Hadronic physics with twisted mass lattice QCD

P.A.M. Guichon (for the ETM coll.)

CEA Saclay, IRFU/SPhN

Plan

- Introduction to twisted mass QCD
- Selected results
  - masses
  - form factors and pdf
- Conclusion

Continuum theory

$$\mathcal{L}_{QCD} = \bar{\psi}(i\gamma_\mu D_\mu - M)\psi - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$$

- fermions part

$$D_\mu = \partial_\mu + igA^a_\mu \frac{\lambda_a}{2}$$

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- $\mathcal{L}_{QCD}$ invariant under local gauge transformations

$$\begin{aligned} \psi(x) &\rightarrow G(x)\psi(x), \\ A_\mu(x) &\rightarrow G(x)(A_\mu(x) - ig\partial)G(x)^{-1} \end{aligned}$$

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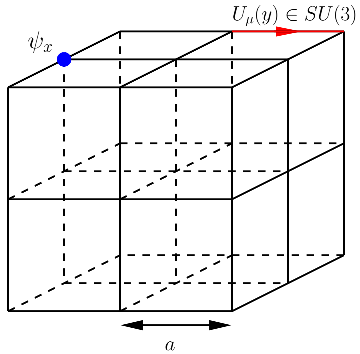
- any approximation should respect this invariance because it is the source of the interactions.

Discrétization

- 4D lattice



- $U_\mu(x) = e^{igaA_\mu(x)}$
- Volume : $24^3 \times 48$ à $48^3 \times 96$
- Lattice spacing : $a \simeq 0.05$ à 0.1 fm

- If S_{QCD} is discretization of the QCD action

$$\langle O[U,\psi,\bar{\psi}]\rangle = \frac{1}{Z}\int \prod_{\text{sommets}} [d\bar{\psi}][d\psi] \prod_{\text{liens}} [dU]\, \mathcal{O}[U,\psi,\bar{\psi}] e^{-S_{QCD}[U,\psi,\bar{\psi}]}$$

Principe of lattice calculation

- A typical corrélator

$$\langle \mathcal{O}(t+\tau)\mathcal{O}(t)\rangle, \tau \geqslant 0$$

- can be evaluated as

$$\langle \mathcal{O}(t+\tau)\mathcal{O}(t)\rangle = \frac{1}{Z}\int [d\psi d\bar{\psi} dU]\, \mathcal{O}(t+\tau)\mathcal{O}(t) e^{-S[U,\psi,\bar{\psi}]}$$

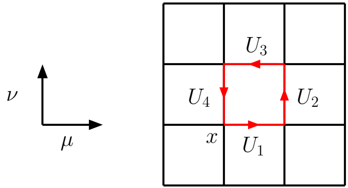
- on the other hand

$$\begin{aligned}\langle \mathcal{O}(t+\tau)\mathcal{O}(t)\rangle &= \langle 0|\mathcal{O} e^{-H\tau}\mathcal{O}|0\rangle \\ &= \sum_n |\langle 0|\mathcal{O}|n\rangle|^2 e^{-M_n \tau}\end{aligned}$$

- More generally the physical information is extracted from the time dependance of some correlator.

Gluon action

- Plaquette



$$U(x,\mu,\nu) = U_4 U_3 U_2 U_1$$

- Wilson action(1974)

$$S_g = \sum_{\text{Plaquettes}} \beta \left[1 - \frac{1}{3} \mathrm{Re}\,\mathrm{Tr}[U(x,\mu,\nu)] \right]$$

- with $\beta = 6/g^2$ then

$$S_g \xrightarrow[a \to 0]{} S_{YM} = \frac{1}{4} \int d^4x F^a_{\mu\nu} F^{\mu\nu}_a$$

Continuum limit

- what we compute is (for instance) $a * M = f(g)$ because there is no mass scale in the action
- g must be a function of a in order that $M = \frac{f(g)}{a}$ be finite when $a \to 0$
- Asymptotic freedom tells that $g(a) \to 0$ when $a \to 0$
- so the continuum theory is at $\beta = \frac{6}{g^2} \to \infty$.

Scale setting

At a given β one needs to know a to compute dimensionfull quantities. One chooses a convenient reference observable :

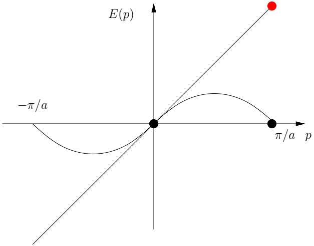
- the Sommer parameter r_0 defined from the static quark antiquark potential

$$r_0^2 \frac{\partial V(r)}{\partial r}|_{r_0} = 1.65$$

- some mass (m_ρ)
- some decay constant (f_π)

Quarks action

- naive discretization naïve : $\partial_\mu \rightarrow$ finite difference
- Doublers



- $|p\rangle = 0$ et $|p\rangle = \pi/a$ are degenerate
- On simulates 2^4 fermions

- add the Wilson term
  - vanishes as $a \rightarrow 0$
  - gives a mass $\propto 1/a$ to the doublers
- explicitly breaks chiral symmetry. (This restored by renormalization)

Twisted quarks

Wilson quarks

$$S_{F,W} = a^4 \sum_x \bar{\psi}(x) \Big[\gamma_\mu \overset{\leftrightarrow}{\nabla}_\mu + m - \frac{a}{2} \sum_\mu \nabla^*_\mu \nabla_\mu \Big] \psi(x)$$

twisted Wilson quarks : $N_f = 2$

$$S_{F,TM} = a^4 \sum_x \bar{\chi}(x) \Big[\gamma_\mu \overset{\leftrightarrow}{\nabla}_\mu + m + i\mu\gamma_5\tau^3 - \frac{a}{2} \sum_\mu \nabla^*_\mu \nabla_\mu \Big] \chi(x)$$

- for the chiral invariant part of the action the twist can be eliminated by a chiral rotation $\tan\omega = \mu/m$

$$\chi \to \exp(-i\gamma_5 \frac{\omega}{2} \tau^3)\psi, \quad \bar{\chi} \to \bar{\psi} \exp(-i\gamma_5 \frac{\omega}{2} \tau^3)$$

- but this would change the Wilson term. So for every twist angle ω one has a different discretization
- on exploit this to improve the continuum approach.

twisted quarks : continuum limit

- The continuum limit of tmQCD **is** équivalent to QCD under the (+ and x) mass renormalization

$$m_R = Z_m(m - m_{cr}), \quad \mu_R = Z_\mu \mu$$

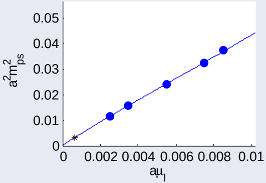
where the critical mass m_{cr} est determined by

$$m_{PCAC} = \frac{\langle \partial_0 A_0^a(x) P^a(0) \rangle}{2\langle P^a(x) P^a(0) \rangle} = 0$$

- Renormalized twist angle $\tan \omega_R = \dfrac{\mu_R}{m_R} = \dfrac{Z_\mu \mu}{Z_m(m - m_{cr})}$

Maximum twist

- Set $\omega_R = \pi/2$, equivalently $m = m_{cr}$
- The pion mass depends only on μ : $m_\pi^2 \propto \mu$



Advantage

- $\mu \neq 0$ provides a cutoff to the zero modes of the Dirac operator
- At maximum twist one has (for free) $O(a^2)$ improvement of the observables.

Drawback

- Explicit breaking (of order a^2) of isospin and parity invariance

Generalization to strange and charmed quarks

- done
- more complicated because s and c are not degenerate
- tuning to maximal twist has to be done every time the s and c masses are changed

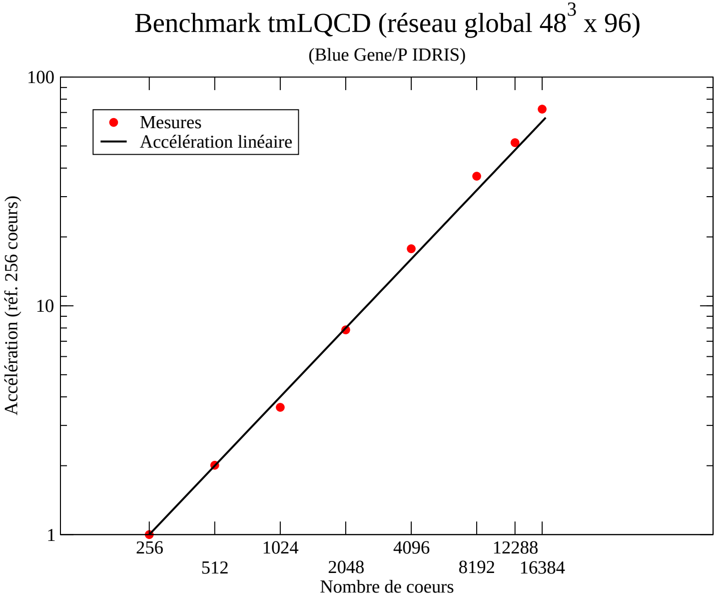
Monte Carlo

- Typical integral to evaluate

$$C(t) = \int \prod_{\text{liens}} [dU] \; \frac{\det \mathcal{D}[U]}{Z} e^{-S_g[U]} \; \Big[\text{propagateurs}\Big]_{\text{U}}$$

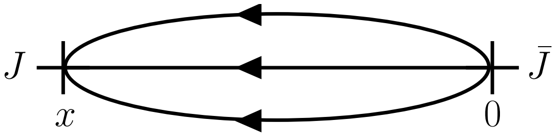
- $\det \mathcal{D}[U]$ is evaluated with pseudo-fermions method
- U is sampled with the hybrid Monte carlo method (molecular dynamics)

Scalabilité



2 points function ($N_f = 2$)

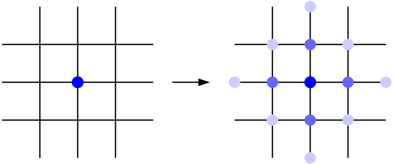
- let $J(t, \vec{x})$ some operator which has the quantum numbers of the nucleon
- $$C(t) = \sum_{\vec{x}} \langle 0 | J(t, \vec{x}) \bar{J}(0) | 0 \rangle \underset{t \to \infty}{\propto} e^{-m_N t}$$



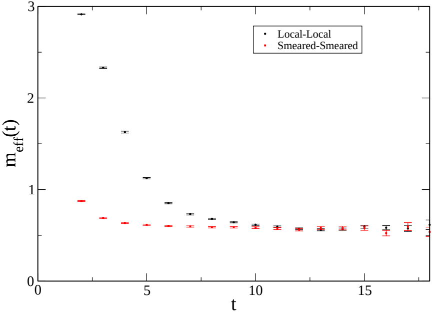
- look for a plateau in $m_{eff}(t) = -\log \frac{C(t)}{C(t-1)}$

Smearing

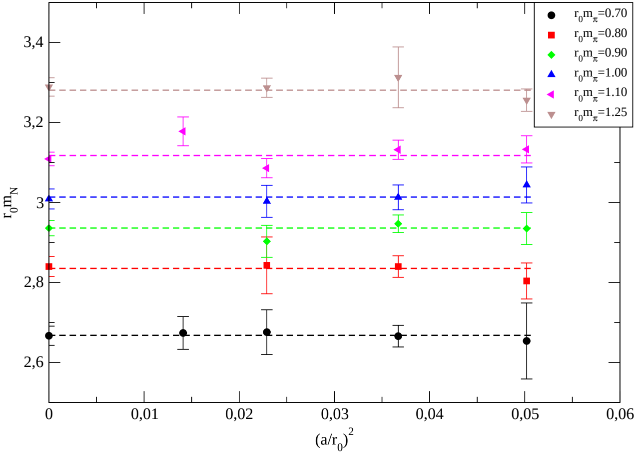
- $\bar{J}|0\rangle$ may have little overlap with the true nucleon
- quark field smearing improves this



- faster approch to the plateau



Continuum limit and finite volume effects



Finite volume effects $m_\pi \simeq 310$ MeV

- L=24 $am_N = 0.5111(58)$
- L=32 $am_N = 0.5126(46)$

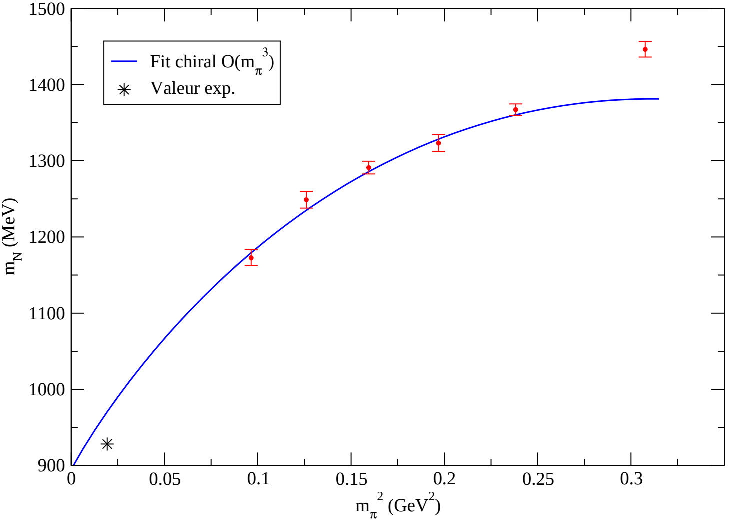
Simple chiral extrapolation

- A order m_π^3 :

$$m_N(m_\pi) = m_N^0 - 4c_N^1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3 + \ldots$$

- No change when higher order terms are included

Chiral extrapolation



Résults

Masses extrapolated to physical point

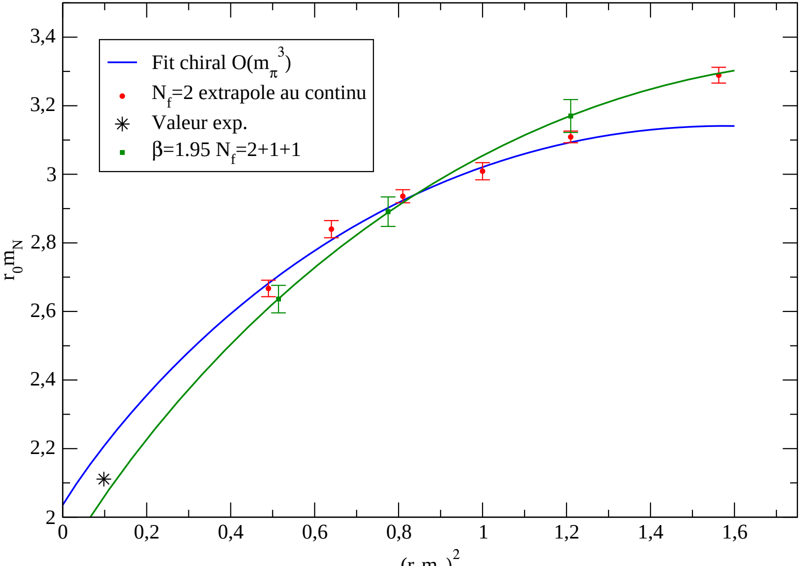
- $m_N = 963(16)$ MeV (exp. $m_N \simeq 938$ MeV)
- $m_\Delta = 1356(33)$ MeV (exp. $m_\Delta \simeq 1232$ MeV)

Terme σ term

$$\sigma_N = \langle N|m_u \bar{u}u + m_d \bar{d}d|N\rangle \simeq m_\pi^2 \frac{\partial m_N}{\partial m_\pi^2}$$

- $\sigma_N = 66.7 \pm 1.3$ MeV. Popular ”experimental” value 45 ± 8 MeV.

Nucléon $N_f = 2+1+1$ (préliminary : one β,one volume)

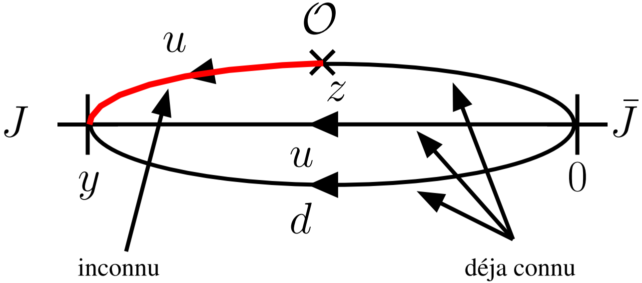


3 points functions

- Needs matrix elements :

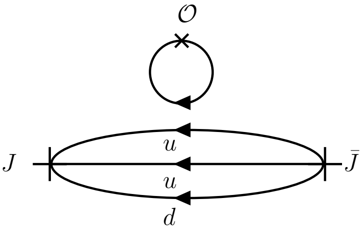
$$\langle N(p_f, s_f)|\mathcal{O}|N(p_i, s_i)\rangle$$

- involves new propagators



Disconnected quark contractions

- The operator can be connected to the valence quarks only through the gluons



- Costly
- Consider only the non singlet combination (p-n)

Operator renormalization

- On the lattice one deals with bare (cutoff dependant) operators.
- They need renormalization to be compared to continuum observables because they do not contain information beyond $1/a$.

$$O_R^i(\mu) = \sum_j Z^{ij}(a\mu,\beta)\, O^j(a)$$

- renormalization constants are evaluated non-perturbatively in the RI-MOM scheme (and possibly converted to $\overline{MS}$ scheme)

Preliminary results

- One value of β ($a = 0.0855$ fm)
- Some renormalisation constants not yet calculated
- p-n combination only

Form factors

Currents

$$V_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$$
$$A_\mu = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$$

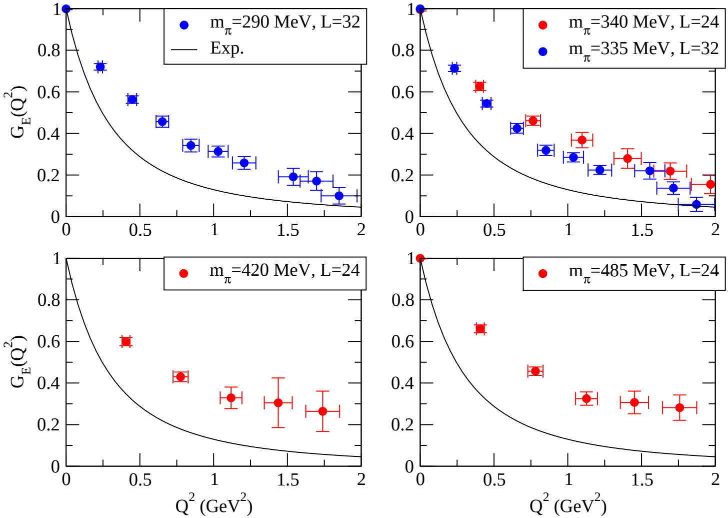
Form factor decomposition

$$\langle N(p_f,s_f)|V_\mu|N(p_i,s_i)\rangle = \bar{u}(p_f,s_f)\left[\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}Q_\nu}{2m}F_2(Q^2)\right]u(p_i,s_i)$$
$$\langle N(p_f,s_f)|A_\mu|N(p_i,s_i)\rangle = \bar{u}(p_f,s_f)\left[\gamma_5\gamma_\mu G_A(Q^2) + \frac{Q_\mu\gamma_5}{2m}G_p(Q^2)\right]u(p_i,s_i)$$

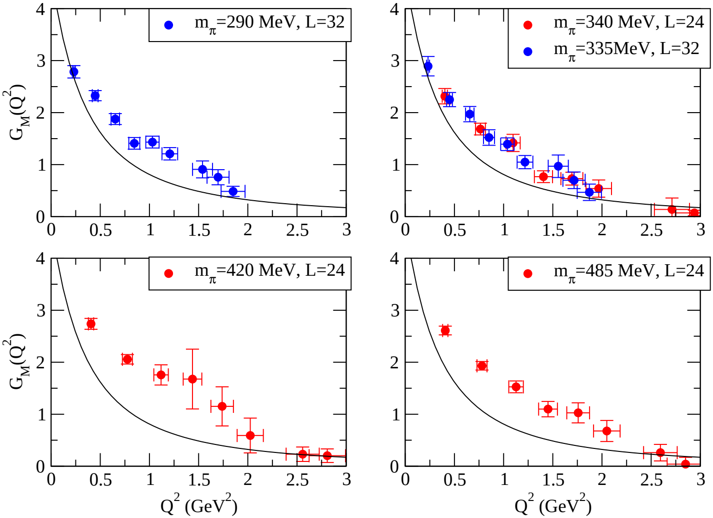
use also

$$G_E = F_1(Q^2) - \frac{Q^2}{4m^2}F_2(Q^2)\,, \qquad G_M = F_1(Q^2) + F_2(Q^2)\,.$$

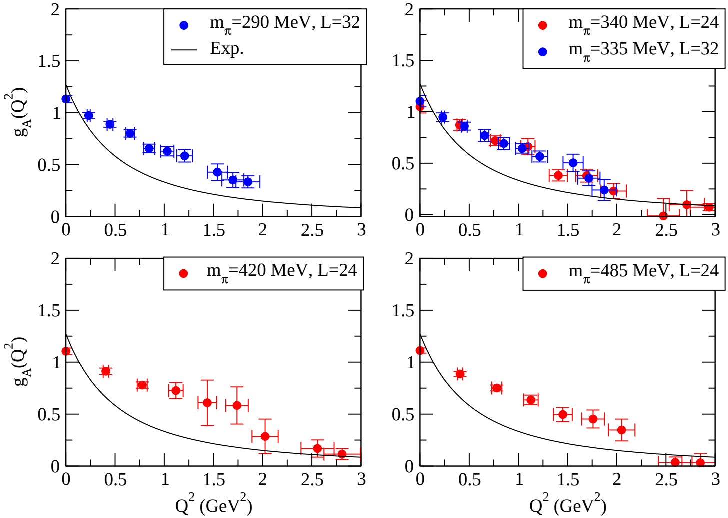
Electric form factor



Magnetic form factor

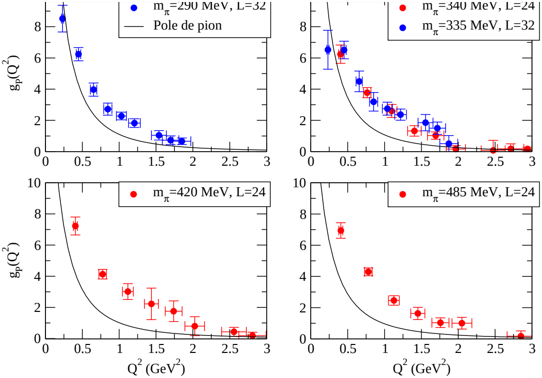


Axial form factor

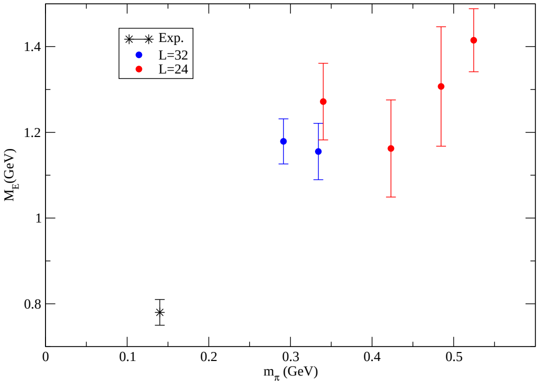


Pseudoscalar form factor

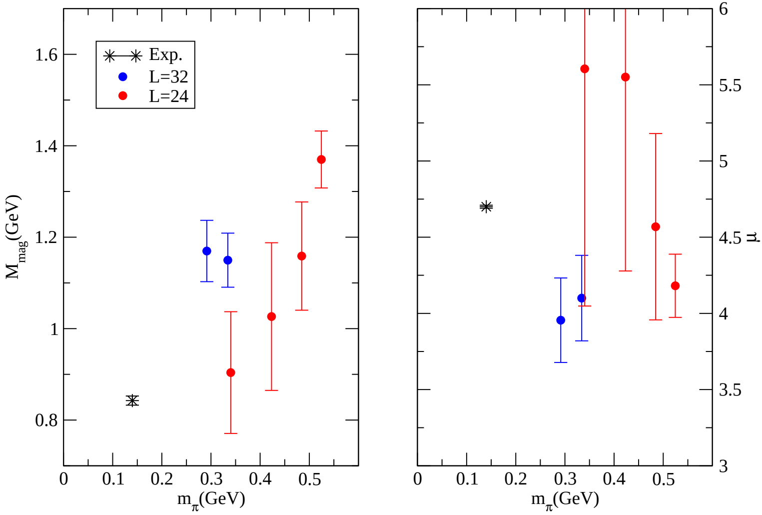
$$G_P^{\text{pion pole}}(Q^2) = \frac{4m^2 G_A(Q^2)}{Q^2 + m_\pi^2}$$



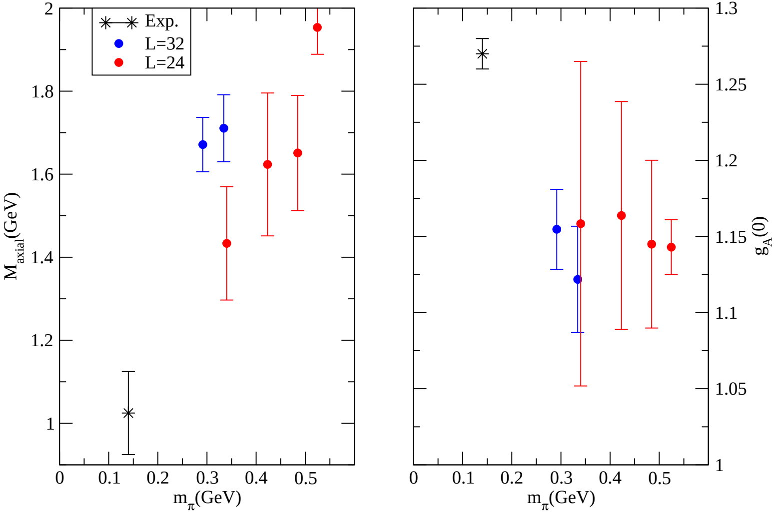
Electric dipole mass (isovector !)



Magnetic dipole mass and $G_M(0)$



Axial dipole mass and $G_A(0)$



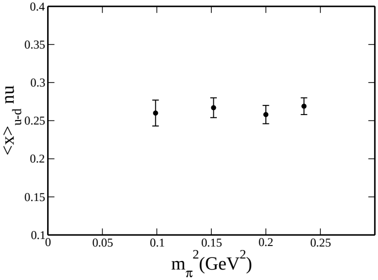
$\langle x \rangle_{u-d}$ et $\langle x \rangle_{\Delta u - \Delta d}$

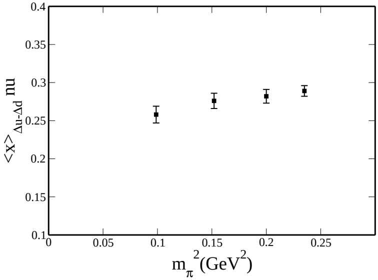
$$\int dx\, xq(x) = \langle x \rangle_q, \quad \int dx\, x\Delta q(x) = \langle x \rangle_{\Delta q}$$

need the operators

$$\begin{aligned} O_q^{00}(x) &= \bar{q}(x)\Big[\gamma_0 \overleftrightarrow{D}_0 - \frac{1}{3}(\gamma_1 \overleftrightarrow{D}_1 + \gamma_2 \overleftrightarrow{D}_2 + \gamma_3 \overleftrightarrow{D}_3)\Big] q(x) \\ \tilde{O}_q^{i0}(x) &= \bar{q}(x)\gamma_5(\gamma_i \overleftrightarrow{D}_0 + \gamma_0 \overleftrightarrow{D}_i) q(x), \quad i = 1, 2, 3 \end{aligned}$$

Résults





- Figures show bare results
- $Z_{00}^{\overline{MS}}(\mu = 2\text{ GeV}) = 1.20(3)$

Result

Calcul : $\langle x \rangle_{u-d} = 0.312(22)$

Experiment (4 GeV^2) : $\langle x \rangle_{u-d} = 0.154(3)$

Final comments

- tmQCD allows simulations at small quark masses at a modest cost.
- m_π=200MeV is running
- tuning 1 single parameter makes the observables $O(a^2)$ improved
- no sign of isospin or parity violation in our results

- Masses come quite well (hyperons too)
- Form factors looks good but need continuum extrapolation. The gauge configuration at $a = 0.65 fm$ and $a = 0.55 fm$ exist. Just do it.
- The pdf pose a serious problem which must be solved before the GPD calculation can be considered.
- How long shall we wait until disconnected diagrams are included ? ? ?

End

End

- $N_f = 2$ Tree level Symanzik
- $N_f = 2 + 1 + 1$ Iwazaki