Travels With Tony- Nucleon Structure Through Our Ages

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Theme- ways of examining nucleon structure have changed with time, '79-'82 was an exciting time and so is now.
Themes- neutron structure and shape of the proton through gpds and tmds

Happy Birthday Tony!

Outline

1. Cloudy bag model

2 Phenomenology: proton is **not** round.

3. Model independent neutron charge density

4. Measure shape of proton on lattice (impact parameter dependent GPD), and in experiment (TMD): IPDGPD is coordinate-space probability and TMD is momentum-space probability

GAM "Transverse Charge Densities" 1002.0355

Experimental progress-several labs

Cloudy Bag Model~1980

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Cloudy bag model of the nucleon

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(Received 28 January 1981)

A previously derived model in which a baryon is treated as a three-quark bag that is surrounded by a cloud of pions is used to compute the static properties of the nucleon. The only free parameter of the model is the bag radius which is fixed by a fit to pion-nucleon scattering in the (3,3)-resonance region to be about 0.8 fm. With the model so determined the computed values of the root-mean-square radii and magnetic moments of the neutron and proton, and g_A, are all in very good agreement with the experimental values. In addition, about one-third of the Δ-nucleon mass splitting is found to come from pionic effects, so that our extracted value of α_s is smaller than that of the MIT bag model.

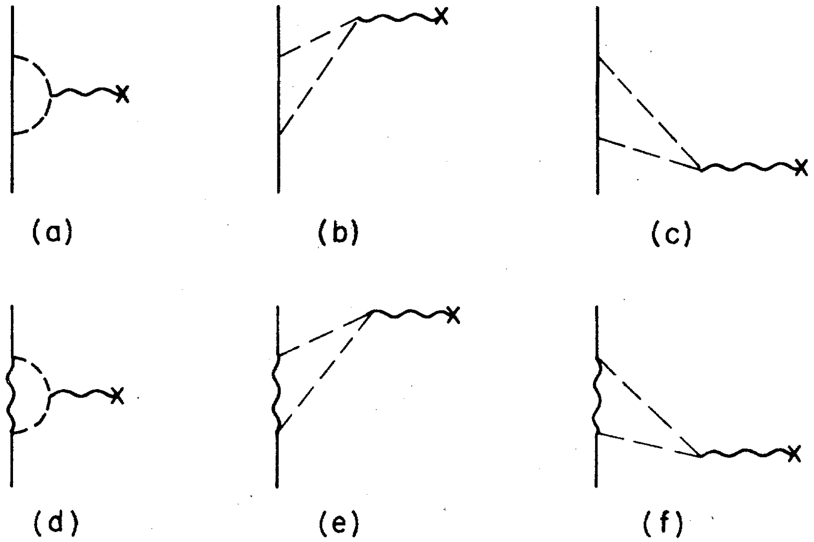
Many successful predictions

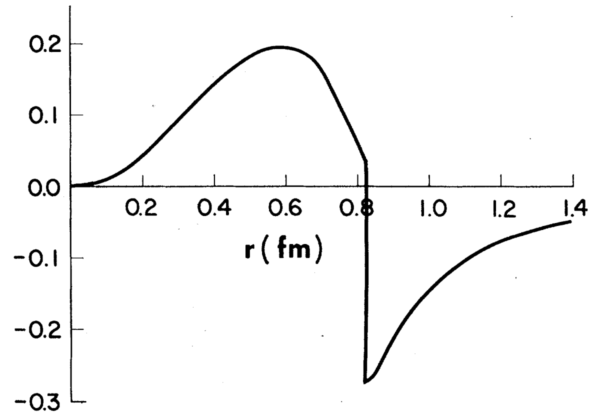
One feature- pion penetrates to the bag interior

n ⟷ **p** π^-

Cloudy Bag Model
1980 TTM

$$4\pi r^2 \int d^3 e^{-i\vec{q}\cdot\vec{r}} G_E^n(\vec{q})$$





Neutron

One gluon exchange gives similar effect, the FT of G_{En} is + near 0.

Meaning of form factor

- $G_E(Q^2)$ is NOT Fourier transform of charge density
- Relativistic treatment needed- wave function is frame-dependent, initial and final states differ, no density
- Light front coordinates, ∞ momentum frame

$$\text{“Time”}\ x^+ = (ct+z)/\sqrt{2} = (x^0+x^3)/\sqrt{2},\ \text{“evolution”}\ \mathrm{p}^- = (\mathrm{p}^0-\mathrm{p}^3)/\sqrt{2}$$

$$\text{“Space”}\ x^- = (x^0-x^3)/\sqrt{2},\ \text{“Momentum”}\ p^+ = (p^0+p^3)/\sqrt{2}$$

$$\text{“Transverse position, momentum,}\ \mathbf{b},\mathbf{p}$$

These coordinates are used to analyze form factors, deep inelastic scattering, GPDs,TMDS

Relativistic formalism- kinematic subgroup of Poincare

- Lorentz transformation –transverse velocity v

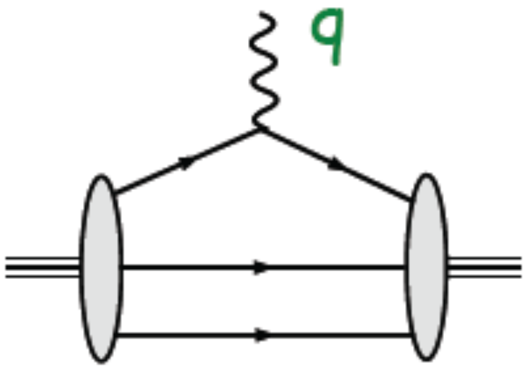
$$k^+ \rightarrow k^+, \ \mathbf{k} \rightarrow \mathbf{k} - k^+\mathbf{v}$$

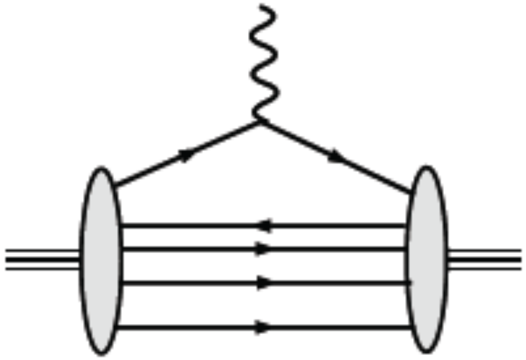
k^- such that k^2 not changed

Just like non-relativistic with k^+ as mass, take momentum transfer in perp direction, then density is 2 Dimensional Fourier Transform, also

$$q^+ = q^0 + q^3 = 0, -q^2 = Q^2 = \mathbf{q}^2$$

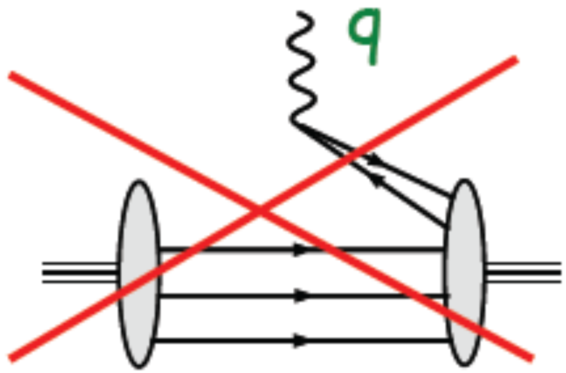
interpretation of FF as quark density





overlap of wave function Fock components with same number of quarks

interpretation as probability/charge density



overlap of wave function Fock components with different number of constituents

NO probability/charge density interpretation

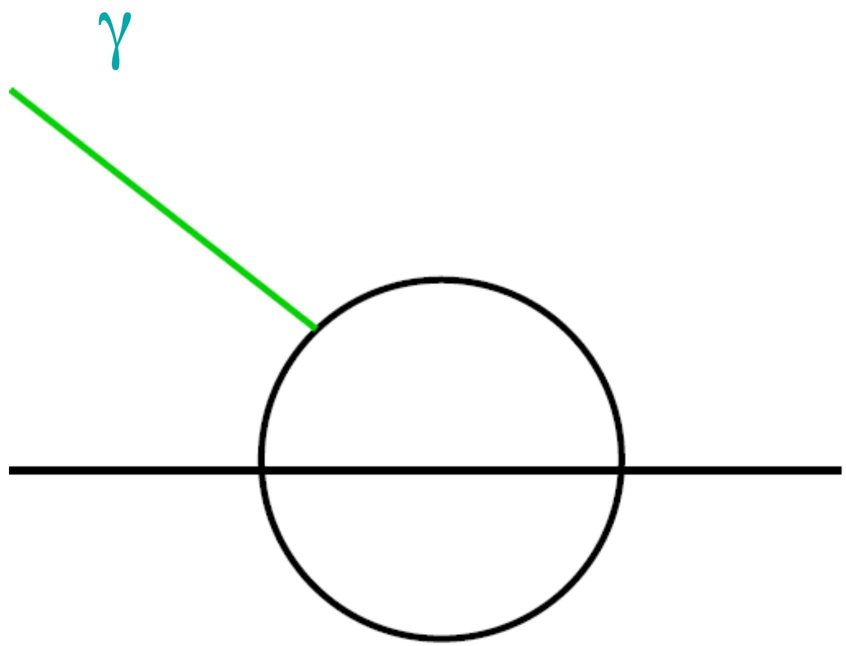
Absent in a Drell-Yan Frame

$$q^+ = q^0 + q^3 = 0$$

From Marc Vanderhaeghen

Model calculation:Frank, Jennings, Miller '95

Model proton wave function $\Psi(\mathbf{k}_\perp, \mathbf{K}_\perp, \xi, \eta)$



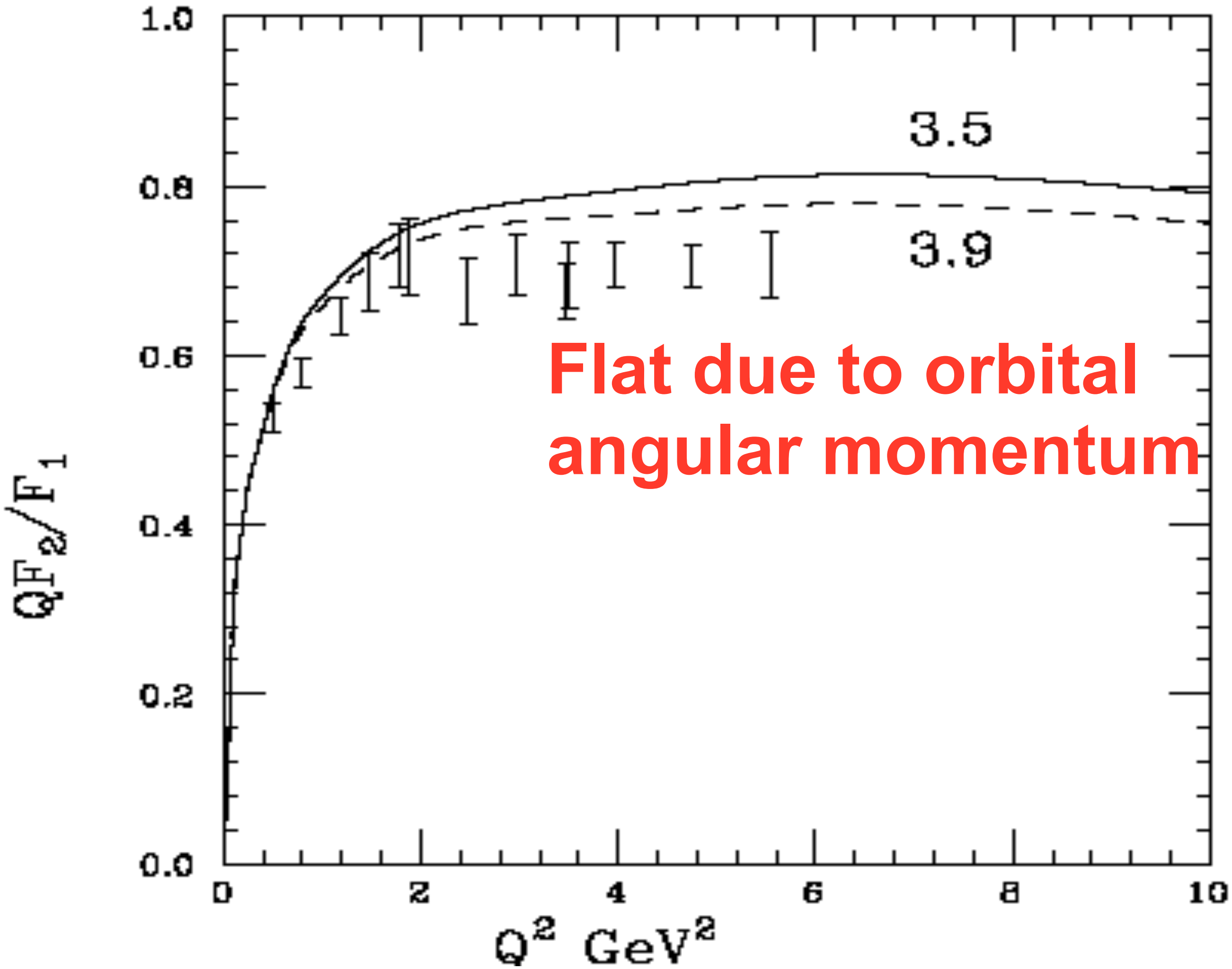
Poincare invariant

Light front variables for boost:

Dirac spinors

carry orbital angular momentum

Ratio of Pauli to Dirac Form Factors 1995 Frank,Jennings, Miller theory, data 2000



Model exists

- **lower components of Dirac spinor**
- **orbital angular momentum**
- **shape of proton?? Wigner Eckart**

no quadrupole moment

- **spin dependent densities SDD**

non-relativistic example

I: Non-Rel. $p_{1/2}$ proton outside 0^+ core

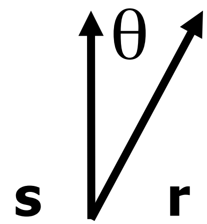
$$\langle \mathbf{r}_p | \psi_{1,1/2s} \rangle = R(r_p) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}}_p | s \rangle$$

$$\rho(r) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) | \psi_{1,1/2s} \rangle = R^2(r)$$

probability proton at $\mathbf{r}$ & spin direction $\mathbf{n}$:

$$\rho(\mathbf{r}, \mathbf{n}) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) \frac{(1+\boldsymbol{\sigma} \cdot \mathbf{n})}{2} | \psi_{1,1/2s} \rangle$$

$$= \frac{R^2(r)}{2} \langle s | \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} (1 + \boldsymbol{\sigma} \cdot \mathbf{n}) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} | s \rangle$$

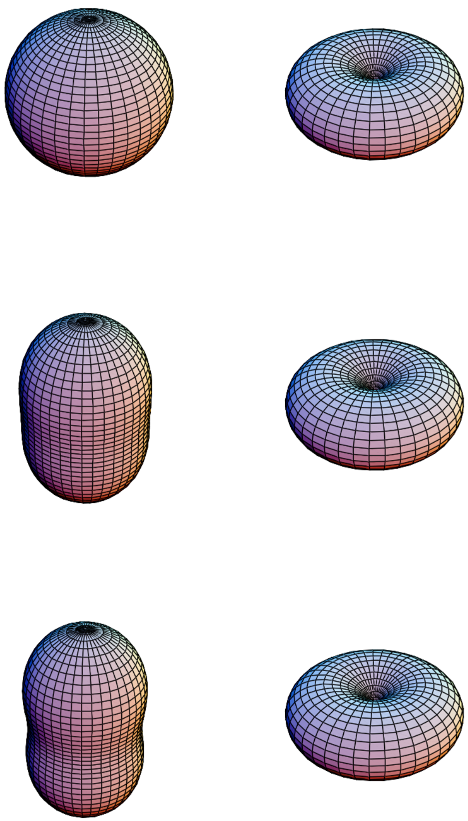


$\mathbf{n} \parallel \hat{\mathbf{s}}$: $\quad \rho(\mathbf{r}, \mathbf{n} = \hat{\mathbf{s}}) = R^2(r) \cos^2 \theta$

$\mathbf{n} \parallel -\hat{\mathbf{s}}$: $\quad \rho(\mathbf{r}, \mathbf{n} = -\hat{\mathbf{s}}) = R^2(r) \sin^2 \theta$

non-spherical shape depends on spin direction

Shapes of the proton



Momentum space

three vectors n, K, S

Relation between coordinate and momentum space densities?

How to measure?-Lattice and/or experiment

Model independent transverse charge density

$$J^+(x^-,\mathbf{b}) = \sum_q e_q q_+^\dagger(x^-,b)q_+(x^-,b)$$ Charge Density

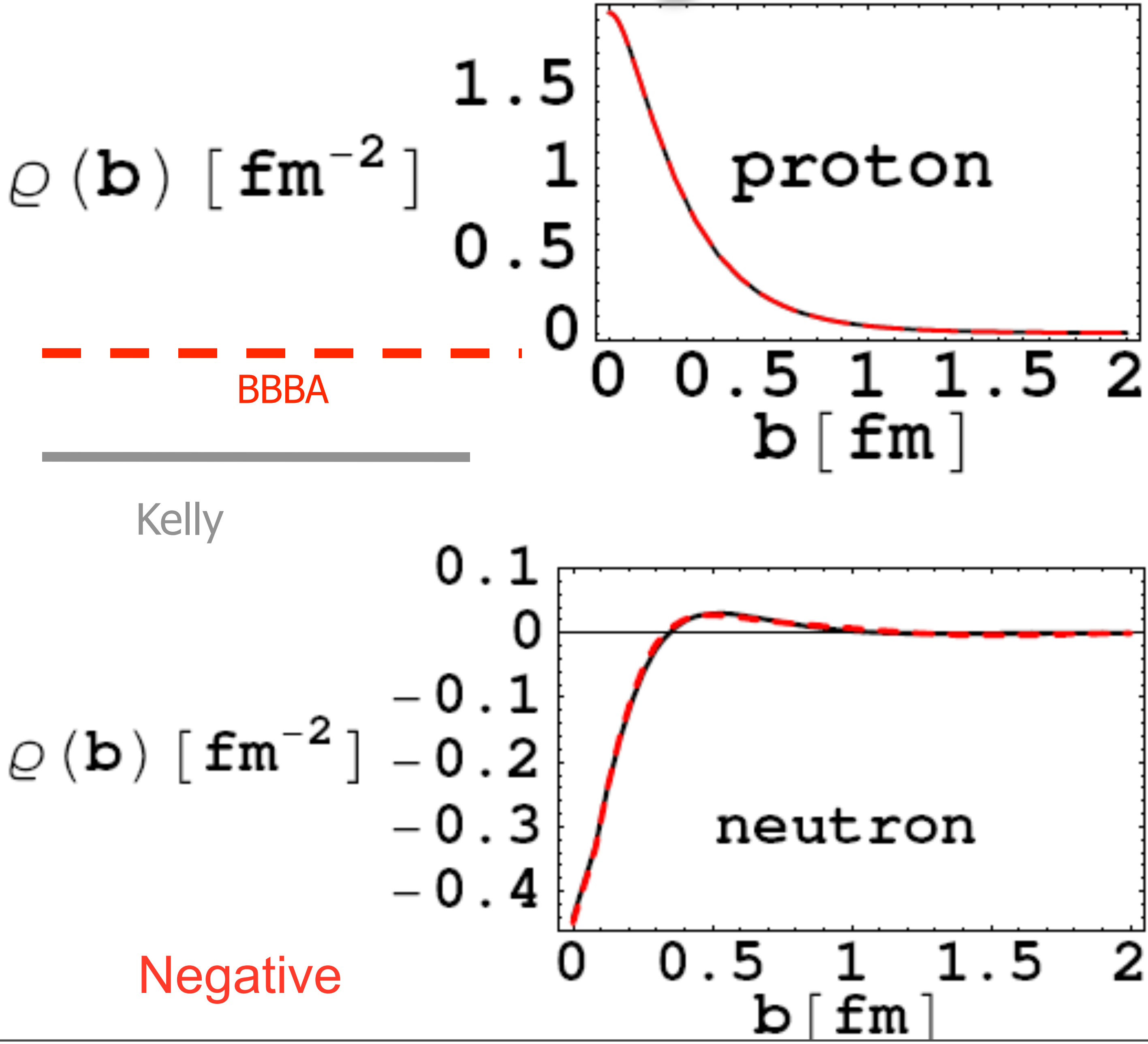
$$\rho_\infty(x^-,\mathbf{b}) = \langle p^+,\mathbf{R}=\mathbf{0},\lambda|\sum_q e_q q_+^\dagger(x^-,b)q_+(x^-,b)|p^+,\mathbf{R}=\mathbf{0},\lambda\rangle$$

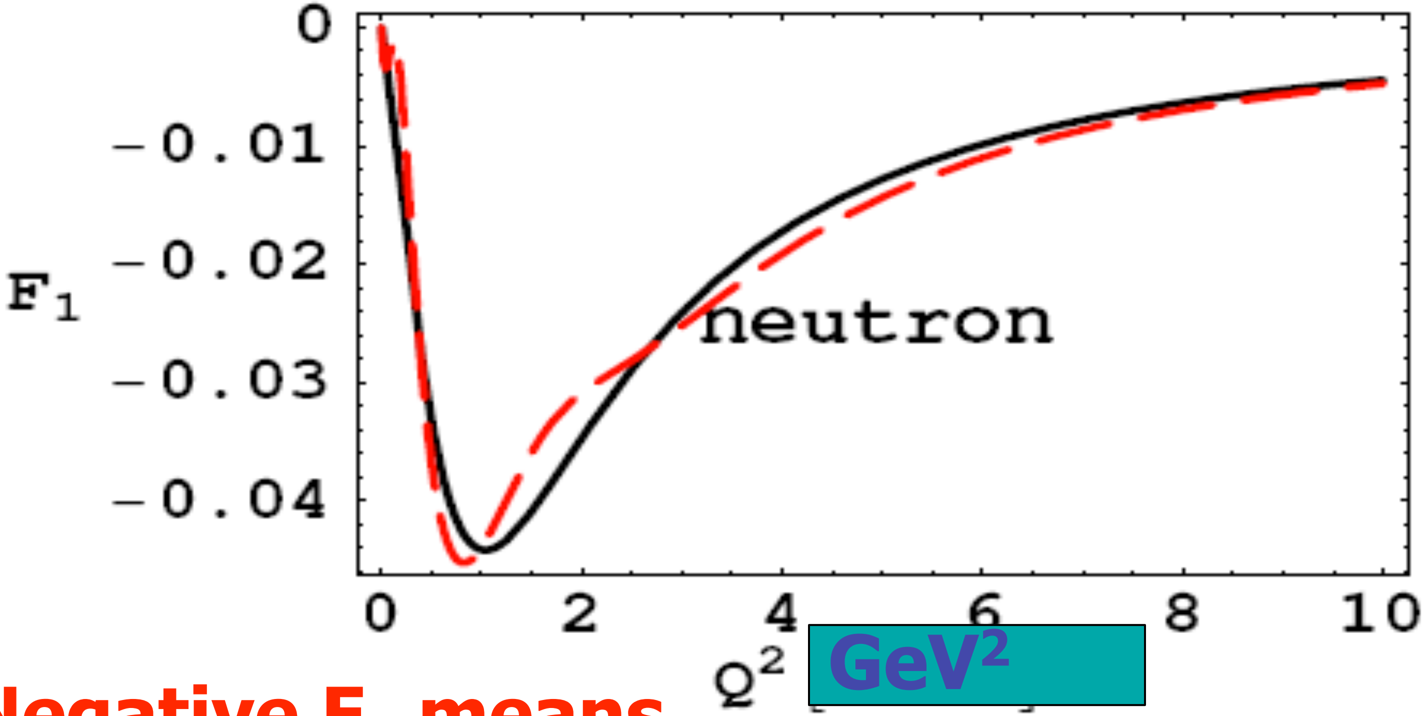
$$F_1 = \langle p^+,\mathbf{p}',\lambda|J^+(0)|p^+,\mathbf{p},\lambda\rangle$$

$$\rho(b) \equiv \int dx^- \rho_\infty(x^-,\mathbf{b}) = \int \frac{QdQ}{2\pi} F_1(Q^2) J_0(Qb)$$

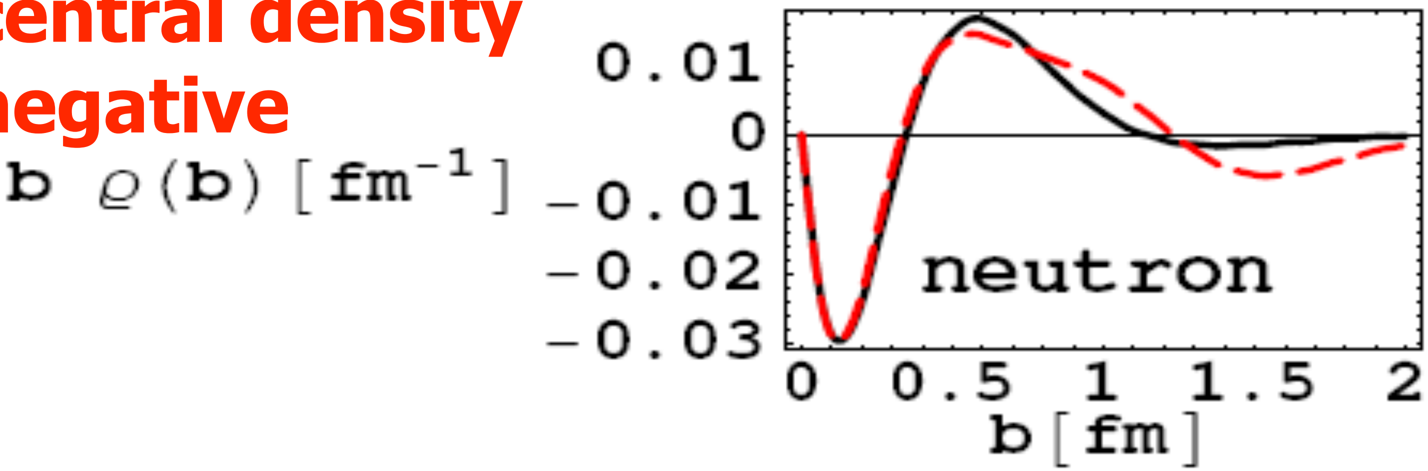
$$\text{Density is } u-\bar{u},\ d-\bar{d}$$

Transverse charge densities

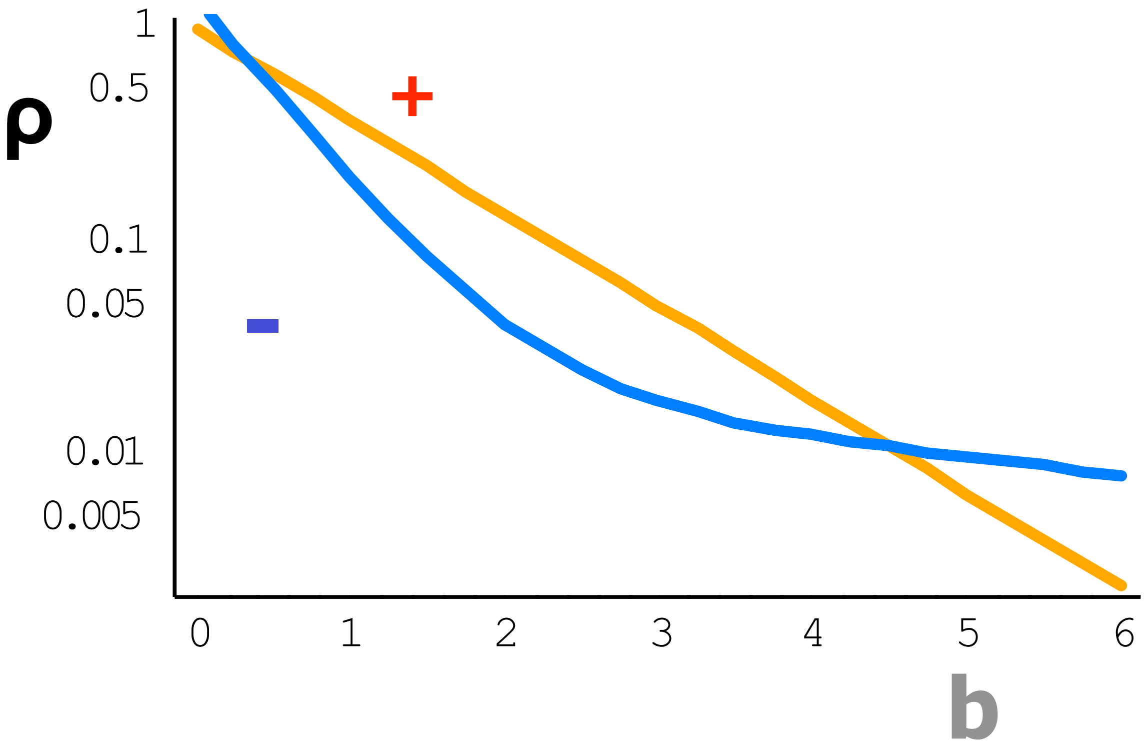




Negative F_1 means central density negative



Neutron Interpretation needed



Why ? What? How? Combine elastic and deep inelastic scattering information.
Generalized parton distribution

Neutron interpretation

- Impact parameter gpd Burkardt $\rho(x, b)$
- From Drell-Yan-West relation between high x DIS and high Q^2 elastic scattering
- High x related to low b, not uncertainty principle
- d quarks dominate DIS from neutron at high x
- d quarks dominate at neutron center, or π^-

$$\text{Density is } u - \bar{u},\ d - \bar{d}$$

$$\pi^- \text{ is } \bar{u}d$$

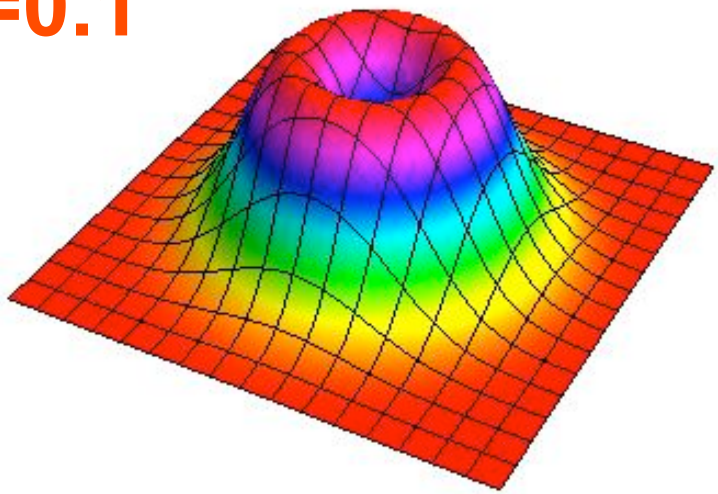
$$\text{decreases } u \text{ contribution}$$

$$\text{enhances } d \text{ contribution}$$

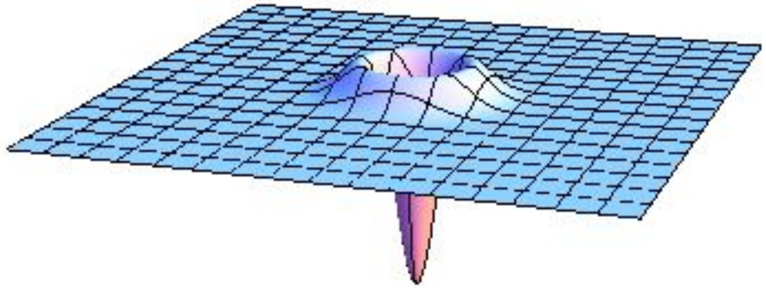
Neutron $\rho(b,x)$ GAM, J. Arrington, PRC78,032201R '08

Using other people's models

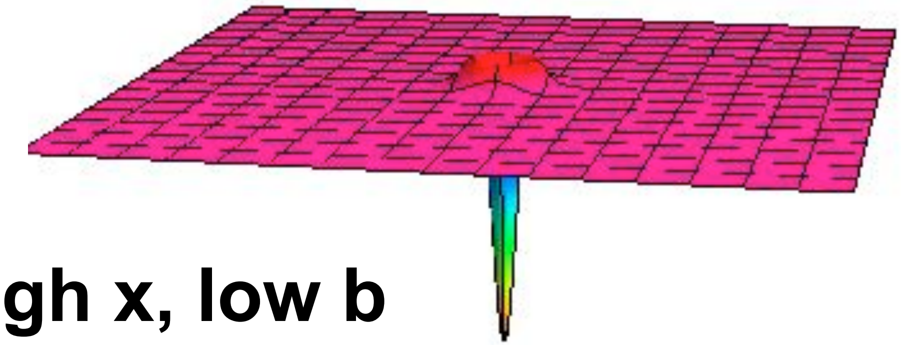
x=0.1



x=0.3



x=0.5



d or π^- dominates at high x, low b

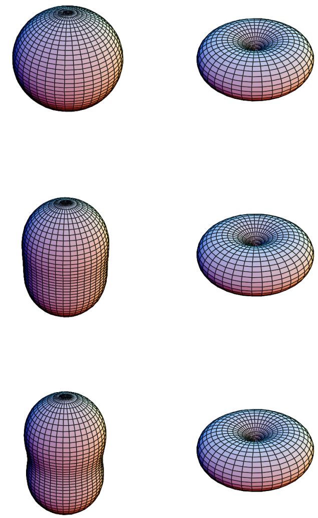
Return of the cloudy bag model

- In a model nucleon:bare nucleon + pion cloud - parameters adjusted to give negative definite F_1, pion at center causes negative central transverse charge density
- Boosting the matrix element of J^0 to the infinite momentum frame changes G_E to F_1

Rinehimer and Miller PRC80,015201, 025206

Shapes of the proton

- Relate spin dependent density to experiment
- **Phys.Rev.C76:065209,2007**



Field-theoretic spin dependent momentum density is related to the transverse momentum distribution $h_{1T}^{\perp}$

$$\Phi^{[\Gamma]}(x, \mathbf{K}_T) = \int \frac{d\xi^- d^2\xi_T}{2\,(2\pi)^3}\, e^{iK\cdot\xi}\, \langle P, S|\overline{\psi}(0)\,\Gamma\,\mathcal{L}(0,\xi;n_-)\,\psi(\xi)|P,S\rangle\Big|_{\xi^+=0}$$

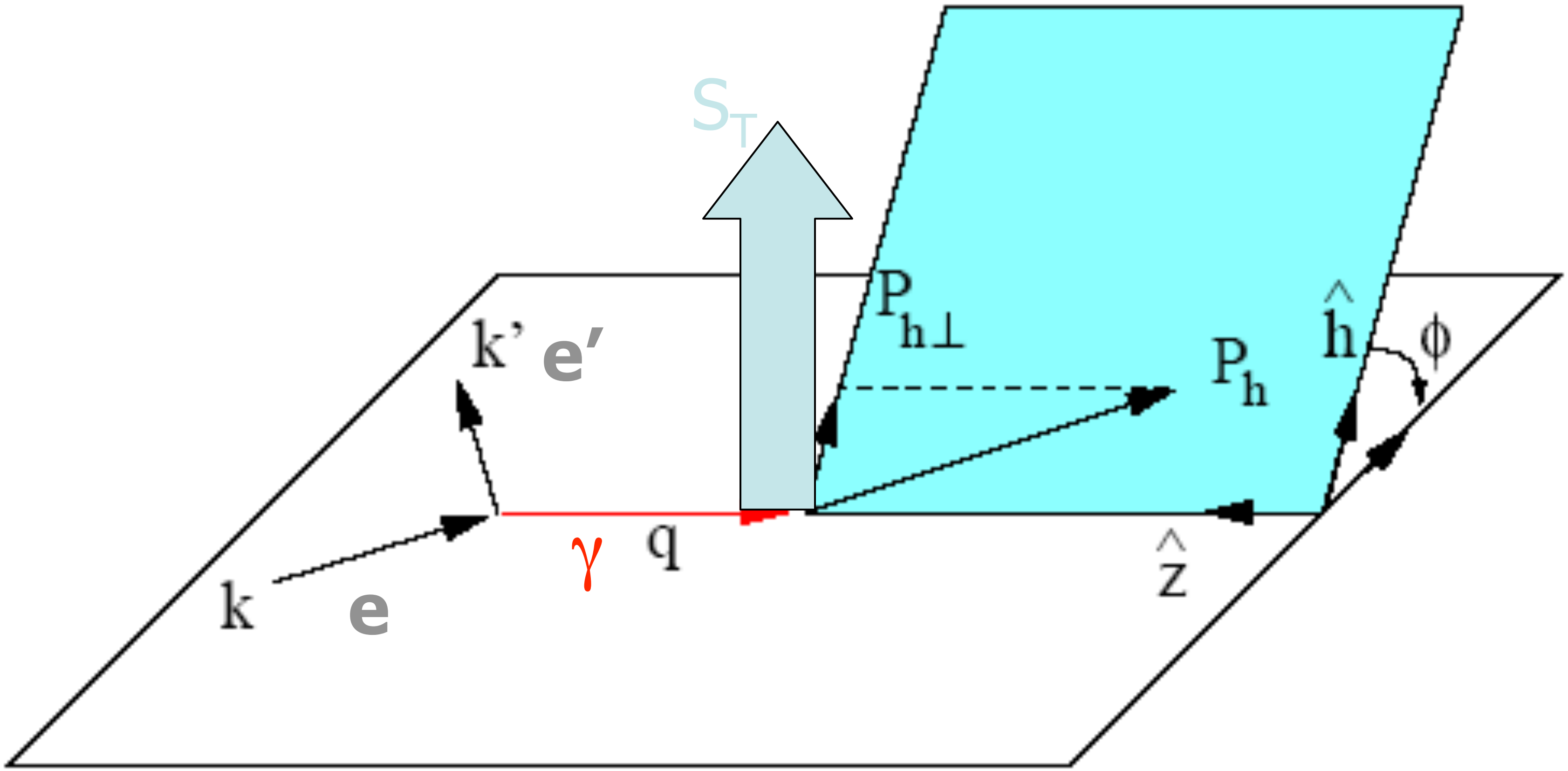
Mulders Tangerman'96

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x, \mathbf{K}_T) = S_T^i\, h_1(x, K_T^2) + \frac{\left(K_T^i K_T^j - \frac{1}{2}K_T^2\delta_{ij}\right) S_T^j}{M^2}\, h_{1T}^{\perp}(x, K_T^2)$$

$$\sigma^{i+}\gamma^5 \sim \gamma^0\gamma^+\sigma^i,$$

then relate equal time to $\xi^+ = 0$ by integration over x

Measure $h_{1T}^{\perp}$:e,↑p → e', π X

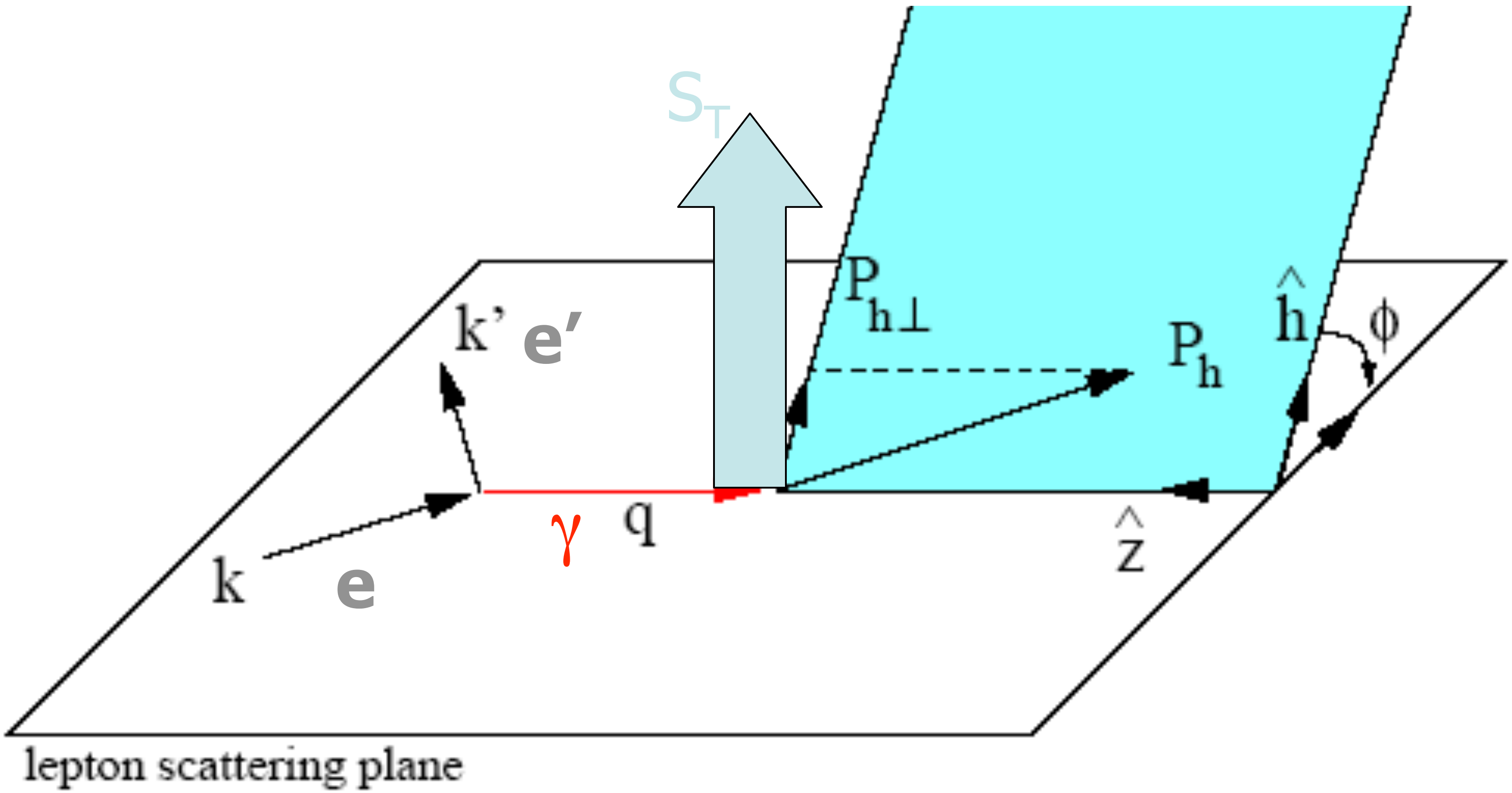


Cross section has term proportional to cos 3ϕ

Boer Mulders '98 there are other ways to see $h_{1T}^{\perp}$

Measure $h_{1T}^{\perp}$:e, ↑p → e', π X

H. Avakian, *et al.* "Transverse Polarization Effects in Hard Scattering at CLAS12 Jefferson Laboratory", LOI12-06-108, and H. Avakian private communication.



Cross section has term proportional to cos 3ϕ

Boer Mulders '98 there are other ways to see $h_{1T}^{\perp}$

Generalized densities

$$\mathcal{O}_q^\Gamma(px, \mathbf{b}) = \int \frac{dx^- e^{ipxx^-}}{4\pi} q_+^\dagger(0, \mathbf{b})\Gamma q_+(x^-, \mathbf{b})$$

$$\rho^\Gamma(b) = \int dx \sum_q e_q \langle p^+, \mathbf{R} = \mathbf{0}, \lambda | \mathcal{O}_q^\Gamma(p^+ x, \mathbf{b}) | p^+, \mathbf{R} = \mathbf{0}, \lambda \rangle$$

$\int dx$ sets $x^- = 0$, get $q_+^\dagger(0, \mathbf{b})\Gamma q_+(0, \mathbf{b})$ **Density!**

$\Gamma = 1/2(1 + \mathbf{n} \cdot \gamma)$ gives spin-dep density

Local operators calculable on lattice Gloeckler et al PRL98,222001 $\widetilde{A}''_{T10} \sim \mathrm{sdd}$ spin-dependent density

Schierholtz, 2009 -this quantity is not zero, proton is not round

Transverse Momentum Distributions - momentum space density

In a state of fixed momentum

$\Phi_q^\Gamma(x, \mathbf{K})$ give probability of quark of given 3-momentum

$h_{1T}^\perp$ gives momentum-space spin-dependent density

measurable experimentally

hard to calculate on lattice because - gauge link

Summary

- **Form factors, GPDs, TMDs, understood from unified light-front formulation**
- **Neutron central transverse density is negative- consistent with Cloudy Bag Model**
- **Proton is not round- lattice QCD spin-dependent-density is not zero**
- **Experiment can whether or not proton is round by measuring** $h_{1T}^{\perp}$

Summary

- Form factors, GPDs, TMDs, understood from unified light-front formulation
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The Proton