The Spin of the Nucleon
- walking in Tony's footprints -

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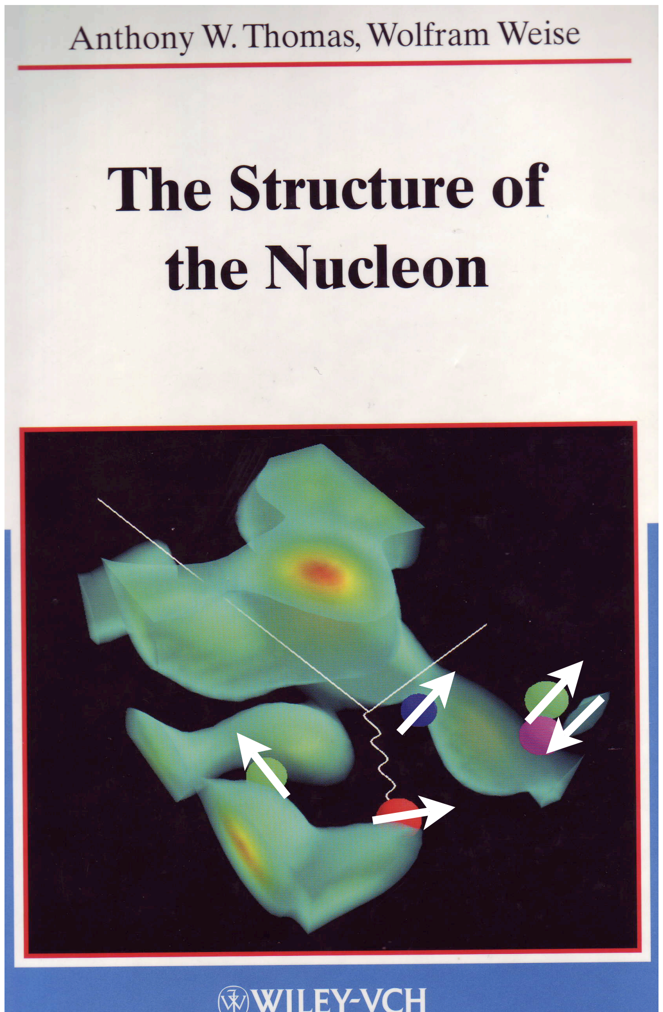

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Interplay of Spin and Orbital Angular Momentum in the Proton

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Abstract

We derive the consequences of the Myhrer-Thomas explanation of the proton spin problem for the distribution of orbital angular momentum on the valence and sea quarks. After QCD evolution these results are found to be in very good agreement with both recent lattice QCD calculations and the experimental constraints from Hermes and JLab.

Nucleon Spin in the Context of QCD

- Basic gauge invariant operators -

... derived from angular momentum tensor $\mathbf{M}^{\lambda\mu\nu} = \mathbf{x}^{\mu}\mathbf{T}^{\lambda\nu} - \mathbf{x}^{\nu}\mathbf{T}^{\lambda\mu}$

- Quark spin: $\vec{\mathbf{S}}_{\mathbf{q}} = \frac{1}{2}\int d^3x\, \bar{\psi}_q(x)\, \vec{\gamma}\, \gamma_5\, \psi_q(x)$
- Quark orbital angular momentum: $\vec{\mathbf{L}}_{\mathbf{q}} = \int d^3x\, \psi_q^{\dagger}(x)\, (\vec{x} \times i\vec{D})\, \psi_q(x)$
- Gluon angular momentum: $\vec{\mathbf{J}}_{\mathbf{g}} = \int d^3x\, (\vec{x} \times (\vec{E} \times \vec{B}))$

- total: $\vec{\mathbf{J}} = \vec{\mathbf{J}}_{\mathbf{q}} + \vec{\mathbf{J}}_{\mathbf{g}} = \vec{\mathbf{S}}_{\mathbf{q}} + \vec{\mathbf{L}}_{\mathbf{q}} + \vec{\mathbf{J}}_{\mathbf{g}}$

- Nucleon Spin Sum:

$$\frac{1}{2} = \langle P\uparrow|\mathbf{J}_3|P\uparrow\rangle = \langle P\uparrow|\mathbf{S}_{\mathbf{q},3} + \mathbf{L}_{\mathbf{q},3} + \mathbf{J}_{\mathbf{g},3}|P\uparrow\rangle$$
$$= \frac{1}{2}\Delta\Sigma(\mu) + L_q(\mu) + J_g(\mu)$$

Gluon contribution to **Nucleon Spin**

- Gauge dependent decomposition: $\vec{\mathbf{J}}_{\mathbf{g}} = \vec{\mathbf{L}}_{\mathbf{g}} + \vec{\mathbf{G}}$

$$\vec{\mathbf{L}}_{\mathbf{g}} = \int d^3x \, [E_i^a (\vec{x} \times \vec{\nabla}) A_i^a - g(\vec{x} \times \vec{A}^a) \, \psi^\dagger \frac{\lambda^a}{2} \psi]$$

$$\vec{\mathbf{G}} = \int d^3x \, (\vec{E}^a \times \vec{A}^a)$$

- Matrix elements:

$$L_g = \langle P \uparrow | \mathbf{L}_{\mathbf{g},3} | P \uparrow \rangle$$

$$\Delta G = \langle P \uparrow | \mathbf{G}_3 | P \uparrow \rangle \quad \leftrightarrow \quad \int_0^1 dx \, \Delta g(x, Q^2)$$

light-cone gauge
$(A^+ = 0)$

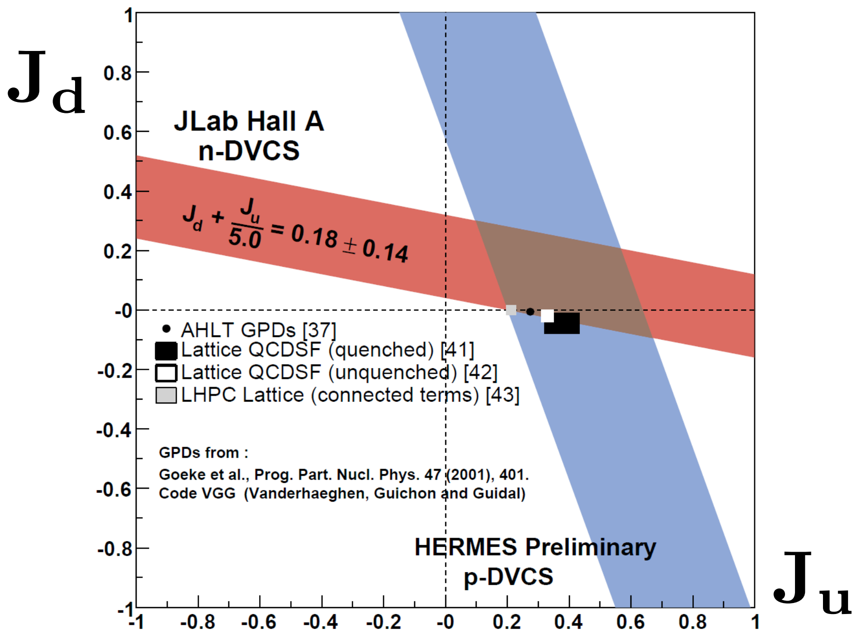
- Spin sum rule: $\boxed{\frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma(\mu)}_{\text{gauge invariant}} + L_q(\mu) + \underbrace{\Delta G(\mu) + L_g(\mu)}_{\text{gauge dependent}}}$

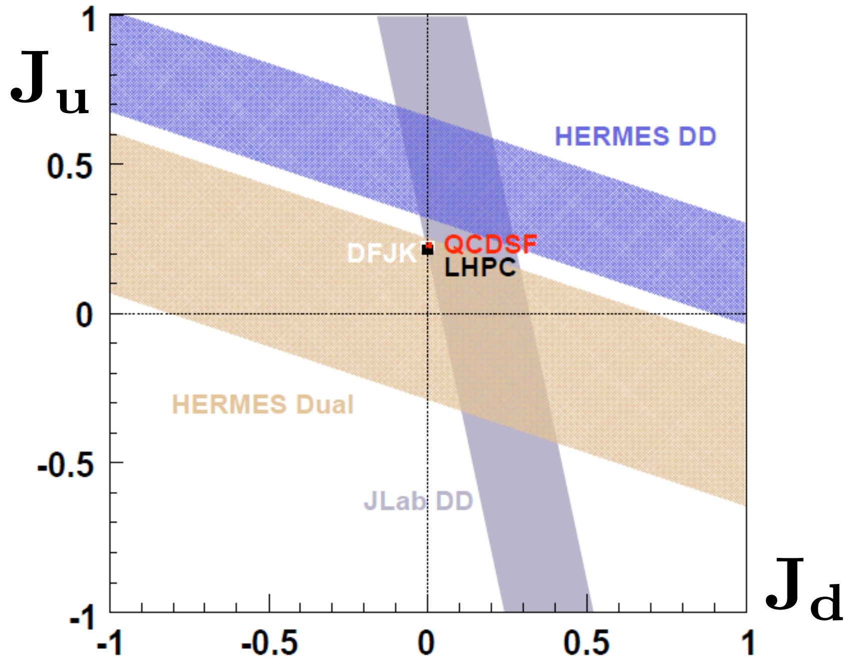
Nucleon Spin: Empirical Facts

- ... from polarized DIS, inclusive and semi-inclusive:
  (**HERMES** PRD 75 (2007) 012007 and **COMPASS** PLB 647 (2007) 8)

$$\boldsymbol{\Delta\Sigma}(5\,GeV^2) = \mathbf{0.33}\ (\pm\,20\,\%) \quad \left\{ \begin{array}{l} \boldsymbol{\Delta\Sigma_u} = \Delta u + \Delta\bar{u} = \mathbf{0.84}\ (\pm\,3\,\%) \\ \boldsymbol{\Delta\Sigma_d} = \Delta d + \Delta\bar{d} = \mathbf{-0.43}\ (\pm\,5\,\%) \\ \boldsymbol{\Delta\Sigma_s} = \Delta s + \Delta\bar{s} = \mathbf{-0.08}\ (\pm\,35\,\%) \end{array} \right.$$

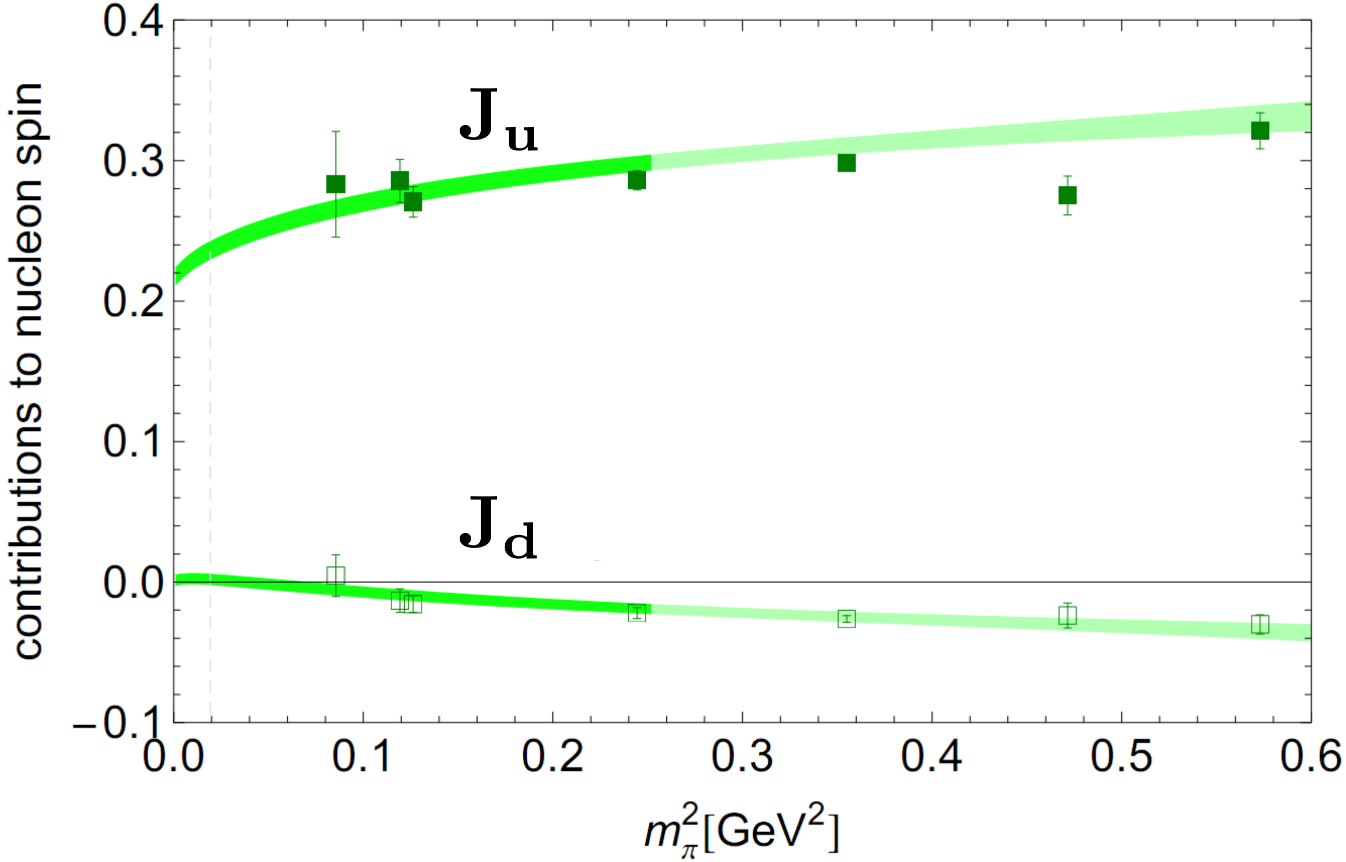
- ... from DVCS: (**JLAB** PRL 99 (2007) 242501 and **HERMES** JHEP 0806:066 (2008))





Nucleon Spin: Lattice QCD

- Recent results from LHPC arXiv:1001.3620 $(\overline{\mathbf{MS}}\ @\ \mathbf{4\,GeV^2})$

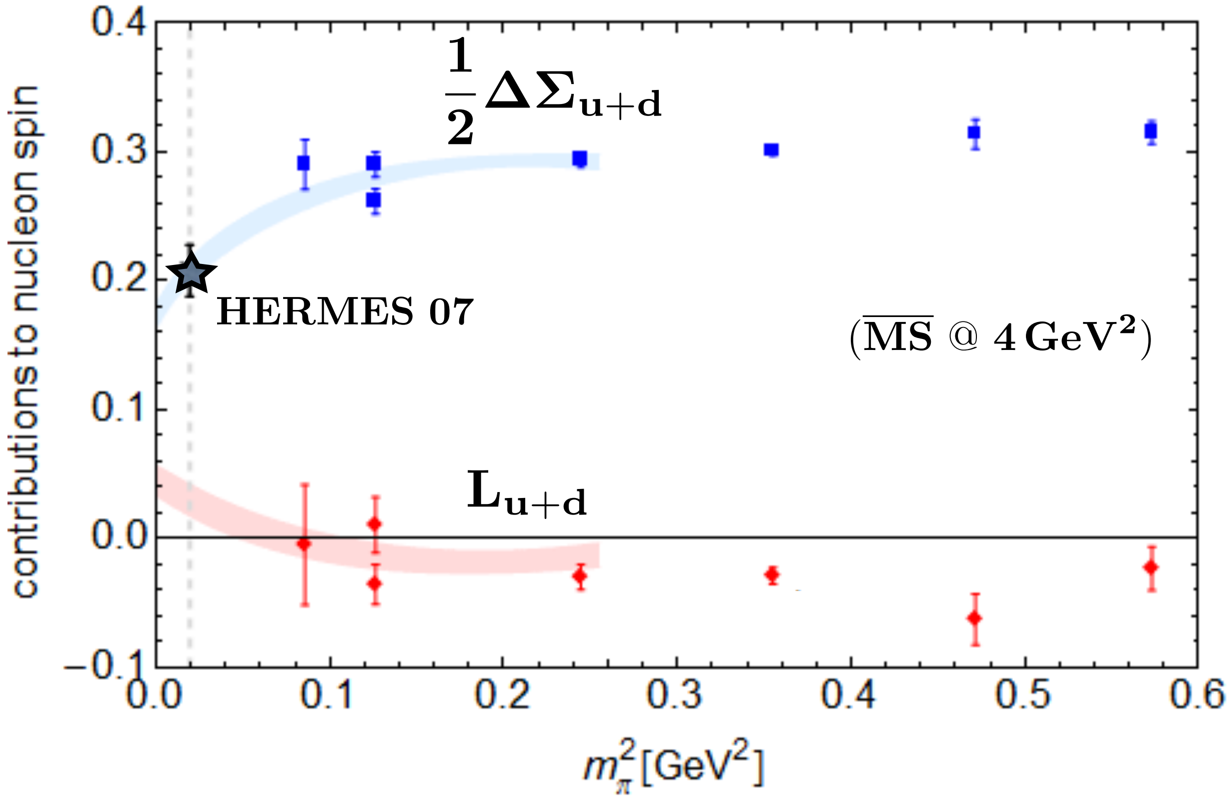


- Chiral extrapolation: covariant heavy-baryon ChPT

$$\left.\begin{array}{l}\mathbf{J_u = 0.236(6)} \\ \mathbf{J_d = 0.002(4)}\end{array}\right\} \quad \mathbf{J_{u+d} = 0.238 \pm 0.008} \quad (\simeq \mathbf{48\%\ of\ 1/2})$$

Nucleon Spin: Lattice QCD (contd.)

- Recent results from LHPC   arXiv:1001.3620, PRD 77 (2008) 094502



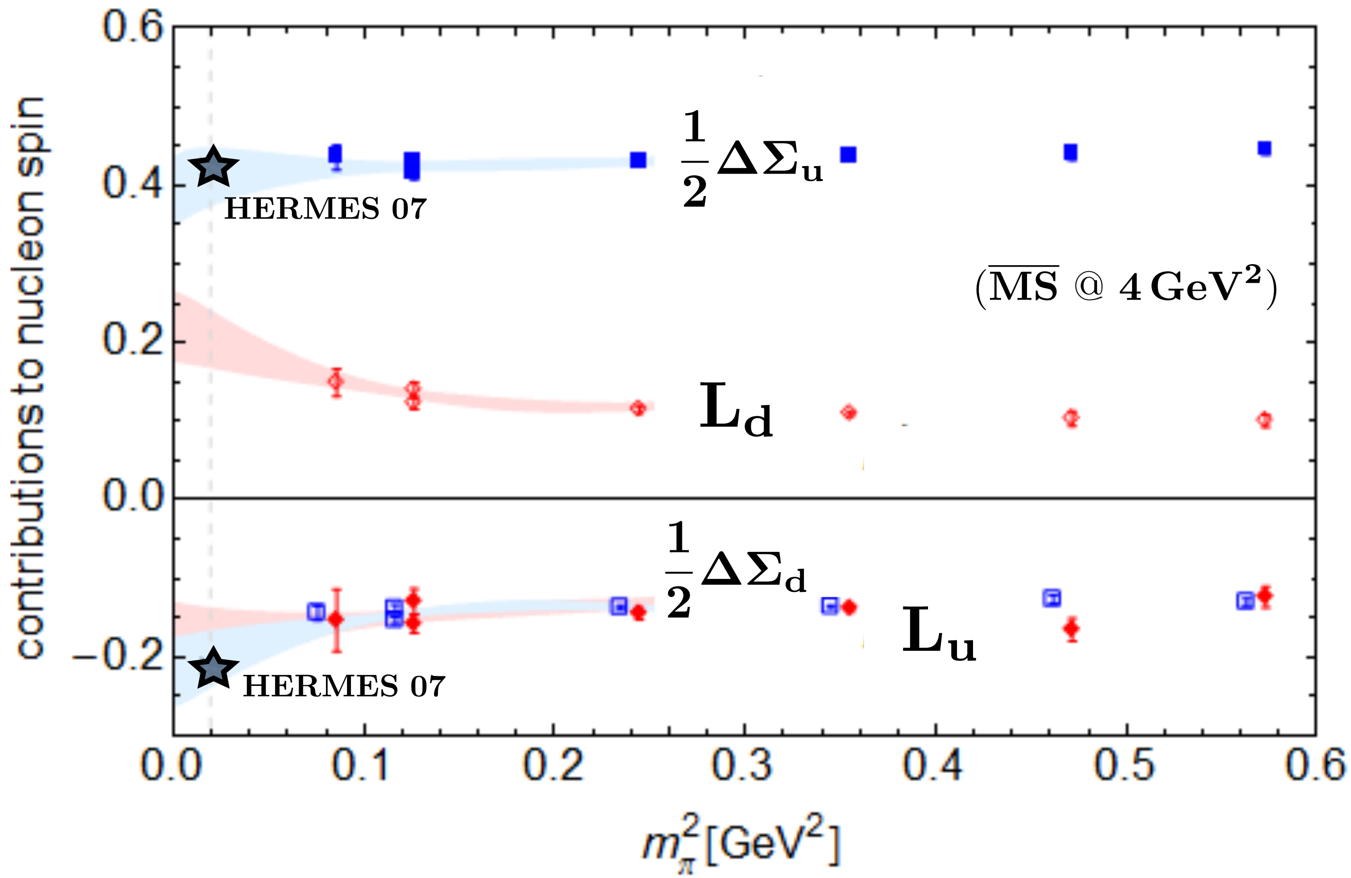
chiral extrapolations:
heavy-baryon ChPT

- $\dfrac{\Delta\Sigma_{u+d}}{2} = 0.21 \pm 0.01$ $L_{u+d} = 0.03 \pm 0.01$!

small orbital angular momentum contribution
seems not compatible with phenomenology / relativistic quark models

Nucleon Spin: Lattice QCD (contd.)

- Recent results from LHPC arXiv:1001.3620, PRD 77 (2008) 094502



- Separate contributions from **u**- and **d**-quarks:

$$\frac{1}{2}\Delta\Sigma_u = 0.41 \pm 0.03 \qquad \frac{1}{2}\Delta\Sigma_d = -0.20 \pm 0.03$$

$$L_d \simeq -L_u = 0.20 \pm 0.04$$

Myhrer - Thomas Scenario

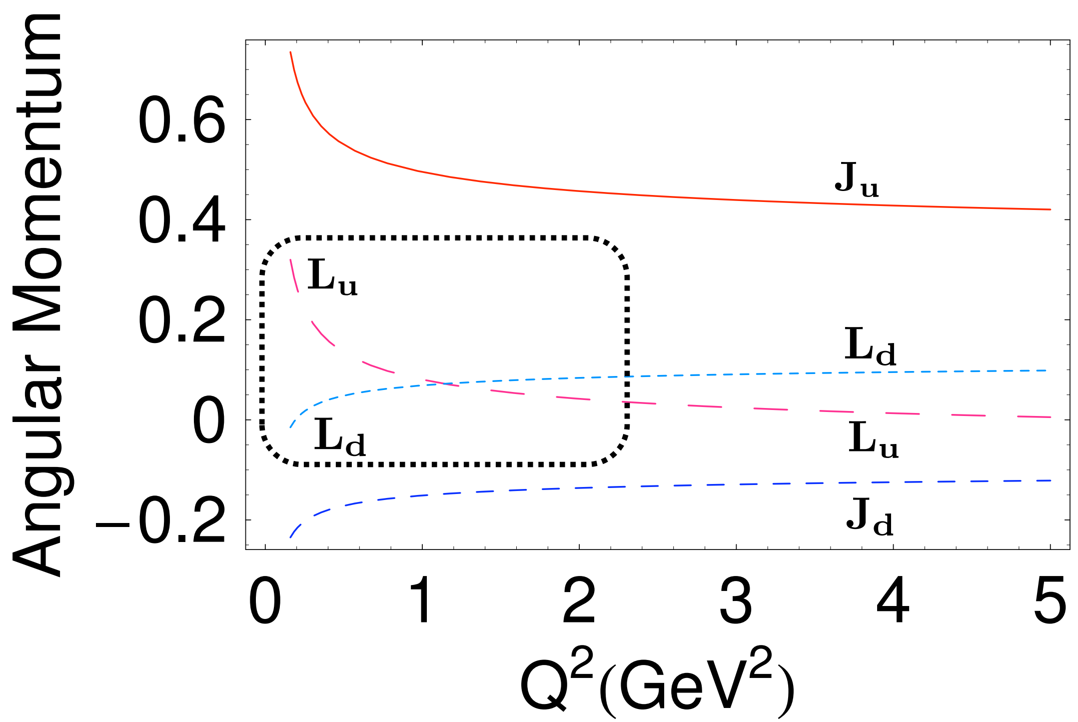
(F. Myhrer and A.W. Thomas, Phys. Lett. B 663 (2008) 302)

- Modelling the nucleon spin:

successively added contributions from

valence quarks
+
one-**gluon** exchange
+
pion cloud effects

	$2L^u$	$2L^d$	$\Delta\Sigma$
Non-relativistic	0	0	1
Relativistic	0.46	-0.11	0.65
OGE	0.67	-0.16	0.49
Pion cloud	0.64	-0.03	0.39



- Scale dependence: QCD evolution

A.W. Thomas, Phys. Rev. Lett. 101 (2008) 102003

see also:
semi-empirical analysis
M. Wakamatsu, arXiv:0908.0972

QCD Evolution: NLO

Evolution equations for $\mathbf{J_q}$ and $\mathbf{J_g}$

- **singlet**:

$$
\begin{aligned}
\frac{d}{d\ln\mu^2}\begin{pmatrix} J_q(\mu^2) \\ J_g(\mu^2)\end{pmatrix} &= \\
&-\frac{1}{2}\frac{\alpha_s}{4\pi}\begin{pmatrix} \frac{64}{9} & -\frac{4}{3}n_F \\ -\frac{64}{9} & \frac{4}{3}n_F \end{pmatrix}\begin{pmatrix} J_q(\mu^2) \\ J_g(\mu^2)\end{pmatrix} \\
&-\frac{1}{2}\left(\frac{\alpha_s}{4\pi}\right)^2\begin{pmatrix} \frac{23488}{243}-\frac{832}{81}n_F & -\frac{1222}{81}n_F \\ -\frac{23488}{243}+\frac{832}{81}n_F & \frac{1222}{81}n_F \end{pmatrix}\begin{pmatrix} J_q(\mu^2) \\ J_g(\mu^2)\end{pmatrix}
\end{aligned}
$$

- **non-singlet**:

$$
\frac{d}{d\ln\mu^2}J_q^{NS}(\mu^2) = -\frac{1}{2}\frac{\alpha_s}{4\pi}\left(\frac{8}{3}\right)^2 J_q^{NS}(\mu^2) - \frac{1}{2}\left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{23488}{243}-\frac{512}{81}n_F\right)J_q^{NS}(\mu^2)
$$

QCD Evolution: NLO (contd.)

Evolution equations for $\mathbf{\Delta\Sigma}$ and $\mathbf{\Delta G}$

- **singlet**:

$$\frac{d}{d\ln\mu^2}\begin{pmatrix}\Delta\Sigma(\mu)\\ \Delta G(\mu)\end{pmatrix} = -\frac{1}{2}\frac{\alpha_s}{4\pi}\begin{pmatrix}0 & 0\\ -8 & -2\beta_0\end{pmatrix}\begin{pmatrix}\Delta\Sigma(\mu)\\ \Delta G(\mu)\end{pmatrix} -\frac{1}{2}\left(\frac{\alpha_s}{4\pi}\right)^2\begin{pmatrix}16n_F & 0\\ -200+\frac{16}{9}n_F & -2\beta_1\end{pmatrix}\begin{pmatrix}\Delta\Sigma(\mu)\\ \Delta G(\mu)\end{pmatrix}$$

- **non-singlet**:

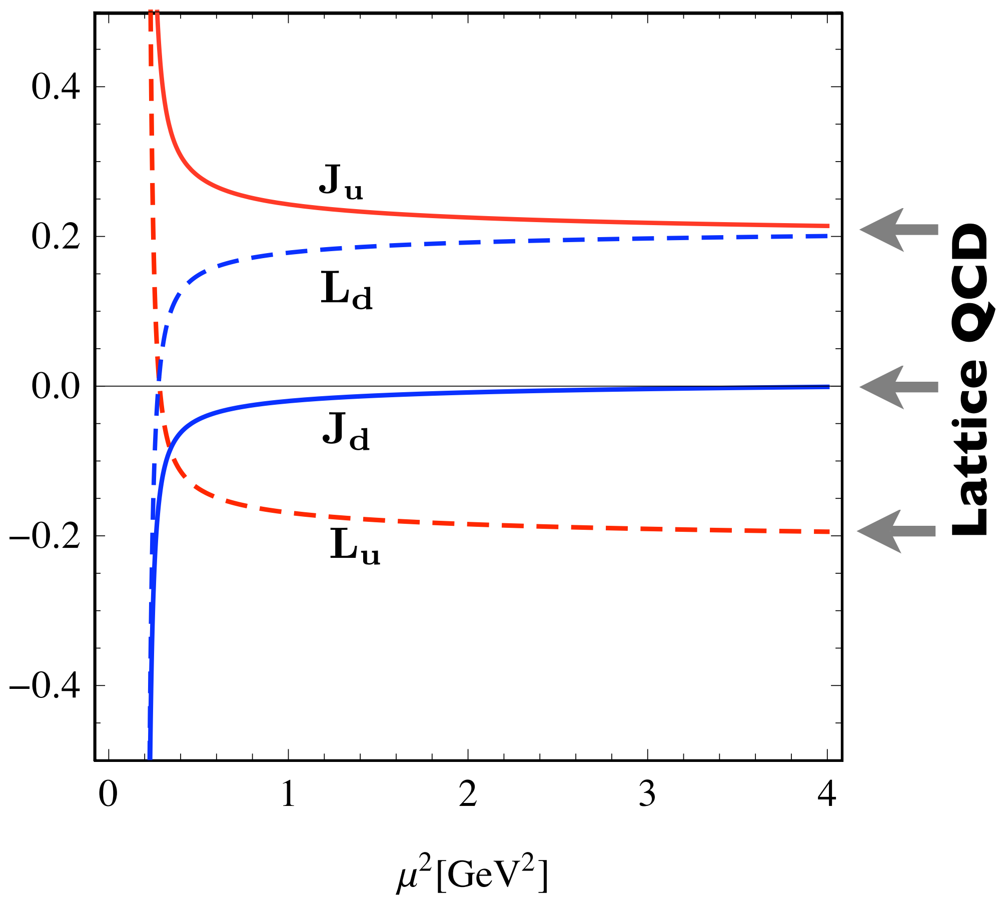
$$\frac{d}{d\ln\mu}\Delta\Sigma^{NS} = 0$$

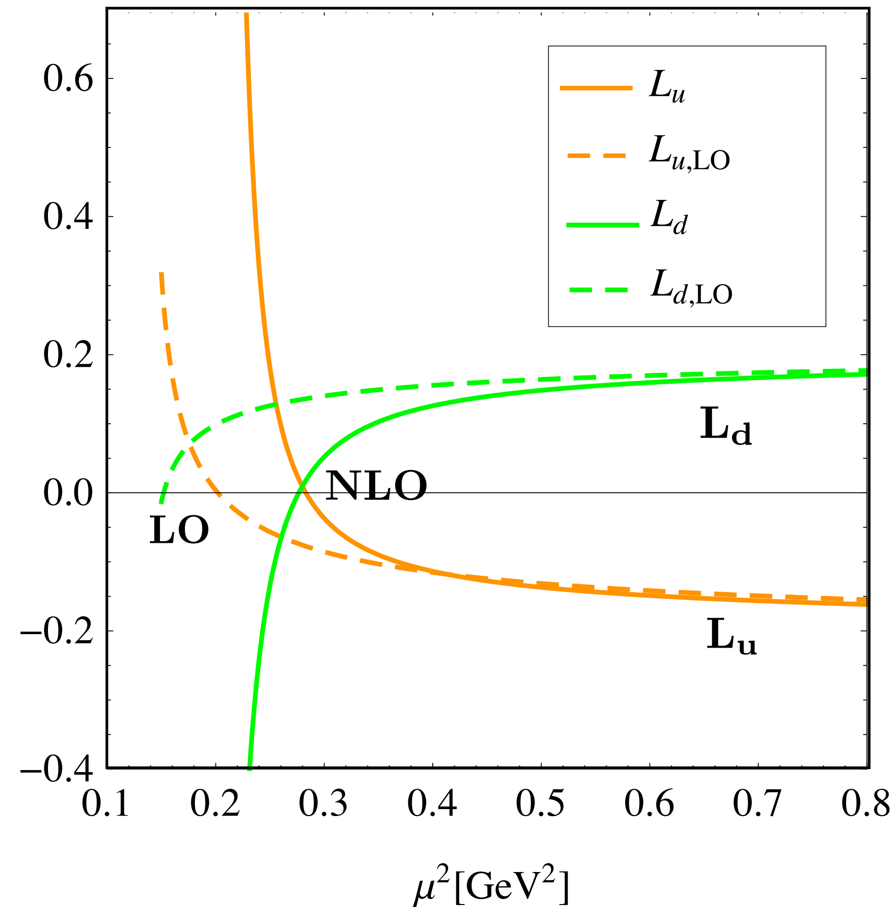
- **orbital angular momenta**:

combine evolution equations for $\mathbf{L_q = J_q - \frac{1}{2}\Delta\Sigma}$ and $\mathbf{L_g = J_g - \Delta G}$

QCD Evolution: NLO (contd.)

M. Altenbuchinger, diploma thesis (2009)





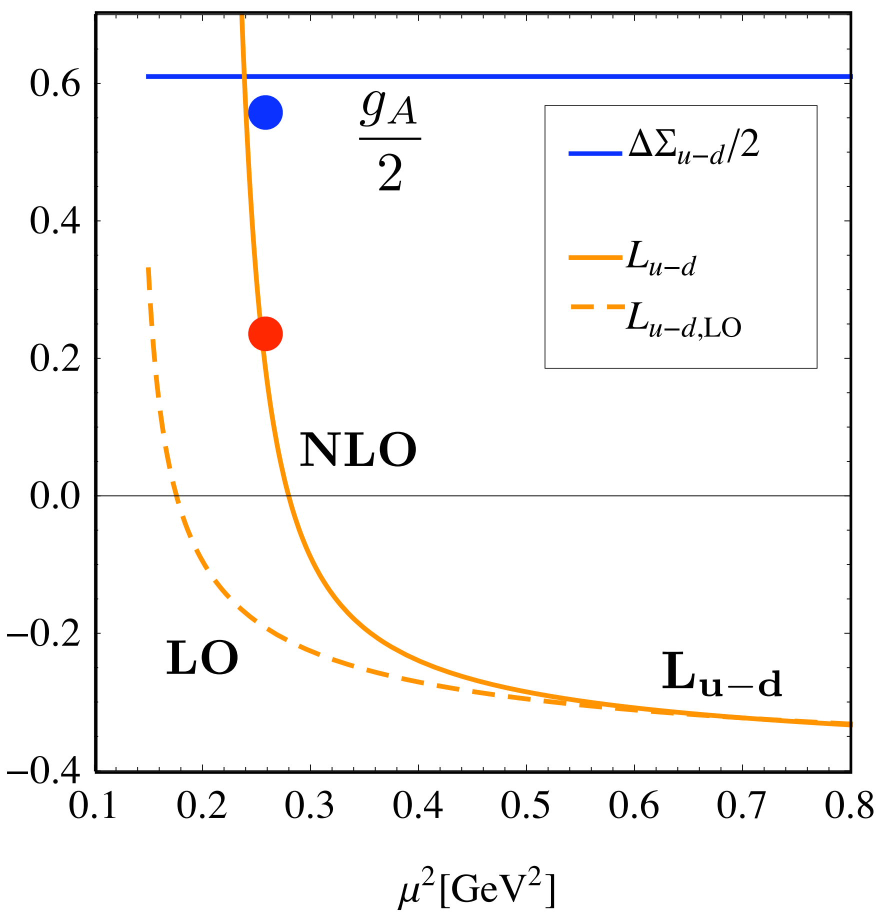
- Input: Lattice QCD at $\mu^2 = 4\,\mathrm{GeV}^2$
- Convergence issues: LO versus NLO

M. Altenbuchinger, Ph. Hägler, W.W. ,
in preparation (2010)

QCD Evolution (contd.)

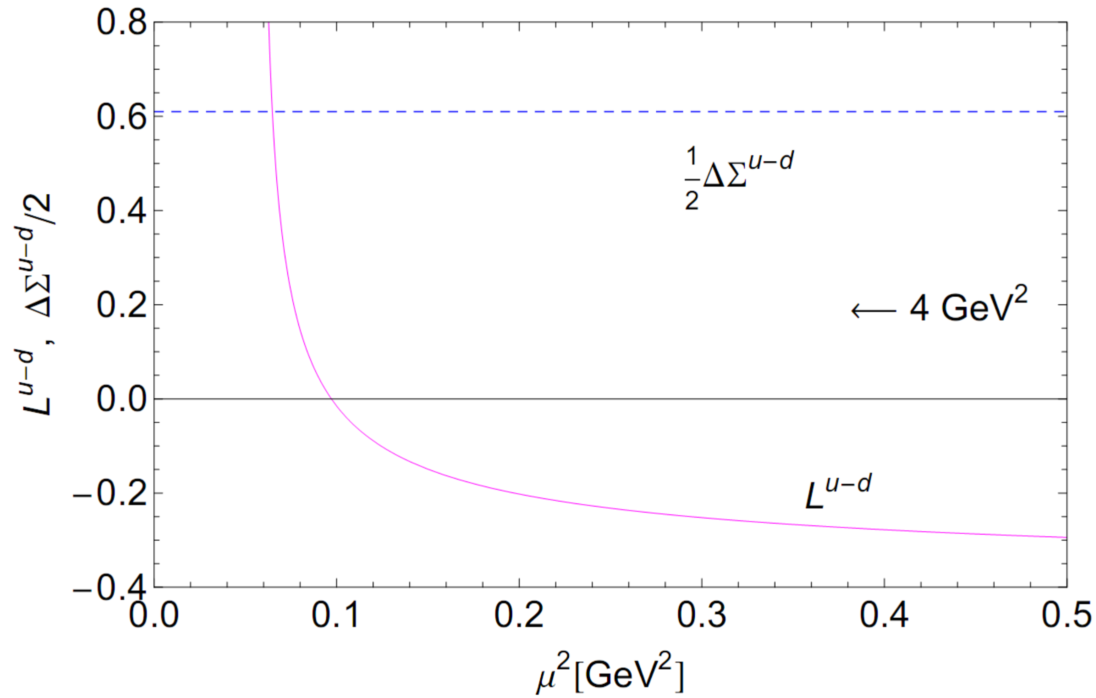
- **non-singlet** (isovector) quantities

Altenbuchinger et al. (2009)



(Wakamatsu 2005; Thomas, PRL 2008)

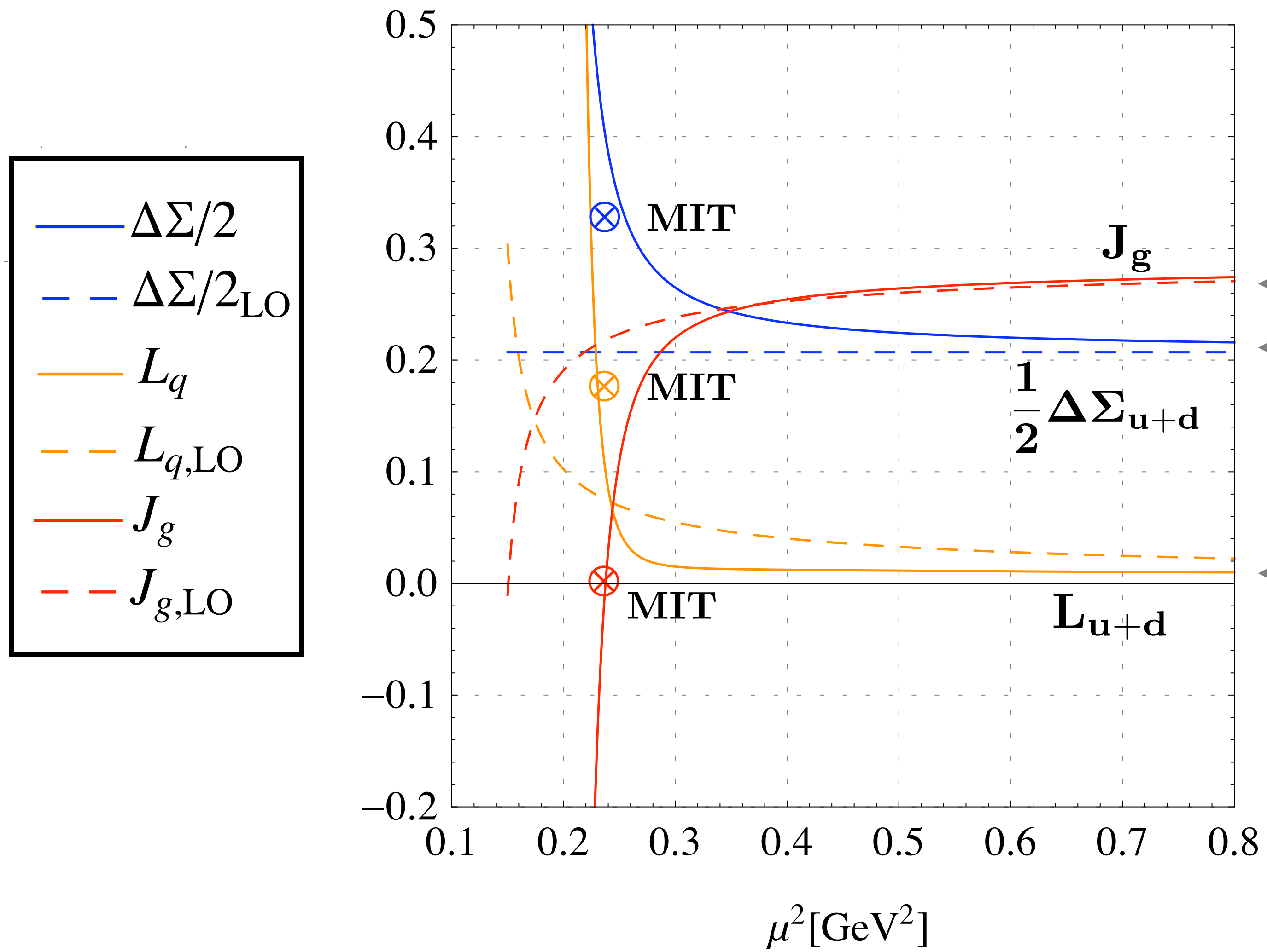
$$L^{u-d}(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{\beta_0}\left(\frac{-16}{9}\right)} \left\{ L^{u-d}(t_0) + \frac{1}{2}\Delta\Sigma^{u-d} \right\} - \frac{1}{2}\Delta\Sigma^{u-d}$$



$\left.\begin{array}{l} g_A/2 \\ L_{u-d} \end{array}\right\}$ from Cloudy (or MIT) bag with one-gluon exchange $(\alpha_s = 0.7)$

From **Lattice QCD** to **Modelling** at Low-Energy Scales

- **QCD evolution** from **lattice** data vs. **relativistic quark model** (MIT bag)



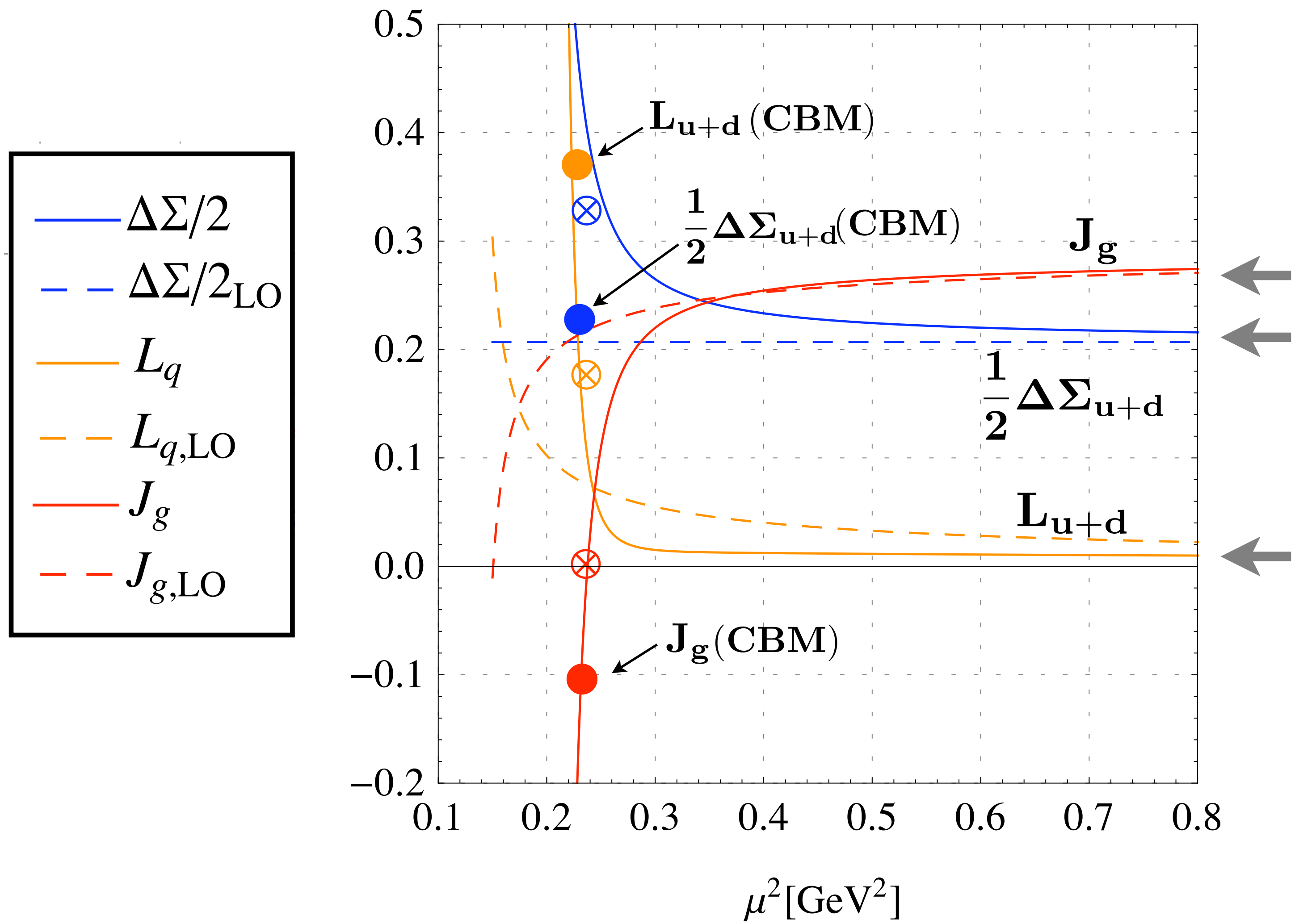
M. Altenbuchinger,
Ph. Hägler,
W.W. (2010)

Lattice
QCD
$\mu^2 = 4\,\mathrm{GeV}^2$

- Model scale "fixed" by NLO evolution to $\mathbf{J_g} = 0$
in comparison with **MIT bag** (no gluon exchange corrections)

From **Lattice QCD** to **Modelling** at Low-Energy Scales

- **QCD evolution** from **lattice** data vs. **relativistic quark model** (MIT bag)



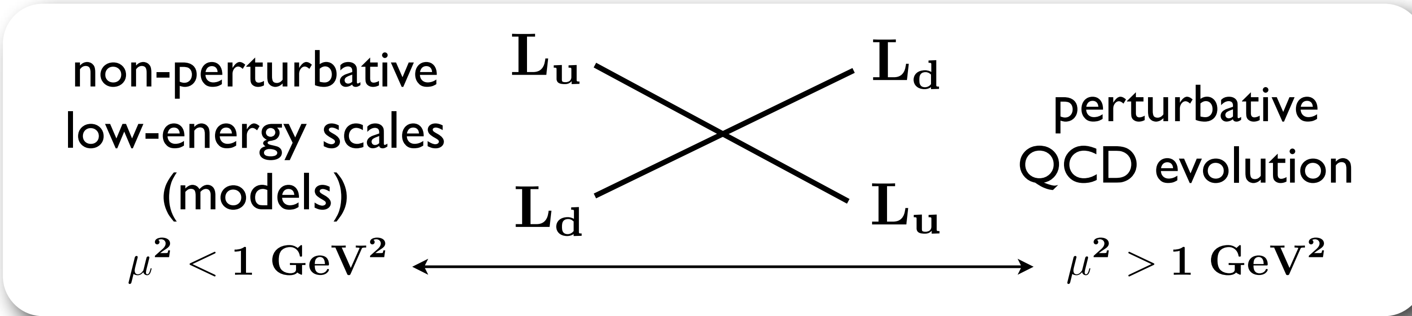
M. Altenbuchinger, Ph. Hägler, W.W. (2009)

Lattice QCD $\mu^2 = 4\,\mathrm{GeV}^2$

- Refinements from **pion cloud** and **one-gluon exchange** $(\alpha_s = 0.7)$ (**C**loudy **B**ag **M**odel)

SUMMARY

- Tony's proposed crossover of quark orbital angular momenta,



is verified at NLO.

- … establishes an impressive qualitative consistency between **lattice QCD** and **relativistic quark models** (e.g. **C**loudy **B**ag **M**odel)

- Tendency o.k. but quantitative uncertainties remain (disconnected terms in lattice QCD still to be computed; convergence of QCD evolution at scales $\mu^2 \leq 1\ \mathrm{GeV}^2$, …)

SUMMARY

- Tony's proposed crossover of quark orbital angular momenta,

non-perturbative low-energy scales (models)	$\mathbf{L_u}$		$\mathbf{L_d}$	perturbative QCD evolution
	$\mathbf{L_d}$		$\mathbf{L_u}$	
$\mu^2 < \mathbf{1\ GeV^2}$	$\longleftrightarrow$			$\mu^2 > \mathbf{1\ GeV^2}$

  is verified at NLO.

- … establishes an impressive qualitative consistency between **lattice QCD** and **relativistic quark models** (e.g. **C**loudy **B**ag **M**odel)

- Tendency o.k. but quantitative uncertainties remain (disconnected terms in lattice QCD still to be computed; convergence of QCD evolution at scales $\mu^2 \leq \mathbf{1\ GeV^2}$, …)

Happy Birthday Tony !