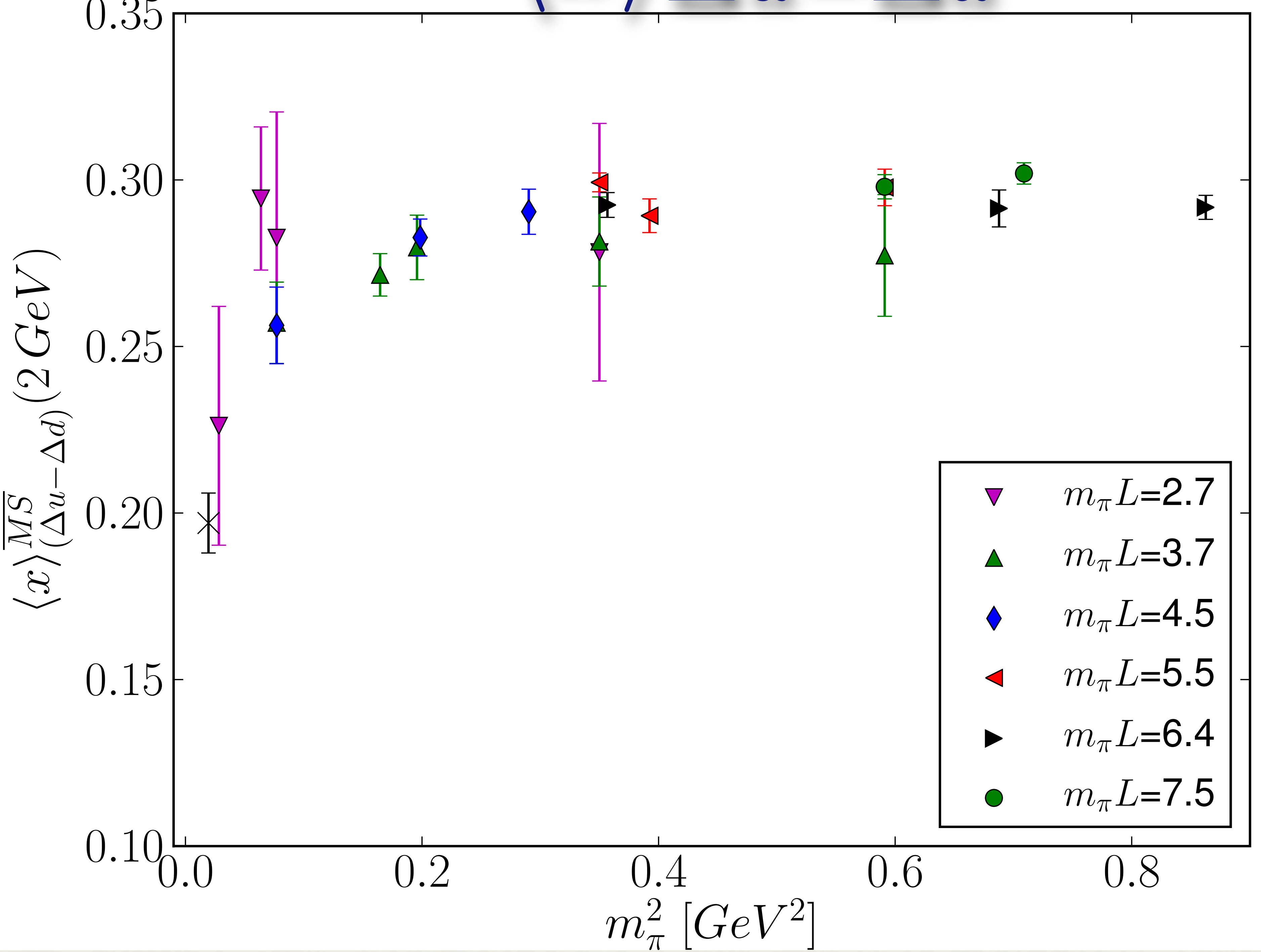
$\langle x \rangle_{\Delta u - \Delta d}$



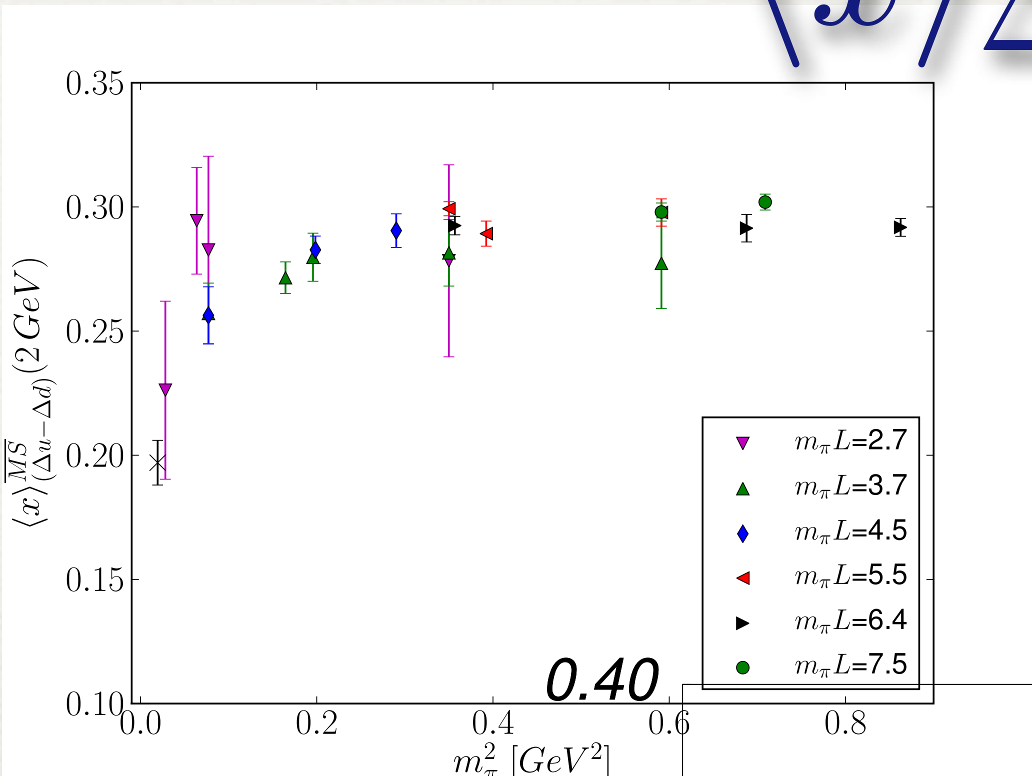
$\langle x \rangle_{\Delta u - \Delta d}$

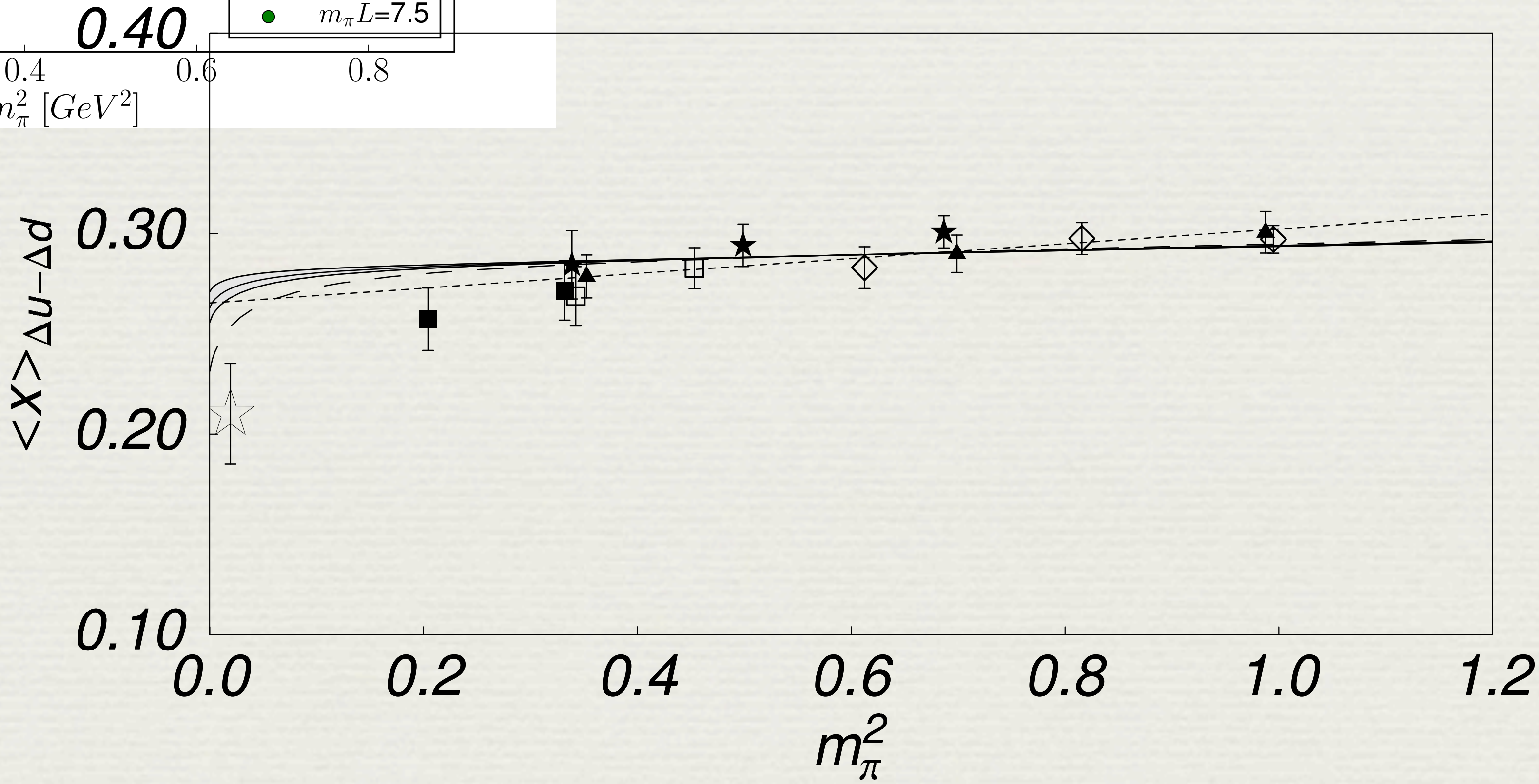


$\langle x \rangle_{\Delta u - \Delta d}$



$\langle x \rangle_{\Delta u - \Delta d}$





Generalised Parton Distributions

Generalised Parton Distributions

Basic properties

M. Diehl (2001): 8 real functions needed for a complete description of the nucleon quark structure at twist 2

$$H(x,\xi,t), E(x,\xi,t) \quad , \quad \tilde{H}(x,\xi,t), \tilde{E}(x,\xi,t)$$

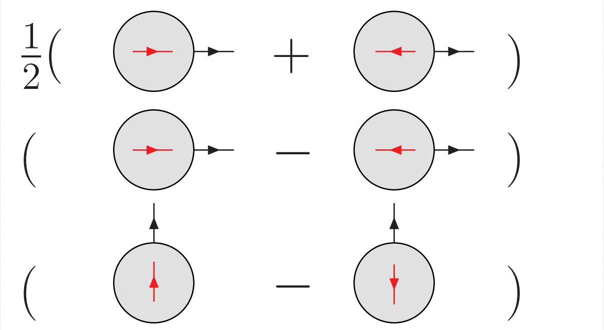
$$H_T(x,\xi,t), E_T(x,\xi,t) \quad , \quad \tilde{H}_T(x,\xi,t), \tilde{E}_T(x,\xi,t)$$

❖ *Forward limit (t=0): reproduces the parton distributions*

$$H(x,0,0) = q(x)$$

$$\tilde{H}(x,0,0) = \Delta q(x)$$

$$H_T(x,0,0) = \delta q(x)$$



Generalised Parton Distributions

Basic properties

M. Diehl (2001): 8 real functions needed for a complete description of the nucleon quark structure at twist 2

$$H(x,\xi,t), E(x,\xi,t) \quad , \quad \tilde{H}(x,\xi,t), \tilde{E}(x,\xi,t)$$

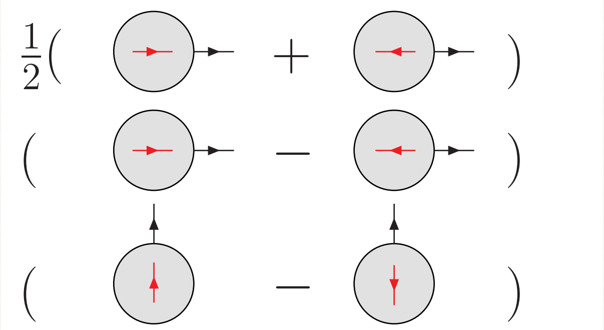
$$H_T(x,\xi,t), E_T(x,\xi,t) \quad , \quad \tilde{H}_T(x,\xi,t), \tilde{E}_T(x,\xi,t)$$

- *Forward limit (t=0): reproduces the parton distributions*

$$H(x,0,0) = q(x)$$

$$\tilde{H}(x,0,0) = \Delta q(x)$$

$$H_T(x,0,0) = \delta q(x)$$



- $\int dx$ *: Form factors*
  - *Dirac:* $\int dx\, H(x,\xi,t) = F_1(t)$
  - *Pauli:* $\int dx\, E(x,\xi,t) = F_2(t)$
  - *Axial:* $\int dx\, \tilde{H}(x,\xi,t) = g_A(t)$
  - *Pseudo-scalar:* $\int dx\, \tilde{E}(x,\xi,t) = g_P(t)$
  - *Tensor:* $\int dx\, H_T(x,\xi,t) = g_T(t)$

Generalised Parton Distributions

$$H(x,\xi,t), E(x,\xi,t) \quad , \quad \tilde{H}(x,\xi,t), \tilde{E}(x,\xi,t)$$

$$H_T(x,\xi,t), E_T(x,\xi,t) \quad , \quad \tilde{H}_T(x,\xi,t), \tilde{E}_T(x,\xi,t)$$

Construct Mellin moments $\int dx\, x^{n-1}$

Non-forward MEs of tower of local twist-2 operators

$$\mathcal{O}_q^{\{\mu_1\cdots\mu_n\}} = \overline{q}\,\gamma^{\{\mu_1}\overleftrightarrow{D}^{\mu_2}\ldots\overleftrightarrow{D}^{\mu_n\}}\,q$$

$$\langle P'|\mathcal{O}_q^{\{\mu_1\cdots\mu_n\}}|P\rangle \propto A_{ni}(t), B_{ni}(t), C_n(t)$$

$$\int_{-1}^{1} dx\, x^{n-1}\, H_q(x,\xi,t) \quad = \quad \sum_{i=0}^{\frac{n-1}{2}} A_{qn,2i}(t)(-2\xi)^{2i} + C_{qn}(t)(-2\xi)^n|_{\text{n even}}$$

$$\int_{-1}^{1} dx\, x^{n-1}\, E_q(x,\xi,t) \quad = \quad \sum_{i=0}^{\frac{n-1}{2}} B_{qn,2i}(t)(-2\xi)^{2i} - C_{qn}(t)(-2\xi)^n|_{\text{n even}}$$

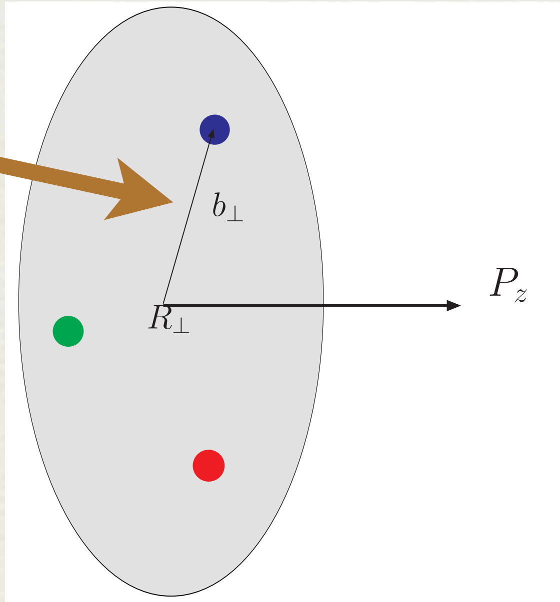
Impact Parameter GPDs (M. Burkardt, 2000)

Quark densities in the transverse plane

Quark (charge) distribution in transverse plane

$$q(b_\perp^2) = \int d^2\Delta_\perp \, e^{-i\vec{b}_\perp \cdot \Delta_\perp} F_1(\Delta^2)$$

Distance of (active) quark to the centre of momentum in a fast moving nucleon



Impact Parameter GPDs (M. Burkardt, 2000)

Quark densities in the transverse plane

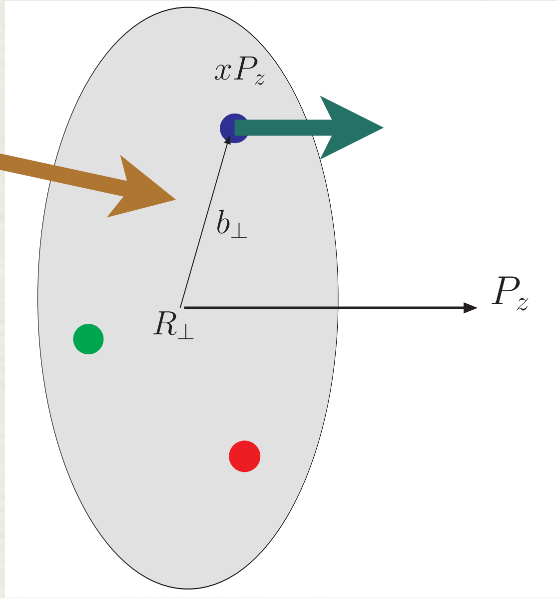
Quark (charge) distribution in transverse plane

$$q(b_\perp^2) = \int d^2\Delta_\perp \, \mathrm{e}^{-i\vec{b}_\perp \cdot \Delta_\perp} F_1(\Delta^2)$$

[Probabilistic interpretation of $H(x,\xi,t)$, $\tilde{H}(x,\xi,t)$, $H_T(x,\xi,t)$ at $\xi = 0$]

$$q(x, \vec{b}_\perp) = \frac{1}{(2\pi)^2} \int d^2\Delta_\perp \, \mathrm{e}^{-i\vec{b}_\perp \cdot \Delta_\perp} H(x, 0, \Delta_\perp^2)$$

Distance of (active) quark to the centre of momentum in a fast moving nucleon



Decompose into contributions from individual quarks with momentum fraction, x

Transverse Spin Structure of the Nucleon

Transverse densities:

$$\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp) = \frac{1}{2}\Bigg\{ A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \Delta_{b_\perp} \widetilde{A}_{Tn0}(b_\perp^2) \right)$$

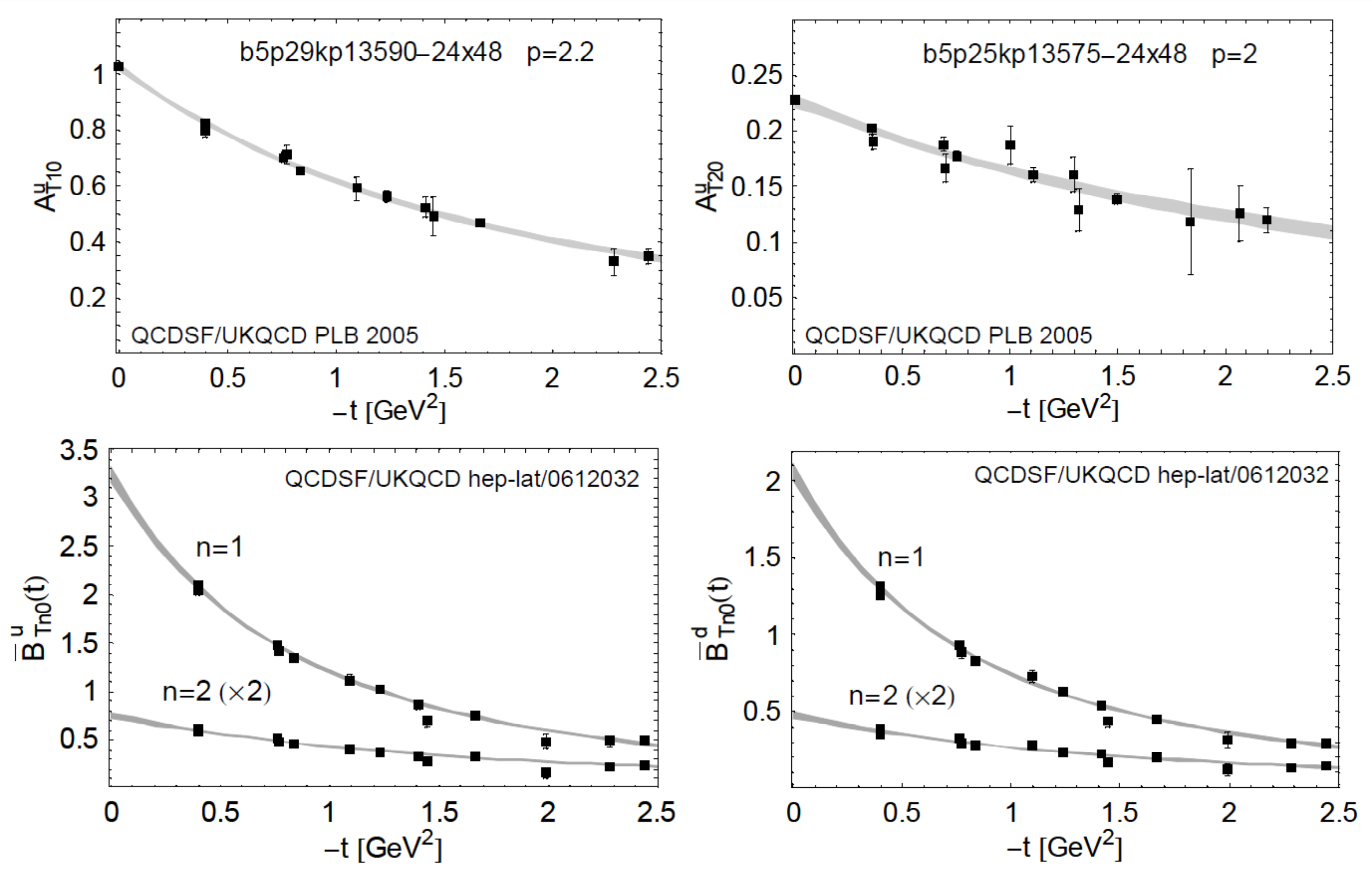
$$+ \frac{b_\perp^j \epsilon^{ji}}{m} \left(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{Tn0}(b_\perp^2) \right) + s_\perp^i (2 b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}) S_\perp^j \frac{1}{m^2} \widetilde{A}''_{Tn0}(b_\perp^2) \Bigg\}$$

[Diehl & Haegler, 2005] [Burkardt, 2005]

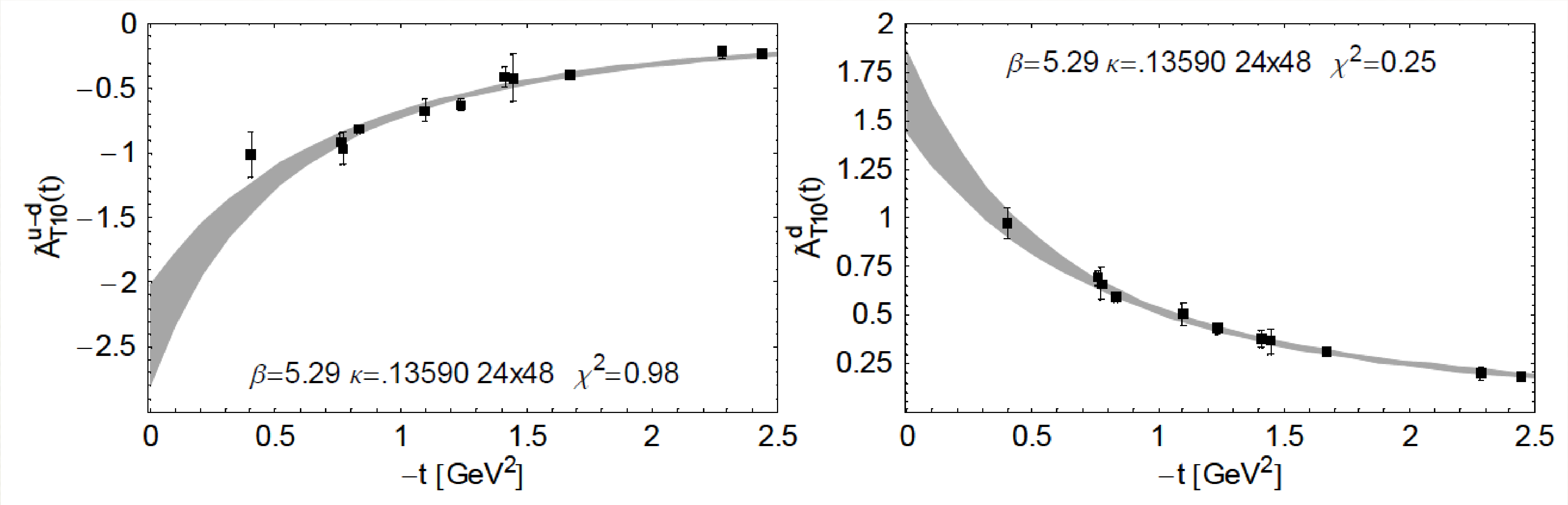
$$F(b_\perp^2) = \int d^2\Delta_\perp\, \mathrm{e}^{-i\vec{b}_\perp \cdot \Delta_\perp} F(\Delta_\perp^2) = \int d^2\Delta_\perp\, \mathrm{e}^{-i\vec{b}_\perp \cdot \Delta_\perp} \frac{F(0)}{(1 - \Delta_\perp^2/M^2)^p}$$

Tensor Form Factors

$\overline{B}_{Tn0}(t)$ *are remarkably large*



Tensor Form Factors



$\overline{A}^d_{Tn0}(t)$ *is sizeable while* $\overline{A}^u_{Tn0}(t) \approx 0$

Anomalous tensor magnetic moment

$$\kappa = \int dx E(x, \xi, 0) = B_{10}(0) = F_2(0)$$

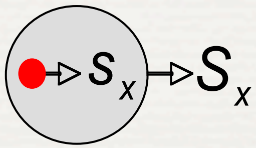
$$\begin{aligned} \kappa_u^{\mathrm{exp}} &\approx 1.67 \\ \kappa_d^{\mathrm{exp}} &\approx -2.03 \end{aligned}$$

$$\kappa_T = \int dx \overline{E}_T(x, \xi, 0) = \overline{B}_{T10}(0)$$

$$\left.\begin{aligned} \kappa_{Tu}^{\mathrm{latt}} &\approx 3.13 \\ \kappa_{Td}^{\mathrm{latt}} &\approx 1.94 \end{aligned}\right\}$$

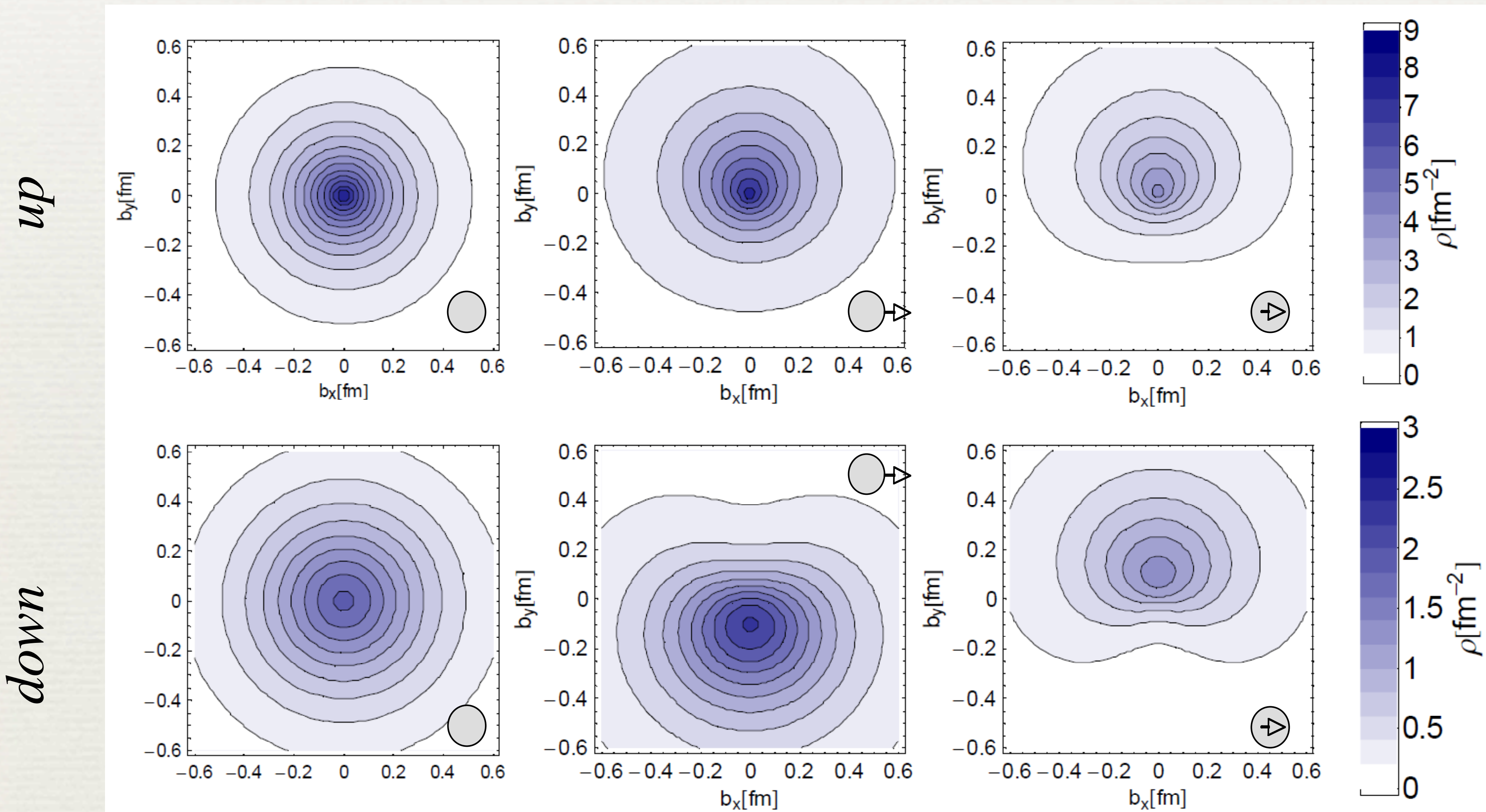
Both positive

Deformed Spin Densities



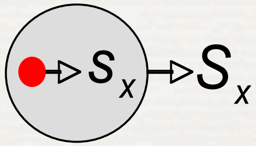
Ph. Hägler (QCDSF) [PRL 98, 222001 (2007)]

(n=1)



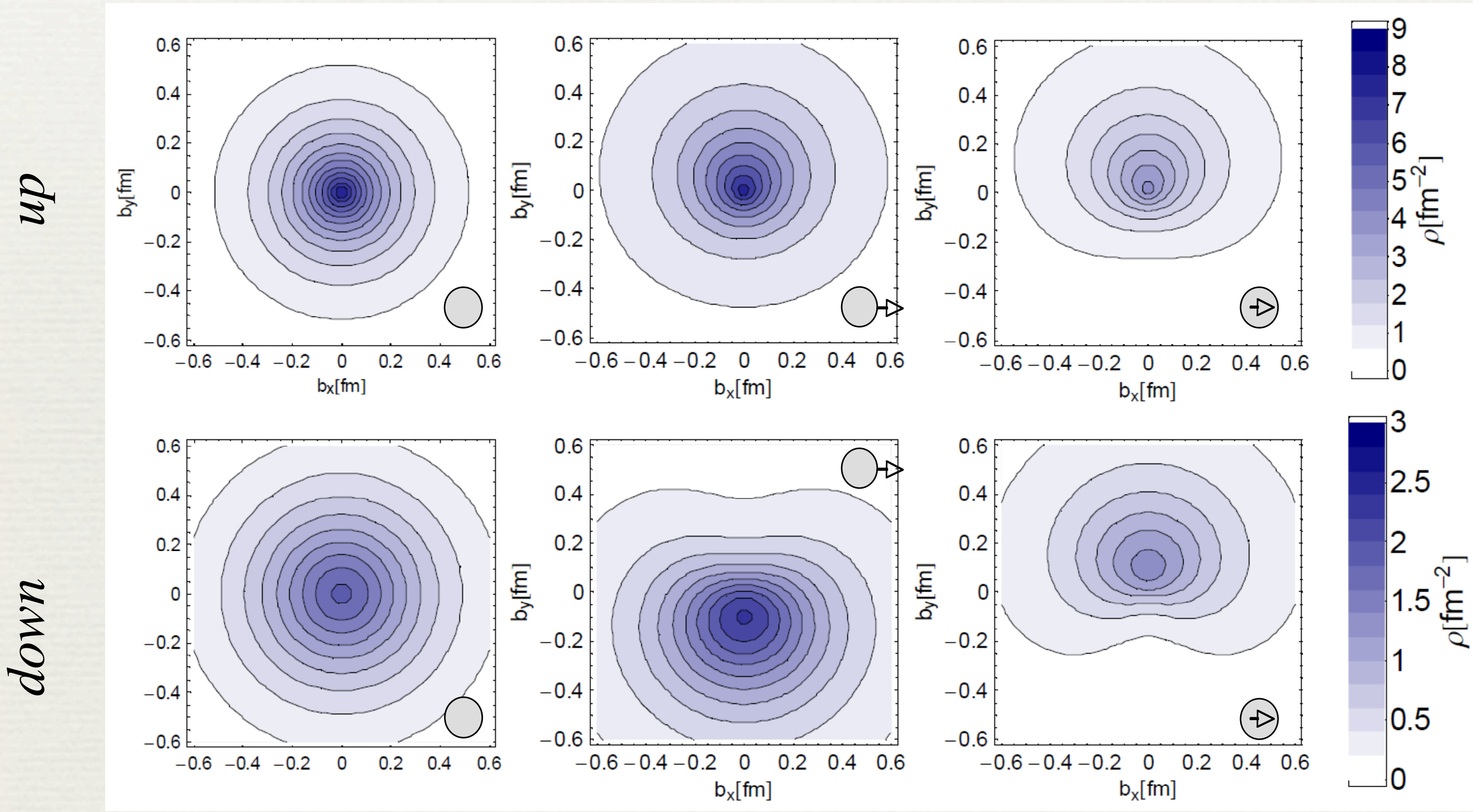
Deformed Spin Densities

Nucleon



Ph. Hägler (QCDSF) [PRL 98, 222001 (2007)]

(n=1)



Conclusion & Outlook

- Calculations of hadronic quantities becoming available close to the physical masses (beware finite size effects)
- Lattice provides a useful tool for investigating FFs/GPDs
- q^2 scaling of F_1, F_1, F_2/F_1, G_E, G_M **(F_1^n negative)**
    - Twisted b.c.s give access to small q^2
- Moments of Generalised Parton Distributions
    - Quark contribution to nucleon spin and angular momentum
    - Non-trivial transverse spin densities in pion and nucleon
- g_A still a challenge.
    - Finite volume effects go in the right direction, but are they enough?
    - Renormalisation? Discretisation?
- $\langle x \rangle$ finite volume analysis at light quark masses indicate results might be going in the right direction indicating "bending down"

Conclusion & Outlook

- Currently improving the 40^3x64 results at the physical point
- Simulations with the same parameters but 64^3x96 volume are starting
- So far have only used N_f=2, now starting simulations with N_f=2+1 flavours of O(a)-improved Clover (Wilson) fermions

Happy Birthday Tony!