Soft & Hard Scale QCD Dynamics in Mesons

Peter Tandy
Kent State University, Ohio USA

Kent State University

Monday, February 15, 2010
Phys Author Rank Algorithm

Rank's evolution

Rank analysis for THOMAS, AW

- Best performance: 0.1154%
- Last performance: 0.1154%
Topics

- Overview of DSE modeling of meson physics—mainly soft scale
  - Masses, decays, form factors

- Including a hard scale:
  - DIS: quark distributions in $\pi$, $K$ mesons
  - Mesons involving a heavy quark

- Summary
Lattice-QCD and DSE-based modeling

Lattice: \[ \langle \mathcal{O} \rangle = \int D\bar{q}qG \mathcal{O}(\bar{q}, q, G) e^{-S[\bar{q}, q, G]} \]

- Euclidean metric, x-space, Monte-Carlo
- Issues: lattice spacing and vol, sea and valence \(m_q\), fermion Det
- Large time limit \(\Rightarrow\) nearest hadronic mass pole

EOMs (DSEs): \[ 0 = \int D\bar{q}qG \frac{\delta}{\delta q(x)} e^{-S[\bar{q}, q, G]+(\bar{n}, q)+(\bar{q}, \eta)+(J, G)} \]

- Euclidean metric, p-space, continuum integral eqns
- Issues: truncation and phenomenology—not full QCD
- Analytic contin. \(\Rightarrow\) nearest hadronic mass pole
- Can be quick to identify systematics, mechanisms, \ldots
DSE-based modeling of Hadron Physics

- Soft physics: truncate DSEs to min: 2-pt, 3-pt fns
- Should be relativistically covariant—-convenient for decays, Form Factors, etc
  - No boosts needed on wavefns of recoiling bound st.
  - \( \propto \) d.o.f. \( \rightarrow \) few quasi-particle effective d.o.f.
- Do not make a 3-dimensional reduction
- Preserve 1-loop QCD renorm group behavior in UV
- Preserve global symmetries, conserved em currents, etc
- Preserve PCAC \( \Rightarrow \) Goldstone’s Thm
- Can’t preserve local color gauge covariance—-just choose Landau gauge [RG fixed pt]
- Parameterize the deep infrared (large distance) QCD coupling
Constraints on Modeling

- Preserve vector WTI, and axial vector WTI
  E.g.
  \[-iP_\mu \Gamma_{5\mu}(k; P) = S^{-1}(k_+) \gamma_5 \frac{\tau}{2} + \gamma_5 \frac{\tau}{2} S^{-1}(k_-) - 2 m_q(\mu) \Gamma_5(k; P)\]

- ⇒ kernels of DSE_q and K_{BSE} are related

- Ladder-rainbow is the simplest implementation

- Goldstone Theorem preserved, ps octet masses good, indep of model details

- DCSB ⇒ π:
  \[\Gamma^0_\pi(p^2) = \frac{i \gamma_5}{f^2_\pi} \left[ \frac{1}{4} \text{tr } S^{-1}_0(p^2) \right] + \cdots\]

- Here, 1-body and 2-body systems are the same
Ladder-Rainbow Model

\[ K_{\text{BSE}} \rightarrow -\gamma_{\mu} \frac{\lambda^{a}}{2} 4\pi \alpha_{\text{eff}}(q^{2}) \ D_{\mu\nu}^{\text{free}}(q) \ \gamma_{\nu} \frac{\lambda^{a}}{2} \]

\[ \alpha_{\text{eff}}(q^{2}) \xrightarrow{\text{IR}} \langle \bar{q}q \rangle_{\mu=1} \text{ GeV} = -(240 \text{MeV})^{3}, \text{ incl vertex dressing} \]

\[ \alpha_{\text{eff}}(q^{2}) \xrightarrow{\text{UV}} \alpha_{s}^{1-\text{loop}}(q^{2}) \]

\[ p \rightarrow_{-1} = p \rightarrow_{-1} + \]

P. Maris & P.C. Tandy, PRC60, 055214 (1999)

\[ M_{\rho}, M_{\phi}, M_{K^*} \text{ good to 5%}, \ f_{\rho}, f_{\phi}, f_{K^*} \text{ good to 10%} \]
### Summary of light meson results

$m_u = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

<table>
<thead>
<tr>
<th>Pseudoscalar $(\text{PM, Roberts, PRC65, 3369})$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Expt.</td>
<td>Calc.</td>
</tr>
<tr>
<td>$\langle \bar{q}q \rangle_\mu^0$</td>
<td>(0.236 GeV)$^3$</td>
<td>(0.241)$^3$</td>
</tr>
<tr>
<td>$m_\pi$</td>
<td>0.1385 GeV</td>
<td>0.138$^\dagger$</td>
</tr>
<tr>
<td>$f_\pi$</td>
<td>0.0924 GeV</td>
<td>0.093$^\dagger$</td>
</tr>
<tr>
<td>$m_K$</td>
<td>0.496 GeV</td>
<td>0.497$^\dagger$</td>
</tr>
<tr>
<td>$f_K$</td>
<td>0.113 GeV</td>
<td>0.109</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Charge radii $(\text{PM, Tandy, PRC62, 055204})$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_\pi^2$</td>
<td>0.44 fm$^2$</td>
<td>0.45</td>
</tr>
<tr>
<td>$r_K^2$</td>
<td>0.34 fm$^2$</td>
<td>0.38</td>
</tr>
<tr>
<td>$r_K^{02}$</td>
<td>-0.054 fm$^2$</td>
<td>-0.086</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma \pi \gamma$ transition $(\text{PM, Tandy, PRC65, 045211})$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{\pi \gamma}$</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>$r_{\pi \gamma}^2$</td>
<td>0.42 fm$^2$</td>
<td>0.41</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weak $K_{l3}$ decay $(\text{PM, Ji, PRD64, 014032})$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_+ (e^3)$</td>
<td>0.028</td>
<td>0.027</td>
</tr>
<tr>
<td>$\Gamma(K_{e3})$</td>
<td>7.6 $\cdot 10^6$ s$^{-1}$</td>
<td>7.38</td>
</tr>
<tr>
<td>$\Gamma(K_{\mu3})$</td>
<td>5.2 $\cdot 10^6$ s$^{-1}$</td>
<td>4.90</td>
</tr>
</tbody>
</table>

---

### Vector mesons $(\text{PM, Tandy, PRC60, 055214})$

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_\rho/\omega$</td>
<td>0.770 GeV</td>
<td>0.742</td>
</tr>
<tr>
<td>$f_\rho/\omega$</td>
<td>0.216 GeV</td>
<td>0.207</td>
</tr>
<tr>
<td>$m_K^*$</td>
<td>0.892 GeV</td>
<td>0.936</td>
</tr>
<tr>
<td>$f_K^*$</td>
<td>0.225 GeV</td>
<td>0.241</td>
</tr>
<tr>
<td>$m_{\phi}$</td>
<td>1.020 GeV</td>
<td>1.072</td>
</tr>
<tr>
<td>$f_{\phi}$</td>
<td>0.236 GeV</td>
<td>0.259</td>
</tr>
</tbody>
</table>

### Strong decay $(\text{Jarecke, PM, Tandy, PRC67, 035202})$

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{\rho\pi\pi}$</td>
<td>6.02</td>
<td>5.4</td>
</tr>
<tr>
<td>$g_{\phi KK}$</td>
<td>4.64</td>
<td>4.3</td>
</tr>
<tr>
<td>$g_{K^* K\pi}$</td>
<td>4.60</td>
<td>4.1</td>
</tr>
</tbody>
</table>

### Radiative decay $(\text{PM, nucl-th/0112022})$

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{\rho\pi\gamma}/m_\rho$</td>
<td>0.74</td>
<td>0.69</td>
</tr>
<tr>
<td>$g_{\omega\pi\gamma}/m_\omega$</td>
<td>2.31</td>
<td>2.07</td>
</tr>
<tr>
<td>$(g_{K^* K\gamma}/m_K)^+$</td>
<td>0.83</td>
<td>0.99</td>
</tr>
<tr>
<td>$(g_{K^* K\gamma}/m_K)^0$</td>
<td>1.28</td>
<td>1.19</td>
</tr>
</tbody>
</table>

### Scattering length $(\text{PM, Cotanch, PRL66, 116010})$

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0^0$</td>
<td>0.220</td>
<td>0.170</td>
</tr>
<tr>
<td>$a_0^0$</td>
<td>0.044</td>
<td>0.045</td>
</tr>
<tr>
<td>$a_1^0$</td>
<td>0.038</td>
<td>0.036</td>
</tr>
</tbody>
</table>

---

bsample
**Qu-lattice** $S(p), D(q)$ *mapped to a DSE kernel*

$$S(p) = Z(p) \left[ i \not p + M(p) \right]^{-1}$$

- Old data
- New 'improved action' data
- $m_q = 0.168 \text{GeV}$
- $m_q = 0.030 \text{GeV}$
- $m_q = 0.225 \text{GeV}$
- $m_q = 0.055 \text{GeV}$
- $m_q = 0.110 \text{GeV}$
- $m_q = 0.0 \text{GeV}$
Quenched lattice $\Rightarrow m_q$ Depn of DSE Kernel

**DSE and Lattice results for** $M_V$ **and** $M_{ps}$

![Graph showing DSE and Lattice results for $M_V$ and $M_{ps}$](attachment:image.png)
Pion electromagnetic form factor

\[ \Lambda_\mu = (P' + P)_\mu F_\pi(Q^2) = N_c \int \frac{d^4 q}{(2\pi)^4} \text{Tr} \left[ \bar{\Gamma}^{\pi} S i \Gamma_\mu S \Gamma^{\pi} S \right] \]
**Pion \( F(Q^2) \): Low \( Q^2 \)**

(P Maris & PCT, PRC 61, 045202 (2000))

(P. Maris & PCT, PRC 62, 0555204 (2000))

\[
\begin{align*}
    r_{\pi}^{\text{DSE}} &= 0.68 \text{ fm} \\
    r_{\pi}^{\text{expt}} &= 0.663 \pm 0.006 \text{ fm}
\end{align*}
\]
Pion electromagnetic form factor

PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]
Pion electromagnetic form factor

JLab data from Volmer \textit{et al.}, PRL86, 1713 (2001) [nucl-ex/0010009]
PM and Tandy, PRC62, 055204 (2000) [nucl-th/0005015]
Pion electromagnetic form factor

PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]
1-loop chiral correction to $r_\pi$ vs $m_\pi$

P. Maris and PCT, in preparation
1-loop chiral correction to $r_\pi$ vs $m_\pi$

P. Maris and PCT, in preparation
$\gamma^* \pi^0 \rightarrow \gamma$ Transition Form Factor

- Abelian axial anomaly + $\pi$ pole in $\Gamma_{5\mu} \Rightarrow G(0, 0)$
- Chiral limit $G(0, 0) = \frac{1}{2}$
  $\Rightarrow$ $\Gamma_{\pi\gamma\gamma}$ to 2%

Graph showing $t(Q^2)g_{\text{rewind}}$ vs. $Q^2$ [GeV$^2$] with different theoretical predictions and experimental data points.
$\gamma^* \pi \gamma^*$ **Asymptotic Limit**

Lepage and Brodsky, PRD22, 2157 (1980): LC-QCD/OPE ⇒

![Graph showing $F(Q^2, Q^2)$ versus $Q^2$ (GeV$^2$) with different lines and labels: DSE results, VMD dipole, bare vertices, and $(4/3) \pi^2 \Gamma_\pi^2 / Q^2$.](image-url)
**LR: Successes, Problems, Resolutions**

**Successes:**
- S-wave mesons, PS and V, light quarks and QQ, no spurious thresholds
- Exact PS mass formula, Goldstone Thm, $\Delta M_{HF}$ from DCSB
- $f_{EW}$, strong decays, radiative decays, form factors, $Q^2 < 5 GeV^2$

**Problems:**
- Axial vector ($L > 0$) mesons ($a_1, b_1, \cdots$) too light
- Physical diquarks, no physical V or PS $qQ$ states
- Excited states are difficult

**Probable Resolution:**
- Quark-gluon vertex: $\Gamma_\mu \Rightarrow \Sigma_q \Rightarrow K_{BSE}$
- Use analysis of spacelike correlators, 3-pt functions
From Gluon vertex to BSE Kernel

- A symmetry-preserving procedure [Bender, Roberts, von Smekal, PLB380, (1996), nucl-th/9602012; Munczek 1995]; Axial vector and vector WTIs, and Goldstone Thm preserved

\[ K_{\text{BSE}}(x', y'; x, y) = -\frac{\delta}{\delta S(x, y)} \Sigma(x', y') \]

- Vertex \( \Gamma_\mu(p, q) = \sum \text{diagrams} \Rightarrow K_{\text{BSE}} = \sum \text{diagrams} \)

- If \( \Sigma \) contains:

- \( K_{\text{BSE}} \) contains:

- Independent of model parameters. Model does not fight chiral symmetry, use light vector mesons to fix parameters
Deep Inelastic Lepton Scattering

- PDFs: $u_\pi(x), u_K(x), s_\pi(x)$
- Drell-Yan data exists
- Pion and Kaon/Pion Ratio
- Employ LR DSE model
- Bjorken limit
DIS is hard and fast—confinement is soft and slow
⇒ \( S(k + q) \rightarrow \frac{\gamma^+}{2(k + P^+x) + i\epsilon} \)

\[ q^+ = q \cdot n = -Mx, \quad |\xi^-| \sim \frac{1}{Mx} \]

\[ q^- = q \cdot p = 2\nu, \quad |\xi^+| \sim 0 \]

\( W^{\mu\nu} \propto \{ T^{\mu\nu}(\epsilon) - T^{\mu\nu}(-\epsilon) \} \Rightarrow \) Euclidean model elements can be continued

\[ f_q(x) = \frac{1}{4\pi} \int d\xi^- e^{iq^+ \xi^-} \langle \pi(P) | \bar{q}(\xi^-) \gamma^+ q(0) | \pi(P) \rangle_c = -f_{\bar{q}}(-x) \]

\[ f_q(x) = \frac{1}{2} \text{tr} \int \frac{d^4k}{(2\pi)^4} \delta(k^+ - P^+x) S(k)\gamma^+ S(k) T(k, P) \]
Deep Inelastic Lepton Scattering

Quark number sum: \[ N_q^V = \int_0^1 dx \{ f_q(x) - f_{\bar{q}}(x) \} = \frac{1}{2P^+} \langle \pi(P)|J^+(0)|\pi(P)\rangle = 1 \]

DSE calculation: \( u_\pi(x), \ u_K(x), \ s_K(x) \) [T. Nguyen, PCT, (2009)]

- BSE \( q\bar{q} \) solutions for \( \pi, K \)
- DSE solns for dressed quark \( S(k) \)
- Constituent mass approx for spectator propagator
- Vertex approx via Ward Id
**DIS on pion: from DSE-BSE solutions**

- Valence quarks, handbag diagram, $\gamma^+, \Gamma_{W I}^+(k)$

- Data: J. S. Conway et al., PRD39, 92 (1989)
  $M_{\bar{l}l} = 4.05$ GeV

- Previous: Hecht, Roberts, Schmidt, PRC63, 025213 (2001)
  $\Gamma_\pi(k, P) \approx i\gamma_5 B_0(k^2)/f_\pi^0 + \cdots$
  $S(p)$ fit to data

- Large $x$ behavior: \((1 - x^{26})^\alpha\), $\alpha =$ ?
**DIS on pion: large $x$ behavior?**

- **Fit:** $a x (1 - x)^{\alpha(x)}$
- **BSE ampls:** pQCD behavior sets in at a larger scale
**DIS on pion: large $x$ behavior?**

- Global fits to (limited) DIS data produce $\alpha \sim 1.5$
- Parton model (F-J), pQCD (Brodsky, Ezawa), DSEs, $\Rightarrow \alpha \sim 2^+$
- Constituent q models, NJL, duality, etc $\Rightarrow \alpha \sim 1$

28
Quark Distributions in $\pi$ and $K$

Evolved to $q = 4.05$ GeV

- Environmental depn of $u(x)$ in accordance with effective quark mass
- $u(x), s(x)$ difference in $K$ in accordance with effective quark mass
\[ \frac{u_K(x)}{u_\pi(x)} \text{ Ratio} \]

Data: (Drell-Yan, CERN-SPS) J. Badier et al., PLB 93, 354 (1980); \( M_{\ell\ell} = 4 - 8 \text{ GeV} \)

\( u \) has greater fraction of \( P_\pi \) than it has of \( P_K \), in accord with effective quark mass
Axial anomaly and $\eta - \eta'$ states

- Ch symm: $\partial_\mu (z) \langle j_{5\mu}^\alpha (z) \, q(x) \bar{q}(y) \rangle$ involves $2 \, \text{tr}_f (F^\alpha) \langle Q_t(z)q(x)\bar{q}(y) \rangle$

- Matrix elements, amputated $\Rightarrow$ AV-WTI

$$P_\mu \Gamma_5^\alpha (k; P) = -2i \, M^{\alpha\beta} \Gamma_5^\beta (k; P) - \delta_{\alpha,0} \Gamma_A (k; P)$$
$$+ S^{-1} (k_+) i \gamma_5 F^\alpha + i \gamma_5 F^\alpha S^{-1} (k_-)$$

- Residues at PS poles $\Rightarrow$ PS mass formula for arbitrary $m_q$, any flavor:

$$m_p^2 f_p  = 2 \, M^{\alpha\beta} \rho_p^{\beta} + \delta^{\alpha,0} n_p$$

$$\rho_p^\alpha (\mu) = \langle 0| \bar{q} \gamma_5 F^\alpha q | p \rangle$$

$p = \text{any PS}$

---[Bhagwat, Chang, Liu, Roberts, PCT, PRC (76), 2007; arXiv:0708.1118]
$\pi^0 - \eta - \eta'$ mixing: 3 flavors

- $m_u - m_d$ causes $\pi^0$ to be mixed in:
  - $135$ MeV: $|\pi^0\rangle \sim 0.72 \bar{u}u - 0.69 \bar{d}d - 0.013 \bar{s}s$
  - $455$ MeV: $|\eta\rangle \sim 0.53 \bar{u}u + 0.57 \bar{d}d - 0.63 \bar{s}s$
  - $922$ MeV: $|\eta'\rangle \sim 0.44 \bar{u}u + 0.45 \bar{d}d + 0.78 \bar{s}s$

- $m_u = m_d \Rightarrow$
  - $455$ MeV: $|\eta\rangle \sim 0.55 (\bar{u}u + \bar{d}d) - 0.63 \bar{s}s$, $\theta_\eta = -15.4^\circ$
  - $924$ MeV: $|\eta'\rangle \sim 0.45 (\bar{u}u + \bar{d}d) + 0.78 \bar{s}s$, $\theta_{\eta'} = -15.7^\circ$

- Chiral limit: $m^2_{\eta'} = (0.852$ GeV$)^2 \equiv 2\text{tr}_f(\mathcal{F}^0) \langle 0 | Q_f | \eta' \rangle / f_{\eta'}^0$

- cf Witten-Veneziano a-v ghost scenario $\Rightarrow m^2_{\eta'} = h^2 + m^2_{GB}$

- It is worth extending to a realistic LR model for $K_N$ with separable $K_A$: one obtains access to decay constants, residues, and details of the mass relations
Quark mass functions from DSE solutions
Constituent Mass Concept for c- and b-quarks

<table>
<thead>
<tr>
<th></th>
<th>All GeV</th>
<th>D(uc)</th>
<th>D*(uc)</th>
<th>D_s (sc)</th>
<th>D_s*(sc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>expt M</td>
<td>1.86</td>
<td>2.01</td>
<td>1.97</td>
<td>2.11</td>
<td></td>
</tr>
<tr>
<td>calc M</td>
<td>1.85(FIT)</td>
<td>2.04</td>
<td>1.97</td>
<td>2.17</td>
<td></td>
</tr>
<tr>
<td>expt f</td>
<td>0.222</td>
<td>?</td>
<td>0.294</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>calc f</td>
<td>0.154</td>
<td>0.160</td>
<td>0.197</td>
<td>0.180</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>All GeV</th>
<th>B(ub)</th>
<th>B*(ub)</th>
<th>B_s (sb)</th>
<th>B_s*(sb)</th>
<th>B_c (cb)</th>
<th>B_c*(cb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>expt M</td>
<td>5.28</td>
<td>5.33</td>
<td>5.37</td>
<td>5.41</td>
<td>6.29</td>
<td>?</td>
</tr>
<tr>
<td>calc M</td>
<td>5.27(FIT)</td>
<td>5.32</td>
<td>5.38</td>
<td>5.42</td>
<td>6.36</td>
<td>6.44</td>
</tr>
<tr>
<td>expt f</td>
<td>0.176</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>calc f</td>
<td>0.105</td>
<td>0.182</td>
<td>0.144</td>
<td>0.20</td>
<td>0.210</td>
<td>0.18</td>
</tr>
</tbody>
</table>

- Fit ⇒ constituent masses: $M_c^{\text{cons}} = 2.0$ GeV, $M_b^{\text{cons}} = 5.3$ GeV
- Consistent with $M^{DSE}(p^2 \sim -M^2)$ generated by $m_c = 1.2 \pm 0.2$, $m_b = 4.2 \pm 0.2$, [PDG, $\mu = 2$ GeV]
- Does heavy quark dressing contribute anything? Too much in this DSE model—no mass shell!
Compare Quark Masses with PDG
Quarkonia

<table>
<thead>
<tr>
<th>All GeV</th>
<th>$M_{\eta_c}$</th>
<th>$f_{\eta_c}$</th>
<th>$M_{J/\psi}$</th>
<th>$f_{J/\psi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>expt</td>
<td>2.98</td>
<td>0.340</td>
<td>3.09</td>
<td>0.411</td>
</tr>
<tr>
<td>calc with $M_c^{\text{cons}}$</td>
<td>3.02</td>
<td>0.239</td>
<td>3.19</td>
<td>0.198</td>
</tr>
<tr>
<td>calc with $\Sigma_c^{\text{DSE}(p^2)}$</td>
<td>3.04</td>
<td>0.387</td>
<td>3.24</td>
<td>0.415</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>All GeV</th>
<th>$M_{\eta_b}$</th>
<th>$f_{\eta_b}$</th>
<th>$M_\Upsilon$</th>
<th>$f_\Upsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>expt</td>
<td>9.4 ?</td>
<td>?</td>
<td>9.46</td>
<td>0.708</td>
</tr>
<tr>
<td>calc with $M_b^{\text{cons}}$</td>
<td>9.6</td>
<td>0.244</td>
<td>9.65</td>
<td>0.210</td>
</tr>
<tr>
<td>calc with $\Sigma_b^{\text{DSE}(p^2)}$</td>
<td>9.59</td>
<td>0.692</td>
<td>9.66</td>
<td>0.682</td>
</tr>
</tbody>
</table>

- QQ and qQ decay constants too low by 30-50% in constituent mass approximation
- Quarkonia decay constants much better for DSE dressed quarks (within 5% of expt.)
- IR sector (gluon $k$ below $\sim 0.8$ GeV) contribute little for bb or cc quarkonia in DSE, BSEs
- QQ states are more point-like than qq or qQ states
Recovery of a qQ Mass Shell

- Suppress gluon $k$ below $\sim 0.8$ GeV in DSE dressing of $b$ propagator
- Retain IR sector for dressed "light" quark and BSE kernel
- Now a mass shell is produced

<table>
<thead>
<tr>
<th>All GeV</th>
<th>B(ub)</th>
<th>$B^*$ (ub)</th>
<th>$B_s$ (sb)</th>
<th>$B^*_s$ (sb)</th>
<th>$B_c$ (cb)</th>
<th>$B^*_c$ (cb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>expt M</td>
<td>5.28</td>
<td>5.33</td>
<td>5.37</td>
<td>5.41</td>
<td>6.29</td>
<td>?</td>
</tr>
<tr>
<td>calc M</td>
<td>4.66</td>
<td>—</td>
<td>4.75</td>
<td>—</td>
<td>5.83</td>
<td>—</td>
</tr>
<tr>
<td>expt f</td>
<td>0.176</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>calc f</td>
<td>0.133</td>
<td>—</td>
<td>0.164</td>
<td>—</td>
<td>0.453</td>
<td>—</td>
</tr>
</tbody>
</table>

Masses are $\sim 10\%$ low

- It makes sense that $R_b < R_{qQ}$ $\Rightarrow$ greater limit on low $k$ in $\Sigma_b$
The V-A Current Correlator

\[ \Pi_{\mu\nu}^V(x) = \langle 0 | T j_\mu(x) j_\nu^\dagger(0) | 0 \rangle, \text{ isovector currents } j_\mu = \bar{u} \gamma_\mu d, \quad j_5^\mu = \bar{u} \gamma_5 \gamma_\mu d \]

\[ \Pi_{\mu\nu}^V(P) = (P^2 \delta_{\mu\nu} - P_\mu P_\nu) \Pi^V(P^2) \]

\[ \Pi_{\mu\nu}^A(P) = (P^2 \delta_{\mu\nu} - P_\mu P_\nu) \Pi^A(P^2) + P_\mu P_\nu \Pi^L(P^2) \]

\[ \Pi_{\mu\nu}^V(P) = -\int_q \gamma_\mu Z_1(\mu, \Lambda) \]

- \( m_q = 0 \) : \( \Pi^V - \Pi^A = 0 \), to all orders in pQCD
- \( \Pi^V - \Pi^A \) probes the scale for onset of non-perturbative phenomena in QCD
Physics from the V-A correlator:

**OPE:**

\[ \Pi^{V-A}(P^2) = \frac{32\pi\alpha_s \langle \bar{q}q\bar{q}q \rangle}{9 P^6} \left\{ 1 + \frac{\alpha_s}{4\pi} \left[ \frac{247}{4\pi} + \ln\left(\frac{\mu^2}{P^2}\right) \right] \right\} + \mathcal{O}\left(\frac{1}{P^8}\right) \]

<table>
<thead>
<tr>
<th>Model</th>
<th>(-&lt;\bar{q}q&gt;_{\mu=19}) (GeV(^3))</th>
<th>(&lt;\bar{q}q\bar{q}q&gt;_{\mu=19}) (GeV(^6))</th>
<th>(R(\mu = 19))</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR DSE</td>
<td>((0.216)^3)</td>
<td>((0.235)^6)</td>
<td>1.65</td>
</tr>
</tbody>
</table>

**Weinberg et al Sum Rules:**

- **I:** \[ \frac{1}{4\pi^2} \int_0^\infty ds [\rho_v(s) - \rho_a(s)] = [P^2 \Pi^{V-A}(P^2)]_{P^2 \to 0} = -f_\pi^2 \]
- **II:** \[ P^2 [P^2 \Pi^{V-A}(P^2)]|_{P^2 \to \infty} = 0 \]
- **DGMLY:** \[ \int_0^\infty dP^2 [P^2 \Pi^{V-A}(P^2)] = -\frac{4\pi f_\pi^2}{3\alpha} [m_{\pi^\pm}^2 - m_{\pi^0}^2] \]

<table>
<thead>
<tr>
<th>Model</th>
<th>(f_\pi^2) (GeV(^2))</th>
<th>(f_\pi) (MeV)</th>
<th>(f_\pi^{exp}/f_\pi^{num})</th>
<th>(\Delta m_\pi) (MeV)</th>
<th>((\Delta m_\pi)_{exp})</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR DSE</td>
<td>0.0081</td>
<td>90.0</td>
<td>1.03</td>
<td>4.88</td>
<td>4.43 ± 0.03</td>
</tr>
</tbody>
</table>
Summary

- Effective ladder-rainbow model based on QCD-DSEs; $\langle \bar{q}q \rangle_\mu \Rightarrow 1$ IR parameter
- Convenient and covariant approach to hadronic form factors: N, $\pi$, various transitions
- Ground state qQ and QQ mesons (V & PS) up to b-quark region
- Dynamical dressing in $S(p)$ at each stage increases the value of the decay constant [factor of 3 for $\bar{b}b$, factor of 2 for $\bar{c}c$]!
- First combination of BSE-DSE solutions for pion and kaon DIS distributions $u(x), s(x)$
- Used $\langle J J \rangle$, V-A, to estimate $\langle \bar{q}q\bar{q}q \rangle$ as $\sim 70\%$ greater than vac saturation, and npQCD enters at scale 0.5 fm.
Collaborators

- Craig Roberts, Argonne National Lab
- Pieter Maris, Iowa State University
- Yu-xin Liu, Lei Chang, Peking University
- Nick Souchlas, Trang Nguyen, Kent State University

Thankyou!