Wave function of the Proton

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Introduction

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- $C = \gamma_2\gamma_4$ is the charge conjugation matrix.
- In order to create the wave function of the proton we fix the position of the u quarks and measure the d quark probability distribution.

u Quark Separation

- Relative to a central spatial point $\vec{x}$, the vectors $\vec{d}_1$ and $\vec{d}_2$ describe the position of the two u quarks respectively, and $\vec{y}$ describes the position of the d quark. So we modify our annhilation operator to be

$$\chi(\vec{x}, \vec{d}_1, \vec{d}_2, \vec{y}) = \epsilon^{abc}(u_a^T(\vec{x}+\vec{d}_1)C\gamma_5 d_b(\vec{x}+\vec{y}))u_c(\vec{x}+\vec{d}_2).$$

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- For even separations, $d_1 = -d_2$, and for odd separations, $d_1 + 1 = -d_2$.

Gauge fixing

- The wave function of the proton on the lattice is then defined to be proportional to the two-point correlation function at zero momentum in position space,

$$G_{2\gamma\rho}(\vec{x}, \vec{d}_1, \vec{d}_2, \vec{y}, t) = \langle\Omega| T\{\chi(\vec{x}, \vec{d}_1, \vec{d}_2, \vec{y}, t)\bar{\chi}(0)\}|\Omega\rangle.$$

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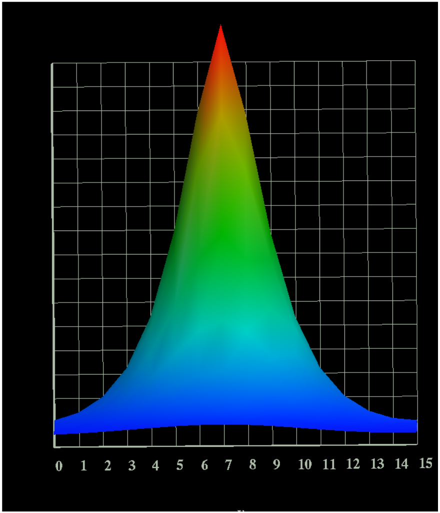
- The two point function we have constructed is no longer gauge invariant.
- The simplest solution is to gauge fix. We study both the Landau and Coulomb gauges.

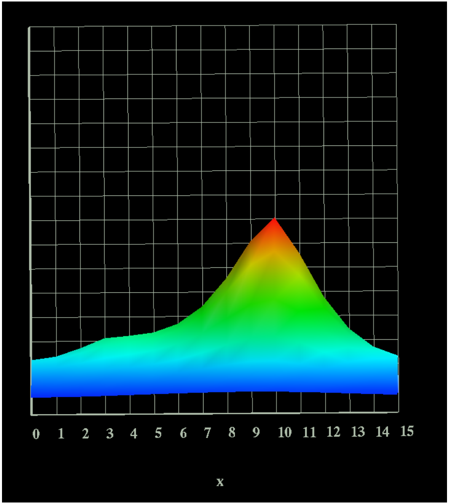
Landau Gauge

- Landau gauge, surface plot, D=0



Landau Gauge

- Landau gauge, surface plot, D=7



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- The wave function should be symmetric around the centre of mass $\vec{x}$.
- The two u quarks should be indistinguishable - but we have explicitly given them different positions.
- Solution: we symmetrise explicitly over d_1 and d_2,

$$\chi(\vec{x},\vec{d}_1,\vec{d}_2,\vec{y}) =$$

$$\epsilon^{abc}\Big[(u_a^T(\vec{x}+\vec{d}_1)C\gamma_5 d_b(\vec{x}+\vec{y}))u_c(\vec{x}+\vec{d}_2)$$

$$+(u_a^T(\vec{x}+\vec{d}_2)C\gamma_5 d_b(\vec{x}+\vec{y}))u_c(\vec{x}+\vec{d}_1)\Big].$$

Landau gauge wave function

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- Diquark clustering clearly present.

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- Different gauge $\implies$ different shape.

Background $\vec{B}$ Field

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Background $\vec{B}$ Field

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- A constant background field in the z direction (with continuum field strength ω) is implemented as a $U(1)$ field, with

$$U_1(x, y) = \exp(-i\omega y),$$
$$U_2(x, y) = 1,$$

except for the boundary,

$$U_2(x, n_y) = \exp(+i\omega n_y x).$$

Background $\vec{B}$ Field

- The plaquettes in the x–y plane are then constant,

$$P_{12}(x,y) = U_1(x,y)U_2(x+1,y)U_1^\dagger(x,y+1)U_2^\dagger(x,y)$$
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- The corner of the lattice requires special treatment,

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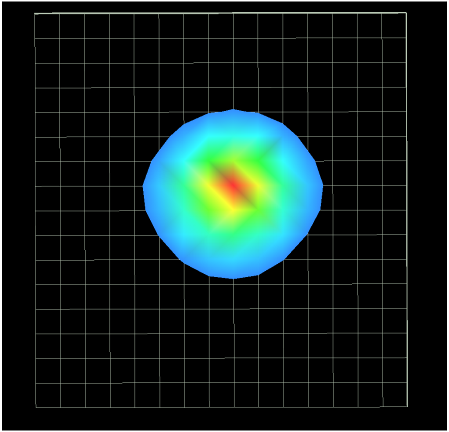
- A quantisation condition on the field strength is induced

$$\exp(+i\omega n_x n_y) = 1 \implies \omega = \frac{2\pi k}{n_x n_y}.$$

Background Field

- Background field, x–y plane.



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  1. Our field description is manifestly asymmetric.
  2. The quark propagator contains the (lattice version of the) inverse of $\partial_\mu + A_\mu$.
  3. We are looking at a gauge dependent quantity.

$\vec{B}$ Asymmetry

- In the continuum it is possible to construct a manifestly (anti-)symmetric background gauge field,

$$F_{12} = \partial_1 A_2 - \partial_2 A_1$$

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- Thats bad...

Gauge Transformation

- If we apply the gauge transformation

$$G(x, y) = \exp(-i\omega xy),$$

using

$$U'_\mu(x, y) = G(x, y)U_\mu(x, y)G^\dagger(x + \delta_{\mu 1}, y + \delta_{\mu 2})$$

we obtain

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- $G(x,y)$ "rotates" the gauge potential 90°.

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- All four gauge potentials produce the same field strength tensor.

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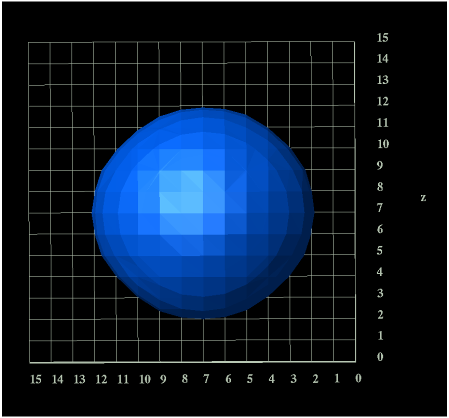

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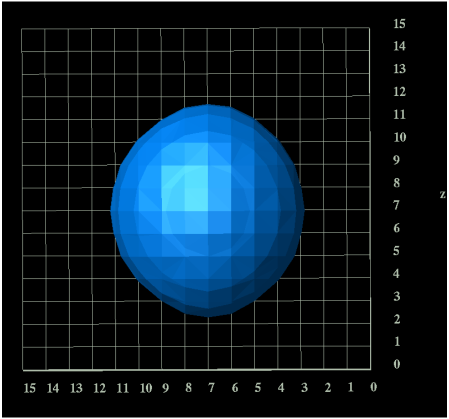

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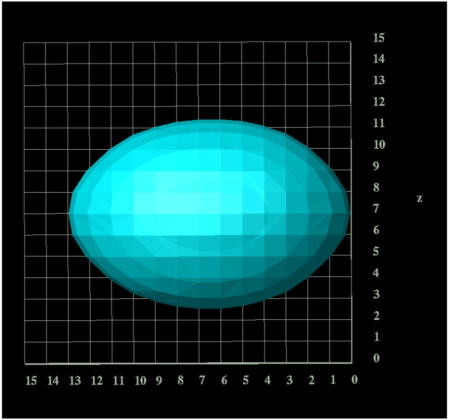

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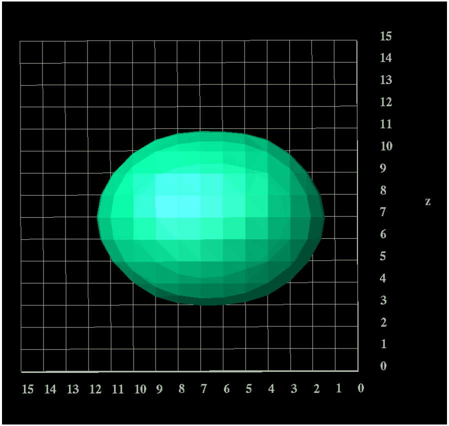

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- Happy Birthday Tony!