## QMC and the nature of dense matter: Written in the stars?

#### J. D. Carroll

( CSSM, University of Adelaide )

Achievements and New Directions in Subatomic Physics: Workshop in Honour of Tony Thomas' 60th Birthday, 2010





















## Outline



- The Nature of Dense Matter
- The Models

#### 2 Simulations:

- Hadronic Matter
- Mixed-Phase Matter



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The Nature of Dense Matter The Models

## What We Know

- Quarks and Gluons are the fundamental degrees of freedom
- At low densities, Baryons (Nucleons) are the effective degrees of freedom
- At high densities...



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- At high densities... ??? = Hyperons? Quarks? Other?



The Nature of Dense Matter The Models

#### Hadronic Models A Brief Overview

## $\mathbf{Q}\text{uantum}\ \mathbf{H}\text{adro}\mathbf{D}\text{ynamics}\ (\mathbf{QHD})$ Model

- Simple description of nucleons immersed in mean-field  $\sigma$ ,  $\omega$ , and  $\rho$  potentials,
- Constructed at the baryon level,
- Issues with large scalar potentials causing negative effective masses.

#### $\mathbf{Q}\text{uark-Meson}\ \mathbf{C}\text{oupling}\ (\mathbf{QMC})$ Model

- Similar final form as QHD, but with self-consistent response to the  $\sigma$  field, despite construction from quark level,
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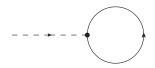
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=  $-g_{\sigma B} \sum_{B'} \frac{g_{\sigma B'}}{m_{\sigma}^2} \frac{(2J_{B'}+1)}{(2\pi)^3} M_{B'}^* \int \frac{\theta(k_{F_{B'}} - |\vec{k}|) d^3k}{\sqrt{\vec{k}^2 + M_{B'}^{*2}}}$ 

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$$B \in \{p, n, \Lambda, \Sigma^{-}, \Sigma^{0}, \Sigma^{+}, \Xi^{-}, \Xi^{0}\} = \{N, Y\}$$
  
•  $\ell \in \{e^{-}, \mu^{-}\}$   
•  $m \in \{\sigma, \omega, \rho\}$ 

$$g_{\omega B} = \frac{(3 - S_B)}{3} g_{\omega N}$$

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The Nature of Dense Matter  $\mathbf{The\ Models}$ 

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### Saturation

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Effective masses from Ref.  $[4]^a$  (previously from Ref.  $[5]^b$ ) derived from the bag model.

<sup>a</sup>Guichon, Thomas, Tsushima: doi:10.1016/j.nuclphysa.2008.10.001
 <sup>b</sup>Rikovska-Stone, Guichon, Matevosyan, Thomas: doi:10.1016/j.nuclphysa.2007.05.011



The Nature of Dense Matter The Models

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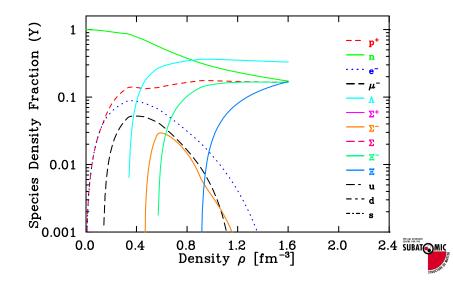
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Hadronic Matter Mixed-Phase Matter

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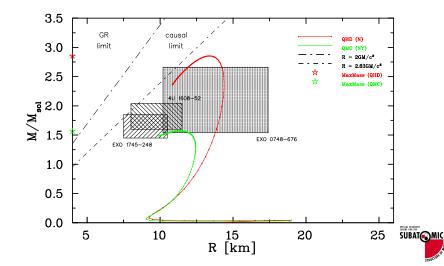
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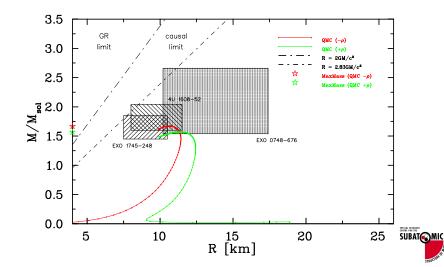
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Hadronic Matter Mixed-Phase Matter

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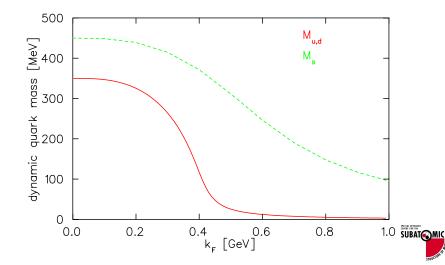
### Nambu-Jona–Lasinio (NJL) Model

# NJL: $k_F = 0$ : $m_u = 350 \text{ MeV}, m_d = 350 \text{ MeV}, m_s = 450 \text{ MeV}$ $k_F = \Lambda$ : $m_u = 5 \text{ MeV}, m_d = 7 \text{ MeV}, m_s = 95 \text{ MeV}$



Hadronic Matter Mixed-Phase Matter

## Quark Models NJL Effective Masses



Hadronic Matter Mixed-Phase Matter

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• $(\mu_i)_H = (\mu_i)_Q$	– Chemical Equilibrium
• $P_H = P_Q$	– Mechanical Equilibrium

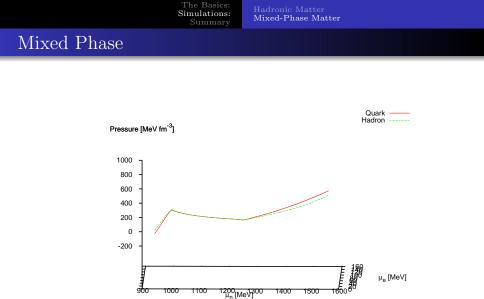


Hadronic Matter Mixed-Phase Matter

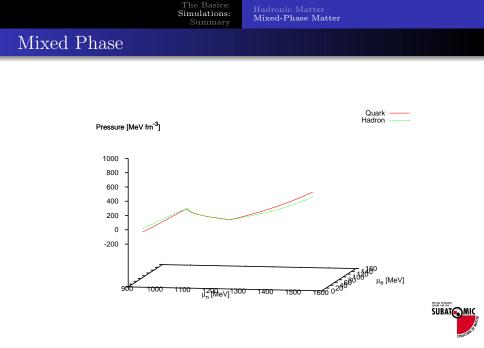
# **Phase Transitions**

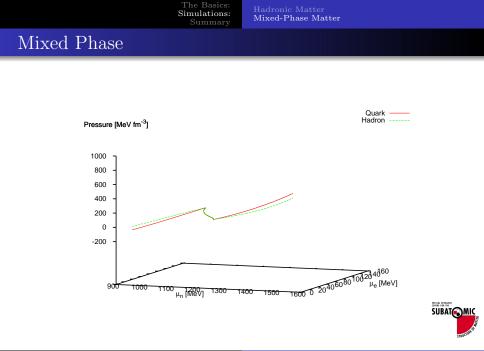
Gibbs Conditions	T = 0
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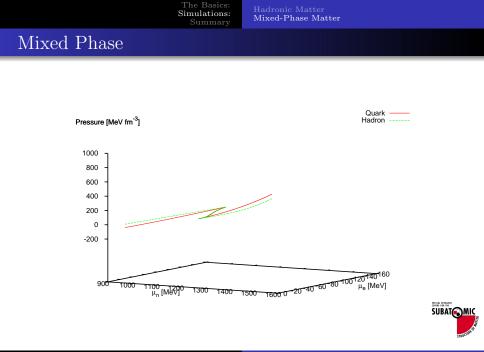


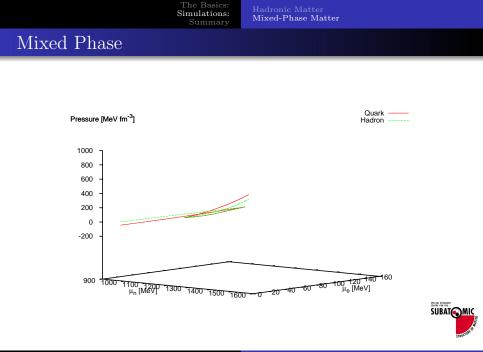


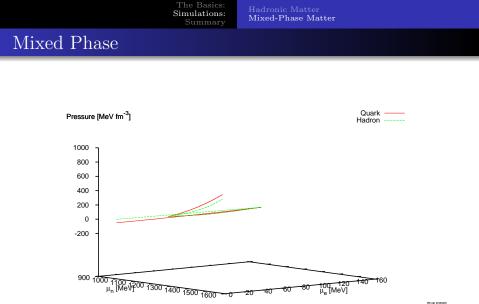






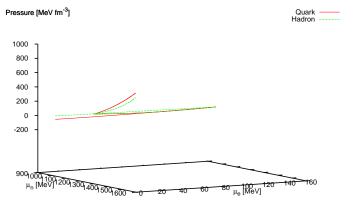








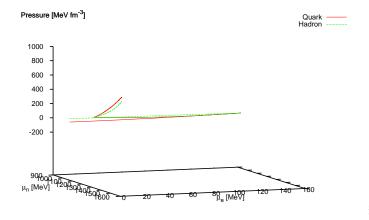






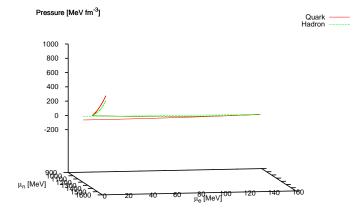
Hadronic Matter Mixed-Phase Matter

# Mixed Phase

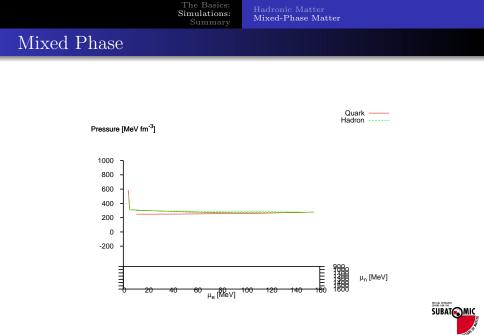


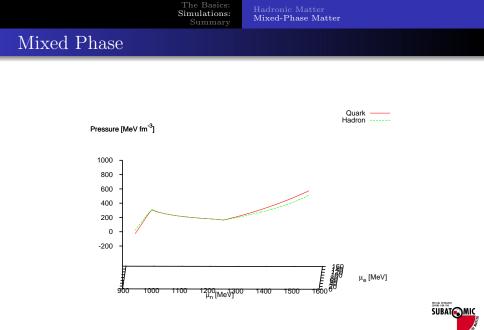




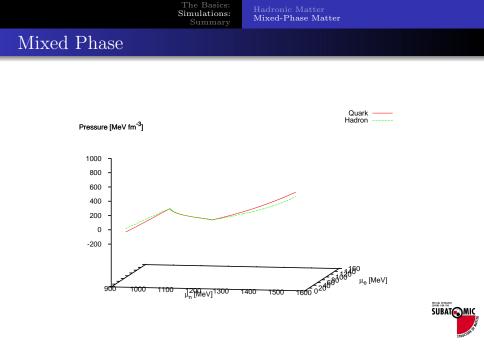


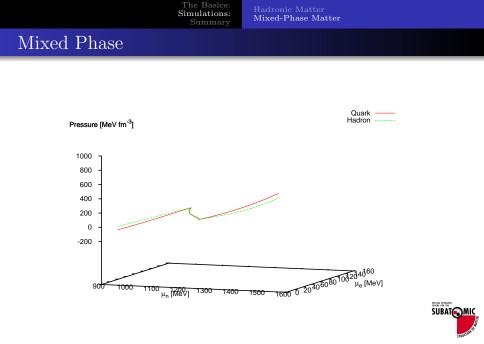


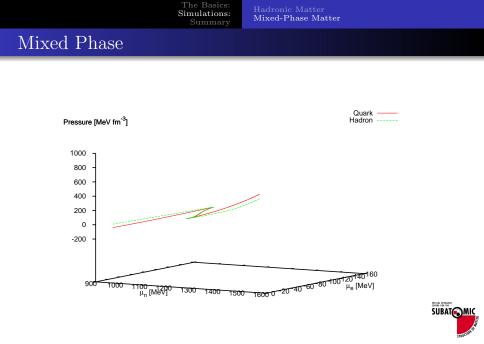


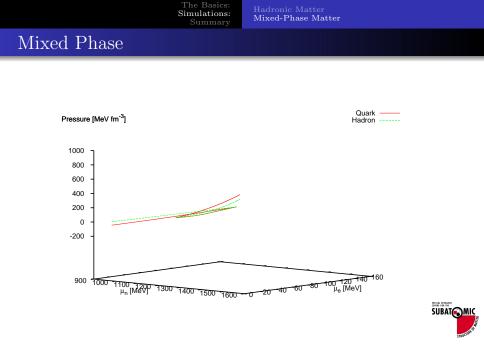


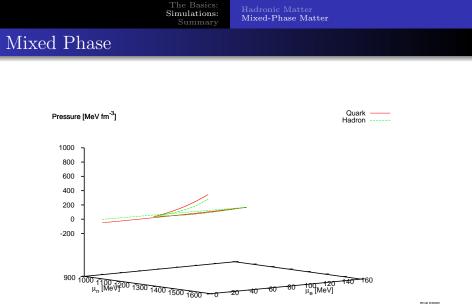
J. D. Carroll QMC: Written in the Stars





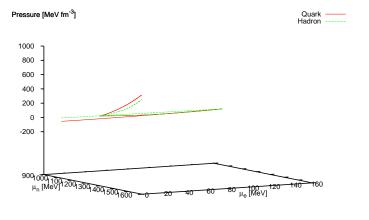








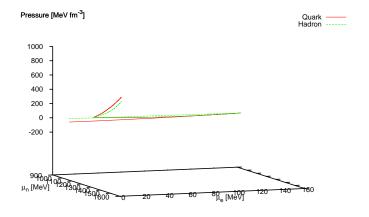






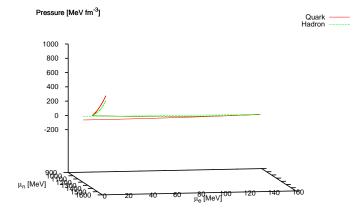
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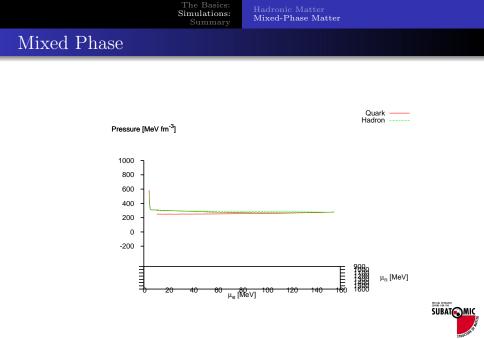






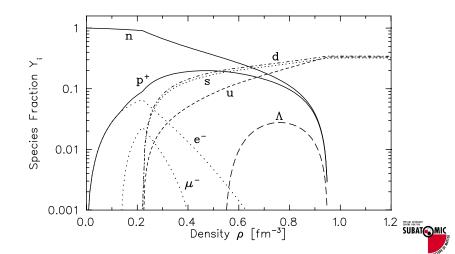






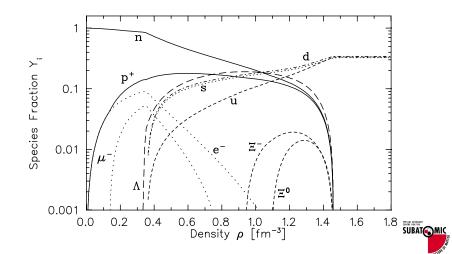
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#### Hyperonic QMC Phase Transition



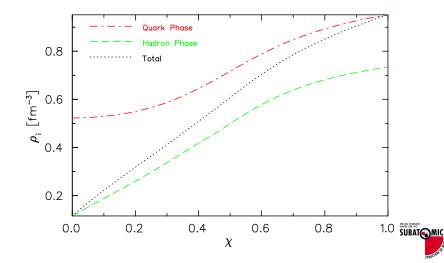
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The Basics: Simulations: Hadronic Matter Mixed-Phase Matter

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Hadronic Matter Mixed-Phase Matter

Phase Transitions Chemical Potentials

Quark Chemical Potentials related to independent chemical potentials;

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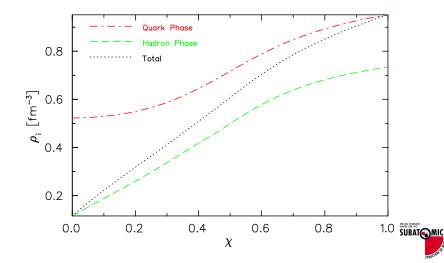
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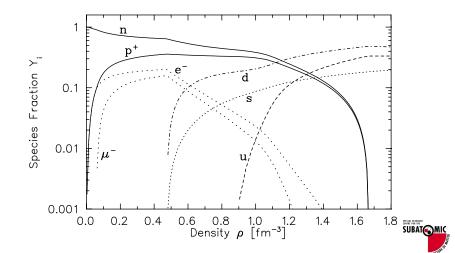
The Basics: Simulations: Hadronic Matter Mixed-Phase Matter

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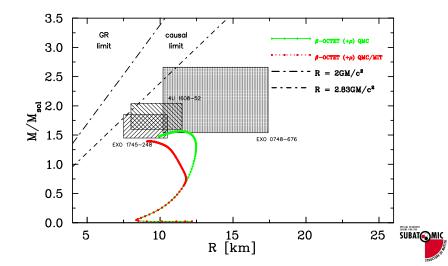
Hadronic Matter Mixed-Phase Matter

### Hyperonic QMC Phase Transition



Hadronic Matter Mixed-Phase Matter

# Mixed-Phase Hyperonic QMC TOV solutions



- The inclusion of  $D\chi SB$  prevents a phase transition to quark matter,
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### Hadronic Models A Brief Overview: Hartree–Fock

At Hartree–Fock level, the scalar self-energy also includes an exchange term, and becomes momentum-dependent:





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Hartree–Fock  $\Sigma_B^s(k)$  (QHD)

$$\begin{split} \Sigma_B^s(k) &= -g_{\sigma B} \langle \sigma \rangle \\ &+ \frac{1}{4\pi^2 k} \int_0^{k_{F_{B'}}} \frac{q \ M_B^*(q)}{E_B^*(q)} \\ &\times \left[ \frac{1}{4} g_{\sigma B'}^2 \Theta_{\sigma}(k,q) - g_{\omega B'}^2 \Theta_{\omega}(k,q) \right] \ dq \end{split}$$



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3WI

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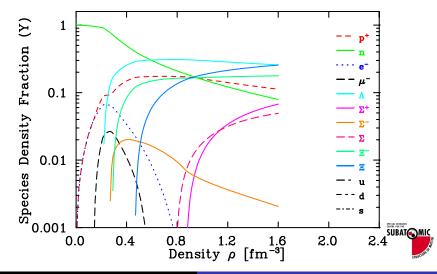
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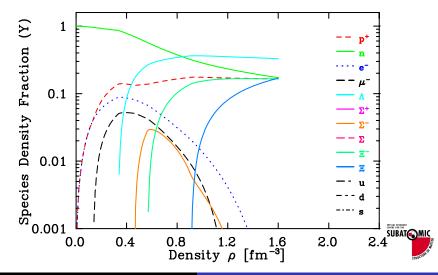


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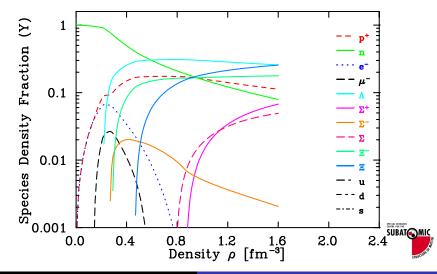
# $\begin{array}{c} \text{Hyperonic QMC} \\ \text{Hartree-Fock} \end{array}$



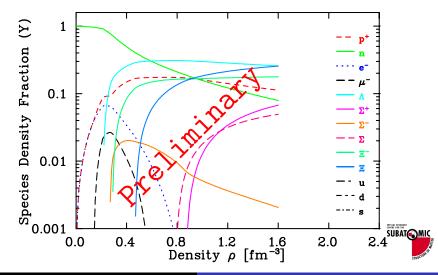
# $\underset{\text{Hartree}}{\text{Hyperonic }QMC}$



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## Further Reading I

### 🍆 Carroll

Applications of the Octet Baryon Quark-Meson Coupling Model to Hybrid Stars (PhD Thesis). arXiv:1001.4318

Carroll, Thomas The Hyperfine, Hyperonic QMC Model - Extension to Hartree–Fock I: Infinite Nuclear Matter. in preparation

 Carroll, Leinweber, Williams, Thomas Phase Transition from QMC Hyperonic Matter to Deconfined Quark Matter. Phys.Rev.C79:045810, 2009 [doi:10.1103/PhysRevC.79.045810]



### Further Reading II

- Guichon, Thomas, Tsushima Binding of hypernuclei in the latest quark-meson coupling model.
   Nucl.Phys.A814:66-73, 2008
   [doi:10.1016/j.nuclphysa.2008.10.001]
- Rikovska-Stone, Guichon, Matevosyan, Thomas Cold uniform matter and neutron stars in the quark-mesons-coupling model. Nucl.Phys.A792:341-369, 2007
   [doi:10.1016/j.nuclphysa.2007.05.011]



























































































































































































# Cheers, Everyone!



