# Effective field theory
# and
# electro-weak processes[1]

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[1]Tony Thomas fest - Adelaide, Feb 15, 2010

A careful and systematic study of low-energy weak- and strong interaction reactions is desirable to better our understanding of astro-physical processes.

The central idea of EFT is that physics probed at
long wave lengths
should be insensitive to details of
the short distance structures.

An effective field theory like Chiral Perturbation Theory (ChPT) allows a
unified approach
to weak- and strong interaction processes.

ChPT allows a model-independent, gauge-invariant evaluation of
radiative QED corrections.

Heavy Baryon Chiral Perturbation Theory (HBChPT) reflects the symmetries and the symmetry breaking of the underlying theory of QCD.

(1) If the $u$ and $d$ quarks are massless, the QCD lagrangian is chirally symmetric.

(2) Chiral symmetry is spontaneously broken generating massless Goldstone Bosons (pions).

(3) The non-zero quark masses, $m_u \simeq m_d \ll \Lambda_{QCD}$, i.e. a perturbative expansion of the chiral symmetry breaking appears reasonable.

(4) In ChPT the hadronic scale $\Lambda_{ch} \simeq m_N$, and the corresponding non-zero pion mass $m_\pi$ $(\propto \sqrt{m_{quark}}) \ll \Lambda_{ch}$.

(5) The (perturbative) expansion parameter is: $\varepsilon = Q/\Lambda_{ch} \ll 1$, where $Q$ is the process' typical 4-momentum or $m_\pi$.

# The HBChPT Lagrangian

$\mathcal{L}_{ch}$ is written as an expansion in powers of $Q/\Lambda_{ch}$

$$\mathcal{L}_{ch} \;=\; \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(3)}_{\pi N} + \cdots$$

where $\mathcal{L}^{(\nu)}$ contains terms of order $(Q/\Lambda_{ch})^{\nu}$.

We assume that the terms in the lowest order $\mathcal{L}$agrangian give the dominant contributions to a process.
The higher order terms presumably give smaller perturbative corrections.

The pions are treated relativistically.

The nucleons are treated non-relativistically.

In reality wee have two simultaneous expansions:

$(Q/\Lambda_{ch})^{\nu}$ and $(Q/m_N)^{\nu}$.

The lowest order pion Lagrangian:

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4} Tr\{\nabla_\mu U^\dagger \nabla^\mu U + \chi^\dagger U + \chi U^\dagger\} ,$$

where $\nabla_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$.

Here $v_\mu$ and $a_\mu$ are external currents, and $\chi \propto \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$ .

The $U$-field is expanded in powers of the pion field:

$$U = uu = exp(i\vec{\tau} \cdot \vec{\phi})/f_\pi \simeq 1 + (i\vec{\tau} \cdot \vec{\phi})/f_\pi + \cdots .$$

The first two terms in $\mathcal{L}_{\pi\pi}^{(2)}$ become:

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{1}{2}\left(\partial_\mu \vec{\phi}\,\right)^2 - \frac{1}{2} m_\pi^2\, \vec{\phi}^{\,2} + \cdots$$

The lowest order heavy nucleon Lagrangian is:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N}\{i\,(v\cdot\mathcal{D}) + g_A\,(S\cdot u)\,\}N$$

If we choose the nucleon velocity $v^\mu = (1,\vec{0})$, then $S^\mu = (0,\frac{1}{2}\vec{\sigma})$.

By expanding the $U$-field we find:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N}\left\{i\frac{\partial}{\partial t} - \frac{\vec{\tau}\cdot\left(\vec{\phi}\times\dot{\vec{\phi}}\right)}{4f_\pi^2} + \frac{g_A}{2f_\pi}\vec{\tau}\left(\sigma\cdot\nabla\vec{\phi}\right)\right\}N + \cdots$$

An effective lagrangian contains Low-Energy-Constants (LECs), which parametrice the short-distance physics not probed by long wave-lengths.

The nucleon axial coupling constant, $g_A \simeq 1.27$ is an example of a LEC.

These LECs must be determined in order for the theory to have predictive power.

The next order heavy nucleon Lagrangian:

$$\mathcal{L}_{\pi N}^{(2)} = \bar{N}\left\{\frac{(v\cdot\partial)^2-\partial^2}{2m_N}+\cdots\right\}N$$

contains the heavy nucleon kinetic operator
("the Schrödinger kinetic operator"):

$$\frac{\vec{\nabla}^{\,2}}{2m_N}$$

This nucleon kinetic operator contributes a
"recoil" correction
to the leading (dominant) terms.
The heavy nucleon expansion is different from the Foldy-Wouthuysen expansion.

In the following we will give some examples of
one- and two-nucleon electroweak processes
which have been evaluated in HBChPT.

First a discussion of a few "well known" one-nucleon processes:

(a) Ordinary muon capture: $\mu^- + p \to \nu_\mu + n$ (OMC)

(b) Radiative muon capture: $\mu^- + p \to \nu_\mu + n + \gamma$ (RMC)

(c) $n \to p + e^- + \nu_e$ and $\bar{\nu}_e + p \to e^+ + n$ and the radiative corrections.

Since in all these processes the mometum transfer $Q \ll m_W$, our effective lagrangian is the "Fermi" Lagrangian:

$$\mathcal{L}_{Fermi} = \frac{G_F}{\sqrt{2}} J_\beta(lepton) \cdot J^\beta(hadron)$$

where

$$J_\beta(lepton) = \bar{u}_\nu \gamma_\beta (1 - \gamma_5) u_l = v_\beta^{lep} - a_\beta^{lep}$$

$$J_\beta(hadron) = v_\beta^{had} - a_\beta^{had}$$

Traditionally the hadronic currents are written as:

$$v_\beta^{had} = \bar{u}\left\{G_V(q^2)\gamma_\beta + G_M(q^2)\frac{i\sigma_{\beta\delta}q^\delta}{2m_N} + 2\text{nd class}\right\}u$$

$$a_\beta^{had} = \bar{u}\left\{G_A(q^2)\gamma_\beta\gamma_5 + G_P(q^2)\frac{q_\beta\ \gamma_5}{2m_N} + 2\text{nd class}\right\}u$$

where, including terms of order $q^2$, the four form-factors are given by the nucleon's (i) r.m.s. radius, (ii) axial radius and (iii) anomalous nucleon magnetic moments, i.e. $G_M(q^2) = \kappa_p - \kappa_n + \cdots$, and

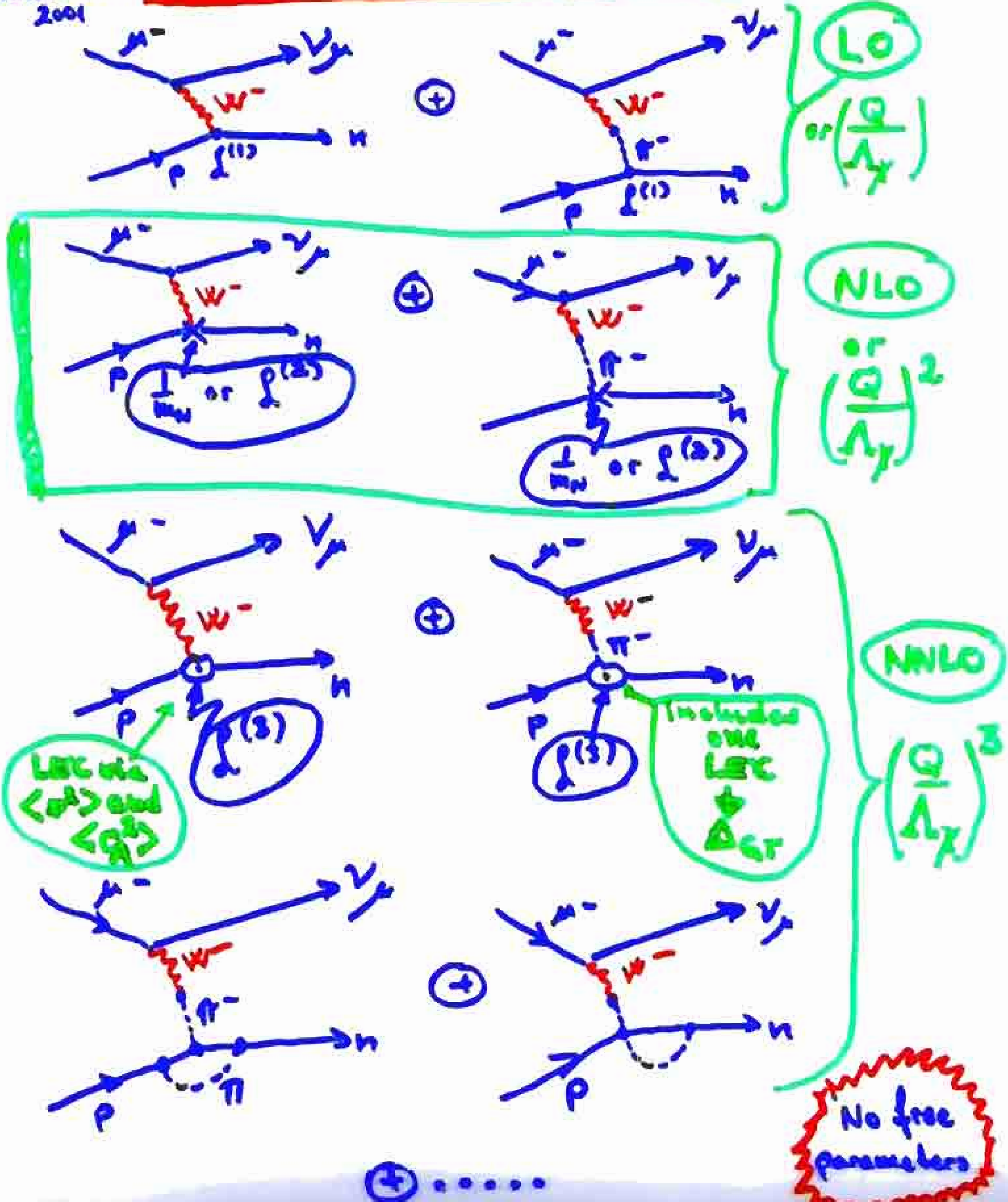
$$\frac{G_P(q^2)}{2m_N} = -\frac{2f_\pi\ g_{\pi NN}}{q^2 - m_\pi^2} - \frac{1}{3}g_A\ m_N < r_A^2 >$$

$G_P(q^2)$ was derived some time ago by Adler and Dothan and Wolfenstein. ChPT including one-loop corrections reproduces this "old" analytic expression. N. Kaiser used ChPT to show that the next order corrections are tiny.

One challenge exists.
Can $G_P(q^2)$ be measured in some process in order to confirm this theoretical prediction?

S. Ando, FM, K. Kubodera 2001

# One-nucleon current illustrated by $\mu^- + p \to \nu_\mu + n$



The expansion converges rapidly for the $\mu^- p$ spin-singlet capture rate [*taken from: V. Bernard et al. (2001)*]:

$$\Gamma = \left(957 - \frac{245\mathrm{GeV}}{m_N} + \left[\frac{30.4\mathrm{GeV}^2}{m_N^2} - 43.17\right]\right) s^{-1}$$

The near cancellation of the "recoil" and $q^2$ form-factor terms illustrate the advantage of the systematic expansion of HBChPT.

The $\mu^- p$ capture rate has been measured at PSI by V.A. Andreev *et al.*.
Initial results in PRL **99** (2007) consistent with ChPT prediction.
Final results expected at 1% accuracy.

The radiative muon capture (RMC) was evaluated in ChPT.
In RMC $q$ changes with the photon energy, $E_\gamma$,
i.e. RMC could determine $G_P$ (pion pole dominance) better than OMC.
A TRIUMF team successfully measured the RMC $\mathrm{d}\Gamma/\mathrm{d}E_\gamma$ for photon energies $E_\gamma > 60$ MeV.

HBChPT is unable to reproduce RMC experimental results.

## Radiative corrections to neutron $\beta$-decay and the CHOOZ process

In the coming decade
$n \to p + e^- + \bar{\nu}_e$ and $\bar{\nu}_e + p \to e^+ + n$
will be measured very precisely.

The CHOOZ process will be used to determine neutrino oscillation parameters.

The precise measurements of neutron $\beta$-decay aim at an accurate value for $V_{ud}$.
To extract $V_{ud}$ requires a better understanding of the radiative corrections (RC).

Why a new evaluation of these RC?

ChPT allows a model-independent, gauge-invariant evaluation of RC.
The short distance physics is well defined by radiative LEC in the lagrangian.

## The two-nucleon processes to be discussed:

(a) Muon capture on the deuteron: $\mu^- + d \rightarrow \nu_\mu + n + n$

(b) The Sudbury Neutrino Observatory (SNO) reactions:
$\nu_e + d \rightarrow e^- + p + p$ and $\nu_x + d \rightarrow \nu_x + p + n$

(c) The radiative pion capture on the deuteron: $\pi^- + d \rightarrow \gamma + n + n$

A ChPT evaluation of (a) - (c) include one unknown two-nucleon axial LEC, $\hat{d}^R$.
The same $\hat{d}^R$ enters in [2]

- $^3H \rightarrow ^3 He + e^+ + \nu_e$
- Solar $pp$ fusion: $p + p \rightarrow d + e^+ + \nu_e$
- Solar Hep process: $^3He + p \rightarrow ^4 He + e^+ + \nu_e$
- Modern three-nucleon potential as one of the two LEC parameters.

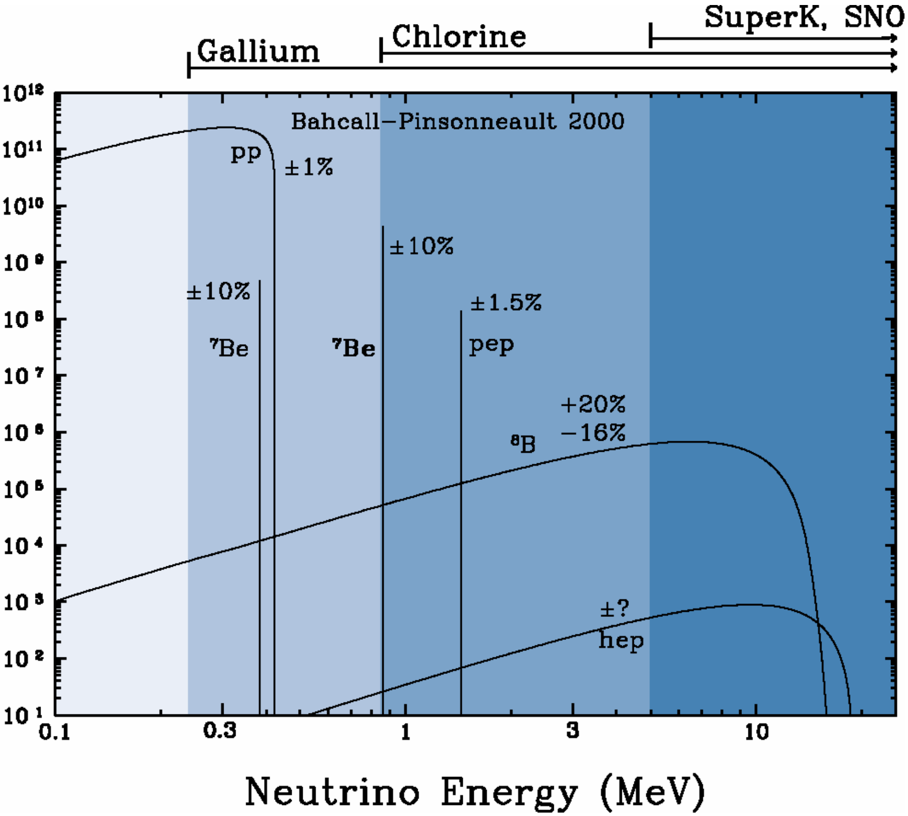
The Hep process produces the highest energy solar neutrinos. A precise evaluation of Hep is difficult since large contributions almost cancel.[3].

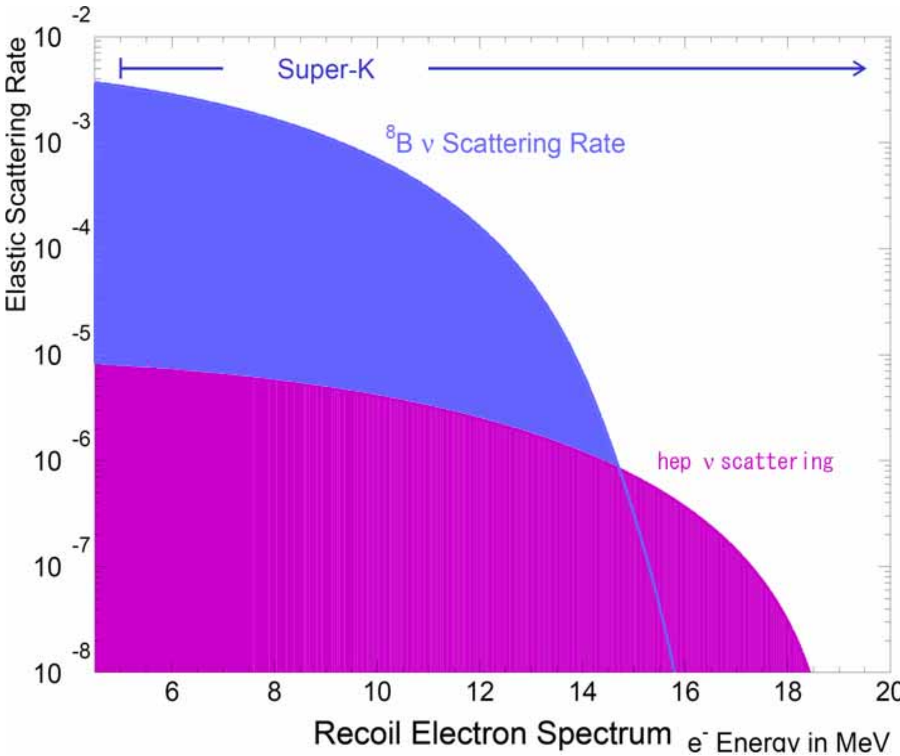
[2]reactions evaluated by T.-S. Park et al., Phys. Rev. C **67**, 055206 (2003).
[3]J. Carlson et al., Phys. Rev. C **44**, 619 (1991)

# Solar neutrino in SK

$\nu e^- \rightarrow \nu e^-$

**Recoil electron spectrum**

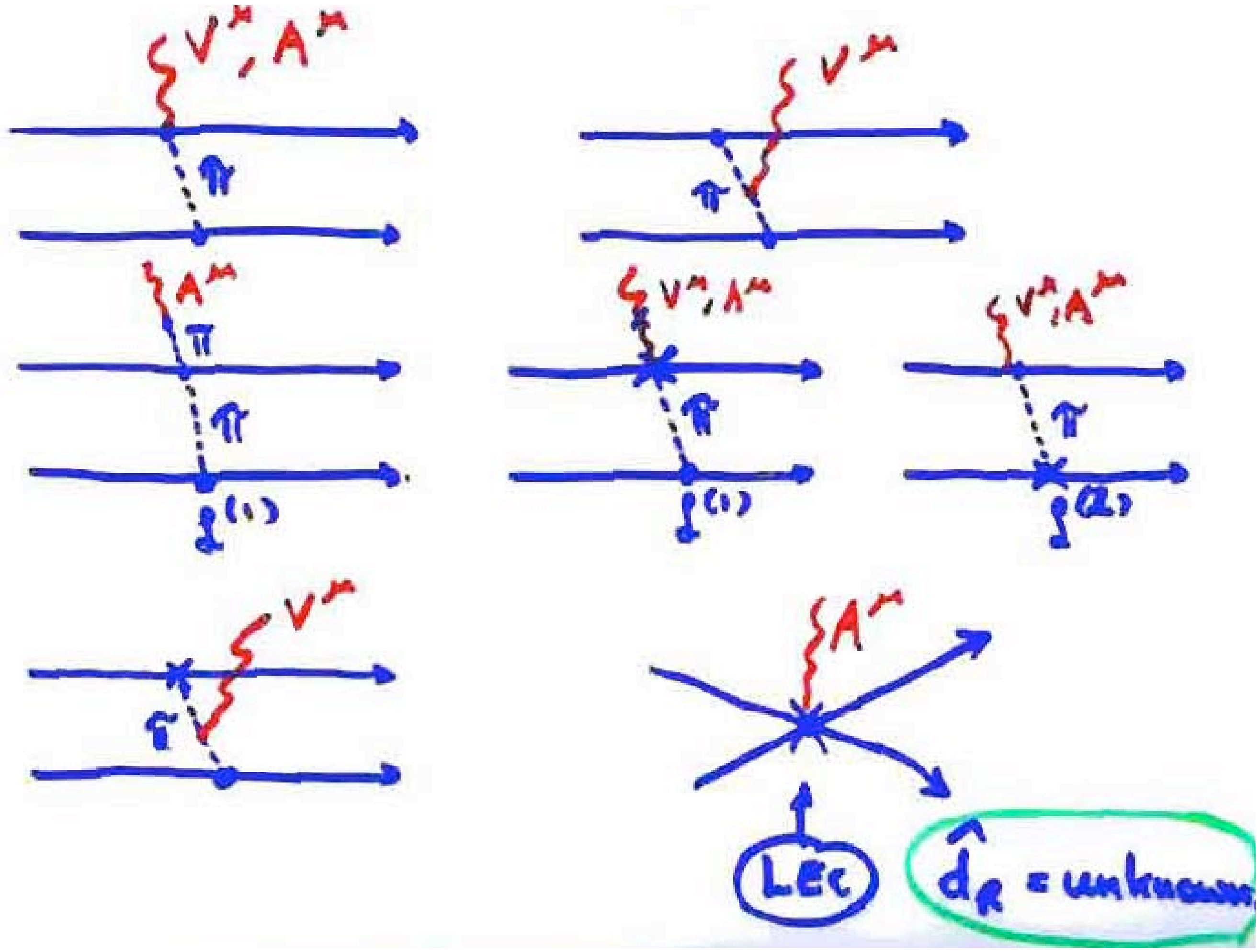



http://www.sns.ias.edu/~jnb/

The two-nucleon diagrams with vertices from the ChPT Lagrangian.

Loop-diagrams which renormalize couplings and give vertex form factors are not shown.




The two-nucleon axial Low-Energy-Constant to be determined.

Ideally these two-nucleon reactions should be evaluated using transition operators and nucleon wave functions from ChPT.

For pragmatic reasons a $hybrid$ ChPT called $EFT^*$ has been used in the two- and more nucleon processes.

- The one- and two-nucleon transition operators from ChPT.
- The wave functions evaluated from modern "high precision" $NN$ potentials.

The reactions are all evaluated within the same formalism.

- At present the two-nucleon axial LEC, $\hat{d}^R$ is determined from tritium $\beta$-decay.

In some $EFT^*$ calculation a Gaussian cut-off $\Lambda_G$ was used in the $NN$ wave functions to limit the contributions from the high momentum components, i.e. $\hat{d}^R$ depends on $\Lambda_G$.

Observables stable (less than 1% variation) for 500 MeV $< \Lambda_G <$ 800 MeV.

Our evaluated neutrino-deuteron CC and NC reaction cross sections were used by the SNO collaboration in their analysis of the solar neutrino data.

Presently $\hat{d}^R$ is is determined from a three-nucleon system: tritium $\beta$-decay rate. It is desireable to have a better determination of the two-nucleon axial coupling $\hat{d}^R$.

A more precise $\hat{d}^R$ value will allow a more accurate evaluation of

(i) the solar $pp$ fusion reaction, the primary energy source in the sun,

(ii) the SNO neutrino-deuteron reactions, which provided convincing evidence for neutrino oscillation.

(iii) The reaction $\pi^- + d \to \gamma + n + n$, which can be used to determine the scattering length $a_{nn}$.

The rate of muon capture on a deuteron ($\mu^- d$) will be measured at PSI by the MuSun collaboration with an expected 1.5% accuracy during 2009-2011. We are presently re-evaluating our $\mu^- d$ ChPT calculation to meet this challenge.

## Supernova Explosion

Computer simulations of the supernova have not been very successful in generating the explosion. This is possibly due to the neutrino luminosity being too small.

We have identify new processes which generate neutrinos in the proto-neutron star at the center of the supernova explosion.
These reactions might affect the explosion-simulation due to an (estimated) increased in the neutrino flux.

## Radiative Corrections

We use HBChPT to evaluate the radiative correction to the CHOOZ reaction

$$\bar{\nu}_e + p \to e^+ + n \ .$$

The LEC appearing are determined by the radiative corrections in neutron $\beta$-decay.

Two other projects are the evaluations of the radiative correction of the $\nu d$ (SNO) and $\mu^- d$ capture reactions.
These corrections are larger or of the order of the experimental errors.

# Conclusions

- The low-energy effective theory, HBChPT, allows a systematic evaluation of electro-weak and strong interaction processes.
- HBChPT predicts accurately $G_P(q^2)$ — confirmed by recent MuCap data.
- HBChPT gives analytic expressions for both the $\mu^- p$ and $\mu^- d$ capture operators. The MuCap experiment at PSI has published a $\mu^- p$ rate compatible with HBChPT
- ChPT permits us to estimate the theoretical uncertainty.
  An observable evaluated at "order" $(Q/\Lambda_{ch})^v$, has an estimated uncertainty given by the next order contribution $(Q/\Lambda_{ch})^{v+1}$.
- Two-nucleons reactions (including the energy dependence of $n+p \to d+\gamma$ which is important in cosmology) are well described by $EFT^*$ (except RMC). A more accurate value for $\hat{d}_R$, determined in a two-nucleon process, is desireable in order to make more accurate predictions for solar $pp$ fusion and $\nu d$ SNO reactions.
- Improved radiative corrections are needed to "compete" with the expected MuCap and MuSun $\mu^-$-capture data.