The role of strange quarks in nucleon structure

Ross Young
University of Adelaide

Achievements and New Directions in Subatomic Physics:
Workshop in Honour of Tony Thomas's 60th Birthday

University of Adelaide
Chiral Extrapolation

**ChPT:** \[ M_N = c_0 + c_2 m_{\pi}^2 + \chi_{\pi} m_{\pi}^3 + \ldots \]

\[ \chi_{\pi} \simeq -0.63 \text{GeV}^{-2} \]
Chiral Extrapolation

ChPT: \[ M_N = c_0 + c_2 m_{\pi}^2 + \chi_{\pi} m_{\pi}^3 + \ldots \]

Model-independent value
\[ \chi_{\pi} = -5.5 \text{GeV}^{-2} \]

\[ \chi_{\pi} \simeq -0.63 \text{GeV}^{-2} \]
Quenched QCD vs QCD

\[ m_{\pi^2} (\text{GeV}^2) \]

\[ \frac{\Pi}{1 \text{ GeV}^2} \]

Dynamical

Quenched Differences described by chiral loops
Quenched QCD vs QCD

\[ m_B = a_0 + a_2 m_{\pi}^2 + a_4 m_{\pi}^4 + \Sigma \]

<table>
<thead>
<tr>
<th></th>
<th>( a_0 )</th>
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Physics!!

- Let’s use these new tools to learn something new
- How about strangeness?
Elastic Form Factors

Electromagnetic form factors characterise the charge and magnetisation distribution in the nucleon

\[ G_E(Q^2) \quad G_M(Q^2) \]
Elastic Form Factors

Electromagnetic form factors characterise the charge and magnetisation distribution in the nucleon

\[ G_E(Q^2), \quad G_M(Q^2) \]

Measure total response from all quarks

Proton

\[ G_{E,M}^p = +\frac{2}{3} G_{E,M}^u - \frac{1}{3} G_{E,M}^d - \frac{1}{3} G_{E,M}^s \]
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Charge symmetry: up quark in proton = down quark in neutron, ...
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**Neutron**

\[ G_{E,M}^n = \frac{1}{3} G_{E,M}^u + \frac{2}{3} G_{E,M}^d - \frac{1}{3} G_{E,M}^s \]
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Neutron

\[ G_{E,M}^n = -\frac{1}{3} G_{E,M}^u + \frac{2}{3} G_{E,M}^d - \frac{1}{3} G_{E,M}^s \]

2 Equations — 3 Unknowns!
Weak Neutral Form Factor

\[ G_{E,M}^{p,Z} = \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) G_{E,M}^u + \left( -1 + \frac{4}{3} \sin^2 \theta_W \right) G_{E,M}^d + \left( -1 + \frac{4}{3} \sin^2 \theta_W \right) G_{E,M}^s \]

Electroweak couplings differ from usual charges

Weak mixing angle: \( \sin^2 \theta_W \)
Weak Neutral Form Factor

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Weak mixing angle: \( \sin^2 \theta_W \)

\[
G^{p,Z}_{E,M} = \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) G^u_{E,M} + \left( -1 + \frac{4}{3} \sin^2 \theta_W \right) G^d_{E,M} + \left( -1 + \frac{4}{3} \sin^2 \theta_W \right) G^s_{E,M}
\]

\[
G^p_{E,M} = \frac{2}{3} G^u_{E,M} - \frac{1}{3} G^d_{E,M} - \frac{1}{3} G^s_{E,M}
\]

\[
G^n_{E,M} = -\frac{1}{3} G^u_{E,M} + \frac{2}{3} G^d_{E,M} - \frac{1}{3} G^s_{E,M}
\]

3 Equations — 3 Unknowns
Can isolate strangeness!
Imposing Charge Symmetry

\[ p = \frac{2}{3} u^p - \frac{1}{3} u^n + O_N \]

\[ n = -\frac{1}{3} u^p + \frac{2}{3} u^n + O_N \]

\[ 3O_N = 2p + n - u^p \]
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\[ \Sigma^+ = \frac{2}{3} u^\Sigma - \frac{1}{3} s^\Sigma + O_\Sigma \]
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\[ \Sigma^+ - \Sigma^- = u^\Sigma \]

\[ 3O_N = 2p + n - \frac{u^p}{u^\Sigma} (\Sigma^+ - \Sigma^-) \]
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\[ \Xi^0 - \Xi^- = u^\Xi \]

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\[ \Xi^0 - \Xi^- = u^{\Xi} \]

\[ 3O_N = p + 2n - \left( \frac{u^n}{u^{\Xi}} \right) (\Xi^0 - \Xi^-) \]

Lattice QCD
Constraint on GMs
$u$-quark in the proton
$u$-quark in the proton

\[ \mu (\mu_N) \quad m_{\pi}^2 \text{ (GeV}^2\text{)} \]
$u$-quark in the proton
$u$-quark in the Sigma
Magnetic strangeness result

\[ \frac{u^p}{u^\Sigma} = 1.092 \pm 0.030 \]
\[ \frac{u^n}{u^\Xi} = 1.254 \pm 0.124 \]

\[ G_M^s = -0.046 \pm 0.022 \mu_N \]
Repeat analysis for electric radius

Valence quarks in proton

$G_E^s (Q^2 = 0.1) = +0.001 \pm 0.004 \pm 0.004$
Strangeness Measurements

SAMPLE @ MIT-Bates
Strangeness Measurements

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PVA4 @ MAMI
Strangeness Measurements

SAMPLE @ MIT-Bates

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JLab
Strangeness Measurements

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JLab

PVA4 @ MAMI

G0
Strangeness Measurements

SAMPLE @ MIT-Bates

HAPPEX

JLab

PVA4 @ MAMI

G0
Experimental Status (2005)

- Conglomerate of world’s strangeness measurements
- Uses constraint from best theoretical estimate of anapole form factor \( \text{Zhu et al.} \)

\[ Q^2 = 0.1 \text{GeV}^2 \]
Global Analysis

• Extract the anapole contribution from experiment
  \[ \tilde{G}^N_A = \tilde{g}^N_A (1 + Q^2/\Lambda^2)^{-2} \]

• Fit strangeness to measured asymmetries

• Consistent treatment of electromagnetic form factors and radiative corrections

• Use all available data for \( Q^2 < 0.3 \text{GeV}^2 \)

• Taylor expansion of strangeness
  \[ G^s_E = \rho_s Q^2 + \rho'_s Q^4 + \ldots \]
  \[ G^s_M = \mu_s + \mu'_s Q^2 + \ldots \]
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  \[ G_E^s = \rho_s Q^2 + \rho_s' Q^4 + \ldots \]
  \[ G_M^s = \mu_s + \mu_s' Q^2 + \ldots \]
Strangeness form factors

\[ Q^2 = 0.1 \text{GeV}^2 \]
New HAPPEX Measurement

- Excellent agreement with global analysis

Strange Electric

Strange Magnetic

HAPPEX, PRL(2007)
Combined Analysis

- Combined constraints on current knowledge of strangeness content.
- Strangeness is small!
- 95% confidence: < 5% charge radius < 6% magnetic moment
- In support of theory estimate
Searching for new physics

Constrained by low-energy data!

\[ \mathcal{L}_{SM}^{PV} = -\frac{G_F}{\sqrt{2}} \bar{e} \gamma_\mu \gamma_5 e \sum_q C_{1q}^{SM} \bar{q} \gamma^\mu q \]
Proton weak charge extrapolation

Proton weak charge

PDG

SM

HAPPEX

SAMPLE

G0

PVA4

Theory estimate for anapole FF

$\overline{A_{LR}^P}$

$Q^2 (\text{GeV}^2)$
Current knowledge and future: Q-weak

\[
\begin{align*}
C_{1u} &- C_{1d} \\
\end{align*}
\]
Q-weak: Assuming SM

\[ \frac{\pi}{2} \left( \frac{3\pi}{2} \right) \]

\[ \frac{\pi}{2} \left( \frac{3\pi}{2} \right) \]

Atomic and others with PVES

95% CL
Q-weak constrains new physics to beyond 2 TeV
Strange condensate in the nucleon

- Gell-Mann–Okubo Relation and Pion-Nucleon sigma term $M_N, M_\Lambda, M_\Sigma, M_\Xi$

\[
m_s \langle N | \bar{s}s | N \rangle \sim 335 \pm 132 \text{ MeV}
\]

Nelson & Kaplan PLB(1987)

\[
\sim M_N^{\text{phys}} - M_N^{SU(3)\text{chiral limit}}
\]
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QCD Lagrangian

\[ \sim \ldots \bar{s}(\mathcal{D} + m_s)s \]

\[ m_s \langle N | \bar{s}s | N \rangle = m_s \frac{\partial M_N}{\partial m_s} \]

evaluated at physical point!
Strange condensate in the nucleon

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evaluated at physical point!

Improved Effective Field Theory estimate

\[ m_s \frac{\partial M_N}{\partial m_s} = 113 \pm 108 \text{ MeV} \]

Borasoy & Meissner (1997)
Fit to 8 LHPC points

\[ m_{s}^{latt} \sim 1.3 m_{s}^{phys} \]

Strange quark-mass correction

Excellent description of lattice results
Accurate prediction of heavier simulation data
Reliable correction for lattice simulation quark mass

Not included in fit

Lattice
Experiment
Fit extrapolation

\[ m_B (GeV) \]
\[ m_{\pi}^2 (GeV^2) \]
Fit to 8 PACS-CS points

PACS-CS: 2+1-flavour simulation; different action discretization to LHPC

Correction in strange quark mass demonstrated to be reliable against numerical simulation

As for LHPC, excellent agreement with observed spectrum

PACS-CS have an additional run with a different strange quark mass
Quantifying Sources of Uncertainty

Nucleon Mass (GeV)

<table>
<thead>
<tr>
<th>Source</th>
<th>MeV</th>
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<tr>
<td>Statistical</td>
<td>23.6</td>
</tr>
<tr>
<td>Discretisation</td>
<td>4.2</td>
</tr>
<tr>
<td>Model</td>
<td>3.1</td>
</tr>
<tr>
<td>Regulator</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.1</td>
</tr>
<tr>
<td>$f_\pi$ (5%)</td>
<td>0.7</td>
</tr>
<tr>
<td>$F$ (15%)</td>
<td>1.3</td>
</tr>
<tr>
<td>$D$ (15%)</td>
<td>1.3</td>
</tr>
<tr>
<td>$C$ (15%)</td>
<td>0.9</td>
</tr>
<tr>
<td>$\Delta_{10-8}$ (15%)</td>
<td>0.4</td>
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Discretisation

LHPC  0.945 ± 0.029
PACS-CS 0.954 ± 0.042

Extrapolated baryon masses and fit parameters (LECs) in agreement — good news for us and lattice practitioners

Regulator

Dipole  0.9410
Sharp  0.9452

Small dependence on choice of regulator — similarly for other functional forms
**Baryon Sigma Terms**

\[
\bar{\sigma}_{Bq} = \frac{m_q}{M_B} \frac{\partial M_B}{\partial m_q}
\]

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<td>(\bar{\sigma}_{Bl})</td>
<td>0.050(9)(1)(3)</td>
<td>0.028(4)(1)(2)</td>
<td>0.0212(27)(1)(17)</td>
<td>0.0100(10)(0)(4)</td>
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<tr>
<td>(\bar{\sigma}_{Bs})</td>
<td>0.033(16)(4)(2)</td>
<td>0.144(15)(10)(2)</td>
<td>0.187(15)(3)(4)</td>
<td>0.244(15)(12)(2)</td>
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\(\bar{\sigma}_{Nl}\) / (GeV)

\(\bar{\sigma}_{Ns}\) / (GeV)

\(\bar{\sigma}_{Bq}\) / (GeV)

\(\bar{\sigma}_{Bq}\) / (GeV)

**πN Sigma Term (Expt):**
- GW: Pavan et al. (2001)

**Octet Masses & Breaking:**
- Gasser (1981)
- BM: Borasoy & Meissner (1997)

**3-flavour Lattice QCD:**
- YT: Young & Thomas (2009)
- TF: Toussaint & Freeman (2009)

We determine precisely *both* the light and strange quark sigma terms.
Spin-independent neutralino cross sections

- Ellis, Olive & Savage, PRD(2008)
  - Constrained Minimal Supersymmetric Standard Model (CMSSM)
  - Neutralino as dark matter candidate

- Scalar contact interaction
  \[ \mathcal{L}_{SI} = \sum_i \alpha_{3i} \bar{\chi} \chi q_i \]

\[ \sigma_{SI}^p \propto |f_p|^2 \]

\[ \frac{f_p}{M_p} = \sum_{q=u,d,s} \bar{\sigma}_{pq} \frac{\alpha_{3q}}{m_q} + \frac{2}{27} f_T^p \sum_{q=c,b,t} \frac{\alpha_{3q}}{m_q} \]

\[ f_{TG}^p = 1 - \sum_{q=u,d,s} \bar{\sigma}_{pq} \]

Trace anomaly:
Shifman, Vainstein & Zakharov, PLB(1978)

Uncertainty dominated by knowledge of light-quark sigma terms
Updated cross sections for benchmark models

Ellis, Olive & Savage

Strong dependence on sigma term from poorly known strangeness

Giedt, Thomas & Young, PRL (2009)

Tremendous advance in precision from new lattice QCD results

Nuclear physics & lattice QCD can help discriminate supersymmetry scenarios
Summary

- Strange quarks play a relatively small role in the nucleon structure
- “Spin-offs”: precision electroweak physics, dark matter scattering
Summary

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- “Spin-offs”: precision electroweak physics, dark matter scattering

HAPPY BIRTHDAY TONY!
Anapole Form Factor

- Backward angle measurements have increased sensitivity to axial form factor.
- Anapole corresponds to an electroweak correction to the proton structure.
- Anapole form factor is not measured in any other process.
Anapole Form Factor

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- Anapole corresponds to an electroweak correction to the proton structure.
- Anapole form factor is not measured in any other process.

Best theory estimate: Zhu et al.
The axial/anapole term

- Global constraint from data

Theory, Zhu et al.