Travels With Tony- Nucleon Structure Through Our Ages

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Theme- ways of examining nucleon structure have changed with time, ’79-’82 was an exciting time and so is now.
Themes- neutron structure and shape of the proton through gpds and tmds

Happy Birthday Tony!
Outline

1. Cloudy bag model

2. Phenomenology: proton is not round.

3. Model independent neutron charge density

4. Measure shape of proton on lattice (impact parameter dependent GPD), and in experiment (TMD): IPDGPD is coordinate-space probability and TMD is momentum-space probability

GAM “Transverse Charge Densities” 1002.0355

Experimental progress—several labs
Cloudy bag model of the nucleon

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A previously derived model in which a baryon is treated as a three-quark bag that is surrounded by a cloud of pions is used to compute the static properties of the nucleon. The only free parameter of the model is the bag radius which is fixed by a fit to pion-nucleon scattering in the (3,3)-resonance region to be about 0.8 fm. With the model so determined the computed values of the root-mean-square radii and magnetic moments of the neutron and proton, and $g_A$, are all in very good agreement with the experimental values. In addition, about one-third of the Δ-nucleon mass splitting is found to come from pionic effects, so that our extracted value of $\alpha_s$ is smaller than that of the MIT bag model.

Many successful predictions

One feature- pion penetrates to the bag interior
One gluon exchange gives similar effect, the FT of $G_{En}$ is $+\text{ near } 0$. 

\[ 4\pi r^2 \int d^3 e^{-i\vec{q} \cdot \vec{r}} G_{En}^n(\vec{q}) \]
Meaning of form factor

• $G_E(Q^2)$ is NOT Fourier transform of charge density

• Relativistic treatment needed—wave function is frame-dependent, initial and final states differ, no density

• Light front coordinates, $\infty$ momentum frame

“Time” $x^+ = (ct + z)/\sqrt{2} = (x^0 + x^3)/\sqrt{2}$, “evolution” $p^- = (p^0 - p^3)/\sqrt{2}$

“Space” $x^- = (x^0 - x^3)/\sqrt{2}$, “Momentum” $p^+ = (p^0 + p^3)/\sqrt{2}$

“Transverse position, momentum, $b, p$

These coordinates are used to analyze form factors, deep inelastic scattering, GPDs, TMDS
Relativistic formalism-kinematic subgroup of Poincare

- Lorentz transformation – transverse velocity $v$

\[ k^+ \rightarrow k^+, \quad k \rightarrow k - k^+v \]

$k^-$ such that $k^2$ not changed

Just like non-relativistic with $k^+$ as mass, take momentum transfer in perp direction, then density is 2 Dimensional Fourier Transform, also

\[ q^+ = q^0 + q^3 = 0, \quad -q^2 = Q^2 = q^2 \]
interpretation of FF as **quark density**

**q**

overlap of wave function Fock components with **different** number of constituents

**NO probability/charge density interpretation**

Absent in a Drell-Yan Frame

\[ q^+ = q^0 + q^3 = 0 \]

From Marc Vanderhaeghen
Model proton wave function $\Psi(k_\perp, K_\perp, \xi, \eta)$

Poincare invariant

Light front variables for boost: 

Dirac spinors carry orbital angular momentum
Ratio of Pauli to Dirac Form Factors 1995
Frank, Jennings, Miller theory, data 2000

Flat due to orbital angular momentum
Model exists

- lower components of Dirac spinor
- orbital angular momentum
- shape of proton?? **Wigner Eckart**
- no quadrupole moment
- spin dependent densities SDD
- non-relativistic example
I: Non-Rel. $p_{1/2}$ proton outside $0^+$ core

$$\langle r_p | \psi_{1,1/2s} \rangle = R(r_p) \sigma \cdot \hat{r}_p | s \rangle$$

$$\rho(r) = \langle \psi_{1,1/2s} | \delta(r - r_p) | \psi_{1,1/2s} \rangle = R^2(r)$$

probability proton at $r$ & spin direction $n$:

$$\rho(r, n) = \langle \psi_{1,1/2s} | \delta(r - r_p) \frac{1 + \sigma \cdot n}{2} | \psi_{1,1/2s} \rangle$$

$$= \frac{R^2(r)}{2} \langle s | \sigma \cdot \hat{r} (1 + \sigma \cdot n) \sigma \cdot \hat{r} | s \rangle$$

$n \parallel \hat{s}$ : $\rho(r, n = \hat{s}) = R^2(r) \cos^2 \theta$

$n \parallel -\hat{s}$ : $\rho(r, n = -\hat{s}) = R^2(r) \sin^2 \theta$

non-spherical shape depends on spin direction
Shapes of the proton

Momentum space
three vectors $n, K, S$

Relation between coordinate and momentum space densities?

How to measure? - Lattice and/or experiment
Model independent transverse charge density

\[ J^+(x^-, b) = \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b) \]  

\[ \rho_\infty(x^-, b) = \langle p^+, \mathbf{R} = 0, \lambda | \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b) | p^+, \mathbf{R} = 0, \lambda \rangle \]

\[ F_1 = \langle p^+, \mathbf{p}', \lambda | J^+(0) | p^+, \mathbf{p}, \lambda \rangle \]

\[ \rho(b) \equiv \int dx^- \rho_\infty(x^-, b) = \int \frac{Q dQ}{2\pi} F_1(Q^2) J_0(Q b) \]

Density is \( u - \bar{u}, d - \bar{d} \)
Transverse charge densities

$\rho(b) \ [fm^{-2}]$

**Proton**

Kelly

**Negative**

$\rho(b) \ [fm^{-2}]$

**Neutron**
Negative $F_1$ means central density negative.
Neutron Interpretation needed

Neutron interpretation

- Impact parameter gpd Burkardt $\rho(x, b)$
- From Drell-Yan-West relation between high $x$ DIS and high $Q^2$ elastic scattering
- High $x$ related to low $b$, not uncertainty principle
- $d$ quarks dominate DIS from neutron at high $x$
- $d$ quarks dominate at neutron center, or

\[
\pi^- \text{ is } \bar{u}d \\
\text{Density is } u - \bar{u}, \ d - \bar{d} \\
\text{decreases } u \text{ contribution} \\
\text{enhances } d \text{ contribution}
\]
Neutron $\rho(b, x)$

Using other people’s models

$d$ or $\pi^-$ dominates at high $x$, low $b$
Return of the cloudy bag model

- In a model nucleon: bare nucleon + pion cloud - parameters adjusted to give negative definite $F_1$, pion at center causes negative central transverse charge density
- Boosting the matrix element of $J^0$ to the infinite momentum frame changes $G_E$ to $F_1$
Shapes of the proton

- Relate spin dependent density to experiment

Field-theoretic spin dependent momentum density is related to the transverse momentum distribution $h_{1T}^{\perp}$

$$\Phi^{[\Gamma]}(x, K_T) = \left. \int \frac{d\xi^- d^2 \xi_T}{2 (2\pi)^3} e^{iK \cdot \xi} \langle P, S | \overline{\psi}(0) \Gamma \mathcal{L}(0, \xi; n_-) \psi(\xi) | P, S \rangle \right|_{\xi^+ = 0}$$

Mulders Tangerman’96

$$\Phi^{[i\sigma^i + \gamma_5]}(x, K_T) = S_T^i h_1(x, K_T^2) + \left( \frac{K_i^j K_T^j - \frac{1}{2} K_T^2 \delta_{ij}}{M^2} \right) S_T^j h_{1T}^{\perp}(x, K_T^2)$$

$$\sigma^i + \gamma^5 \sim \gamma^0 \gamma^+ \sigma^i,$$

then relate equal time to $\xi^+ = 0$ by integration over $x$
Cross section has term proportional to \( \cos 3\phi \).

Boer Mulders ‘98 there are other ways to see \( h_{1T} \).

\[ \text{Measure } h_{1T} : e, p \rightarrow e', \pi X \]

Cross section has term proportional to $\cos 3\phi$

Boer Mulders ’98 there are other ways to see $h_{1T}^\perp$
Generalized densities

\[ \mathcal{O}_q^\Gamma (px, b) = \int \frac{dx^- e^{ipxx^-}}{4\pi} q_+^\dagger (0, b) \Gamma q_+ (x^-, b) \]

\[ \rho^\Gamma (b) = \int dx \sum_q e_q \langle p^+, \mathbf{R} = 0, \lambda | \mathcal{O}_q^\Gamma (p^+ x, b) | p^+, \mathbf{R} = 0, \lambda \rangle \]

\[ \int dx \] sets \( x^- = 0 \), get \( q_+^\dagger (0, b) \Gamma q_+ (0, b) \) Density!

\[ \Gamma = 1 / 2 (1 + \mathbf{n} \cdot \gamma) \] gives spin-dep density

Local operators calculable on lattice Gloeckler et al
PRL98,222001

\( \tilde{A}_{T10}'' \sim \text{sdd} \) spin-dependent density

Schierholtz, 2009 -this quantity is not zero, proton is not round
Transverse Momentum Distributions -
momentum space density

In a state of fixed momentum

\[ \Phi_{q}^{\Gamma}(x, \mathbf{K}) \] give probability of quark of given 3-momentum

\[ h_{1T}^{\bot} \] gives momentum-space spin-dependent density

measurable experimentally

hard to calculate on lattice because - gauge link
Summary

• Form factors, GPDs, TMDs, understood from unified light-front formulation
• Neutron central transverse density is negative-consistent with Cloudy Bag Model
• Proton is not round- lattice QCD spin-dependent-density is not zero
• Experiment can whether or not proton is round by measuring $h_{1T}$
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The Proton