Fermions, Scalars and Dark Energy

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Outline

1. Background
   - The ultimate Copernican revolution
   - 14 Years ago in Adelaide ....
   - Is this related to Dark Energy?
2. Calculation of w
   - w as a function of fermion number density
   - Connecting data and parameters
3. Summary

Type Ia Supernovae

- Discovery of the re-acceleration of the universe in 1998
- The acceleration of the time evolution of the scale parameter is given by the Friedmann Equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad (1)$$

- Introduce the parameter w
  where $p = w\rho_E$
  and ρ_E = total energy density
- Positive acceleration requires $w < -1/3$
- $w = -1$ is Einstein's cosmological constant Λ
- Most recent data and analysis (arXiv:0901.4804)
  $1 + w = 0.013 \pm 0.067 \pm .11$
  assuming constant w

Dark Energy

- The *stuff* with $w \approx -1$ is called dark energy, and it is about 70% of the present energy density of the universe.
- Only 5% is baryons, and the remaining 25% is dark matter
- Most of the universe is not the stuff of which we are made

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invited lecture at the Topical Workshop on Neutrino Physics, National Institute for Theoretical Physics, Adelaide University, Oct-Nov 1996

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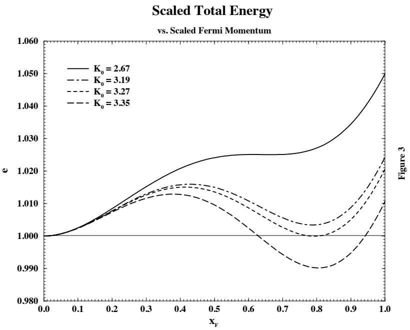

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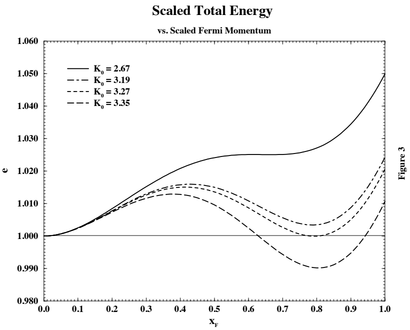

- The total energy of a sea of interacting neutral fermions (neutrinos), mass m_0, Fermi energy $k_F = m_0 x_F$, as a function of x_F or density.

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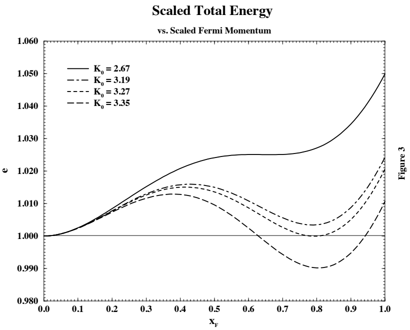

- The total energy of a sea of interacting neutral fermions (neutrinos), mass m_0, Fermi energy $k_F = m_0 x_F$, as a function of x_F or density.
- Note the region of negative pressure.

The basic fermion-scalar system

- Lagrangian:
  Majorana Fermions (2 component) Φ and scalars ζ

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2\imath}[\Phi^\dagger\sigma^\mu\partial_\mu\Phi - \partial_\mu\Phi^\dagger\sigma^\mu\Phi] + m_0\frac{1}{2}[\Phi^T\sigma^2\Phi + \Phi^\dagger\sigma^2\Phi^*] \\
&+ \frac{1}{2}\left[\zeta(\partial^2 - m_\zeta^2)\zeta\right] \\
&+ \frac{1}{2}g[\Phi^T\sigma^2\Phi + \Phi^\dagger\sigma^2\Phi^*]\zeta
\end{aligned}
$$

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- the equations of motion

$$\begin{aligned}\left[\partial^2 + m_\zeta^2\right]\zeta &= -\frac{1}{2}g[\Phi^T\sigma^2\Phi + \Phi^\dagger\sigma^2\Phi^*] \\ [i\sigma^\mu\partial_\mu]\,\Phi &= m_0\sigma^2\Phi^* + g\zeta\sigma^2\Phi^*.\end{aligned}$$

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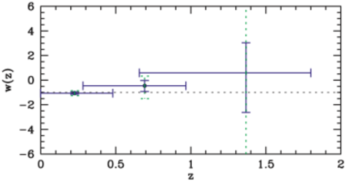
Is w constant?



From Reiss et al, Ap J **659** (2007) 98

w does not vary rapidly with z

Is it flat or is it rising?

Does the data constrain our model?

In our model we find

- w is not constant

Does the data constrain our model?

In our model we find

- w is not constant
- But the variation of w can be made consistent with the present data

The effective mass

- remember the equations of motion

$$\begin{aligned}\left[\partial^2 + m_\zeta^2\right]\zeta &= -\frac{1}{2}g[\Phi^T\sigma^2\Phi + \Phi^\dagger\sigma^2\Phi^*] \\ [i\sigma^\mu\partial_\mu]\,\Phi &= m_0\sigma^2\Phi^* + g\zeta\sigma^2\Phi^*.\end{aligned}$$

- Clearly the scalar field induces an effective mass of the fermion $m^* = m_0 + g\zeta$
- For a homogeneous filled Fermi sea, the expectation value of the Lorentz scalar density $\mathcal{S} = \frac{1}{2}\{\Phi^T\sigma^2\Phi + \Phi^\dagger\sigma^2\Phi^*\}$ is simply related to the number density of the fermions and, with $y = m^*/m_0$,

$$y = 1 - \{yK_0/2\}\left[e_F x_F - y^2 \ln\left(\{e_F + x_F\}/y\right)\right],$$

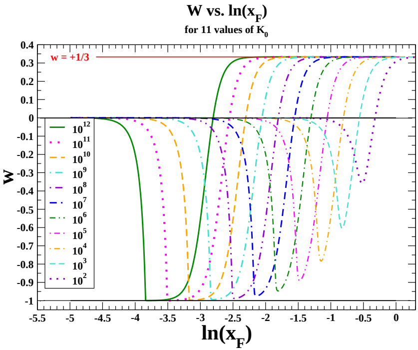
with $e_F = \sqrt{x_F^2 + y^2}$, and $K_0 = \{g^2(m_0)^2\}/\{\pi^2 m_\zeta^2\}$.

Calculating w

- Using the equations of the FLRW metric, with a scale parameter a, $1 + w = -\frac{1}{3}\frac{\partial \ln \rho_E}{\partial \ln a}$.
- In our model, with the only contribution to ρ_E from the neutral fermions and the scalars, and $\rho_E = e m_n^{(0)} \rho_n$, where ρ_n is the fermion number density, $w = \frac{1}{3}\frac{\partial \ln(e)}{\partial \ln(x_F)}$.
- $$w = \frac{1}{3}\frac{e_F K_0 x_F^3 - 3(2-y)(1-y)}{e_F K_0 x_F^3 + (2-y)(1-y)}$$

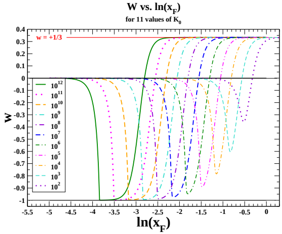
  It follows immediately that $w > -1$ in our model.

The result



- For very small x_F (in fact $x_F^3 \ll K_0^{-1}$), $w \approx x_F^2/5 \rightarrow 0$
- For large x_F, the effective mass goes to zero, and $w \rightarrow 1/3$, the value for a relativistic gas of Fermions.
- There is a minimum of w, $w_{\rm min}$, which gets closer to -1 as K_0 increases. It occurs at $x_{\rm min} \approx (3.83/K_0)^{1/3}$ for $K_0 > 10^6$

How to fit the observed $w(z)$



- For $x_F > x_{\min}$ and large K_0, $w(x_F)$ varies slowly with x_F. Push the "flat" region out to $z \approx 1.0$
- For $x_F < x_{\min}$ and large K_0, $w(x_F)$ rapidly approaches 0. Put this near or after now.
- For $K_0 > 10^6$, $1 + w_{\min} \approx 2x_{\min}$, and then the observed limit on $1 + w$ requires $x_{\min} < 0.06$

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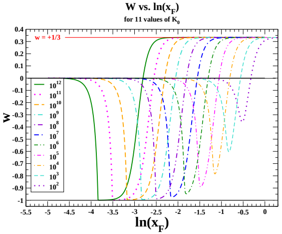

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- For $K_0 > 10^6$, $1 + w_{\min} \approx 2x_{\min}$, and then the observed limit on $1 + w$ requires $x_{\min} < 0.06$
- Can we find a set of parameters for our model that do this?

The parameters

The parameters of the model are

- the Fermion mass m_0
- the Scalar mass m_ζ
- the coupling constant g
- which enter in the dimensionless parameter $K_0 = \{g^2(m_0)^2\}/\{\pi^2 m_\zeta^2\}$
- the density of the fermions, parameterized by the Fermi momentum in units of the Fermion mass, x_F, at some value of the LFRW scale parameter $a = a_1$, of the red shift $z = z_1$

Data

We know

- The limit on $1 + w$, if assumed constant: $1 + w \leq 0.13$ at the 1σ level
- The total dark energy density, if assumed constant: $\rho_E^{(\mathrm{obs})} = (2.4 meV)^4$
- IF we assume that the neutral fermions which generate dark energy are one of the species of active neutrinos, then their present number density is 100 cm^{-3}

Satisfying constraints

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- whence $m_0 \geq 160$ meV
- with the above, $x_F \approx (3.83/K_0)^{1/3} \Longrightarrow k_F < 2.49$ meV

With more analysis

- As $x_F \propto (1 + z)$, and we want w to stay near -1 as $z = 0.2 \rightarrow z = 1$ we need the minimum of w to be flat at its minimum for a range of a factor of 2 in x_F.

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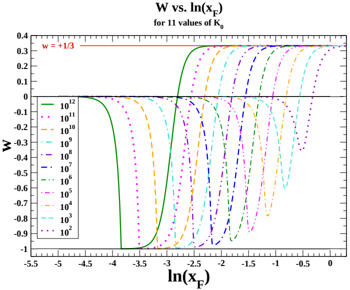

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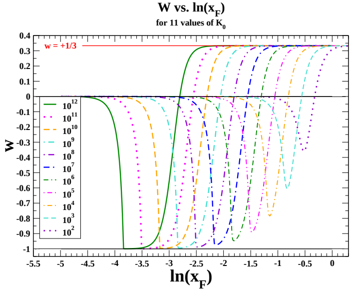

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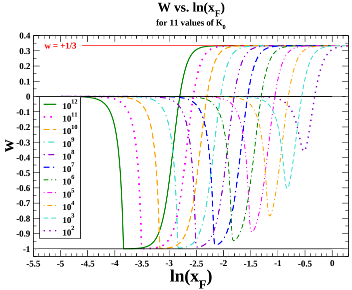
- $m_0 > 0.89$ eV and $k_F < 1.40$ meV



Accommodating LSP

- Take $K_0 = 10^{54}$
- Then $m_0 = 160$ GeV, and $k_F = 25$ μeV
- Remember $K_0 \ = \ \{g^2(m_0)^2\}/\{\pi^2 m_\zeta^2\}$
  Take $m_\zeta = (7 \times 10^9 \text{ lightyears})^{-1} \sim 3 \times 10^{-30}$ meV
  Then $g^2/(4\pi) = 2.8 \times 10^{-34}$
- far too small to be constrained by terrestrial experiments

Summary

- Our old neutrino cloud model finds a new life as a natural model for dark energy
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- Next questions
    - What is the extension of the standard model to include the new interactions?
    - Do "clouds" form in this version of the model?

Thank you

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- To Tony, with best wishes for many happy returns from all the authors