The NJL-jet model
for quark fragmentation functions

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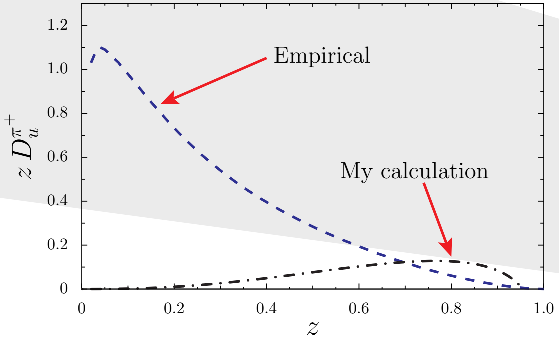
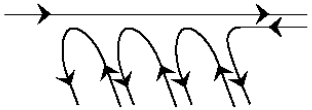
CSSM Workshop in Honour of Tony Thomas
Adelaide, Feb. 15-19 2010

Some background . . .

In 2008 I visited Tony at JLab for 6 months.
Sometimes he was very pleased . . .

but sometimes he worried about my results on fragmentation functions, and drew diagrams on the whiteboard, which I could not understand . . .

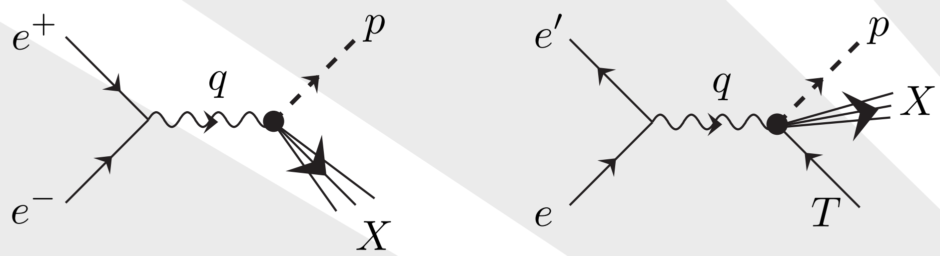




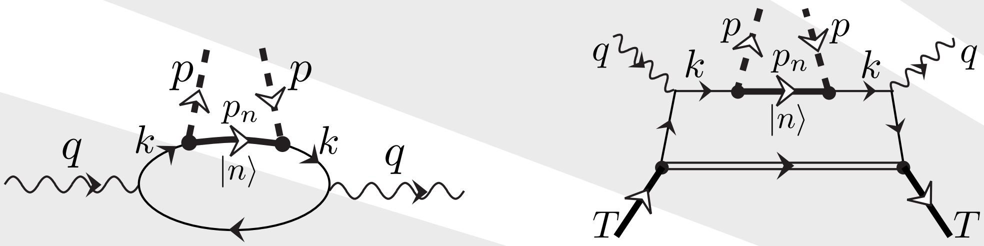
(Tony's diagram)

Introduction

Fragmentation function $D_q^h(z)$ $(z = \frac{2p\cdot q}{q^2})$ describes **semi-inclusive hadron production** in e^+e^- annihilation and (e, e') DIS processes:



Parton model diagrams for cross sections:



- Recently, much effort was made to extract **empirical fragmentation** functions from data.
  See, for example: M. Hirai et al: PRD **75** (2007) 094009.

Introduction

- Much effort was also made to describe **fragmentation functions in effective quark theories**.

  However, almost all calculations introduced artificial "normalization factors" (or other ad-hoc parameters) to enlarge the calculated fragmentation functions.

Definitions and interpretation

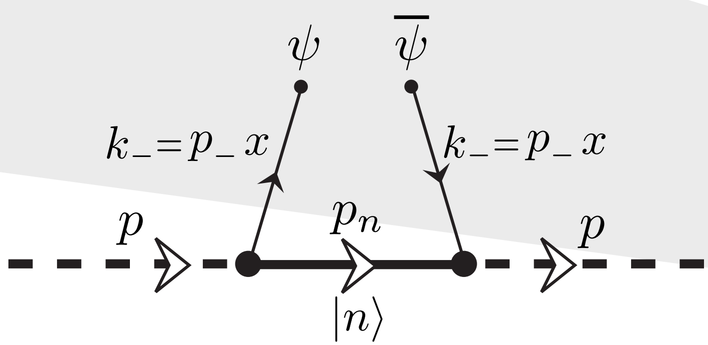
Compare definitions of **distributions** and **fragmentations**:

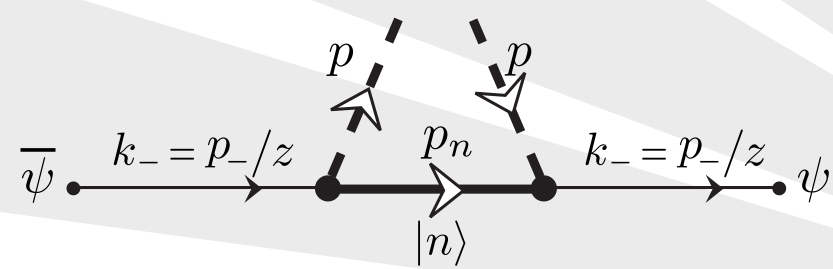
$$f_q^h(x) = \frac{1}{2}\sum_n \delta(p_- x - p_- + p_{n-})\langle p|\overline{\psi}|p_n\rangle\gamma^+\langle p_n|\psi|p\rangle$$

$$= p_- \int \mathrm{d}^2 k_T \sum_\alpha \frac{\langle p|b_\alpha^\dagger(k) b_\alpha(k)|p\rangle}{\langle p|p\rangle} \quad : \ \textbf{quarks in hadron.}$$

$$D_q^h(z) = \frac{z}{12}\sum_n \delta\left(\frac{p_-}{z} - p_- - p_{n-}\right)\langle p, p_n|\overline{\psi}|0\rangle\gamma^+\langle 0|\psi|p, p_n\rangle$$

$$= \frac{k_-}{6}\int \mathrm{d}^2 p_\perp \sum_\alpha \frac{\langle k(\alpha)|a_h^\dagger(p) a_h(p)|k(\alpha)\rangle}{\langle k(\alpha)|k(\alpha)\rangle} \quad : \ \textbf{hadrons in quark!}$$





Momentum sum rules

- Interpretation of $D_q^h(z)$: **Probability that a hadron (p) in the cloud of a virtual quark (k) has fraction z of the quark's light cone momentum: $\mathbf{p_- = z k_-}$.**
- Formal relation between distribution and fragmentation (from crossing and charge conjugation): $D_q^h(z) = (-1)^{2(s_q+s_h)+1} \frac{z}{6} f_q^h(x = \frac{1}{z})$. However, in practice this "**Drell-Levy-Yan**" relation is (almost) useless.
- Momentum sum rules (sums include antiparticles):

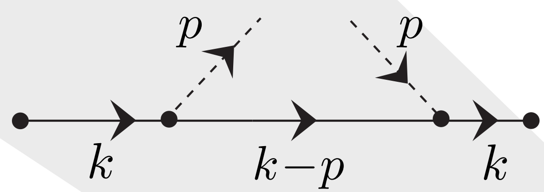
$$\sum_q \int_0^1 x\, \mathrm{d}x\, f_q^h(x) = 1 : \textbf{hadron consists of quarks.}$$

$$\sum_h \int_0^1 z\, \mathrm{d}z\, D_q^h(z) = 1 : \textbf{quark hadronizes completely!}$$

Derivation of sum rule for D_q^h assumes that **quark is an eigenstate of the momentum operator** $\hat{P}_- = \sum_h \int_0^\infty \mathrm{d}p_- \int \mathrm{d}^2 p_\perp \left(p_- a_h^\dagger(p) a_h(p)\right)$ **expressed in terms of hadrons!**

Elementary NJL $q \to \pi$ fragmentation: $d_q^\pi(z)$

Simplest approximation: Truncate $|n\rangle$ to **one-quark state**:



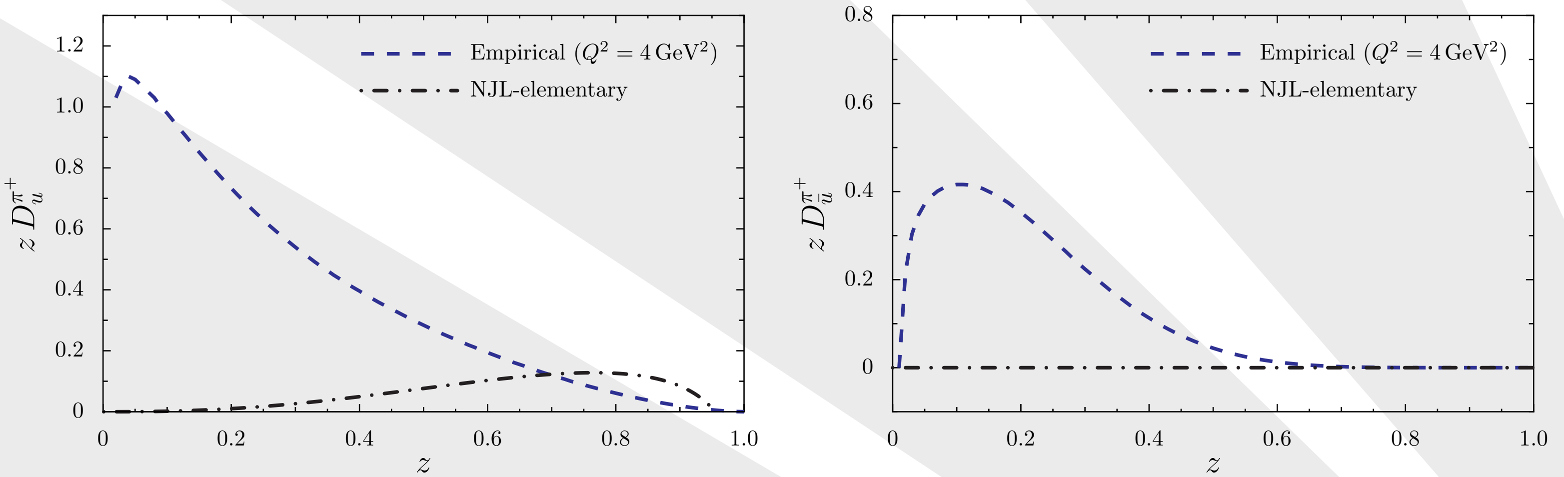
$$d_q^\pi(z) = \frac{1}{2}\,(1+\tau_\pi\tau_q)\; g_\pi^2\,\frac{z}{2}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}_D$$

$$\left[S_F(k)\gamma^+ S_F(k)\gamma_5\,(\not{k}-\not{p}+M)\,\gamma_5\right]\;\delta(k_- - p_-/z)\,\delta\big((p-k)^2-M^2\big)$$

$$= \frac{1}{2}\,(1+\tau_\pi\tau_q)\;z\,g_\pi^2\int\frac{d^2p_\perp}{(2\pi)^3}\,\frac{\mathbf{p}_\perp^2+M^2z^2}{\left[\mathbf{p}_\perp^2+M^2z^2+(1-z)\,m_\pi^2\right]^2}$$

- Isospins: $(\tau_u,\tau_d)=(1,-1)\,,\quad(\tau_{\pi^+},\tau_{\pi^0},\tau_{\pi^-})=(1,0,-1)$.
- The second form is obtained by the substitution $\mathbf{k}_T=-\mathbf{p}_\perp/z$, which follows from the Lorentz transformation.
- Coupling g_π is defined by residue of qq t-matrix at pion pole.
- Calculation performed using invariant mass cut-off and $M=300$ MeV.
- Formally this is the continuation of distribution $f_q^\pi(x)$ to $x=1/z>1$, but the relation is violated in any sensible regularization scheme.

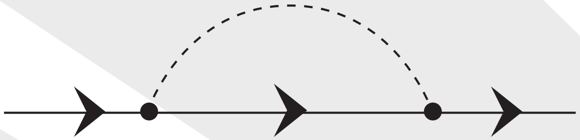
Elementary NJL fragmentation: Results



- **Evolution** of NJL result to $Q^2 = 4$ GeV^2 does not help - the curve on left figure would almost disappear on this scale!
- **These are disastrous results!** To avoid this, previous calculations using effective quark models introduced "normalization constants" or other ad-hoc parameters.
- **Puzzling**: The lowest order fragmentation process $q \to q\pi$ is completely inadequate to describe fragmentation functions, although the "crossed" process $\pi \to q\bar{q}$ describes distribution functions well!

Reason for failure

In constituent - like quark models: **Large probability ($Z_Q \simeq 0.85$) to have a quark "without its pion cloud"**. Here Z_Q is the residue of quark propagator including pion-loop self energy:



- Elementary NJL fragmentation function corresponds to the following "**number of pions per quark**":

$$\int_0^1 \mathrm{d}z \sum_\pi d_q^\pi(z) = 1 - Z_Q \simeq 0.15$$

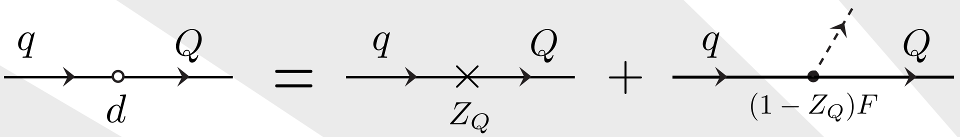
  Therefore the **pion momentum sum is small**:

$$\int_0^1 z\,\mathrm{d}z \sum_\pi d_q^\pi(z) \simeq 0.1 < 1 - Z_Q.$$

- On the other hand, empirical functions show that $\simeq 74\%$ of the initial quark momentum is converted to pions!
- $\Rightarrow$ Expect: **High-energy quark may radiate a large number of pions, and we must sum up the momenta of *all* pions!**

Product ansatz for multifragmentations

Auxiliary quantity: $d_q^Q(\eta)$ = **fragmentation function for: quark (q) $\to$ quark (Q).** [Same as: **distribution of Q inside q.**]



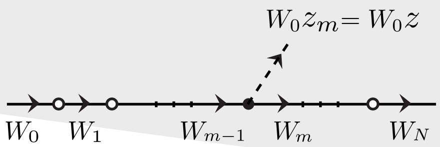
$$6\, d_q^Q(\eta) = Z_Q \delta(\eta - 1) + d_q^\pi(1-\eta) \equiv Z_Q \delta(\eta - 1) + (1 - Z_Q) F(\eta)$$

(Isospin indices omitted.) **Here $F(\eta)$ is normalized to 1.**

d_q^Q describes the elementary $q \to Q$ splitting $\Rightarrow$ **Product ansatz** for $D_q^\pi(z)$: **If a quark can produce a maximum of N pions, then**

$$D_q^\pi(z) = \int_0^1 \mathrm{d}\eta_1 \ldots \int_0^1 \mathrm{d}\eta_N \; 6d(\eta_1) \cdot 6d(\eta_2) \cdot \cdots \cdot 6d(\eta_N) \left(\sum_{m=1}^{N} \delta(z - z_m) \right)$$

$$D_q^\pi(z) = \sum_{m=1}^{N}$$



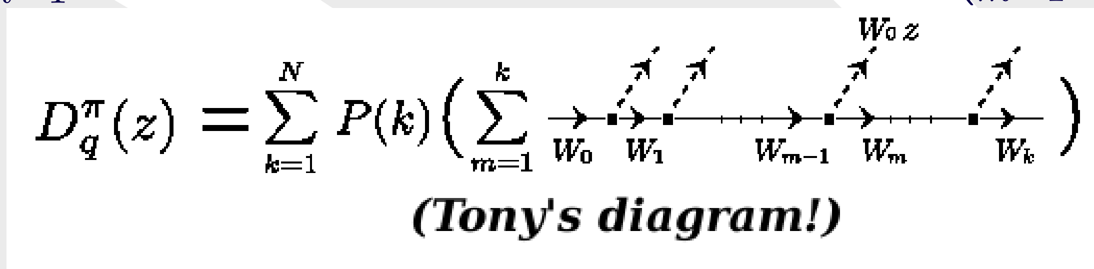
$$\left(z_m = \frac{W_{m-1} - W_m}{W_0} = \eta_1 \eta_2 \ldots \eta_{m-1}(1 - \eta_m) \right)$$

Quark cascades (NJL-jet model)

What is the **physical meaning of this ansatz**?

- Rewrite the product *identically* as follows:

$$D_q^\pi(z) = \sum_{k=1}^{N} P(k) \int_0^1 \mathrm{d}\eta_1 \ldots \int_0^1 \mathrm{d}\eta_k \, F(\eta_1) \ldots F(\eta_k) \left(\sum_{m=1}^{k} \delta(z - z_m) \right)$$



$P(k)$ is the **probability that k pions are produced**:

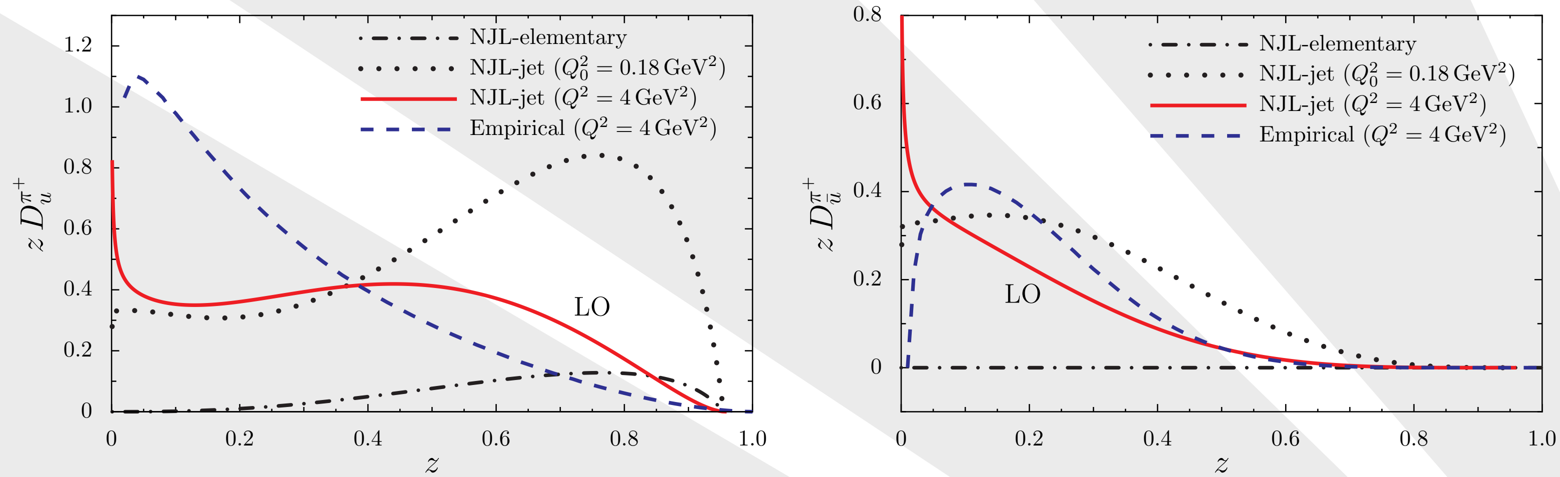
$$P(k) = \binom{N}{k} (1 - Z_Q)^k \, Z_Q^{N-k} \quad \Rightarrow \quad \sum_{k=0}^{N} P(k) = 1.$$

In the limit $N \to \infty$, $P(k)$ becomes a **normal distribution** with mean number (multiplicity) $\langle k \rangle = N(1 - Z_Q)$.

- In each elementary process, a fraction $\alpha \equiv \langle z F(z) \rangle < 1$ is left to the quark $\Rightarrow$ Fraction left to the final quark remainder is:

$$\sum_{k=0}^{N} P(k) \alpha^k \xrightarrow{N\to\infty} 0.$$ $\Rightarrow$ **For $\mathrm{N} \to \infty$, 100$\%$ of quark momentum is converted to pions!** **(Price: Divergent multiplicity.)**

NJL-jet model: Results



- **Cascade-like processes enhance the fragmentation functions tremendously!**
- Calculated functions are still **too stiff** because:
  - Q^2 evolution should be performed in **NLO**. (At present, codes are not available for the public . . .)
  - Some of observed pions are **secondary pions** (from decay of vector mesons).
  - Other fragmentation channels (**nucleons, kaons**) should be included.

Summary

- **Cascade - type multifragmentation processes are extremely important to describe fragmentation functions.**
- **The "NJL-jet model" describes qualitatively the empirical fragmentation functions without any new parameters.**
- **Straight forward extensions will improve the description: NLO effects in Q^2 evolution; inclusion of vector mesons; inclusion of nucleon and kaon channels.**
- **Important: The product ansatz should be derived from field theory ($\Leftrightarrow$ rainbow-ladder approximation for quark self energy).**