A high-accuracy extraction of the isoscalar πN scattering length from pionic deuterium data

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Tony and πd/πN scattering

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Faddeev approach to pion production and pion-deuteron scattering*

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PION–NUCLEON SCATTERING IN THE CLOUDY BAG MODEL

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πN scattering lengths

- In isospin limit:

$$t^{ab}_{\pi N}(0) = 4\pi(1 + M_\pi/M_N)(a^+\delta^{ab} + a^- i\epsilon^{abc}\tau_c)$$

- a^- and a^+, isoscalar and isovector scattering lengths

$$a^- = \frac{M_\pi}{8\pi F_\pi^2}; \quad a^+ = 0$$ Weinberg (1969)

- Leading-order in an expansion in M_π, an expansion around the chiral limit of QCD
- a^+ encodes extent of chiral-symmetry breaking
- Connected to other issues in strong interactions, πN sigma term, πNN coupling constant (via GMO sum rule)

Hadronic atoms

$$E^{ns}_{\pi^- A} = -\frac{\alpha^2_{em}\mu_{\pi A}}{2n^2} - |\psi_n(0)|^2 \frac{2\pi a_{\pi A}}{\mu_{\pi A}}$$

- Atom is bound by Coulomb force, so characteristic distances are ~ $M_\pi \alpha_{em}$.
- Forms basis for expansion in powers of α_{em}.
- In this theory πA scattering length is high-energy information.

$$E_{1s} = E^{QED}_{1s} - 2\alpha^3_{em}\mu^2_{\pi A} a_{\pi^- A}(1 + 2\alpha(1 - \log\alpha)\mu_{\pi A} a_{\pi^- A} + \delta^{\text{vac}}_A),$$

e.g. Lyubovitsky, Rusetsky (2000)

- Difference E_{1s}-E_{1s}^{QED} can now be measured to 1% or better

Hadronic-atom observables

- Here focus will be on $\pi^- p$ and $\pi^- d$ atoms
- Can measure "strong level shift", $E_{1s}-E_{1s}^{QED}$, in both
- Information on scattering length for $\pi^- p$, real part of scattering length for $\pi^- d$
- $\text{Re } a_{\pi d} \approx 2a^+ + a_{\text{three-body}}$,
- Can measure width in both atoms too
- Proton gives $\pi^- p \rightarrow \pi^0 n$, pionic deuterium width gives little information on threshold πN scattering.
- Three constraints on a^- and a^+
- Test consistency, improve accuracy of extraction

How accurate?

- Fits to low-energy πN data give: Fettes, Meissner (2000)

$$a^+ = 0 \pm 10 \times 10^{-3}\ M_\pi^{-1}$$

- C.f. $a^-_{LO} = 88 \times 10^{-3} M_\pi^{-1}$
- and $\mathrm{Re}\ a_{\pi d} \approx 20 \times 10^{-3} M_\pi^{-1}$
- For a^+ to accuracy 1×10^{-3} need theory that addresses few-body dynamics on 5% level or better
- At this level of accuracy we need to also worry about isospin violation: from both m_u-m_d and electromagnetic effects

Claim: the calculation of $a_{\pi d}$ I will present is accurate at the 5% level, i.e. $\Delta a_{\pi d} \sim 1 \times 10^{-3} M_\pi^{-1}$

Plan

- Introduction
- A theory of the πd scattering length
- Including isospin violation
- Results

Wanted: theory of $a_{\pi d}$

- One approach: multiple-scattering series. Expansion in

$$\frac{4\pi a^-}{r_d} \approx 0.35$$

- Here, χPT: nominally an expansion in $M_\pi/(4\,\pi\,F_\pi) \approx M_\pi/M_N$
- We incorporate isospin violation in both the "two-body" (πN) and "three-body" (πNN) sector.
- Counting $e \sim M_\pi/M_N \equiv P$
- Challenge: additional momentum scales
- E.g. $1/r_d \sim (M_N\,B_d)^{1/2}$, $(M_\pi\,B_d)^{1/2}$

Tony's favourite quote(s):

"A foolish consistency is the hobgoblin of small minds"

Ralph Waldo Emerson
"Self-reliance"

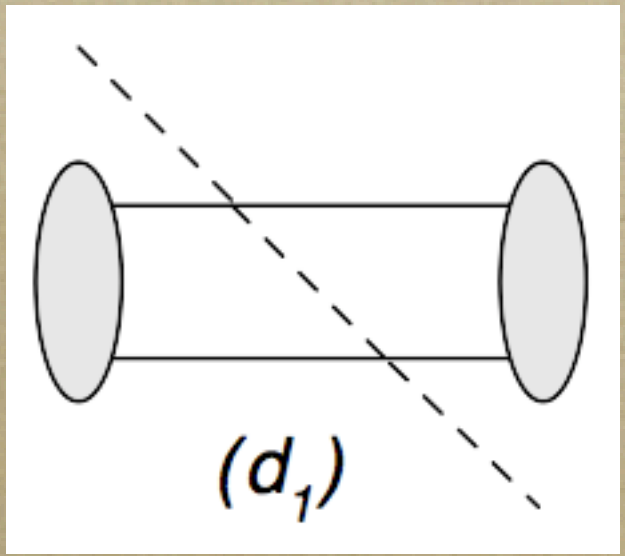
Goya, from Wikipedia Commons
modified by Phillips

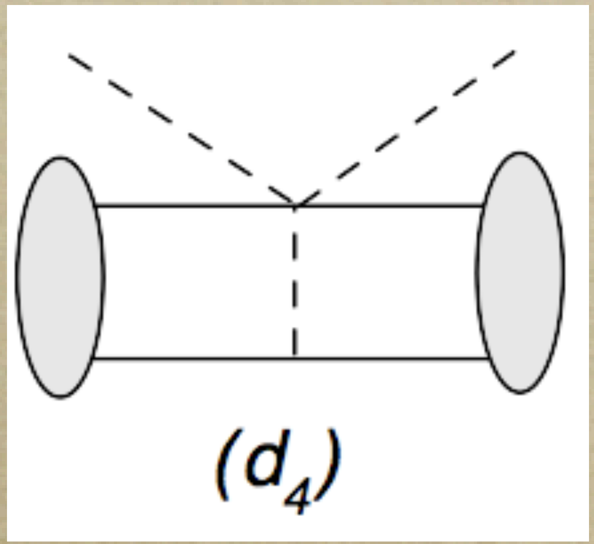
"The nucleon has a finite size, and if you'd just use the cloudy-bag model...."

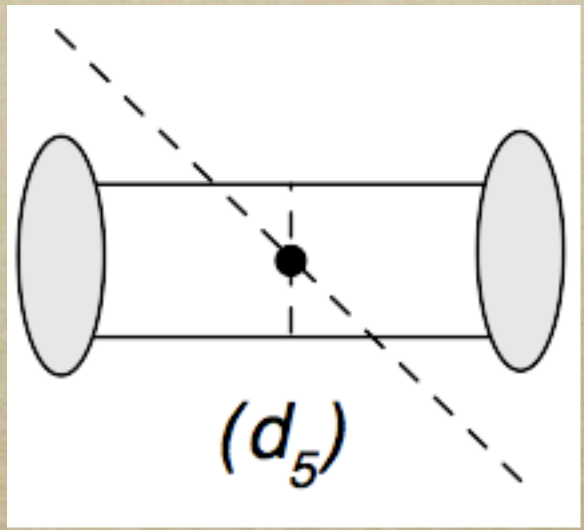
The simple bit....

Weinberg (1992), Beane et al. (1998)

Up to O(P³) $$a_{\pi d} = 2\frac{1+M_\pi/M_N}{1+M_\pi/M_d}a^+ + a_{(d_1)} + a_{(d_4)} + a_{(d_5)}$$







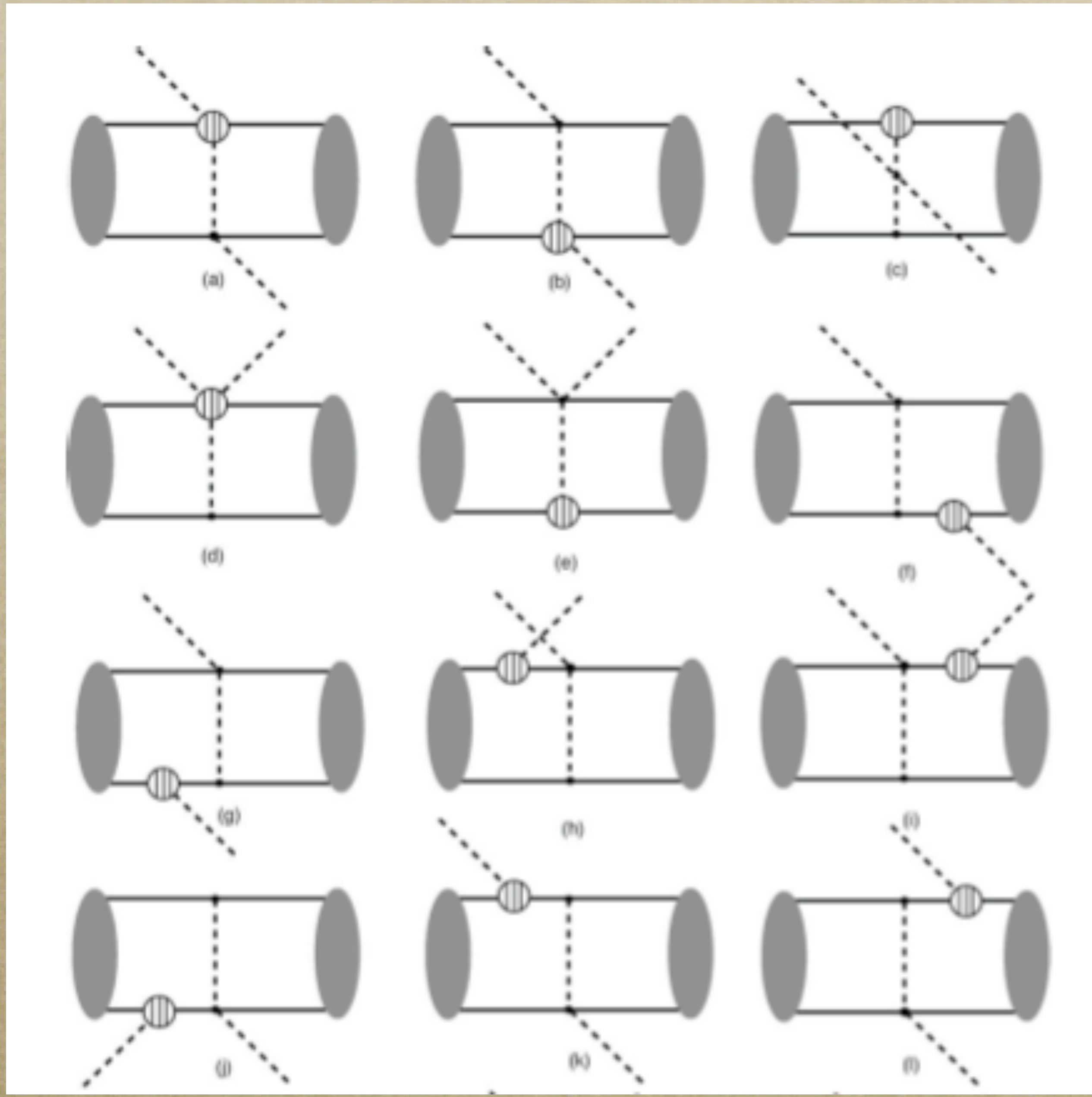
$$a_{(d_1)} = \frac{M_\pi^2}{32\pi^4 F_\pi^4(1+M_\pi/M_d)}\left\langle \frac{1}{\mathbf{q}^2} \right\rangle$$

- $a_{(d1)} \approx 20 \times 10^{-3}\, M_\pi^{-1}$=almost all of experimental $a_{\pi d}$. Means that we must look for a^+ in what's left.

Going further

Beane et al. (2003)

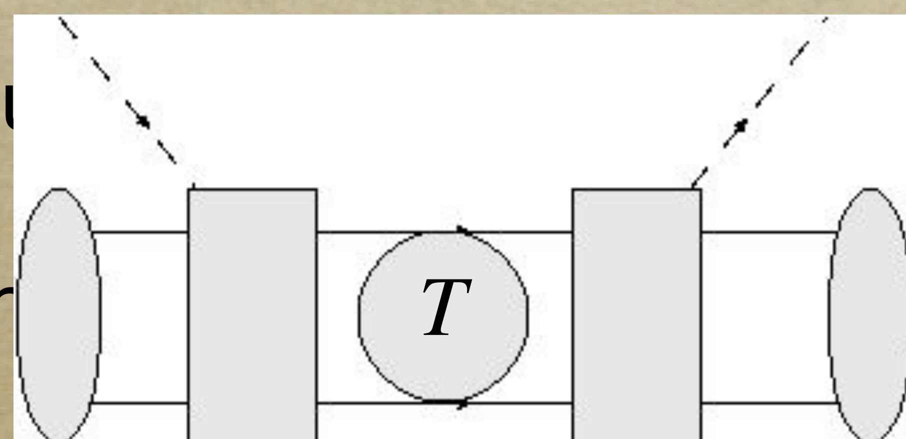
- Diagrams of $O(P^4)$ contributing to three-body part of $a_{\pi d}$:



Sum to zero!

Tony’s favourite questions:

- “But you haven’t accounted for the NN intermediate state, and it’s *known* that the dispersive contribution is an important effect in this reaction.
- Use successful χPT calculation of $\pi^{-}d \rightarrow nn$ to calculate dispersive contribution. (Explains Im $a_{\pi d}$.)
- “You have a hu … attering only a few hundred MeV … your calculation completely ign…



- Include effects of Delta(1232) intermediate states

$$a^{\mathrm{disp}+\Delta} = (-0.6 \pm 1.5) \times 10^{-3} M_{\pi}^{-1}.$$

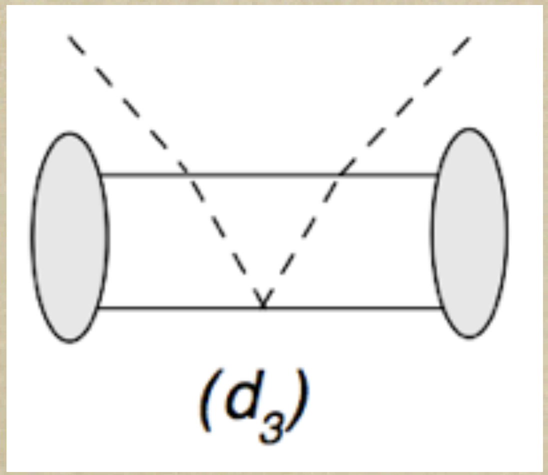
- Nominally these pieces are of order $P^{9/2}$

Baru et al. (2006, 2007)

More on πNN mechanisms

Triple scattering:

$$a_{(d_3)} = (2.6 \pm 0.5) \times 10^{-3} M_\pi^{-1}$$



- Nominally of $O(P^5)$, but enhanced by a factor of π^2
- Therefore must be included to achieve desired accuracy
- Also need to include recoil corrections to double-scattering graph, and impact of embedding πN amplitude in πNN system.

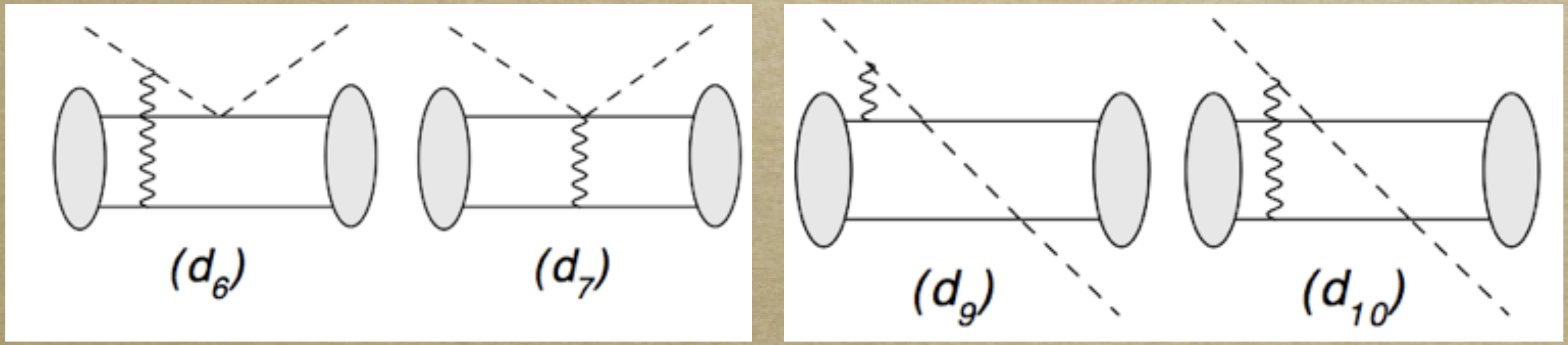
$$a^{\rm str} = (-22.7 \pm 1.1 \pm 0.4) \times 10^{-3} M_\pi^{-1}.$$

CD-Bonn, AV18, NNLO χET

- Difficult to go further: contact term at $O(P^5)$~1 x 10^{-3} M_π^{-1}

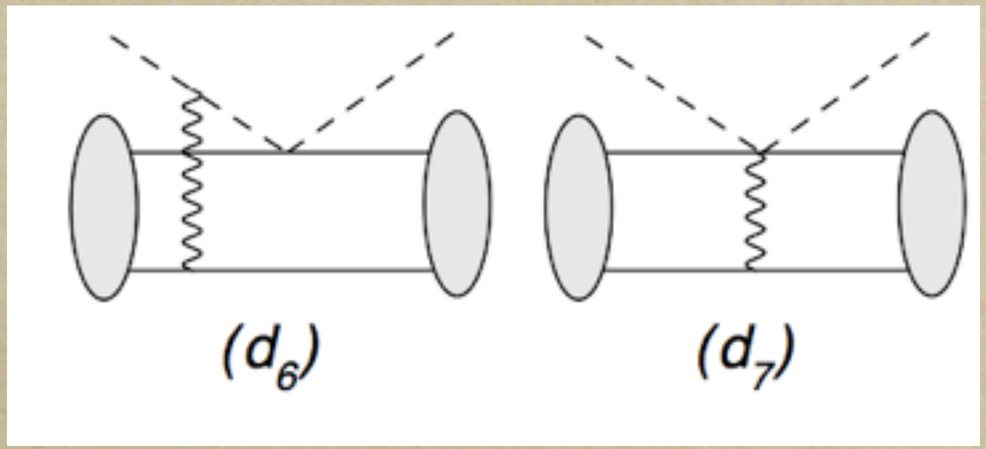
Isospin violation in πNN parts

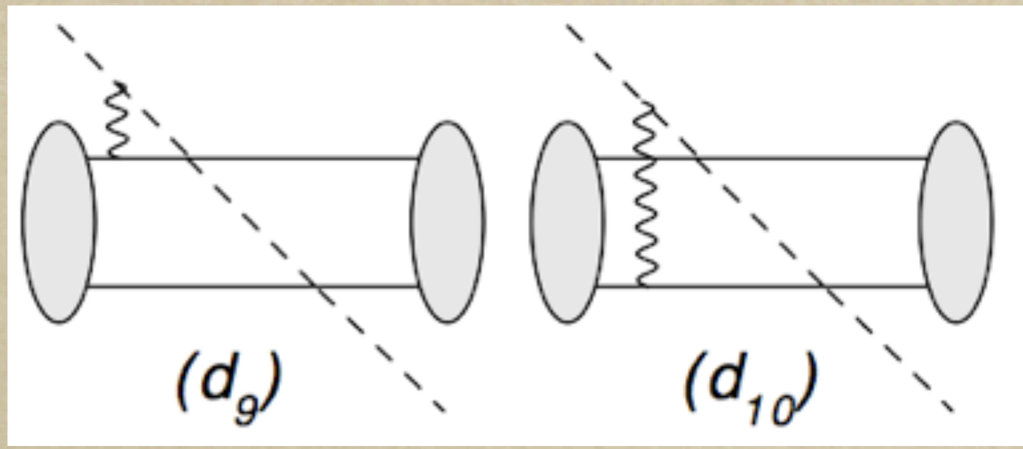
- Pion-mass differences already incorporated in this number
- As were IV differences in the πN scattering lengths (only give 1% of $a_{(d1)}$)
- Isospin violation in $O(P^4)$ below our accuracy threshold
- But still need to compute photon exchange in three-body system, e.g.



- Dominant effect from Coulomb photons
- Need to worry about NN interaction in intermediate state

Scales, scales everywhere...



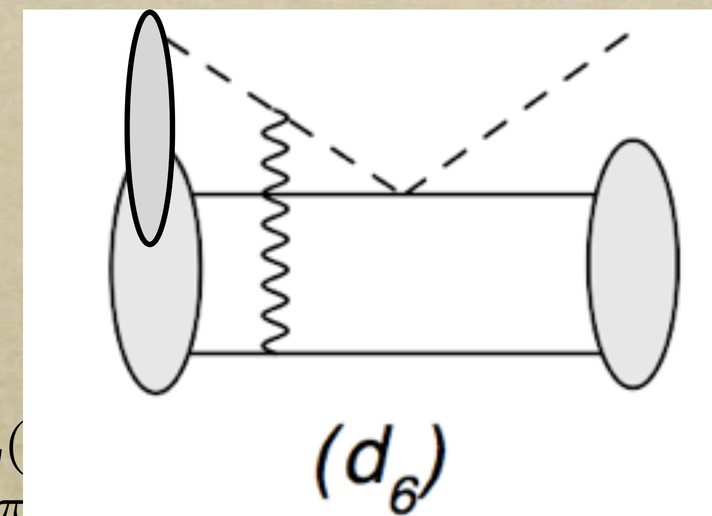


- d_6, d_9, and d_{10} potentially infrared divergent, $<1/|\mathbf{q}|^4>$
- But regulated at hadronic atom scale: give part of level shift
- Need to compute part not already included in modified Deser formula
- $(M_\pi B_d)^{1/2}$ appears thanks to three-body dynamics
- $(M_N B_d)^{1/2}$ from deuteron wave function
- So, in the infra-red enhanced diagrams, does χPT counting apply?

A very useful theorem

Baru, Epelbaum, Rusetsky (2009)

- Consider isoscalar part of, e.g.:

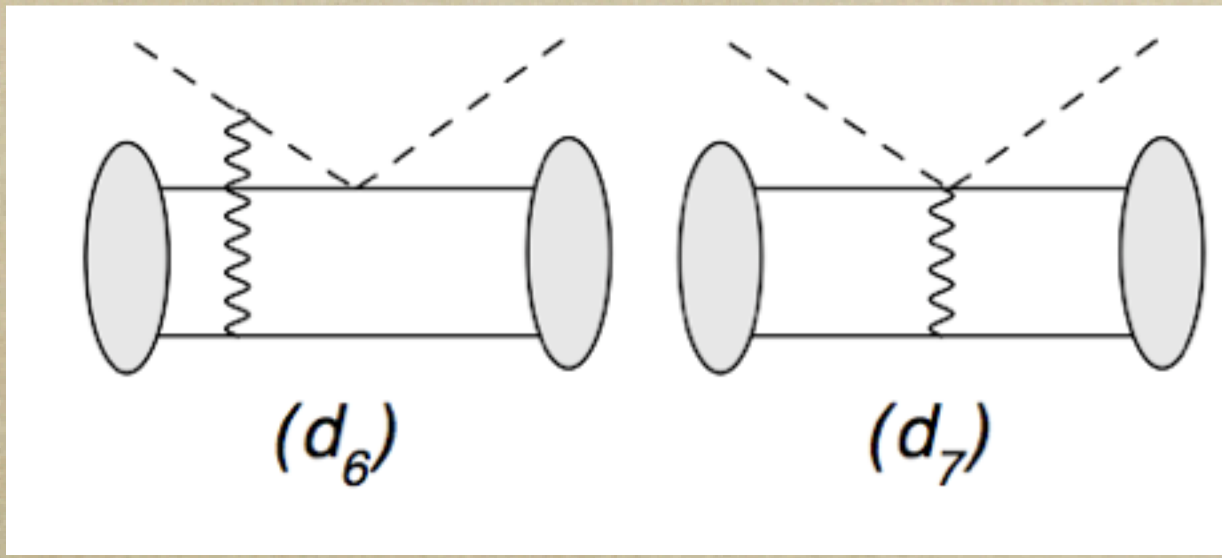


Add NN interactions in int. state:

$$\langle\psi_d|T_{\pi N}G_{NN}(-B_{at})\left(\frac{Q}{\mathbf{k}^2}\right)|\psi_d\rangle = \sum_n \frac{\langle\psi_d|T_{\pi}^{(}\cdots}{-B_{at}-E_n} - \frac{T_{\pi N}^{(s)}}{B_d-B_{at}}\frac{1}{\mathbf{k}^2}$$

- Since $T_{\pi N}$ and Q are spin and isospin independent, if no momentum transfer, orthogonality gives zero
- So no contribution from momenta of order $(M_\pi B_d)^{1/2}$
- Momenta of order $(M_N B_d)^{1/2}$ give shift in $a_{\pi d}$ of < 1% a^+
- Momenta of order M_π give smaller contribution still from this isoscalar part

P waves



- Isovector part ends up giving main contribution
- Only effect from momenta of order M_π

$$a^{\mathrm{EM}} = (0.95 \pm 0.01) \times 10^{-3} M_\pi^{-1}$$

- Consistent with power-counting estimate
- This is largest IV πNN mechanism
- So no need to consider more complex photon exchanges

Now the πN isospin violation...

Hoferichter, Kubis, Meissner (2009)

- In fact, a^+ cannot be measured directly in either πd or πN scattering. Can only be measured in combination:

$$\tilde{a}^+ \equiv a^+ + \frac{1}{4\pi(1+M_\pi/M_p)}\left\{\frac{4(M_\pi^2 - M_{\pi_0}^2)}{F_\pi^2}c_1 - 2e^2 f_1\right\}$$

- a^+ is defined in the isospin limit: this corrects for m_u-m_d and presence of hard photons

- In $a_{\pi d}$ what is measured is actually, up to $O(P^3)$

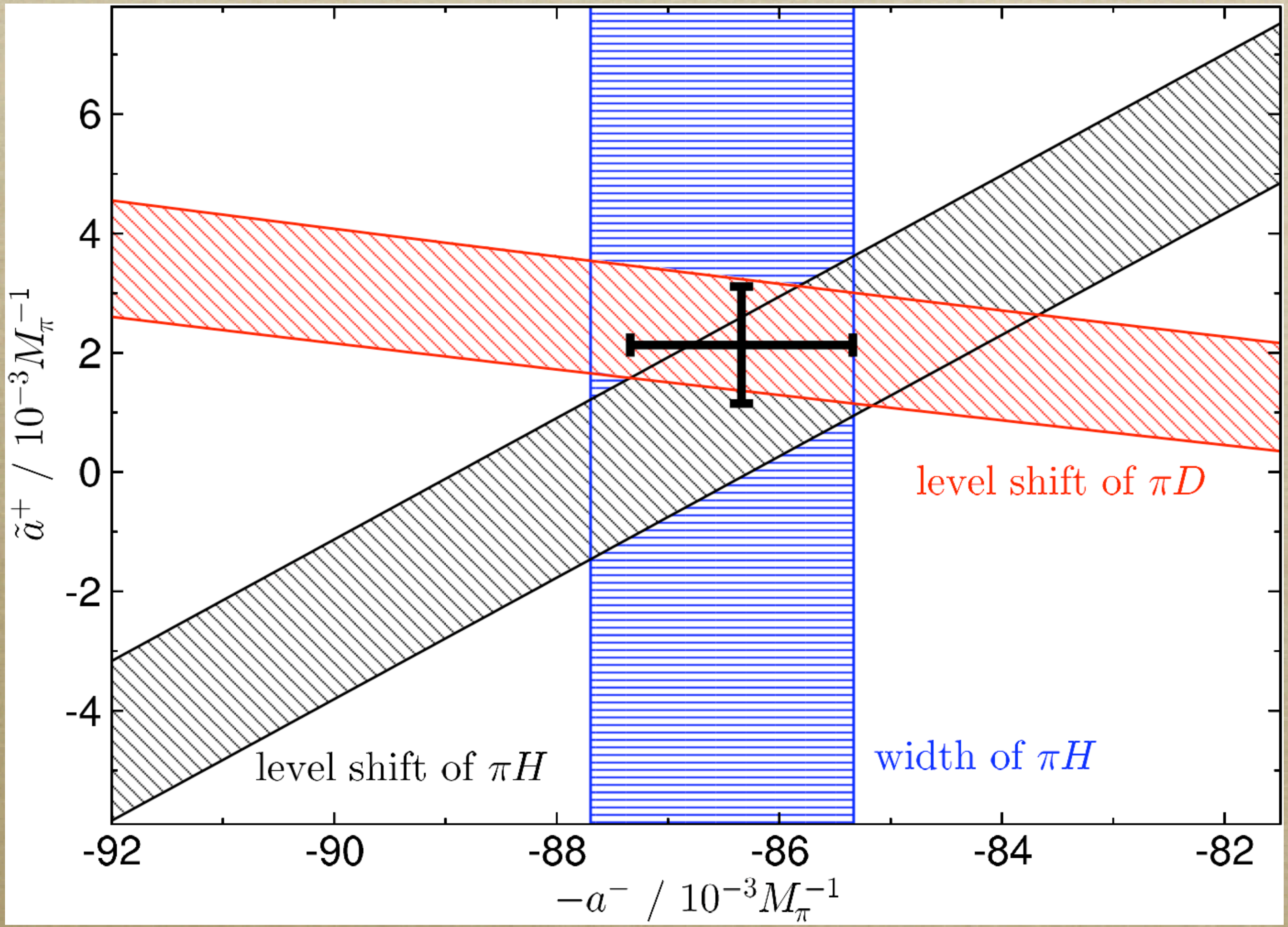
$$a^{(2)}_{\pi^- d} = \frac{2(1+M_\pi/M_p)}{1+M_\pi/M_d}\left(\tilde{a}^+ + \Delta\tilde{a}^+\right),$$

$$\Delta\tilde{a}^+ = \frac{1}{4\pi(1+M_\pi/M_p)}\left[\frac{g_A^2 M_\pi}{32\pi F_\pi^2}\left(\frac{33(M_\pi^2 - M_{\pi_0}^2)}{4F_\pi^2} + e^2\right) + e^2\left(2g_6^{\mathrm{r}} + g_8^{\mathrm{r}}\right)\right]$$

- Estimate: $\Delta\tilde{a}^+ = (-3.3 \pm 0.3) \times 10^{-3} M_\pi^{-1}$

Numbers

Experiment: Gotta et al. (2005, 2010)



$\Delta\epsilon_{1s}^D$	$\Delta a^-, \Delta a_{\pi^- p}^{\text{cex}}$	$\Delta a^{\text{disp}+\Delta}$	$\Delta\tilde{a}^+$	Wave-function averages
16 %	21 %	75 %	30 %	53 %

Conclusions

$$\tilde{a}^+ = (2.1 \pm 1.0) \times 10^{-3} M_\pi^{-1}, \quad a^- = (86.3 \pm 1.0) \times 10^{-3} M_\pi^{-1}$$

- With estimates for LECs we get:

$$a^+ = 7.3^{+3.9}_{-2.9} \times 10^{-3} M_\pi^{-1}$$

- Note theory error still larger than experiment, but difficult to do better without better knowledge of (other) short-distance physics in both πNN and πN sectors
- Need to treat three-body dynamics and isospin violation carefully to achieve this level of accuracy
- Systematicity of χPT useful
- Lots of modifications to power-counting estimates

"The counting is more like what you'd call guidelines than actual rules"

Pirates of the Cloudy Bag:
The Curse of the Blocked Pion

Pirates of the Cloudy Bag:
On Stranger Quarks

Pirates of the Cloudy Bag:
At World's End

Happy Birthday Tony!