



Electromagnetic currents of the pion-nucleon system

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Introduction

- ▶ New approach to **pion-nucleon scattering** and related **EM currents** that is “taylor made” for **Tony’s Cloudy Bag Model**



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 - ▶ **Good:** such approaches respect (at least) 2-body **unitarity**
 - ▶ **Bad: symmetries broken** (crossing, gauge invariance, PCAC...) in the process
- ▶ Here I present an approach that sums the perturbation series in a different way, so that **symmetries are preserved**



Gauging equations method

- ▶ To generate equations for **EM** currents of strong processes:



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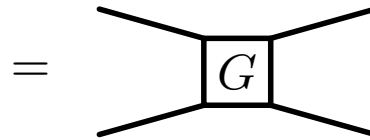
- ▶ Resulting **EM** currents are **gauge invariant**



Gauging the two-body Green function

- ▶ Given the Bethe-Salpeter equation

$$G = G_0 + G_0 V G \quad (1)$$

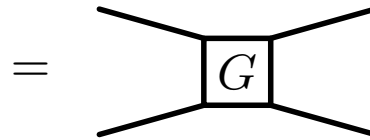




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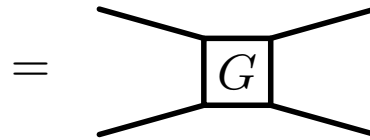
- ▶ add μ everywhere in Eq. (1):

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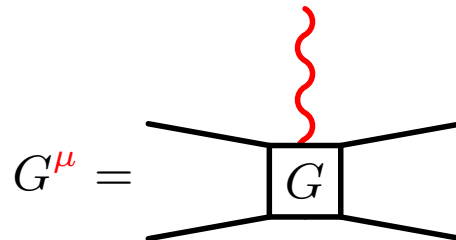
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- ▶ add μ everywhere in Eq. (1):

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- ▶ this attaches a **photon everywhere** in G and gives the 5-point function

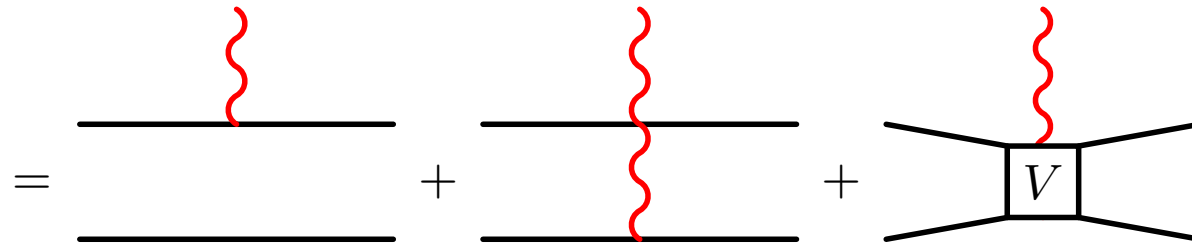


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▶ simple algebra gives

$$G^\mu = G \Gamma^\mu G$$

$$\Gamma^\mu = G_0^{-1} G_0^\mu G_0^{-1} + V^\mu$$

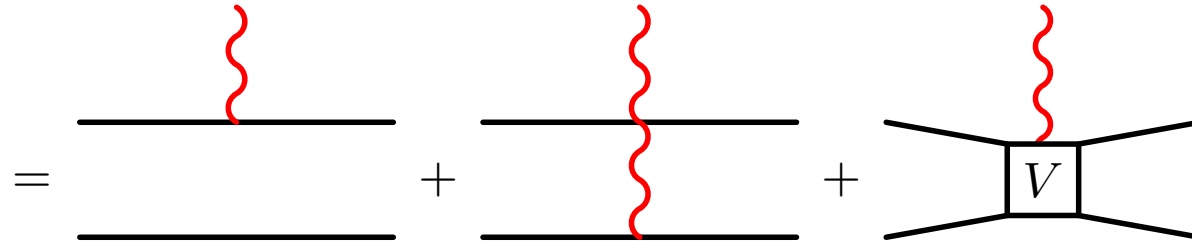


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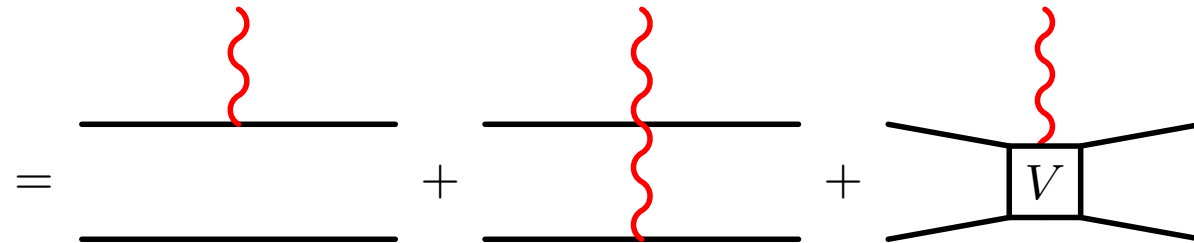
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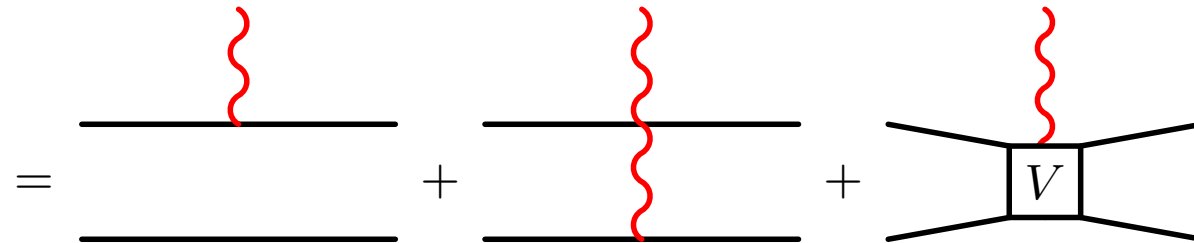
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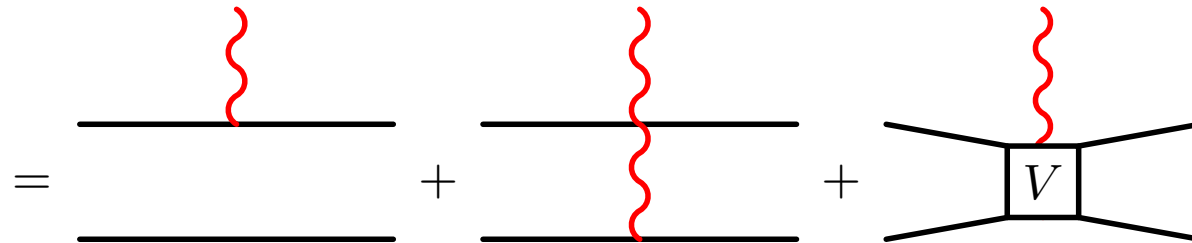
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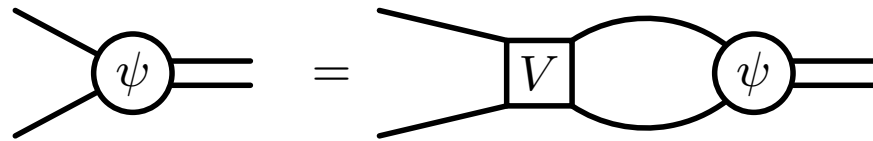


- ▶ Gauge invariance is guaranteed since photon is attached everywhere
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Gauging the bound state wave function

▶ The two-body bound state equation:

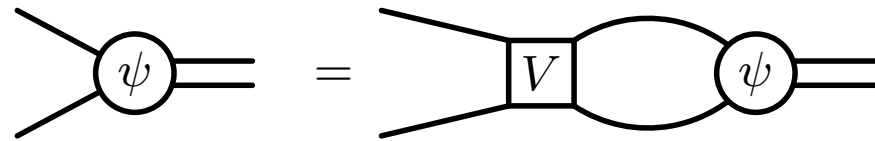
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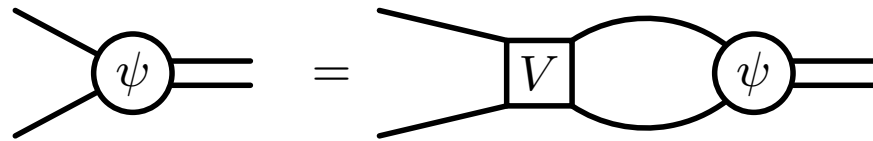
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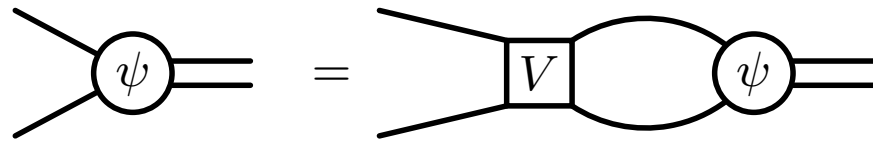


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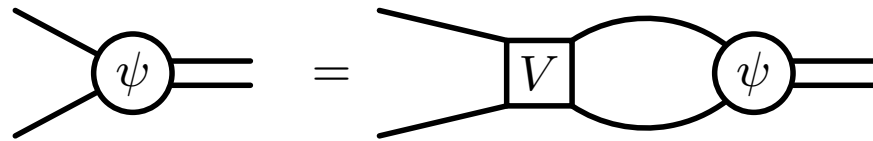
- ▶ Gauging Eq. (2):

$$\psi^\mu = G_0^\mu V \psi + G_0 V^\mu \psi + G_0 V \psi^\mu$$

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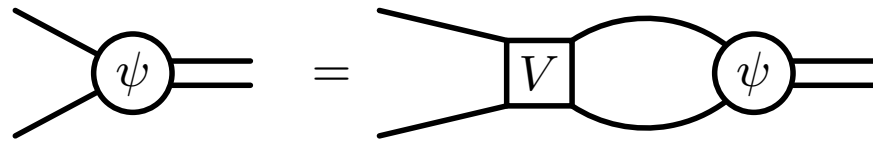
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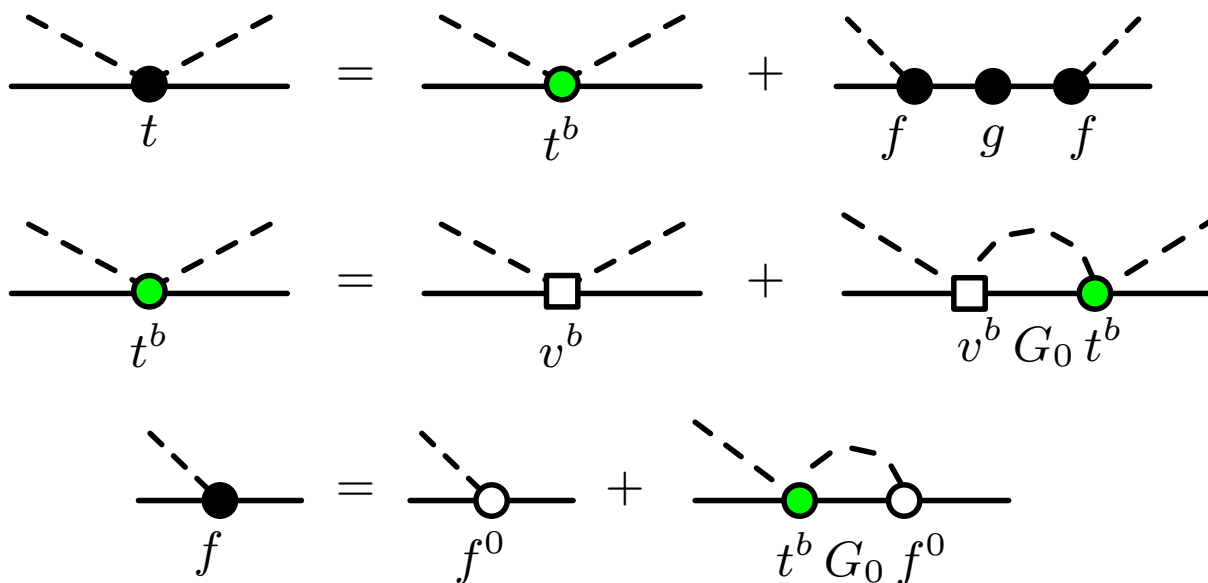
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- ▶ The "deuteron" photodisintegration amplitude is then $G_0^{-1} \psi^\mu \epsilon_\mu$

Standard description of πN scattering

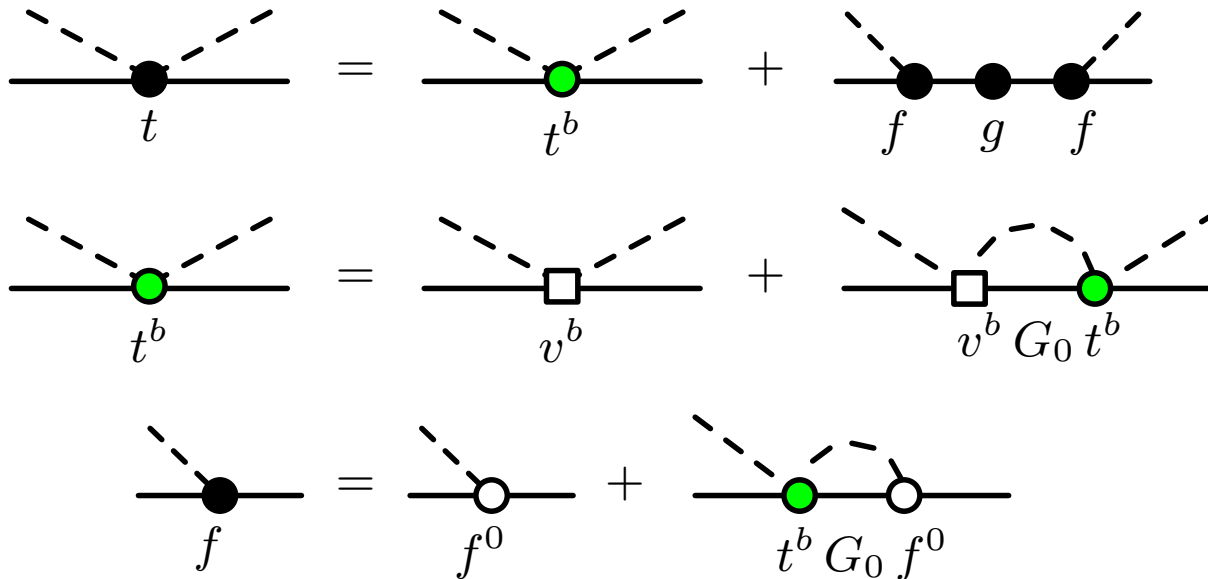


$$g^{-1} = g_0^{-1} - \Sigma$$

$$\Sigma = \bar{f} - f_0$$

► Good: two-body unitarity

Standard description of πN scattering



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- ▶ Good: two-body unitarity
- ▶ Bad: **No crossing symmetry** once v^b approximated, no PCAC



An new (formal) description of πN scattering

- ▶ Start with the dressed propagator g of the standard description



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- ▶ Then “double gauge” g , but with pions



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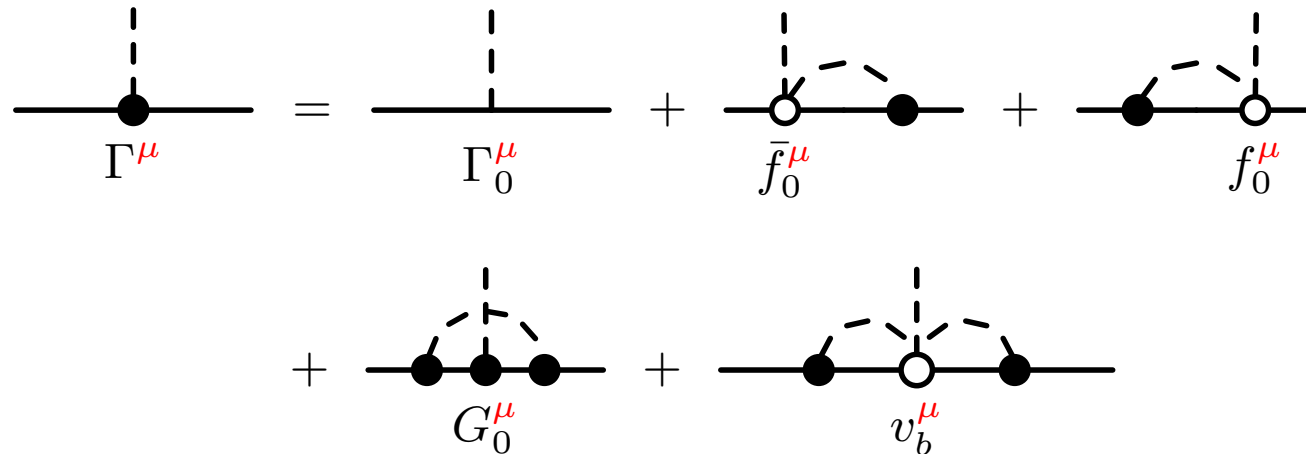
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$$g^\mu = g \Gamma^\mu g$$

$$\Gamma^\mu = \Gamma_0^\mu + \Sigma^\mu$$

$$\Gamma^\mu = \Gamma_0^\mu + \bar{f}_0^\mu G_0 f + \bar{f} G_0 f_0^\mu + \bar{f} G_0^\mu f + \bar{f} G_0 v_b^\mu G_0 f$$



An new (formal) description of πN scattering

- ▶ Identifying Γ^μ with f and comparing with the standard expression

$$\begin{array}{c} \text{---} \bullet \text{---} \\ f \end{array} = \begin{array}{c} \text{---} \circ \text{---} \\ f^0 \end{array} + \begin{array}{c} \text{---} \circ \text{---} \bullet \text{---} \\ v^b G_0 f \end{array}$$

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yields a noteworthy result

$$\begin{array}{c} \diagup \\ \text{---} \\ \bullet \\ f_0 \end{array} = \Gamma_0^\mu + \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ \bullet \quad \circ \\ \bar{f} \quad f_0^\mu \end{array}$$

$$v_b = \begin{array}{c} \diagdown \quad \diagup \\ \text{---} \\ \circ \\ \bar{f}_0^\mu \end{array} + \begin{array}{c} \diagdown \quad \diagup \\ \text{---} \\ \bullet \quad \bullet \\ \bar{f} \quad \Gamma^\mu \end{array} + \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ \bullet \quad \circ \\ \bar{f} \quad v_b^\mu \end{array}$$



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▶ $g^{\mu\nu} = g(\Gamma^\mu g\Gamma^\nu + \Gamma^\nu g\Gamma^\mu + \Gamma^{\mu\nu})g$

$$\Gamma^{\mu\nu} = \Gamma_0^{\mu\nu} + \Sigma^{\mu\nu}$$

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$$T_{\pi N} = \begin{array}{c} \diagup \quad \diagdown \\ \bullet \quad \bullet \\ \Gamma^\mu \quad \Gamma^\nu \end{array} + \begin{array}{c} \diagdown \quad \diagup \\ \bullet \quad \bullet \\ \Gamma^\nu \quad \Gamma^\mu \end{array} + \begin{array}{c} \diagup \quad \diagdown \\ \circ \\ \Gamma_0^{\mu\nu} \end{array} + \Sigma^{\mu\nu}$$

$$\begin{aligned} \Sigma^{\mu\nu} &= \bar{f}_0^{\mu\nu} G_0 f + \bar{f} G_0 f_0^{\mu\nu} + \bar{f} G_0 \Delta^{\mu\nu} G_0 f \\ &+ \bar{f}_0^\mu G \Delta^\nu G_0 f + \bar{f} G_0 \Delta^\nu G f_0^\mu + (\mu \leftrightarrow \nu) \\ &+ \bar{f}_0^\mu G f_0^\nu + \bar{f} G_0 \Delta^\mu G \Delta^\nu G_0 f + (\mu \leftrightarrow \nu) \end{aligned}$$

$$\Delta^\mu = \Gamma^\mu g_\pi^{-1} + v_b^\mu, \quad G = G_0 + G_0 t_b G_0$$

An new (formal) description of πN scattering

$$\begin{aligned}
 \Sigma^{\mu\nu} = & \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \\
 & + \text{Diagram 4} + \text{Diagram 5} + (\mu \leftrightarrow \nu) \\
 & + \text{Diagram 6} + \text{Diagram 7} + (\mu \leftrightarrow \nu)
 \end{aligned}$$

The diagrams are represented as follows:

- Diagram 1:** A horizontal line with a white circle on the left and a black circle on the right. A dashed arc connects the white circle to the black circle. Below the white circle is $\bar{f}_0^{\mu\nu}$ and below the black circle is f . The label G_0 is between them.
- Diagram 2:** A horizontal line with a black circle on the left and a white circle on the right. A dashed arc connects the white circle to the black circle. Below the white circle is $f_0^{\mu\nu}$ and below the black circle is \bar{f} . The label G_0 is between them.
- Diagram 3:** A horizontal line with three black circles. A dashed arc connects the first and second circles, and another dashed arc connects the second and third circles. Below the first circle is \bar{f} , below the second is $G_0 \Delta^{\mu\nu} G_0$, and below the third is f .
- Diagram 4:** A horizontal line with a white circle on the left and two black circles on the right. A dashed line with a 'v' vertex connects the white circle to the first black circle. A dashed line with an 'x' vertex connects the first black circle to the second black circle. A box labeled G is between the two black circles. Below the white circle is \bar{f}_0^μ , below the first black circle is Δ^ν , and below the second is f .
- Diagram 5:** A horizontal line with two black circles on the left and a white circle on the right. A dashed line with an 'x' vertex connects the first black circle to the second black circle. A dashed line with a 'v' vertex connects the second black circle to the white circle. A box labeled G is between the two black circles. Below the first black circle is \bar{f} , below the second is Δ^ν , and below the white circle is f_0^μ .
- Diagram 6:** A horizontal line with a white circle on the left and a white circle on the right. A dashed line with a 'v' vertex connects the white circle to a box labeled G . A dashed line with a 'v' vertex connects the box G to the white circle on the right. Below the left white circle is \bar{f}_0^μ and below the right white circle is f_0^ν .
- Diagram 7:** A horizontal line with two black circles on the left and two black circles on the right. A dashed line with an 'x' vertex connects the first black circle to the second black circle. A dashed line with an 'x' vertex connects the third black circle to the fourth black circle. A box labeled G is between the second and third black circles. Below the first black circle is \bar{f}_0 , below the second is Δ^μ , below the third is Δ^ν , and below the fourth is f .

► Bad: ?

An new (formal) description of πN scattering

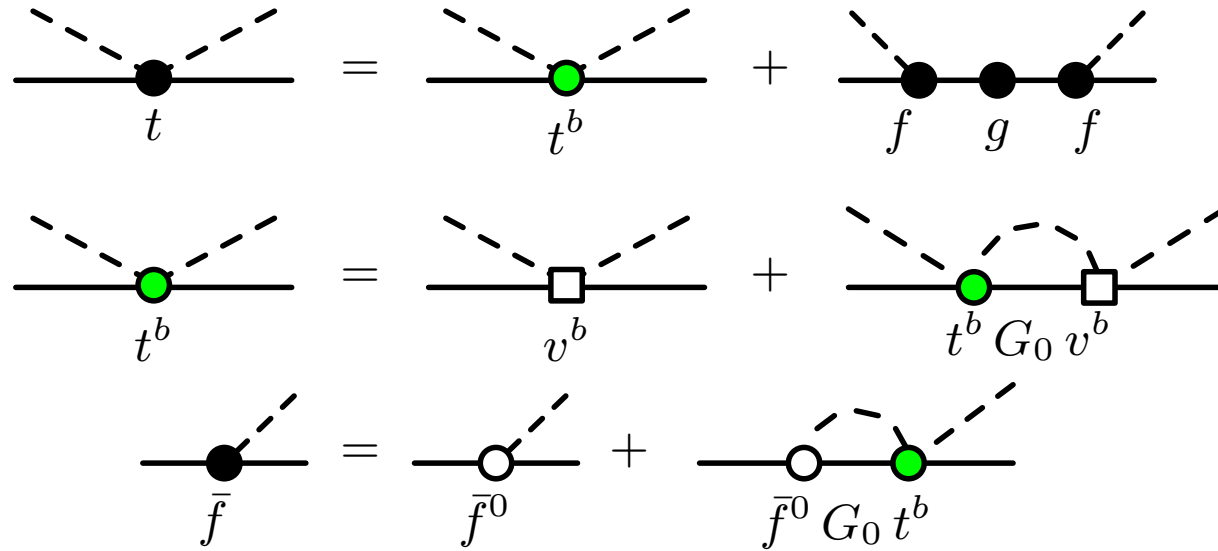
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The diagrams are Feynman diagrams for pion-nucleon scattering.
 - Diagram 1: $\bar{f}_0^{\mu\nu} G_0 f$
- Diagram 2: $\bar{f} G_0 f_0^{\mu\nu}$
- Diagram 3: $\bar{f} G_0 \Delta^{\mu\nu} G_0 f$
- Diagram 4: \bar{f}_0^μ (left), Δ^ν (middle), f (right)
- Diagram 5: \bar{f} (left), Δ^ν (middle), f_0^μ (right)
- Diagram 6: \bar{f}_0^μ (left), f_0^ν (right)
- Diagram 7: \bar{f}_0 (left), Δ^μ (middle), Δ^ν (middle), f (right)

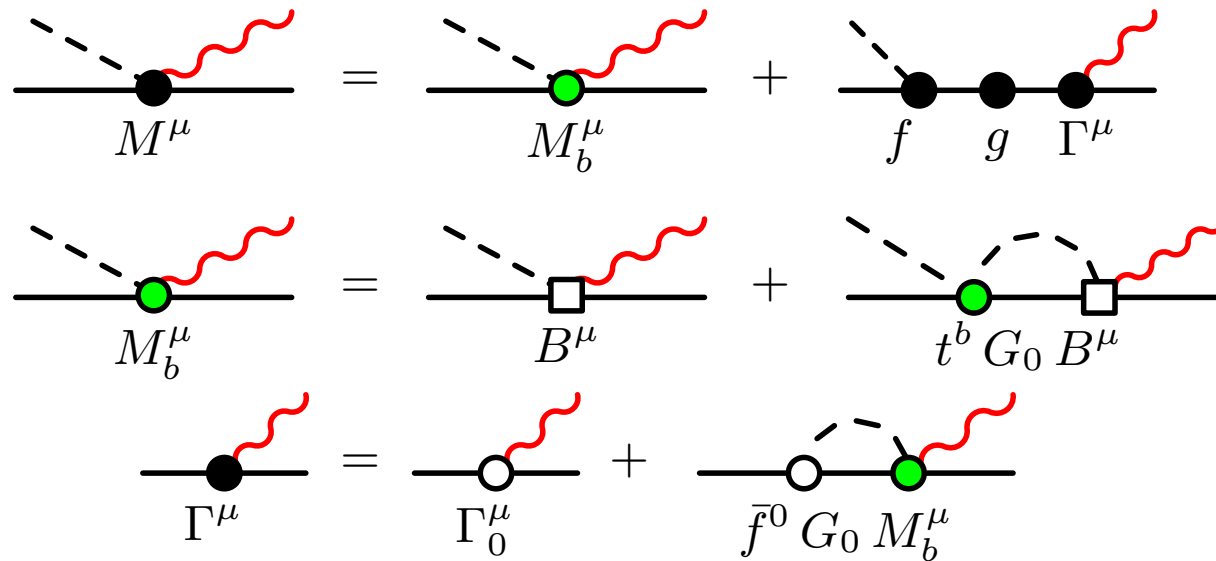
▶ Bad: ?

▶ Good: Obeys crossing symmetry, PCAC a possibility

Standard description of pion photoproduction

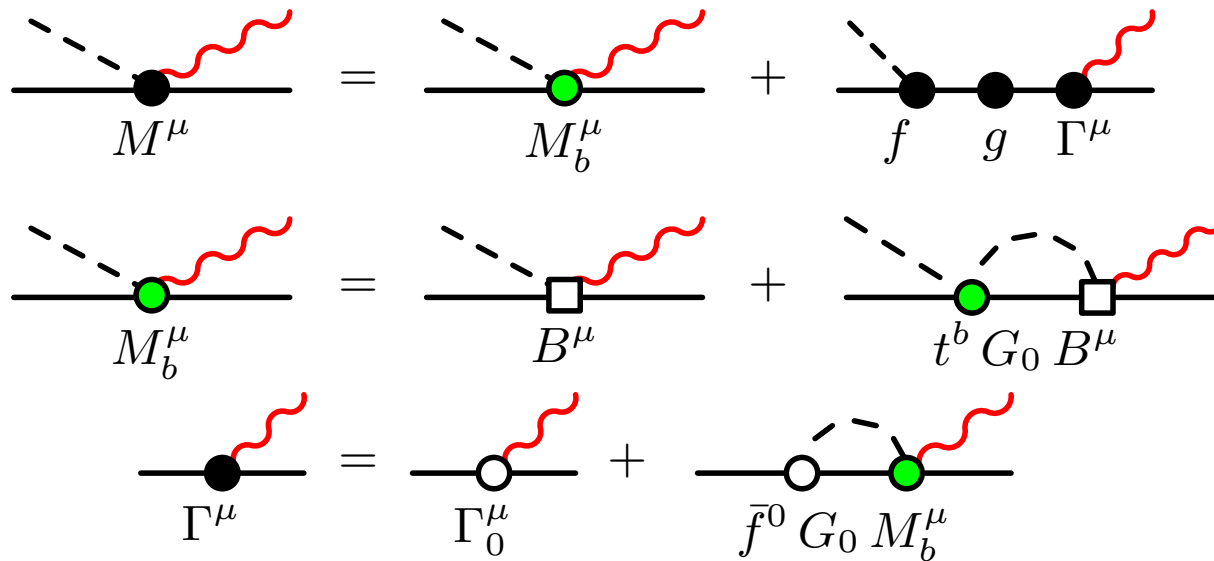


Standard description of pion photoproduction



- ▶ Good: two-body unitarity guaranteed (Watson's theorem)

Standard description of pion photoproduction



- ▶ Good: two-body unitarity guaranteed (Watson's theorem)
- ▶ Bad: not gauge invariant

Gauge invariant pion photoproduction

- ▶ Start with the dressed πNN vertex  of the standard πN description

Gauge invariant pion photoproduction

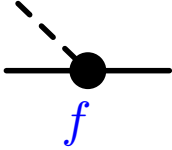
▶ Start with the dressed πNN vertex  of the standard πN description

▶ Then gauge the equation for $G_0 f g$ since

$$M^\mu = G_0^{-1} (G_0 f g)^\mu g^{-1}$$

is the pion photoproduction amplitude

Gauge invariant pion photoproduction

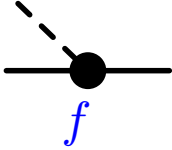
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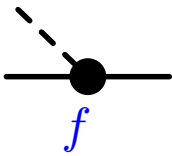
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- ▶ $f = f_0 + v_b G_0 f$:
- ▶ $(G_0 f g)^\mu = (G_0 f_0 g)^\mu + (G_0 v_b G_0 f g)^\mu$
 $= (G_0 f_0 g)^\mu + (G_0 v_b)^\mu + (G_0 f g)^\mu$

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- ▶ $(G_0 f g)^\mu = (G_0 f_0 g)^\mu + (G_0 v_b G_0 f g)^\mu$
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$$M^\mu = f g \Gamma^\mu + (1 + t G_0) (f_0^\mu + \Gamma_0^\mu G_0 f + v_b^\mu G_0 f)$$

Gauge invariant pion photoproduction

- ▶ The derived pion photoproduction amplitude is:

The diagram shows the decomposition of the pion photoproduction amplitude M^μ into several terms. The first equation shows M^μ as the sum of a term with vertices f , g , and Γ^μ , and a term $(1 + tG_0) B^\mu$. The second equation decomposes B^μ into four terms: a contact term f_0^μ , a term with vertices Γ^μ , g , and f , a term with vertex f , and a term with vertices v_b^μ , G_0 , and f .

$$M^\mu = f \quad g \quad \Gamma^\mu + (1 + tG_0) B^\mu$$

$$B^\mu = f_0^\mu + \Gamma^\mu \quad g \quad f + f + v_b^\mu \quad G_0 \quad f$$

Gauge invariant pion photoproduction

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$$\begin{aligned}
 M^\mu &= f-g-\Gamma^\mu + (1+tG_0)B^\mu \\
 B^\mu &= f_0^\mu + \Gamma^\mu-g-f + f + v_b^\mu G_0 f
 \end{aligned}$$

- ▶ The amplitude M^μ is gauge invariant as long as the gauged inputs Γ^μ , f_0^μ , and v_b^μ satisfy Ward-Takahashi identities.



Gauge invariant pion photoproduction

- ▶ f_0^μ and v_b^μ can be constructed either from models of substructure, or if phenomenological, from a minimal substitution prescription.



Gauge invariant pion photoproduction

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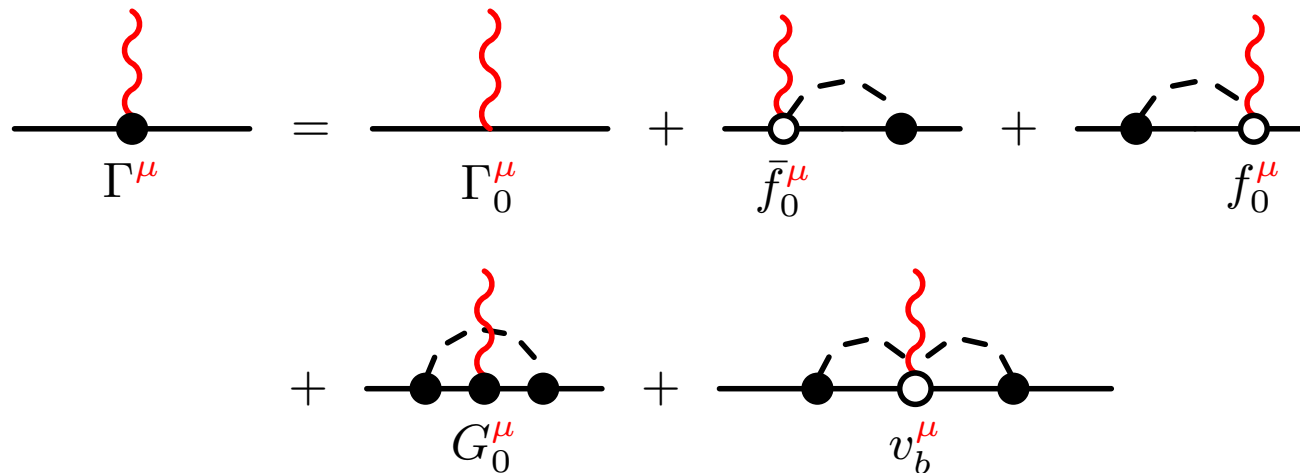
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$$g^\mu = g \Gamma^\mu g$$

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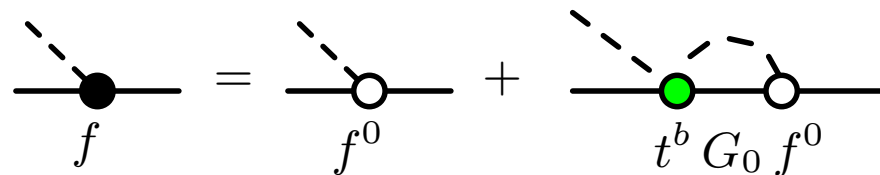
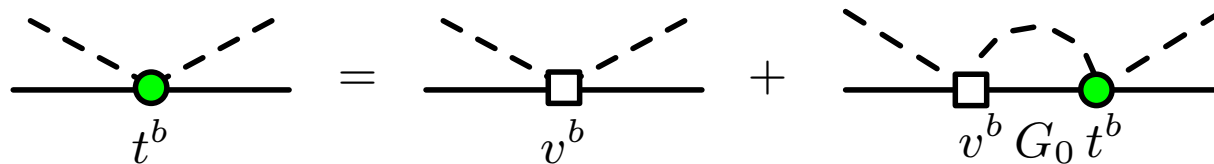
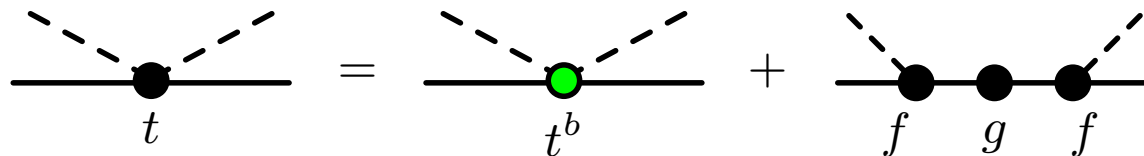
$$\Gamma^\mu = \Gamma_0^\mu + \bar{f}_0^\mu G_0 f + \bar{f} G_0 f_0^\mu + \bar{f} G_0^\mu f + \bar{f} G_0 v_b^\mu G_0 f$$



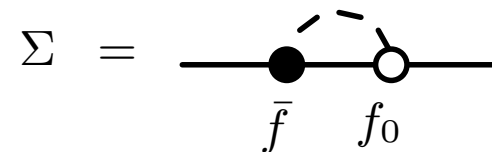
Pion photoproduction in the *spectator model*

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$$g_\pi(k) = \frac{i}{k^2 - \mu^2 + i\epsilon} \quad \rightarrow \quad \delta_\pi(k) = 2\pi\delta^+(k^2 - \mu^2)$$



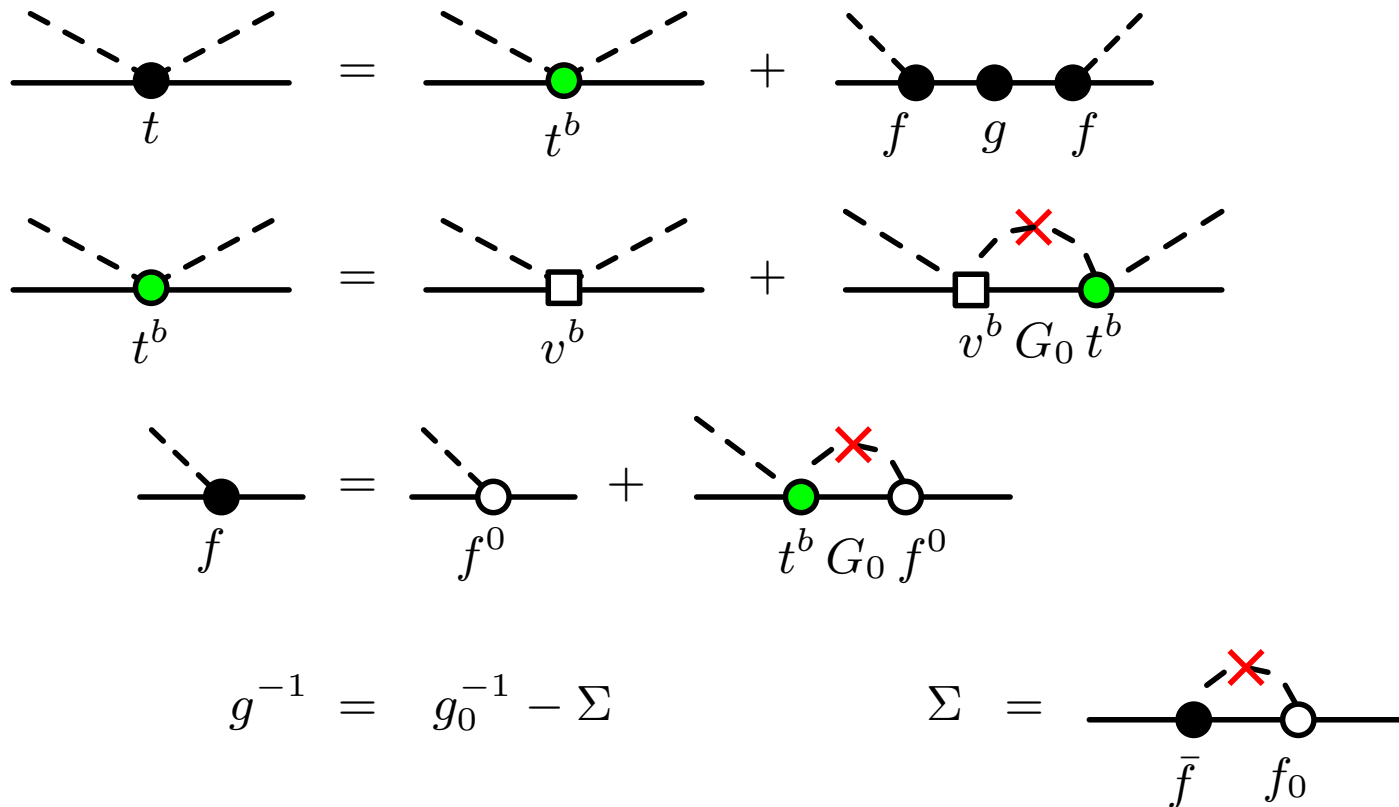
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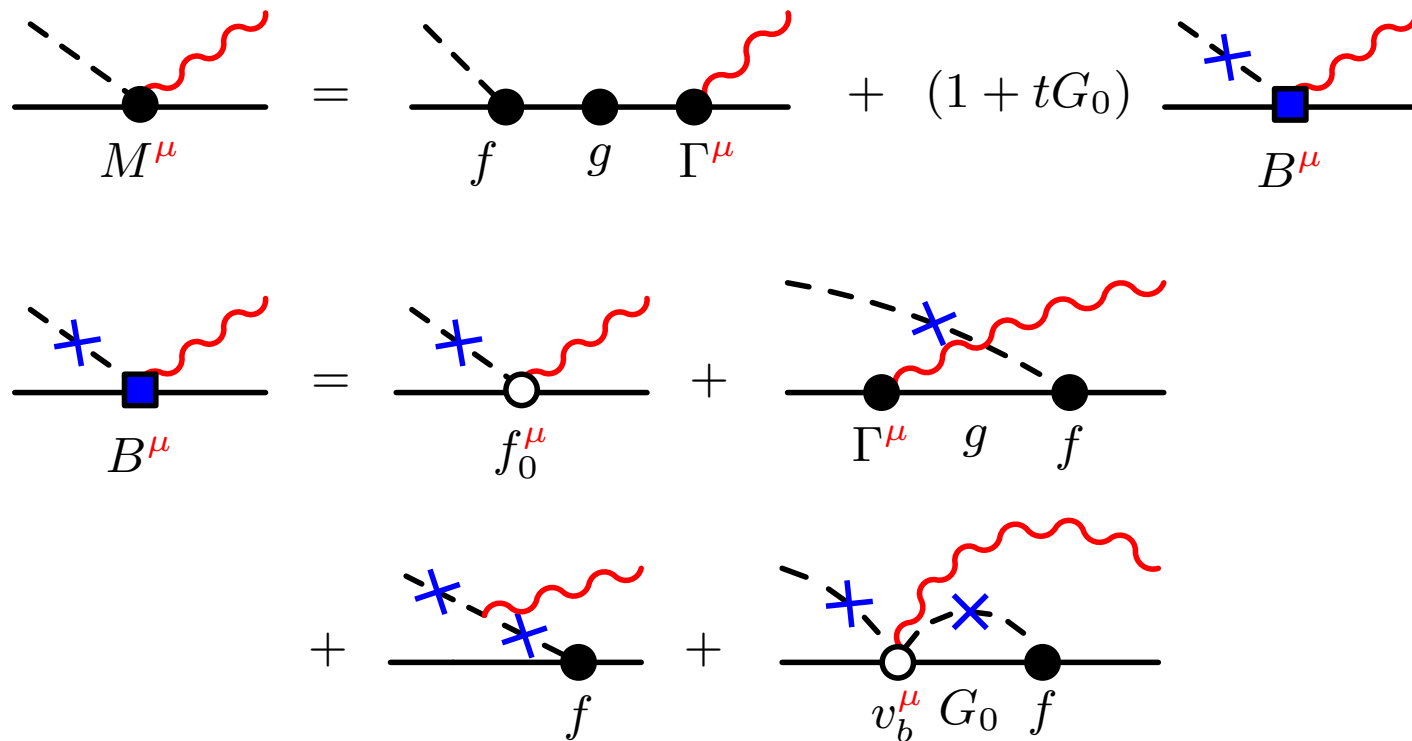
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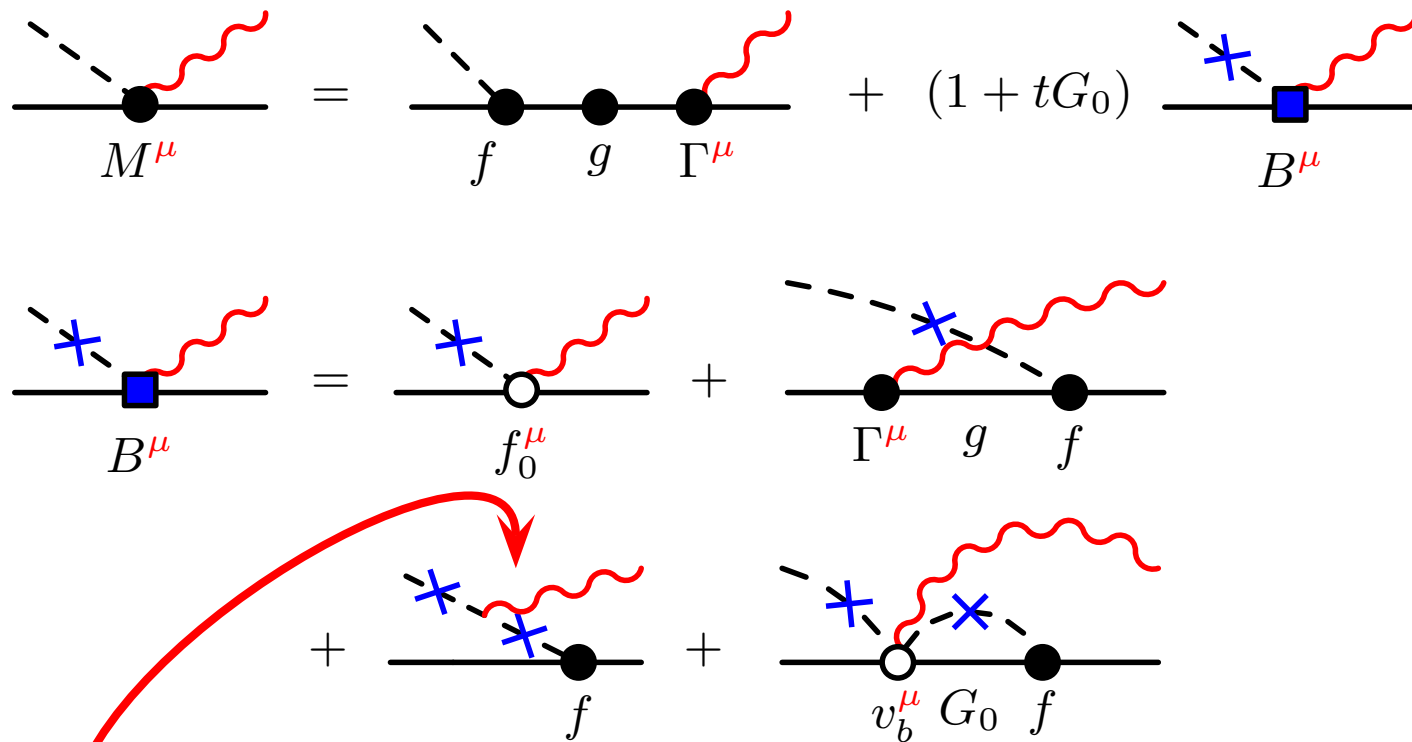
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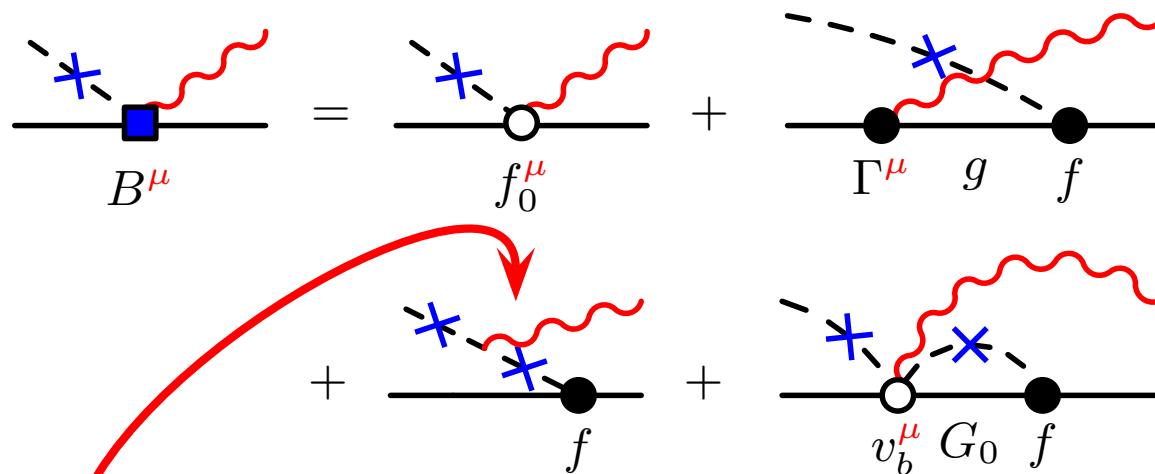
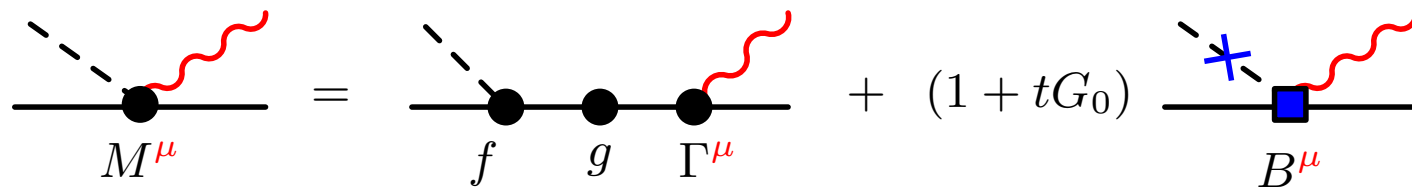
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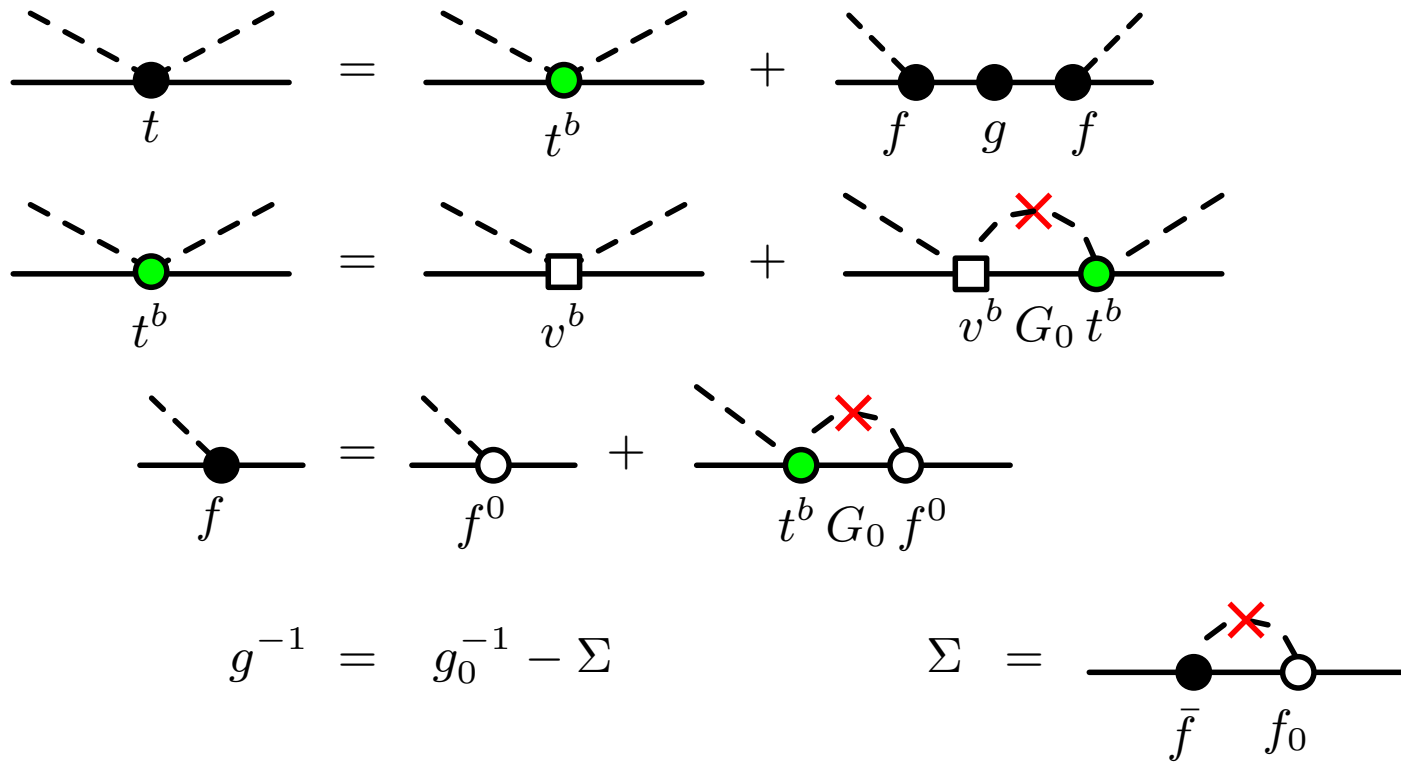


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In any case, gauge invariance will be lost

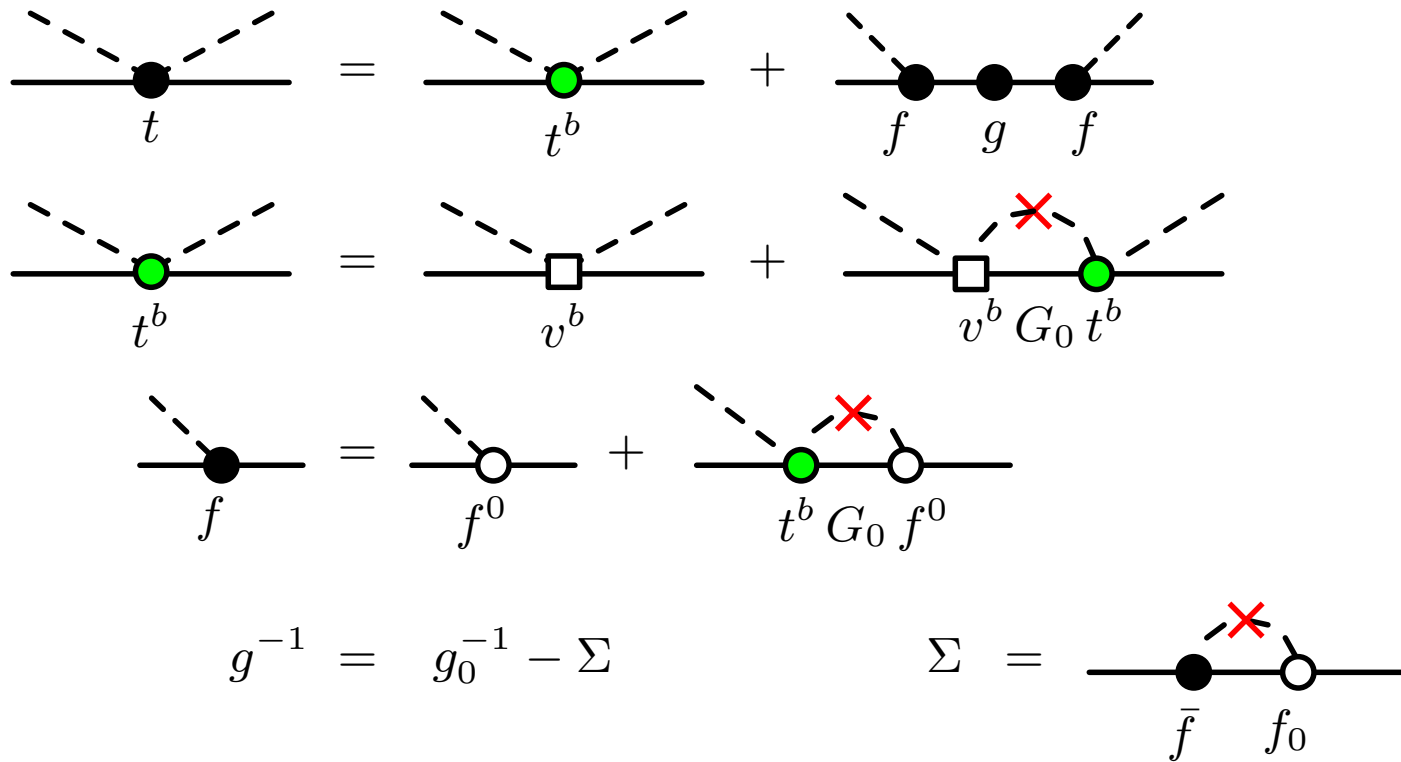
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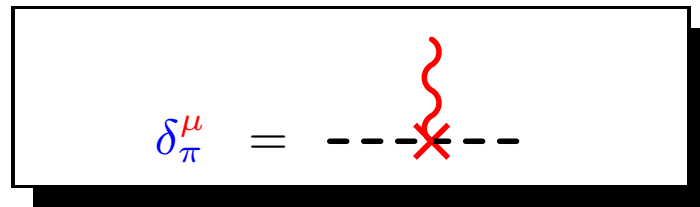


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▶ In this case one is faced giving meaning to δ_π^μ :





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- ▶ Such a δ_π^μ is provided by the Ansatz

$$\delta_\pi^\mu(k', k) = 2\pi ie_\pi (k'^\mu + k^\mu) \frac{\delta^+(k'^2 - \mu^2) - \delta^+(k^2 - \mu^2)}{k^2 - k'^2}$$

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πN equations:

- ▶ Gauging the dressed nucleon propagator g with **one pion** leads to an interesting “self consistency” equation for the πN potential v_b .
- ▶ Gauging g with **two pions** gives a πN amplitude that is **crossing symmetric**
- ▶ Complete pion attachment is a necessary first step towards PCAC



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- ▶ **Happy Birthday Tony!**