Crystalline Condensates in the Chiral Symmetry Breaking Phase Diagram

Gerald Dunne

University of Connecticut

G.Başar & GD, arxiv:0803.1501, PRL **100**, 200404 (2008)
arxiv:0806.2659, PRD **78**, 065022 (2008)
G.Başar, GD & M.Thies, arxiv:0903.1868, PRD **79**, 105012 (2009)
F. Correa, GD & M.Plyushchay, arxiv:0904.2768, Ann. Phys. 324, 2522 (2009)
G.Başar, GD & D.Kharzeev, in preparation

Happy Birthday Tony!

motivation: QCD phase diagram

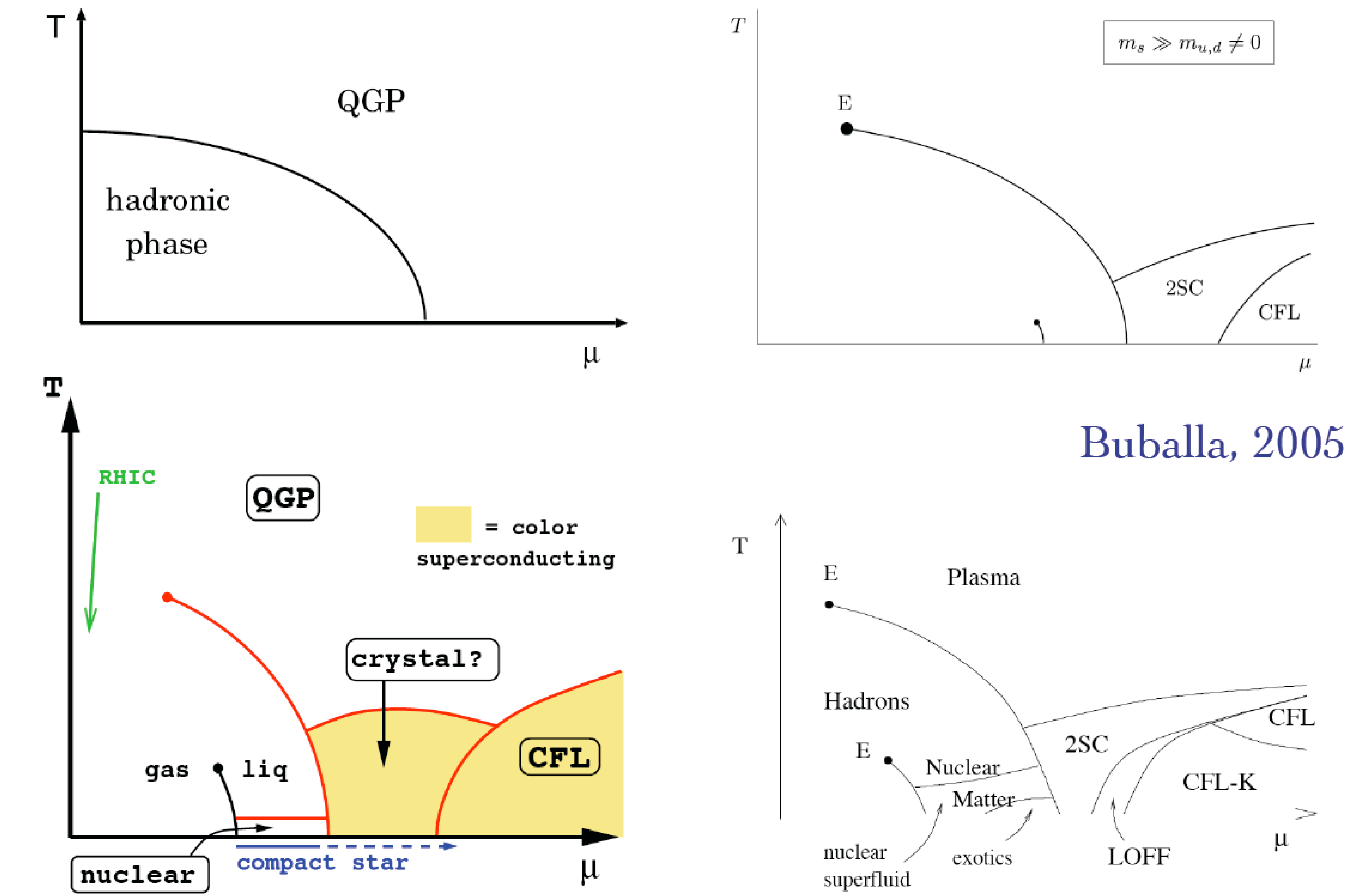
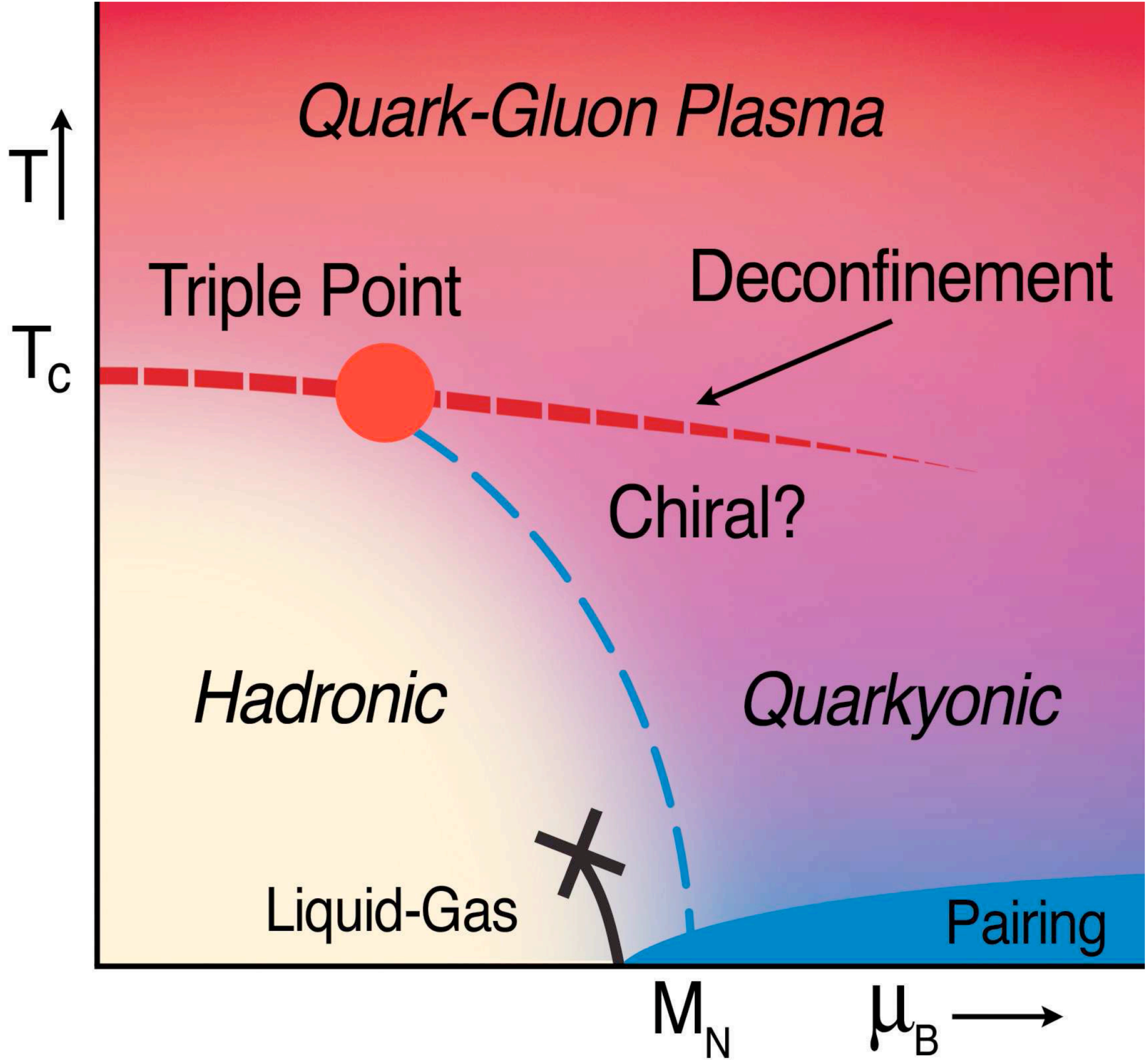


Figure 1.1: Schematic QCD phase diagrams in the chemical potential–temperature plane. Upper left: generic phase diagram of the "pre-color superconductivity era", see, e.g., Refs. [3, 4]. The other diagrams are taken from the literature. Upper right: Rajagopal (1999) [17]. Lower left: Alford (2003) [23]. Lower right: Schäfer (2003) [14].



Andronic et al, 2009

Hidaka, Kojo, McLerran, Pisarski, 2009

Outline

- Gross-Neveu Models : GN_2 and NJL_2
- gap equation -> nonlinear Schrödinger equation
- GN_2 and NJL_2 phase diagrams
- Ginzburg-Landau expansion and AKNS hierarchy
- conclusions and outlook

Gross-Neveu Models

Gross/Neveu, 1974

GN$_2$ $$\mathcal{L}_{\rm GN} = \bar\psi\, i\,\partial\!\!\!/ \psi + \frac{g^2}{2}\left(\bar\psi\psi\right)^2 \qquad \psi \to \gamma^5\,\psi$$

χGN$_2$ NJL$_2$ $$\mathcal{L}_{\rm NJL} = \bar\psi\, i\,\partial\!\!\!/ \psi + \frac{g^2}{2}\left[\left(\bar\psi\psi\right)^2 + \left(\bar\psi i\gamma^5\psi\right)^2\right] \qquad \psi \to e^{i\alpha\,\gamma^5}\,\psi$$

- renormalizable at large N_f
- asymptotically free
- chiral symmetry breaking
- dynamical mass generation $\quad m = \Lambda\, e^{-\frac{\pi}{N_f\, g^2}}$
- self-bound baryonic states $\quad m_B = \frac{2}{\pi}\, m \quad$ DHN, 1975

Gross-Neveu Models

Gross/Neveu, 1974

GN$_2$ $$\mathcal{L}_{\rm GN} = \bar{\psi}\, i\, \partial\!\!\!/ \psi + \frac{g^2}{2}\left(\bar{\psi}\psi\right)^2$$ $$\psi \to \gamma^5\, \psi$$

χGN$_2$
NJL$_2$ $$\mathcal{L}_{\rm NJL} = \bar{\psi}\, i\, \partial\!\!\!/ \psi + \frac{g^2}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi} i\gamma^5 \psi\right)^2\right]$$ $$\psi \to e^{i\alpha\,\gamma^5}\, \psi$$

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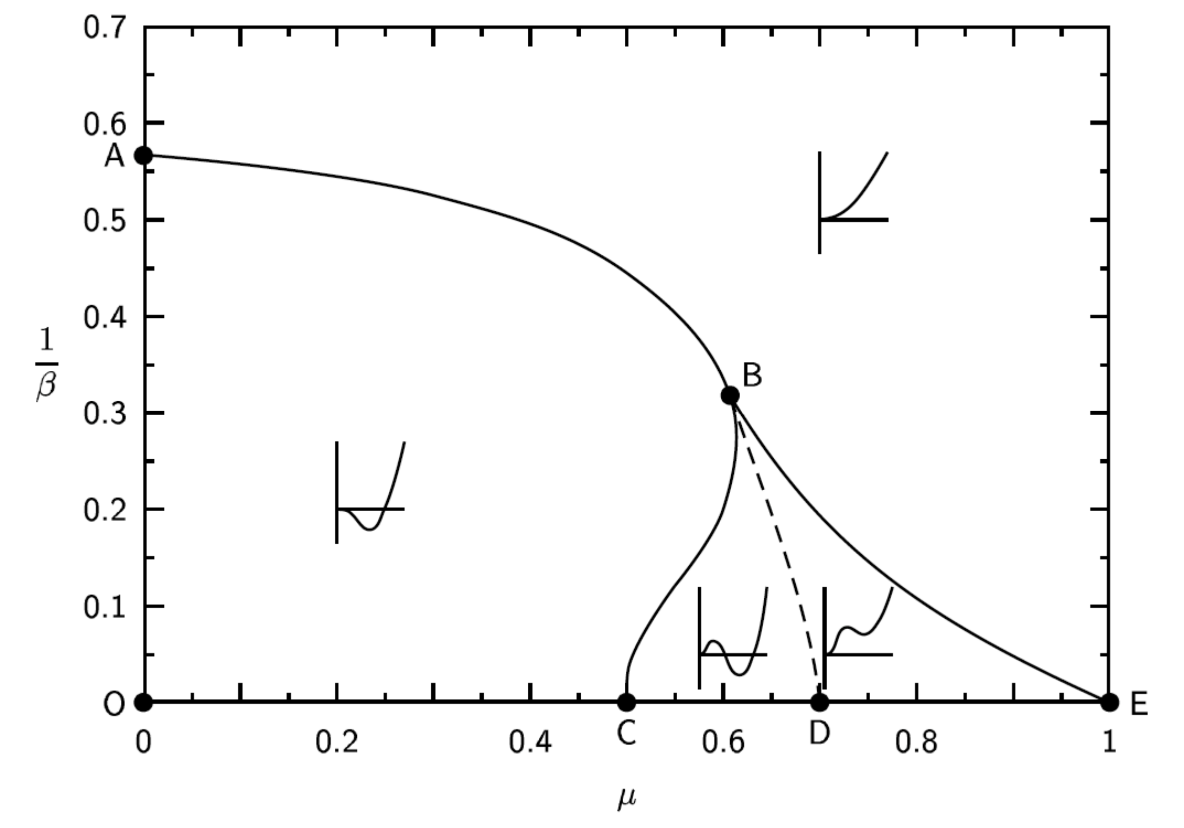
(T, μ) phase diagram?

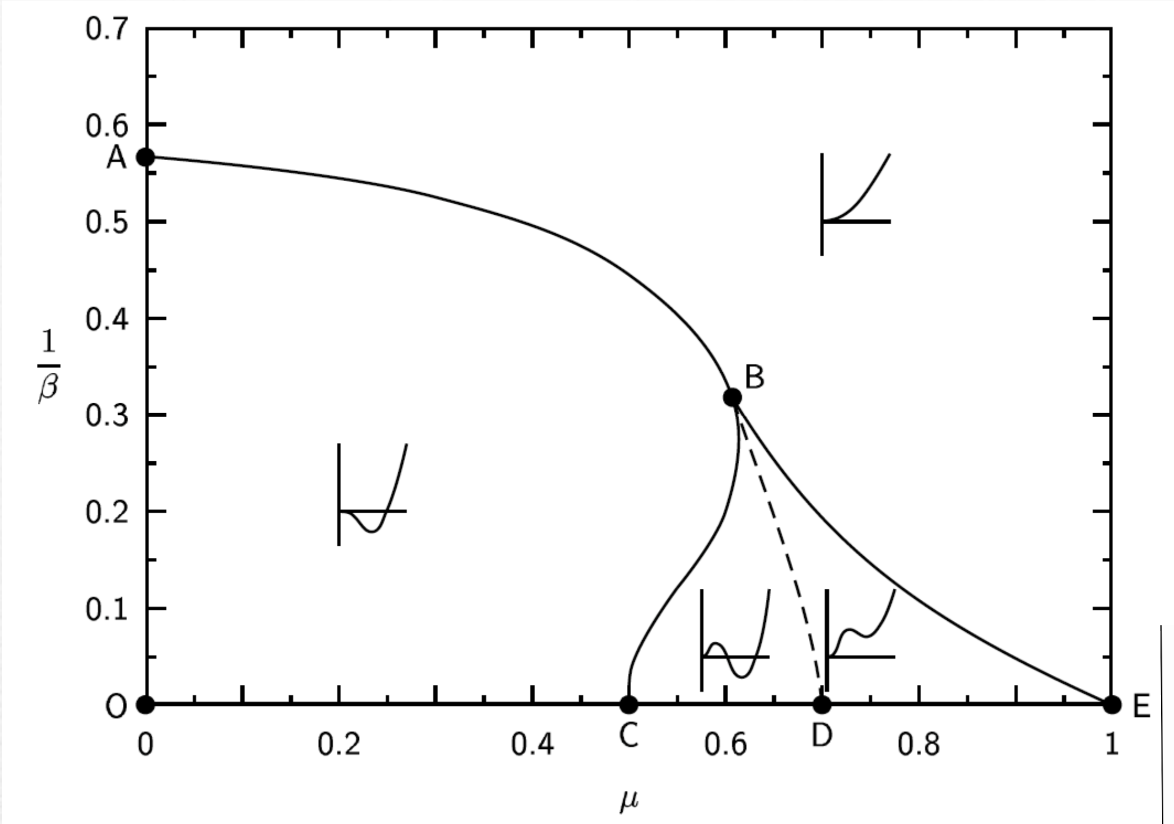


Phase diagram of Gross-Neveu model

uniform condensate

Wolff, 1985

Phase diagram of Gross-Neveu model



uniform condensate

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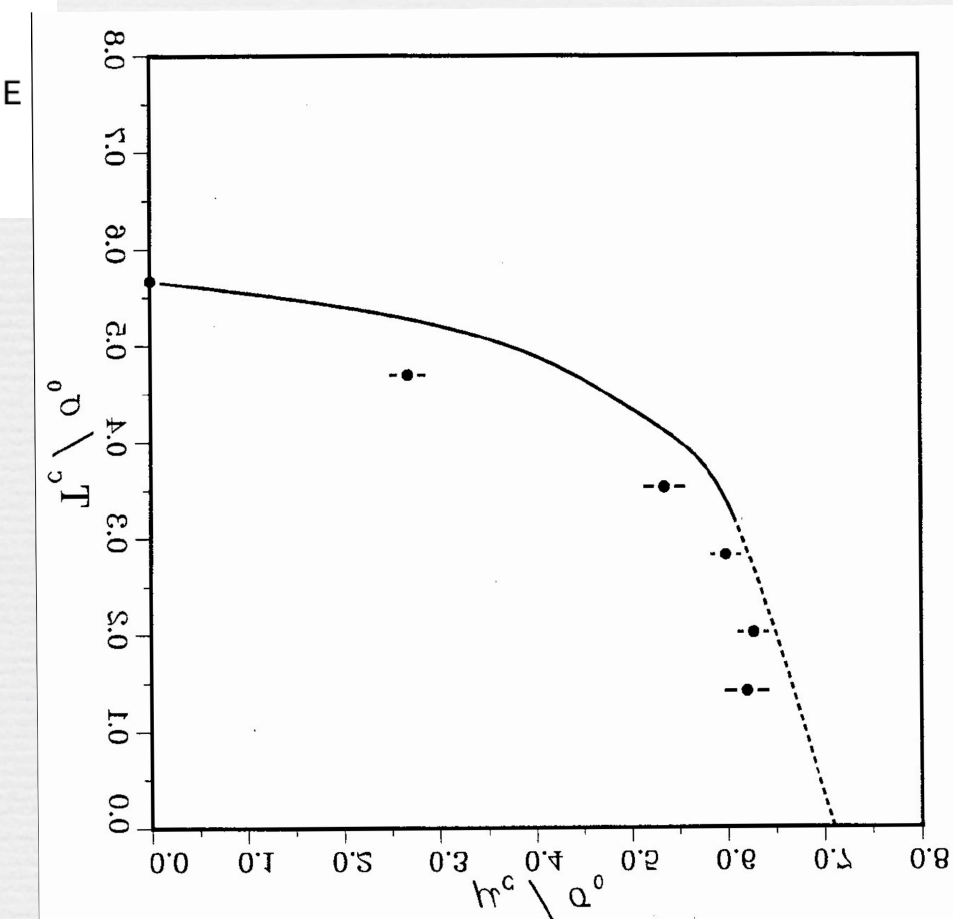
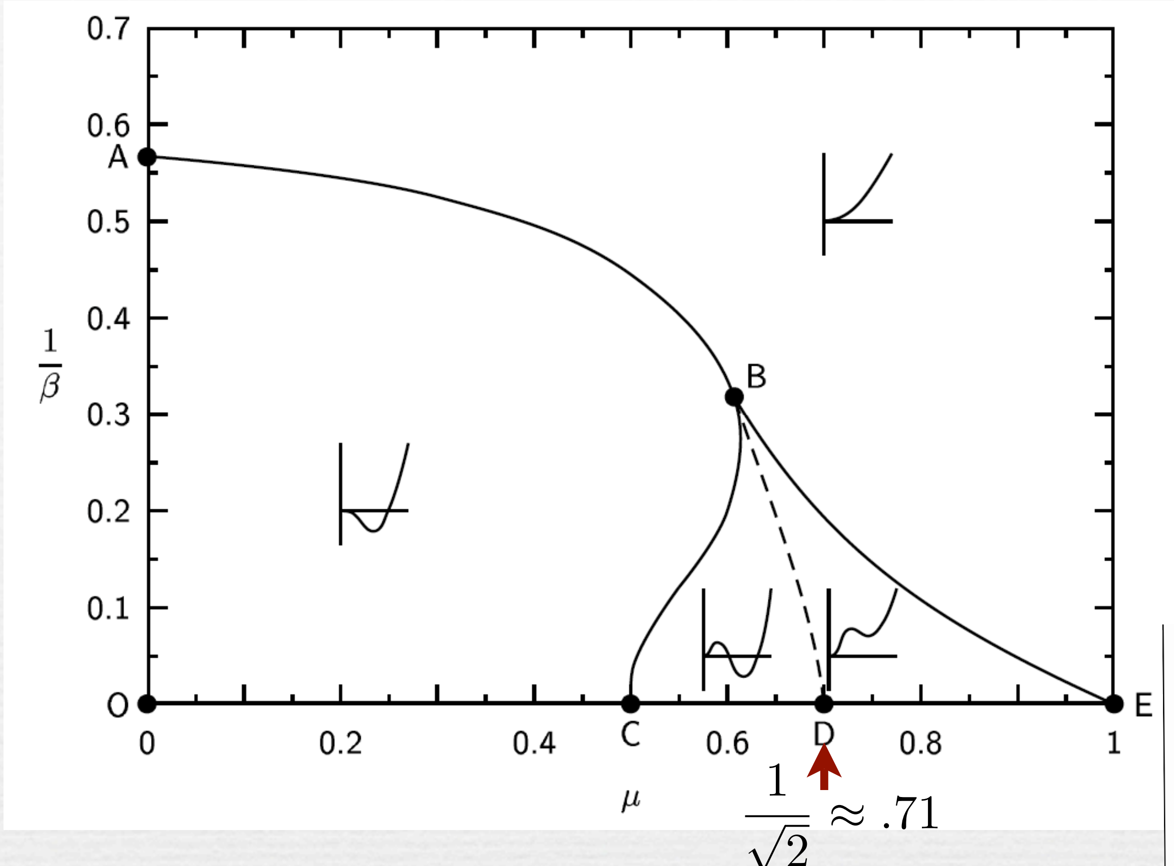


lattice analysis of GN_2

Karsch et al 1986

Phase diagram of Gross-Neveu model



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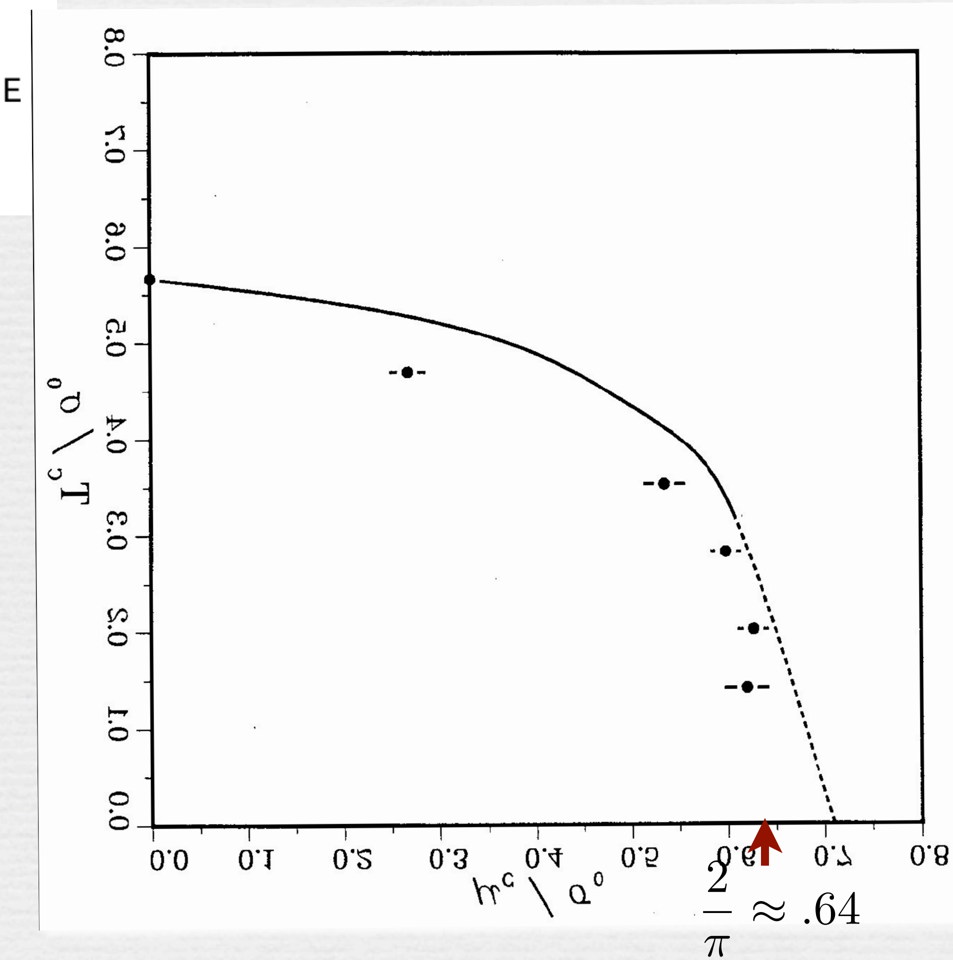

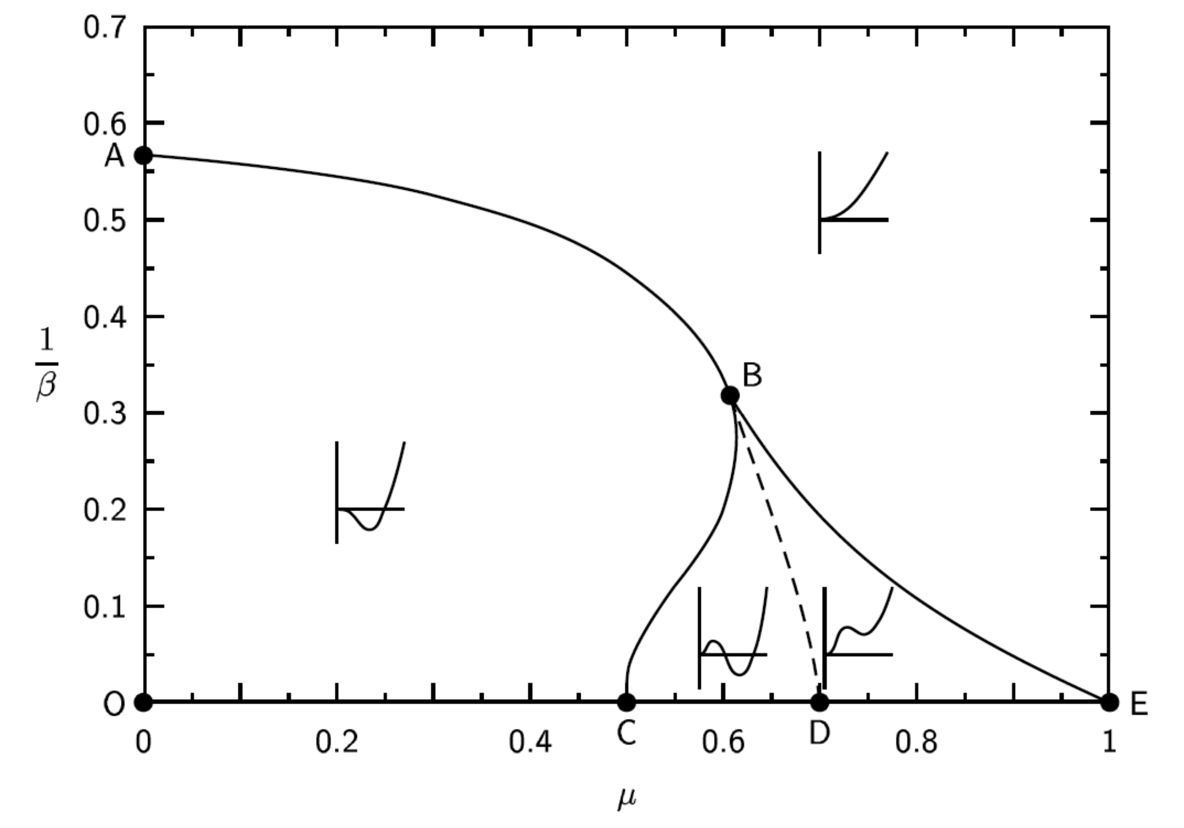

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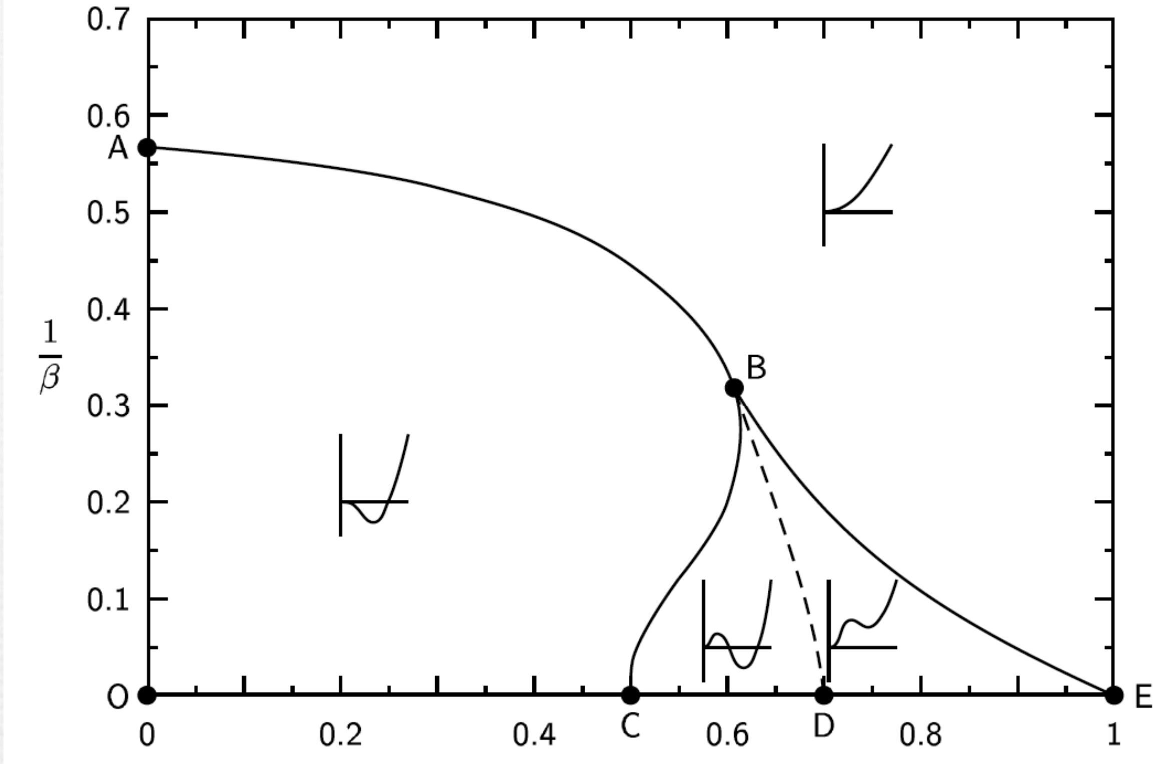

uniform condensate

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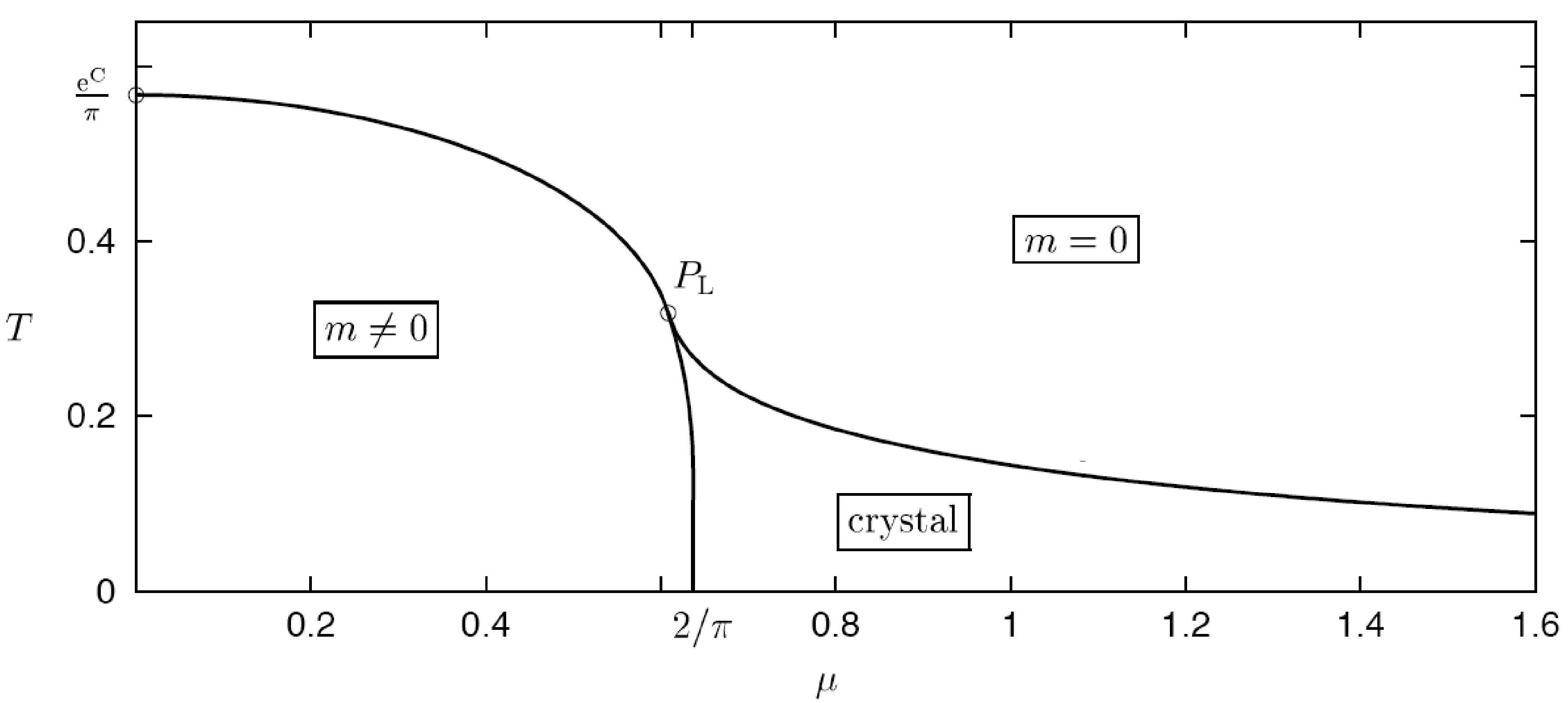


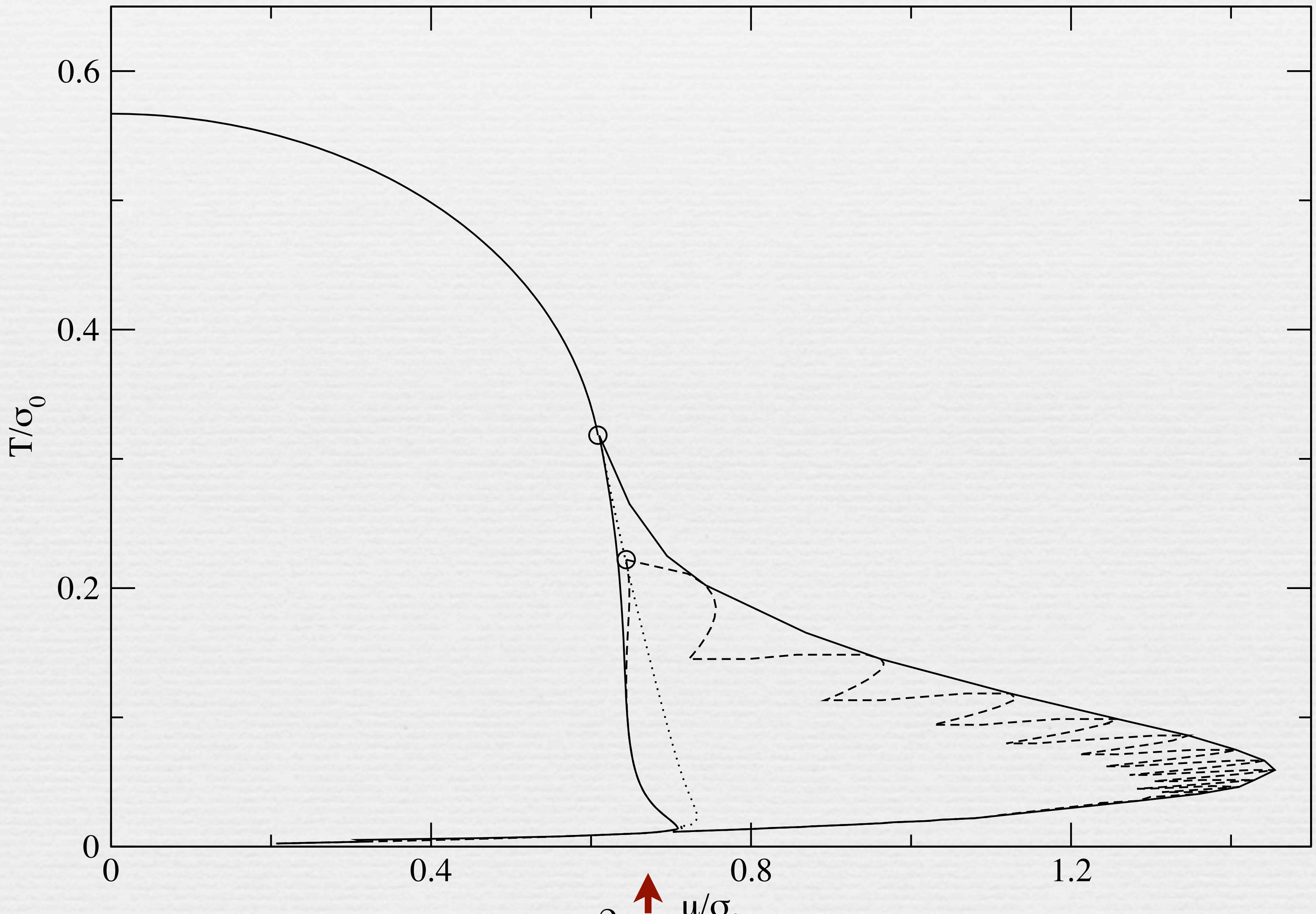
Thies & Urlichs, 2005

Basar, GD, Thies, 2009

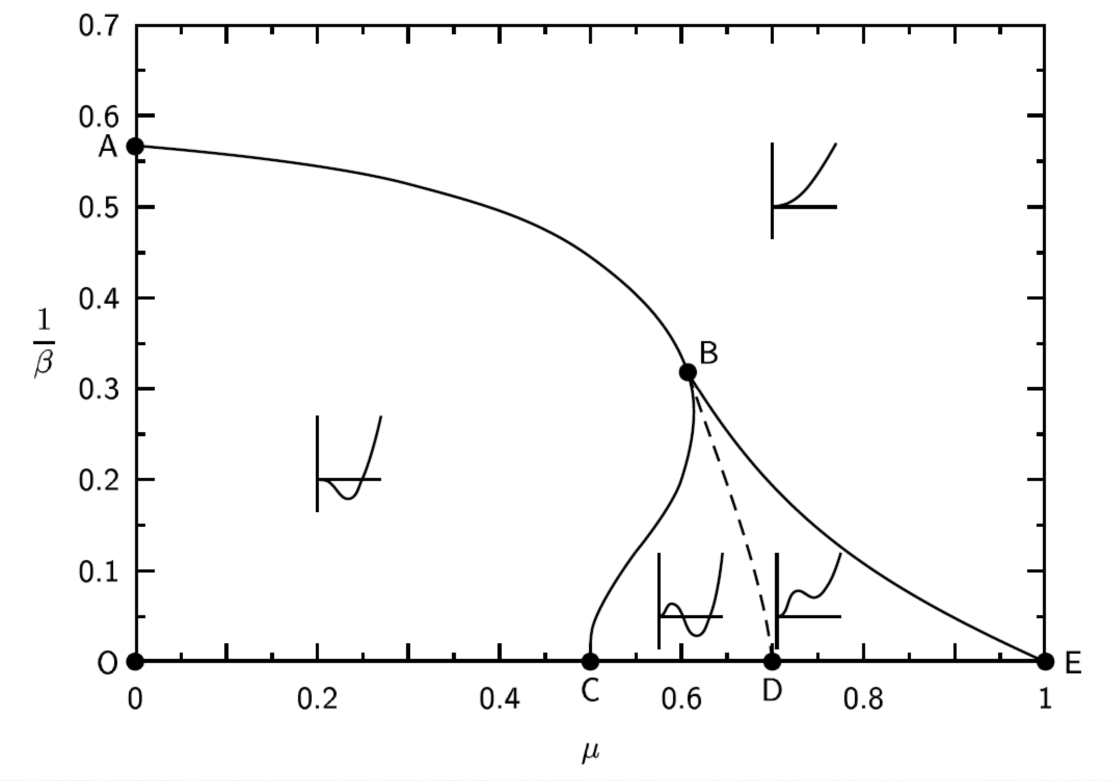
periodic, crystalline, phase

lattice GN_2 model

de Forcrand/Wenger 2006



$\frac{2}{\pi} \approx .64$

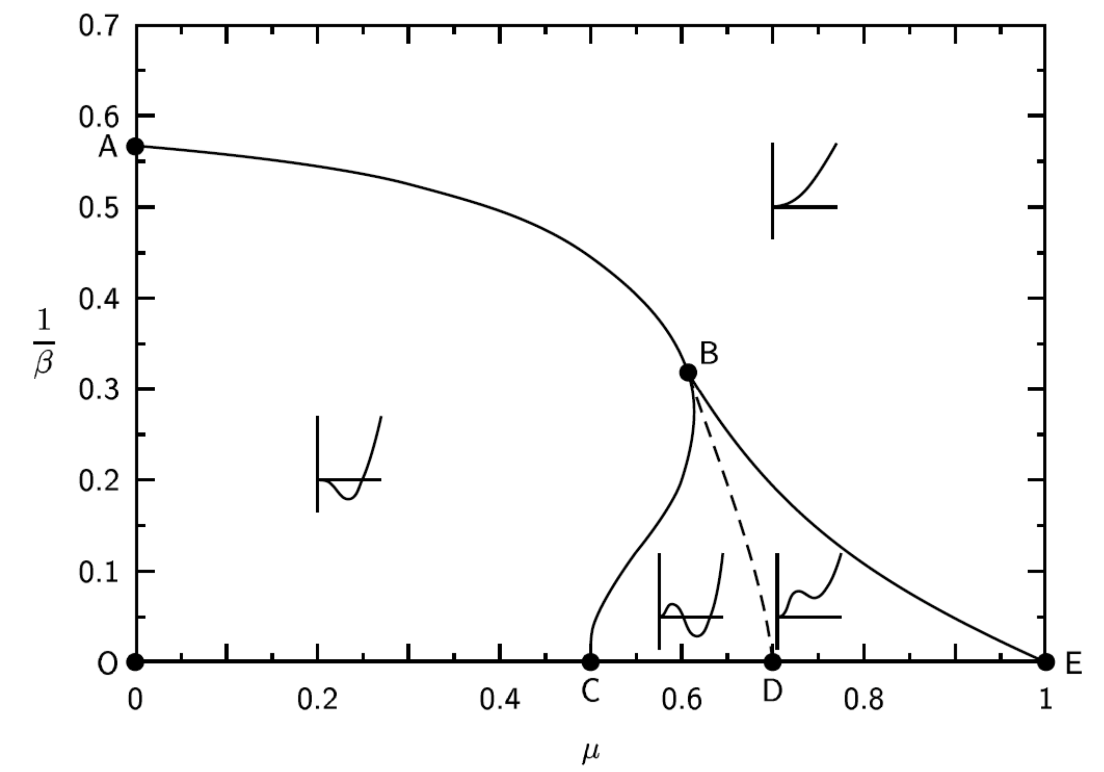


Phase diagram of NJL$_2$ model

uniform condensate
(same as GN$_2$)

Wolff, 1985
Barducci et al, 1995

Phase diagram of NJL$_2$ model



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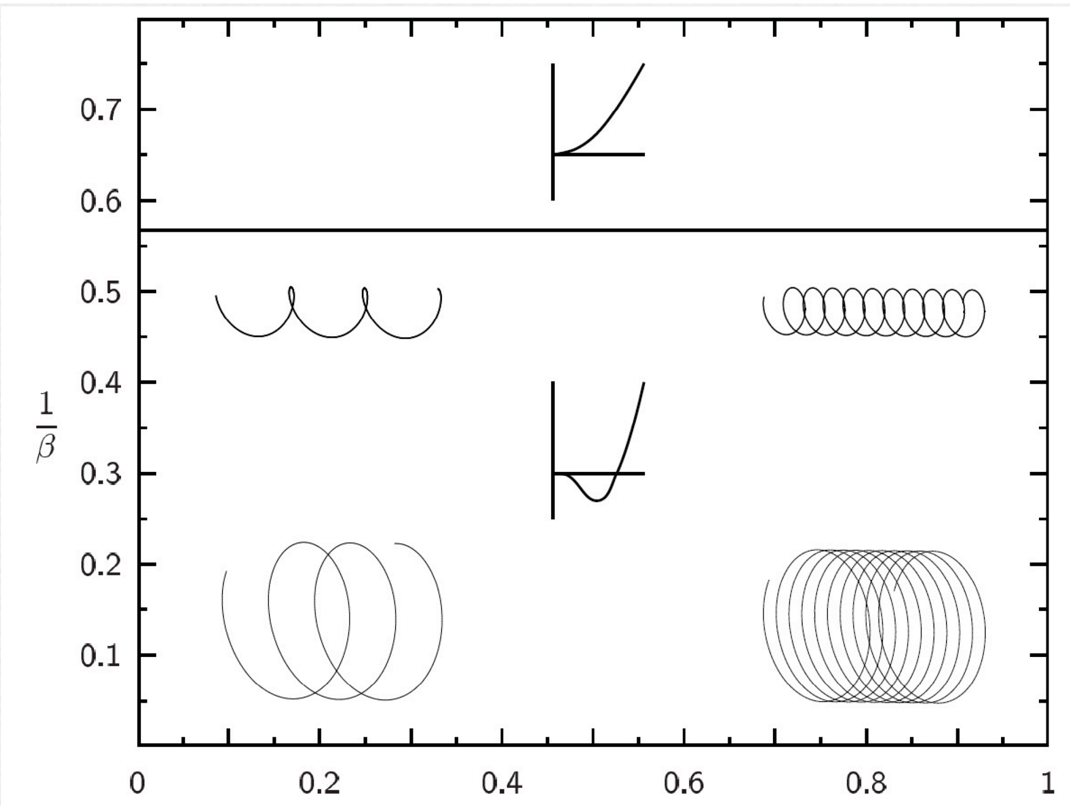

"chiral spiral"

$$\sigma(x) - i\pi(x) = A\, e^{2i\mu x}$$

Schön, Thies, 2000

Basar, GD, Thies, 2009

Gap Equation Approach

partition function

$$Z = \int \mathcal{D}\psi \, \exp\left\{ - \int \left[\bar{\psi}\partial\!\!\!/\psi - \frac{g^2}{2}\left(\bar{\psi}\psi\right)^2 \right] \right\}$$

Gap Equation Approach

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scalar condensate σ

Gap Equation Approach

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scalar condensate σ

$$\mathcal{L}_{\text{eff}} = \bar{\psi} \not{\partial} \psi + \sigma \left(\bar{\psi}\psi \right) + \frac{1}{2g^2} \sigma^2$$

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effective potential

$$V[\sigma] = \frac{1}{2g^2 N} \sigma^2 - \ln \det \left[\partial\!\!\!/ + \sigma \right]$$

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$$\frac{\sigma(x)}{g^2 N} = \frac{\delta}{\delta\sigma(x)} \ln \det \left[\partial\!\!\!/ + \sigma(x) \right]$$

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at finite T, μ

gap equation : GN_2

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gap equation : GN_2

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DHN(1975): reflectionless potentials: $V_\pm = \sigma^2 \pm \sigma'$

single kink: $\sigma(x) = m \tanh(m\, x)$

gap equation : GN_2

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DHN(1975): reflectionless potentials: $V_\pm = \sigma^2 \pm \sigma'$

single kink: $\sigma(x) = m \tanh(m\,x)$

Thies/Urlichs (2005): finite-gap potentials: $V_\pm = \sigma^2 \pm \sigma'$

kink crystal: $\sigma(x) = m\,\nu\,\frac{\mathrm{sn}(m\,x;\nu)\,\mathrm{cn}(m\,x;\nu)}{\mathrm{dn}(m\,x;\nu)}$

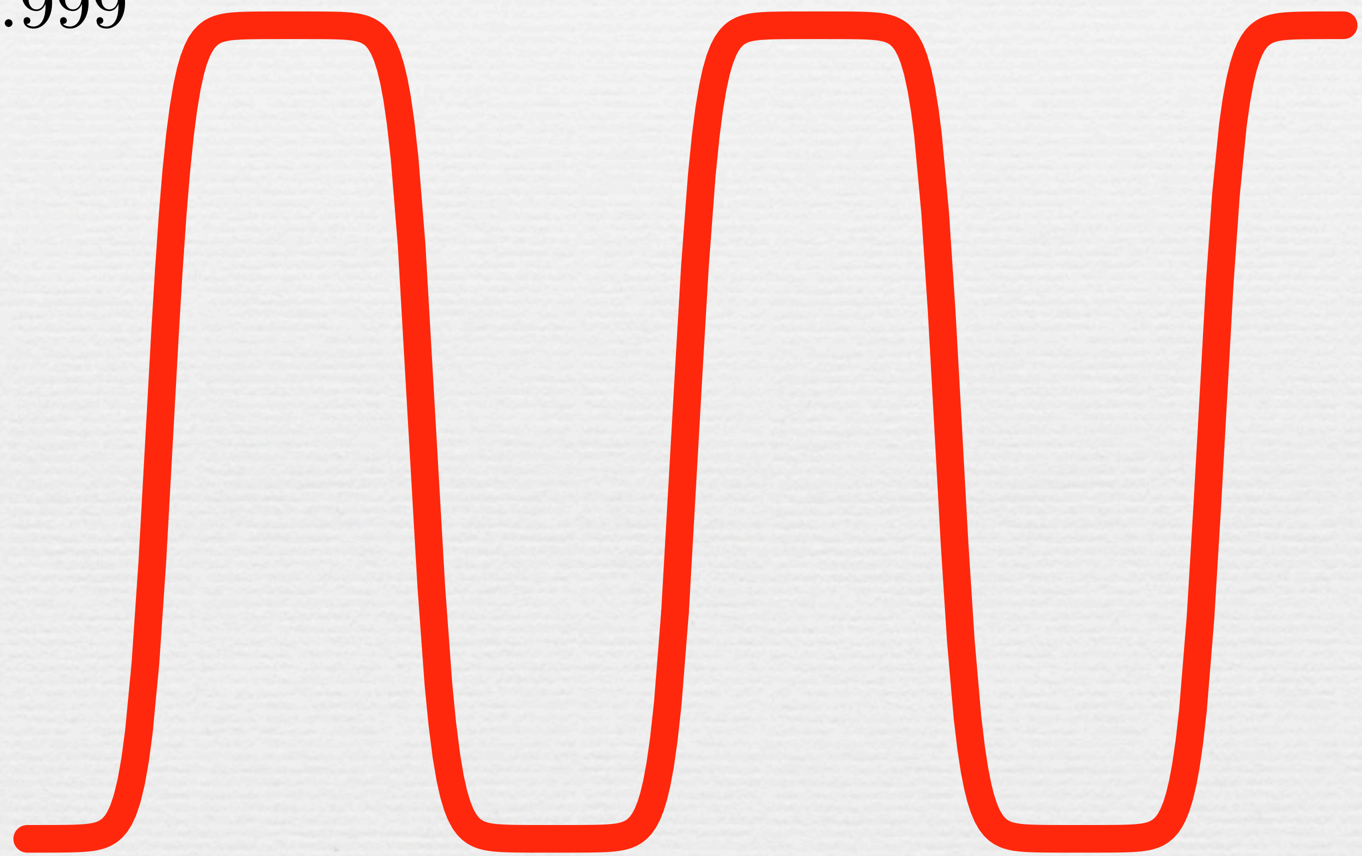
kink crystal

$$\sigma(x) = \nu\, m\, \frac{\mathrm{sn}(mx;\nu)\mathrm{cn}(mx;\nu)}{\mathrm{dn}(mx;\nu)}$$

kink crystal

$$\sigma(x) = \nu\, m\, \frac{\mathrm{sn}(mx;\nu)\,\mathrm{cn}(mx;\nu)}{\mathrm{dn}(mx;\nu)}$$

$\nu = 0.999$



kink crystal

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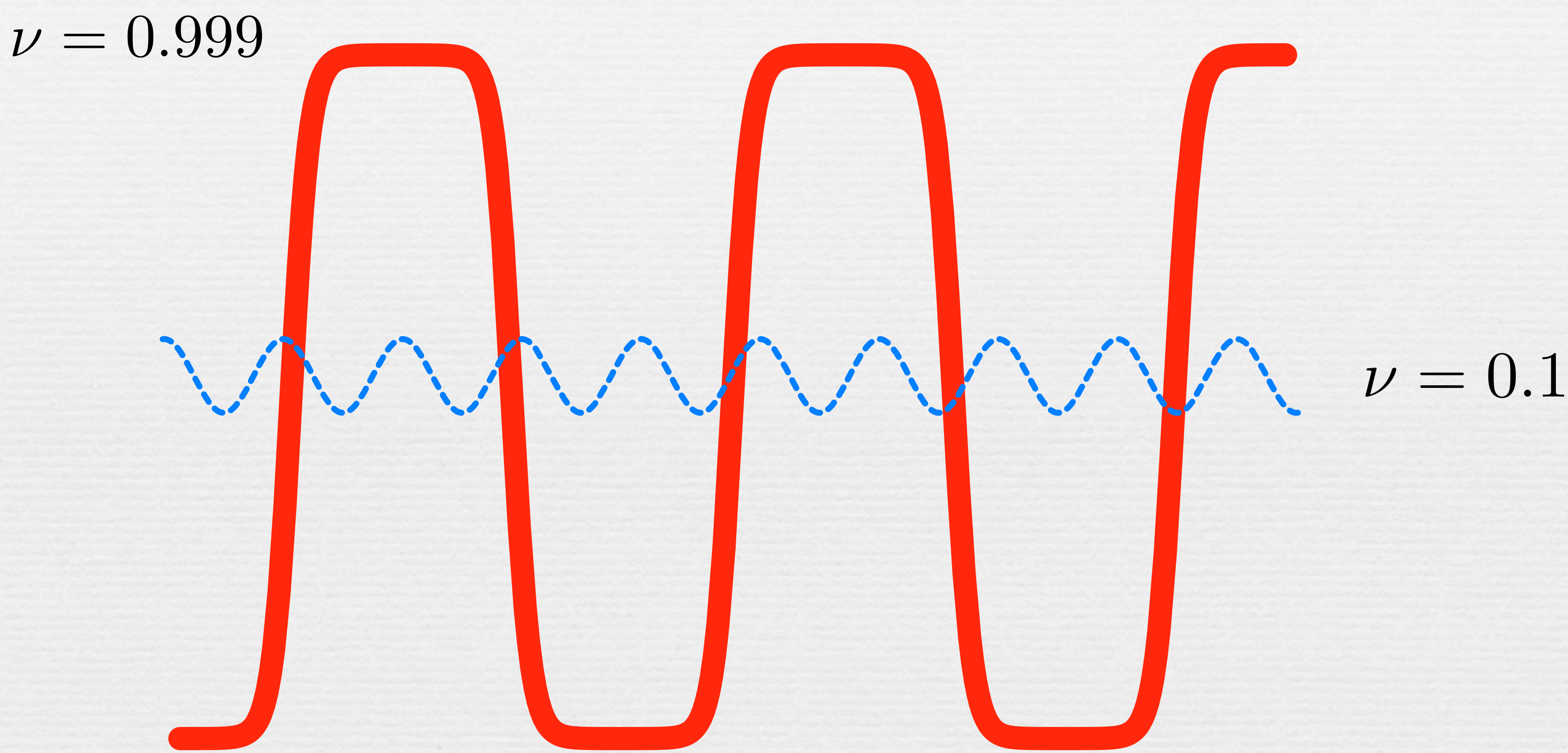

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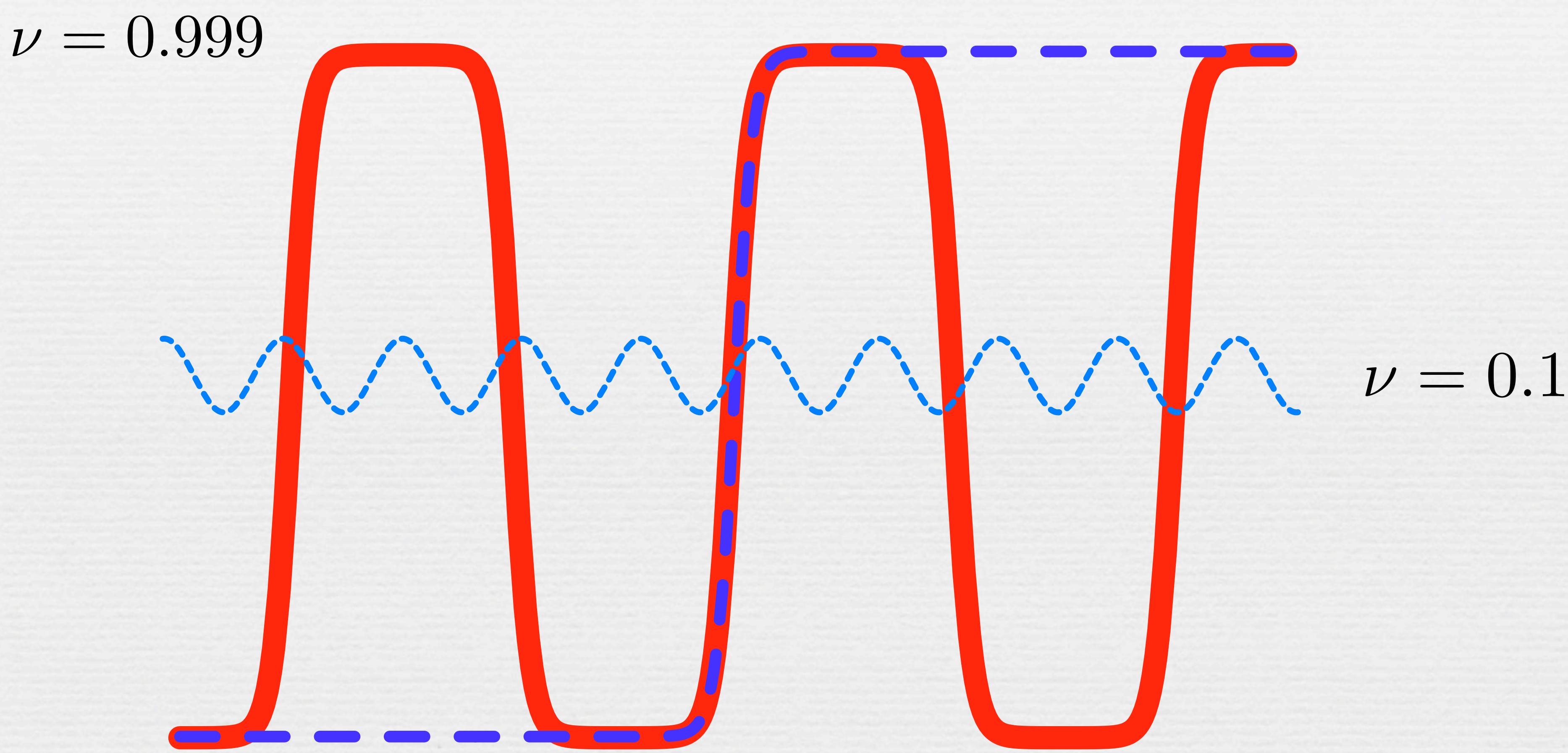

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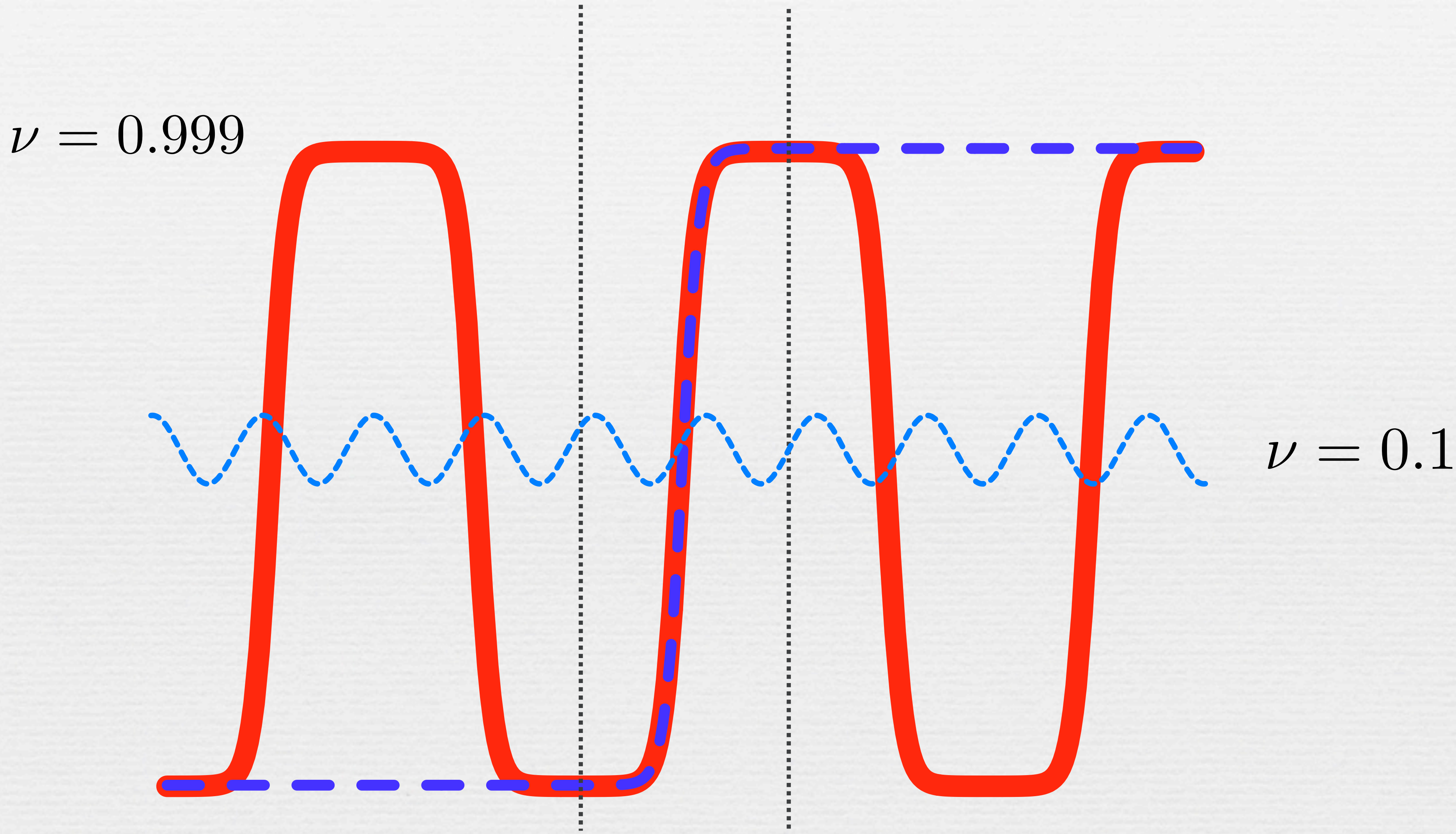


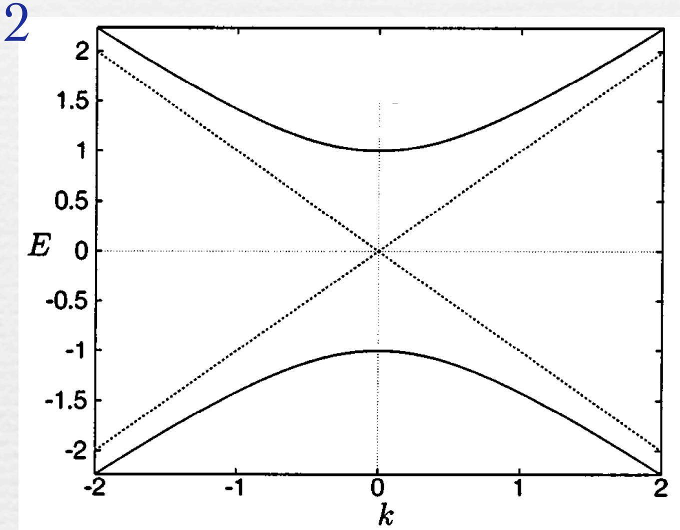
Peierls Instability

one dimension: gap formation at the Fermi surface can lead to breakdown of translational symmetry

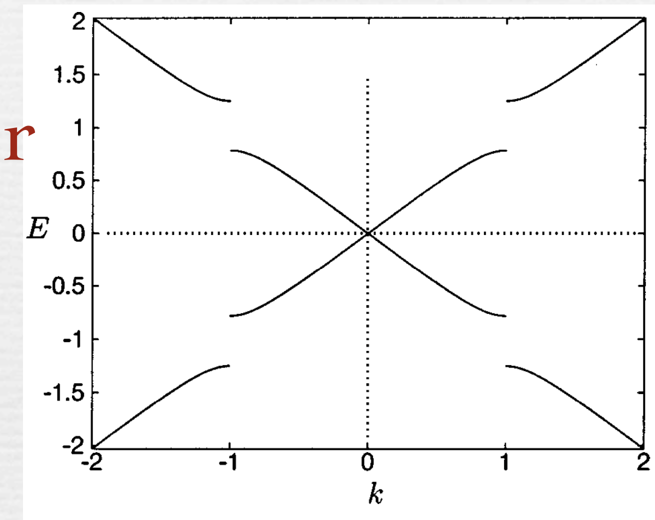
Peierls Instability

one dimension: gap formation at the Fermi surface can lead to breakdown of translational symmetry

GN_2



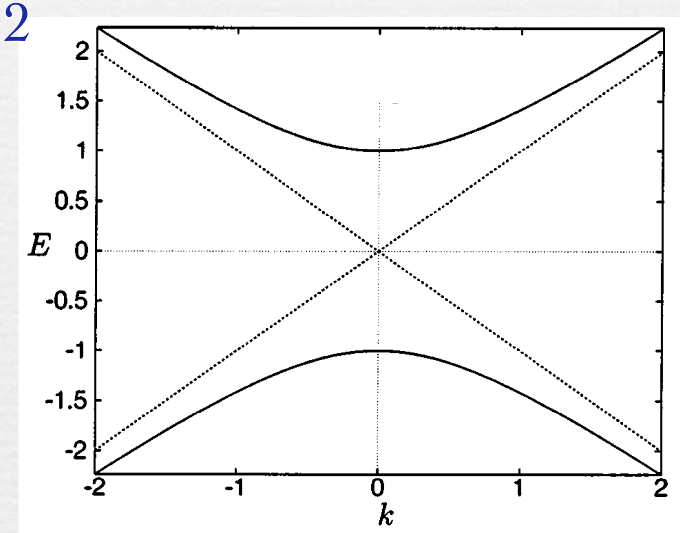
or



Peierls Instability

one dimension: gap formation at the Fermi surface can lead to breakdown of translational symmetry

GN_2



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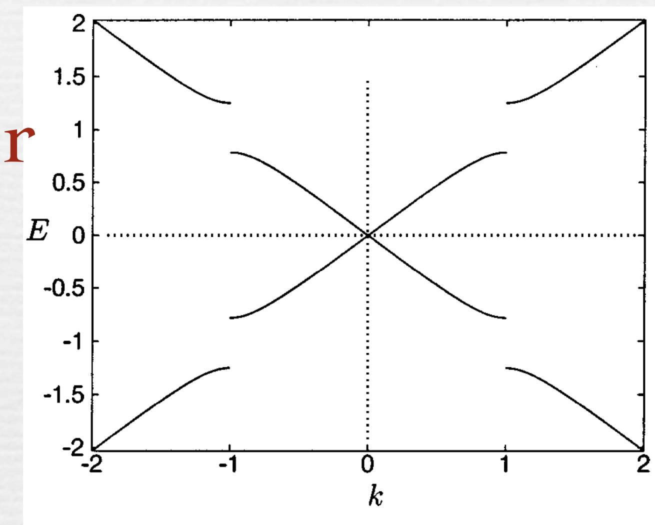
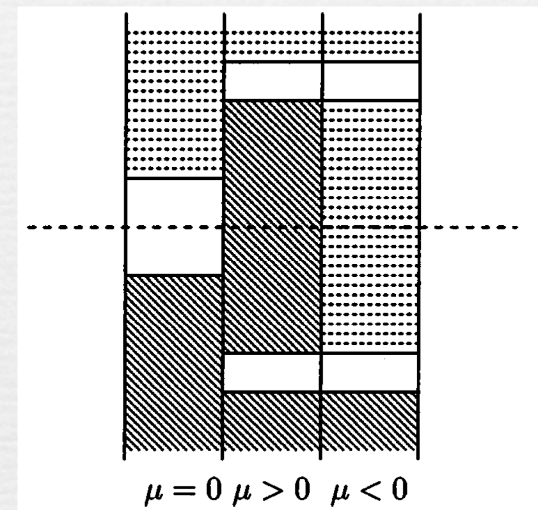
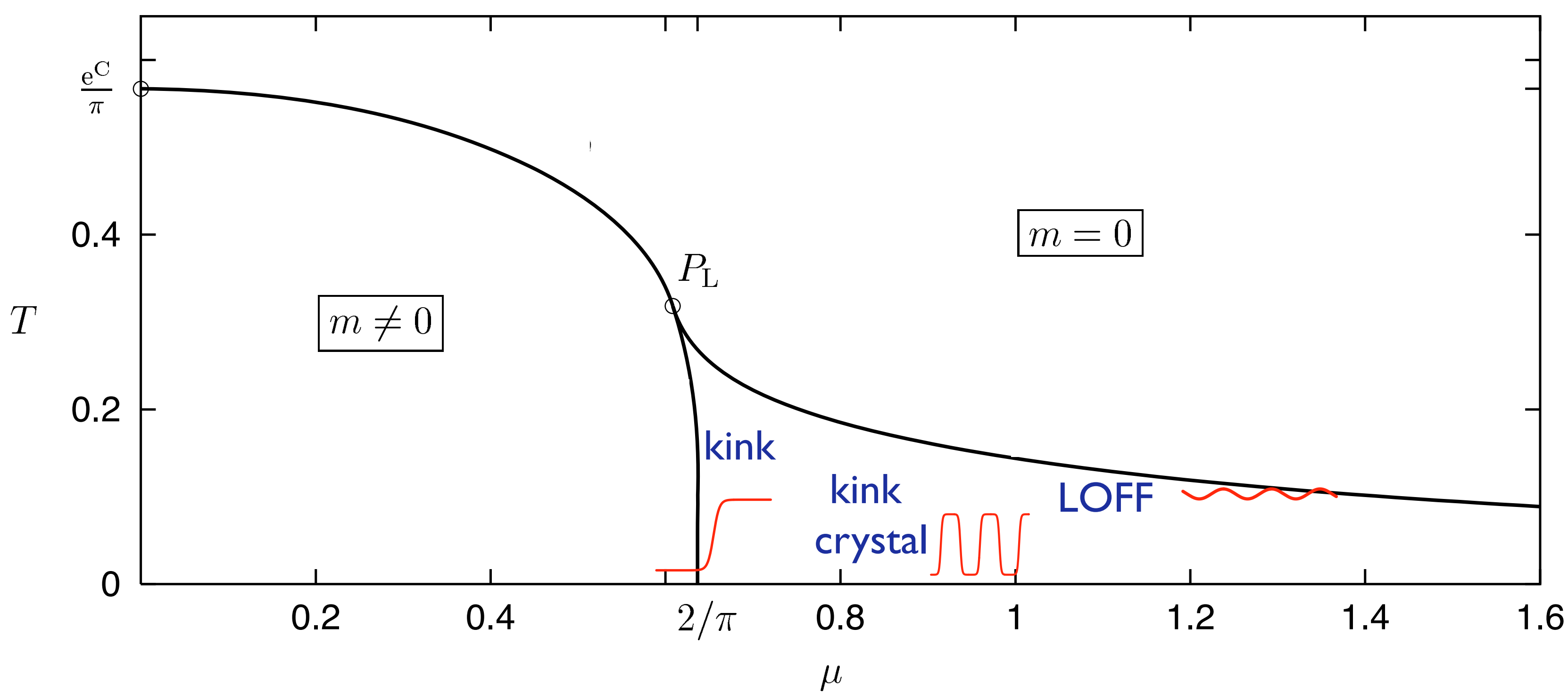


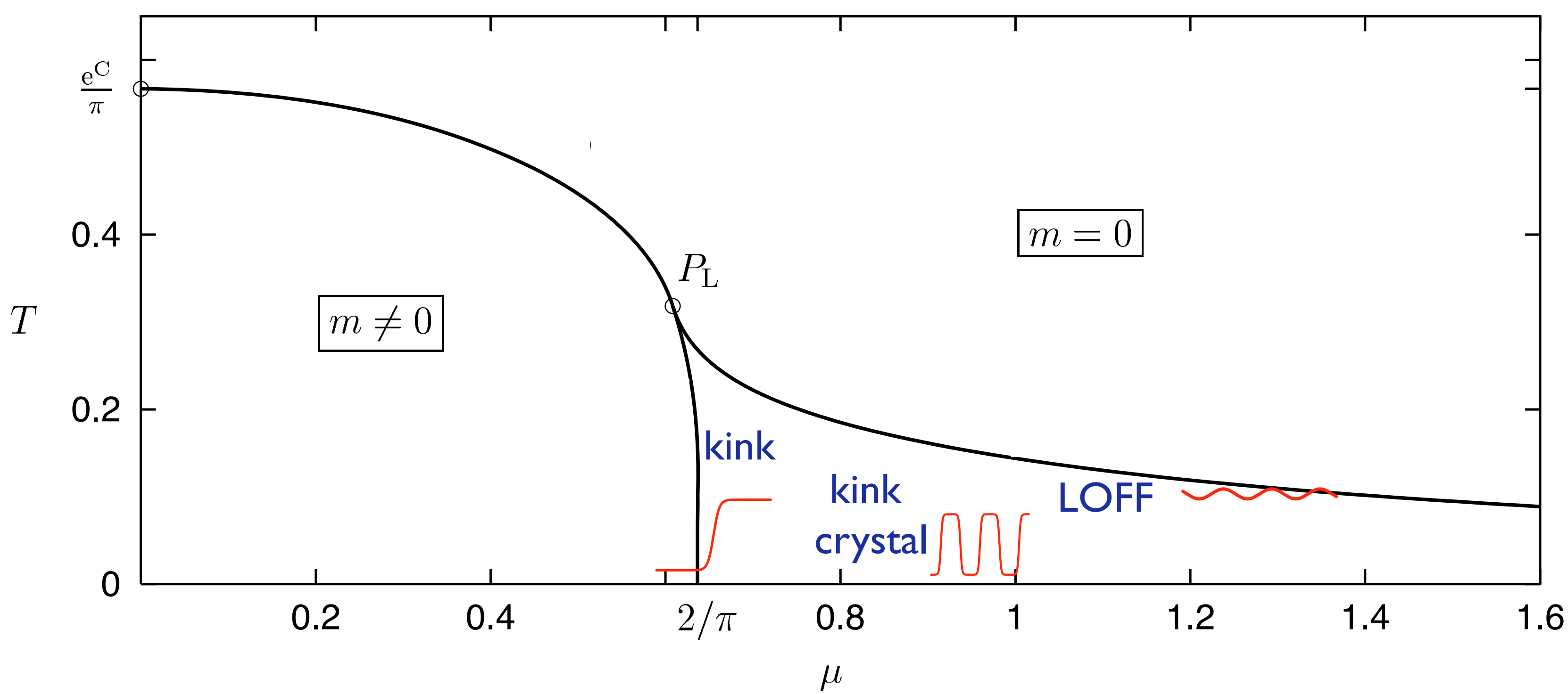


Thies, Urlichs, 2003



$\frac{e^C}{\pi}$
0.4
0.2
0
T
0.2
0.4
$2/\pi$
0.8
1
1.2
1.4
1.6
μ
$m \neq 0$
$m = 0$
P_L
kink
kink
crystal
LOFF


all gap
no gap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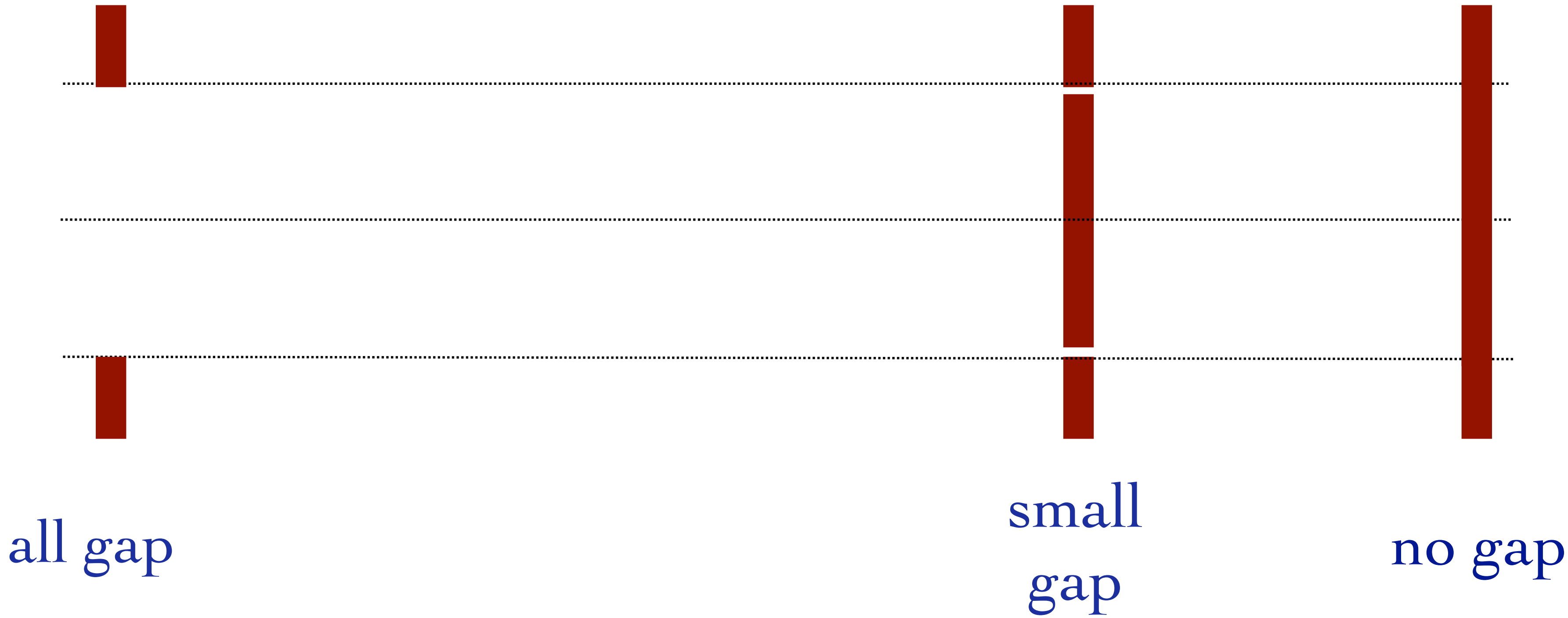
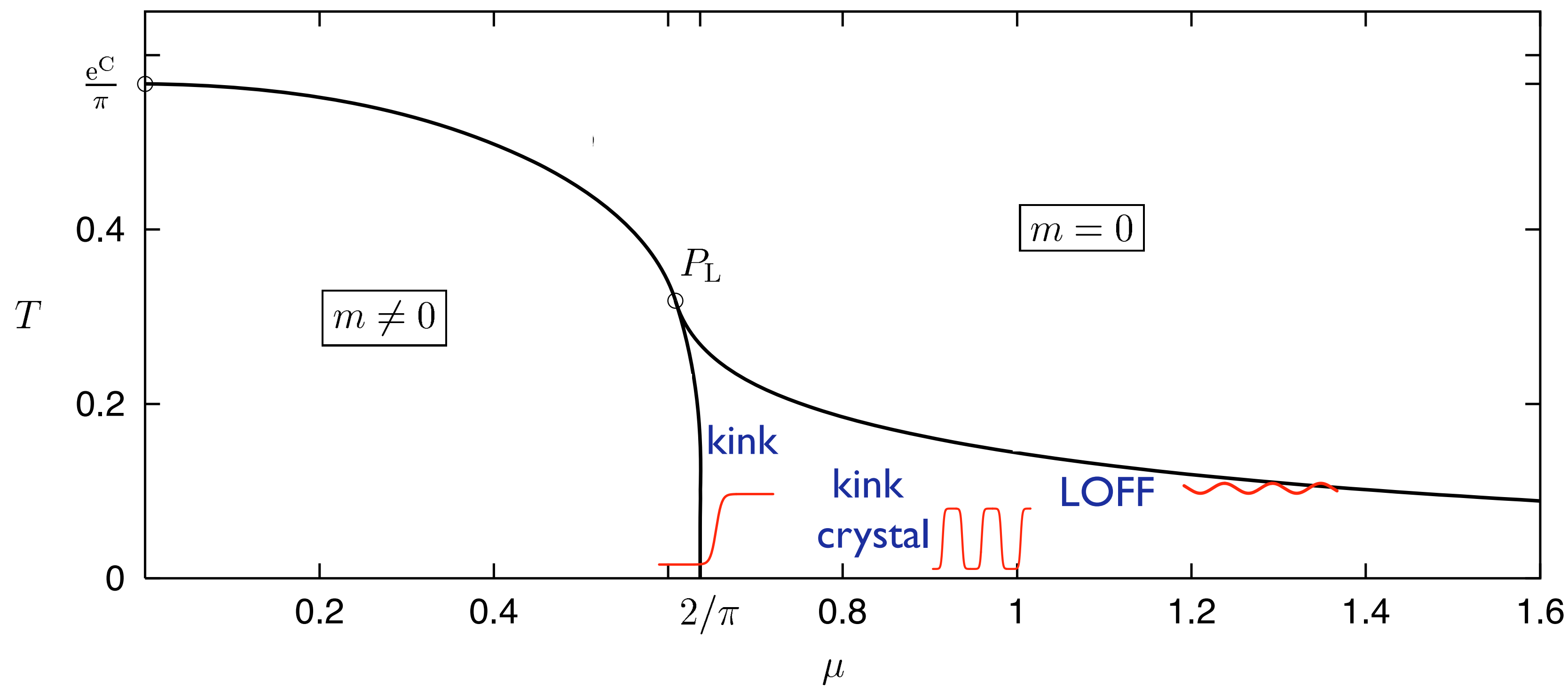

all gap
small gap
no gap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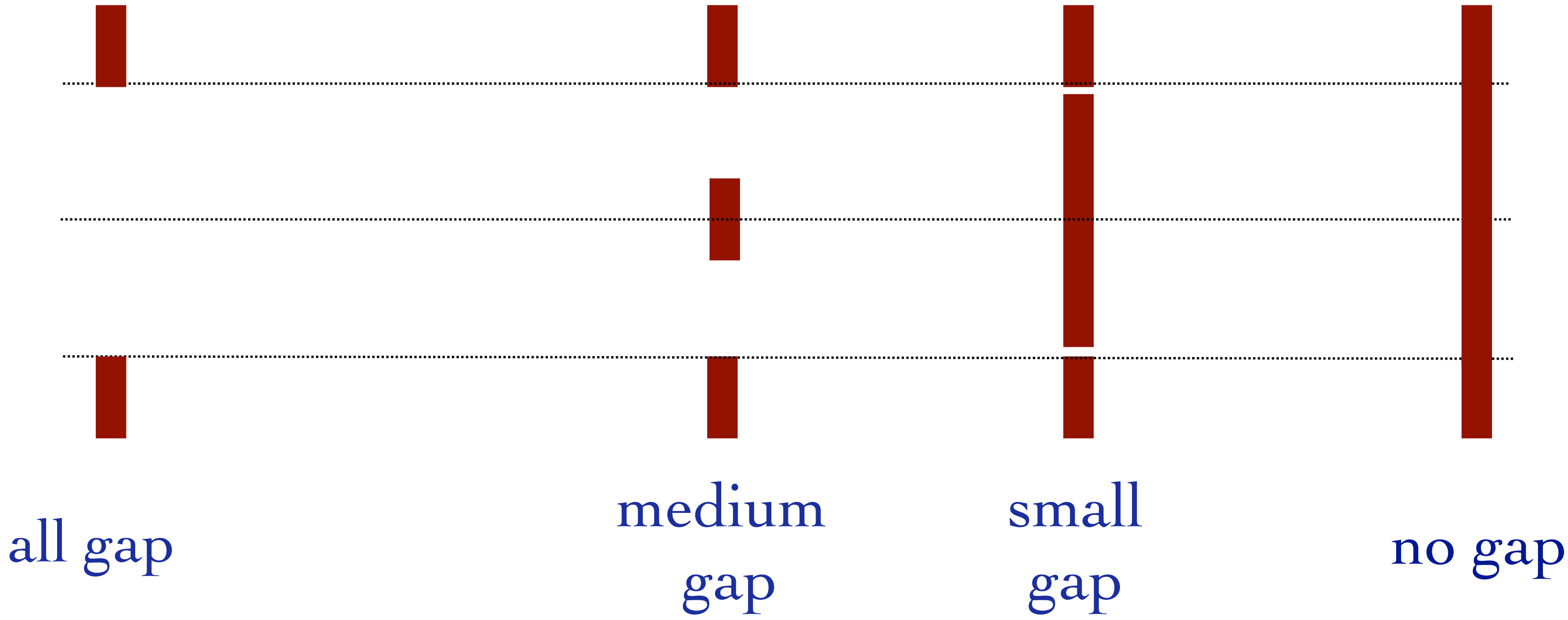
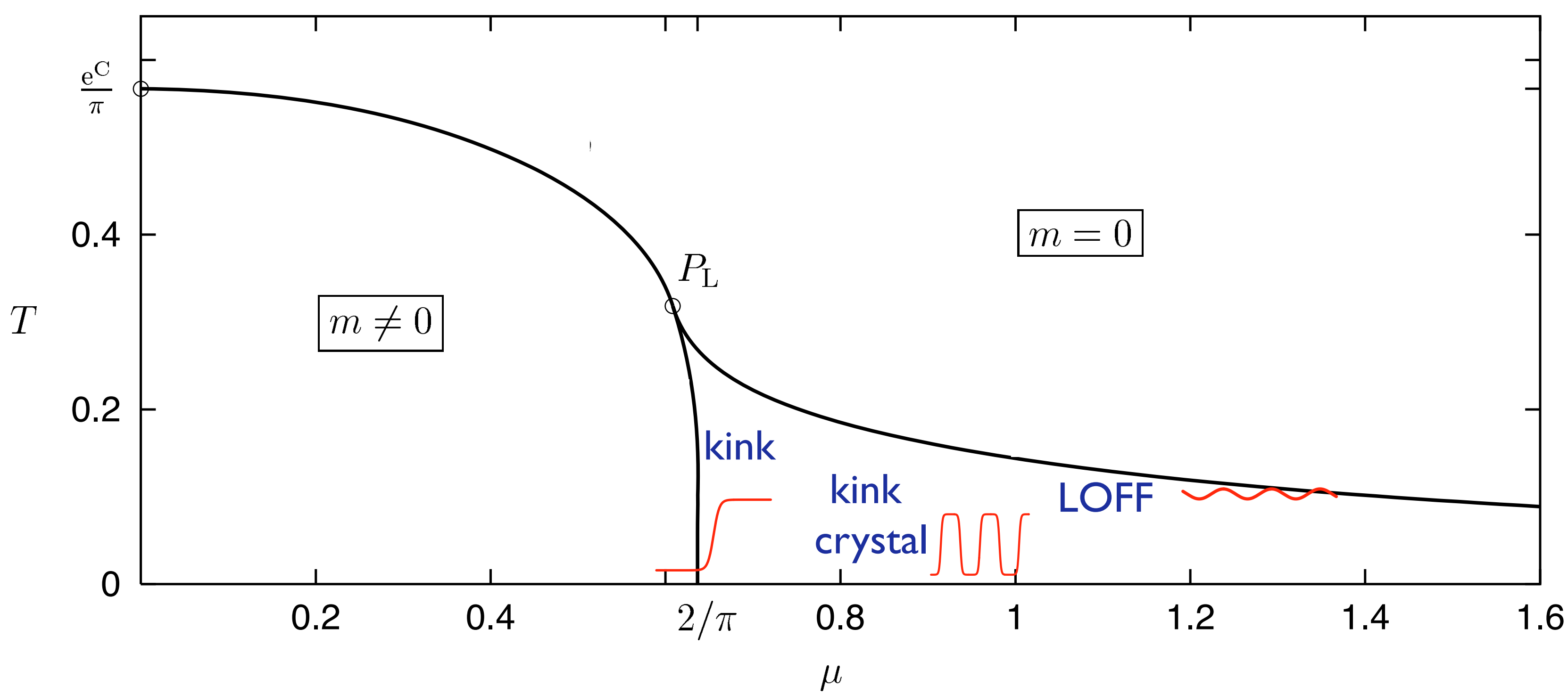

all gap
medium gap
small gap
no gap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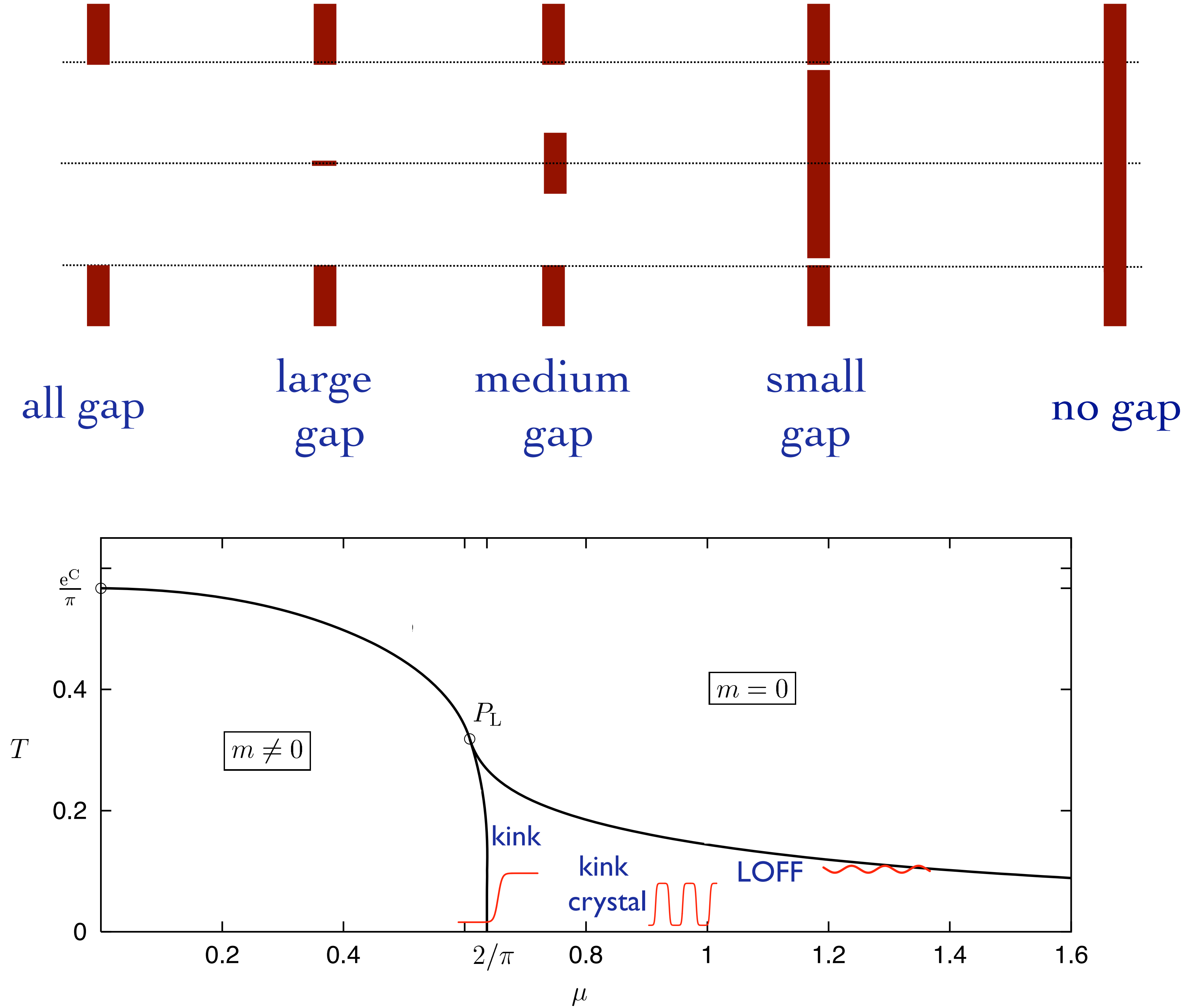
all gap
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chiral Gross-Neveu or NJL_2

$$\mathcal{L} = \bar{\psi} \partial\!\!\!/ \psi - \frac{g^2}{2} \left[\left(\bar{\psi} \psi \right)^2 + \left(\bar{\psi} i \gamma^5 \psi \right)^2 \right]$$

chiral Gross-Neveu or NJL_2

$$\mathcal{L} = \bar{\psi}\partial\!\!\!/\psi - \frac{g^2}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\psi\right)^2\right]$$

scalar condensate σ

pseudoscalar condensate π

chiral Gross-Neveu or NJL$_2$

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scalar condensate σ

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$$\mathcal{L}_{\text{eff}} = \bar{\psi} \partial\!\!\!/ \psi + \bar{\psi} \left(\sigma - i\pi\gamma^5\right) \psi + \frac{1}{2g^2} \left(\sigma^2 + \pi^2\right)$$

chiral Gross-Neveu or NJL$_2$

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scalar condensate σ pseudoscalar condensate π

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gap equation(s)

$$\frac{\sigma(x)}{g^2 N} = \frac{\delta}{\delta\sigma(x)} \ln \det\left[\partial\!\!\!/ + \left(\sigma(x) - i\gamma^5\pi(x)\right)\right]$$

$$\frac{\pi(x)}{g^2 N} = \frac{\delta}{\delta\pi(x)} \ln \det\left[\partial\!\!\!/ + \left(\sigma(x) - i\gamma^5\pi(x)\right)\right]$$

complex gap equation : NJL_2

$$\frac{\Delta(x)}{g^2 N} = \frac{\delta}{\delta \Delta^*(x)} \ln \det \left[\partial\!\!\!/ + \left(\sigma(x) - i\gamma^5 \, \pi(x)\right)\right]$$

$$\Delta = \sigma - i\pi$$

complex gap equation : NJL_2

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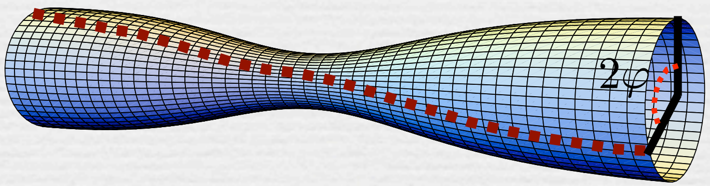
Shei (1976): reflectionless Dirac system

$$\Delta(x) = m \, \frac{\cosh\left(m \sin(\frac{\theta}{2})\, x - i \frac{\theta}{2} \right)}{\cosh\left(m \sin(\frac{\theta}{2})\, x \right)}$$

twisted kink



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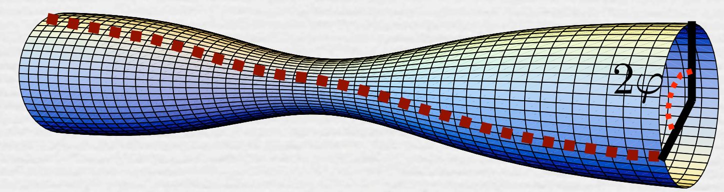
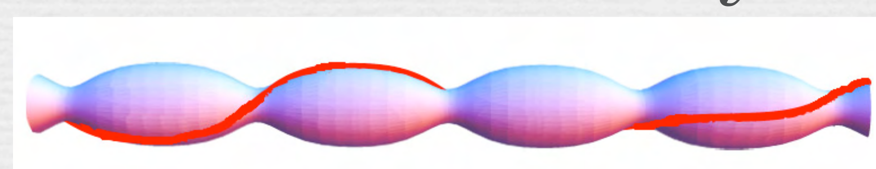
twisted kink



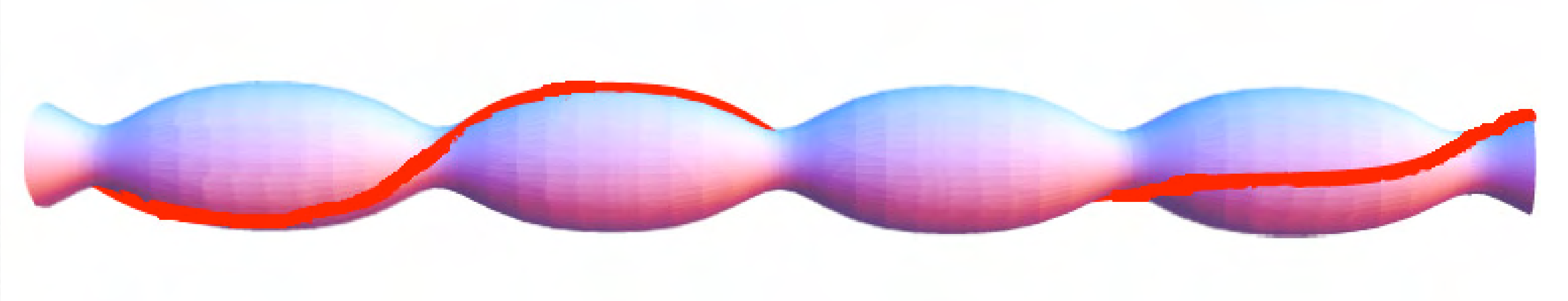
GD & Basar (2008): finite-gap Dirac system

$$\Delta(x) = A \frac{\sigma\left(A\, x + i\mathbf{K}' - i\frac{\theta}{2}\right)}{\sigma\left(A\, x + i\mathbf{K}'\right) \sigma\left(i\frac{\theta}{2}\right)} e^{iQx}$$

twisted kink crystal



twisted kink crystal: general solution of NJL_2 gap equation



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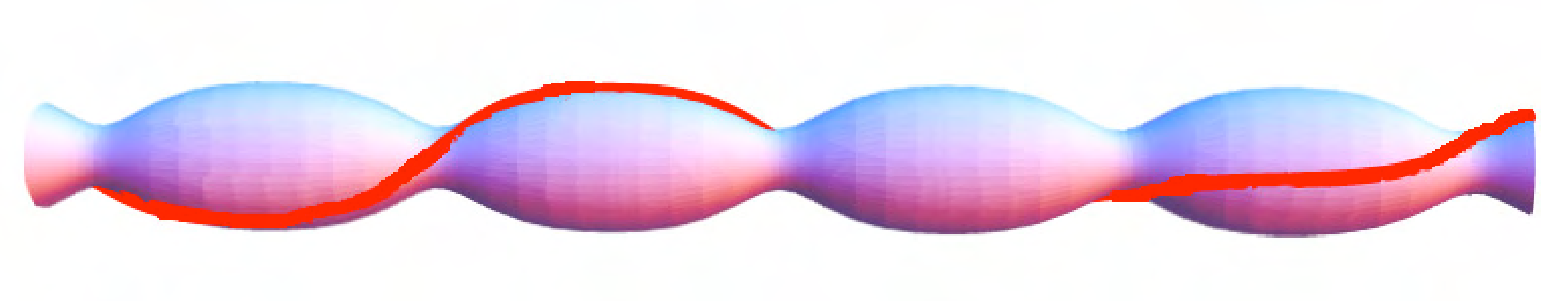
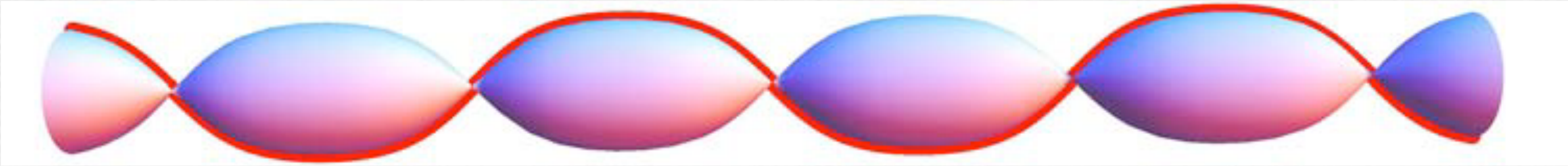
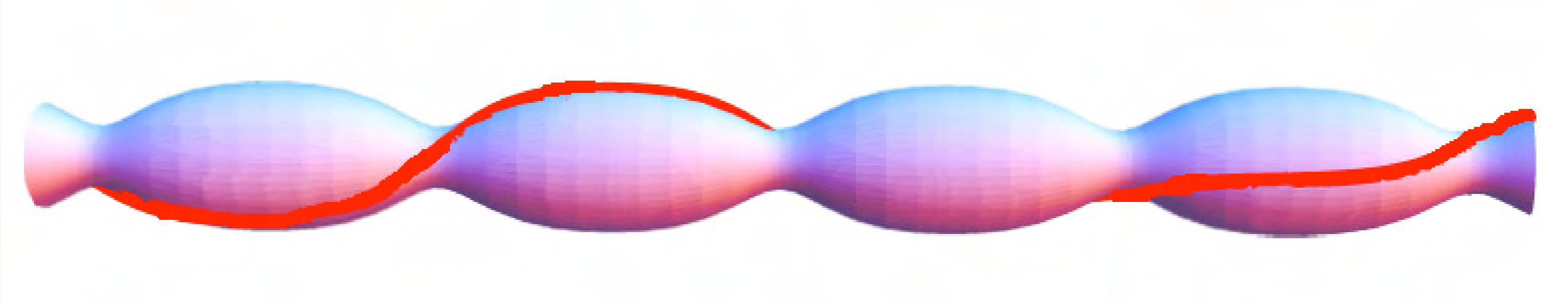


real kink crystal



twisted kink crystal: general solution of NJL_2 gap equation



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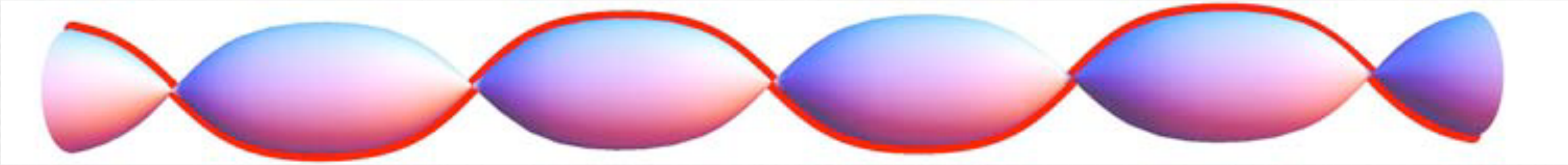
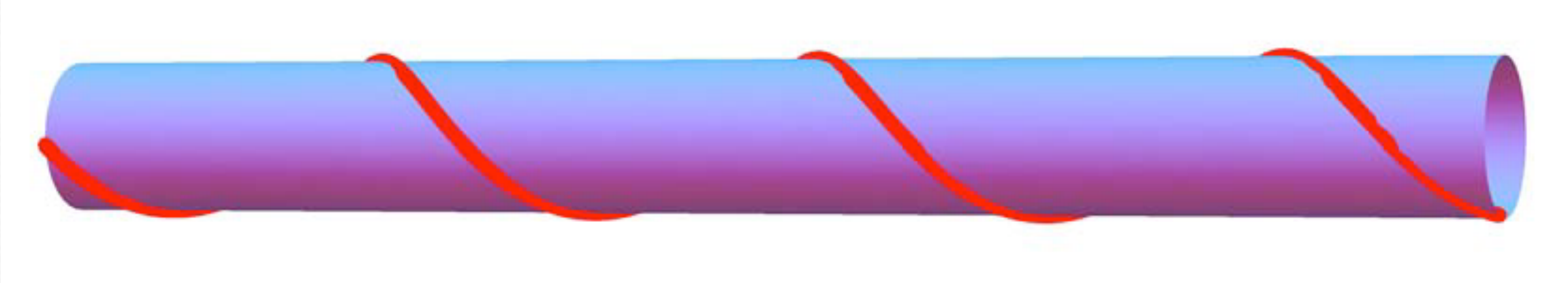


spiral crystal



phase diagram of chiral Gross-Neveu (NJL_2)

gap equation solution has 4 parameters

grand potential:

$$\Psi = -\frac{1}{\beta}\int dE\,\rho(E)\,\ln\left(1+e^{-\beta(E-\mu)}\right)$$

minimize Ψ w.r.t. parameters, as function of T and μ

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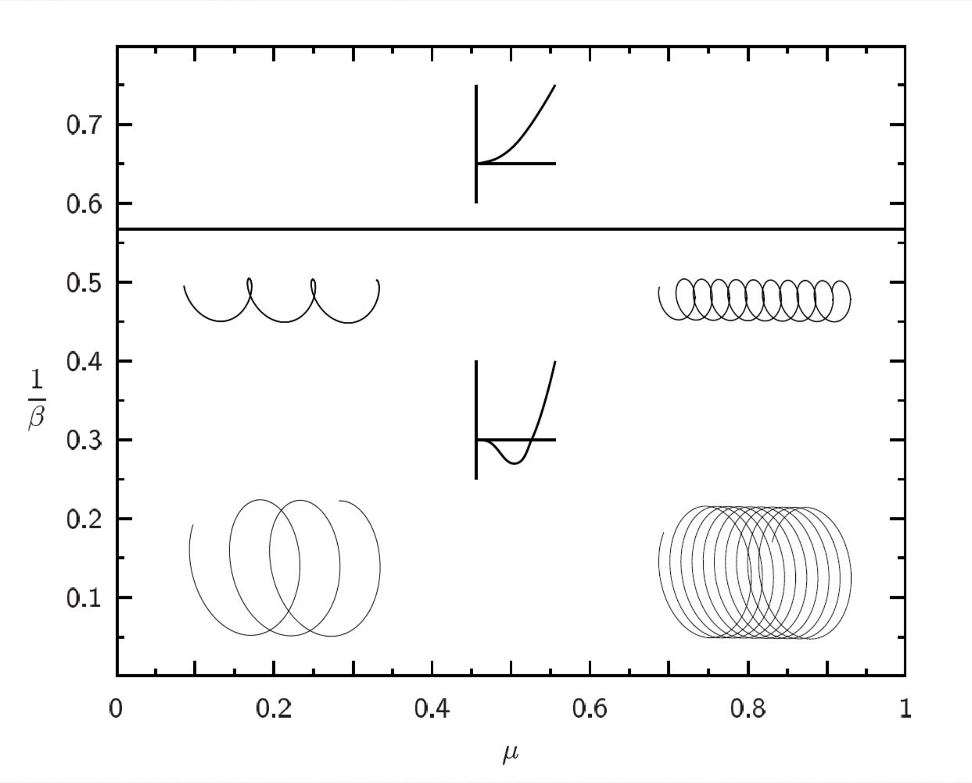
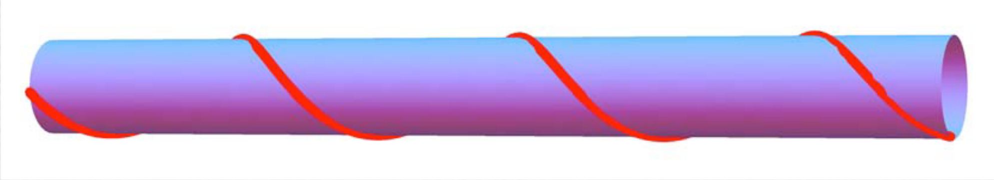


chiral spiral

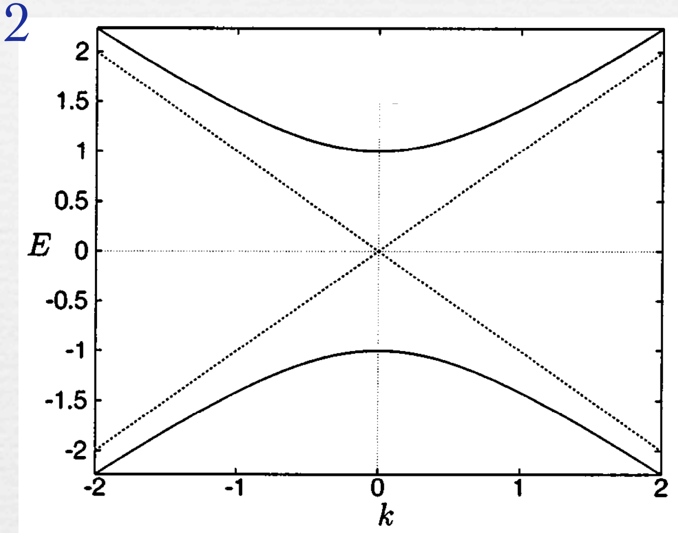


$$\sigma(x) - i\,\pi(x) = A(T)\,e^{2i\mu\,x}$$

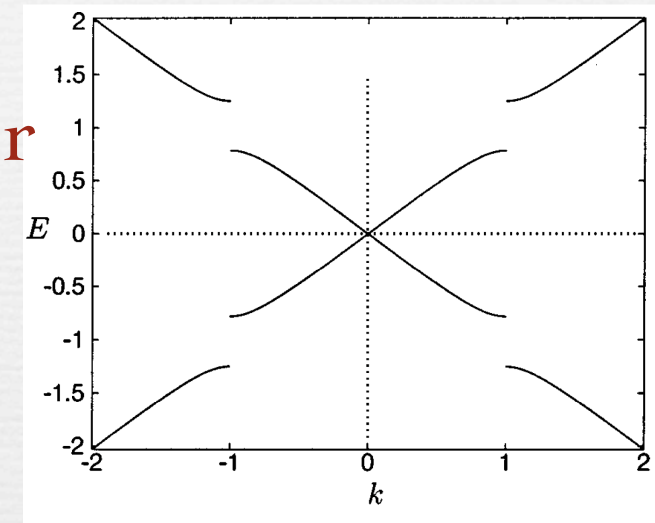
Peierls Instability

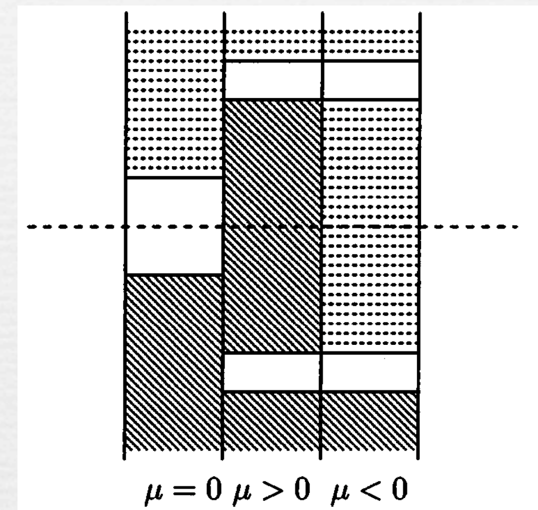
one dimension: gap formation at the Fermi surface
can lead to breakdown of translational symmetry

GN_2

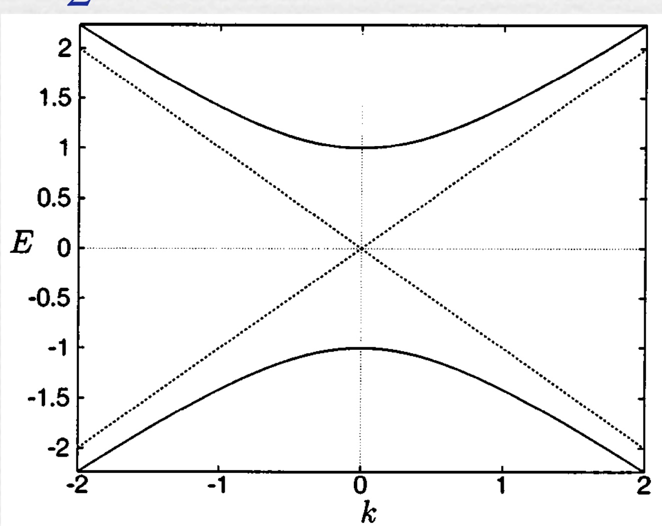


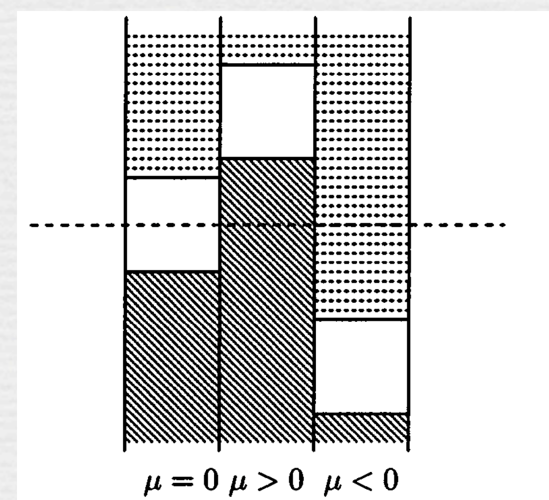
or





NJL_2





Thies, Urlichs, 2003

why can these gap equations be solved?

why can these gap equations be solved?

Ginzburg-Landau expansion is the mKdV (GN_2) and AKNS (NJL_2) integrable hierarchy

Başar, GD, Thies, 2009

Correa, GD, Plyushchay, 2009

Ginzburg-Landau expansion for GN_2 (real condensate)

$$\Psi = \alpha_2 \int \sigma^2 + \alpha_4 \int \left[\sigma^4 + (\sigma')^2\right] + \alpha_6 \int \left[2\sigma^6 + 10\sigma^2(\sigma')^2 + (\sigma'')^2\right] + \ldots$$

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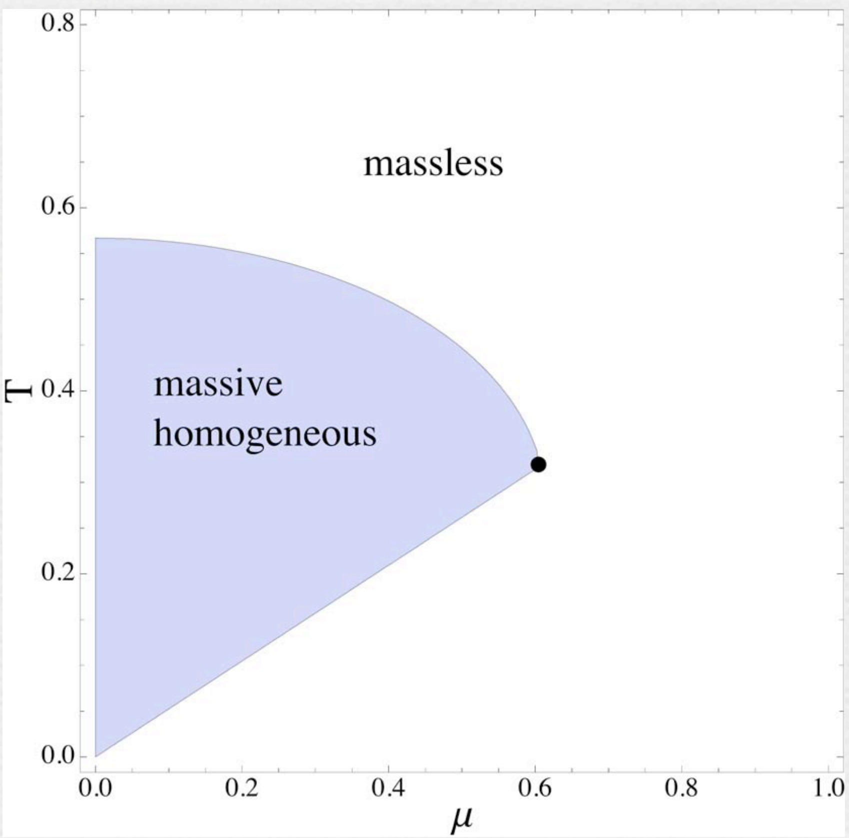

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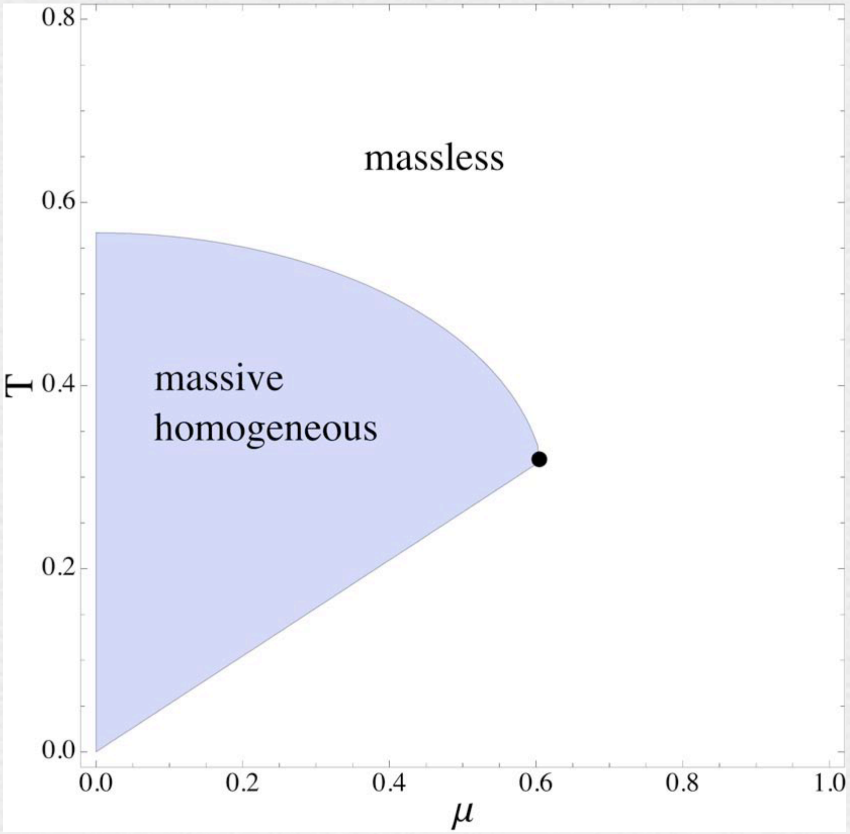

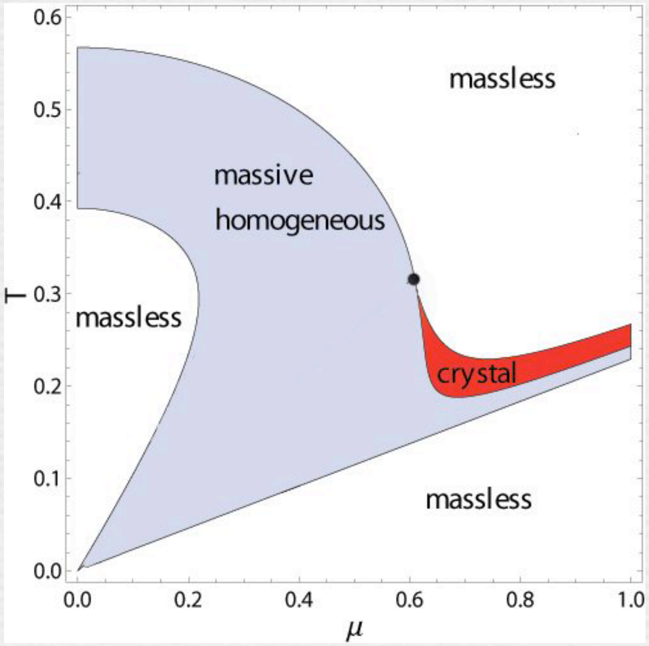

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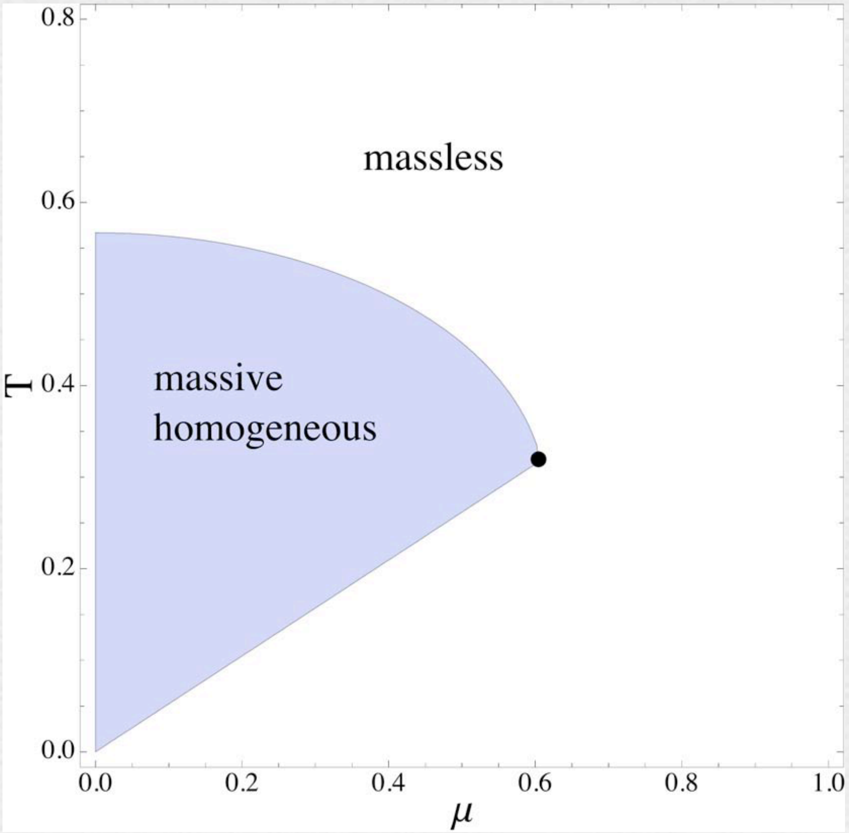

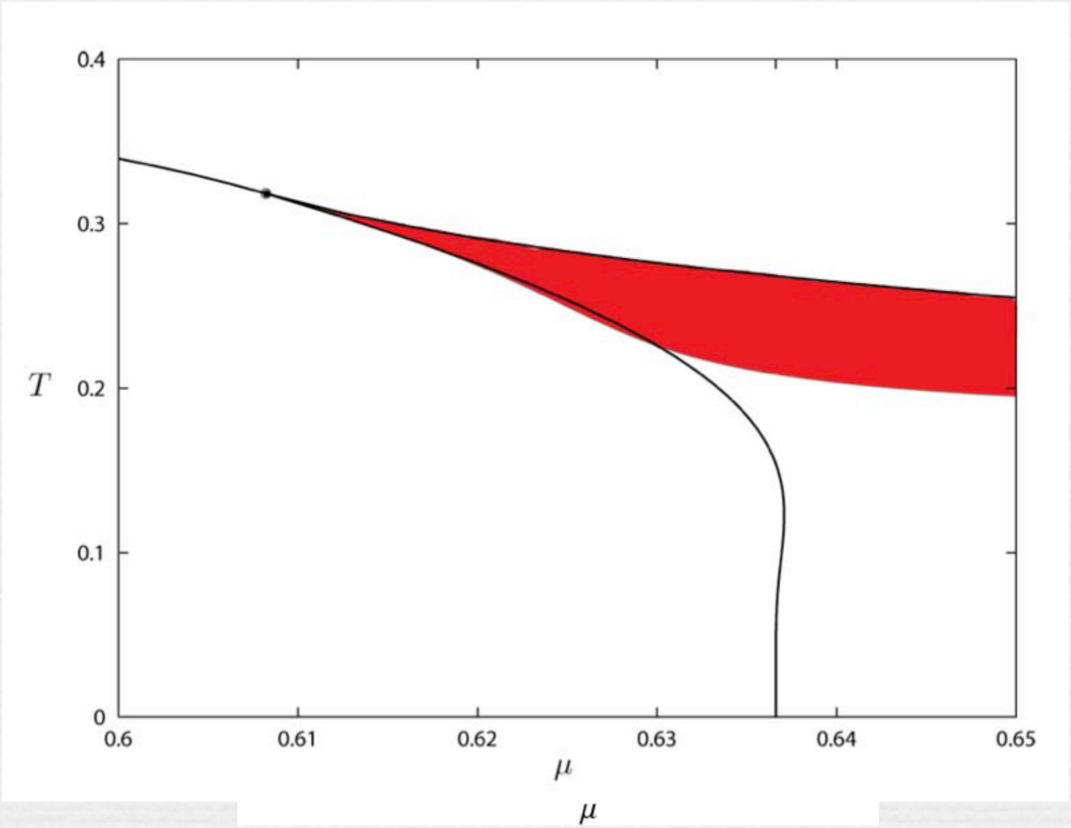

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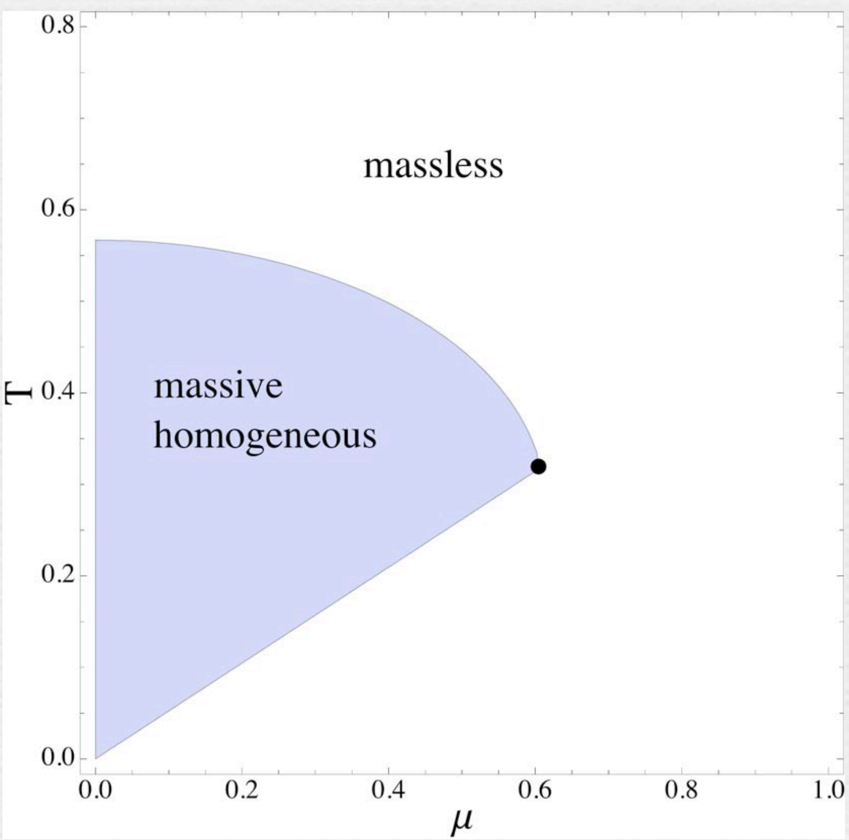

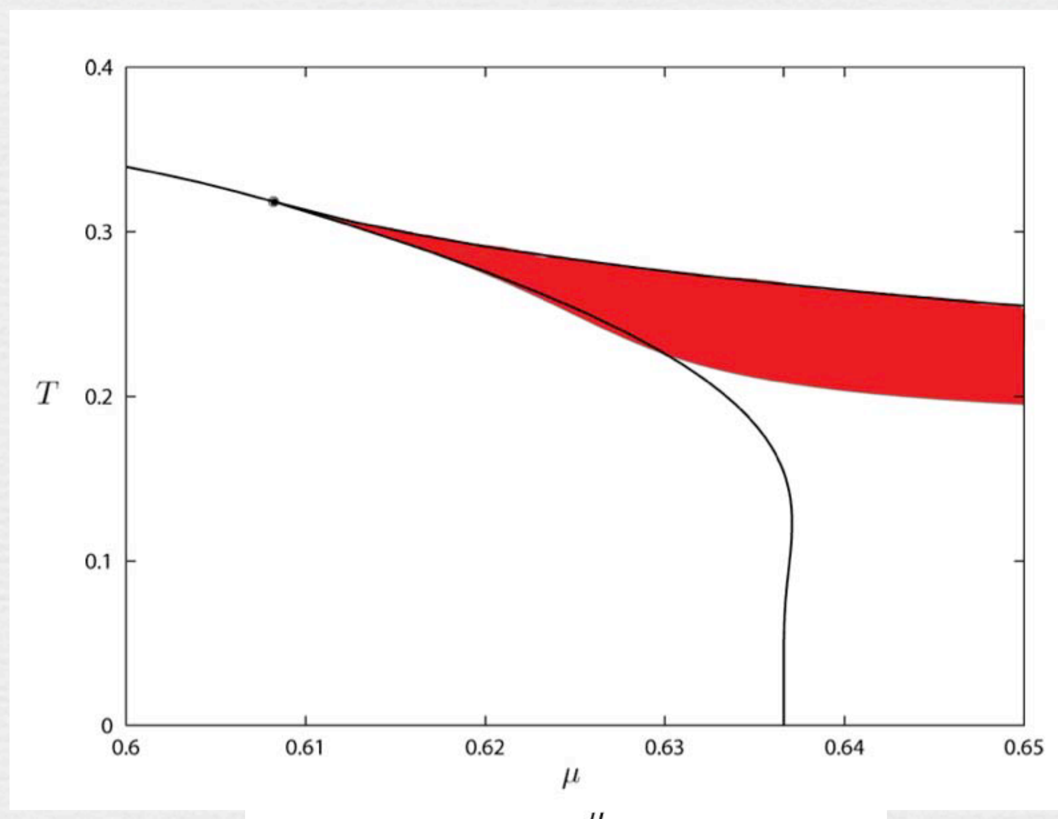

...

Ginzburg-Landau for NJL_2 (complex condensate)

$$\Psi \quad = \quad \alpha_2 \int |\Delta|^2 + \alpha_3 \int \mathrm{Im}[\Delta(\Delta')^*] + \alpha_4 \int \left[|\Delta|^4 + |\Delta'|^2\right]$$
$$+\alpha_5 \int \mathrm{Im}\left[(\Delta'' - 3|\Delta|^2\Delta)(\Delta')^*\right]$$
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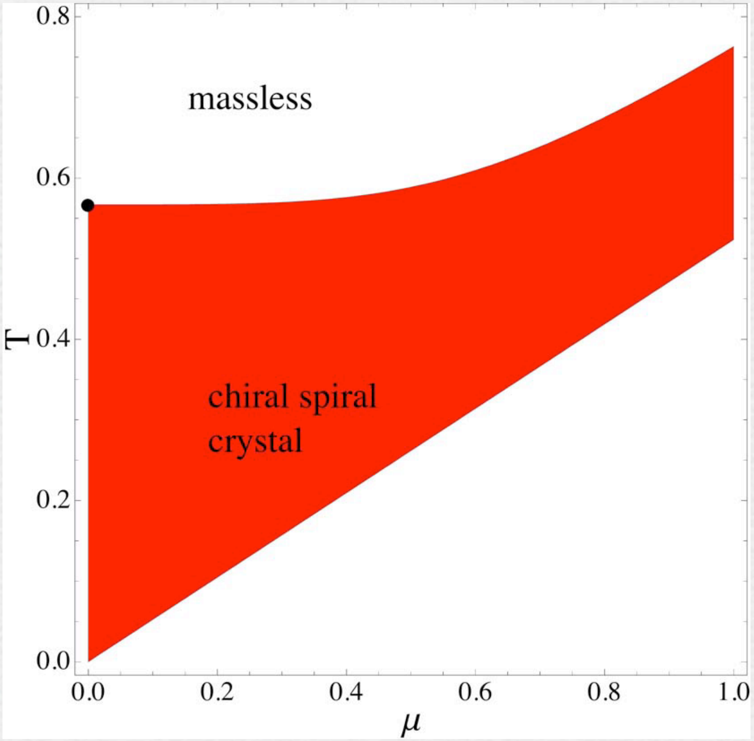

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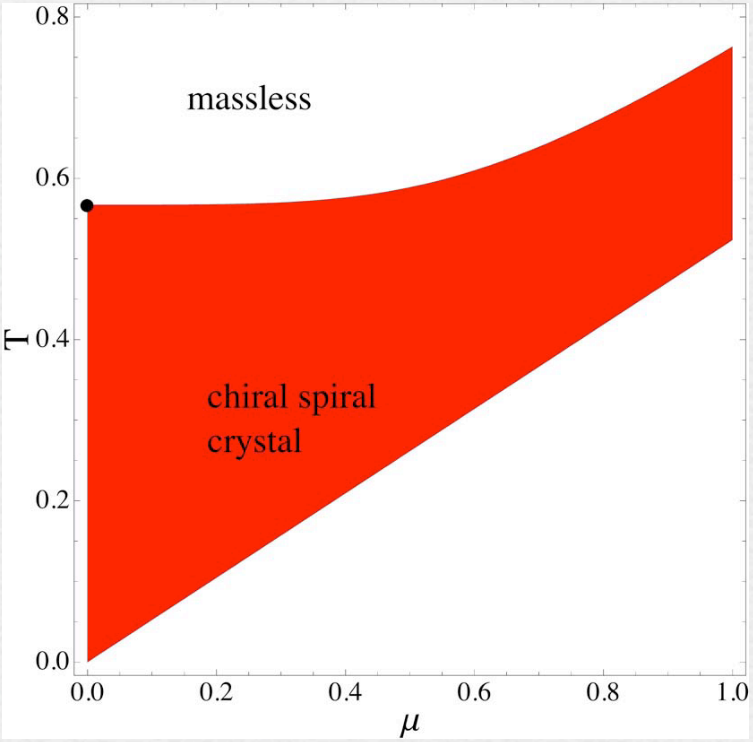


...

gap equation to all-orders in Ginzburg-Landau

$$\mathcal{L}_{GL} = \sum_n \alpha_n(T,\mu) \int \hat{g}_n(x)$$

gap equation :

$$\sum_n \alpha_n(T,\mu) \frac{\delta}{\delta \Delta^*(x)} \left(\int \hat{g}_n \right) = \Delta(x)$$

amazing fact ("integrability"):

NLSE → entire hierarchy satisfied, for ALL n

beyond 1 dimension ?

beyond 1 dimension ?

finite gap/reflectionless ? : no

beyond 1 dimension ?

finite gap/reflectionless ? : no

integrable hierarchy: unlikely

beyond 1 dimension ?

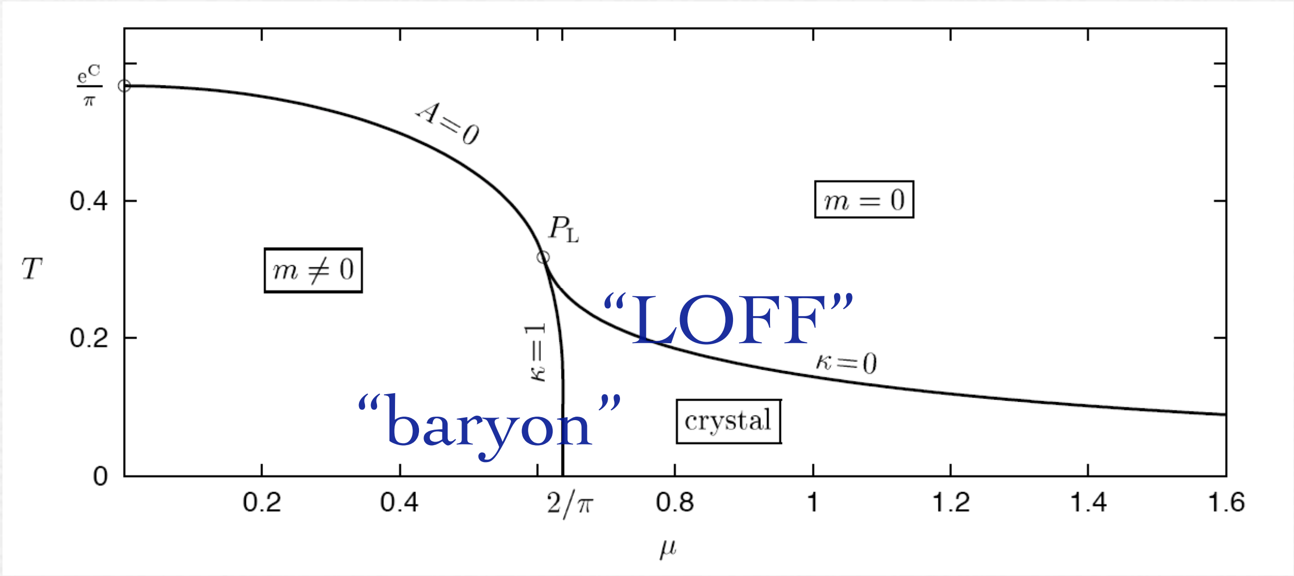
finite gap/reflectionless ? : no

integrable hierarchy: unlikely

"Skyrme crystal" : possibly

crystal phase boundaries - beyond LOFF

crystal phase (usually) has two boundaries



"LOFF edge":

$$\sigma(x) = A \sin(q\, x)$$

"baryon edge":

$$\sigma(x) = A \tanh(q\, x)$$

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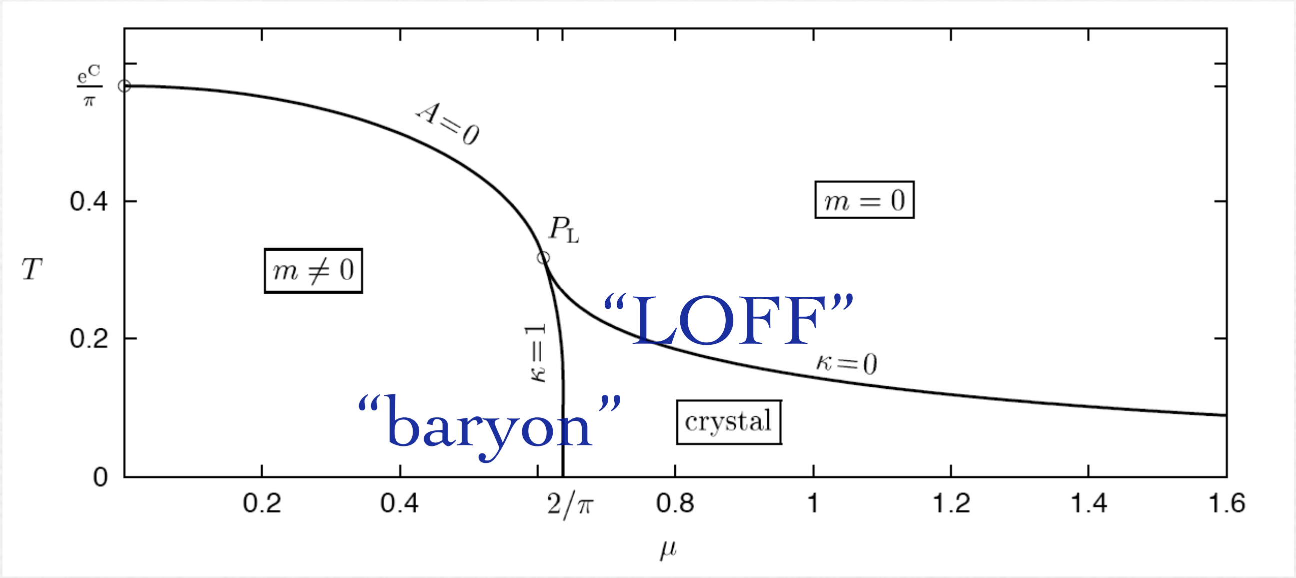

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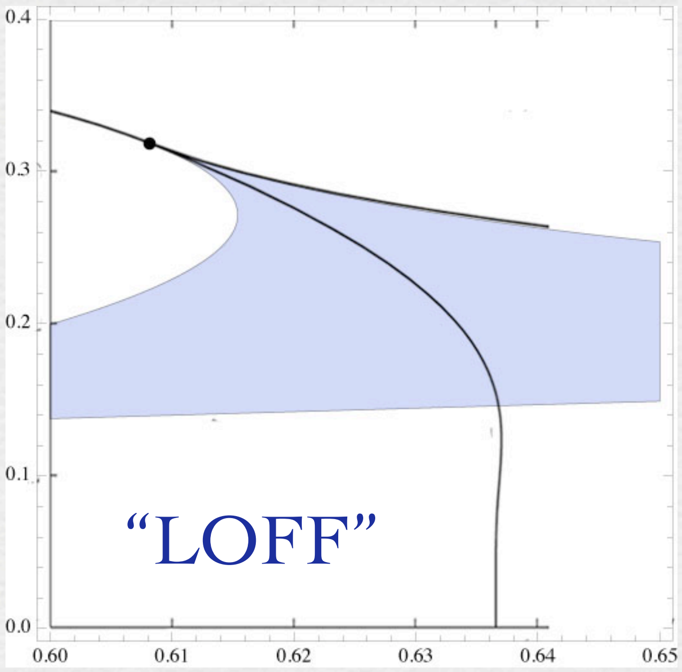

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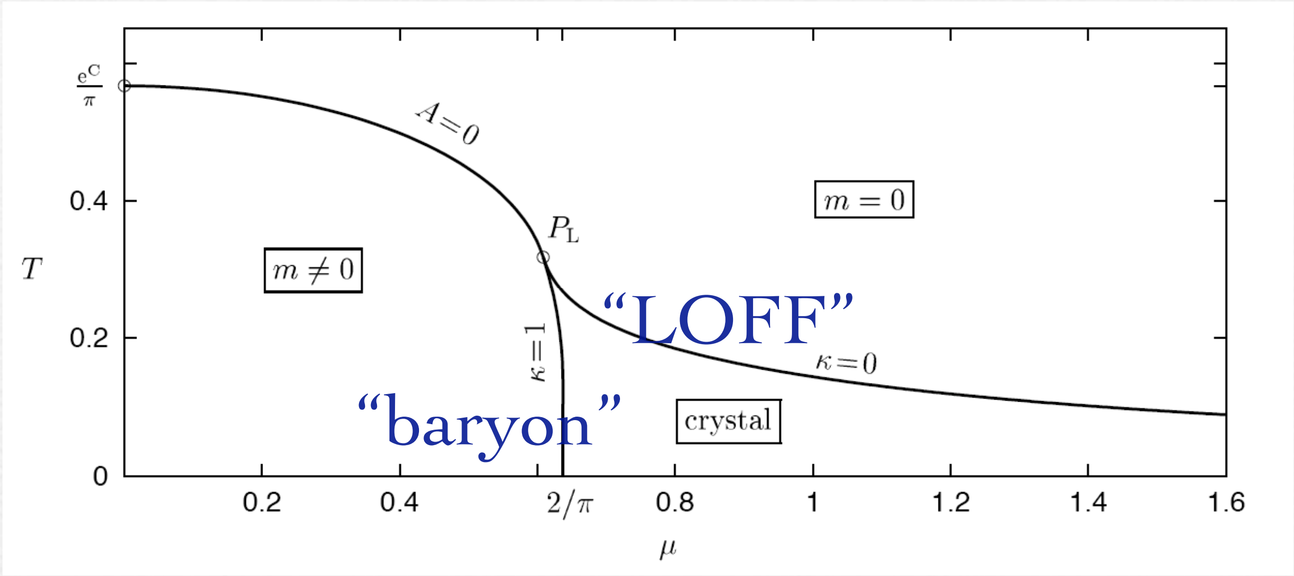

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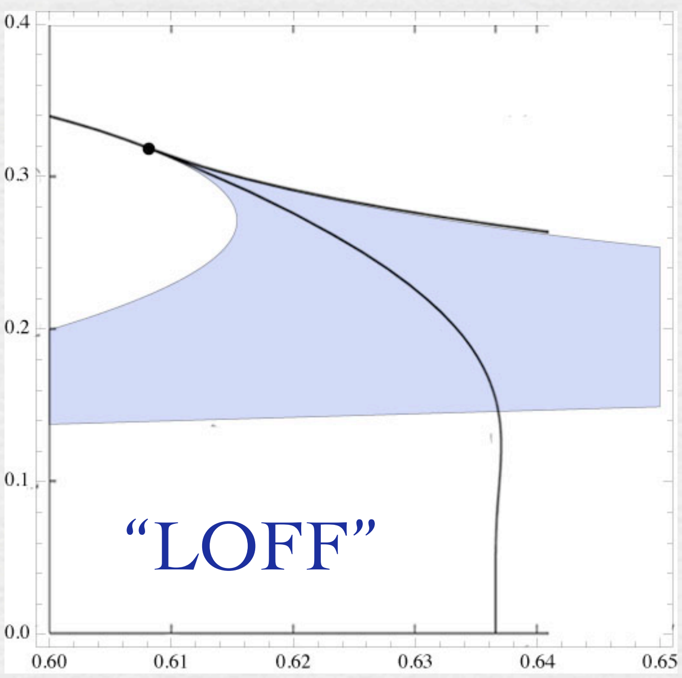

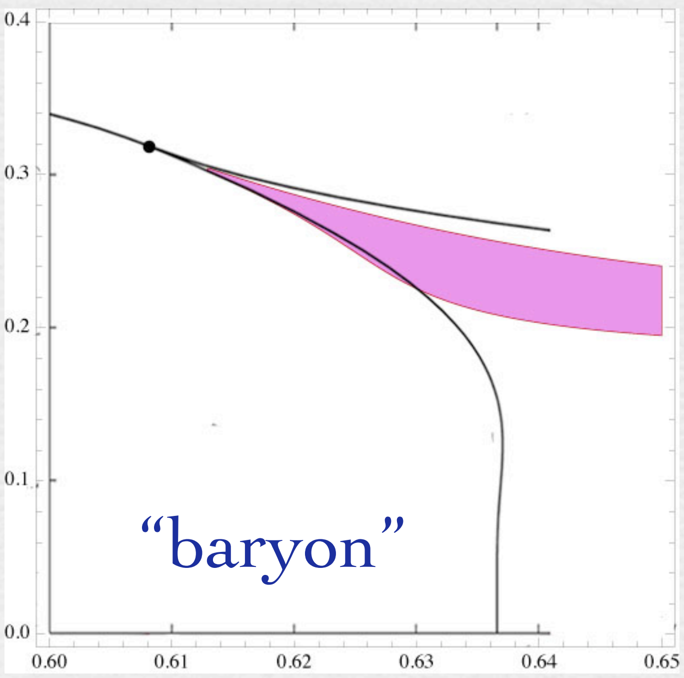

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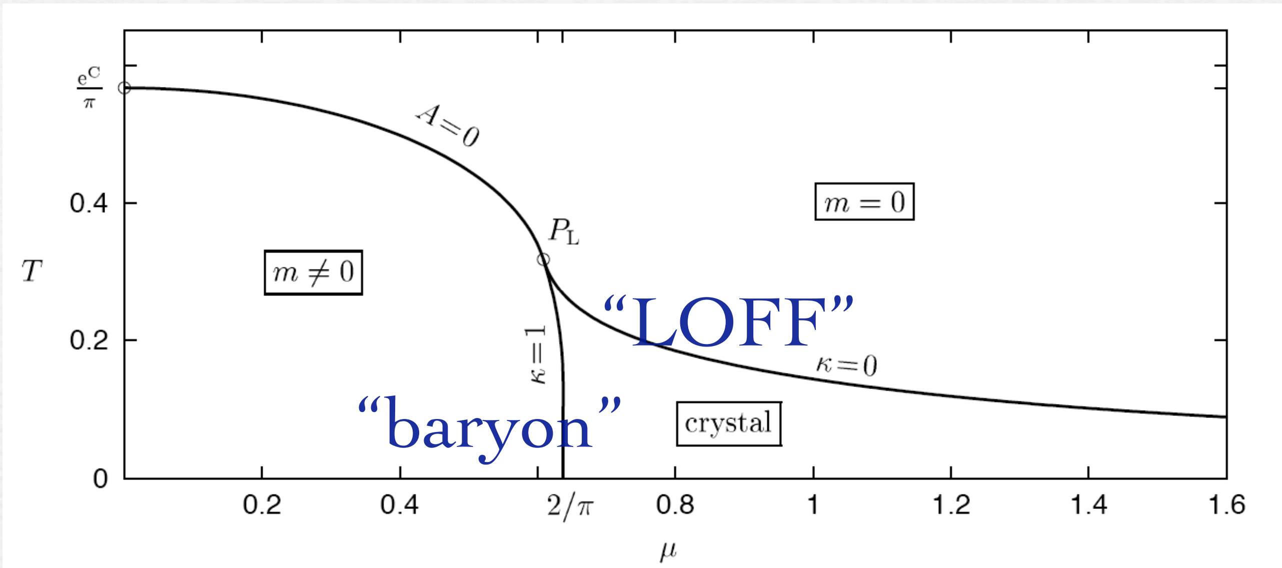

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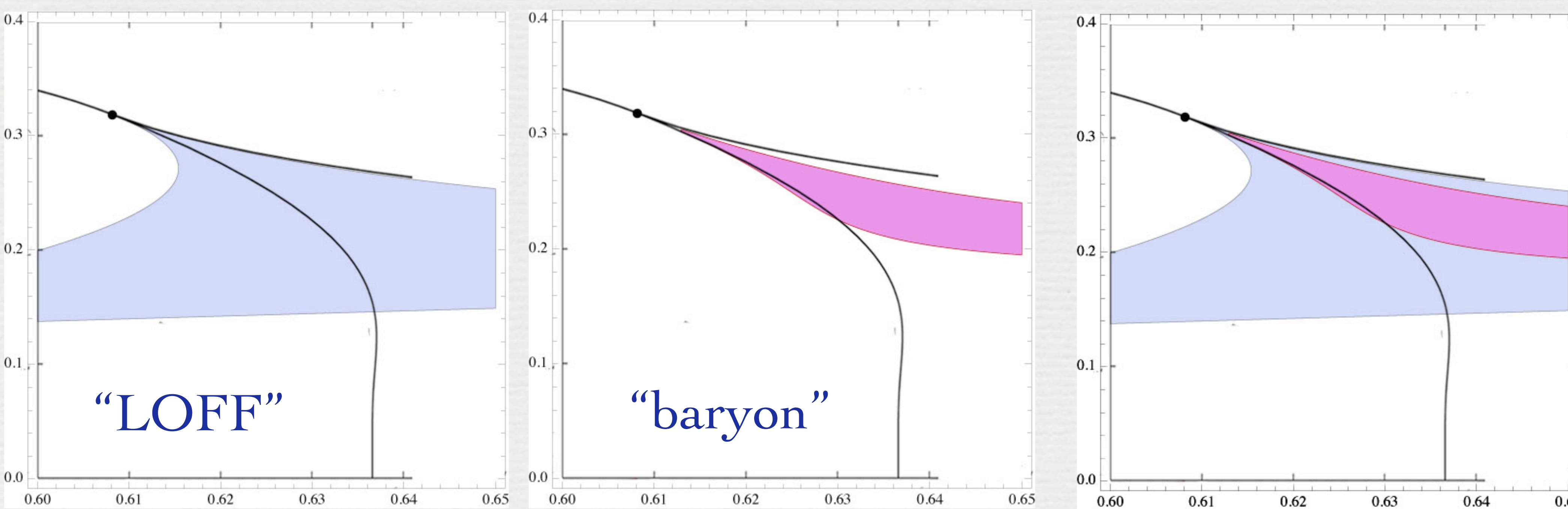

recent appearances of these 1+1 dim solutions
in other models ...

Nickel & Buballa, 2008

$$\Delta_{3+1}(z) = \Delta_{1+1}(x_\theta(z))$$

$$\Delta_{1+1}(x_\theta(z)) = \kappa_\theta \sqrt{\nu}\, \mathrm{sn}\left(\kappa_\theta(x_\theta - x_{\theta,0}); \nu\right)$$

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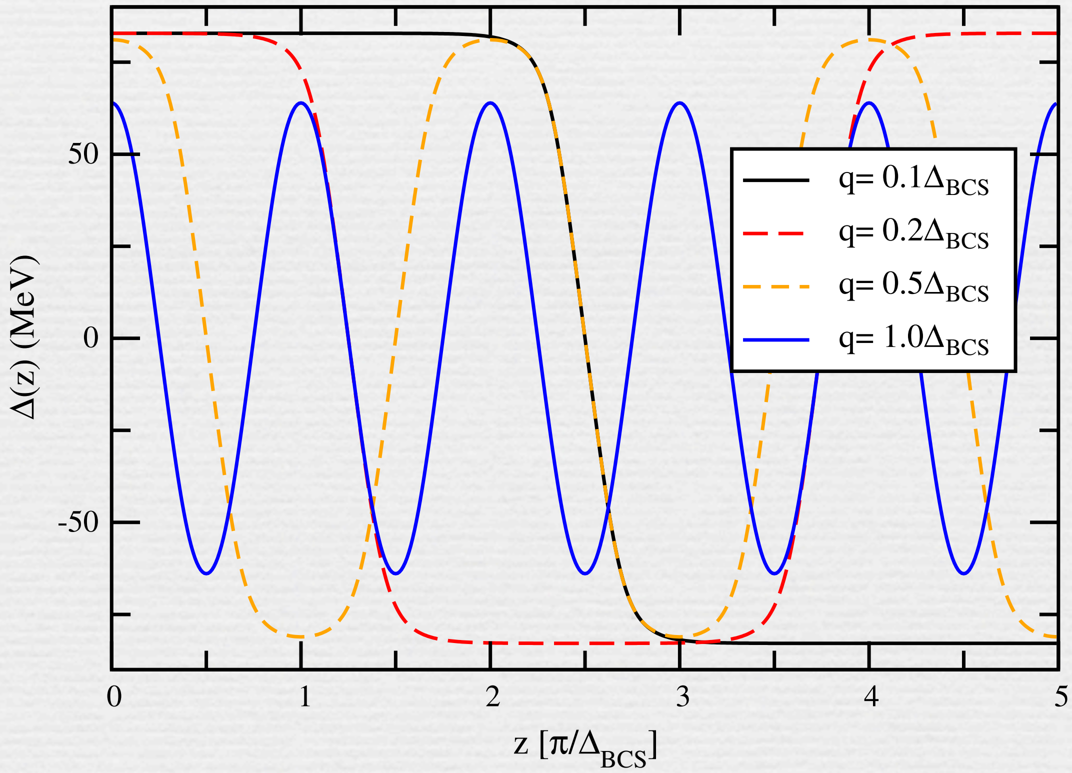
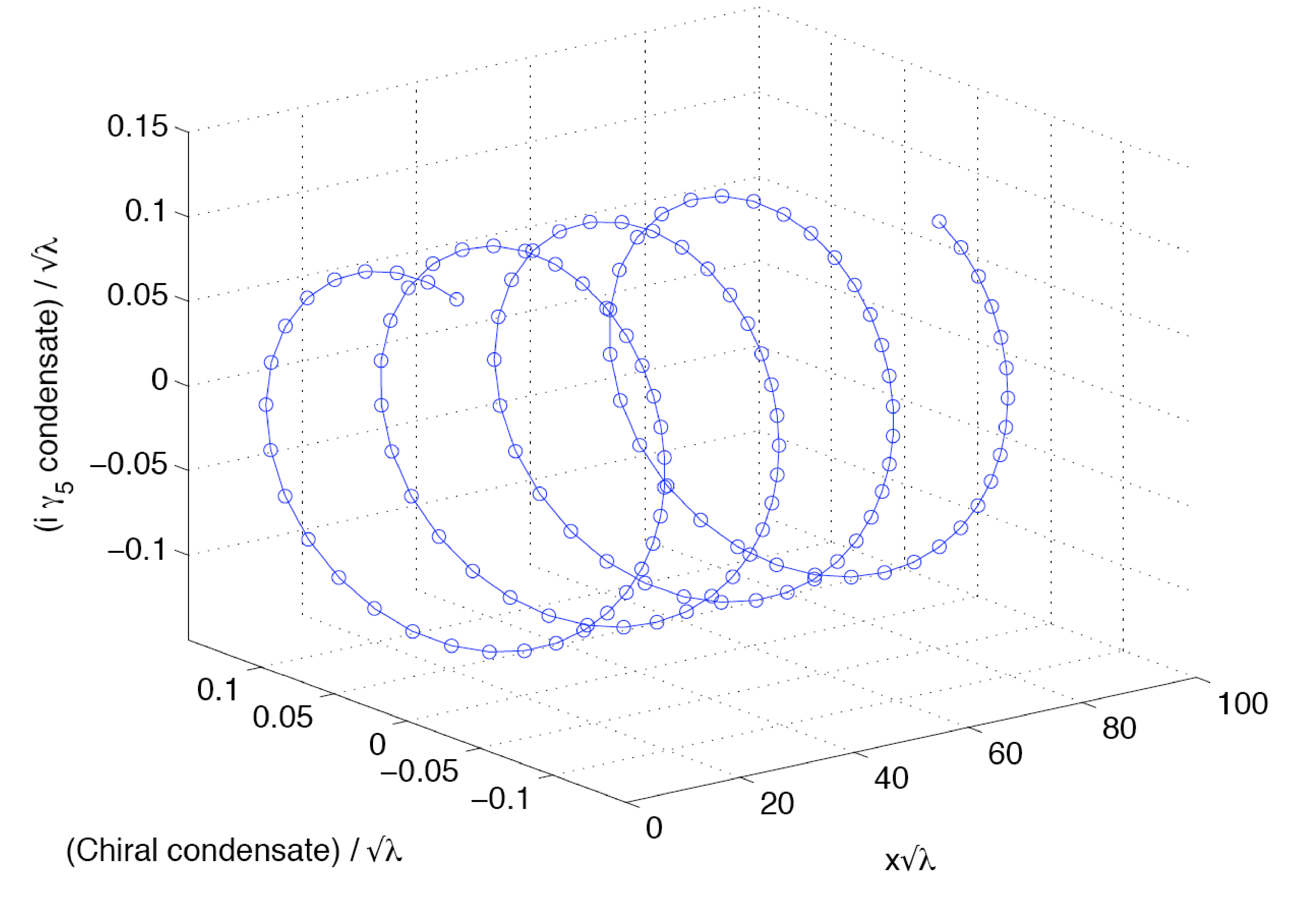
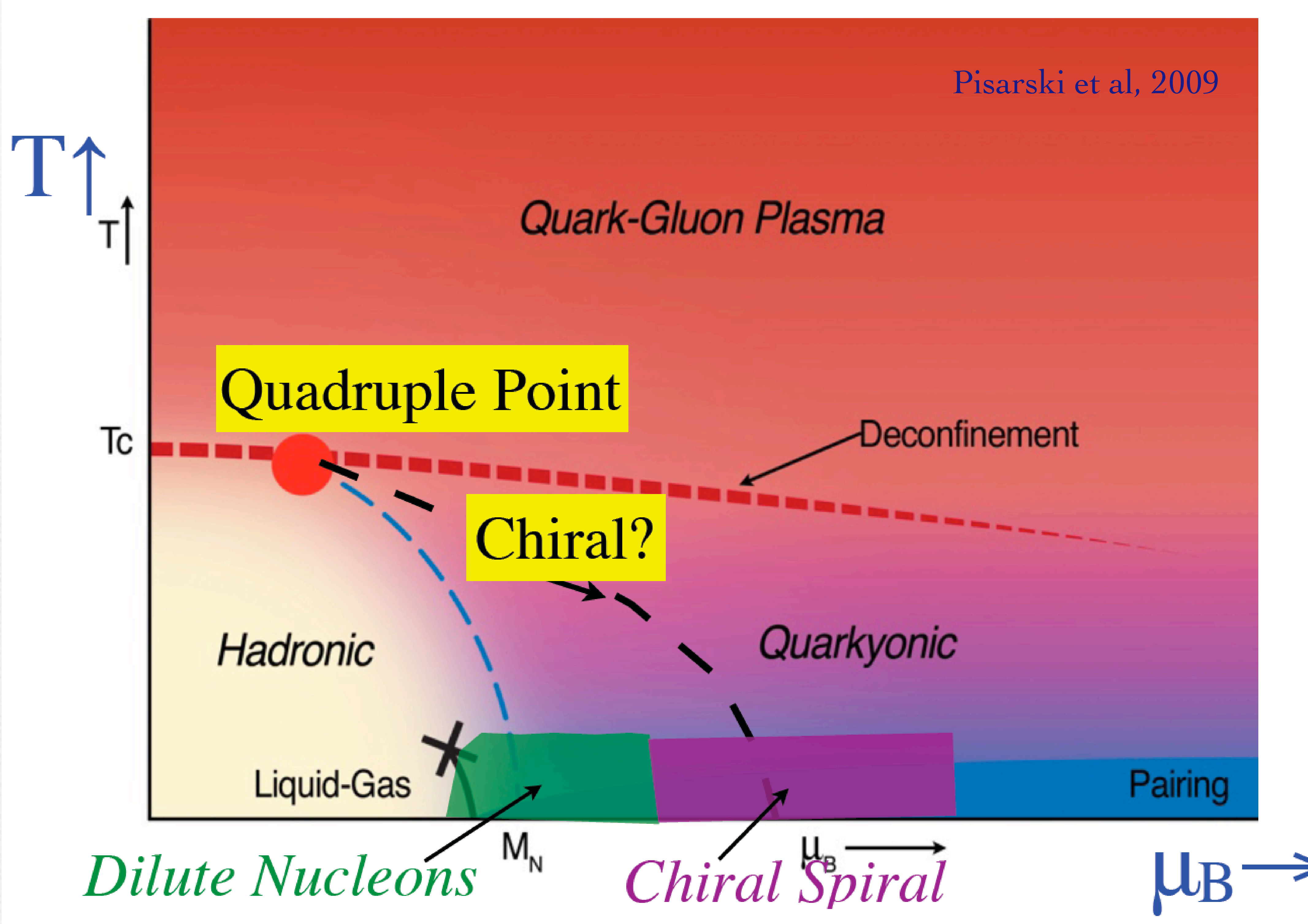


FIG. 2 (color online). The gap function in coordinate space at $\delta\mu = 0.7\Delta_{\mathrm{BCS}}$ for different fixed values of q.



1+1 't Hooft model

Bringoltz, PRD 2009


Pisarski et al, 2009
T↑
T
Quark-Gluon Plasma
Quadruple Point
Tc
Deconfinement
Chiral?
Hadronic
Quarkyonic
Liquid-Gas
Pairing
M_N
μ_B
Dilute Nucleons
Chiral Spiral
μ_B

Conclusions

- complete solution of gap equation for chiral GN/NJL_2
- gap equation reduced to NLSE
- full, exact, thermodynamics & phase diagram for chiral symmetry breaking model(s)
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- higher dimensional models ?
- confinement vs chiral symmetry breaking?

Happy Birthday Tony!

many more ...

solving the (complex) gap equation

$$\frac{\Delta(x)}{g^2 N} = \frac{\delta}{\delta \Delta^*(x)} \ln \det \left[\partial\!\!\!/ + \left(\sigma(x) - i\gamma^5 \, \pi(x) \right) \right]$$

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Bogoliubov/de Gennes hamiltonian

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Bogoliubov/de Gennes hamiltonian

resolvent : Gorkov Green's function $R(x; E) \equiv \langle x | \frac{1}{H - E} | x \rangle$

gap equation, and basic properties of resolvent, essentially determine the resolvent

GD & Basar (2008)

$$R(x;E) = \mathcal{N}(E) \begin{pmatrix} a(E) + |\Delta(x)|^2 & b(E)\Delta(x) - i\Delta'(x) \\ b(E)\Delta^*(x) + i\Delta'^*(x) & a(E) + |\Delta(x)|^2 \end{pmatrix}$$

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$$\Delta'' - 2|\Delta|^2\,\Delta + i\,(b - 2E)\,\Delta' - 2\,(a - Eb)\,\Delta = 0$$

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