Chiral Effective Field Theory
Inspired by Tony

Derek Leinweber

Key Contributors

Ian Cloet, Ding Lu, Tony Thomas, Kazuo Tsushima,

Ping Wang, Stewart Wright, Ross Young

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Overview

1. "Baryon masses from lattice QCD: Beyond the perturbative chiral regime"
   D. B. Leinweber, A. W. Thomas, K. Tsushima and S. V. Wright
   Phys. Rev. D **61**, 074502 (2000) [arXiv:hep-lat/9906027]
   102 Citations
2. "Nucleon magnetic moments beyond the perturbative chiral regime"
   D. B. Leinweber, D. H. Lu and A. W. Thomas
   Phys. Rev. D **60**, 034014 (1999) [arXiv:hep-lat/9810005]
   89 Citations
3. "Physical nucleon properties from lattice QCD"
   D. B. Leinweber, A. W. Thomas and R. D. Young
   Phys. Rev. Lett. **92**, 242002 (2004) [arXiv:hep-lat/0302020]
   86 Citations

Overview

4. “Chiral analysis of quenched baryon masses”
   R. D. Young, D. B. Leinweber, A. W. Thomas and S. V. Wright
   Phys. Rev. D **66**, 094507 (2002) [arXiv:hep-lat/0205017]
   83 Citations

5. “Precise determination of the strangeness magnetic moment of the nucleon”
   D.B. Leinweber, S. Boinepalli, I.C. Cloet, A.W. Thomas, A.G. Williams, R.D. Young, J.M. Zanotti, J.B. Zhang,
   Phys. Rev. Lett. **94**, 212001 (2005) [arXiv:hep-lat/0406002]
   73 Citations

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- Early Ideas – The Cloudy Bag Model and the Padé
  - Both small and large m_π limits are important!

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  - Incorporation of light η' meson
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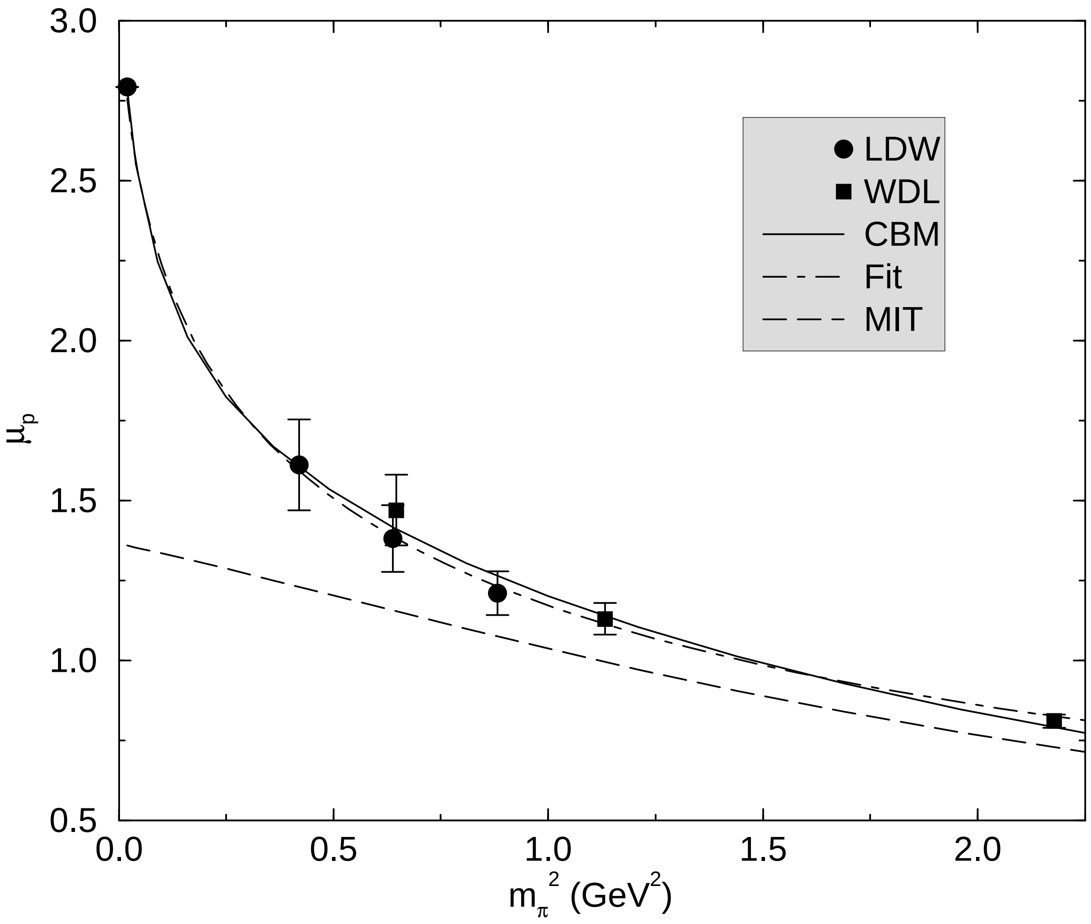
- Quenched Chiral Perturbation Theory
  - Modification of meson-baryon vertices
  - Incorporation of light η' meson
  - Correcting the Quenched Approximation
- Fascinating aspects of baryon structure.

Early Ideas – Proton Magnetic Moment



D.B. Leinweber, D.H. Lu, A.W. Thomas, Phys. Rev. **D60**, 034014 (1999)

Early Ideas – The Padé

- Series expansion of $\mu_{p(n)}$ in powers of m_π is not a useful approximation for m_π larger than the physical mass.
- The simple Padé approximant:

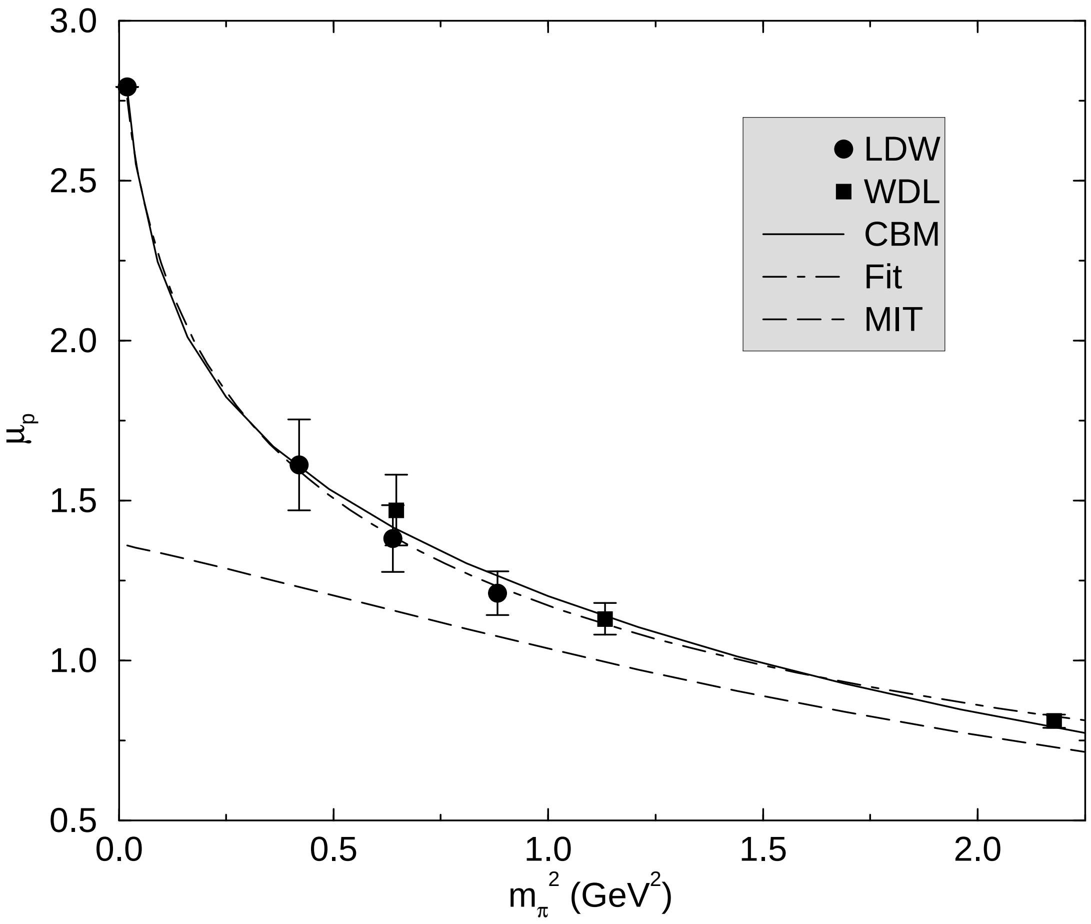
$$\mu_{p(n)} = \frac{\mu_0}{1 - \chi\, m_\pi/\mu_0 + \beta\, m_\pi^2}\,,$$

- Builds in the Dirac moment at moderately large m_π^2
- Has the correct LNA behavior of chiral perturbation theory

$$\mu = \mu_0 + \chi m_\pi\,,$$

  with χ a model independent constant, as $m_\pi^2 \to 0$.
- Two-parameter fits to lattice results proceed by
  - Fixing χ at the value given by chiral perturbation theory,
  - Optimizing μ_0 and β.

Proton Magnetic Moment



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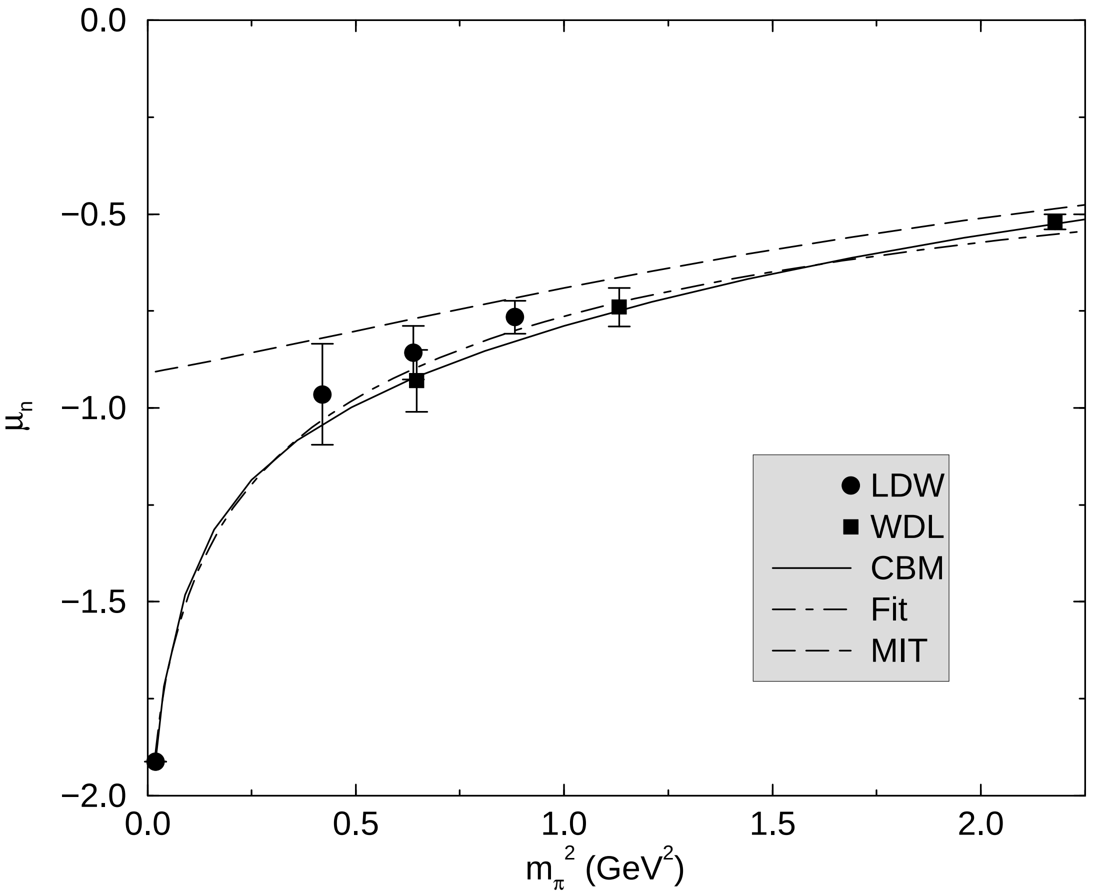
Neutron Magnetic Moment



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Chiral Effective Field Theory

- General low-energy expansion about chiral limit ($m_q = 0$)

$$M_N = \{\text{Terms Analytic in } m_q\} + \{\text{Chiral loop corrections}\}$$

Chiral Effective Field Theory

- General low-energy expansion about chiral limit ($m_q = 0$)

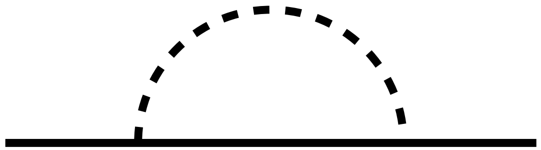
$$M_N = \{\text{Terms Analytic in } m_q\} + \{\text{Chiral loop corrections}\}$$

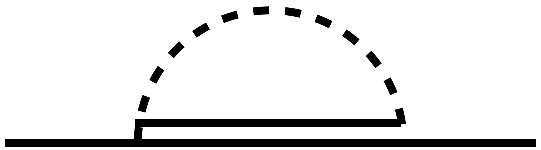
- Analytic terms
  - Coefficients are not constrained by chiral symmetry
  - To be determined via analysis of Lattice QCD results
  - Related to the Low Energy Constants of χPT
- Chiral loops
  - Predict nonanalytic behaviour in the quark mass
  - Coefficients are known and are model independent

Chiral Effective Field Theory

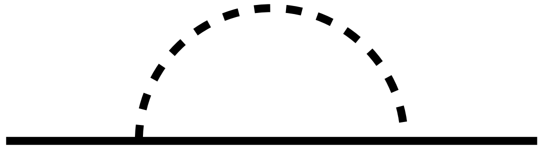
- General low-energy expansion about chiral limit ($m_q = 0$)
- Common to formulate the expansion in terms of $m_\pi^2 \sim m_q$

$$
\begin{aligned}
M_N &= \{a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + a_6 m_\pi^6 + \cdots\} \\
&\quad + \{\chi_\pi \, I_\pi(m_\pi) + \chi_{\pi\Delta} \, I_{\pi\Delta}(m_\pi) + \cdots\}
\end{aligned}
$$

$I_\pi \;=$ 

$I_{\pi\Delta} \;=$ 

Regularisation of Loop Integrals

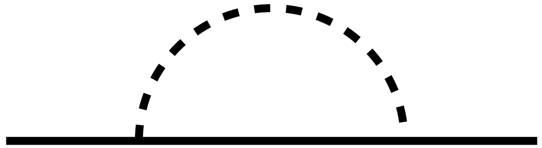


- Consider the self-energy of the nucleon in heavy-baryon χPT

$$\chi_\pi I_\pi(m_\pi) = -\frac{3\,g_A^2}{32\,\pi\,f_\pi^2}\,\frac{2}{\pi}\int_0^\infty dk\,\frac{k^4}{k^2+m^2}$$

with $g_A = 1.26$ and $f_\pi = 0.093$ GeV.

Regularisation of Loop Integrals



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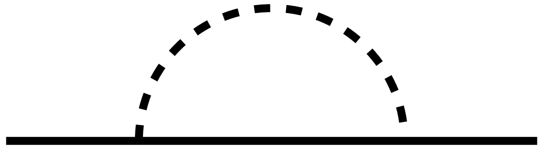
- Standard approach: dimensional regularisation, $\epsilon \to 0$

$$I_\pi \to \infty + \infty m_\pi^2 + m_\pi^3$$

- a_0 and a_2 undergo an infinite renormalisation

$$\begin{aligned} M_N &= \{a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + a_6 m_\pi^6 + \cdots\} \\ &\quad + \{\chi_\pi I_\pi(m_\pi) + \chi_{\pi\Delta} I_{\pi\Delta}(m_\pi) + \cdots\} \end{aligned}$$

Regularisation of Loop Integrals



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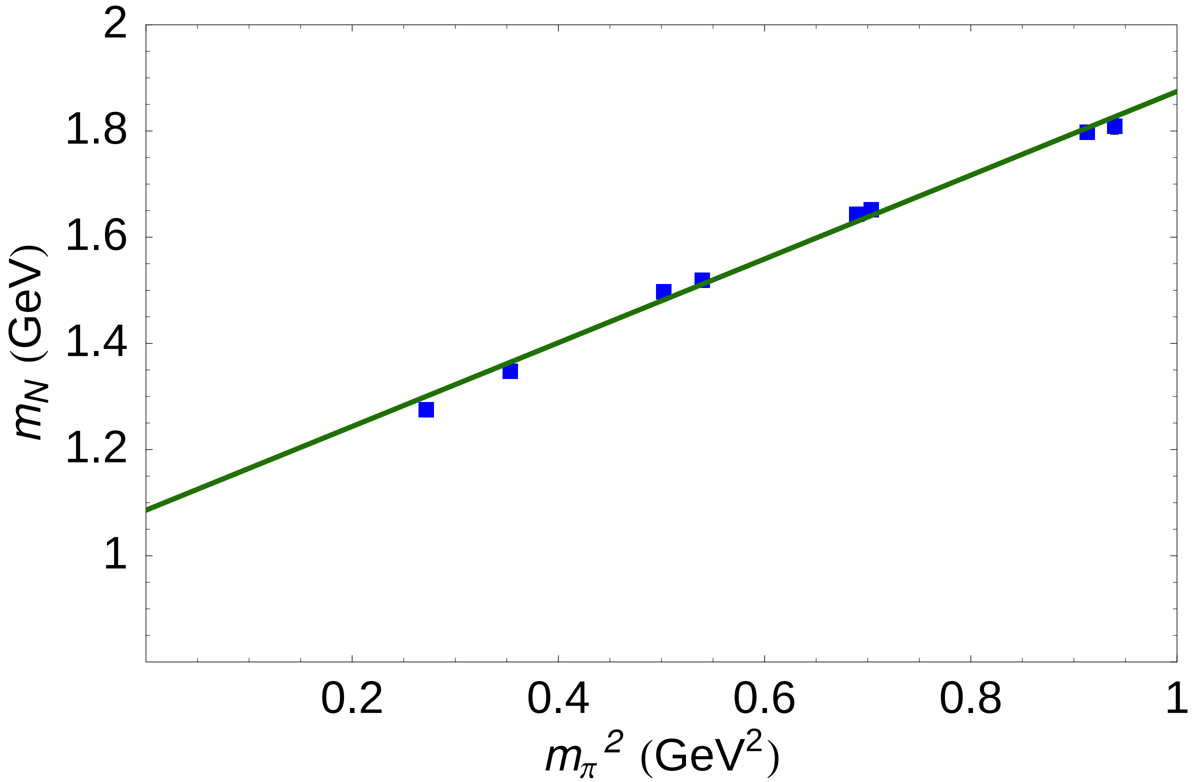
$$I_\pi \to \infty + \infty m_\pi^2 + m_\pi^3$$

- Nucleon expansion $\longrightarrow$

$$M_N = c_0 + c_2 m_\pi^2 + \chi_\pi m_\pi^3 + c_4 m_\pi^4 + \cdots$$

Lattice QCD and Dim Reg χPT

- CP-PACS collaboration results Phys. Rev. D65 (2002) 054505



- A: $c_0 + c_2 m_\pi^2$

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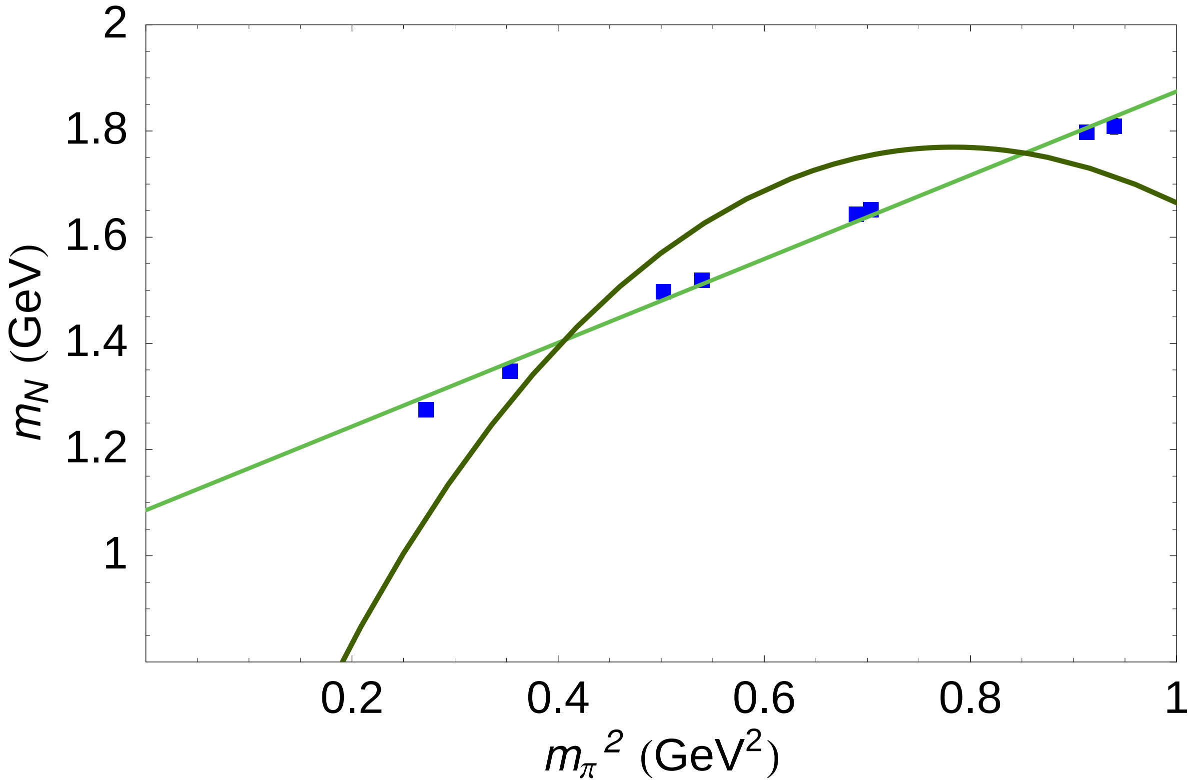

- B: $c_0 + c_2 m_\pi^2 + \chi_\pi \, m_\pi^3$

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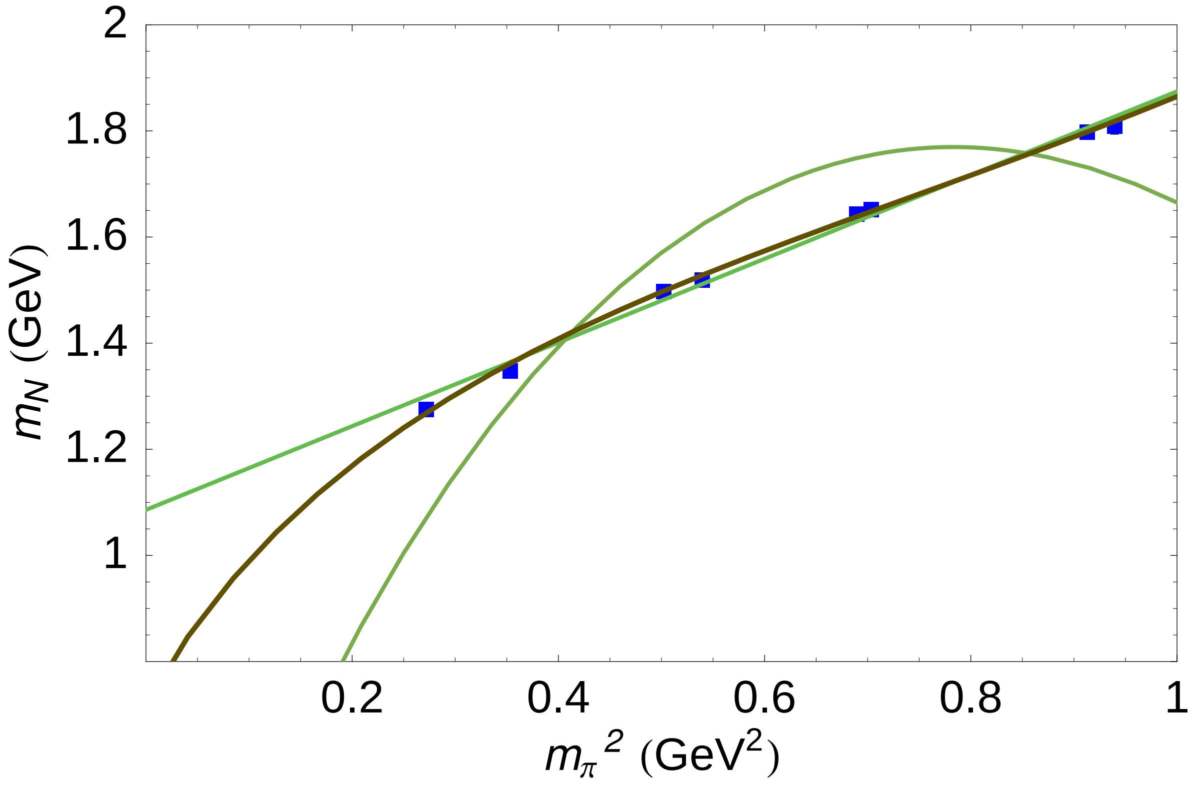
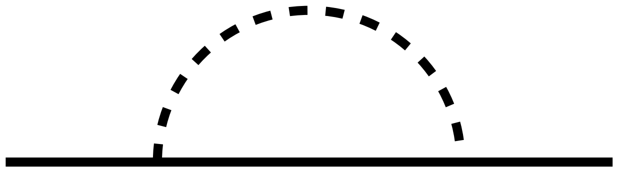


- C: $c_0 + c_2 m_\pi^2 + \chi_\pi \, m_\pi^3 + c_4 m_\pi^4$

Slow RATE of Convergence

- Origin lies in regularisation prescription
- DR: Large contributions to integral from $k \to \infty$ portion of integral



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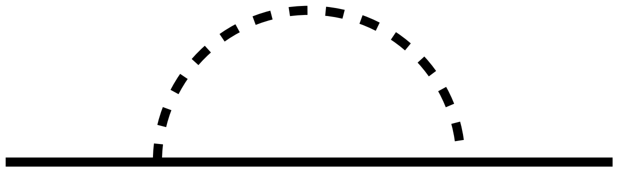

- Short distance physics is highly overestimated!
- Always require large analytic terms at next order
  - no sign of convergence

Overcoming This Problem

- KEEP low-energy (infrared) structure of χPT
- REMOVE the incorrect short-distance contributions associated with ultraviolet behaviour of loop integrals

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- KEEP low-energy (infrared) structure of χPT
- REMOVE the incorrect short-distance contributions associated with ultraviolet behaviour of loop integrals
- INTRODUCE *“separation-scale”* to identify short- and long-distance physics
- Natural scale to be associated is the physical size of the pion source
  - Axial-vector form factor of the nucleon

Regularisation: Revisited

- Use a Finite-Range Regulator (FRR)

$$M_N = \{a_0^\Lambda + a_2^\Lambda m_\pi^2 + a_4^\Lambda m_\pi^4 + a_6^\Lambda m_\pi^6 + \cdots\} + \{\chi_\pi I_\pi(m_\pi, \Lambda) + \chi_{\pi\Delta} I_{\pi\Delta}(m_\pi, \Lambda) + \cdots\}$$

- Loop integral is cutoff in momentum space at mass scale Λ

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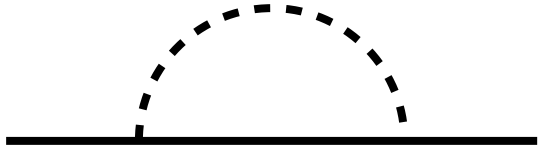
$$

- Loop integral is cutoff in momentum space at mass scale Λ
- Different from standard QFT
  - Λ remains finite for EFT
  - Ultraviolet suppression for loop momenta $k > \Lambda$

Finite-Range Regularisation



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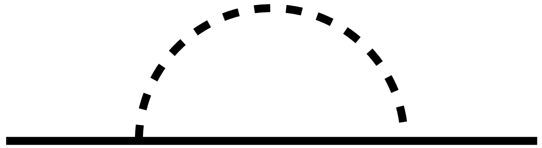
$$I_\pi(m_\pi) = \frac{2}{\pi}\int_0^\infty dk\, \frac{k^4\, u^2(k)}{k^2 + m^2}$$

- with a dipole regulator (on each $NN\pi$ vertex)

$$u(k) = \left(\frac{\Lambda^2}{\Lambda^2 + k^2}\right)^2$$

Finite-Range Regularisation



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$$I_\pi = \frac{1}{16}\frac{\Lambda^5(m_\pi^2+4m_\pi\Lambda+\Lambda^2)}{(m_\pi+\Lambda)^4}$$

Model-Independent Nonanalytic Behavior

- Taylor expand

$$I_\pi = \frac{1}{16}\frac{\Lambda^5(m_\pi^2 + 4m_\pi\Lambda + \Lambda^2)}{(m_\pi + \Lambda)^4}$$

about $m_\pi = 0$

$$I_\pi \to \frac{\Lambda^3}{16} - \frac{5\Lambda}{16}m_\pi^2 + {\color{red}m_\pi^3} - \frac{35}{16\Lambda}m_\pi^4 + \frac{4}{\Lambda^2}m_\pi^5 + \ldots$$

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- I_π contains a resummation of the chiral expansion such that

$I_\pi \to 0$ as m_π becomes large.

- In accord with the lattice simulation results.

Renormalised Expansion Coefficients

- Combine the analytic terms of

$$M_N^{\mathrm{LNA}} = a_0 + a_2 m_\pi^2 + \chi_\pi I_\pi(m_\pi) + a_4 m_\pi^4$$

and

$$I_\pi^{\mathrm{DIP}} \to \frac{\Lambda^3}{16} - \frac{5\Lambda}{16} m_\pi^2 + m_\pi^3 - \frac{35}{16\Lambda} m_\pi^4 + \dots$$

- Recover the renormalized expansion coefficients c_i

$$\begin{aligned} M_N^{\mathrm{LNA}} &= \left(a_0 + \chi_\pi \frac{\Lambda^3}{16}\right) + \left(a_2 - \chi_\pi \frac{5\Lambda}{16}\right) m_\pi^2 + \chi_\pi m_\pi^3 \\ &\quad + \left(a_4 - \chi_\pi \frac{35}{16\Lambda}\right) m_\pi^4 + \cdots \\ &= c_0 + c_2 m_\pi^2 + \chi_\pi m_\pi^3 + c_4 m_\pi^4 \end{aligned}$$

Renormalised Expansion (FRR)

- Any value of Λ is allowed!

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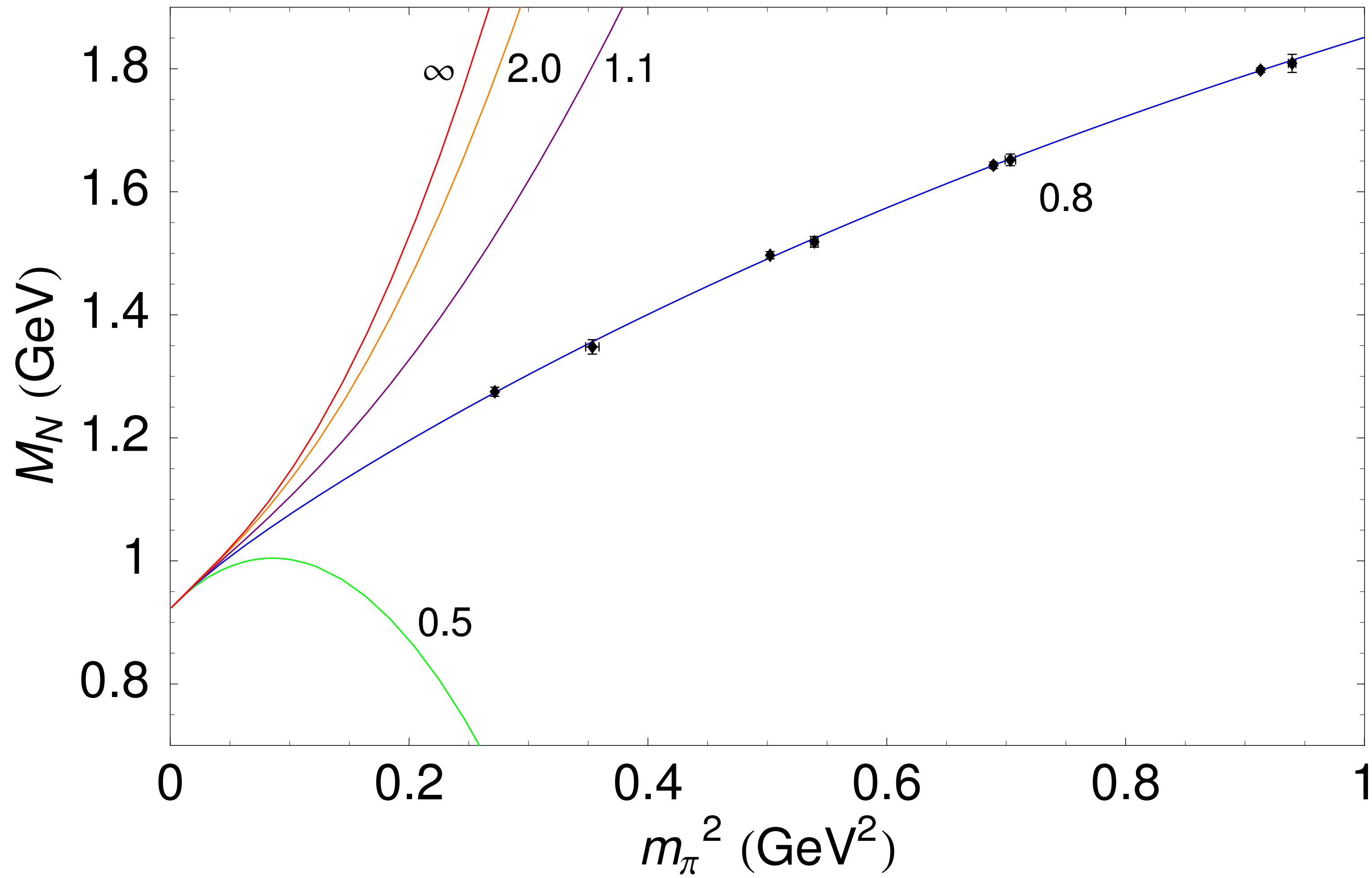
$$= c_0 + c_2 m_\pi^2 + \chi_\pi m_\pi^3 + c_4 m_\pi^4$$

- To any finite order, FRR is mathematically equivalent to Dimensional Regularisation.
- Within the power-counting regime of χPT
  - FRR EFT is not a model
  - Higher-order terms are truly negligible.

The Power Counting Regime



- Renormalised coefficients c_0, c_2 and c_4 are fixed.

Application of FRR Result

- Fit the resummed expression to lattice QCD results

$$M_N = a_0^\Lambda + a_2^\Lambda m_\pi^2 + \chi_\pi I_\pi(m_\pi, \Lambda) + a_4^\Lambda m_\pi^4$$

with

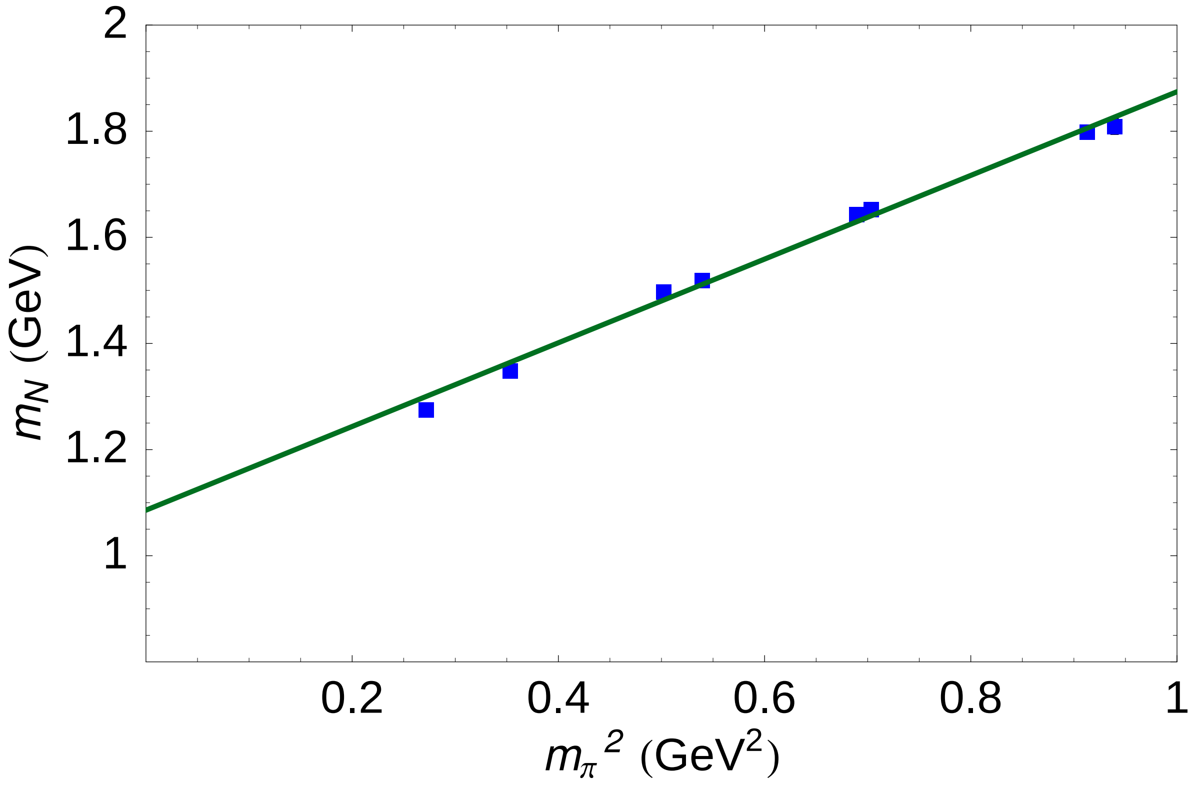
$$I_\pi = \frac{1}{16} \frac{\Lambda^5 (m_\pi^2 + 4 m_\pi \Lambda + \Lambda^2)}{(m_\pi + \Lambda)^4}$$

and

$$\Lambda = 0.8 \text{ GeV}$$

Lattice QCD and FRR EFT

- Dipole Regularisation



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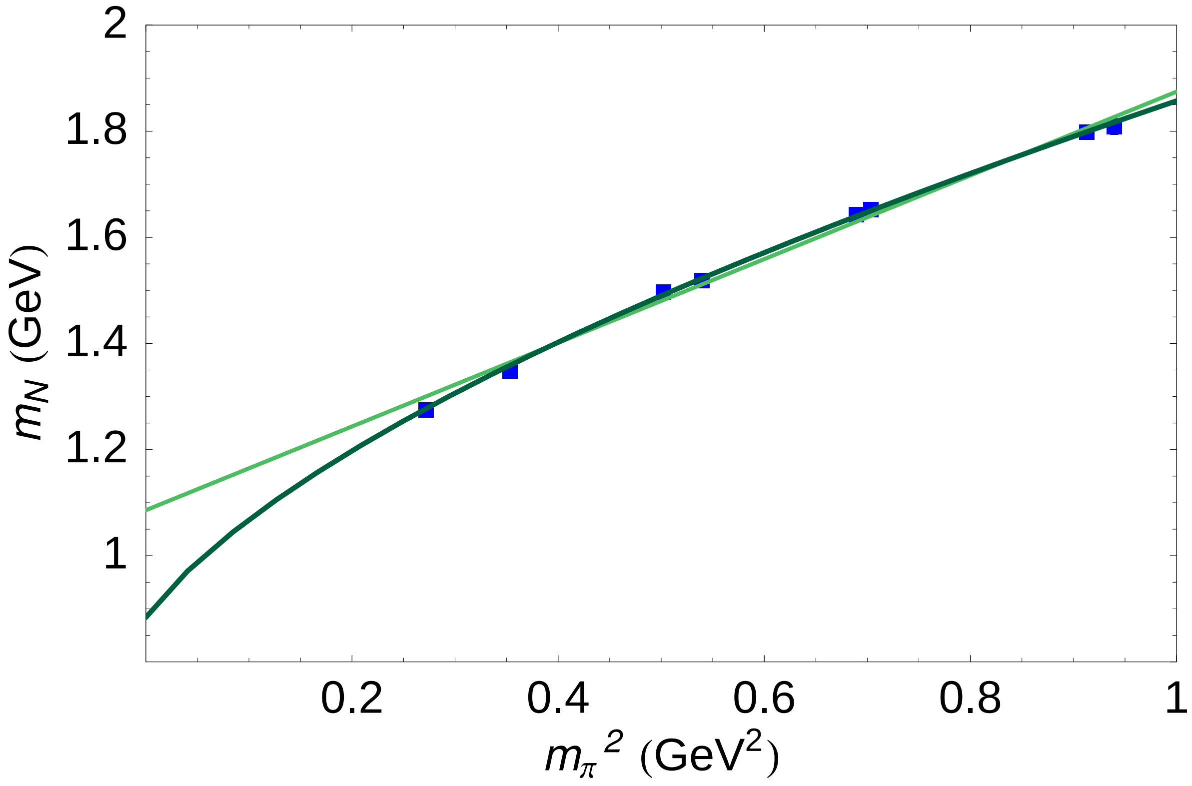

- B: $a_0 + a_2 m_\pi^2 + \chi_\pi I_\pi$

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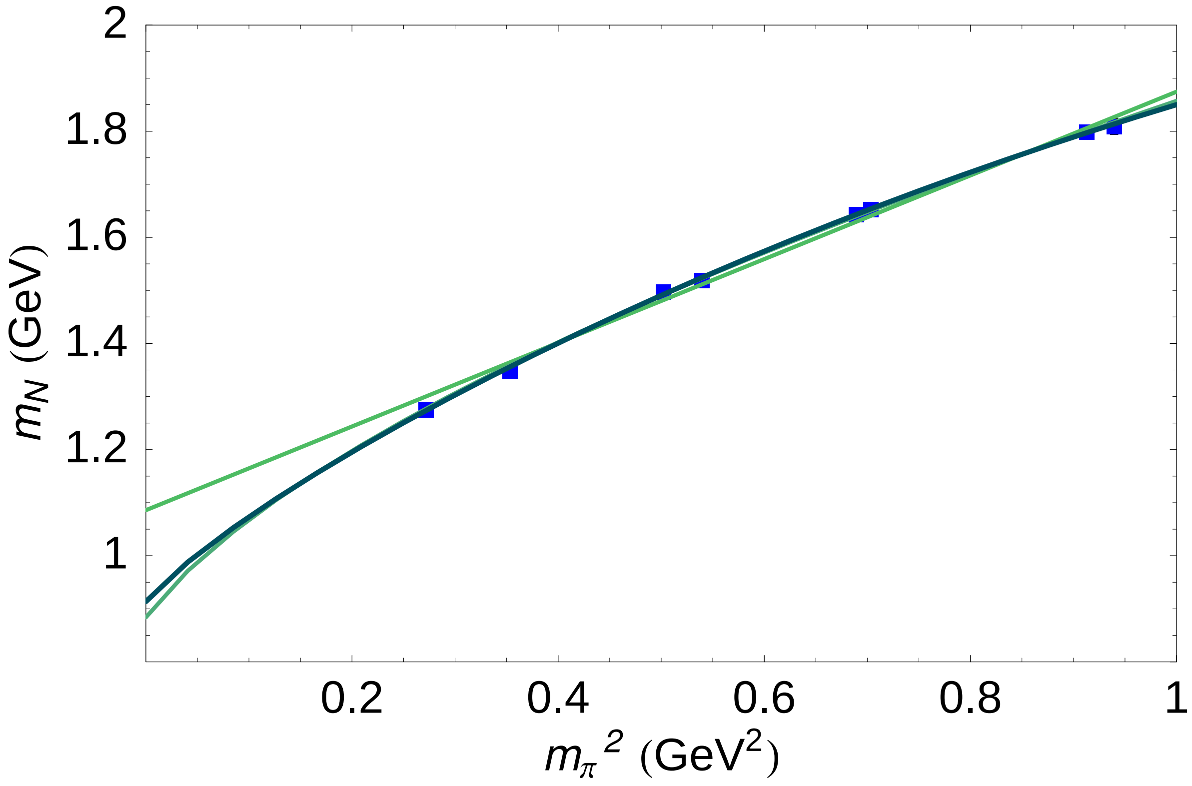
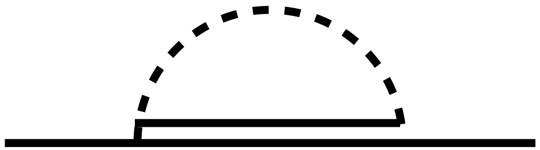
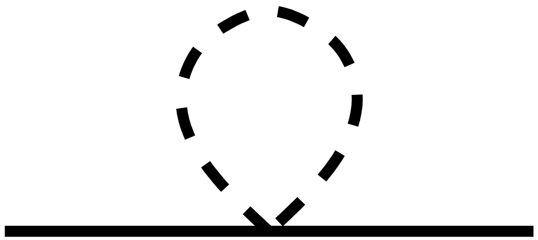


- C: $a_0 + a_2 m_\pi^2 + \chi_\pi I_\pi + a_4 m_\pi^4$

Next Leading Order

$$
\begin{aligned}
M_N^{\mathrm{NLNA}} &= a_0^\Lambda + a_2^\Lambda m_\pi^2 + \chi_\pi I_\pi(m_\pi, \Lambda) + a_4^\Lambda m_\pi^4 \\
&\quad + \chi_{\pi\Delta} I_{\pi\Delta}(m_\pi, \Lambda) + \chi_\pi^{\mathrm{tad}} I_\pi^{\mathrm{tad}}(m_\pi, \Lambda) + a_6^\Lambda m_\pi^6
\end{aligned}
$$

$I_{\pi\Delta} =$  $\sim m_\pi^4 \ln m_\pi$

$I_\pi^{\mathrm{tad}} =$  $\sim m_\pi^4 \ln m_\pi$

FRR Regulators

- Alternatives:
  - Sharp cut-off

$$\theta(\Lambda - k)$$

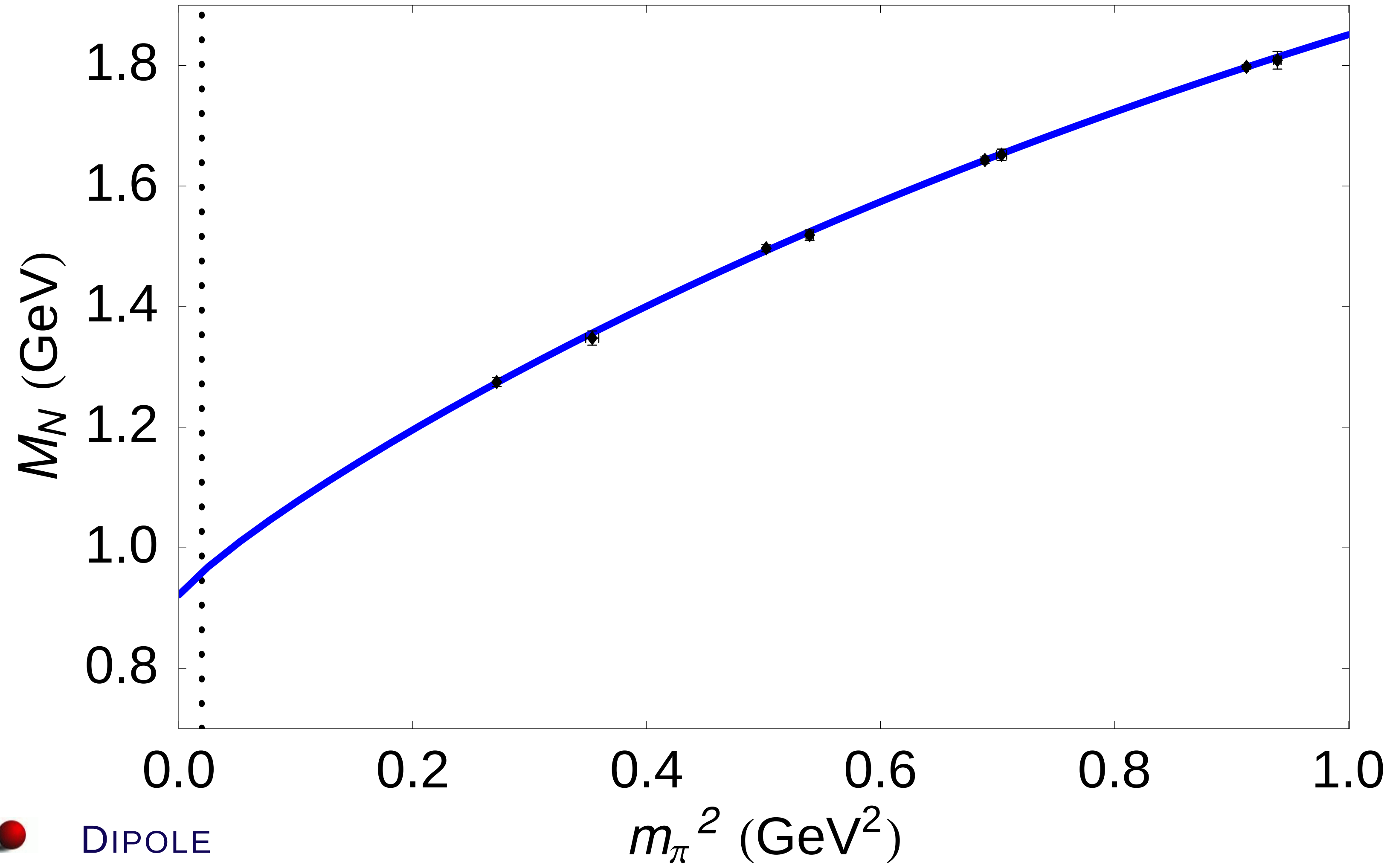
  - Monopole

$$\left(\frac{\Lambda^2}{\Lambda^2 + k^2}\right)$$

  - Gaussian

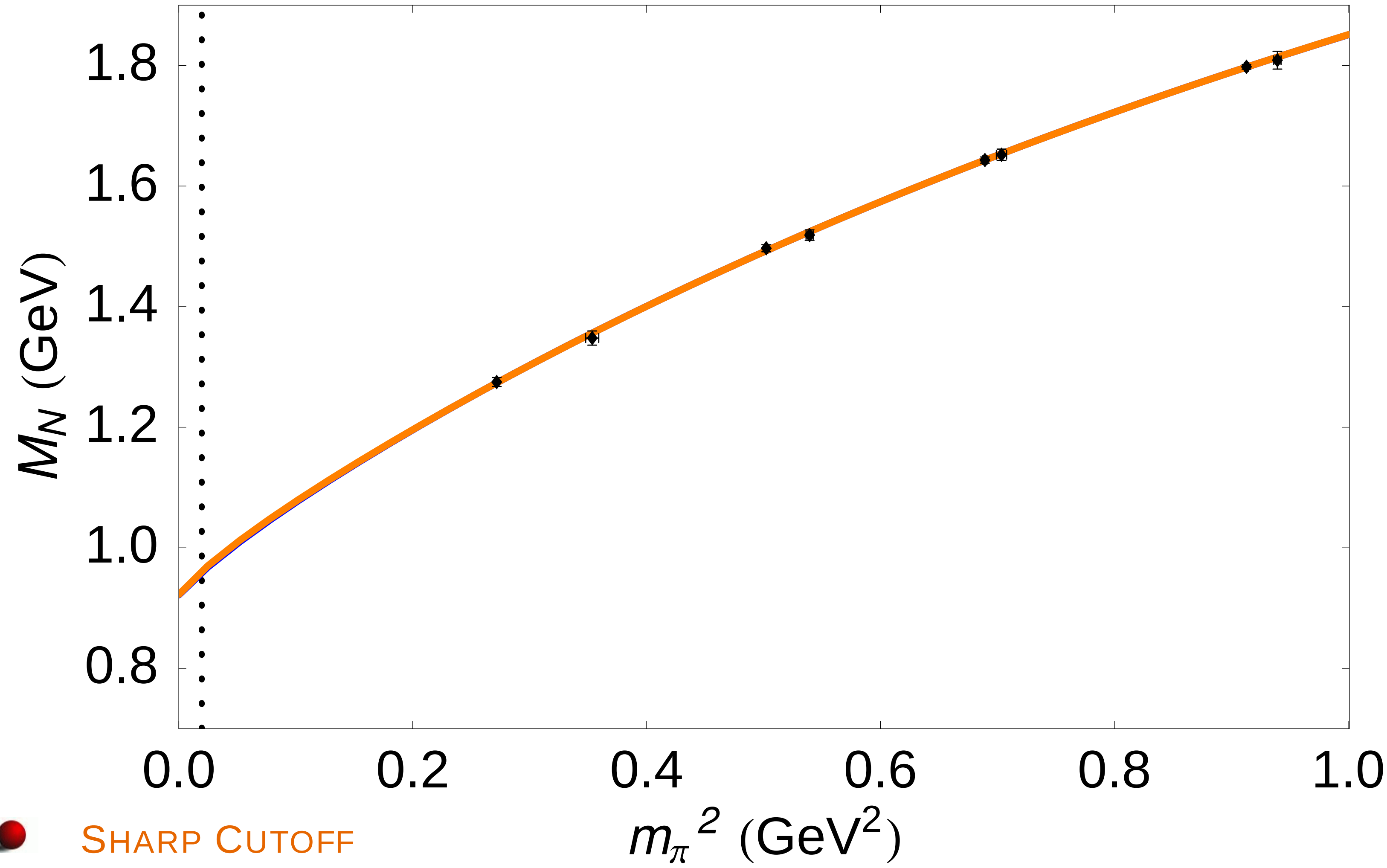
$$\exp(-\frac{k^2}{\Lambda^2})$$

M_N^{NLNA} Extrapolation



- DIPOLE

M_N^{NLNA} Extrapolation



- SHARP CUTOFF

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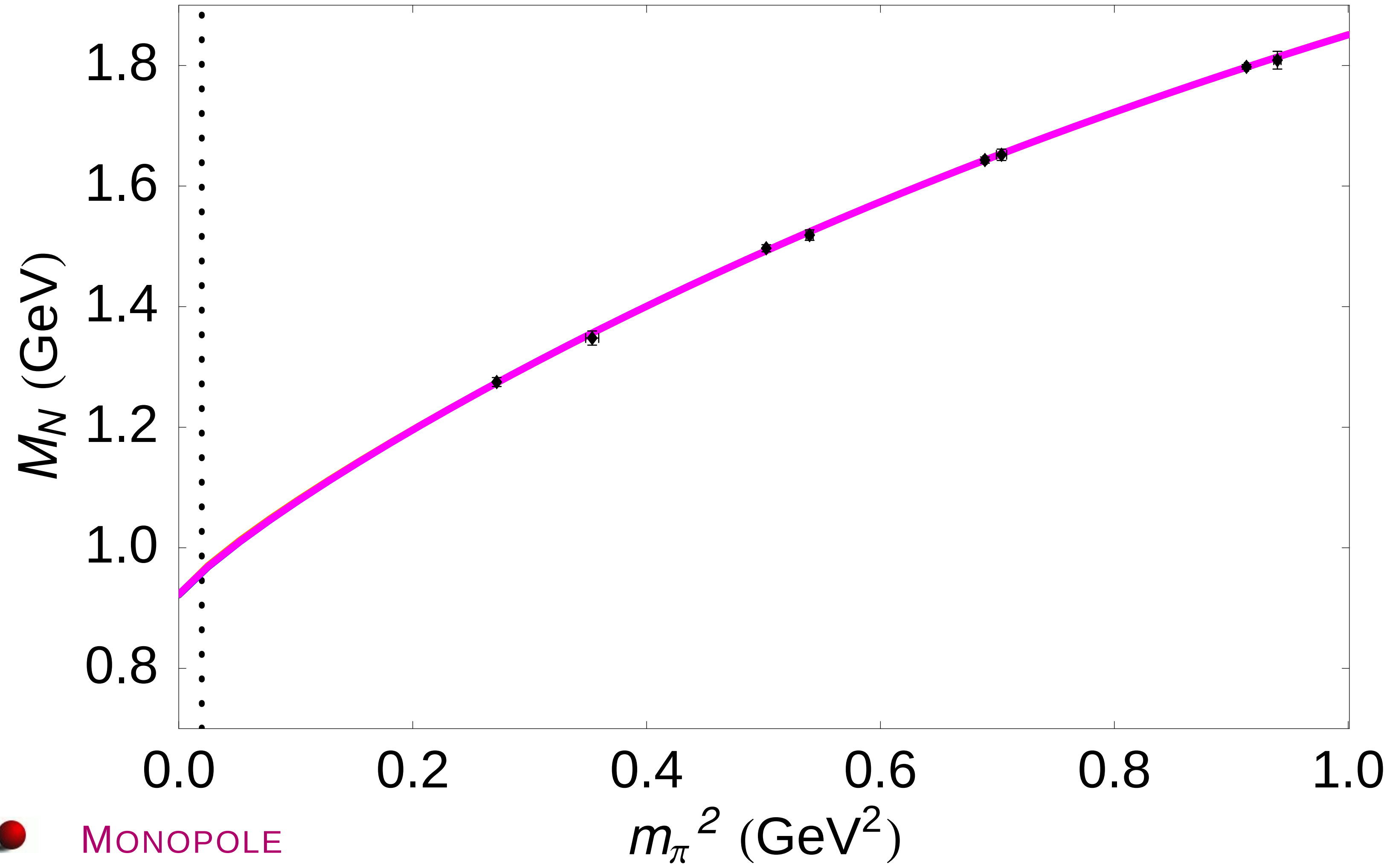
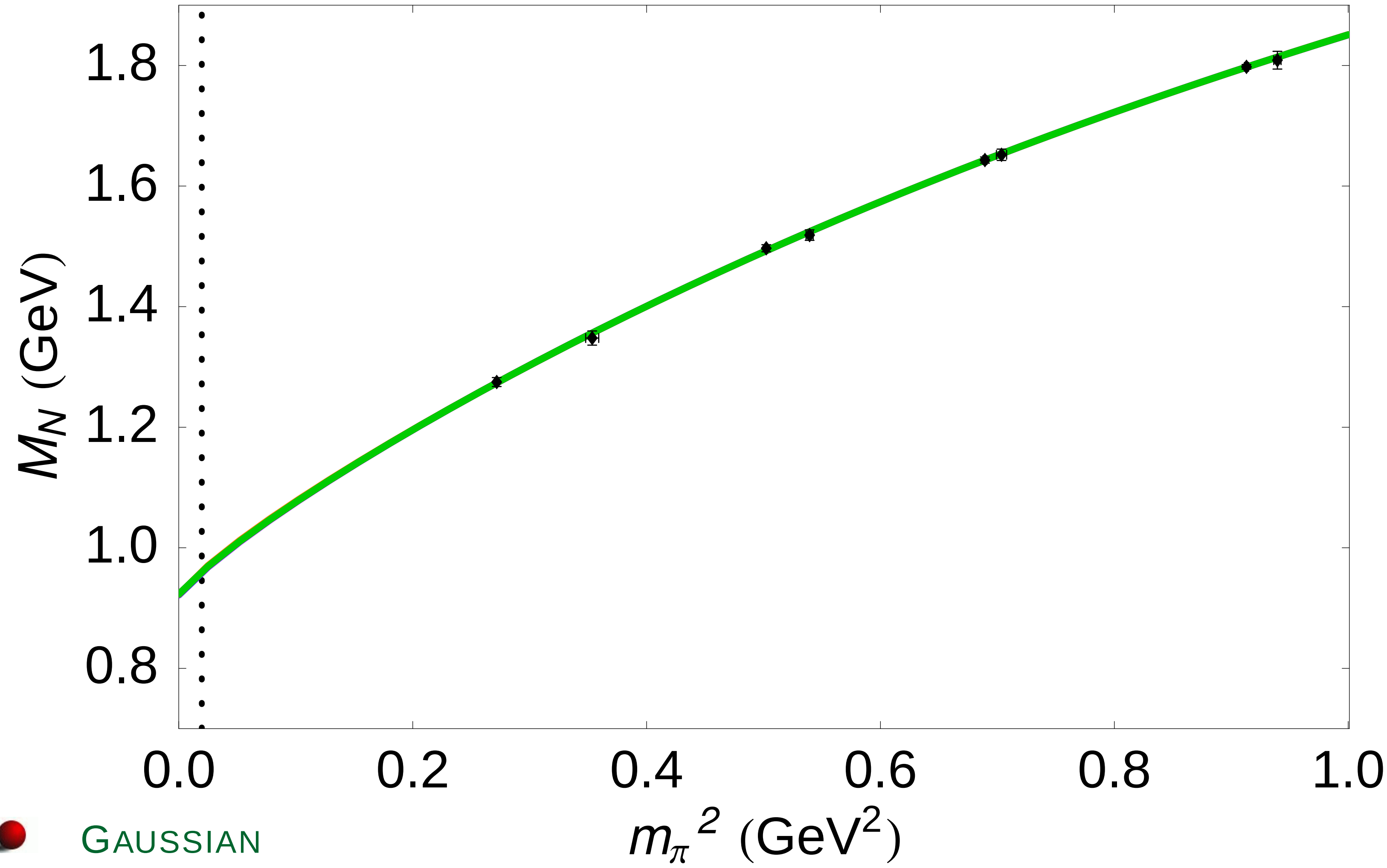


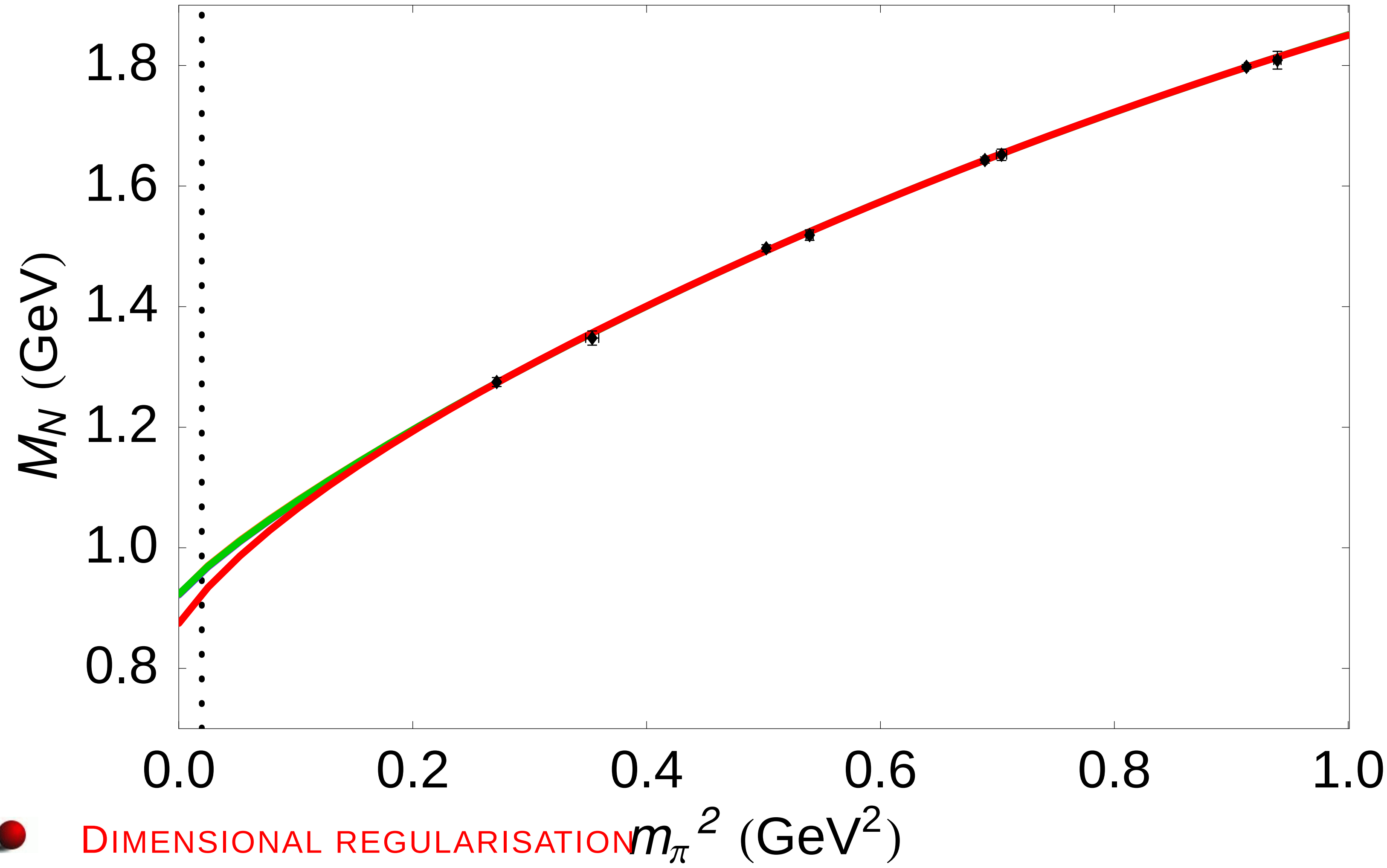
- MONOPOLE

M_N^{NLNA} Extrapolation



- GAUSSIAN

M_N^{NLNA} Extrapolation



- DIMENSIONAL REGULARISATION

Low Energy Coefficients

- NLNA results are largely independent of the model!

Regulator	c_0	c_2	c_4
Dipole	0.922	2.49	18.9
Sharp cutoff	0.923	2.61	15.3
Monopole	0.923	2.45	20.5
Gaussian	0.923	2.48	18.3
Dim. reg.	0.875	3.14	7.2

Series Truncation

- Residual series coefficients

Regulator	a_4 (GeV^{-3})	a_6 (GeV^{-5})
Dipole	-0.49	0.09
Sharp cutoff	-0.55	0.12
Monopole	-0.49	0.09
Gaussian	-0.50	0.10
Dim. reg.	8.9	0.38

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- Lattice QCD simulation results are generally smooth slowly varying functions of the quark mass.
  - Higher-order terms of the DR expansion must sum approximately to zero.
- Finite-range regularisation resums the chiral expansion of DR.
  - Linear combinations of higher order DR terms appear already in one-loop calculations.

FRR Summary

- To any finite order, FRR is mathematically equivalent to Dimensional Regularisation.
- Lattice QCD simulation results are generally smooth slowly varying functions of the quark mass.
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- Higher order DR terms obtained in FRR EFT sum such that loop contributions vanish as the quark mass becomes large.

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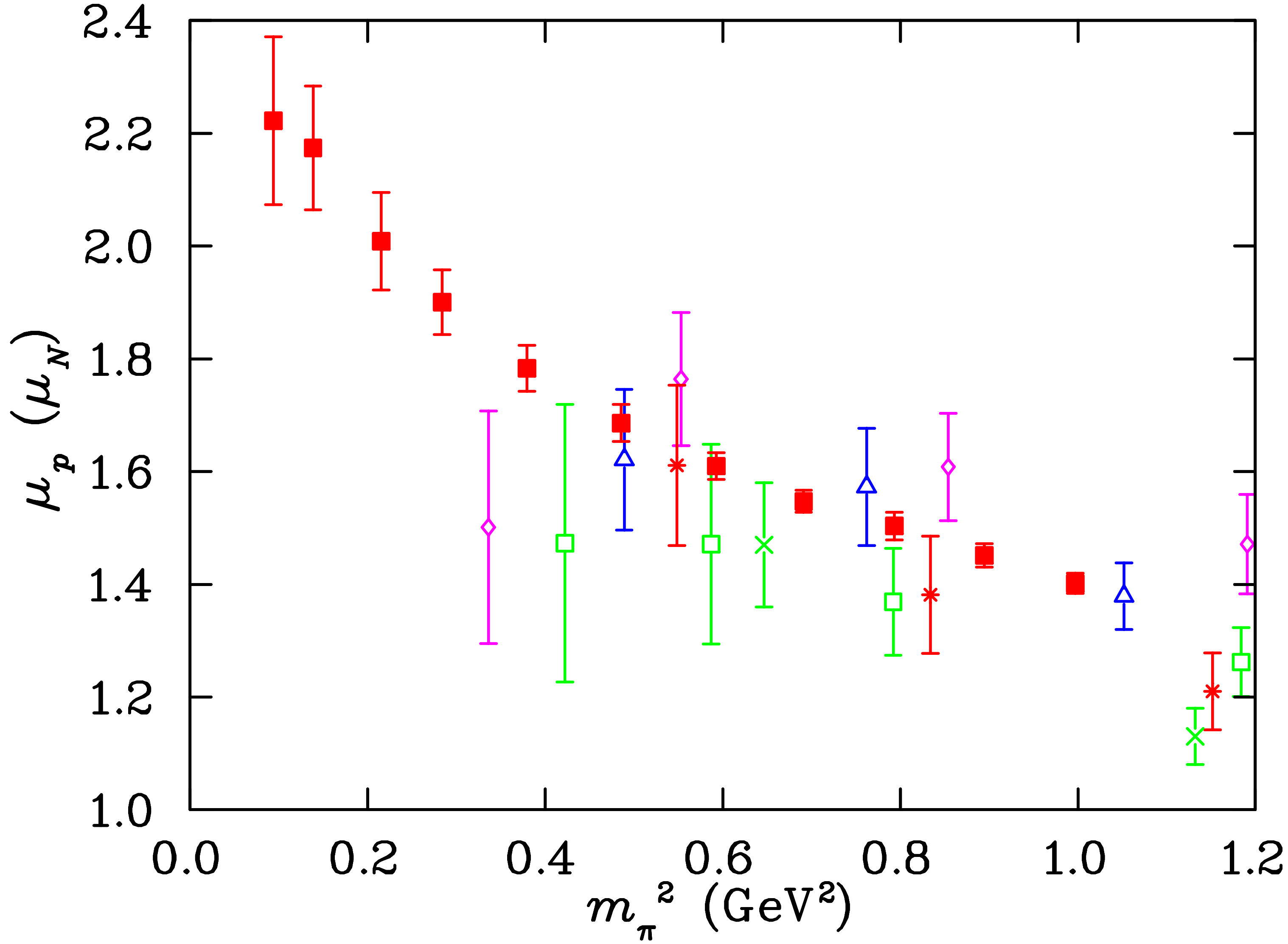
  - Higher-order terms of the DR expansion must sum approximately to zero.
- Higher order DR terms obtained in FRR EFT sum such that loop contributions vanish as the quark mass becomes large.
- Regulator parameter, Λ, shifts strength between FRR loop integrals and the residual expansion of terms analytic in the quark mass.
  - Provides a new mechanism to optimize the convergence properties of the chiral expansion.

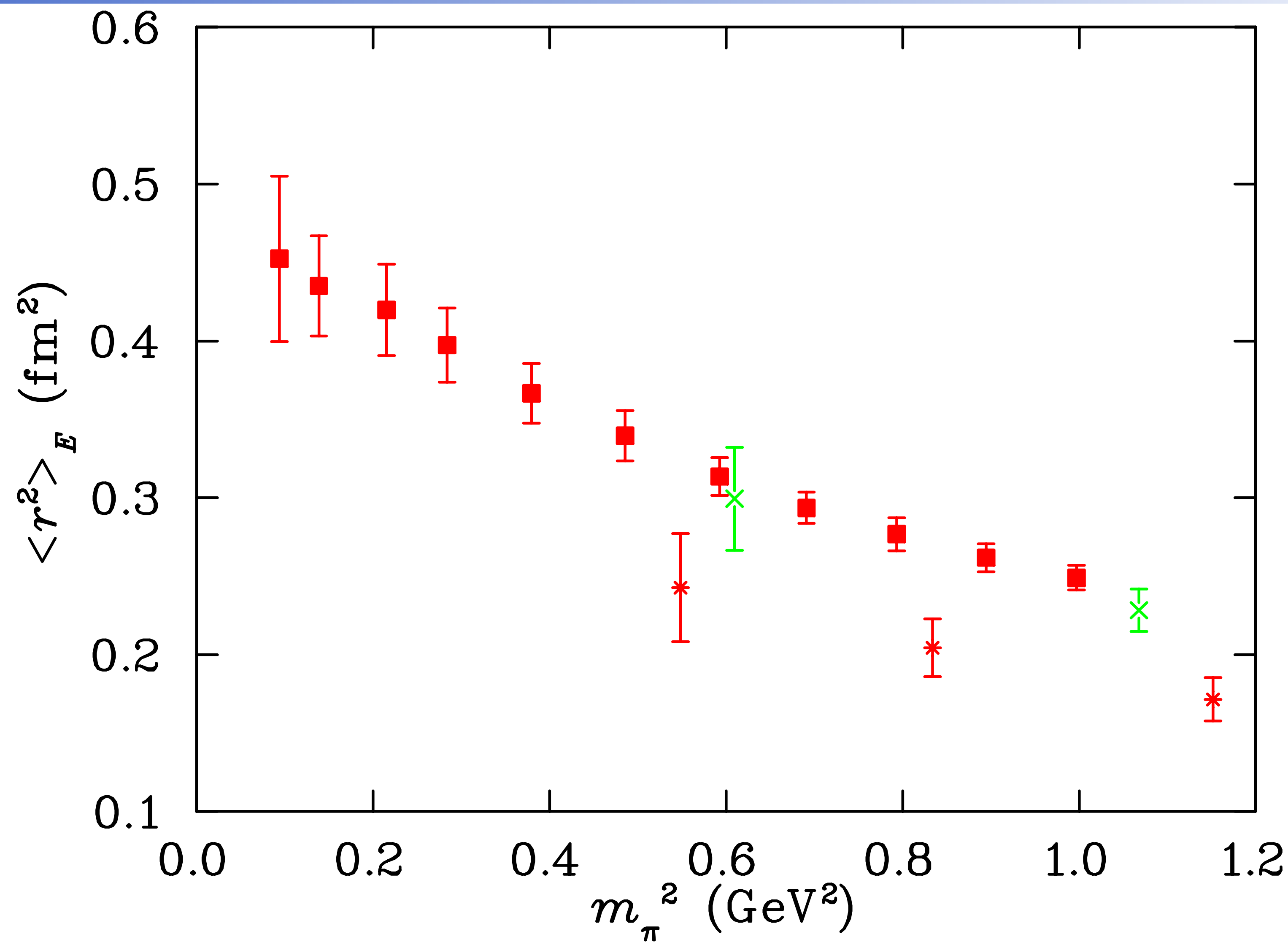
Optimal Regularisation?

- Regulator parameter Λ should be constrained by lattice QCD results.
- Several criteria were under investigation.
- See Jonathan Hall's poster tomorrow for the solution.

Proton Moment in Quenched QCD

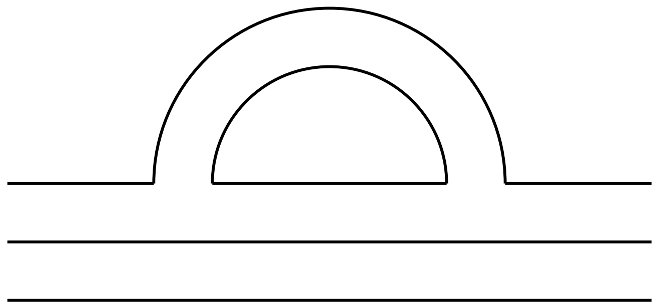


Proton Radius in Quenched QCD

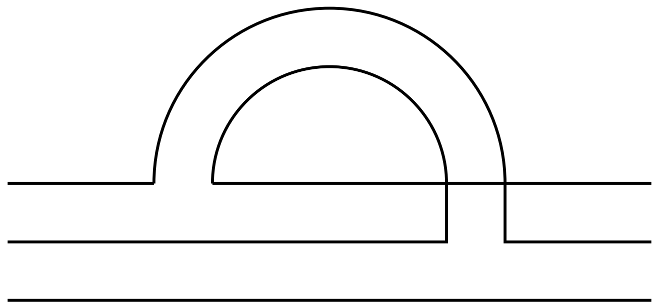


Quenched Chiral Nonanalytic Behavior

- “Disconnected” sea-quark loops are absent, modifying vertices.



(a)



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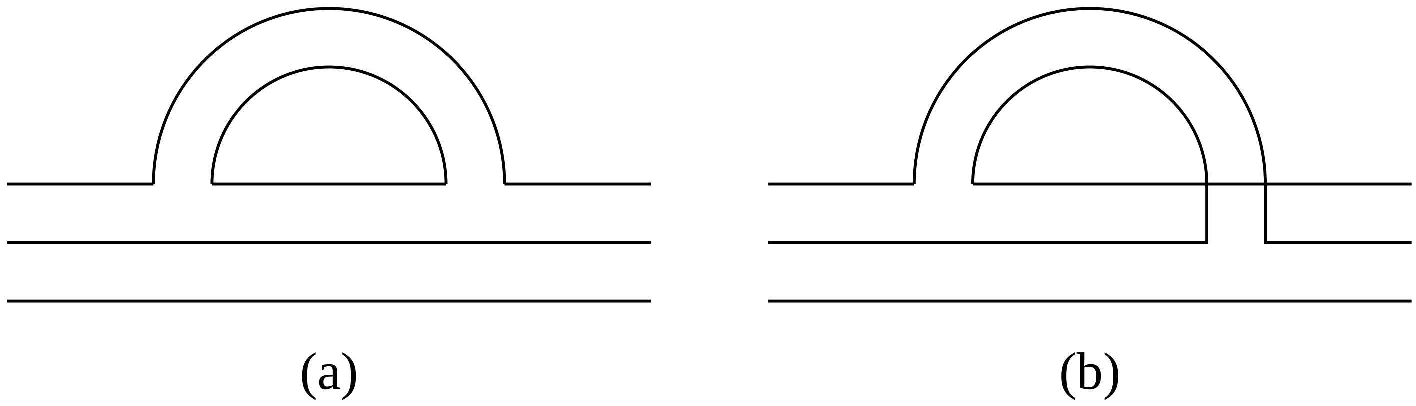
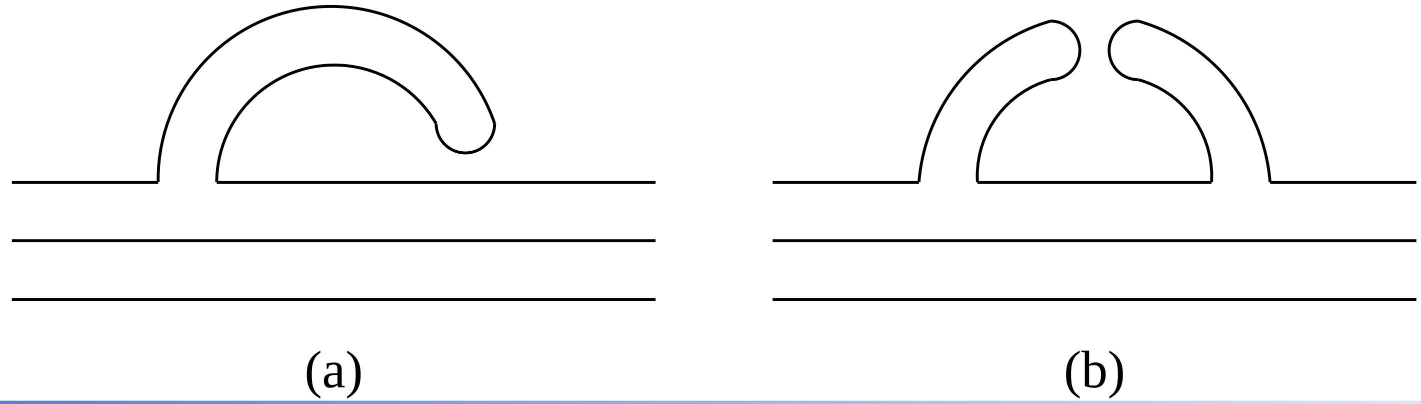
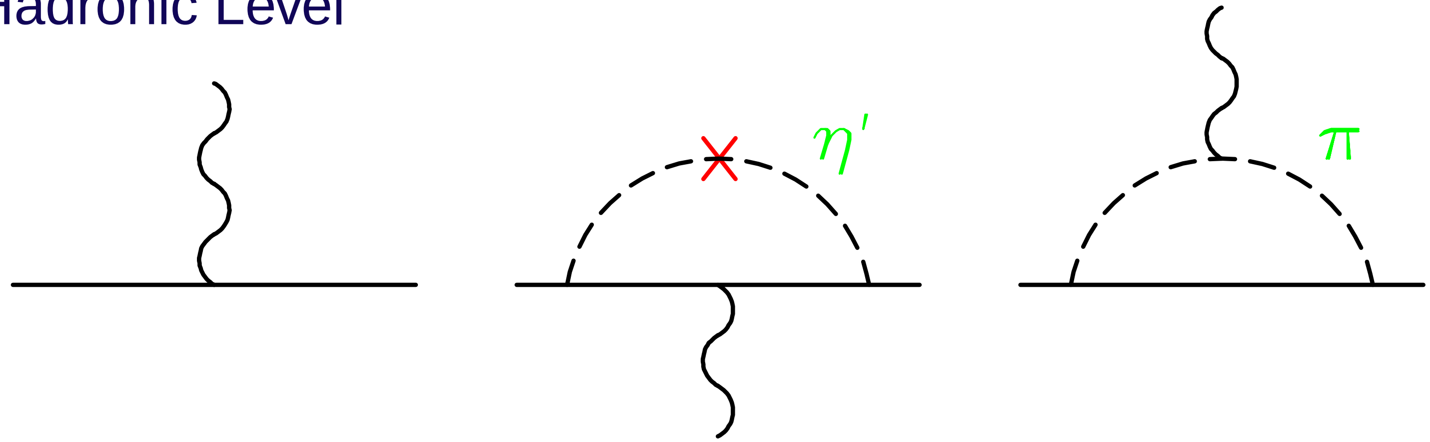


- η'-meson mass remains degenerate with the pion and can contribute new nonanalytic terms to the chiral expansion.

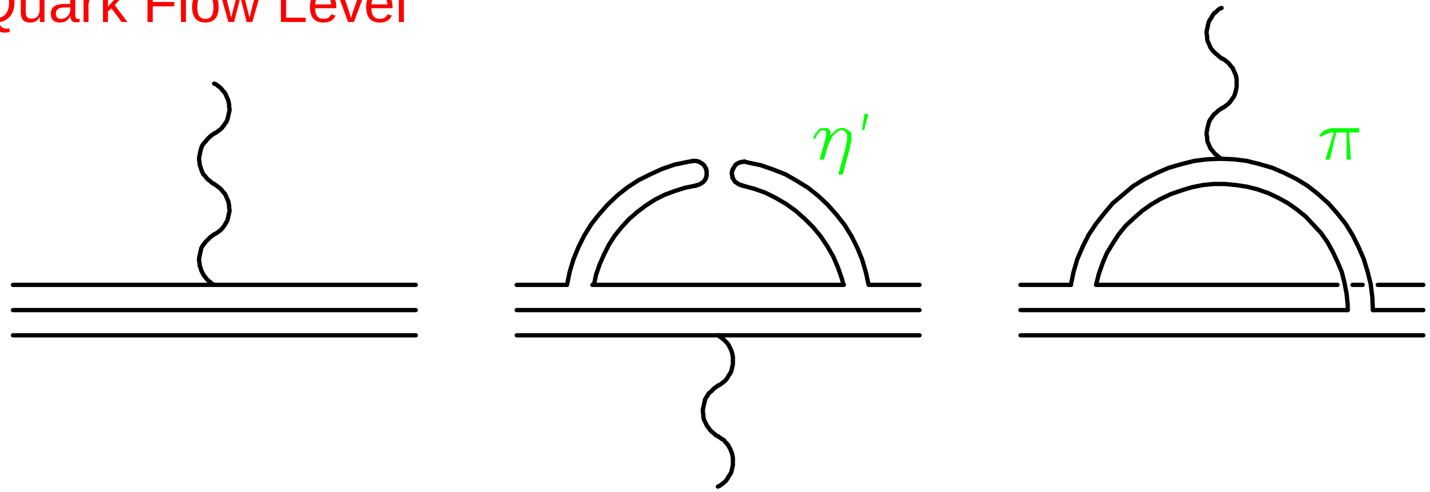


Quenched Quark Flow for Form Factors

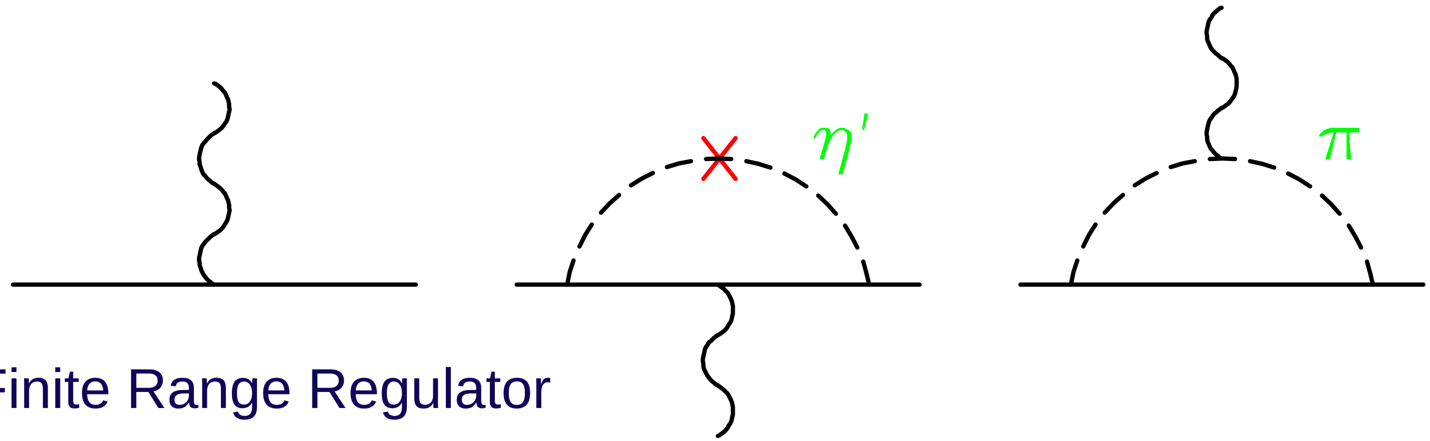
- Hadronic Level



- Quark Flow Level



Finite-Range Regularisation



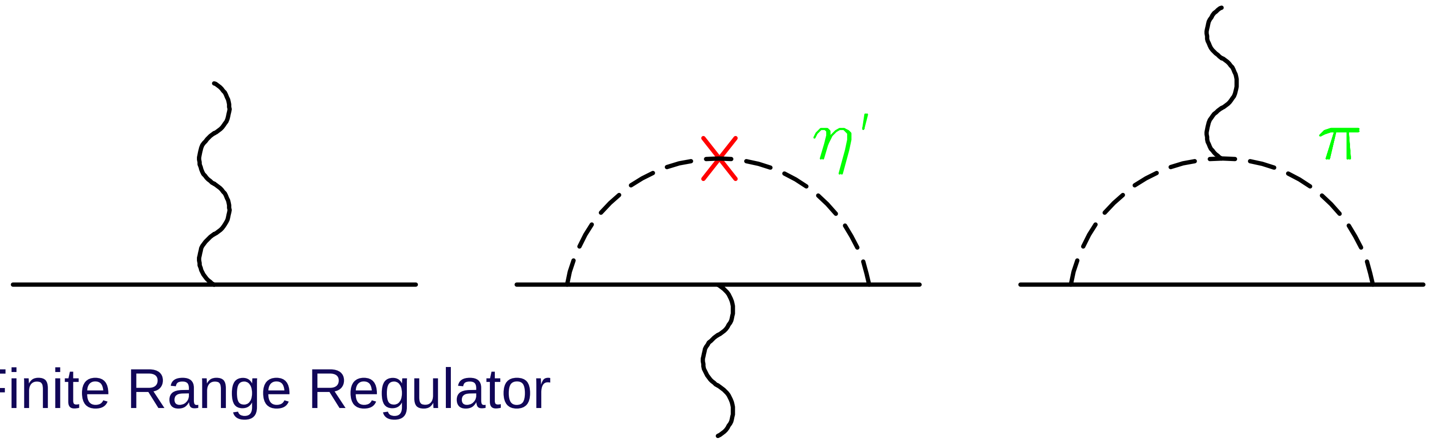
- Finite Range Regulator

$$\mu_p = a_0^\Lambda + \mu_p\, \chi_\eta\, I_\eta(m_\pi, \Lambda) + \chi_\pi\, I_\pi(m_\pi, \Lambda) + \chi_K\, I_K(m_K, \Lambda) + a_2^\Lambda\, m_\pi^2 + \cdots$$

- Kaon mass relation

$$m_K^2 = m_K^{(0)\,2} + \frac{1}{2}\, m_\pi^2$$

Finite-Range Regularisation



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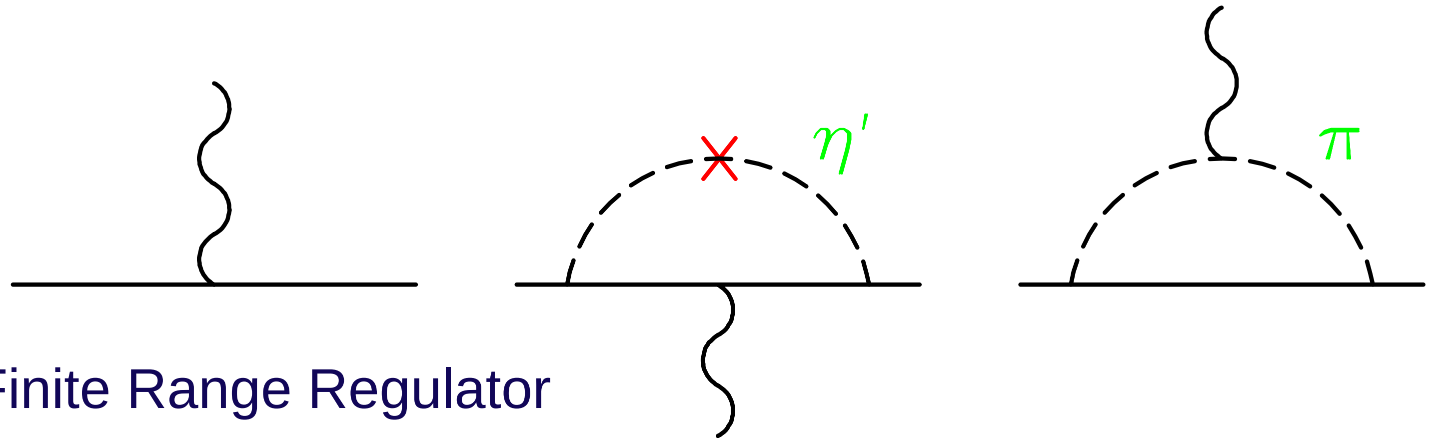
- Limit $m_\pi \to 0$

$$\mu_p = c_0 + \mu_p\,\chi_\eta\left[l_0 + \log\left(\frac{m_\pi^2}{\Lambda^2}\right)\right] + \chi_\pi\, m_\pi + \chi_K\, m_K + c_2\, m_\pi^2 + \cdots$$

Finite-Range Regularisation



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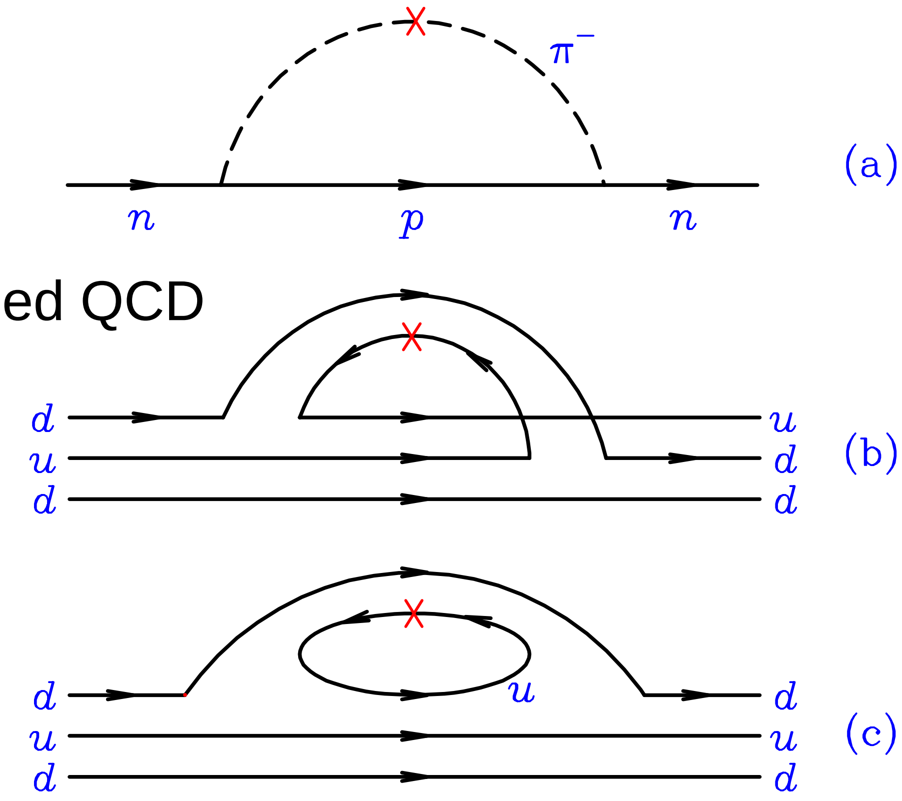
- Dimensional Regularisation

$$\mu_p = c_0 + \mu_p\,\chi_\eta\,\left[\log\frac{\Lambda^2}{\Lambda'^2} + \log\left(\frac{m_\pi^2}{\Lambda^2}\right)\right] + \chi_\pi\, m_\pi + \chi_K\, m_K + c_2\, m_\pi^2 + \cdots$$

Direct Loop Contributions

- (a) Full QCD = (b) + (c)
- (b) Valence in Full & Quenched QCD
- (c) Direct sea-quark loop



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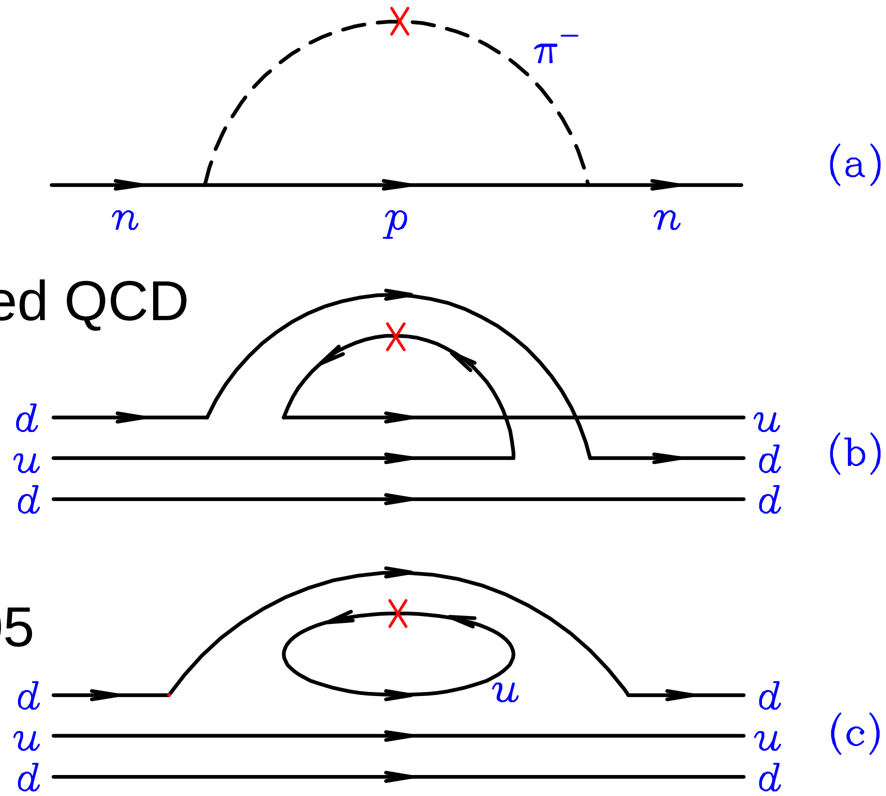
- Phys. Rev. D69 (2004) 014005 [arXiv:hep-lat/0211017].



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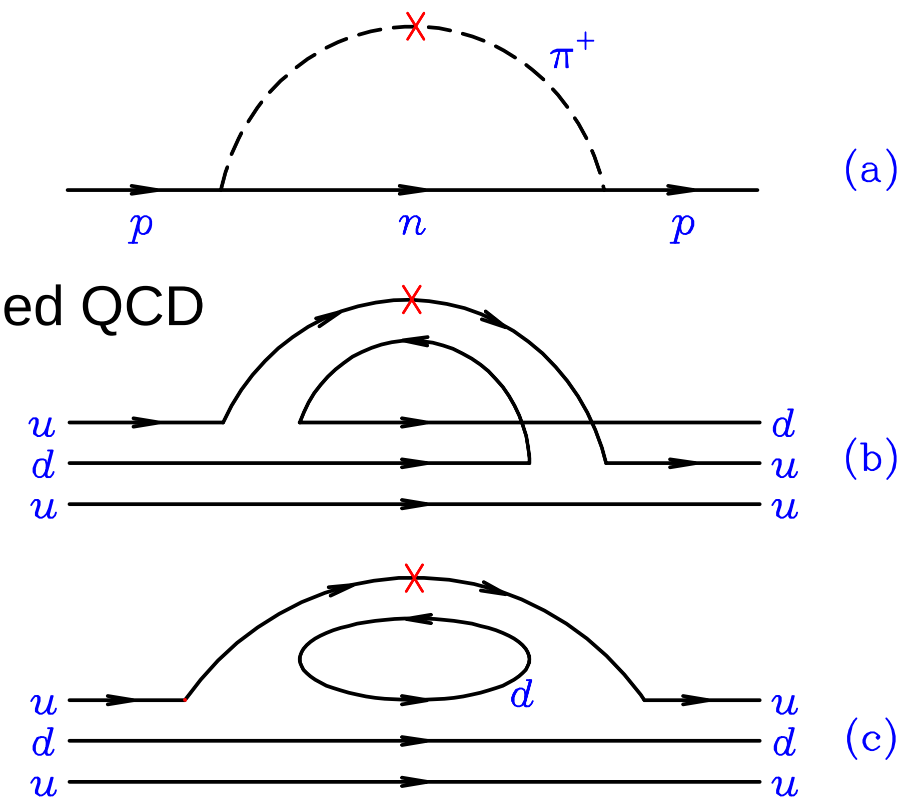

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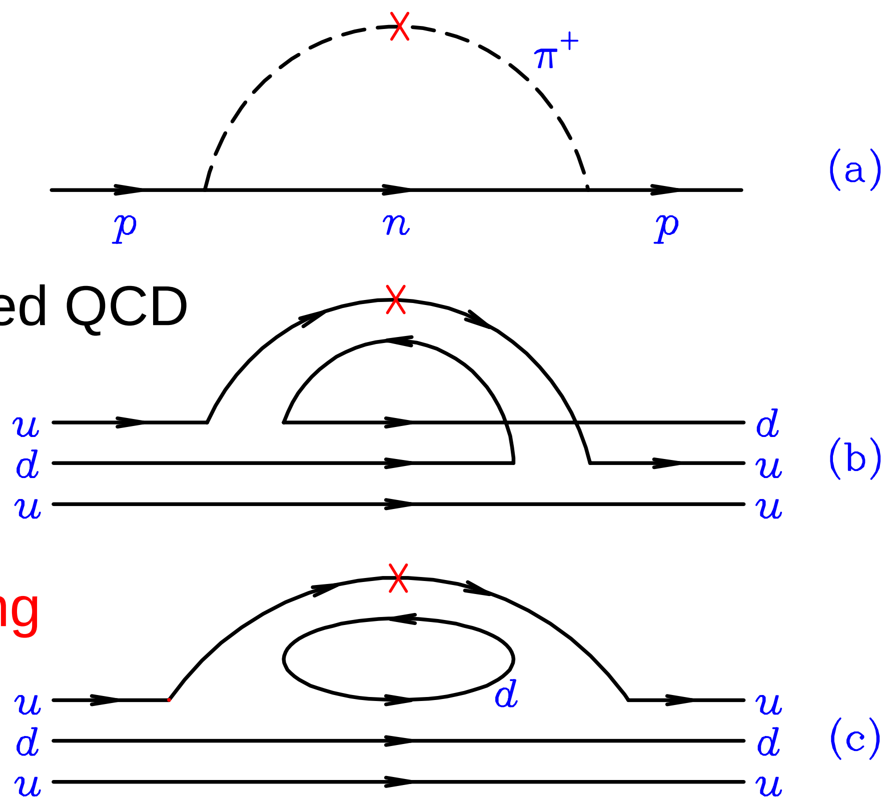
- (c) Indirect sea-quark loop
- (c) is removed upon quenching



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Coefficients χ_π and χ_K (μ_N/GeV)

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	ΛK	-3.68	0	-3.68	0
	ΣK	-0.15	0	-0.15	0
d_p	$N\pi$	$+6.87$	$+4.12$	$+2.75$	$+3.33$
	ΣK	-0.29	0	-0.29	0
s_p	ΛK	$+3.68$	$+3.68$	0	0
	ΣK	$+0.44$	$+0.44$	0	0
$2\,u_{\Sigma^+}$	$\Sigma\pi$	-2.16	$+2.16$	-4.32	0
	$\Lambda\pi$	-1.67	$+1.67$	-3.33	0
	NK	0	$+0.29$	-0.29	-0.29
	ΞK	-6.87	0	-6.87	-3.04

Finite Volume Artifacts

- Directly incorporate finite-volume effects into the chiral expansion.
- General expansion for the small parameters m_π and $1/L$

$$\begin{aligned} M_N &= \{\text{Terms Analytic in } m_\pi^2 \text{ and } 1/L\} \\ &\quad + \{\text{Volume-modified Chiral loop corrections}\} \end{aligned}$$

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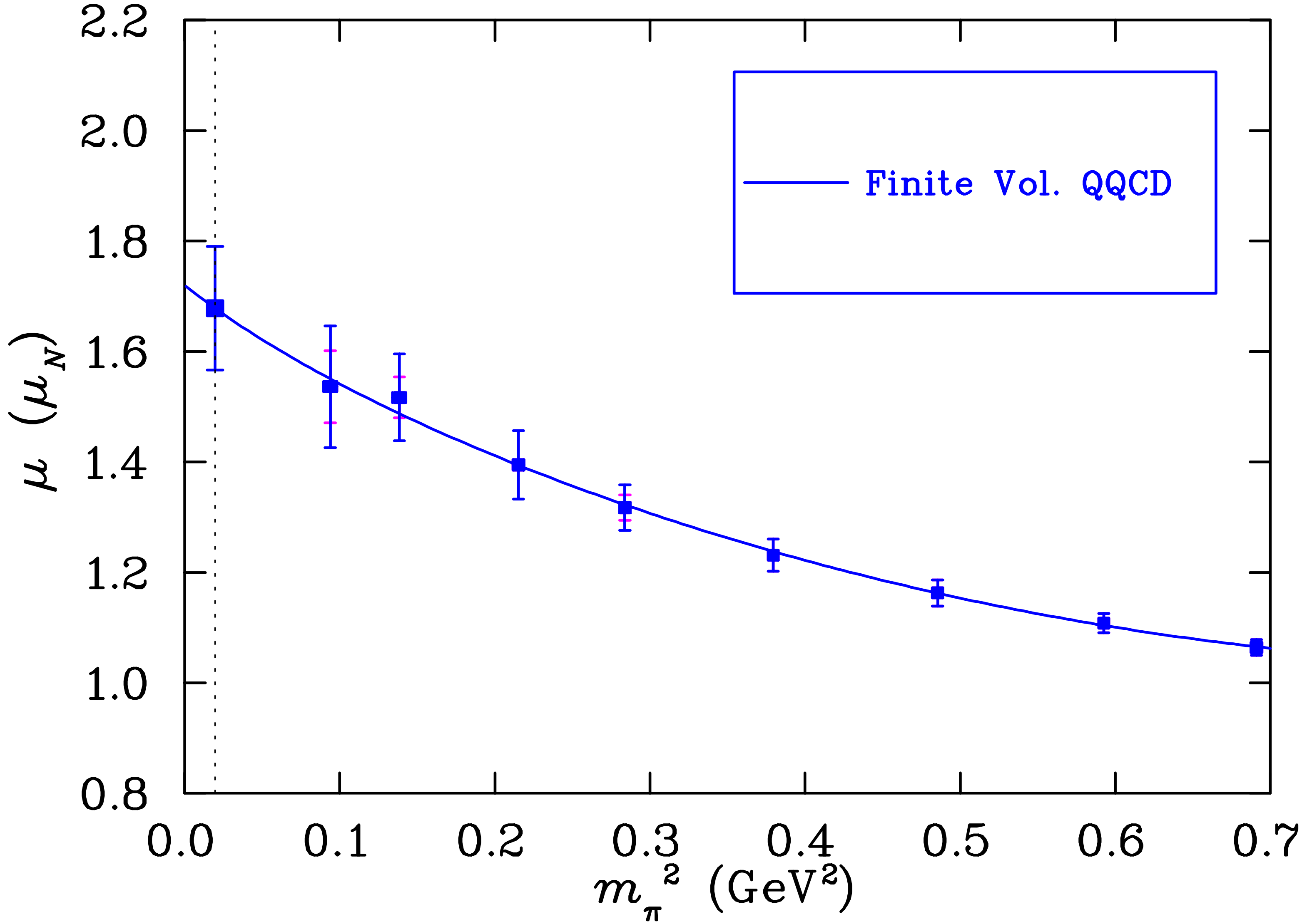
  - Analytic terms in $1/L$ are small by constraint.
- The finite periodic volume of the lattice modifies integrals

$$\int d^3k \to \left(\frac{2\pi}{L^3}\right)^3 \sum_{k_x,k_y,k_z} .$$

u quark in the Proton: Quenched QCD



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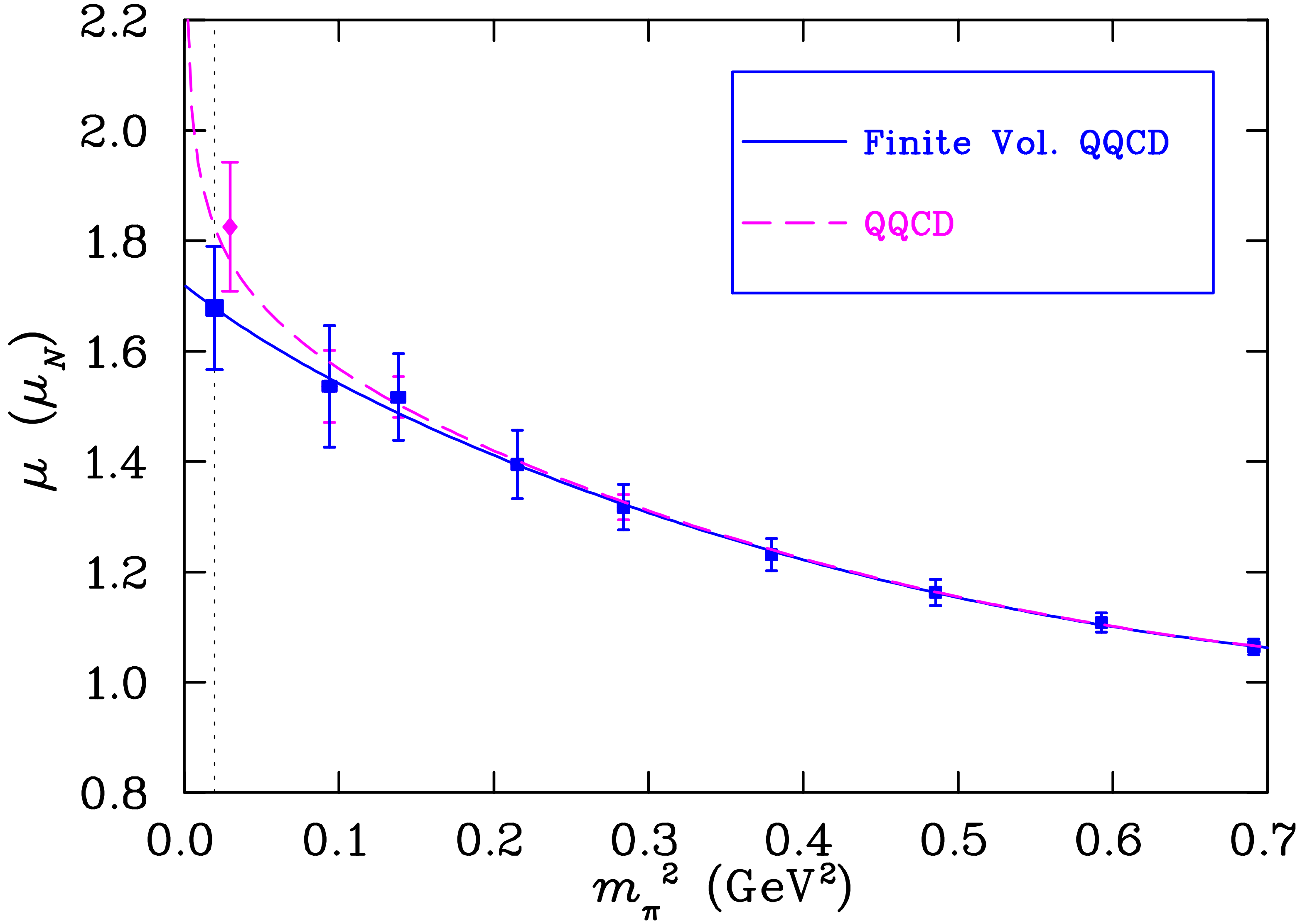
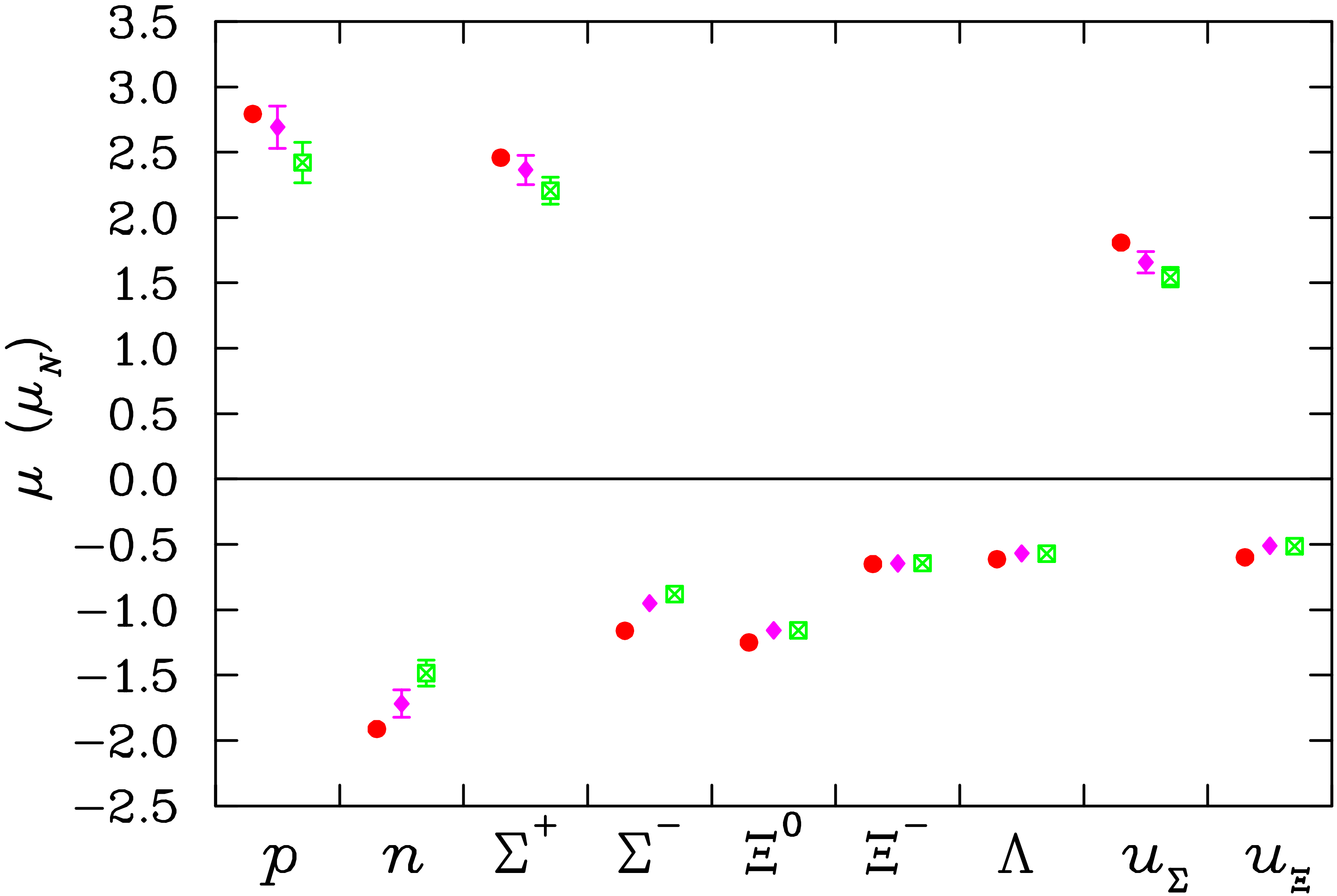


Quenched Finite Volume Moments



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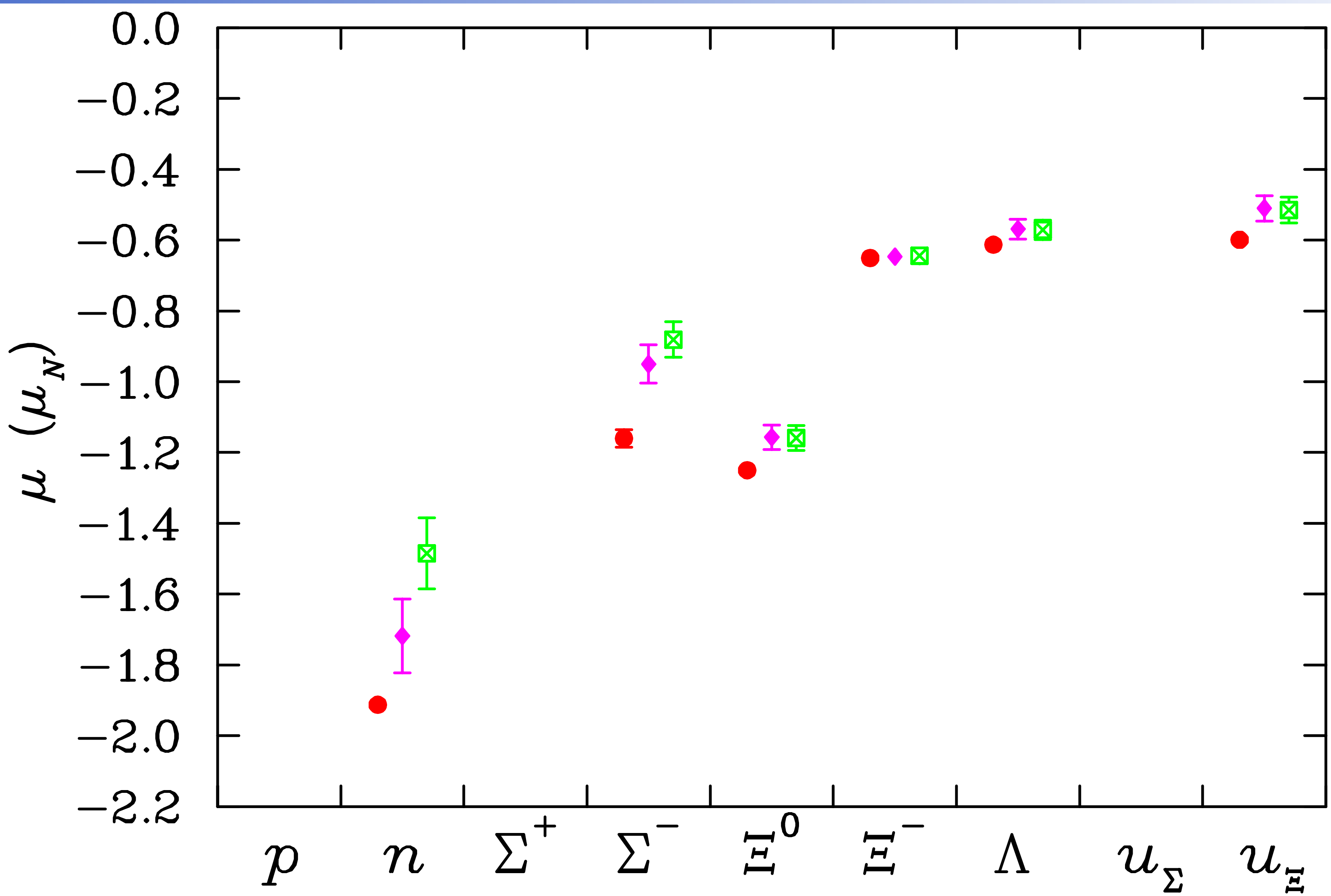

Correcting the Quenched Approximation

- Studied a matched set of quenched QCD and full QCD gauge configurations from the MILC Collaboration
- Fit the nucleon mass in quenched QCD and in full QCD
  - With Finite-Range Regularised quenched EFT and full EFT
  - Regulator Parameter $\Lambda = 0.8$

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  - Are the same within errors

$$
\begin{aligned}
M_N &= \{a_0^\Lambda + a_2^\Lambda m_\pi^2 + a_4^\Lambda m_\pi^4 + a_6^\Lambda m_\pi^6 + \cdots\} \\
&\quad + \{\chi_\pi \, I_\pi(m_\pi, \Lambda) + \chi_{\pi\Delta} \, I_{\pi\Delta}(m_\pi, \Lambda) + \cdots\}
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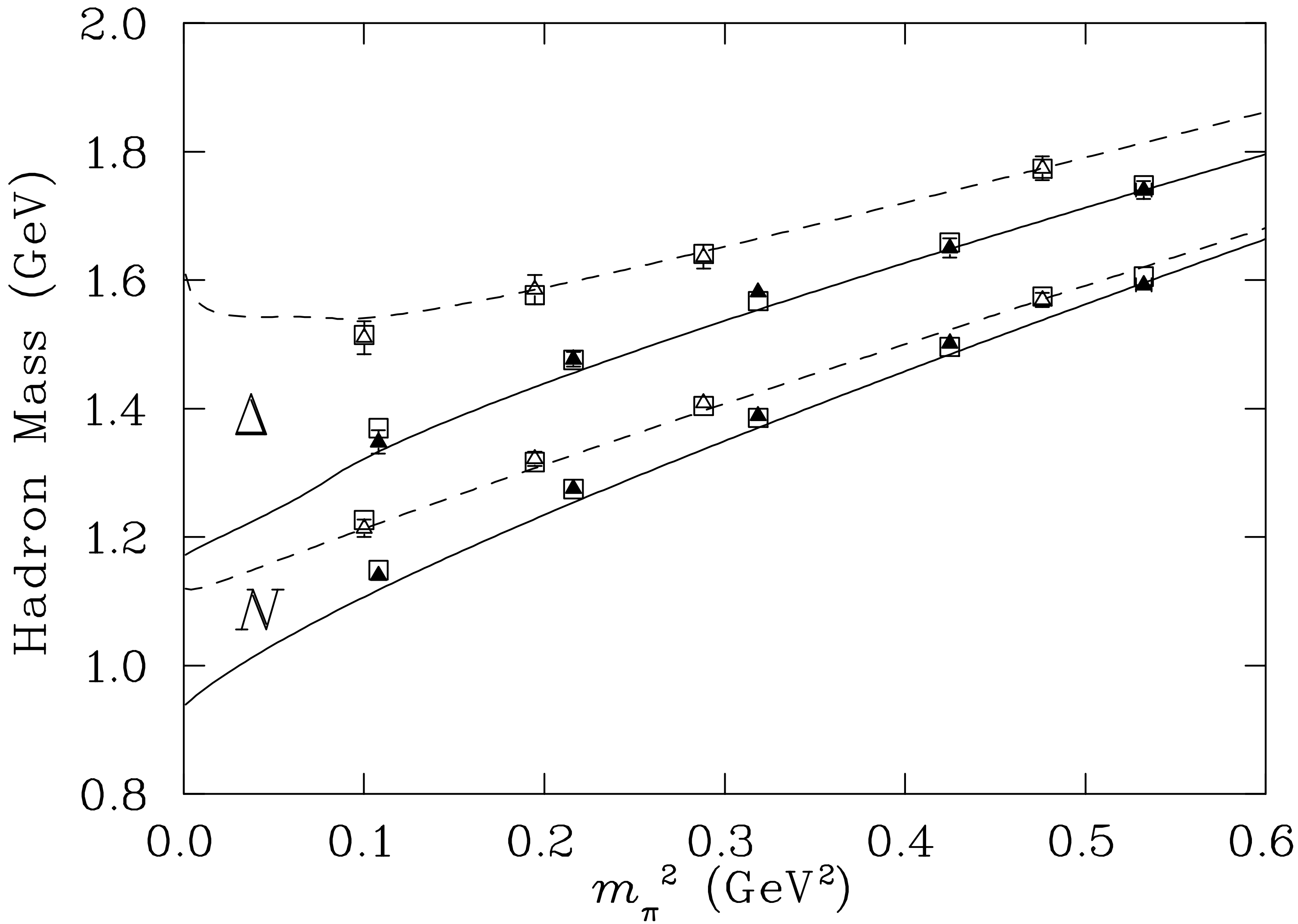
- Leads to the concept of separating
  - The pion cloud
    - Affected by quenching and finite volume
  - The core (the source of the pion cloud)
    - Invariant to quenching and finite volume artifacts.

MILC Collaboration Simulations



Coefficients of Analytic Terms

- For case of Regulator Parameter $\Lambda = 0.8$
- Nucleon

	a_0	a_2	a_4
N (Dynamical)	$1.23(1)$	$1.13(8)$	$-0.4(1)$
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Units are in appropriate powers of GeV.

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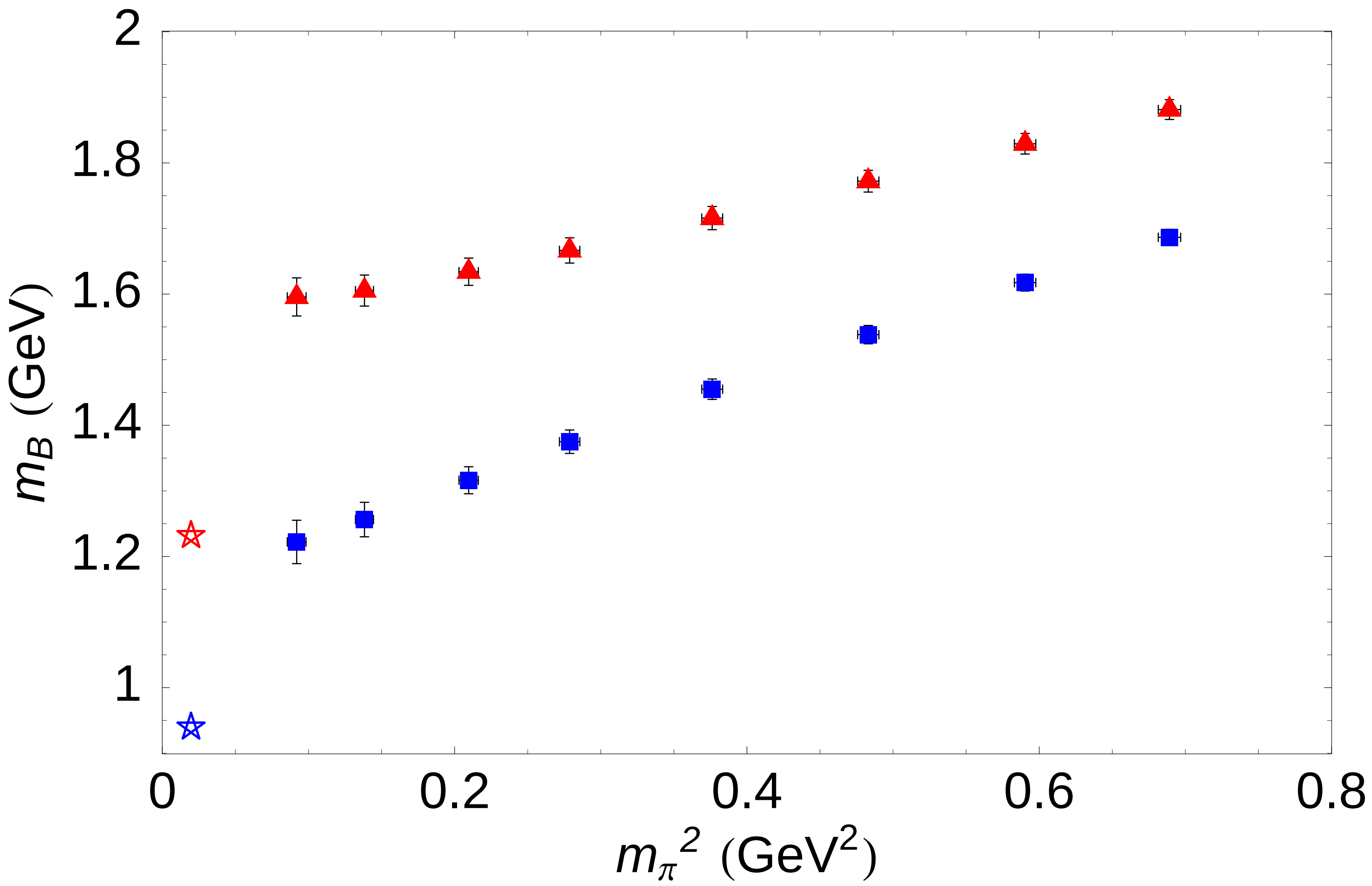
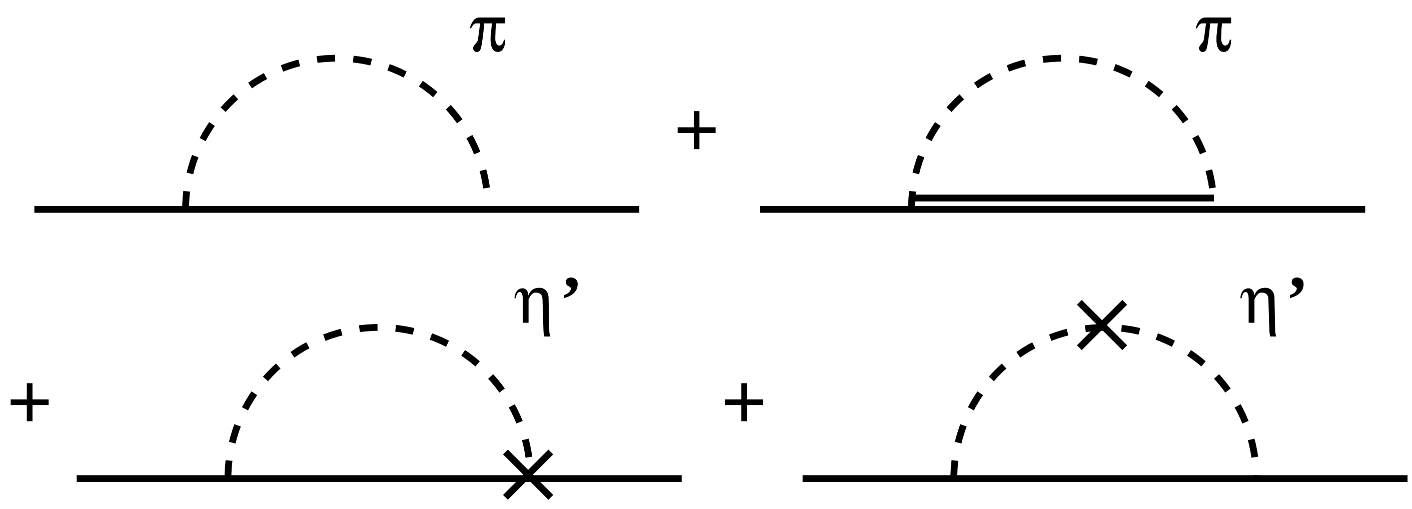
- Delta

	a_0	a_2	a_4
Δ (Dynamical)	$1.40(3)$	$1.1(2)$	$-0.6(3)$
Δ (Quenched)	$1.43(3)$	$0.8(2)$	$-0.1(3)$

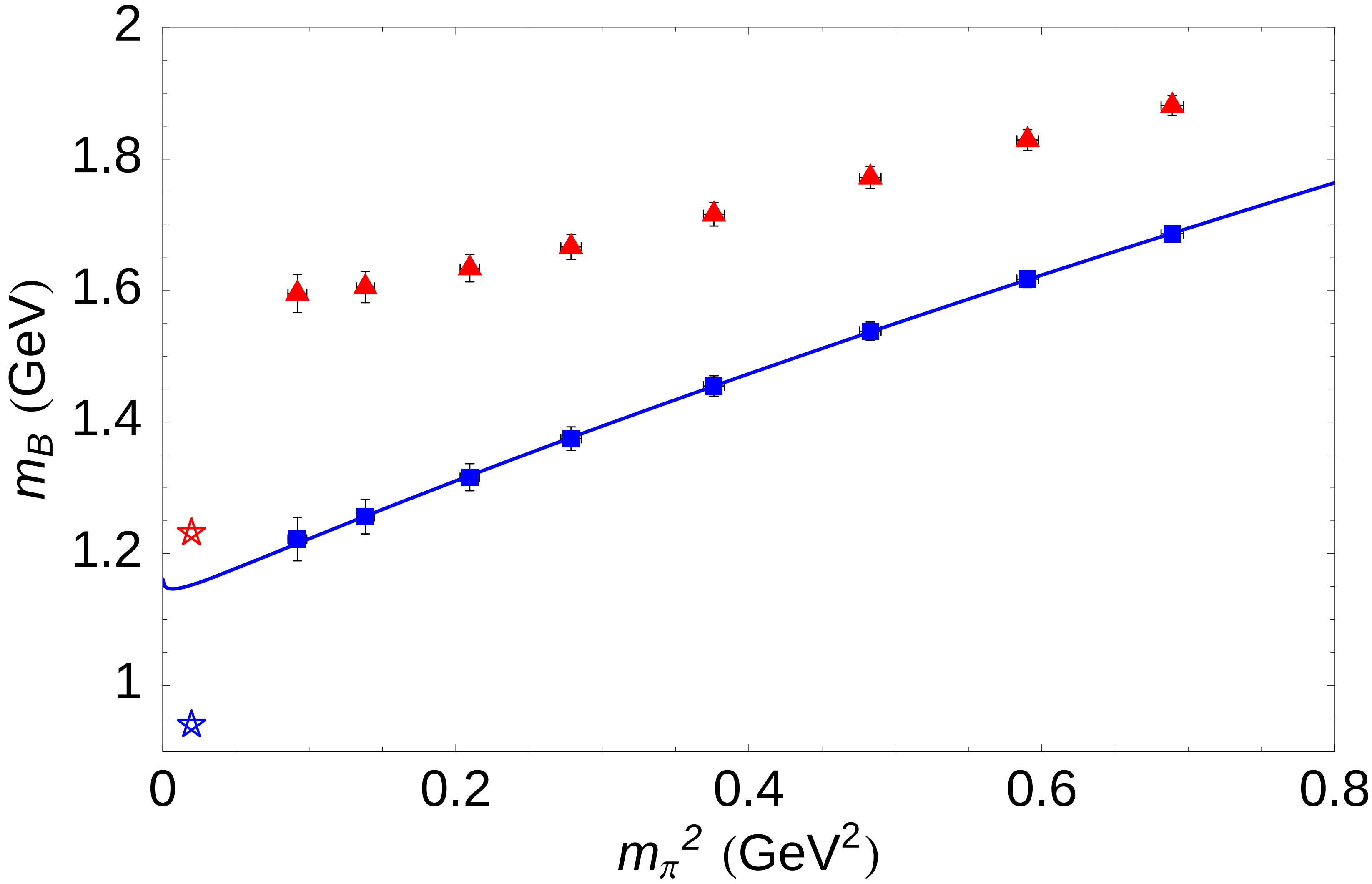
Nucleon and Delta Masses



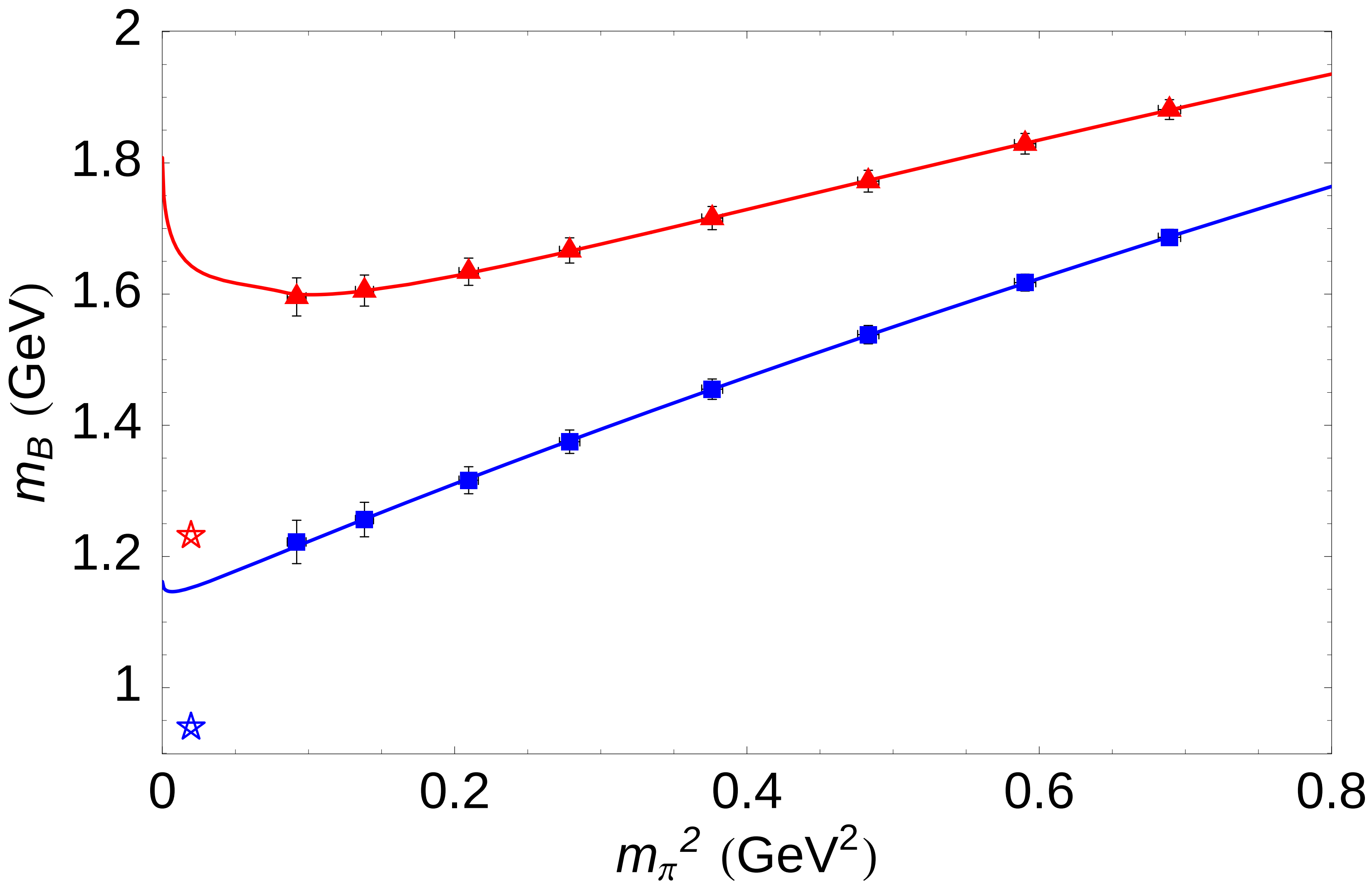
Quenched Baryon Masses



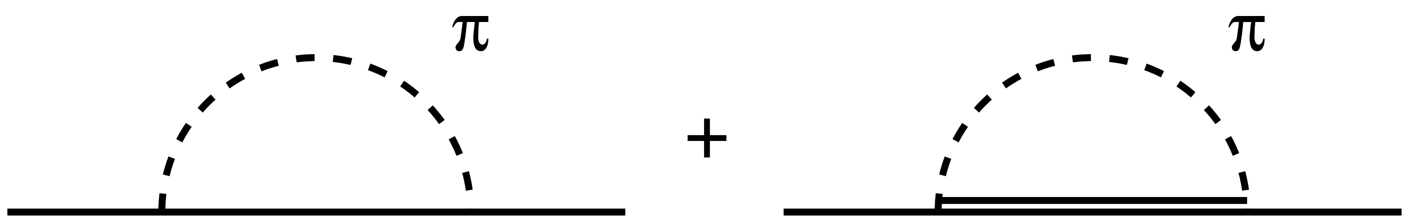
Nucleon Quenched χPT Fit



Delta Quenched χPT Fit



Correct Chiral Nonanalytic Behavior



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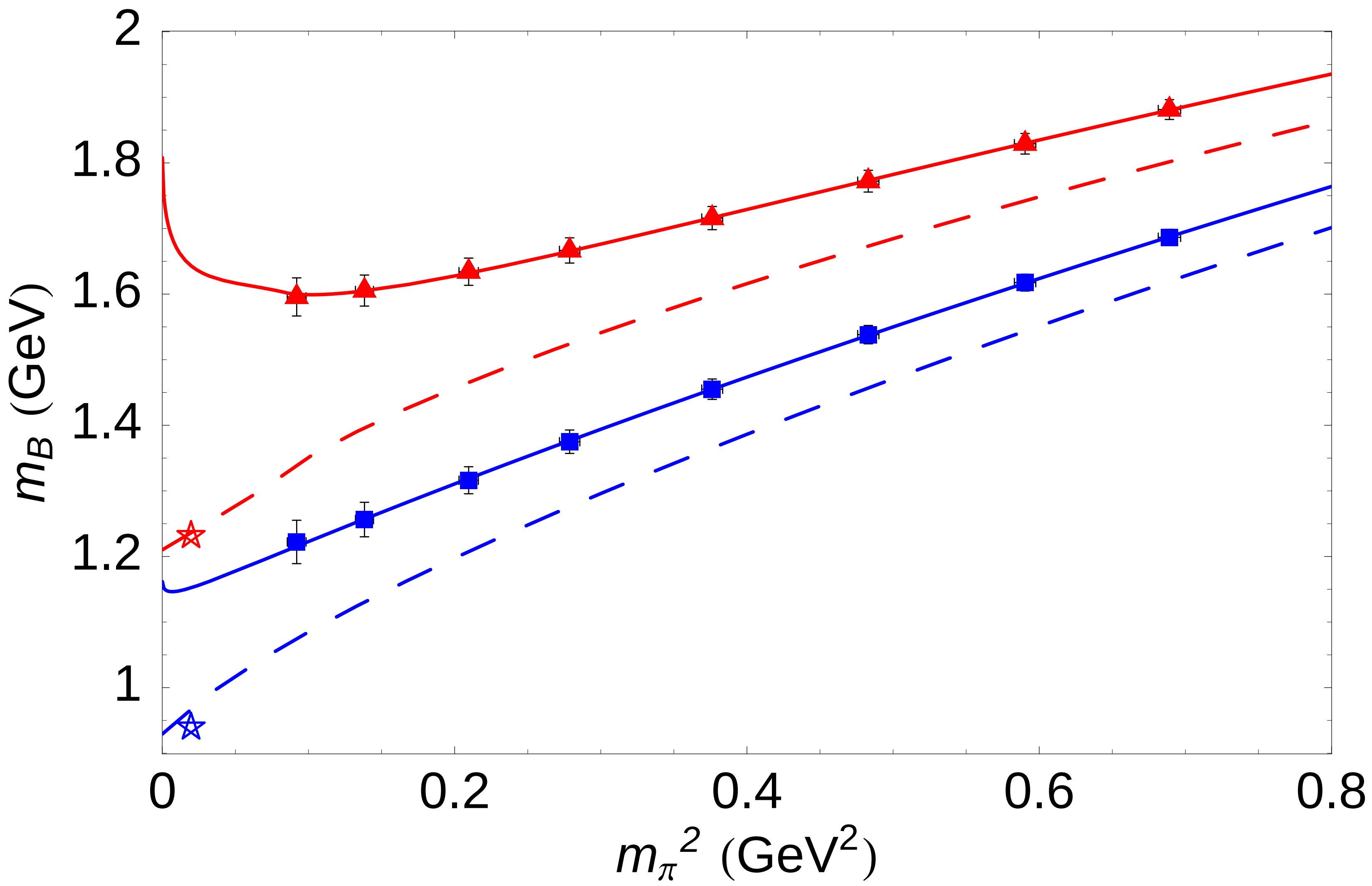
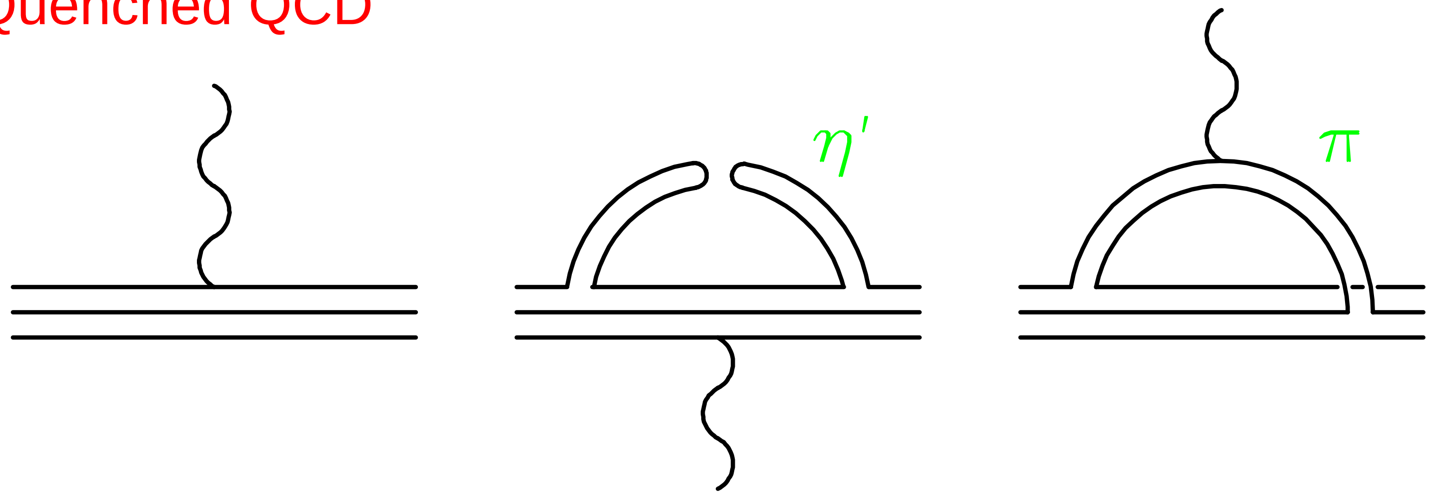


Correct Moments to Full QCD

- Quenched QCD



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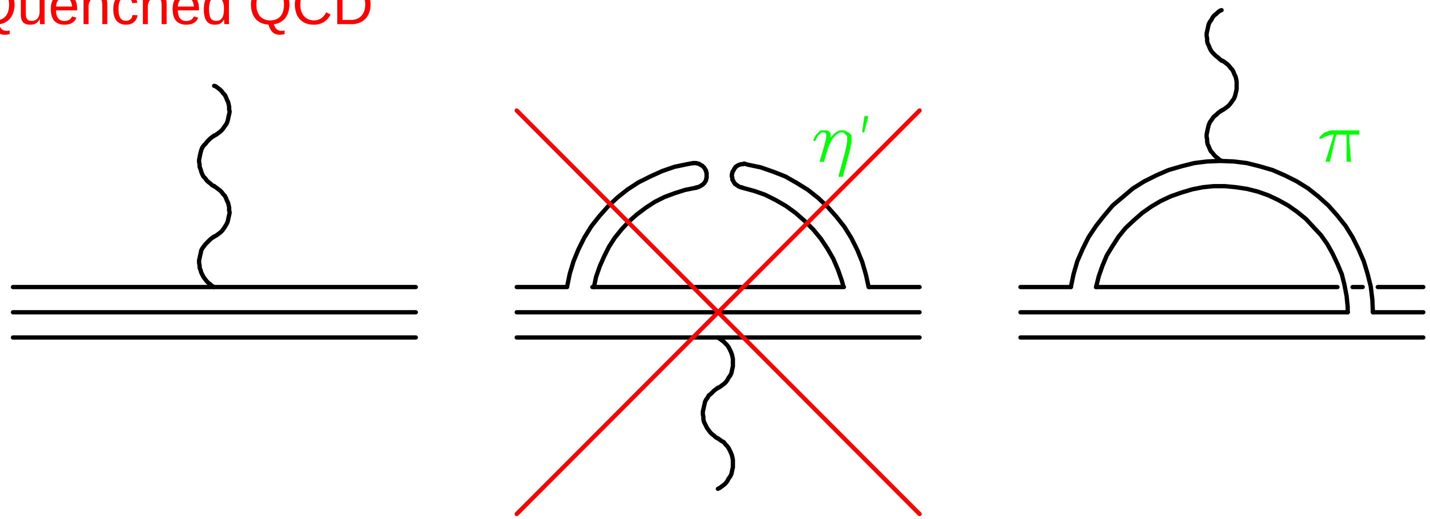

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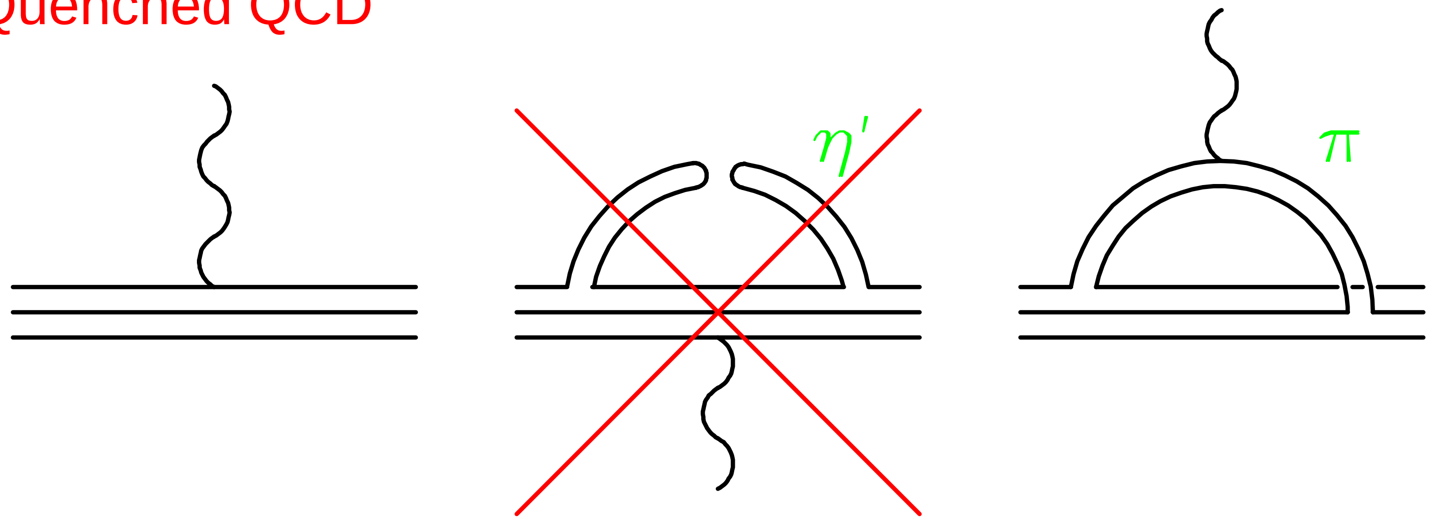

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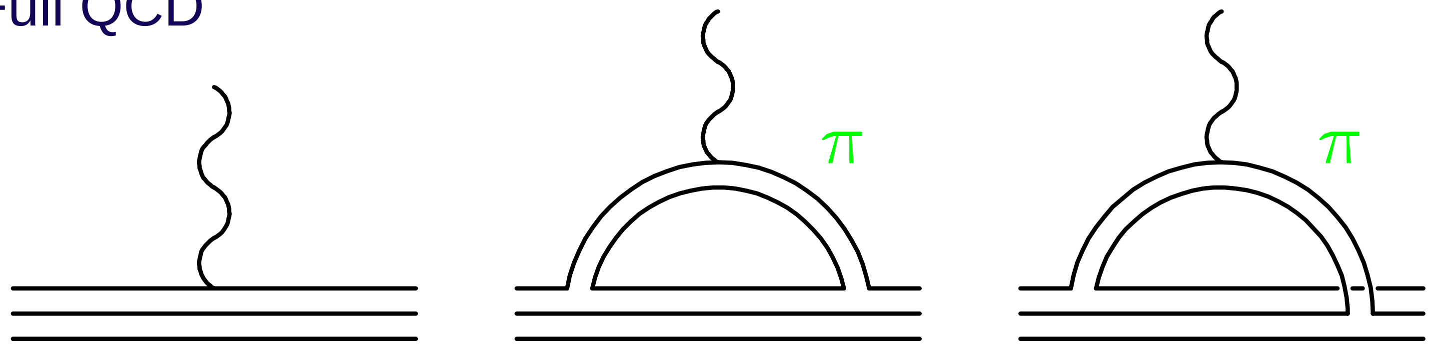

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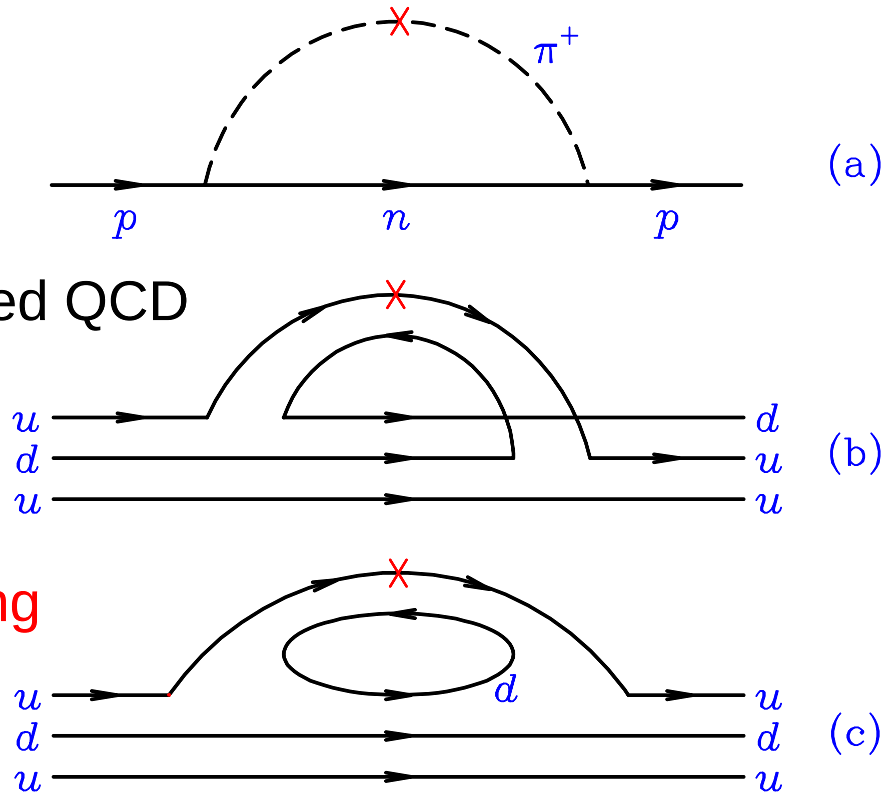

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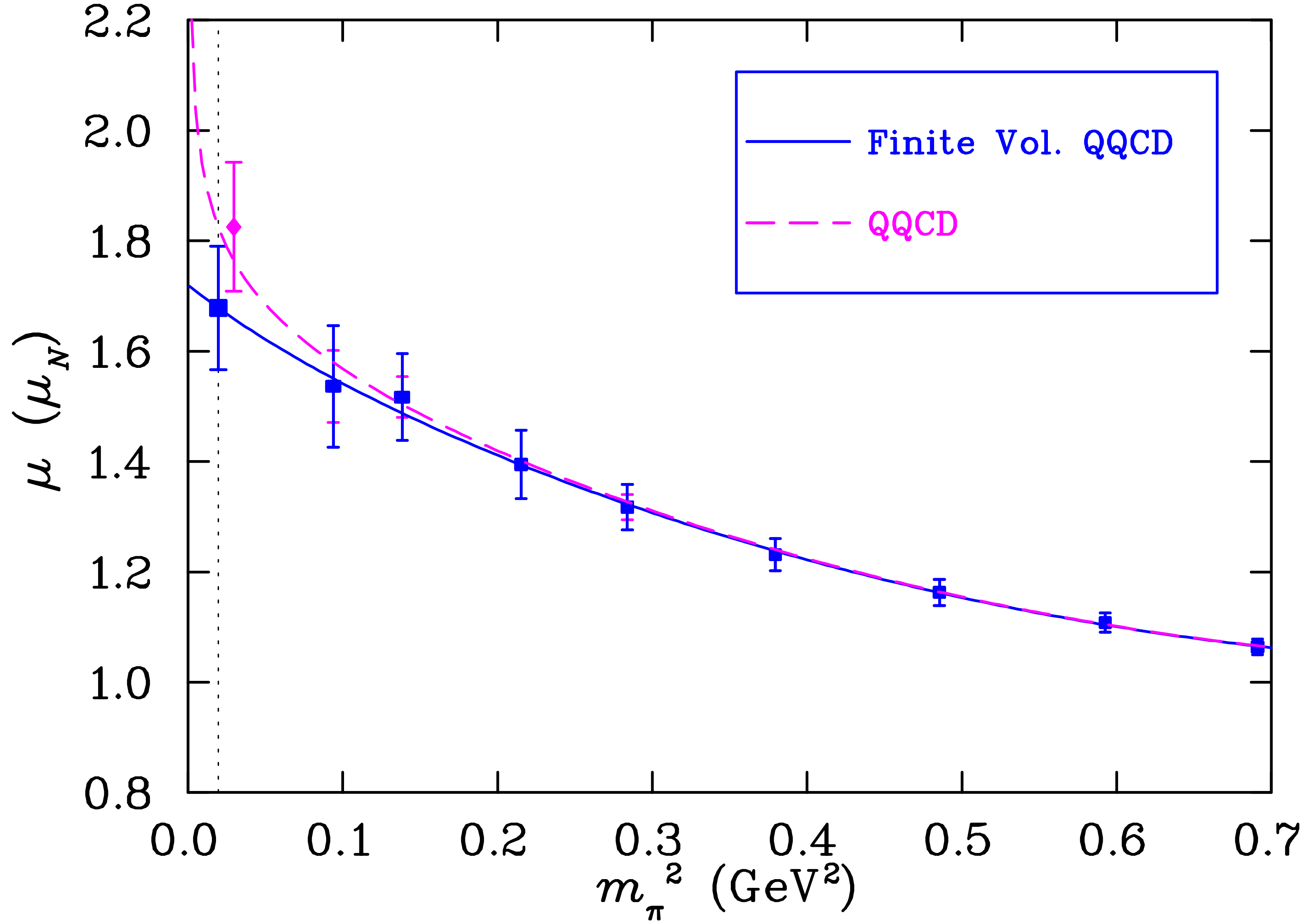
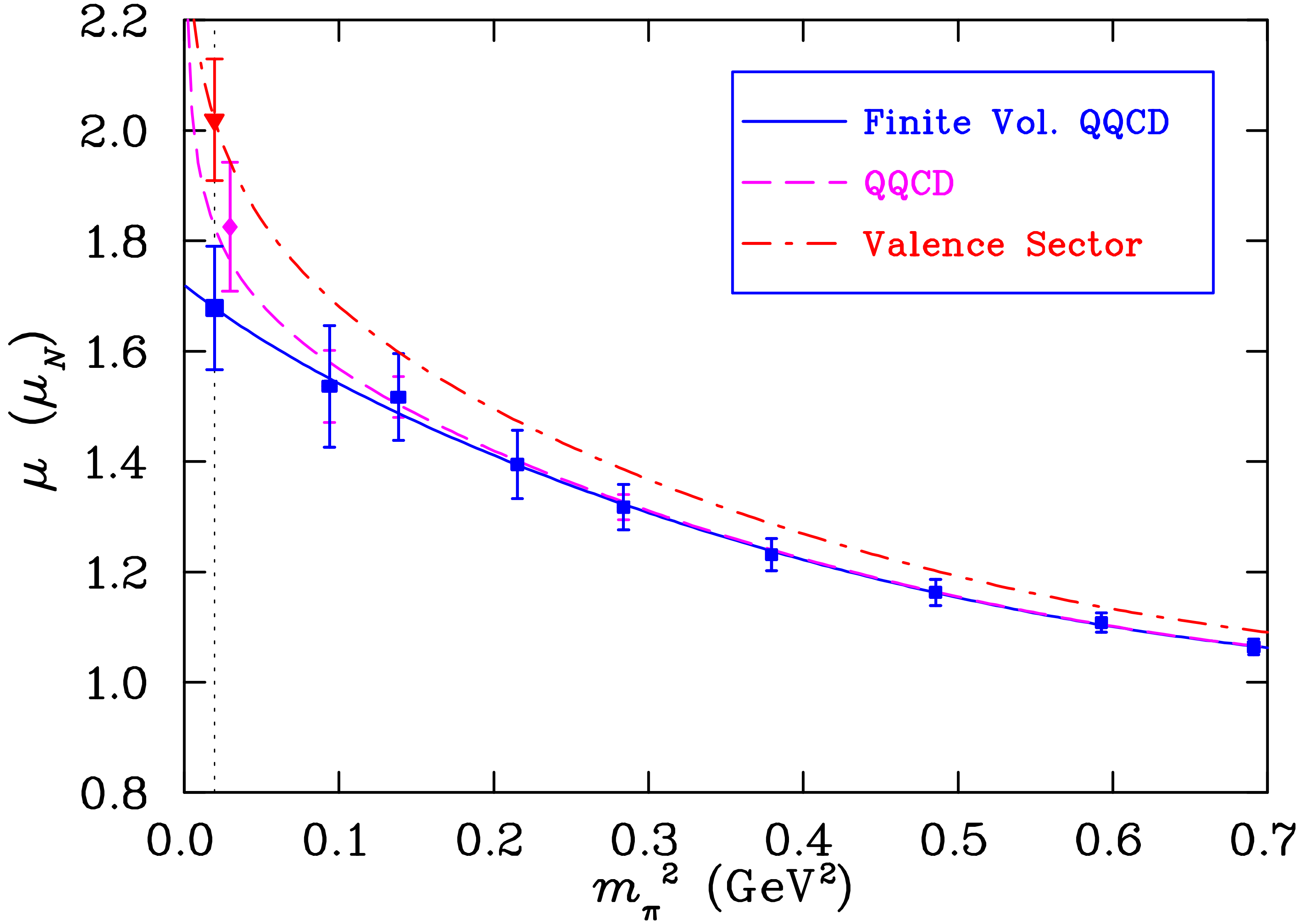
u quark in the Proton: Quenched QCD



Valence u quark in the Proton: Full QCD



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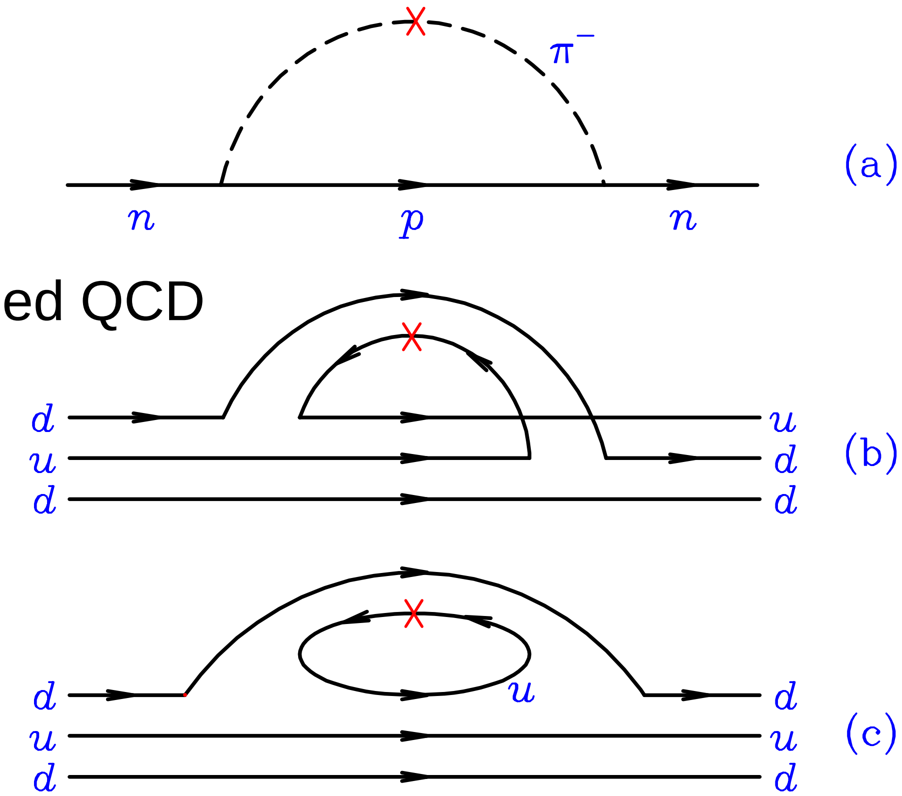

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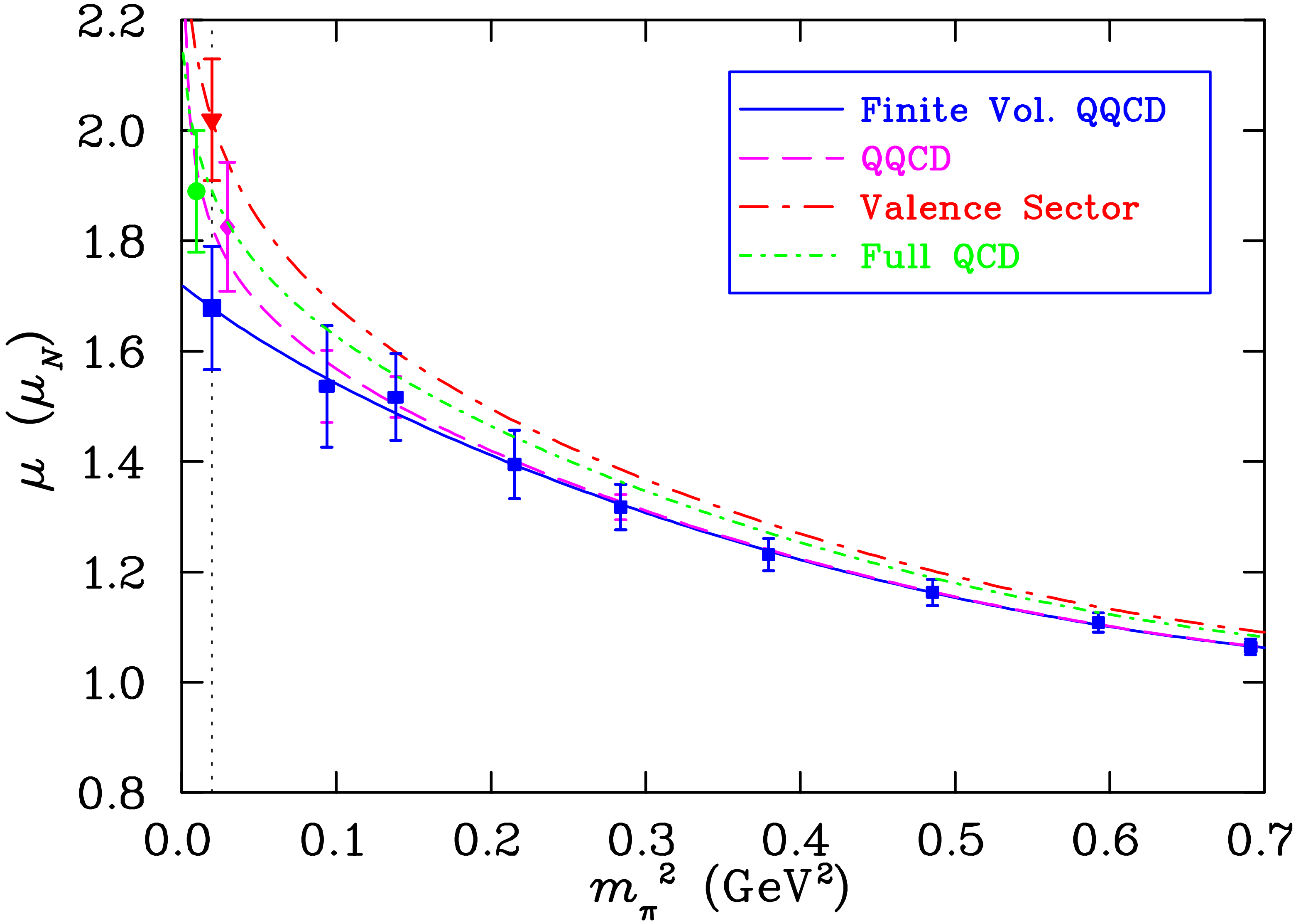
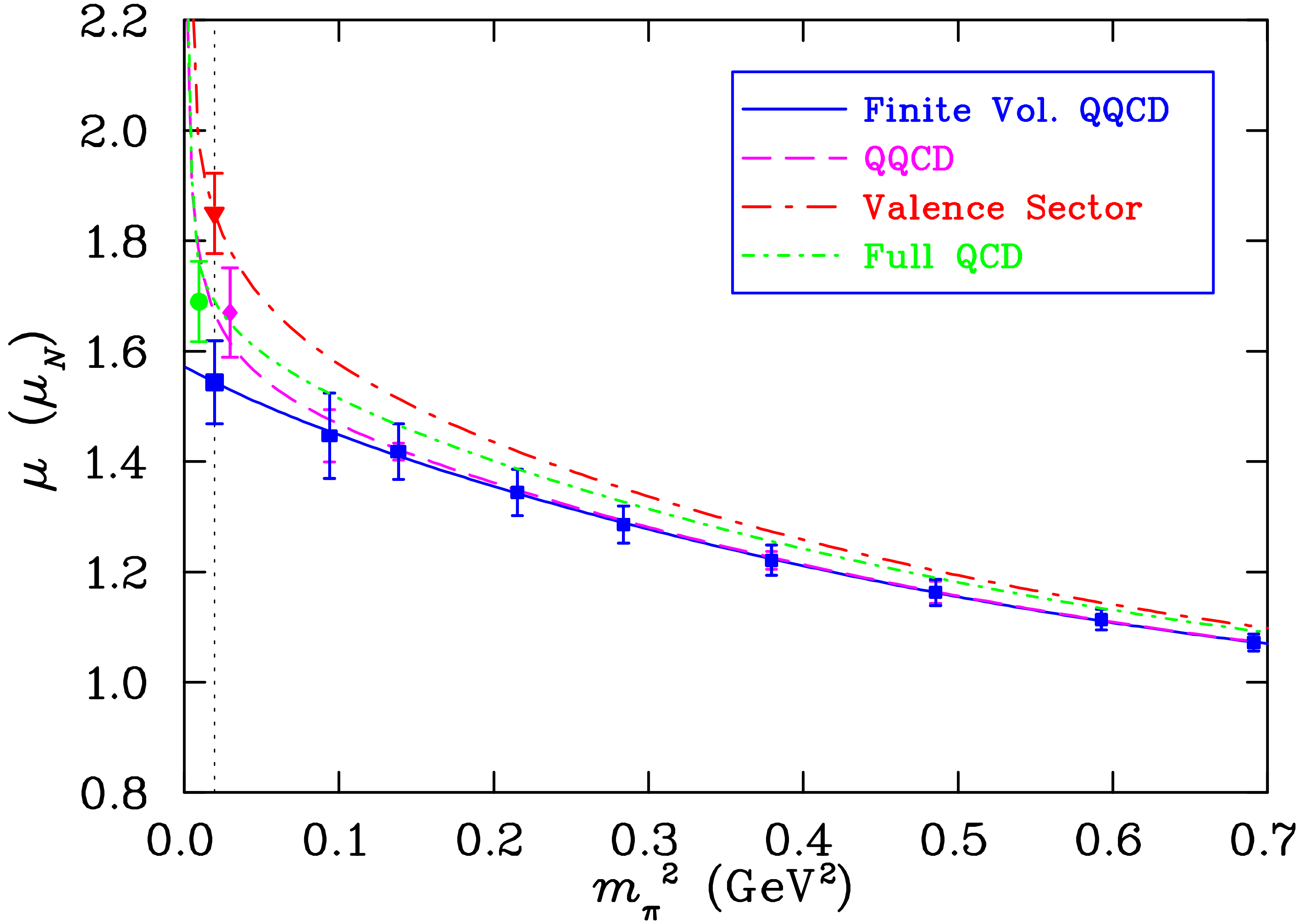
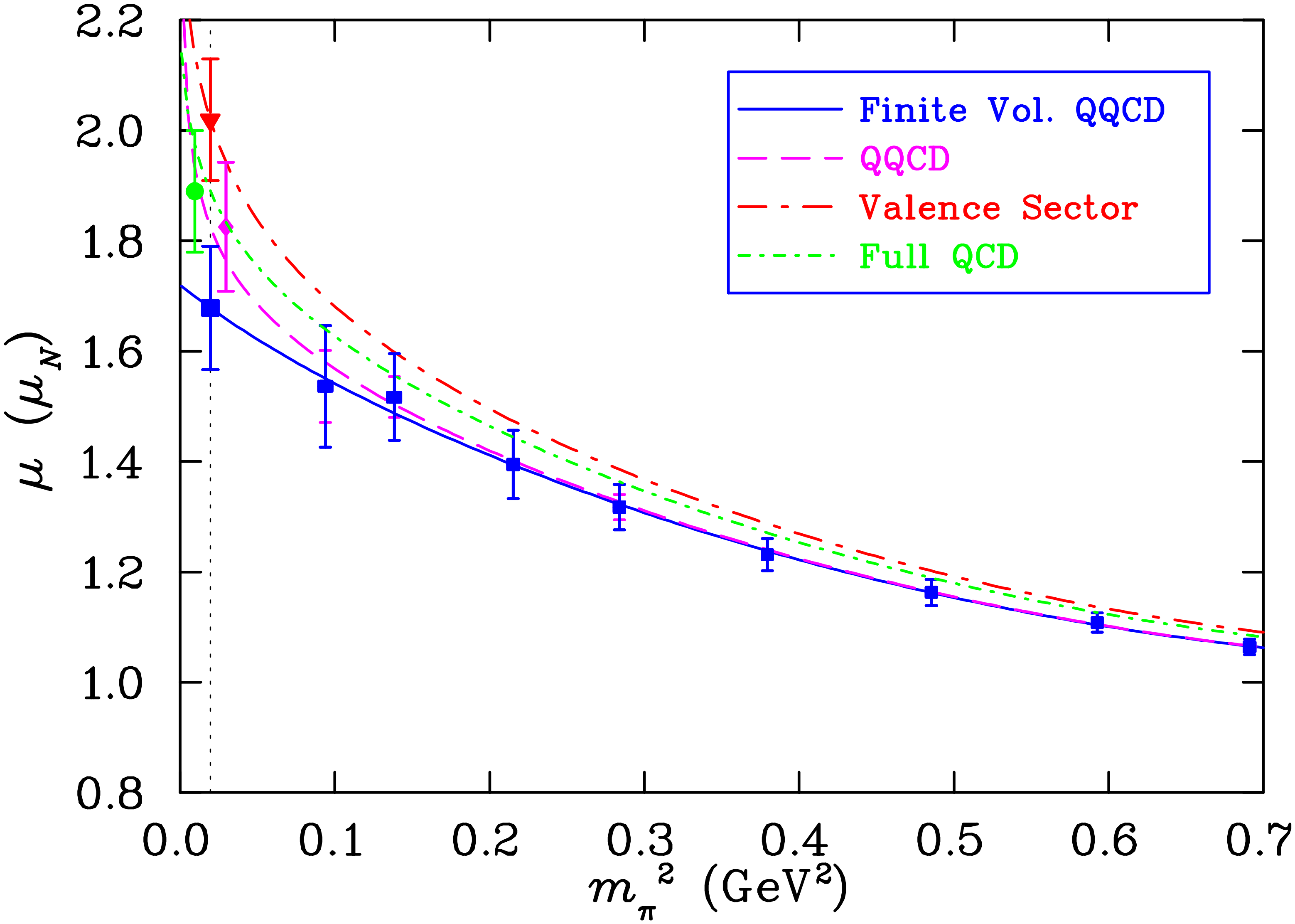
u quark in the Proton: Full QCD



u quark in Σ^+

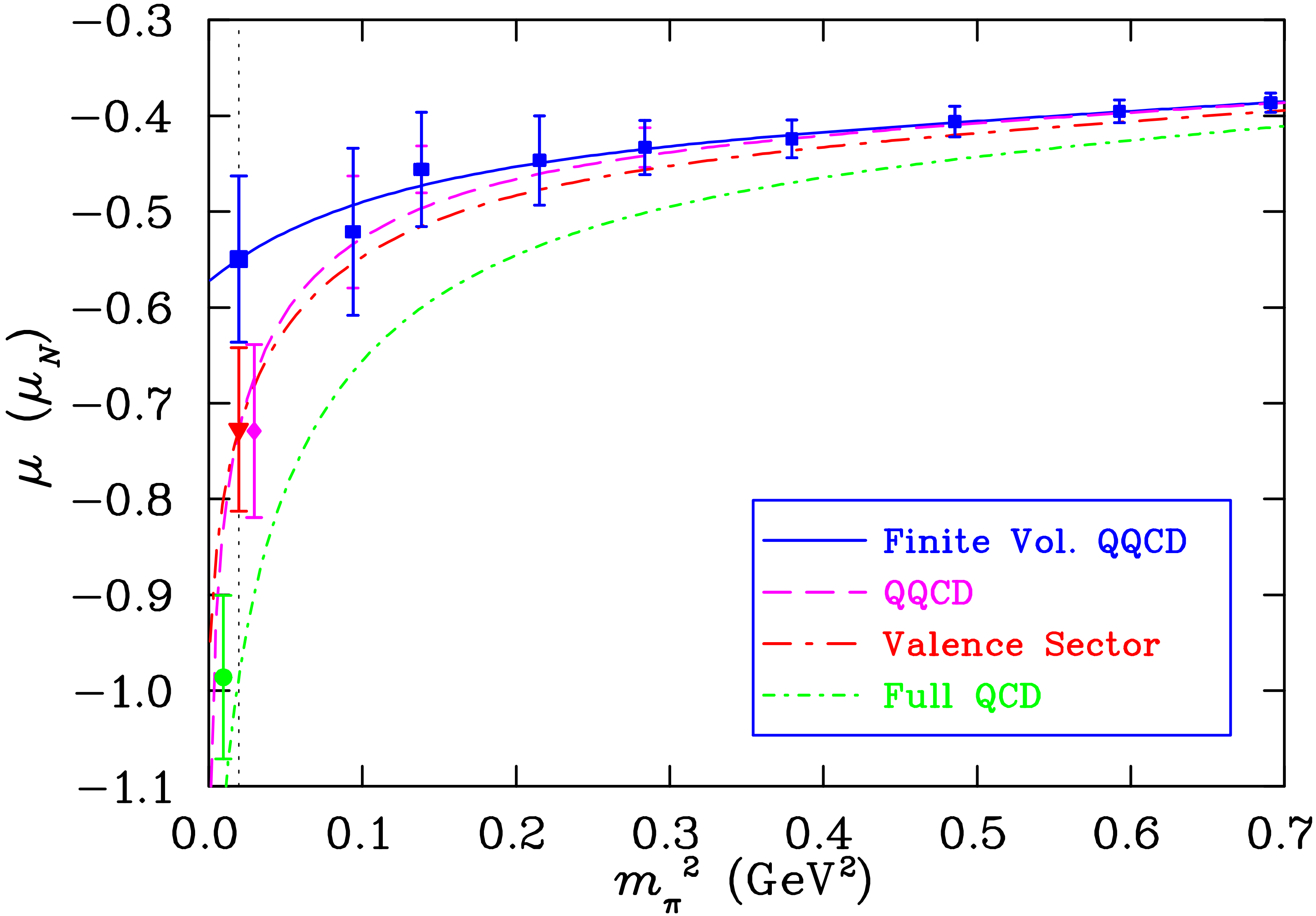


u quark in p



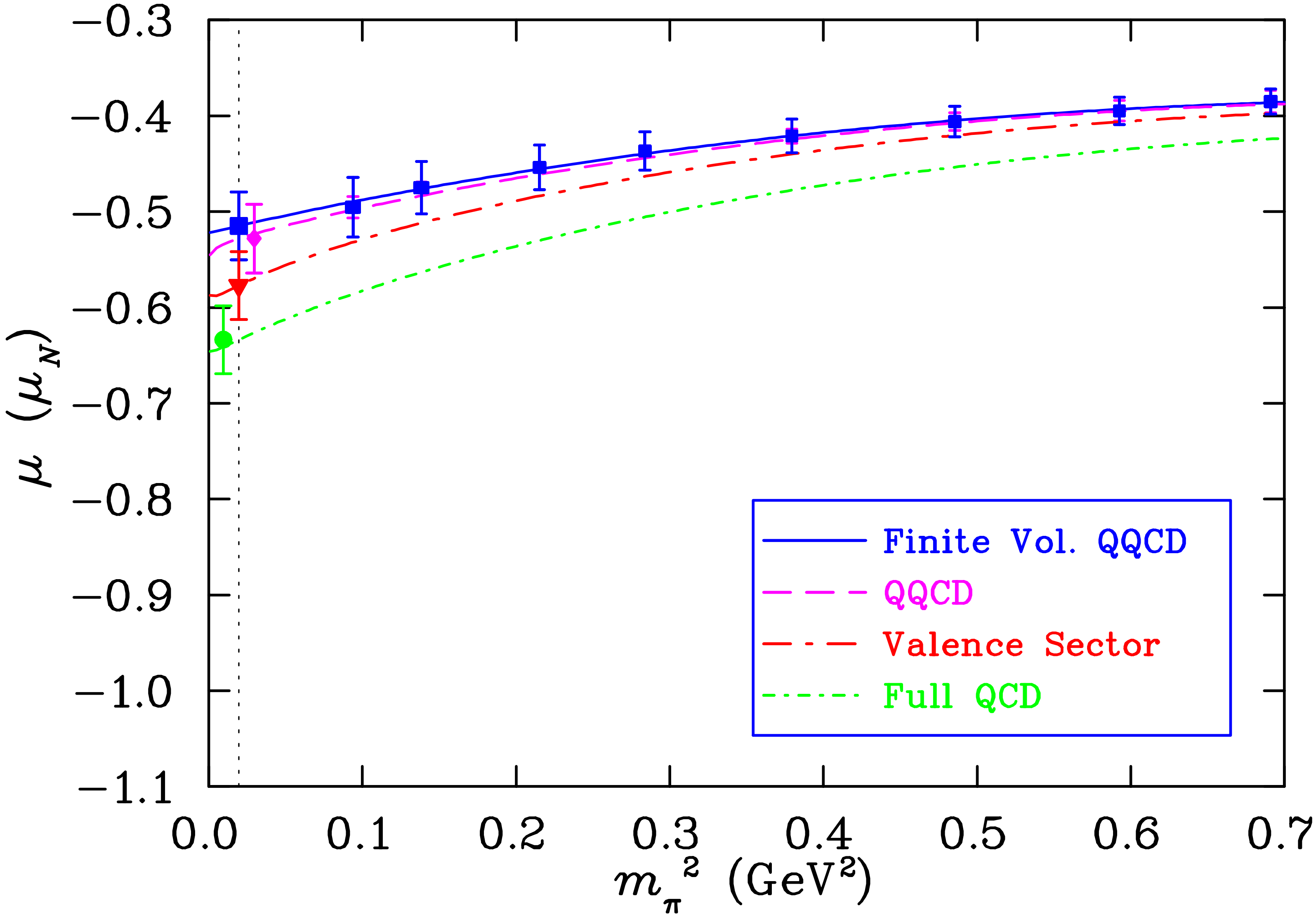
Finite Vol. QQCD
QQCD
Valence Sector
Full QCD
μ (μ_N)
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
m_π² (GeV²)

u quark in n

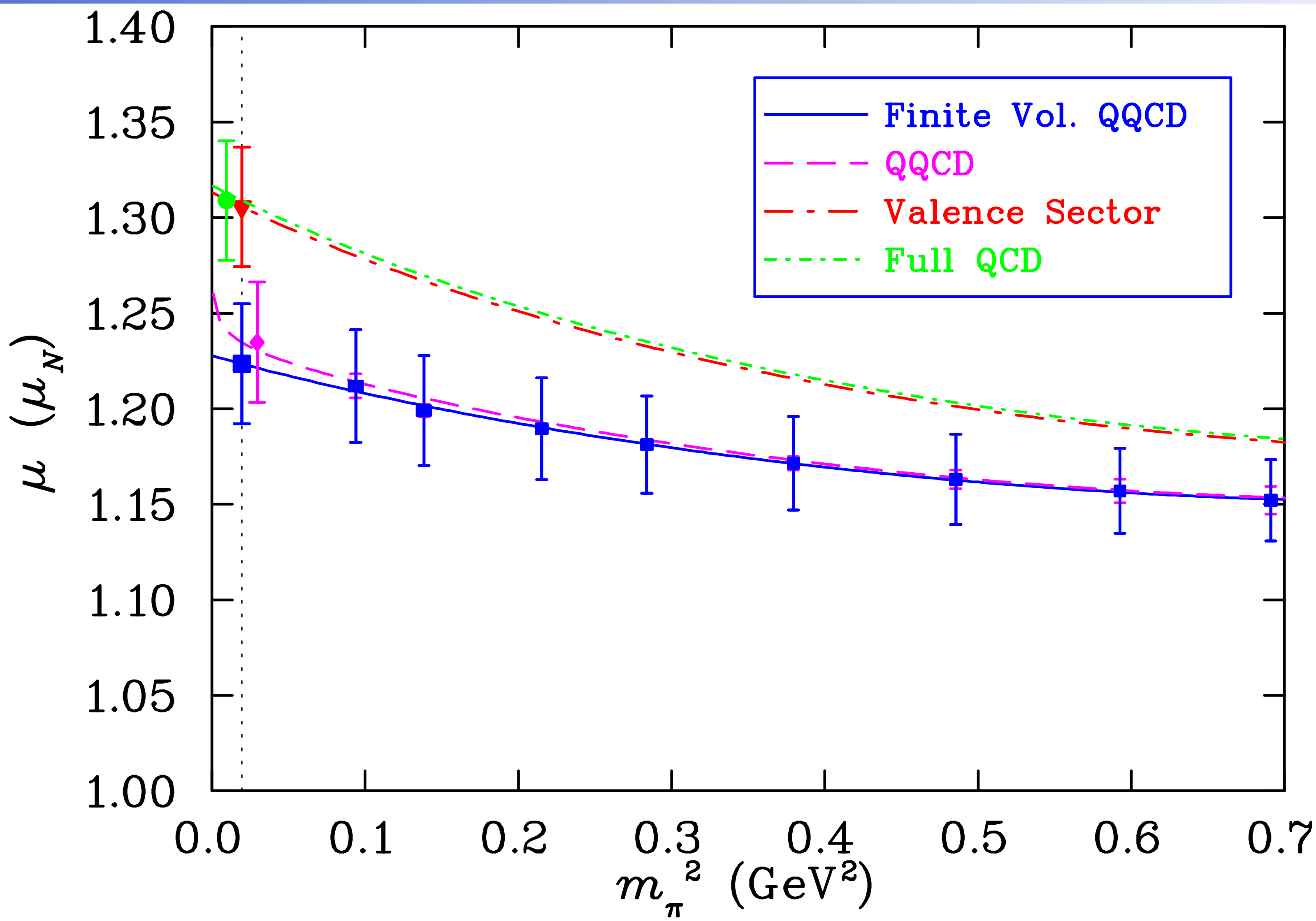


−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
−1.0
−1.1
μ (μN)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
mπ² (GeV²)
Finite Vol. QQCD
QQCD
Valence Sector
Full QCD

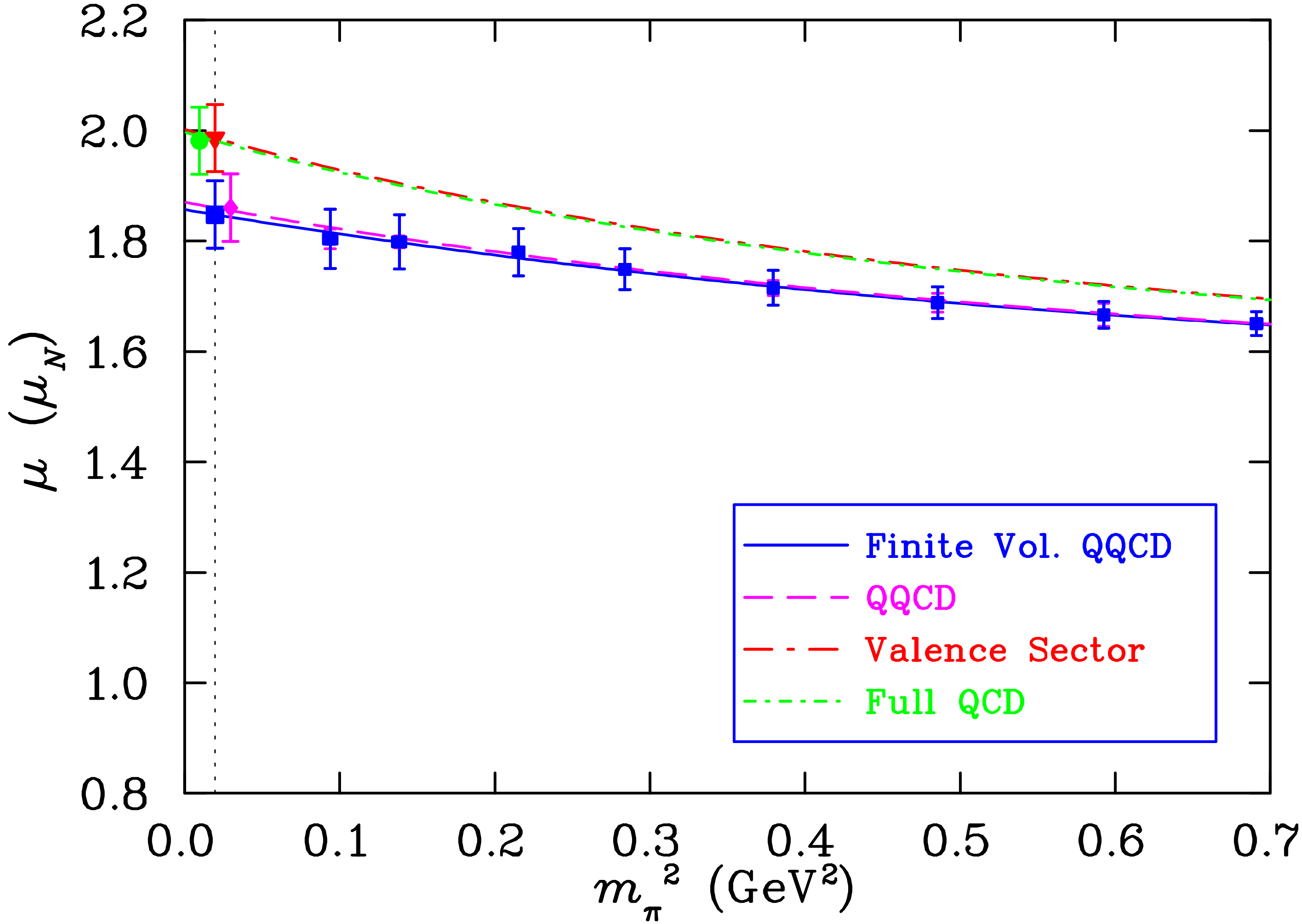
u quark in Ξ^0



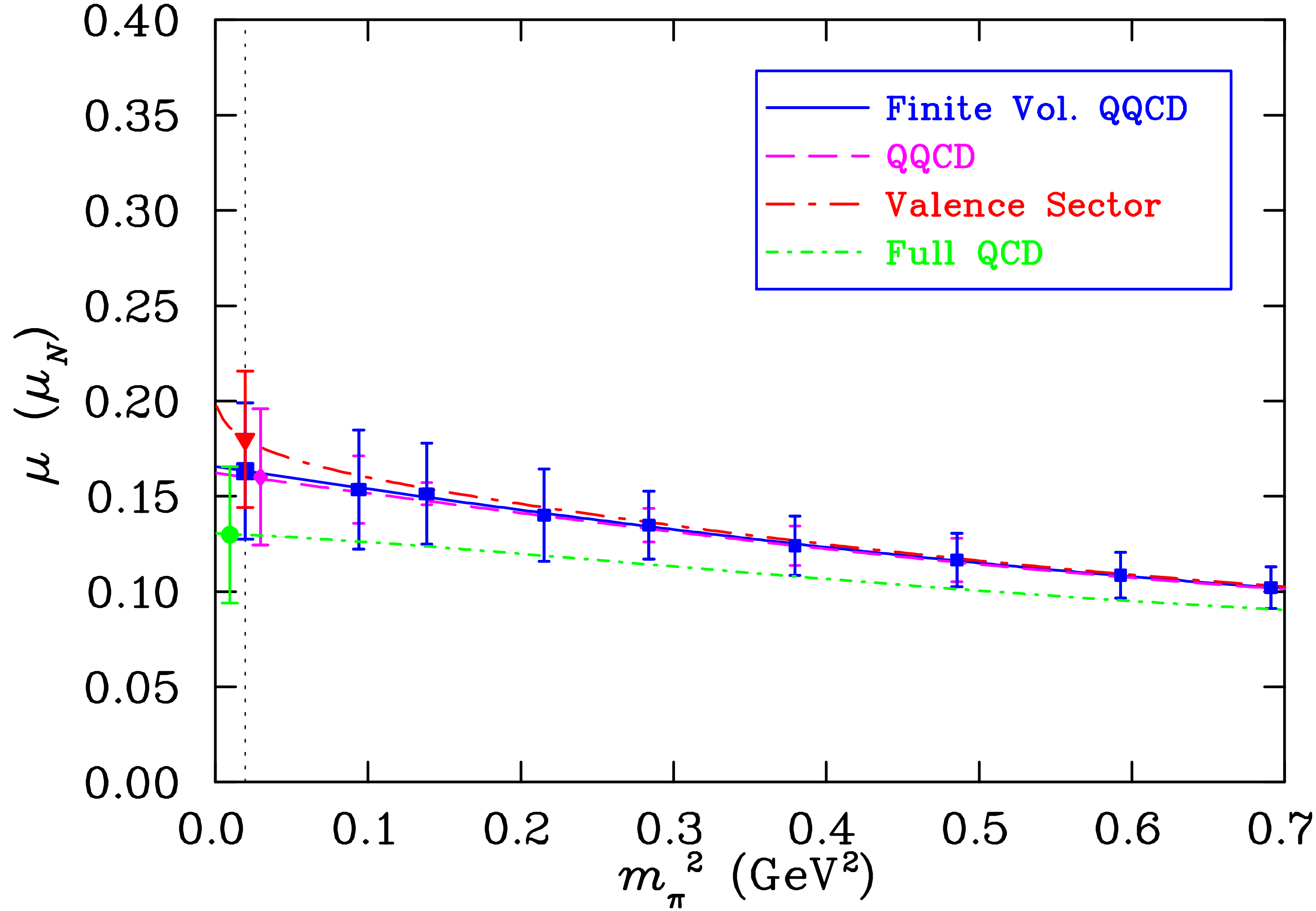
s quark in Ξ^0



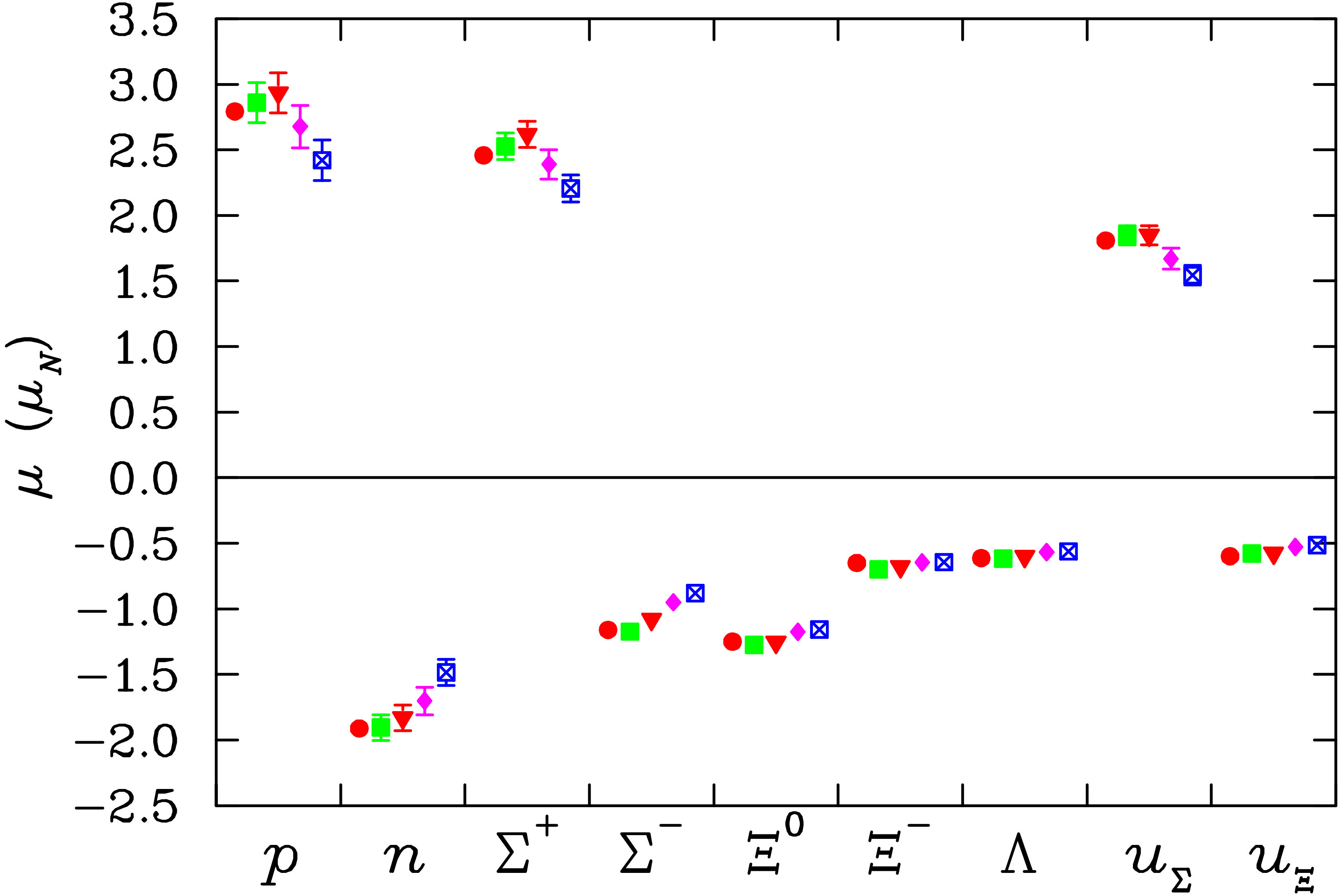
s quark in Λ



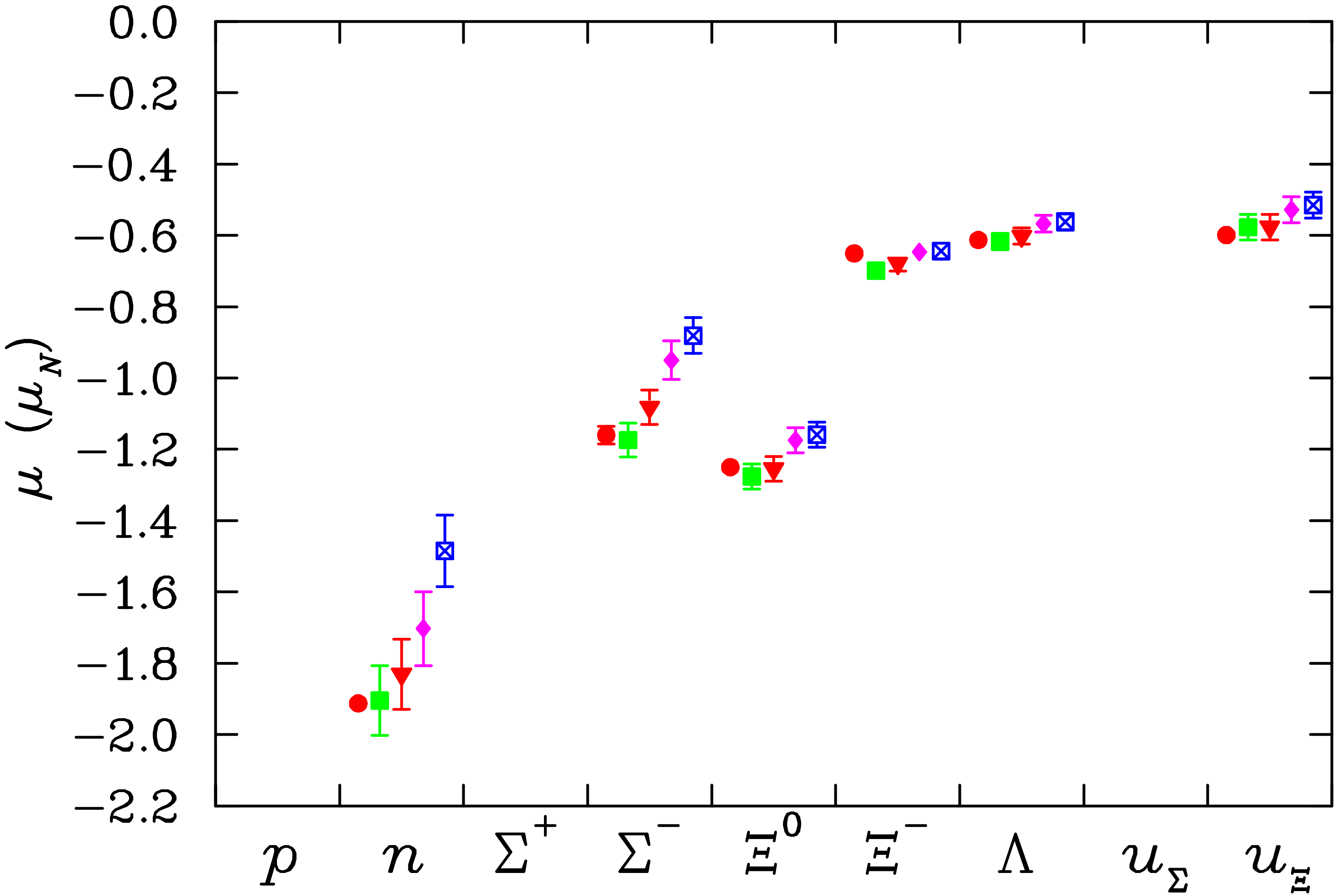
u or d quark in Λ



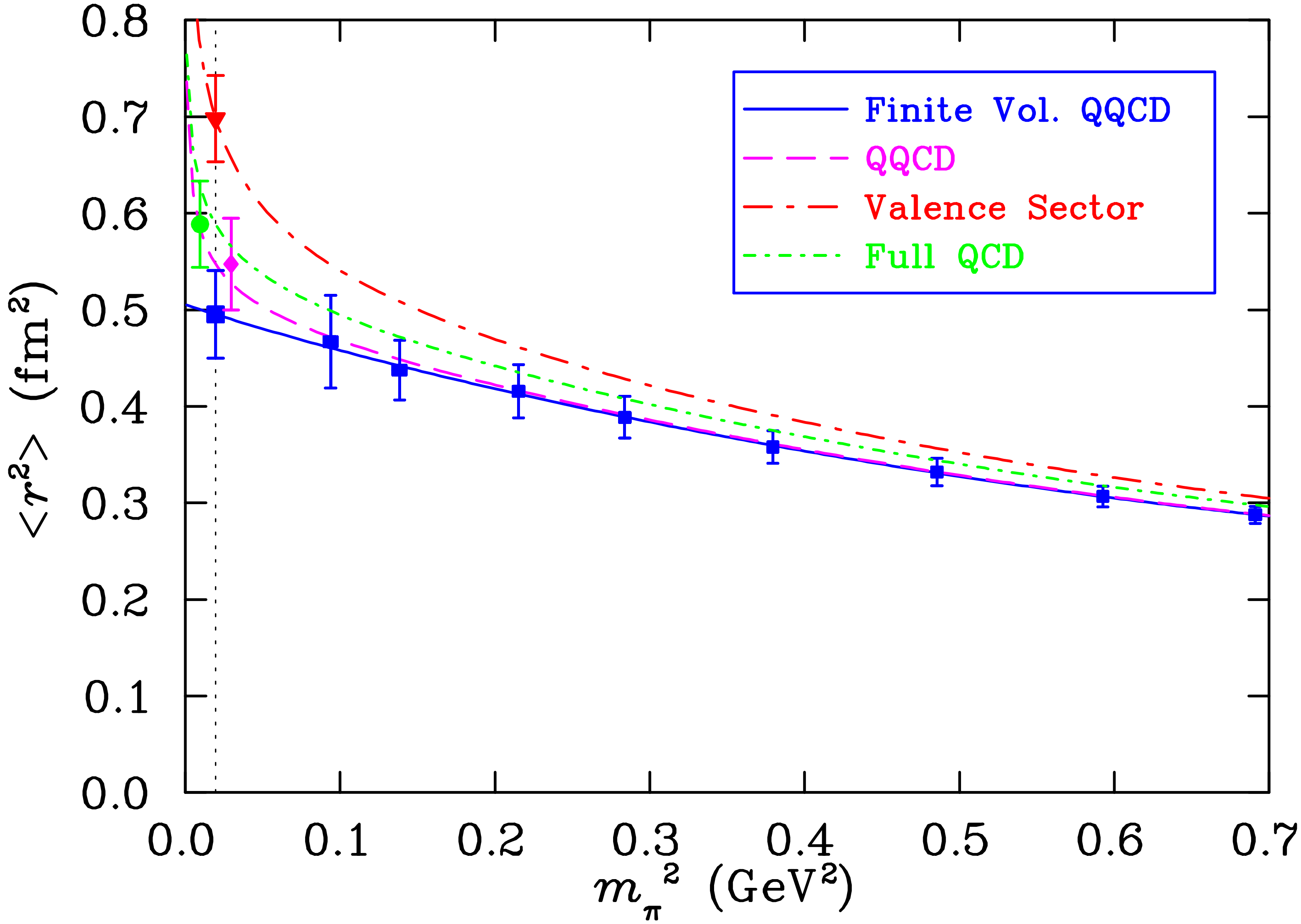
Octet Baryon Magnetic Moments



Octet Baryon Magnetic Moments

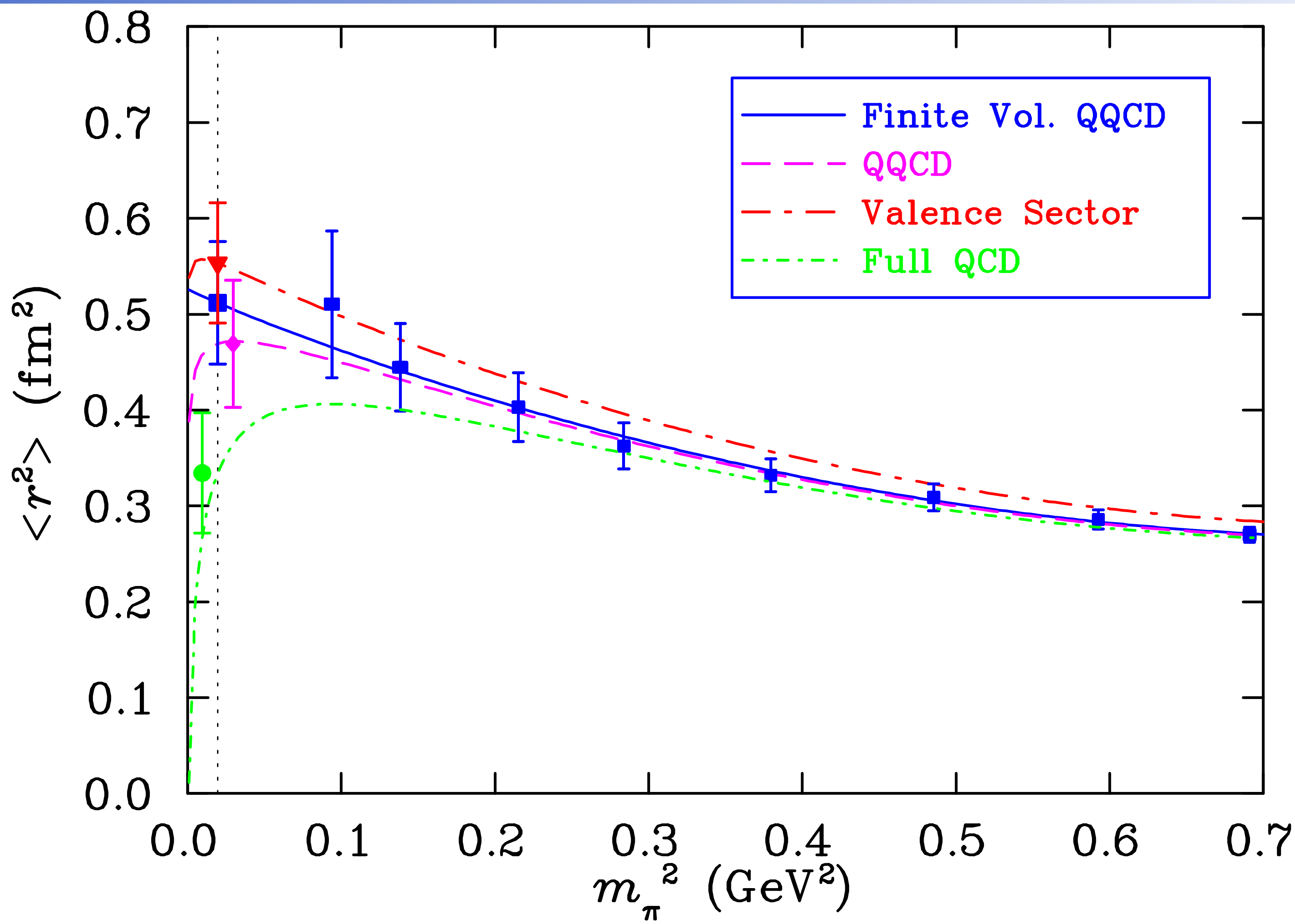


u quark in p



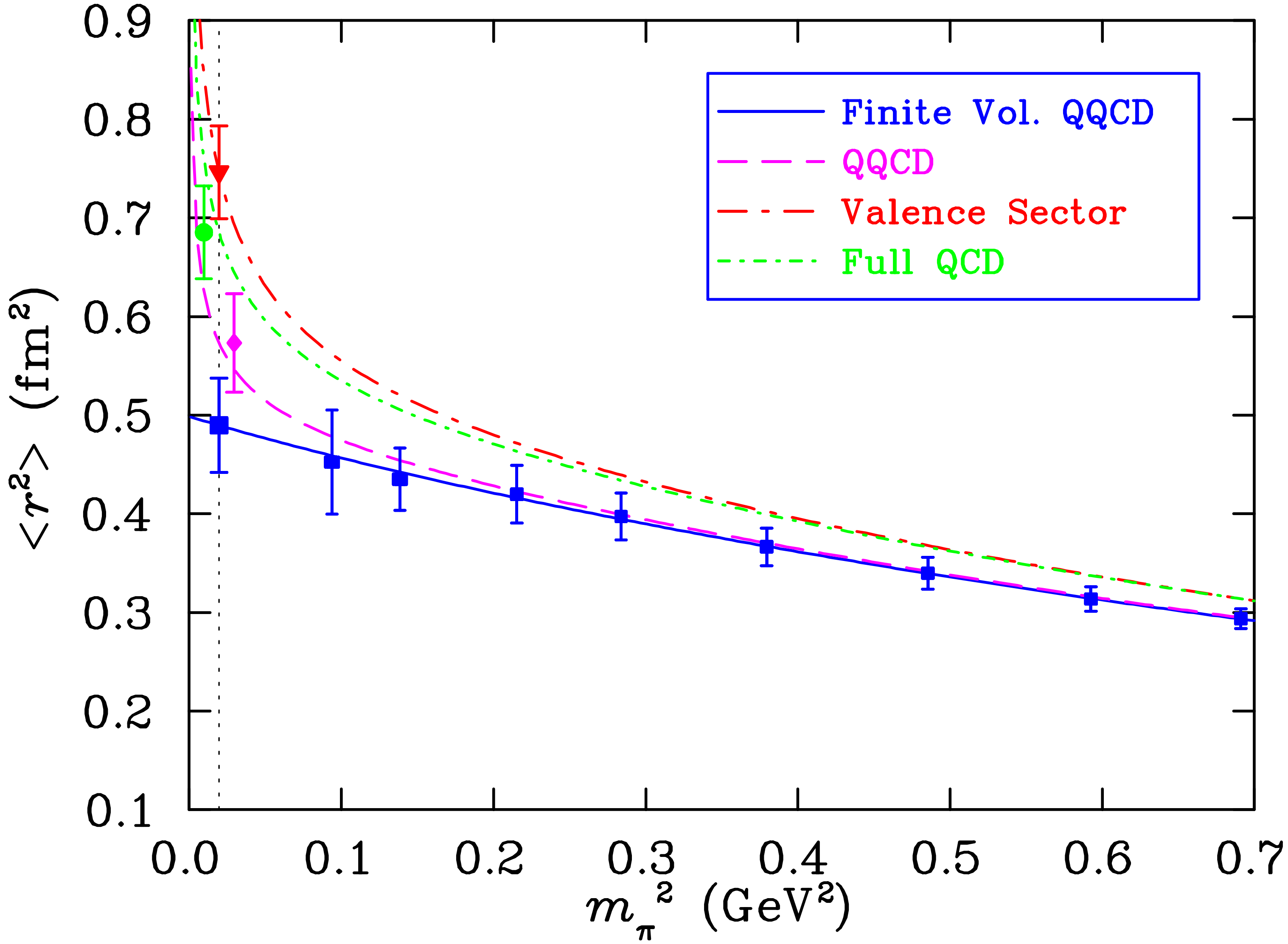
Finite Vol. QQCD
QQCD
Valence Sector
Full QCD
$\langle r^2 \rangle$ (fm^2)
$m_\pi{}^2$ (GeV^2)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

u quark in n

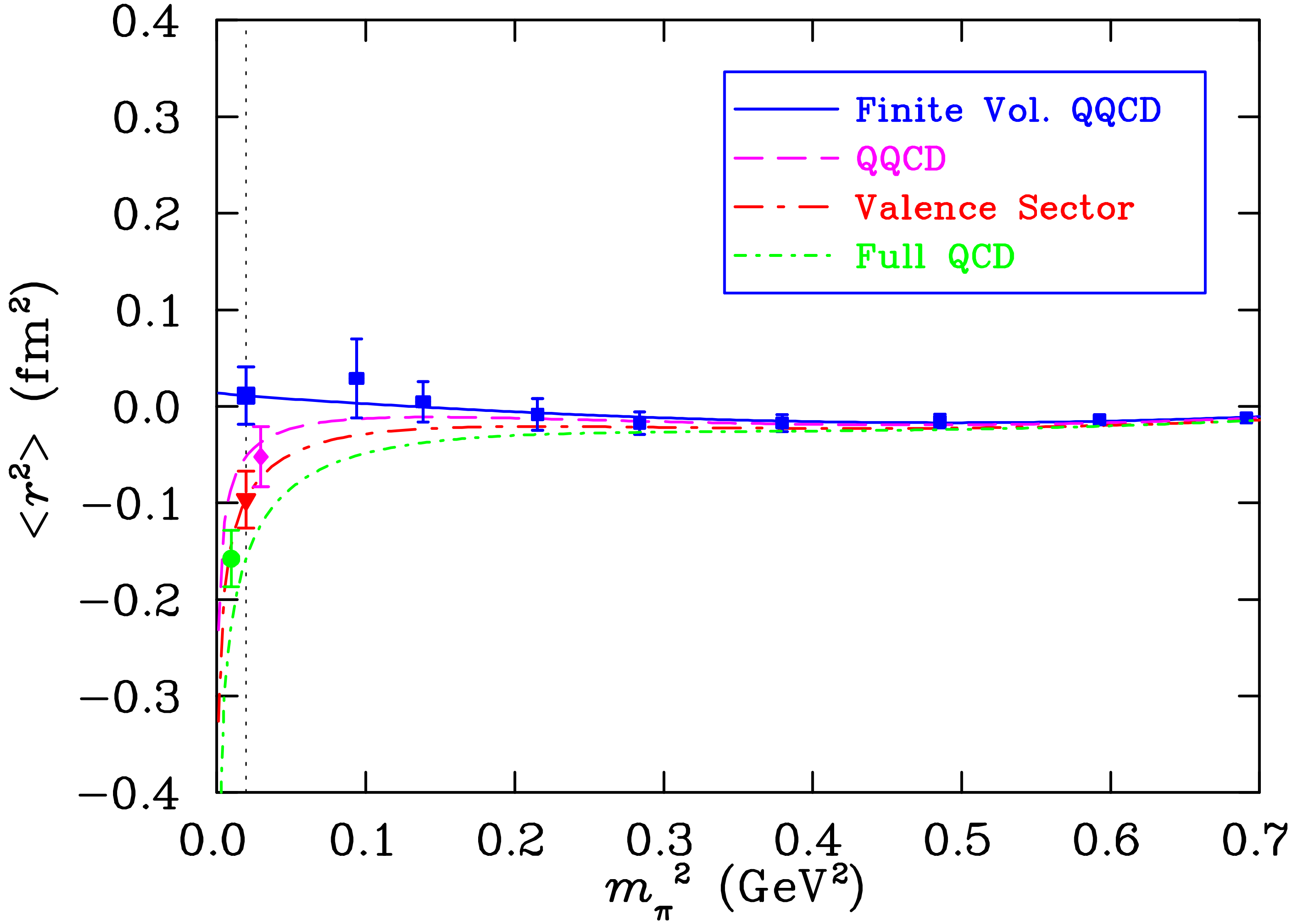


Finite Vol. QQCD
QQCD
Valence Sector
Full QCD
$<r^2>$ (fm^2)
m_π^2 (GeV^2)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

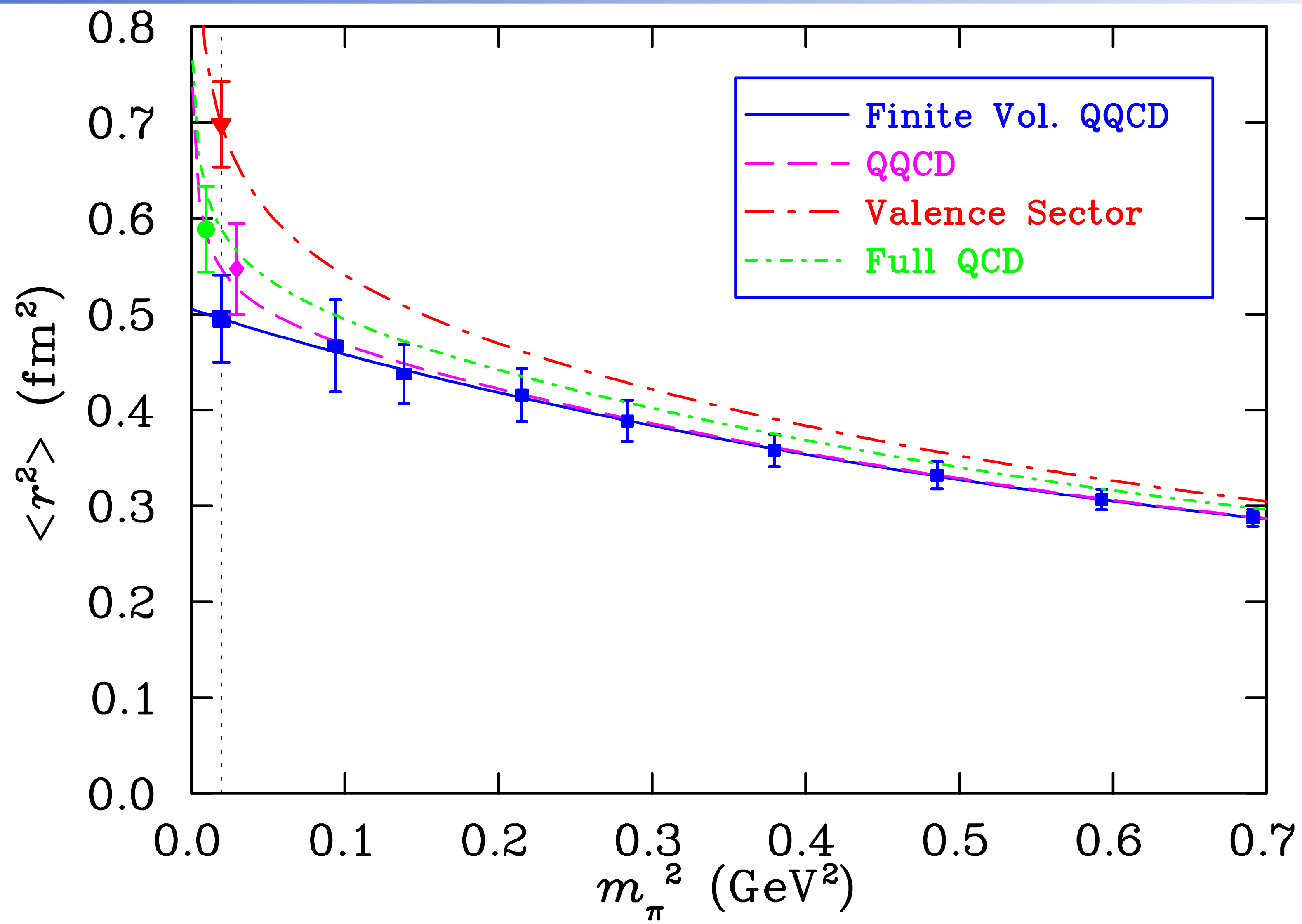
Proton Charge Radius



Neutron Charge Radius

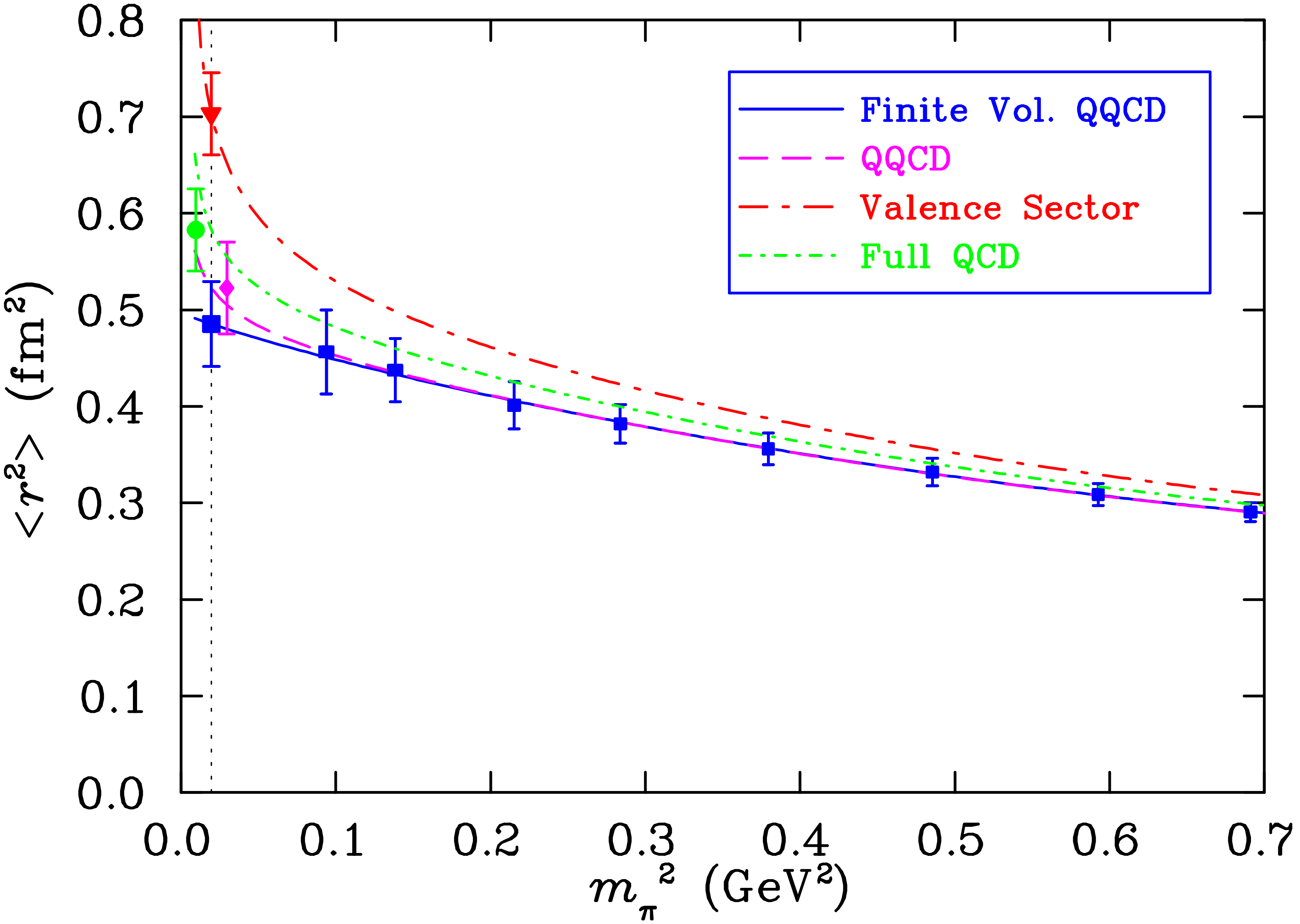


u quark in p

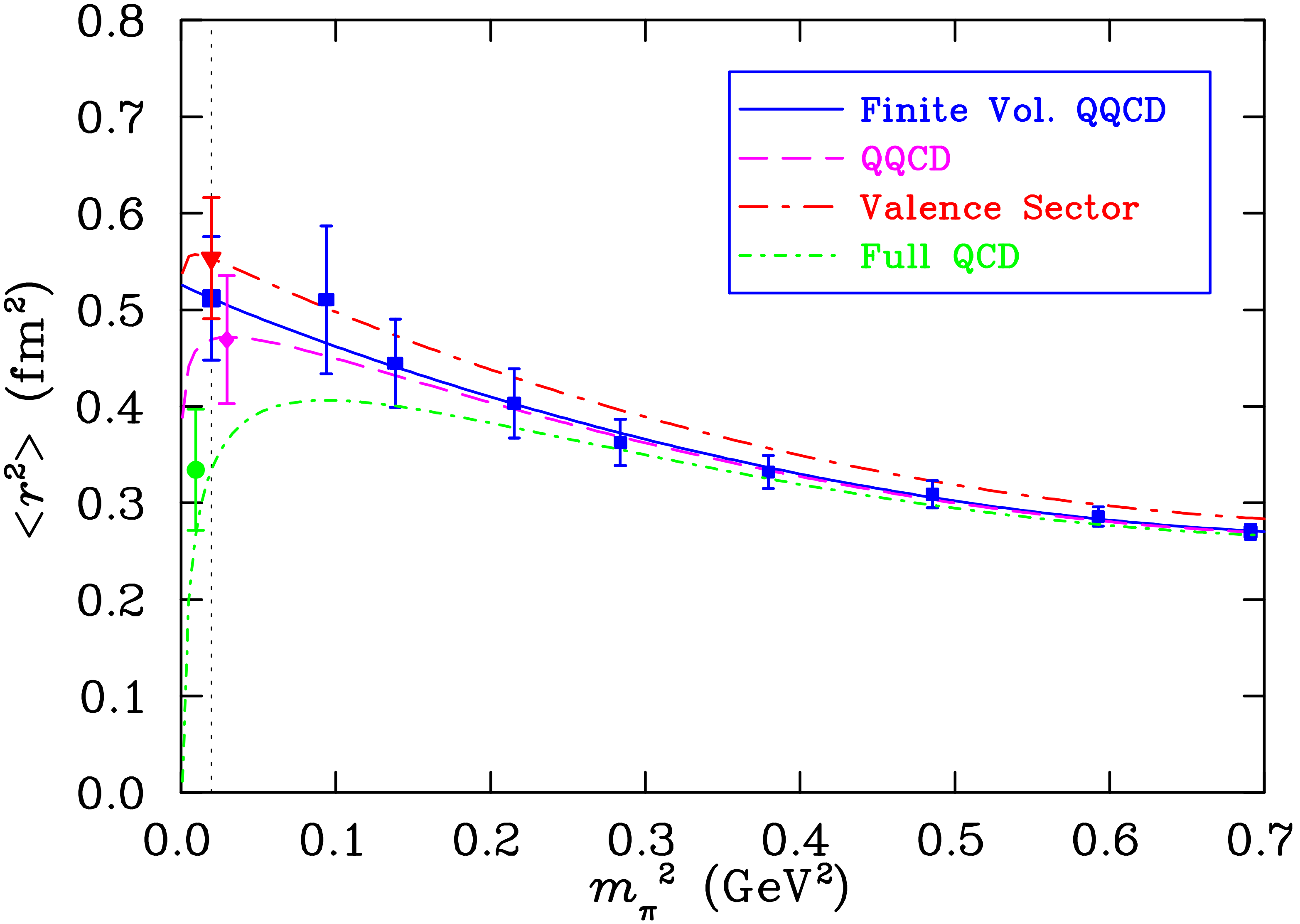


Finite Vol. QQCD
QQCD
Valence Sector
Full QCD
$\langle r^2 \rangle$ (fm^2)
m_π^2 (GeV^2)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

u quark in Σ^+

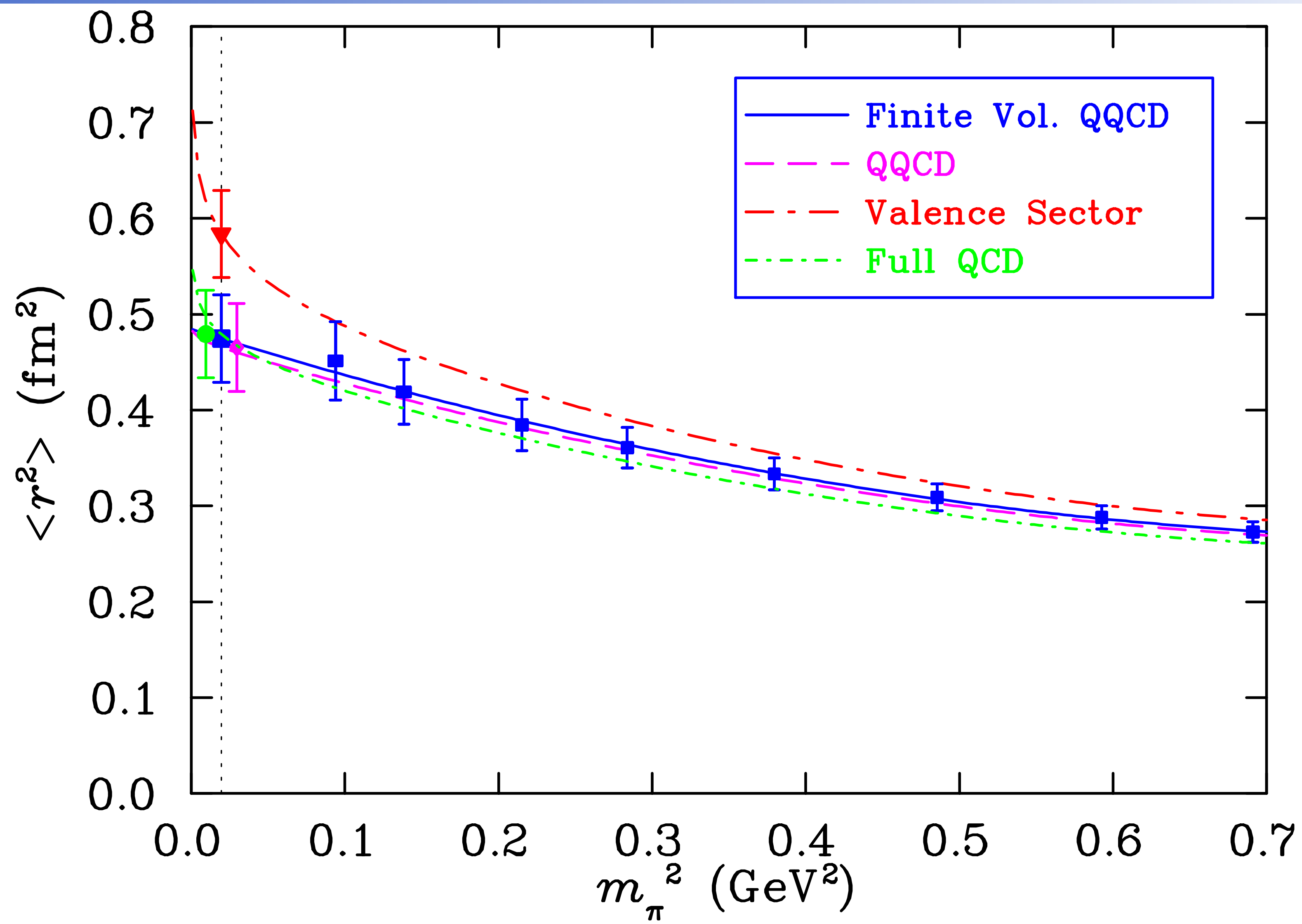


u quark in n

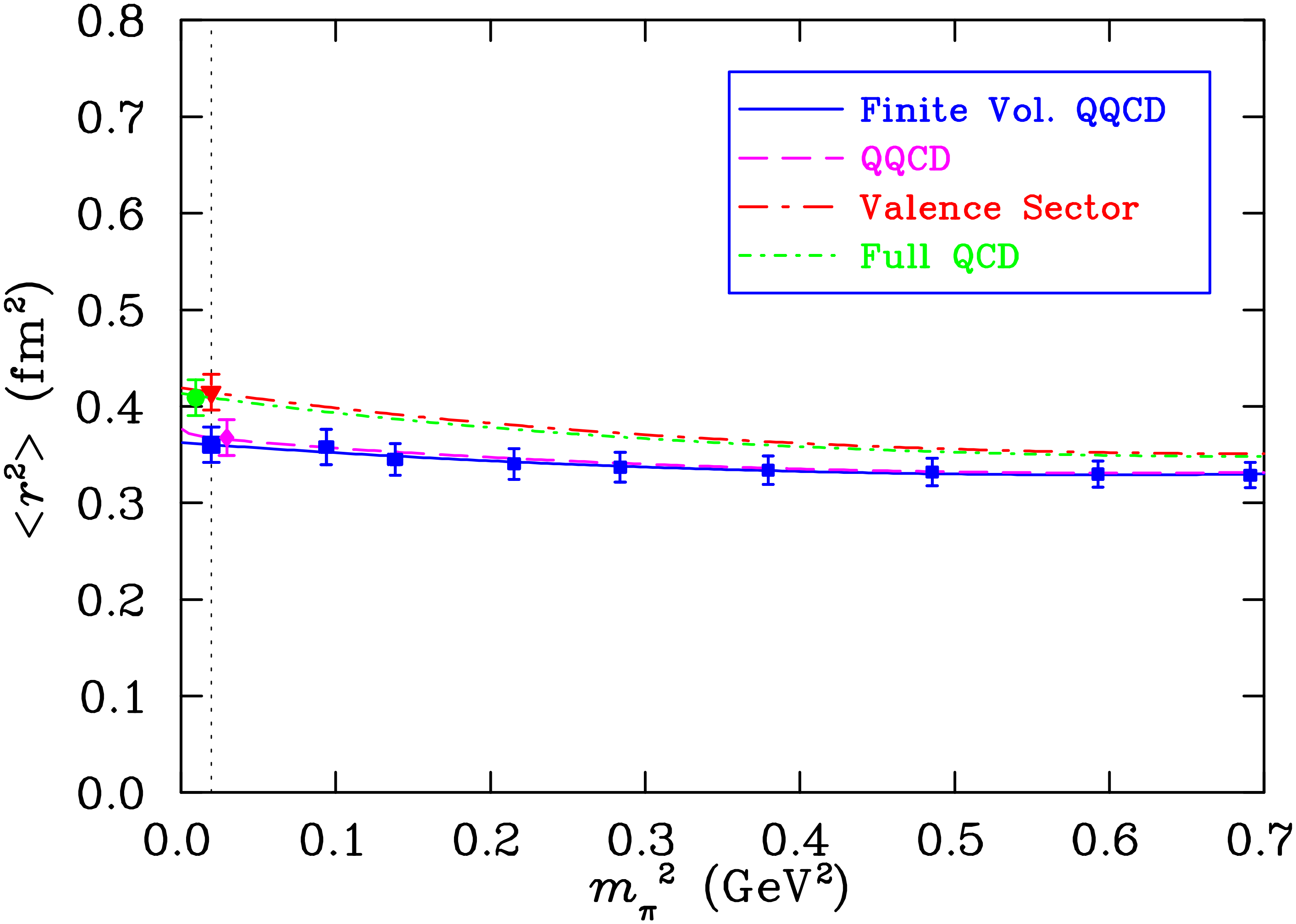


Finite Vol. QQCD
QQCD
Valence Sector
Full QCD
<r²> (fm²)
m_π² (GeV²)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

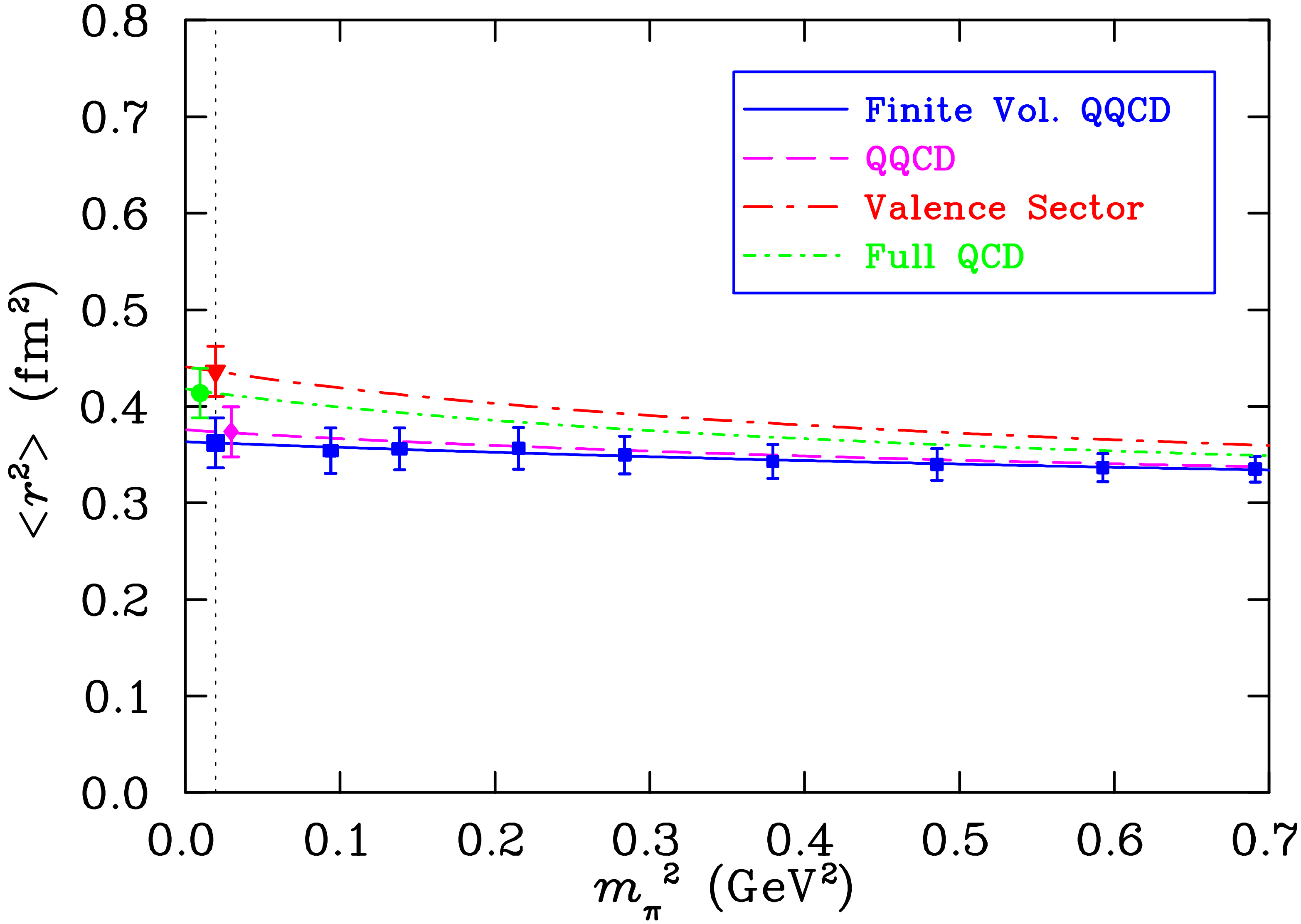
u quark in Ξ^0



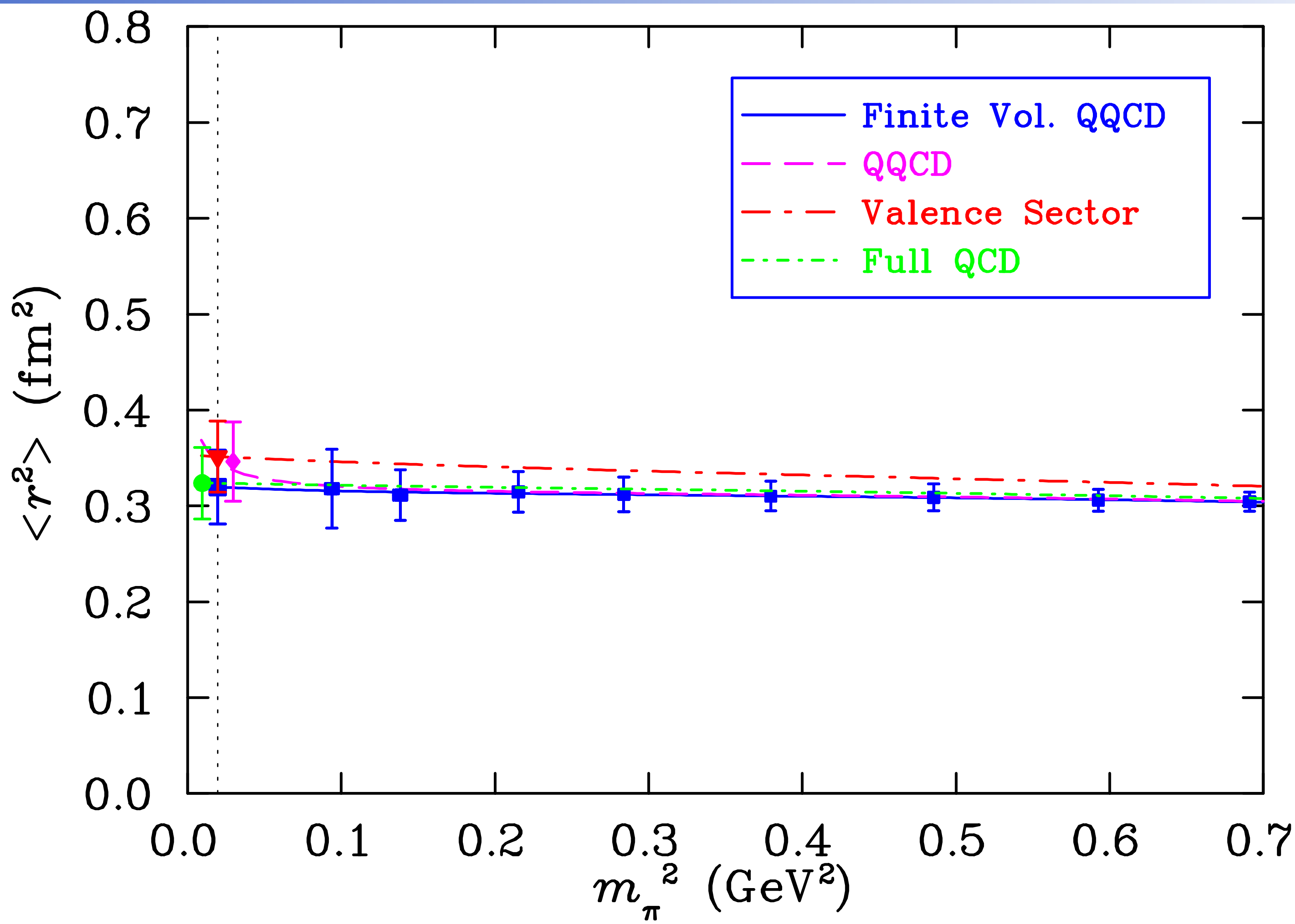
s quark in Ξ^0



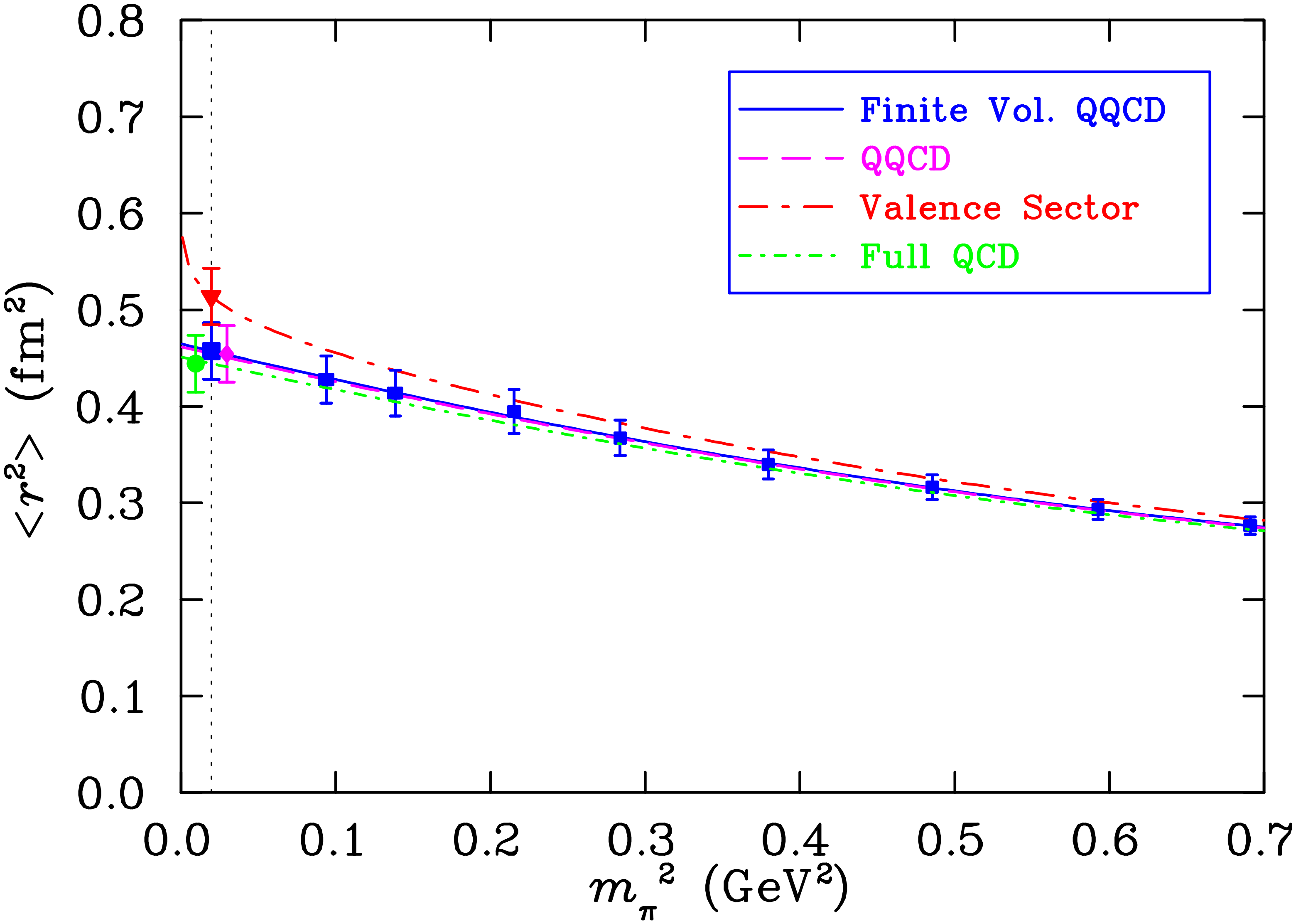
s quark in Λ



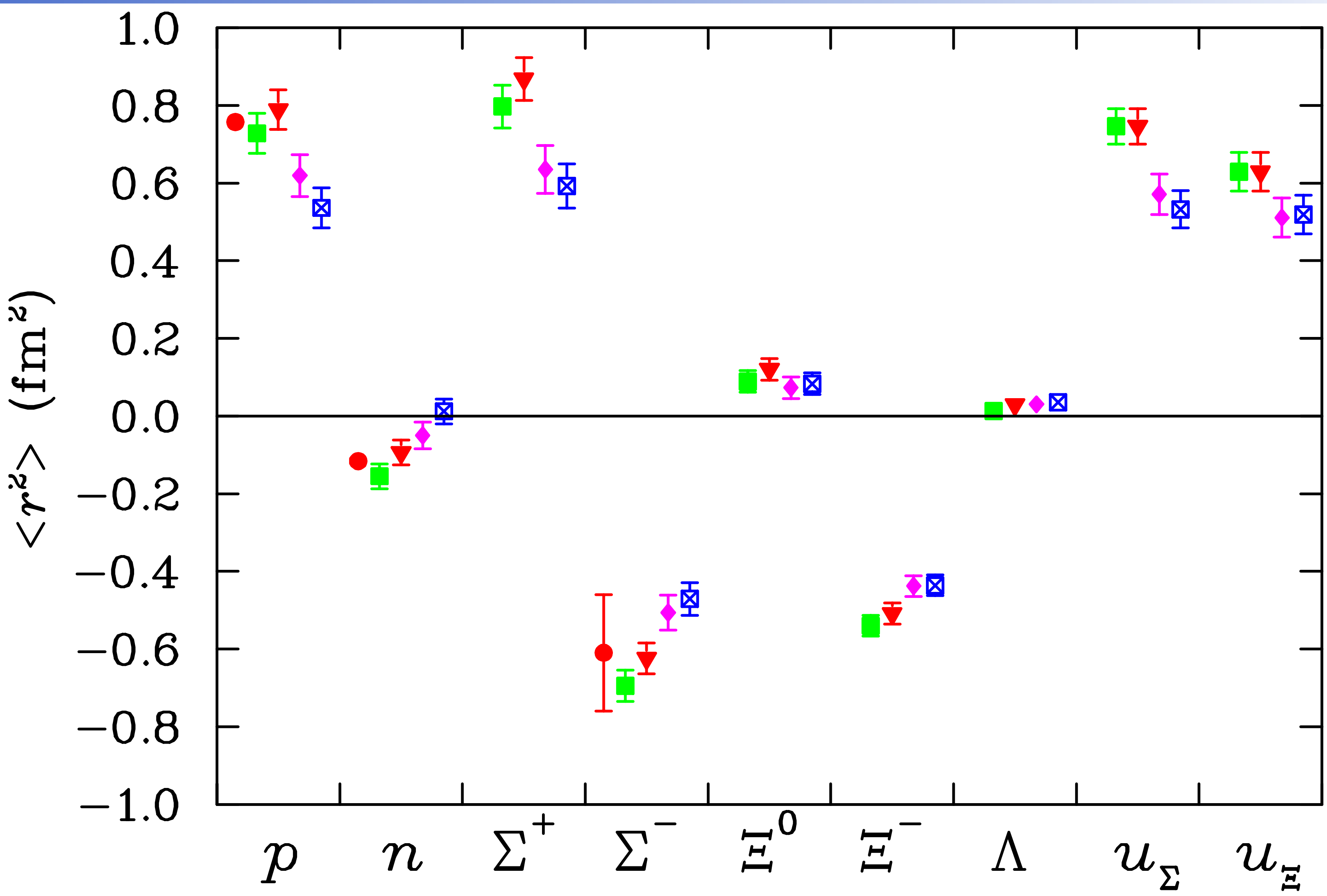
s quark in Σ



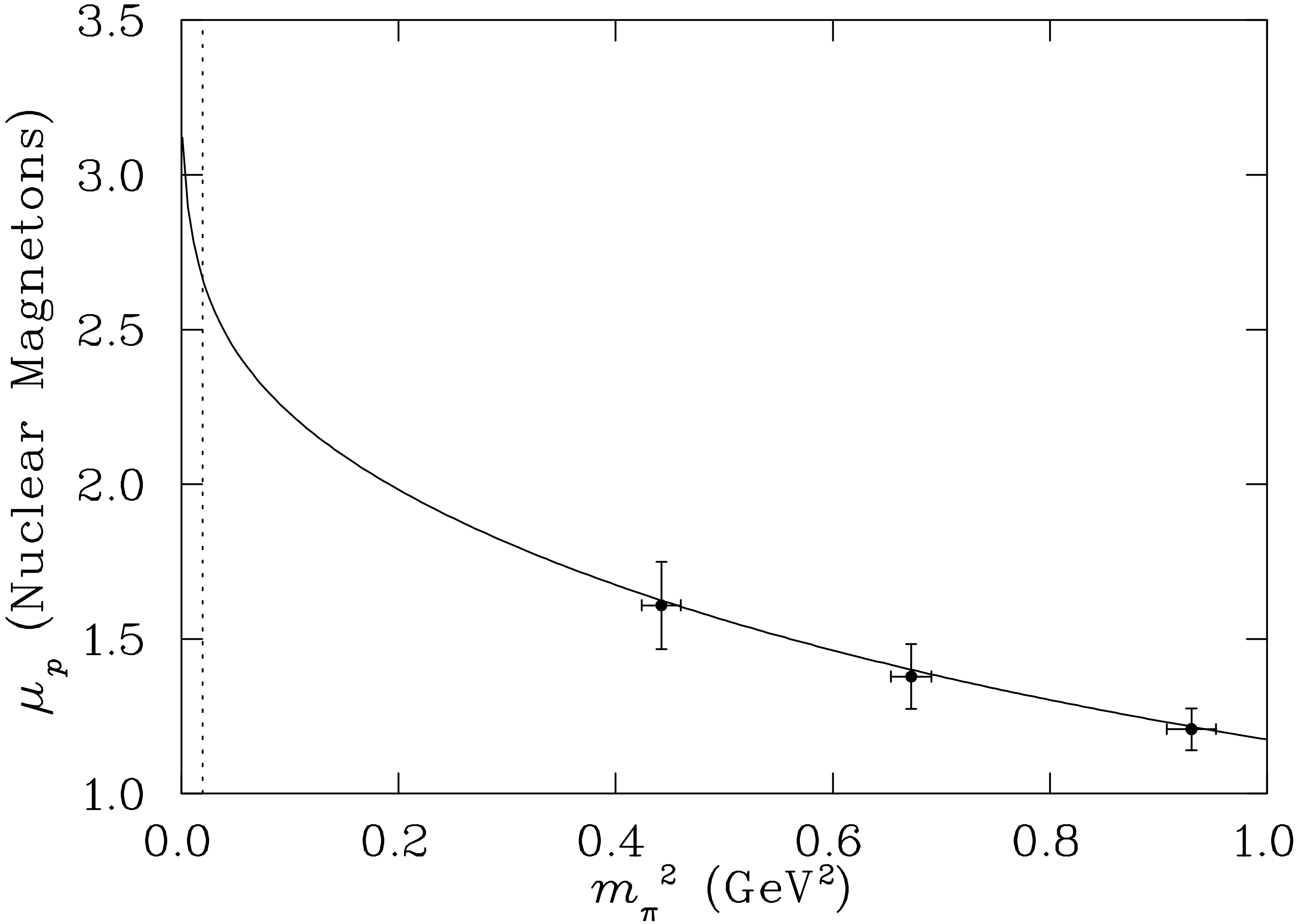
u or d quark in Λ



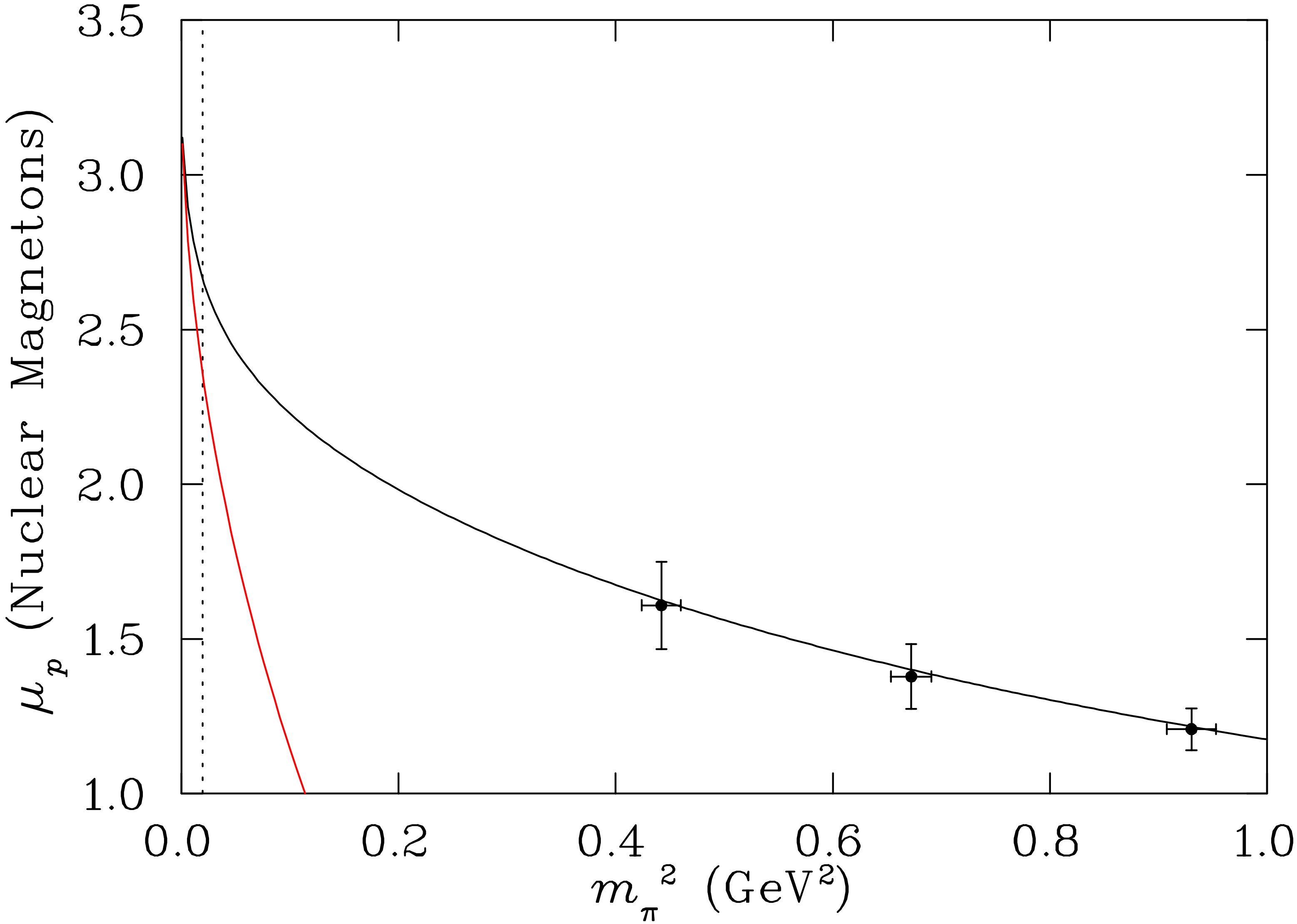
Octet Baryon Charge Radii



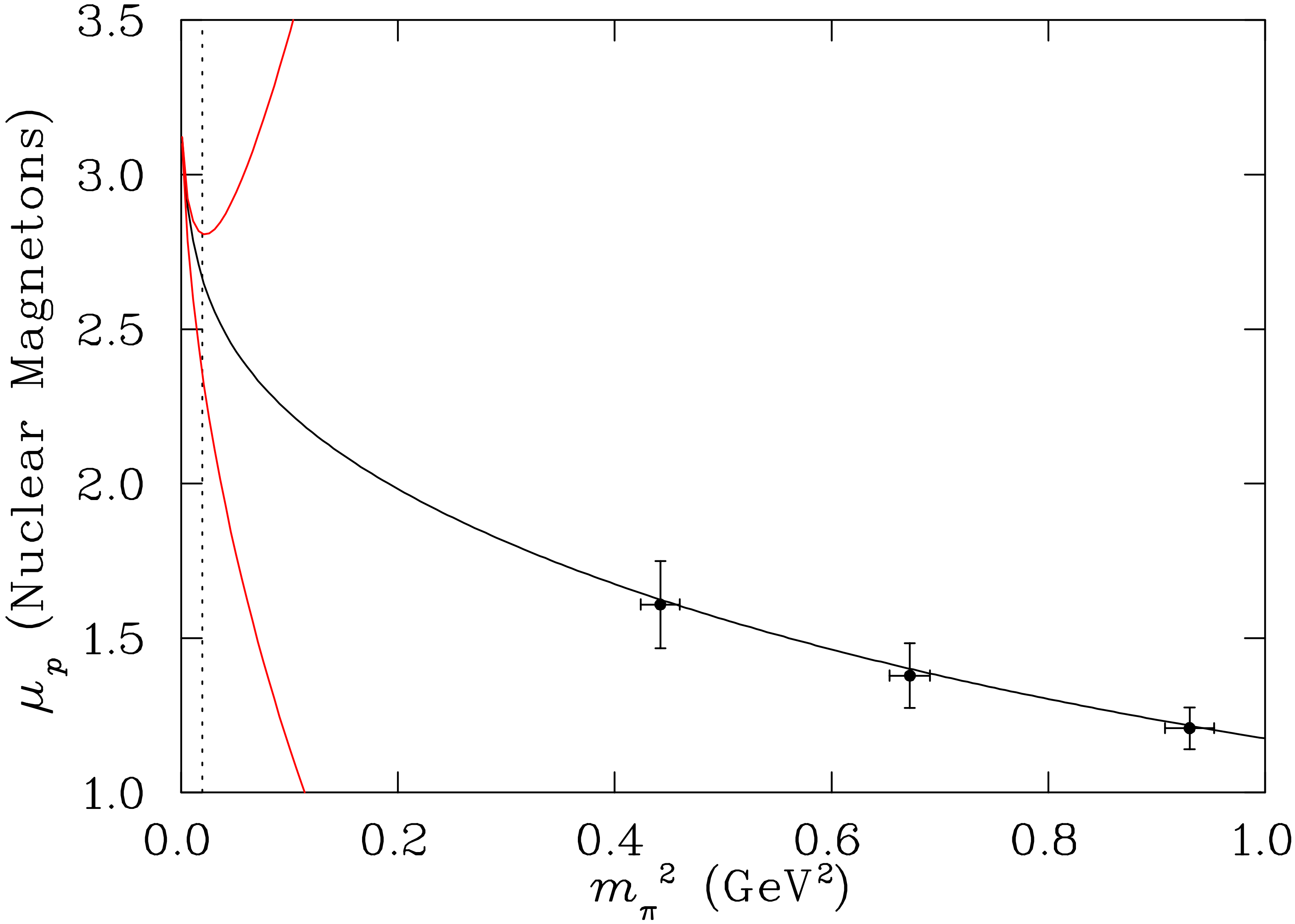
Proton Moment in Full QCD



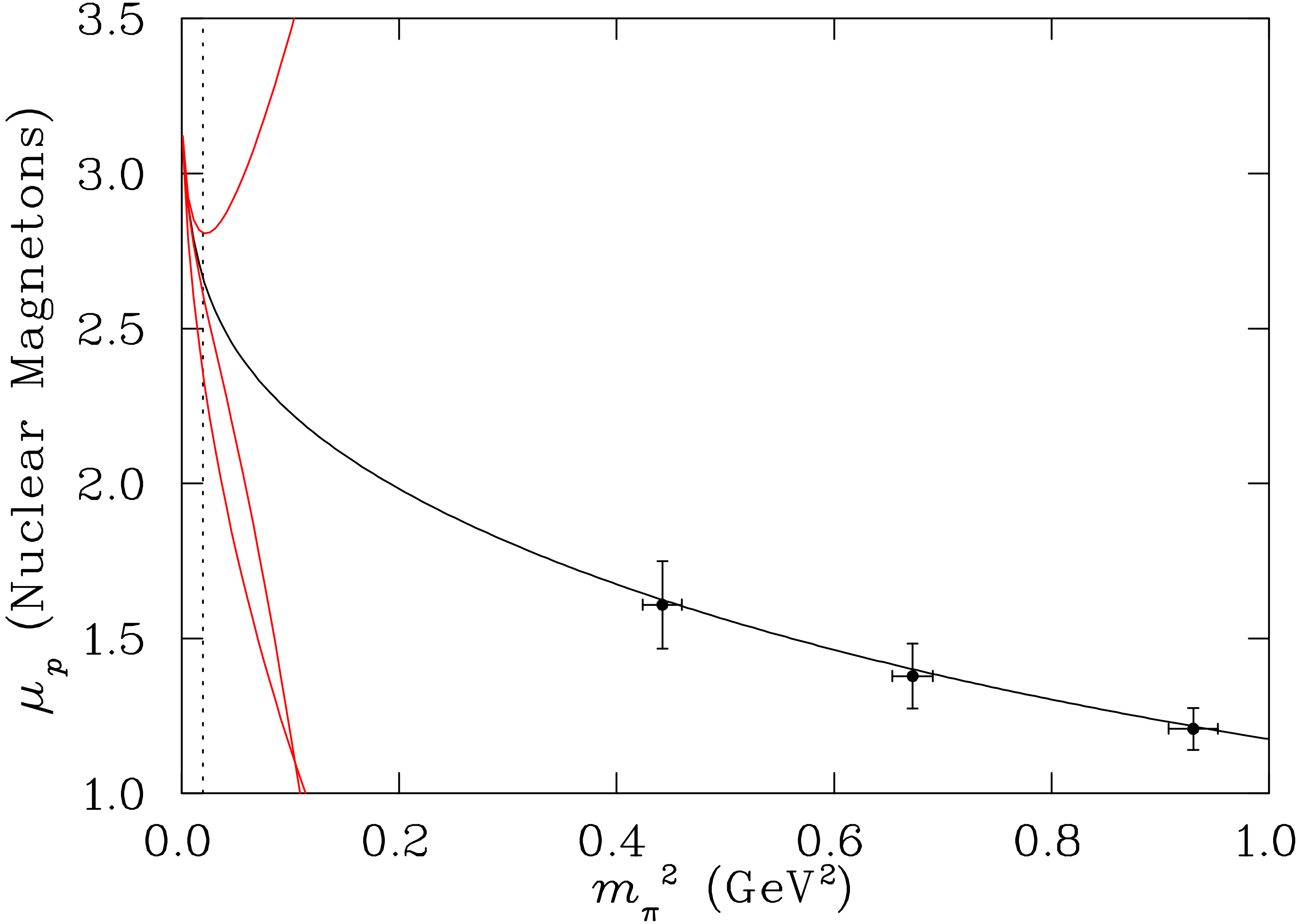
Power Counting: $\mathcal{O}(m_\pi^1)$



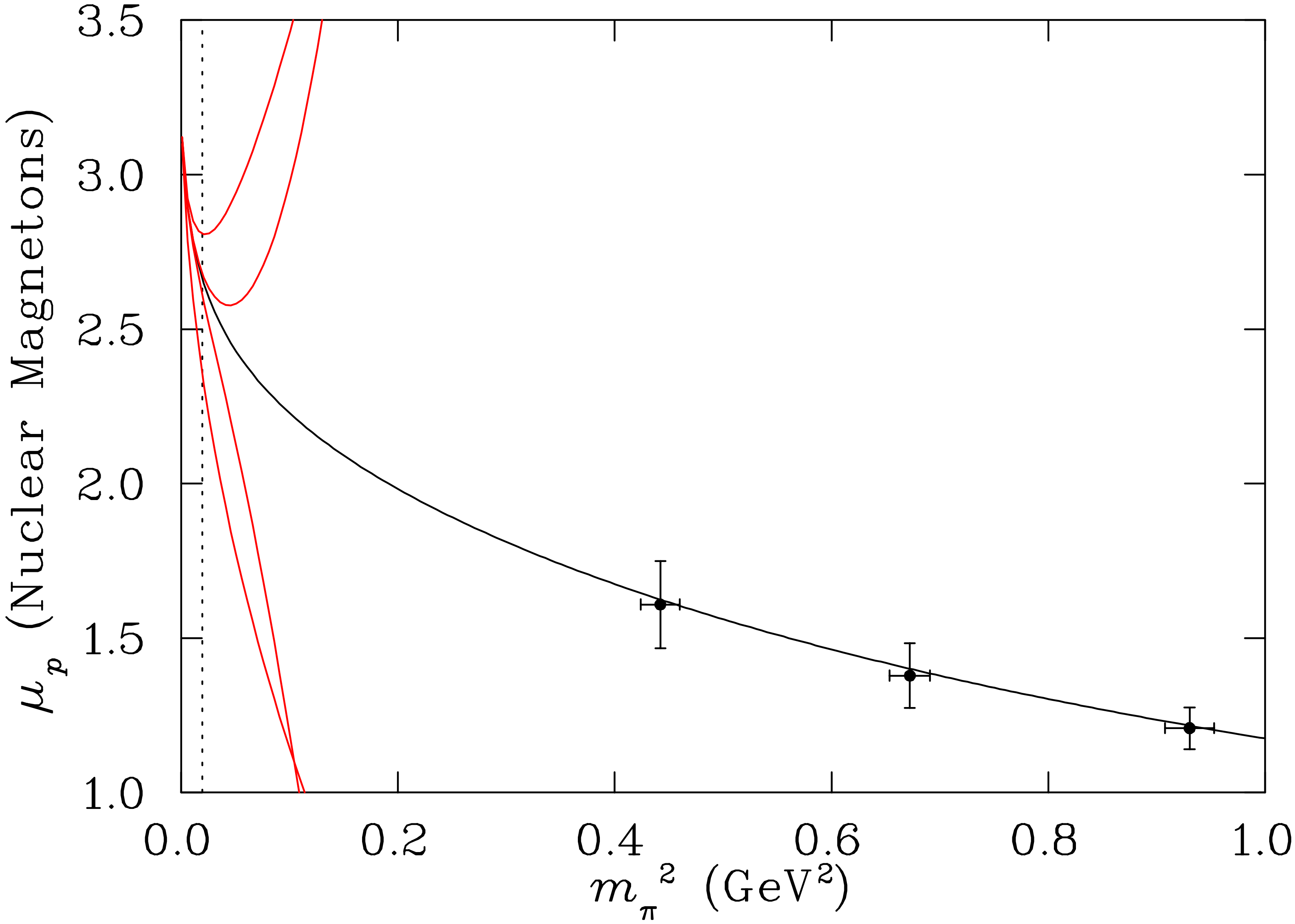
Power Counting: $\mathcal{O}(m_\pi^2)$



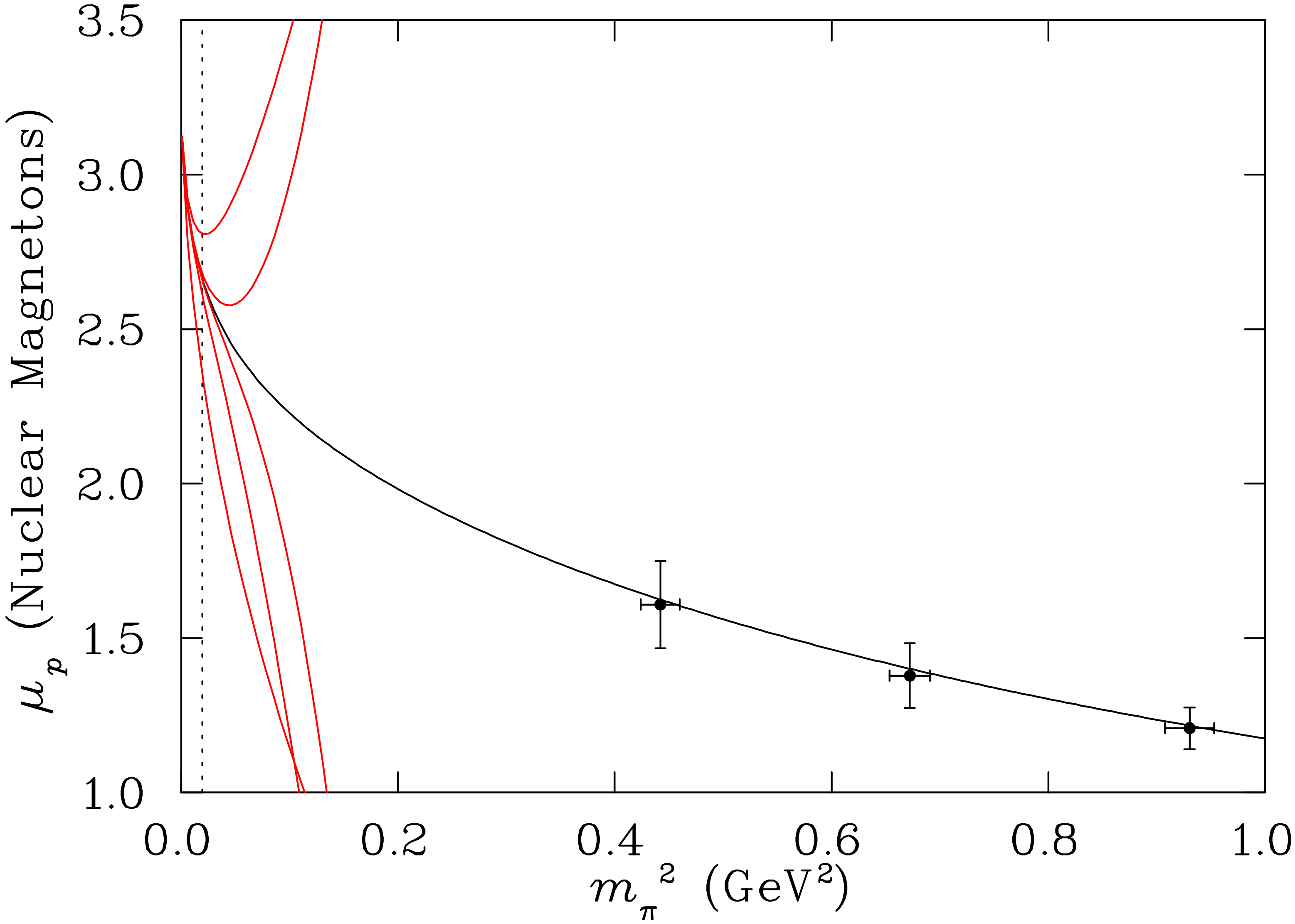
Power Counting: $\mathcal{O}(m_\pi^3)$



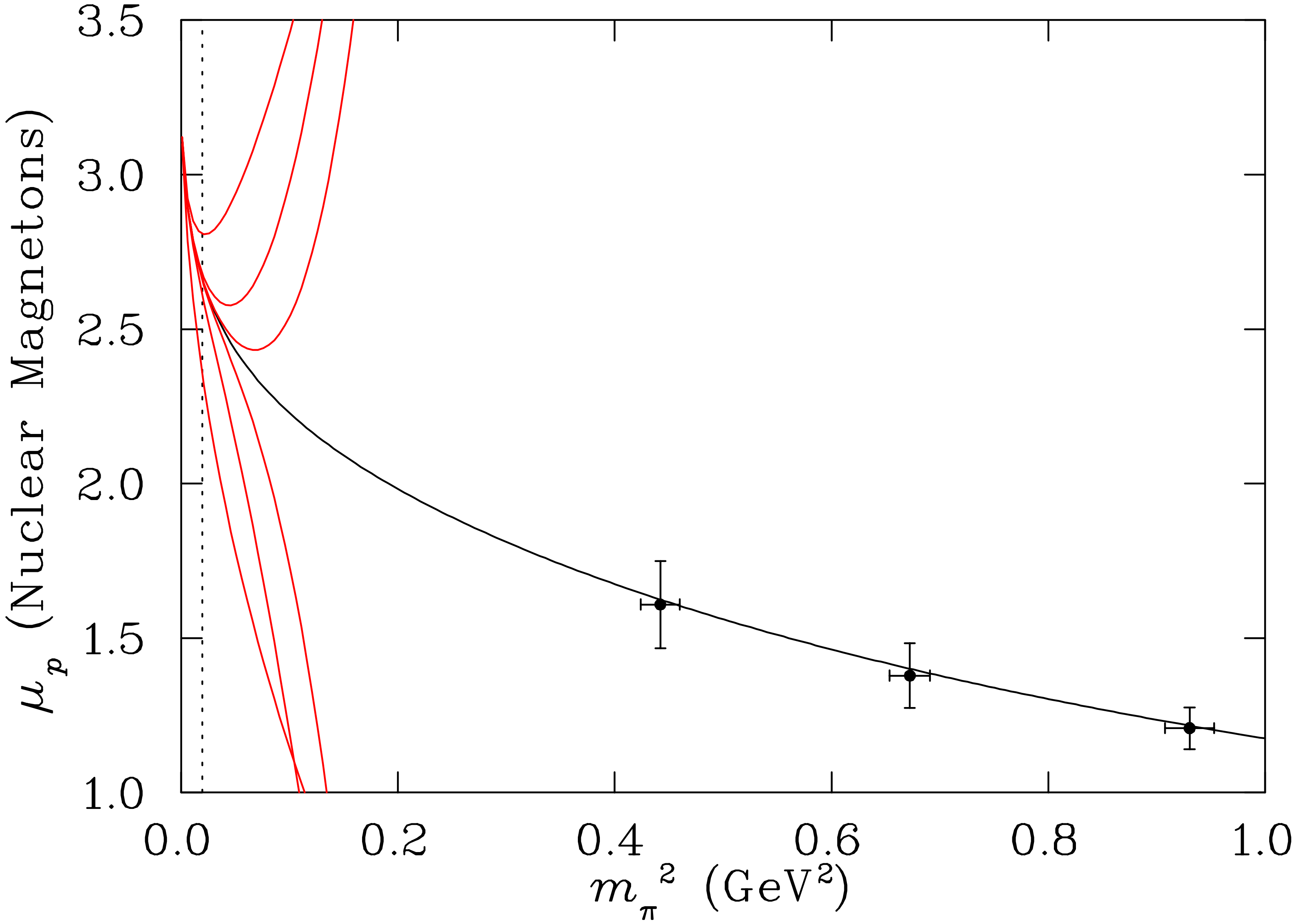
Power Counting: $\mathcal{O}(m_\pi^4)$



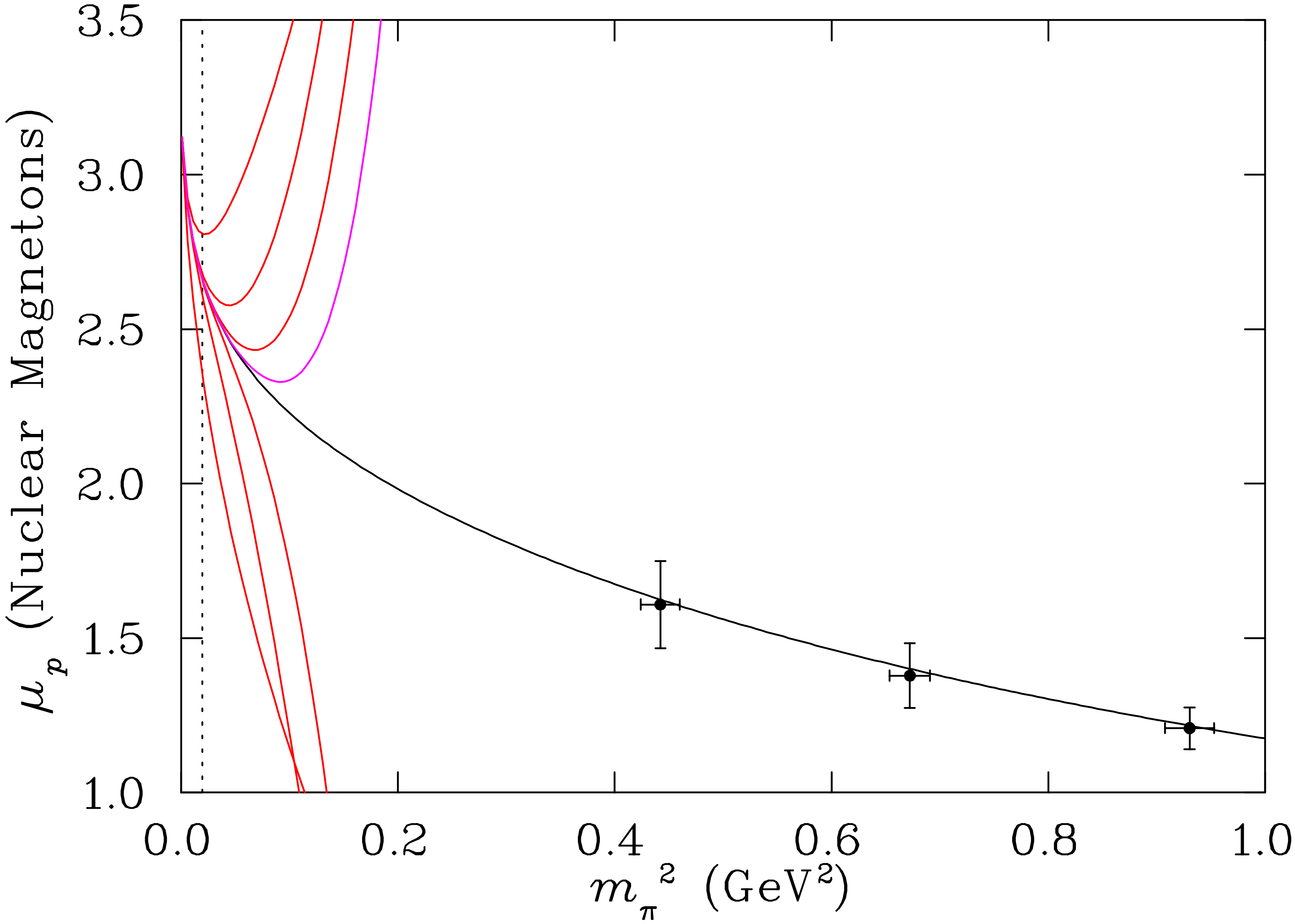
Power Counting: $\mathcal{O}(m_\pi^5)$



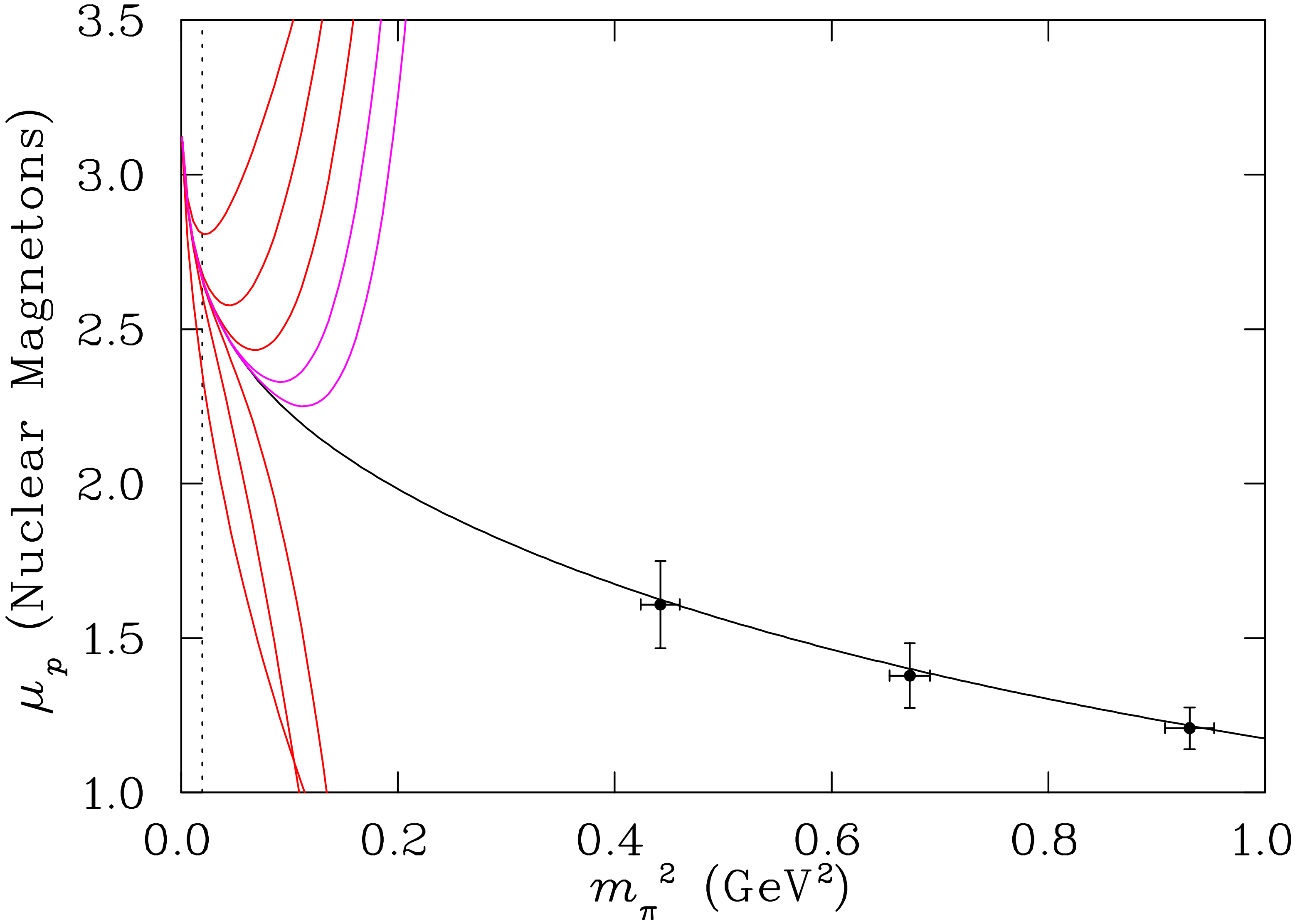
Power Counting: $\mathcal{O}(m_\pi^6)$



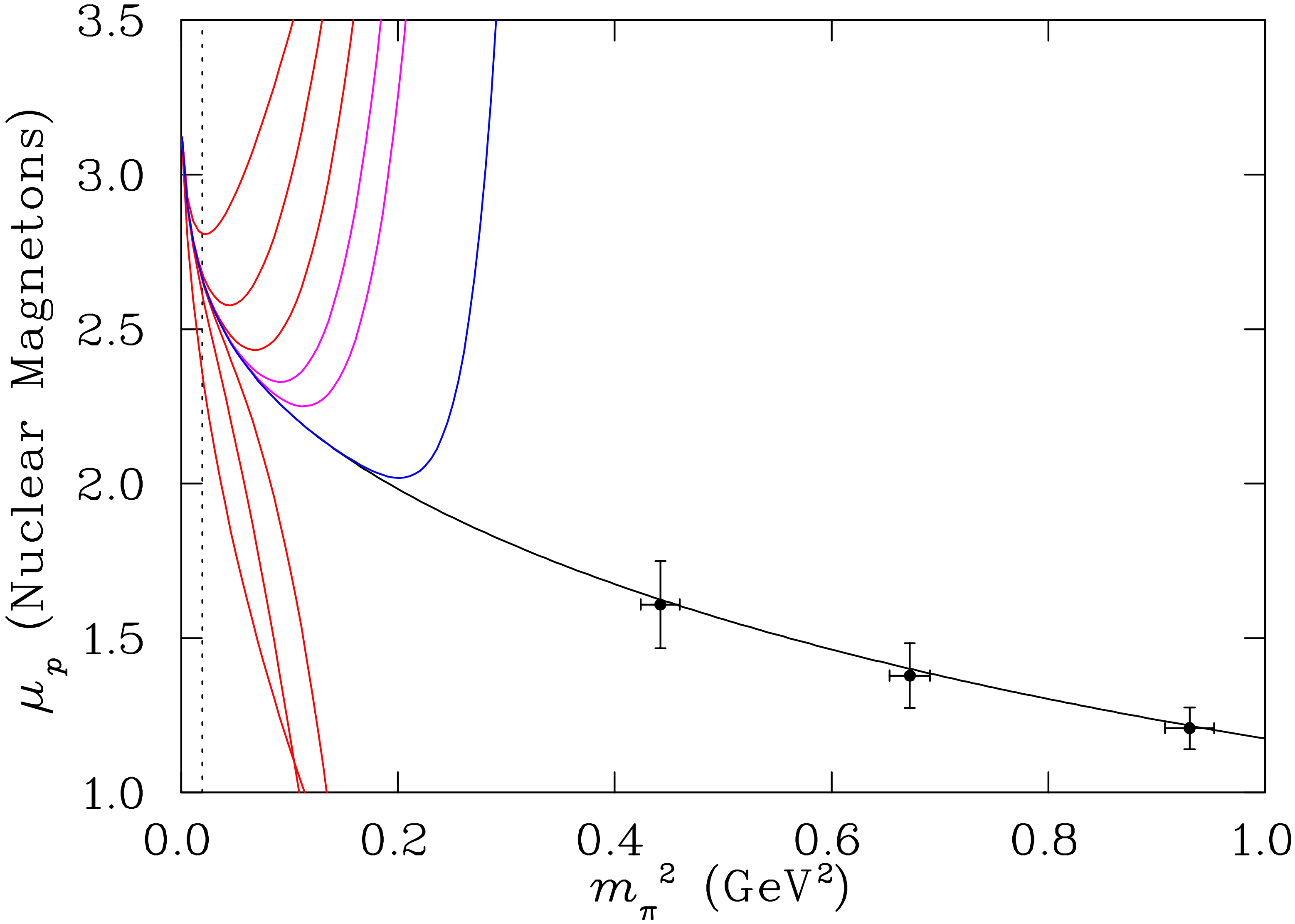
Power Counting: $\mathcal{O}(m_\pi^8)$



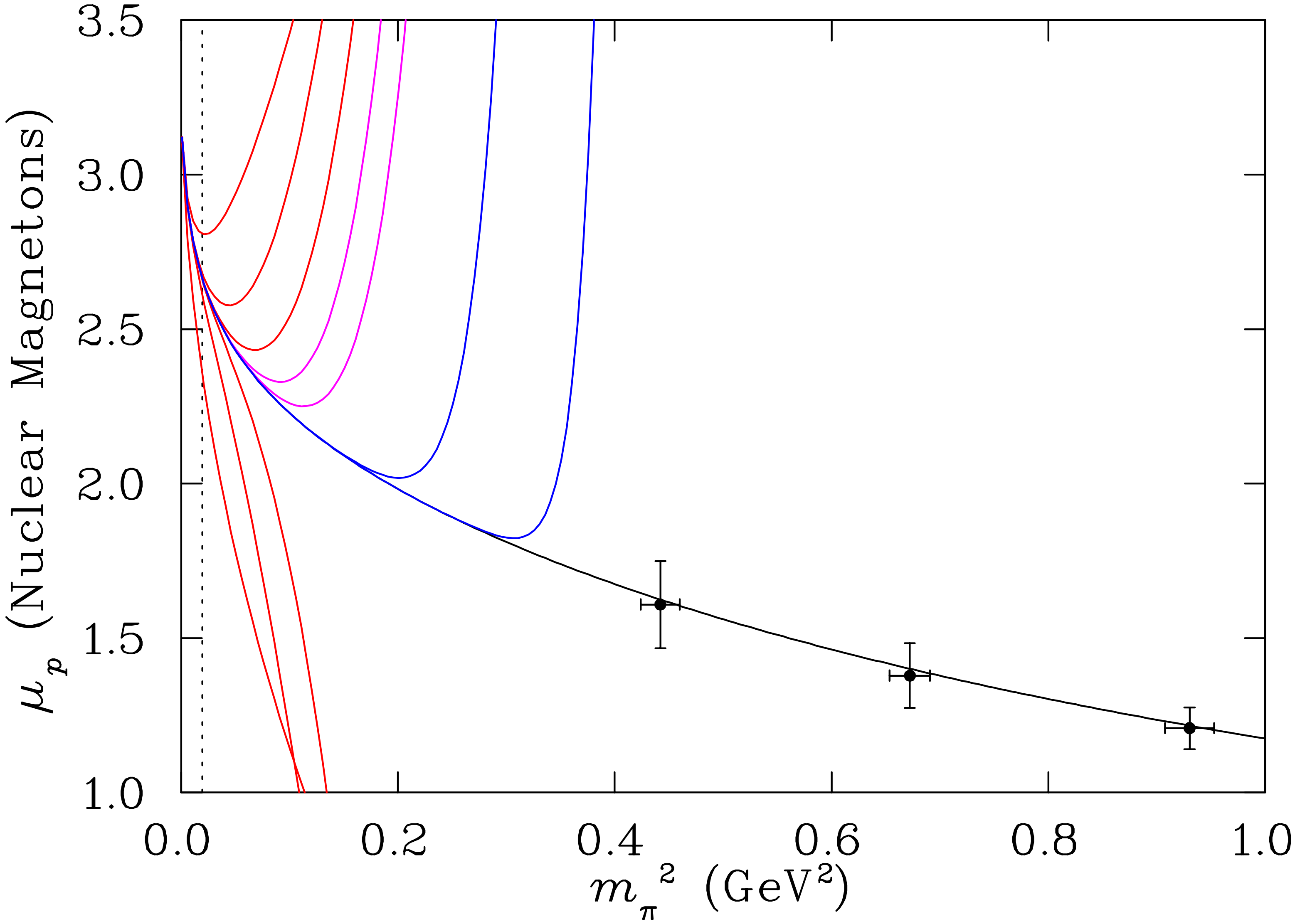
Power Counting: $\mathcal{O}(m_\pi^{10})$



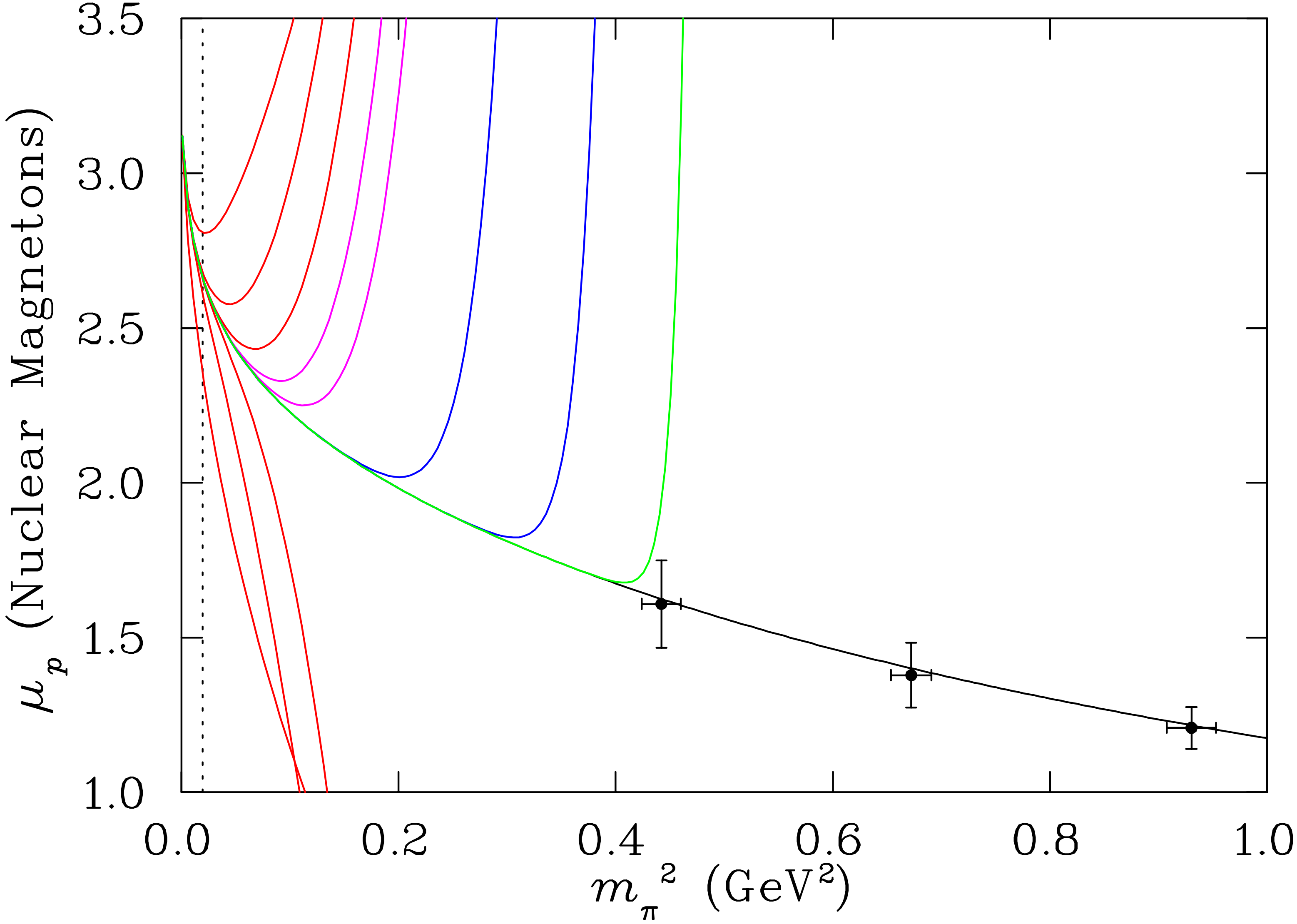
Power Counting: $\mathcal{O}(m_\pi^{20})$



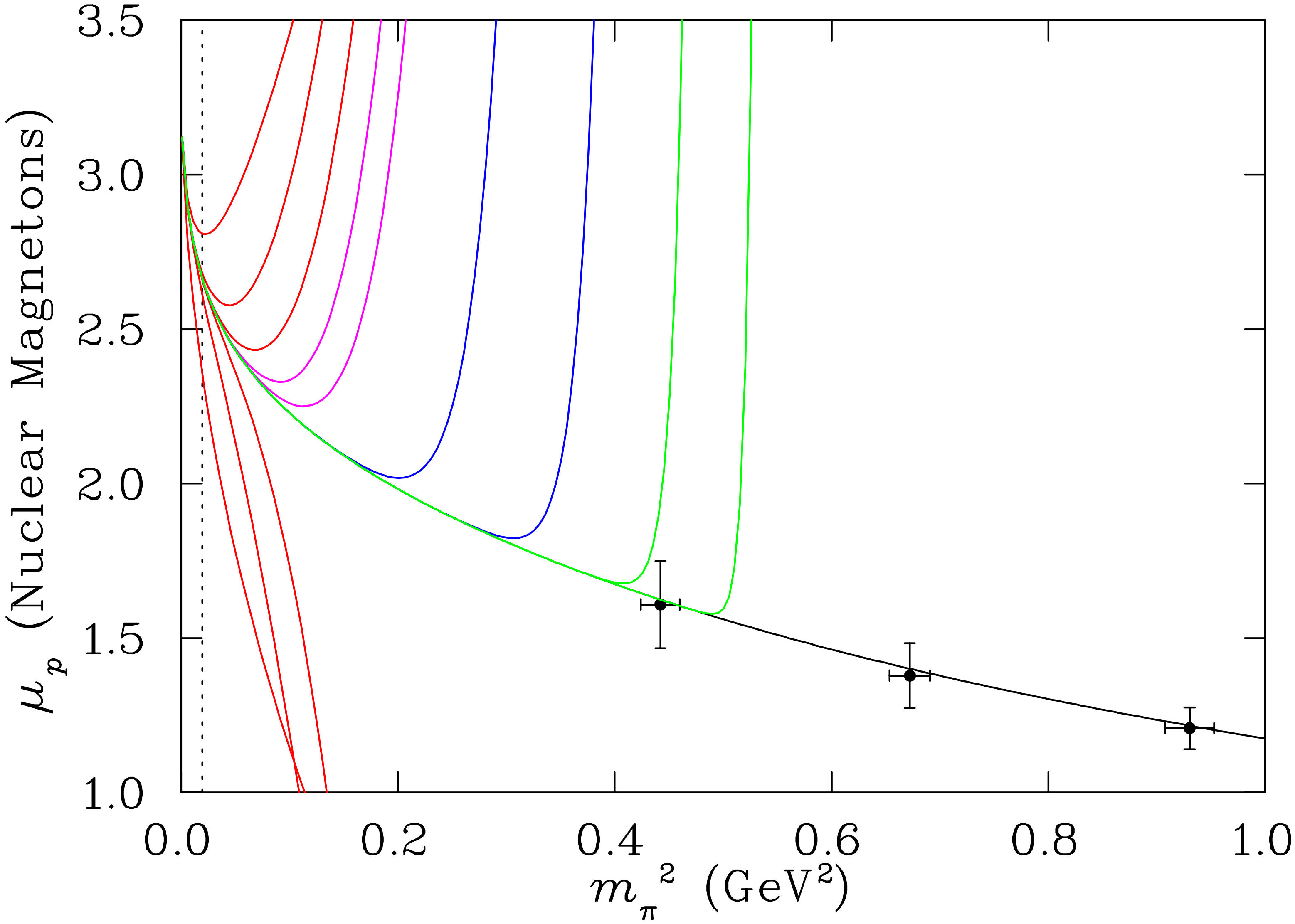
Power Counting: $\mathcal{O}(m_\pi^{40})$



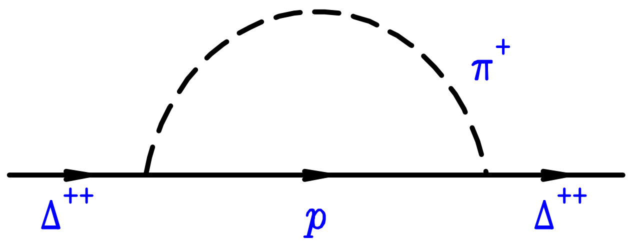
Power Counting: $\mathcal{O}(m_\pi^{80})$

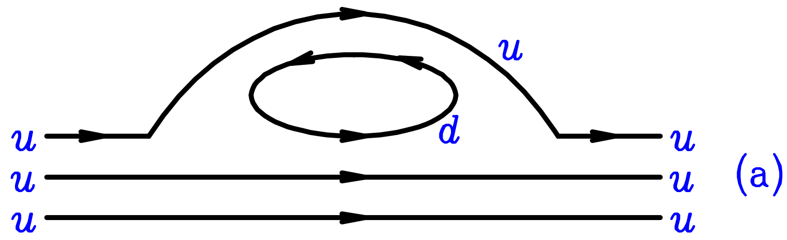


Power Counting: $\mathcal{O}(m_\pi^{160})$

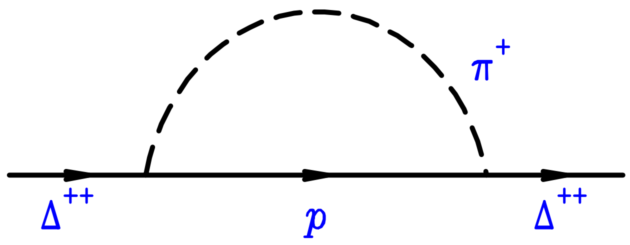


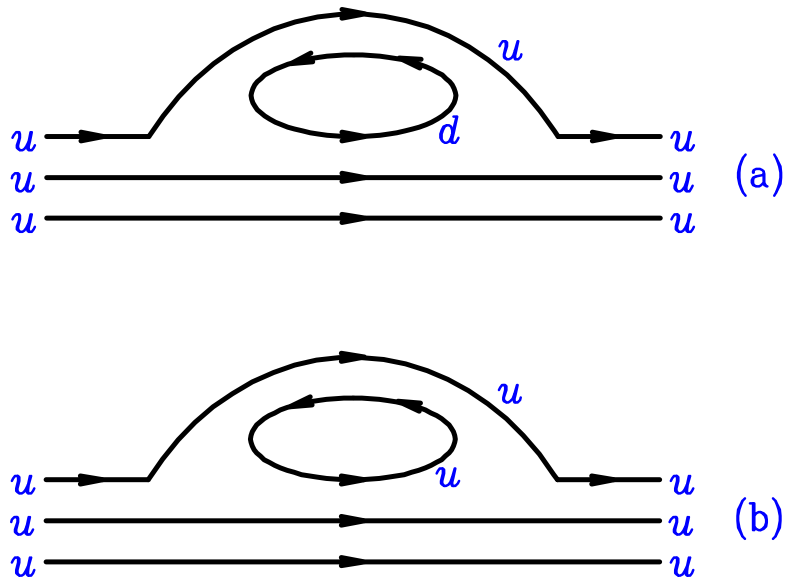
Δ^{++} Decay in Full QCD





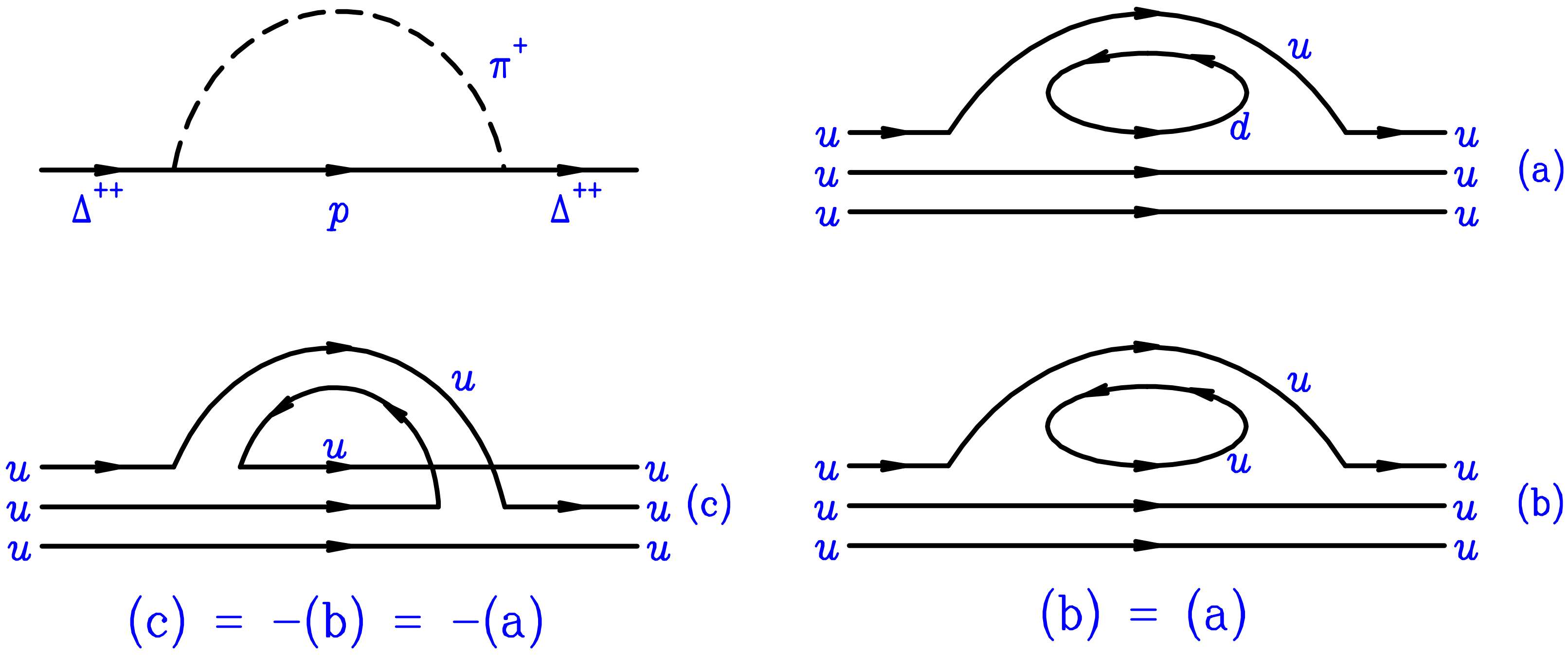
QCD is Flavour Blind



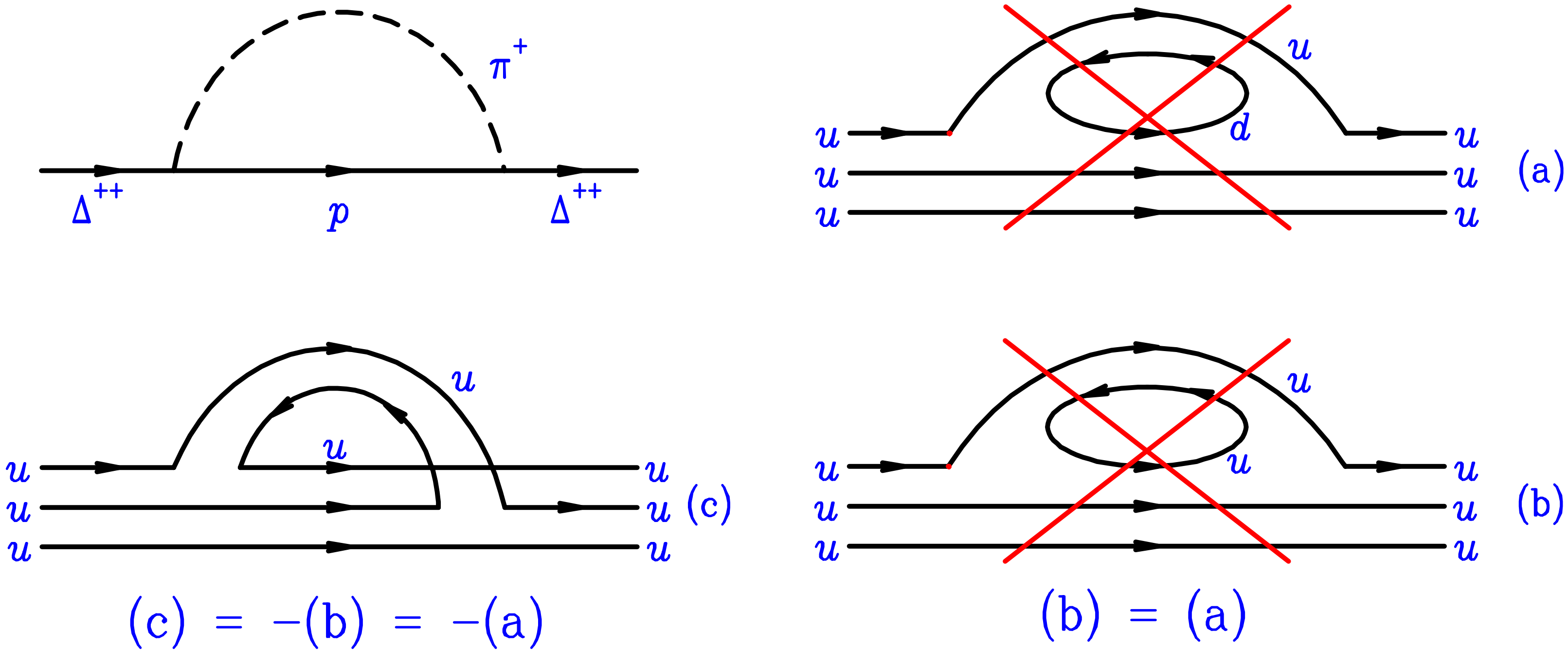


(b) = (a)

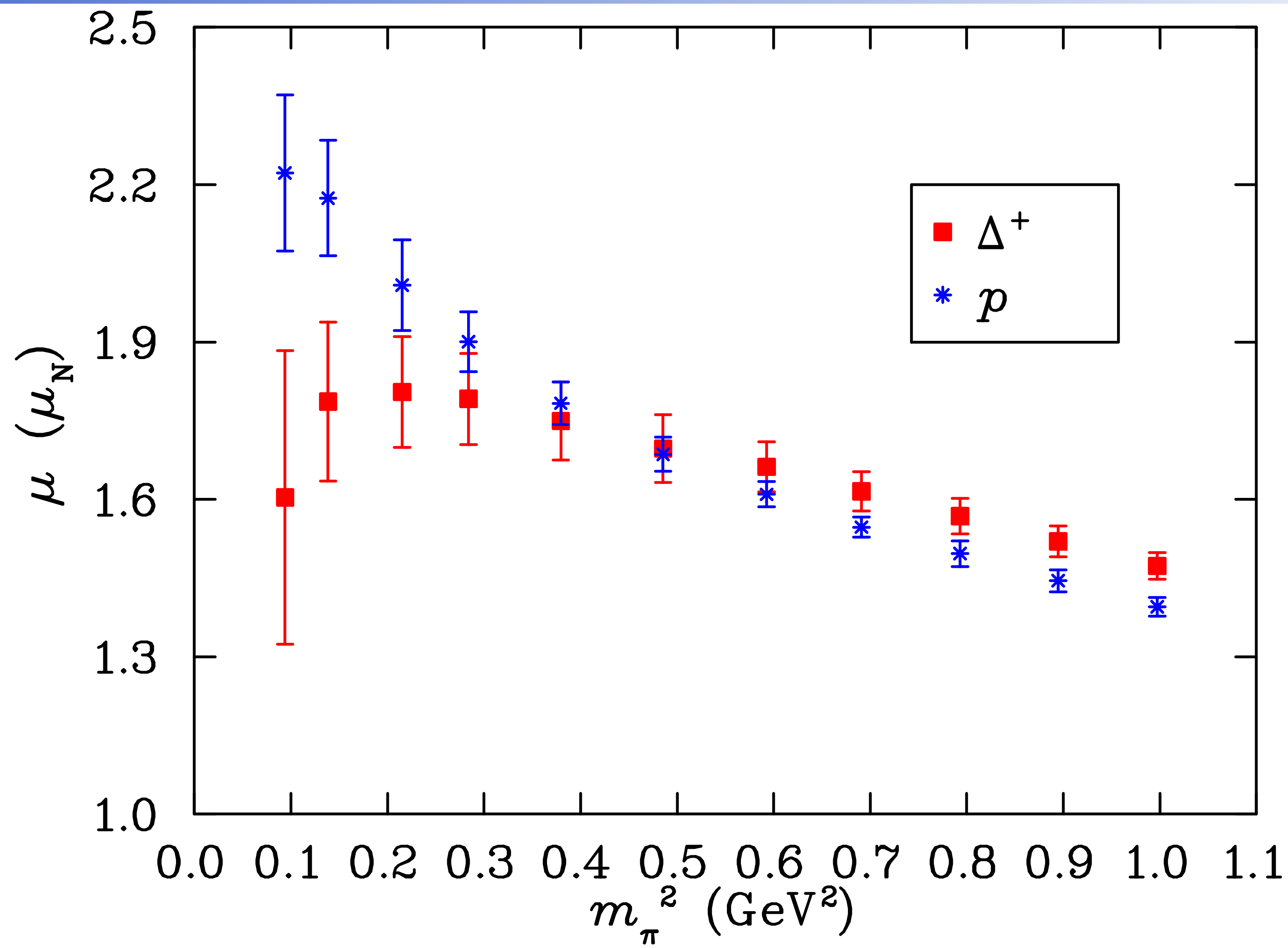
But there is no uuu proton!



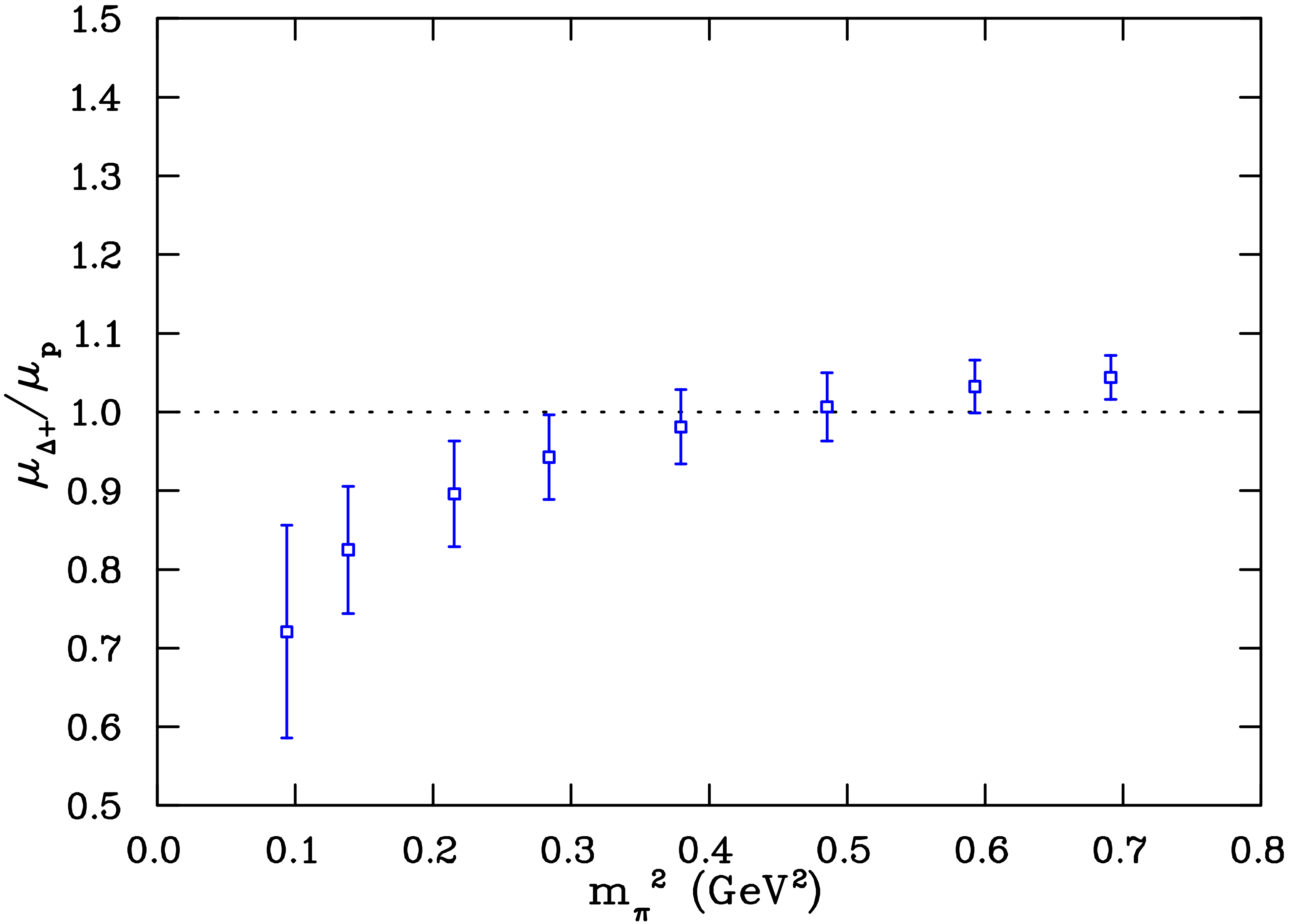
Quenched Δ: Negative Metric Contribution



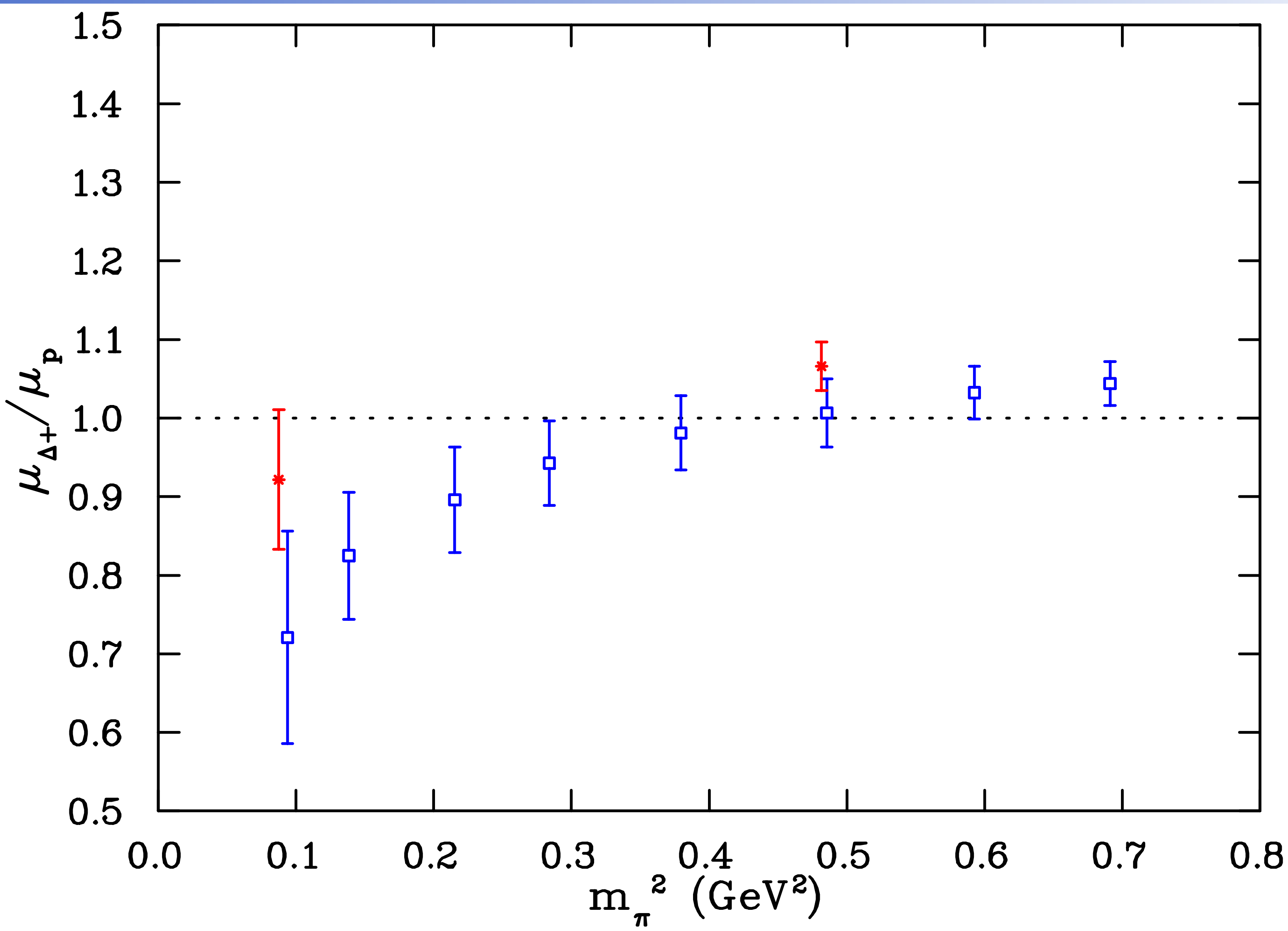
p and Δ^+ Magnetic Moments



p/Δ^+ Magnetic Moment Ratio in QQCD



p/Δ^+ Magnetic Ratio in Full QCD



The Structure of the Nucleon