QCD$_{light}$

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– QCDSF Collaboration –

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- Since the cost of full QCD computations in a volume large enough to contain the pion grows with a large inverse power of the pion mass, initial calculations were restricted to relatively heavy pions

- In order for lattice calculations to capture the physics of quarks and gluons in captivity, and reach the needed accuracy requested by the experiments, simulations at physical quark masses, on suitably (!) large volumes and at small lattice spacings are required

- In this talk I shall report on Achievements and New Directions in Lattice QCD simulations at Physical Quark Masses

$\Longrightarrow$ Talk by James Zanotti

Interest: $m_q \longrightarrow V$ dependence

Lattice

Action

$O(a)$ improved

$$S \equiv S_G + S_F \longrightarrow S + O(a^2)$$

Lattice

Clover
Domain wall
Overlap
} fermions

The Simulation

- Generate sequence of configurations $\{U_\mu^{(i)} | i = 1, \cdots, N\}$ with probability

$$\mathcal{P}\{U_\mu^{(i)}\} \propto \int \prod_x \mathcal{D}\bar{\psi}(x)\mathcal{D}\psi(x)\, \exp\{-S_F - S_G\} = \det\left(\not{D}(U_\mu^{(i)}) + am\right)\, \exp\{-S_G\}$$

(R)HMC

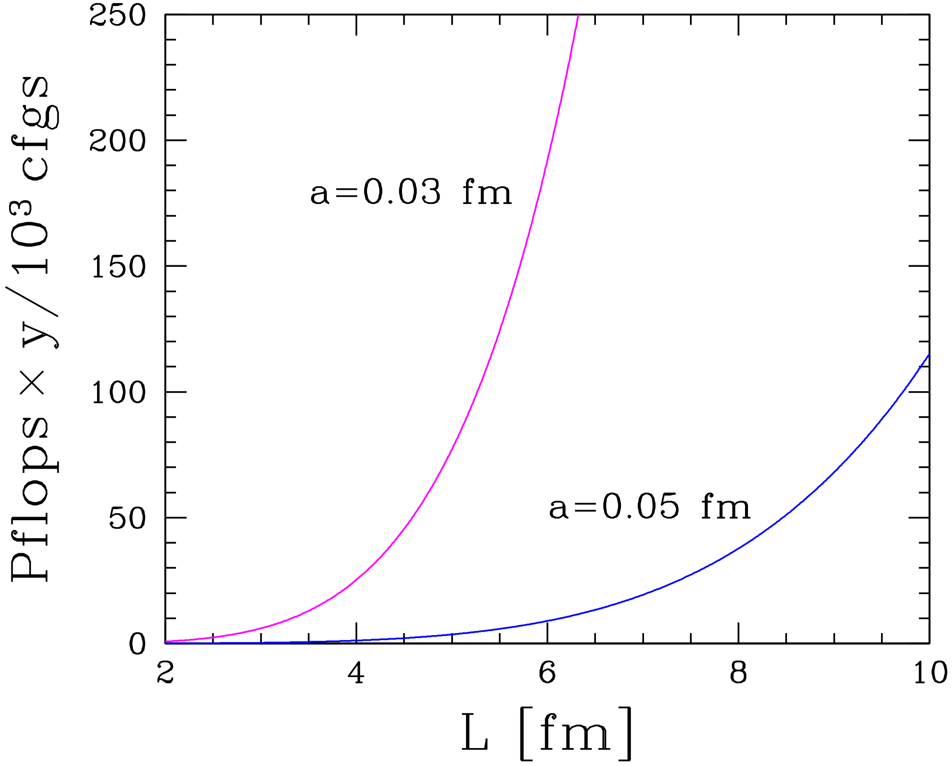
- Compute observable

$$\langle \mathcal{O} \rangle = \frac{1}{N}\sum_{i=1}^{N} \mathcal{O}(U_\mu^{(i)})$$

$N_f = 2$

Costs

$m_\pi = 140$ MeV



10^3 independent configurations

QPACE

SFB TR55

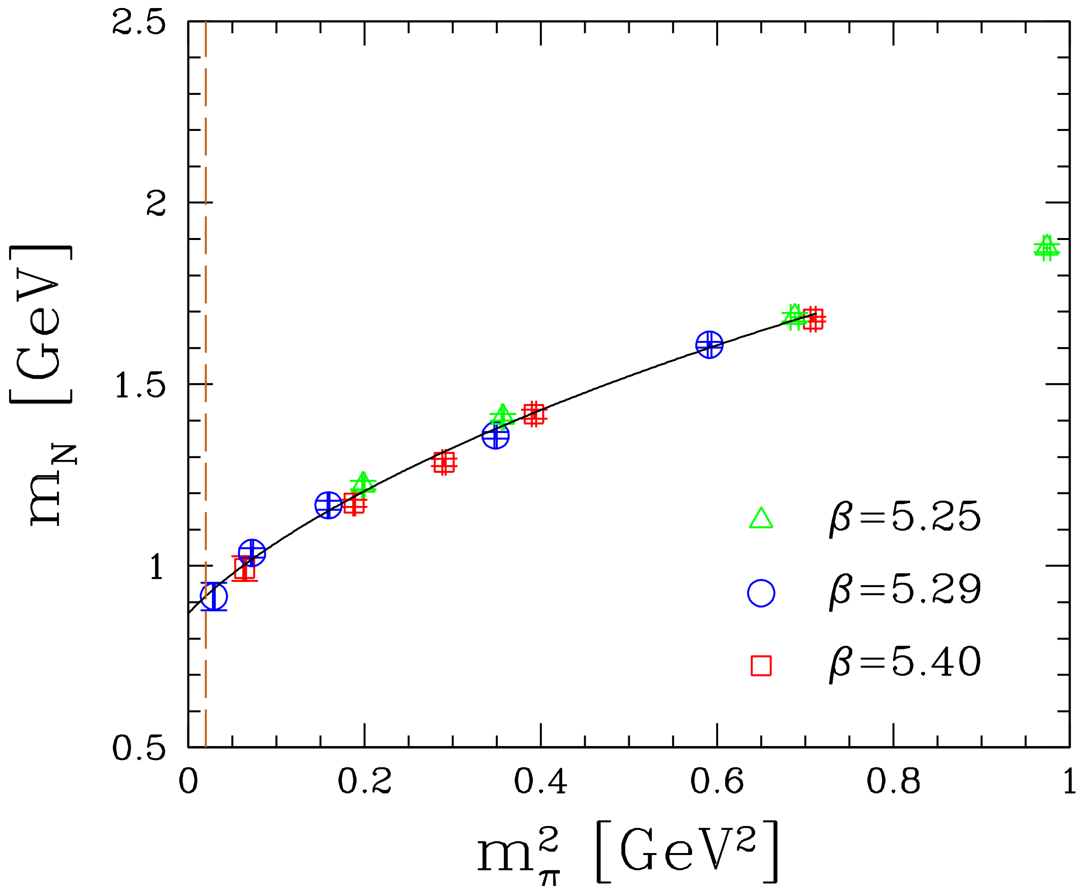
$$(4+4) \times 52 \text{ TFlops} = 416 \text{ TFlops} \quad \text{(SP)}$$

Nucleon

Scale

Mass

FS corrected



Scale: $r^2 \left.\frac{\partial V(r)}{\partial r}\right|_{r=r_0} = 1.65$ $\quad r_0 = 0.467(15)$ fm

$$m_N = m_0 - 4c_1 m_{PS}^2 - \frac{3g_A^{0\,2}}{32\pi f_0^2} m_{PS}^3 + \left[e_1(\mu) - \frac{3}{64\pi^2 f_0^2}\left(\frac{g_A^{0\,2}}{m_0} - \frac{c_2}{2}\right) - \frac{3g_A^{0\,2}}{32\pi^2 f_0^2}\left(\frac{g_A^{0\,2}}{m_0} - 8c_1 + c_2 + 4c_3\right) \ln\frac{m_{PS}}{\mu}\right] m_{PS}^4 + \frac{3g_A^{0\,2}}{256\pi f_0^2 m_0^2} m_{PS}^5 + O(m_{PS}^6)$$

Procura et al.

$$m_N - m_N(L) = -\frac{3g_A^{0\,2} m_0 m_{PS}^2}{16\pi^2 f_0^2} \sum_{|\vec{n}|\neq 0} \int_0^\infty dz K_0\left(\sqrt{m_0^2 z^2/m_{PS}^2 + (1-z)}\,\lambda\right) - \frac{3m_{PS}^4}{4\pi^2 f_0^2} \sum_{|\vec{n}|\neq 0} \left[(2c_1 - c_3)\frac{K_1(\lambda)}{\lambda} + c_2 \frac{K_2(\lambda)}{\lambda^2}\right] + O(m_{PS}^5)$$

$\mu = 1$ GeV , $c_1,\ c_2,\ c_3$ fit parameters $\lambda = m_{PS}|\vec{n}|L$

QCDSF

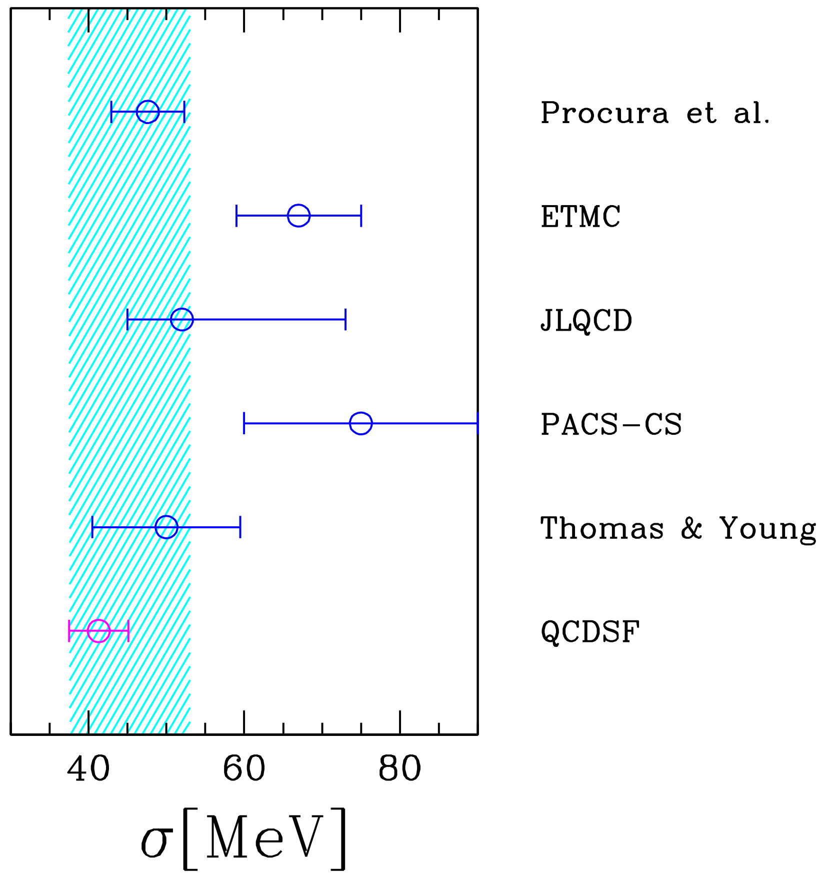
Nucleon Sigma Term

$$\sigma_N = m_\ell \frac{d\, m_N(m_\ell)}{d\, m_\ell} \stackrel{!}{=} m_\pi^2 \frac{d\, m_N(m_\pi)}{d\, m_\pi^2} = -\,4\, c_1 m_\pi^2 - \frac{9\, {g_A^0}^2}{64\, \pi\, f_0^2} m_\pi^3 + O(m_\pi^4) \Big|_{m_\pi = m_\pi^{\mathrm{phys}}}$$

$$\boxed{\sigma_N = 41.3 \pm 2.7 \pm 1.1\, \mathrm{MeV}}$$

$$\sigma(0) = 45 \pm 8\, \mathrm{MeV}$$

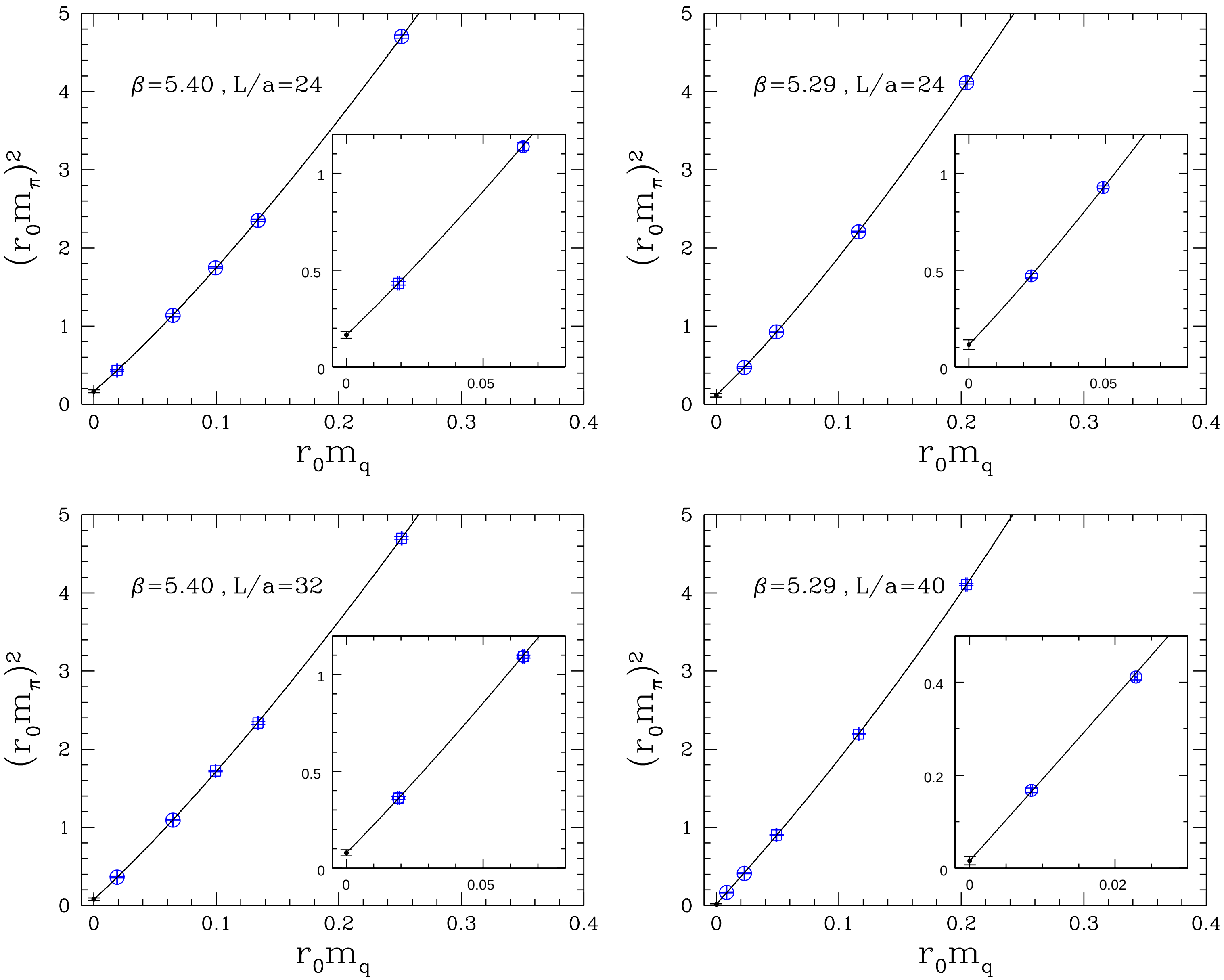
Comparison



Procura et al.
ETMC
JLQCD
PACS−CS
Thomas & Young
QCDSF
40
60
80
σ[MeV]

Pion

Chiral Extrapolation



δ Regime

$m_\pi L \ll 1\,,\ L \ll T$

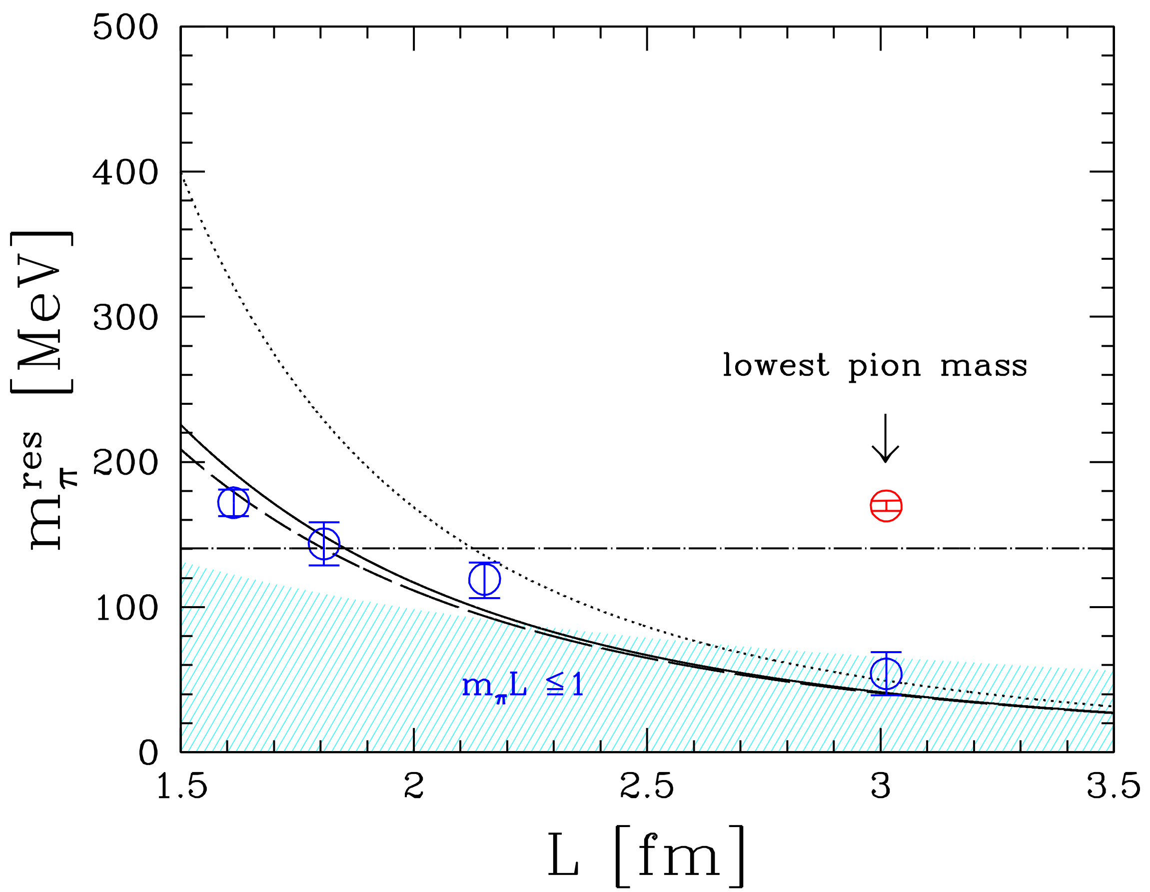
$$m_\pi^{\rm res} = \frac{3}{2F_\pi^2 L^3 (1+\Delta)}$$

with

$$\Delta = \frac{2}{F_\pi^2 L^2}\, 0.2257849591$$
$$+ \frac{1}{F_\pi^4 L^4}\left[0.088431628 - \frac{0.8375369106}{3\pi^2}\left(\frac{1}{4}\ln\left(\Lambda_1 L\right)^2 + \ln\left(\Lambda_2 L\right)^2\right)\right]$$

Leutwyler, Niedermayer & Hasenfratz

Residual Mass



$$F_0 = F_\pi|_{m_\pi=0} = 78^{+14}_{-10}\,\text{MeV}$$

Rho

The ρ meson is practically a two-pion resonance. It has isospin 1, and the two pions form a p-wave state

We denote the pion momentum in the center-of-mass frame by $k = |\vec{k}|$. Phenomenologically, the scattering phase shift $\delta_{11}(k)$ is very well described by the effective range formula

$$\frac{k^3}{E} \cot\, \delta_{11}(k) = \frac{24\pi}{g_{\rho\pi\pi}^2} \left(k_\rho^2 - k^2\right)$$

where $E = 2\sqrt{k^2 + m_\pi^2}$ and $k_\rho = \frac{1}{2}\sqrt{m_\rho^2 - 4m_\pi^2}$. The width of the ρ is given by

$$\Gamma_\rho = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{k_\rho^3}{m_\rho^2}$$

Experimentally, $\Gamma_\rho = 146\,\text{MeV}$, which translates into

$$g_{\rho\pi\pi} \;=\; 5.9$$

The physical ρ mass (at any given m_π) is obtained from the momentum k, at which the phase shift $\delta_{11}(k)$ passes through $\pi/2$

In the case of noninteracting pions, the possible energy levels in a periodic box of length L are given by

$$E = 2\sqrt{k^2 + m_\pi^2} \qquad\qquad k = \frac{2\pi|\vec{n}|}{L}\ , \quad \vec{n} \in \mathbb{N}^3$$

In the interacting case, k is the solution of a nonlinear equation involving the phase shift

$$\delta_{11}(k) = \arctan\left\{\frac{\pi^{3/2}q}{\mathcal{Z}_{00}(1,q^2)}\right\} \bmod \pi\ , \quad q = \frac{kL}{2\pi}$$

Task

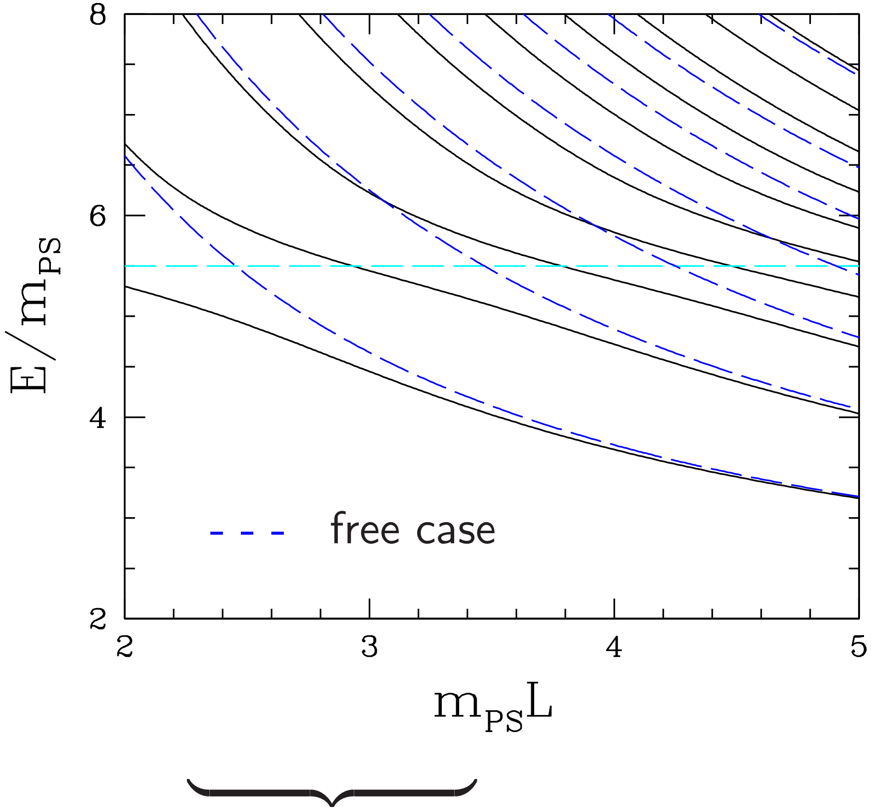
$$E\,|_{m_\pi,L} \quad\longrightarrow\quad k \quad\longrightarrow\quad \delta_{11}(k) \quad\longrightarrow\quad m_\rho\ ,\ \Gamma_\rho$$

by fitting $\delta_{11}(k)$
to effective range
formula

Lüscher, Wiese

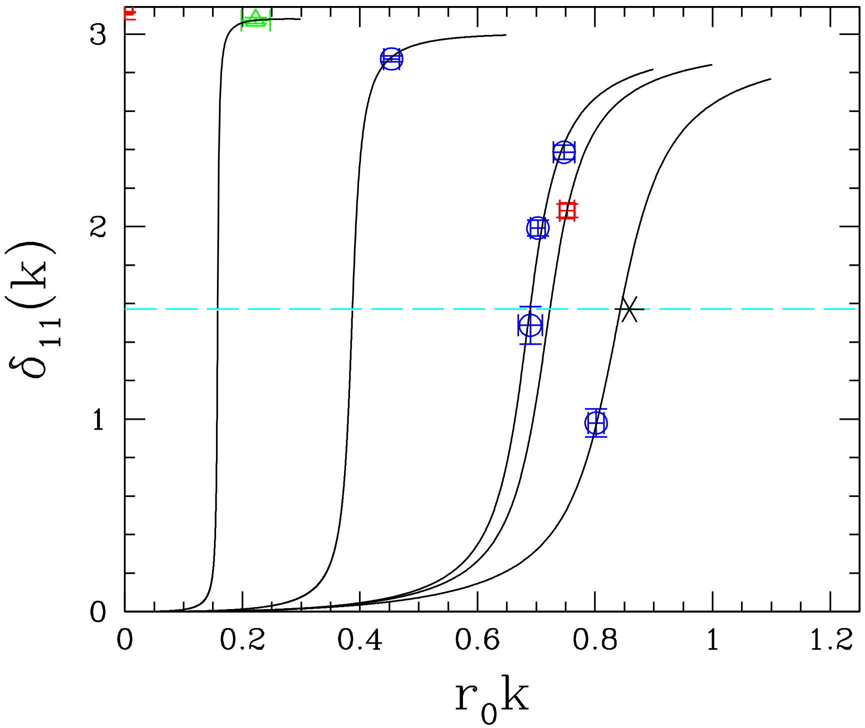
Energy Levels

Physical m_π, m_ρ and Γ_ρ



Useful region

Phase Shift



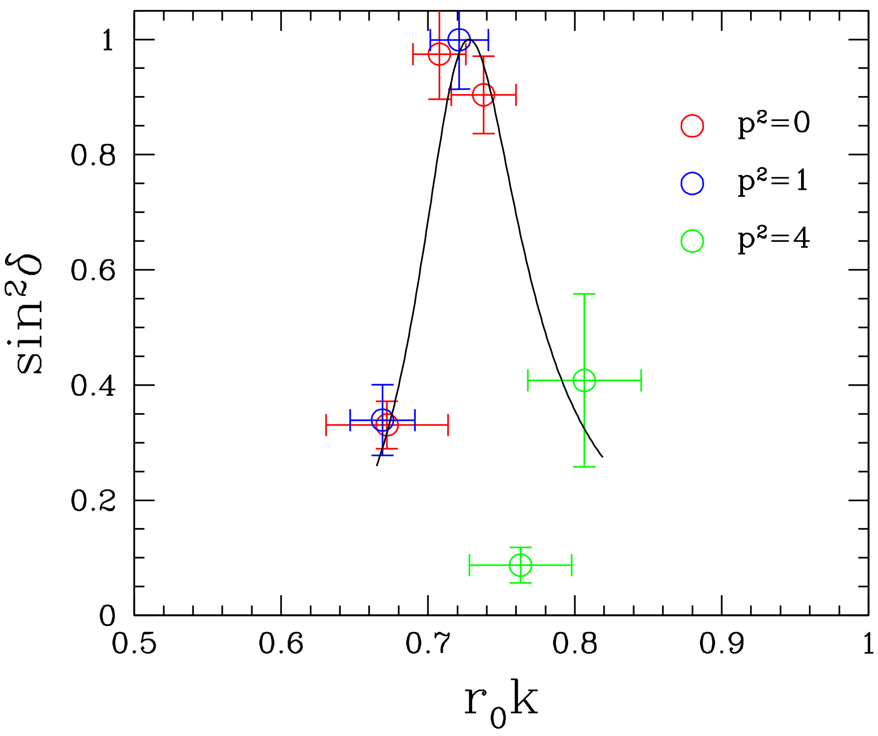
$m_\pi =$ 430 390 250 150 240 MeV

↑
Fit

$g_{\rho\pi\pi} = 5.1 \pm 0.4$

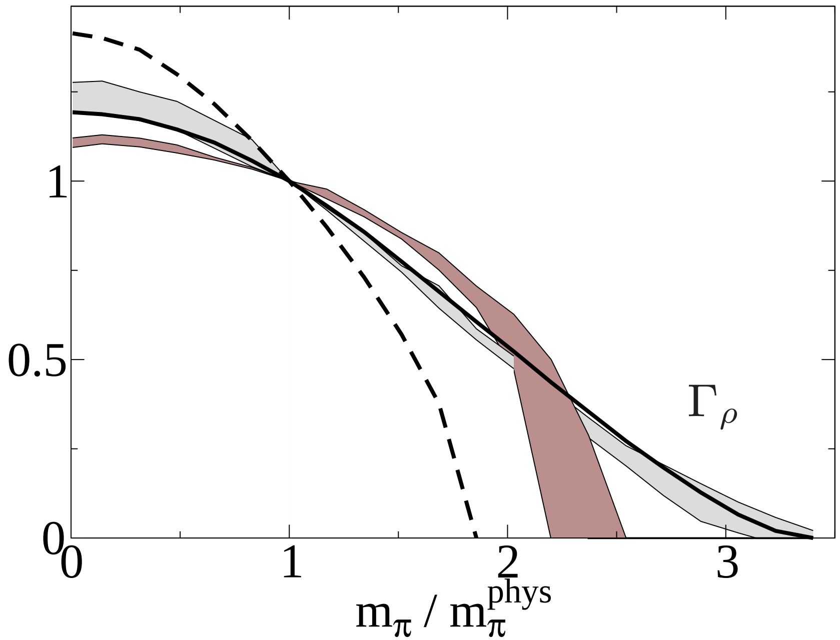
$p \neq 0$

250 MeV



Extrapolation to physical pion mass

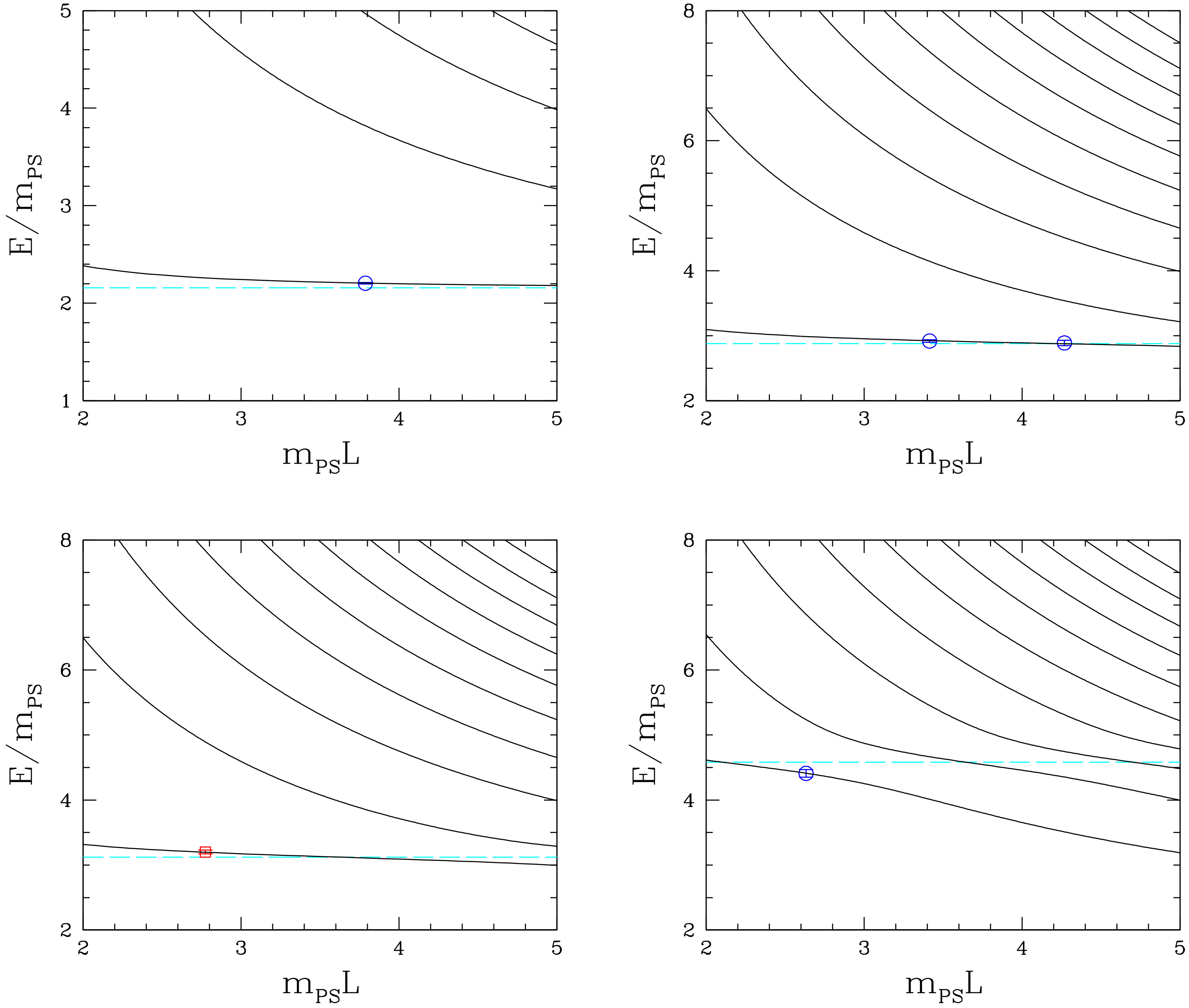
ChPT



Ríos Márquez et al.

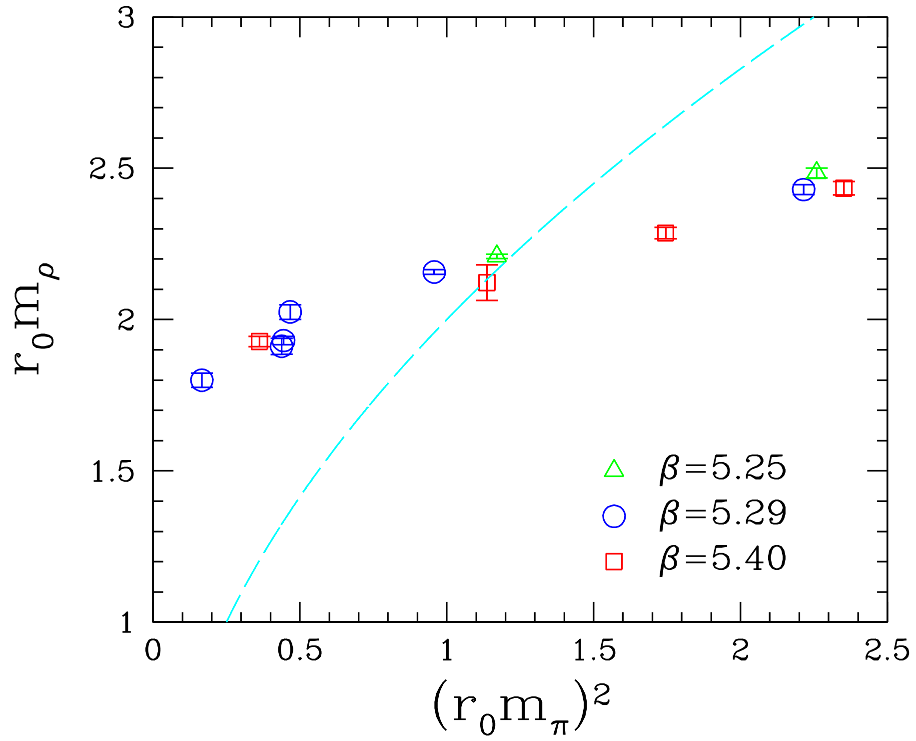
$$g_{\rho\pi\pi}\,\big|_{m_\pi=140\,\mathrm{MeV}} = 5.9 \pm 0.5$$

Actual levels

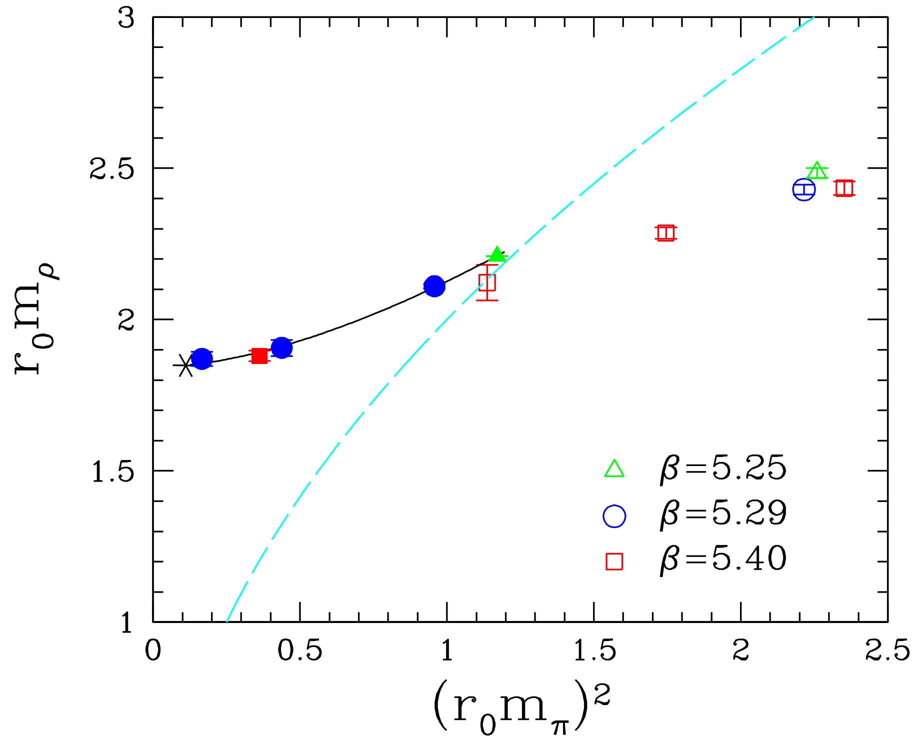


Rho Mass

Lowest energy levels



True ρ mass



Chiral fit: $m_\rho = m_\rho^0 + c_1 m_\pi^2 + c_2 m_\pi^3 + c_3 m_\pi^4 \ln(m_\pi^2)$

Kink ? Bruns & Meißner

$\hookrightarrow$ Armour et al.

Delta

The $\Delta(1232)$ baryon is an elastic p-wave pion-nucleon resonance with isospin 3/2. Its scattering phase shift $\delta_{3/2\,1}(k)$ is very well described by the effective range formula

$$\frac{k^3}{E} \cot \delta_{3/2\,1}(k) = \frac{24\pi}{g^2_{\Delta N\pi}} \left(m^2_\Delta - E^2\right)$$

Here

$$E = \sqrt{k^2 + m_\pi^2} + \sqrt{k^2 + m_N^2}\,, \quad m_\Delta = \sqrt{k_\Delta^2 + m_\pi^2} + \sqrt{k_\Delta^2 + m_N^2}$$

$$\Gamma_\Delta = \frac{g^2_{\Delta N\pi}}{6\pi}\frac{k^3_\Delta}{m^2_\Delta}$$

Experimentally: $\Gamma_\Delta = 118\,\text{MeV} \Longrightarrow \frac{g^2_{\Delta N\pi}}{4\pi} = 14.6$

Free case

$$k = \frac{2\pi|\vec{n}|}{L}\,, \quad \vec{n} \in \mathbb{N}^3$$

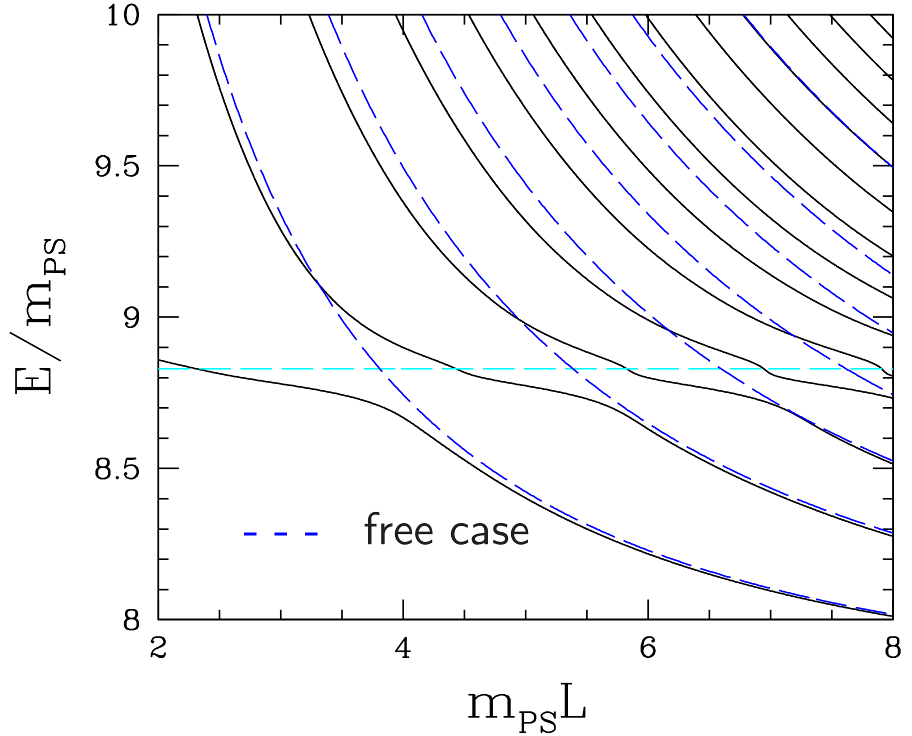
Interacting case

$$\delta_{11}(k) = \arctan\left\{\frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1, q^2)}\right\} \bmod \pi\,, \quad q = \frac{kL}{2\pi}$$

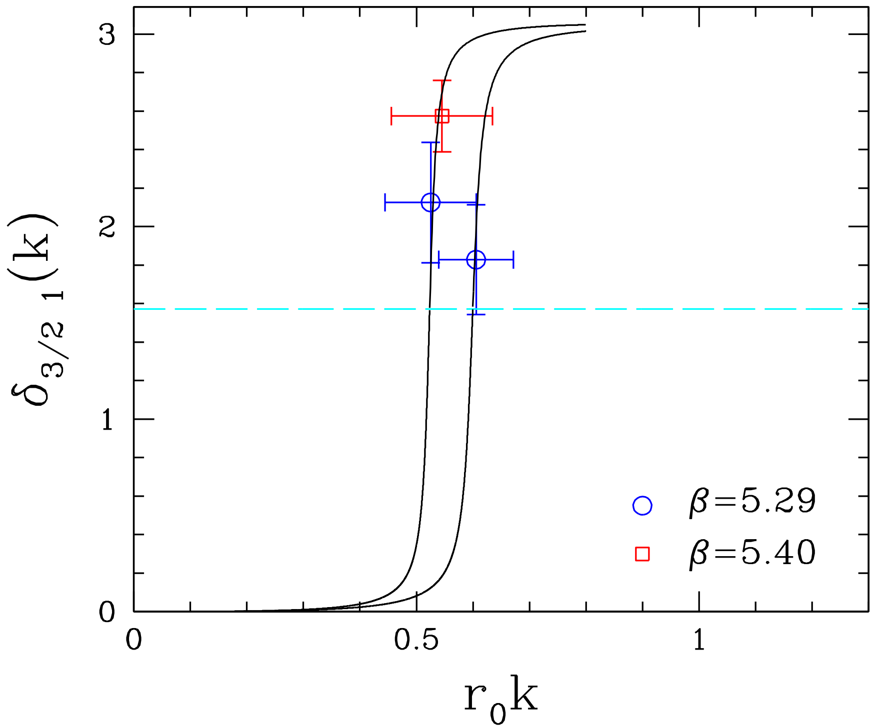
Bernard et al.

Energy Levels

Physical m_π, m_Δ and Γ_Δ

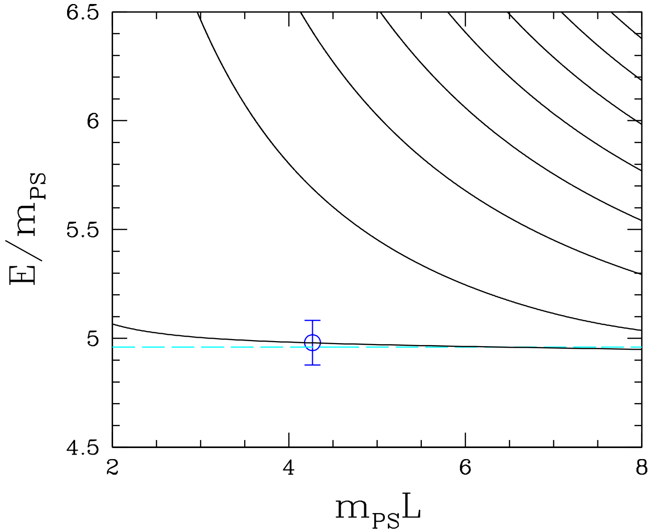


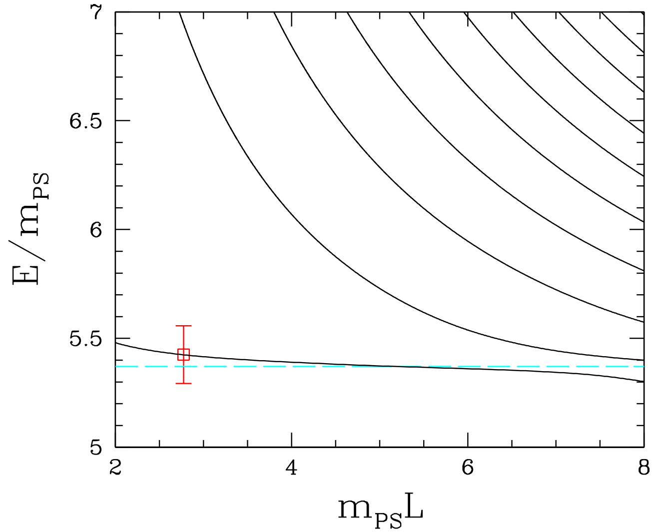
Phase Shift

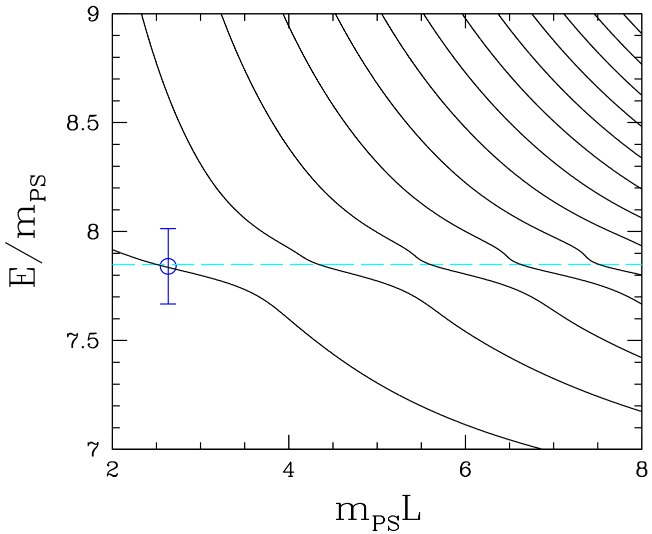


$m_\pi =$ 250 150 MeV

Actual levels

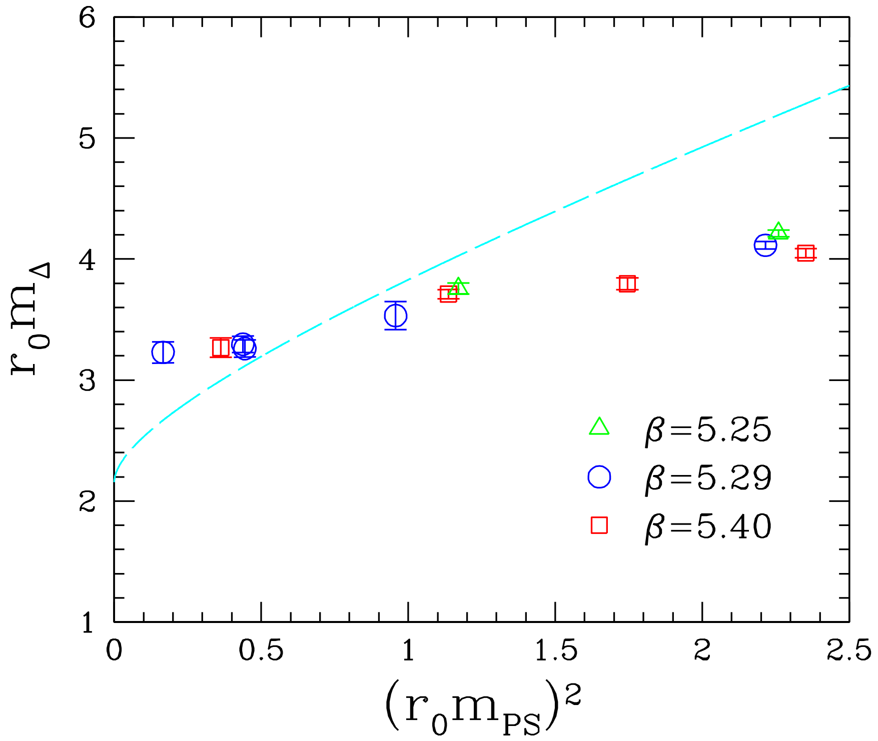




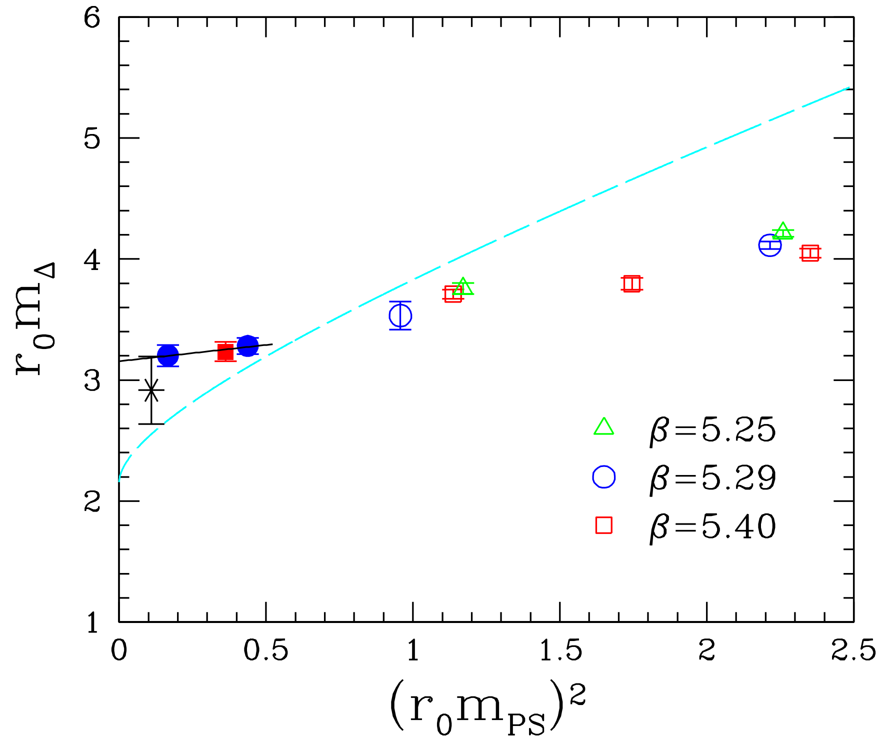


Delta Mass

Lowest energy levels



True Δ mass



Chiral fit: $m_\Delta = m_\Delta^0 - 4c_1 m_\pi^2 + c_2 m_\pi^3$

Bernard et al.

Beyond Delta

$$
\left.
\begin{aligned}
N(1440) \rightarrow\ & N\pi \qquad \text{Roper} \\
& \Delta\pi \\
& N\eta \\
N^*(1535) \rightarrow\ & N\pi \\
& N\eta \\
\vdots\ &
\end{aligned}
\right\} \Longrightarrow \text{Multichannel Lüscher formalism}
$$

Lage, Meißner & Rusetsky

Conclusions & Outlook

- Simulations at the physical pion mass with Wilson-type fermions progressing

- Improvement of algorithms
- Increase of computing power
- QPACE

- Interest is shifting from chiral extrapolation to volume dependence

Ideal volume : $m_\pi L = 2 - 4$

$\approx 3 - 6$ fm

- Computation of phase shifts and resonance masses of ρ and Δ in chiral regime progressing

Lowest energy level E sufficient

- To exploit the full potential of lattice calculations, a major investment in finite volume corrections is needed

- Finite volume corrections of hadron observables
- Inelastic scattering in finite volume