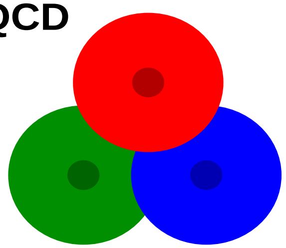
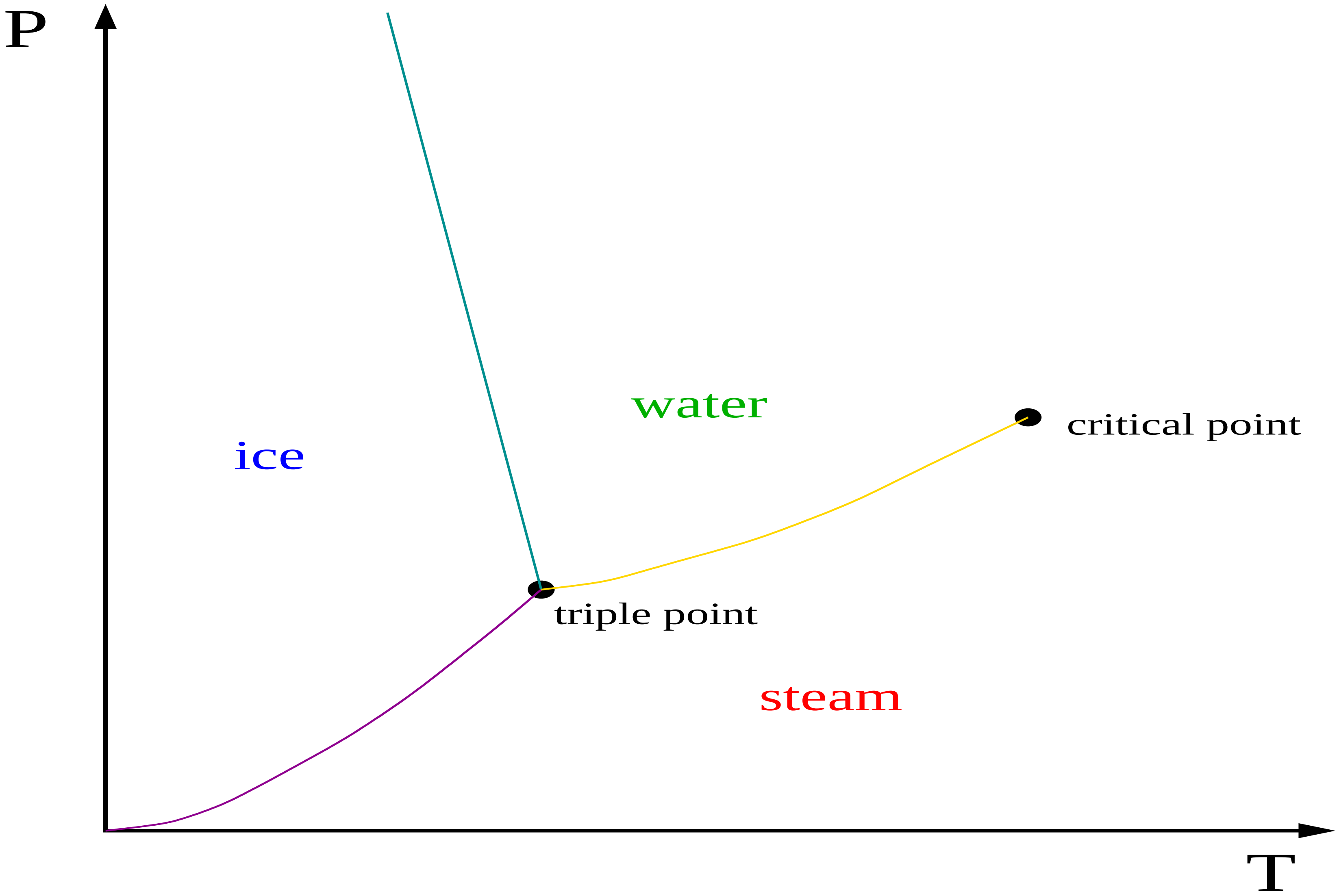
Charmonium melting in the quark-gluon plasma phase of QCD

Chris Allton

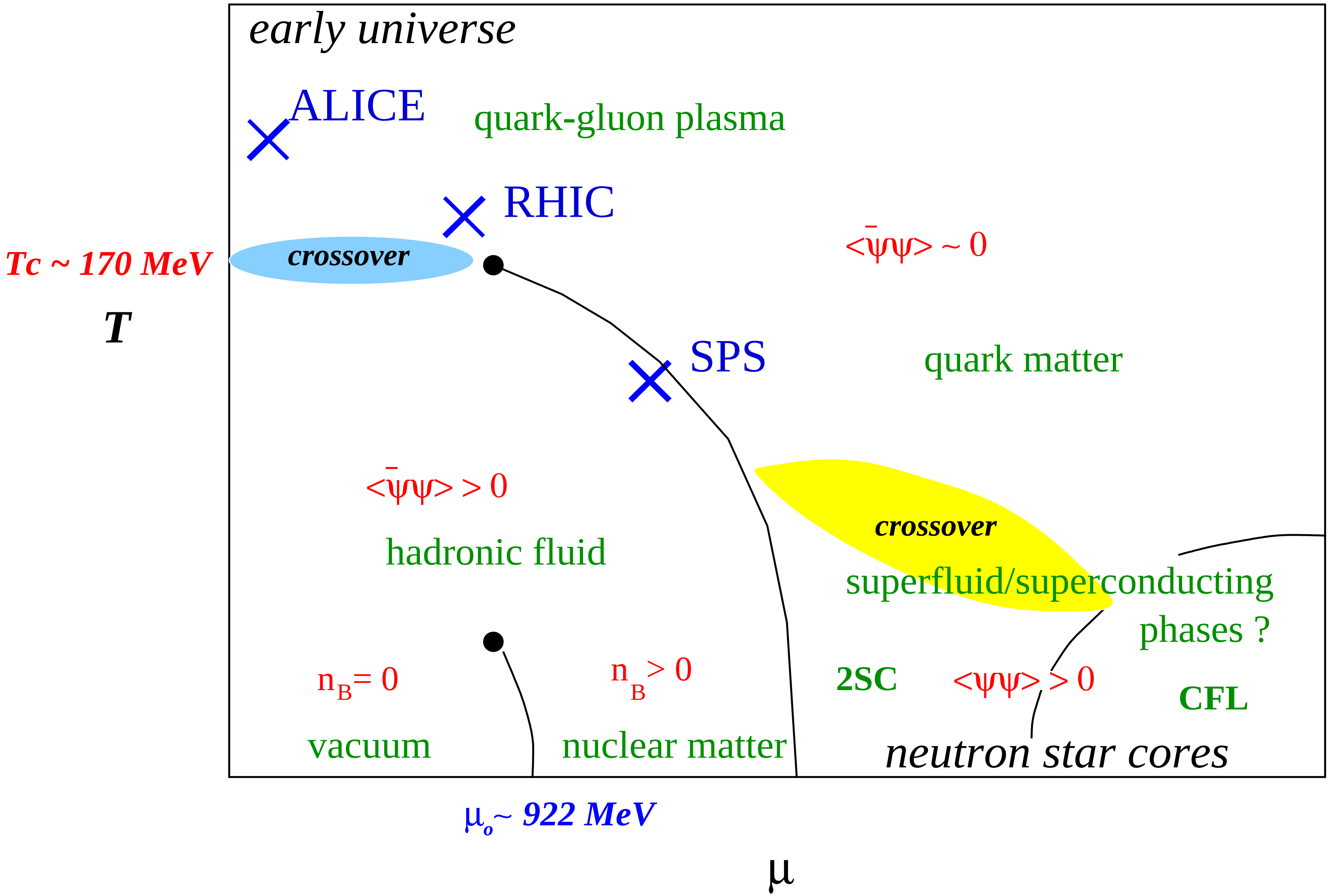
Swansea University

	Continuum	Lattice
$T \neq 0$		
$T = 0$	**Ordinary QCD** 	

H_2O phase diagram



QCD phase diagram



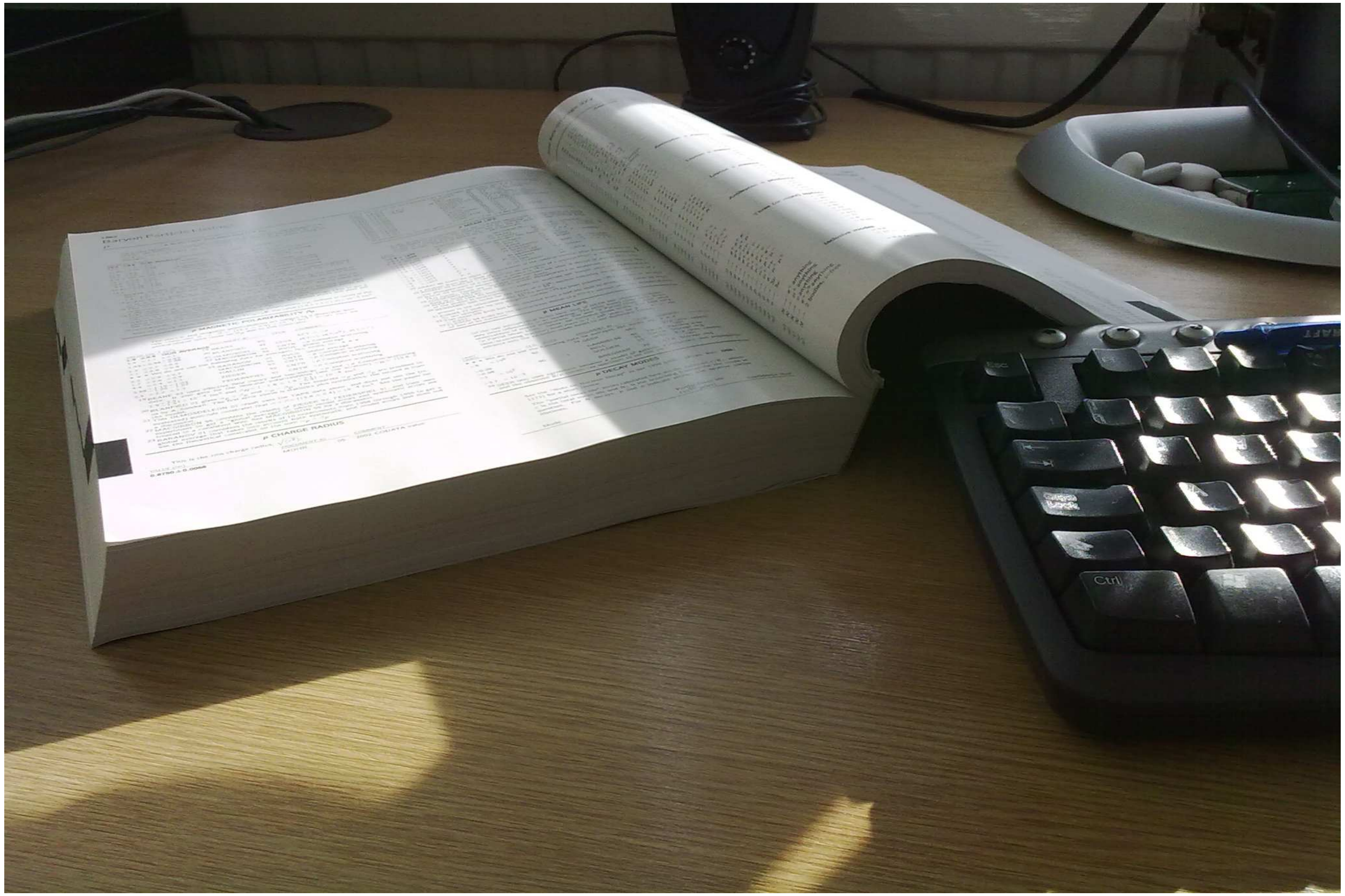
Overview of QCD phases

	$T = \mu = 0$
quarks are	confined
accuracy of predictions:	< 5%
has similarities with:	atomic physics (bound states)
fundamental properties:	*masses & transition mx els*

Overview

	$T = \mu = 0$	T or μ large
quarks are	confined	*de-confined*
accuracy of predictions:	< 5%	$\sim 20\%$
has similarities with:	atomic physics (bound states)	plasma/fluid (spectral functions)
fundamental properties:	*masses & transition mx els*	*pressure, transport coefficients*

Particle Data Book

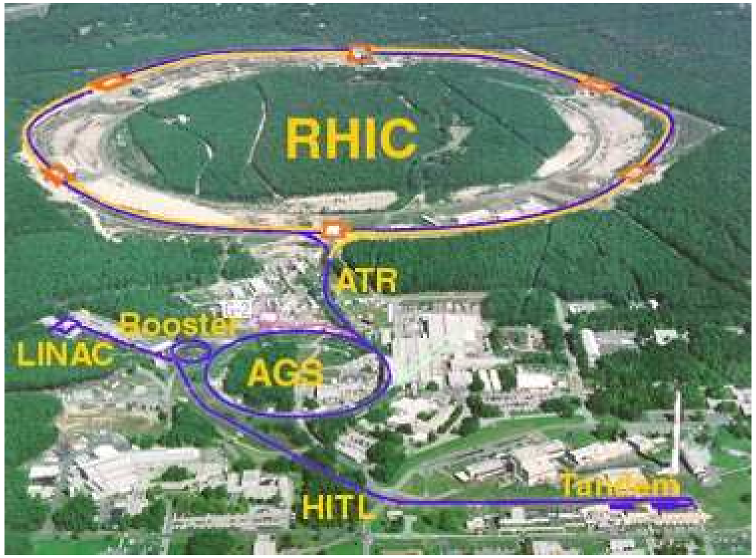


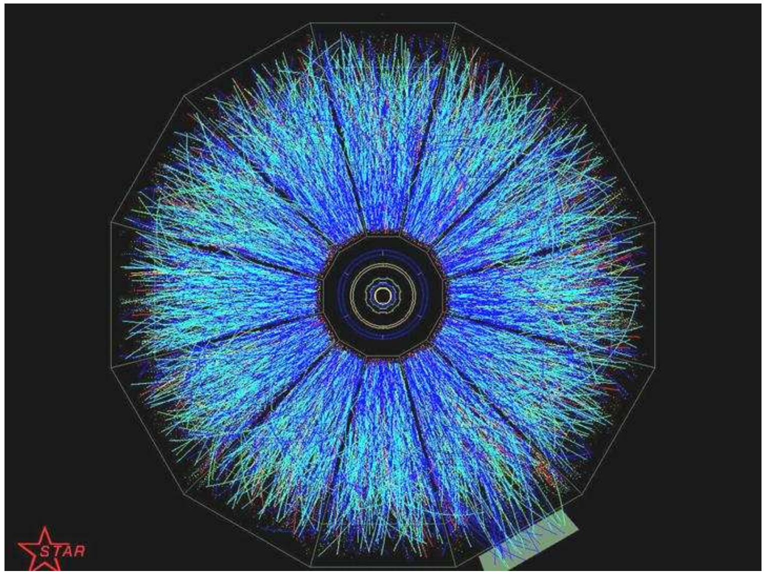
$\sim 1.5 \times 10^3$ pages

zero pages on Quark-Gluon Plasma...

Experiments of QCD at $T \neq 0$

RHIC Experiment @ BNL





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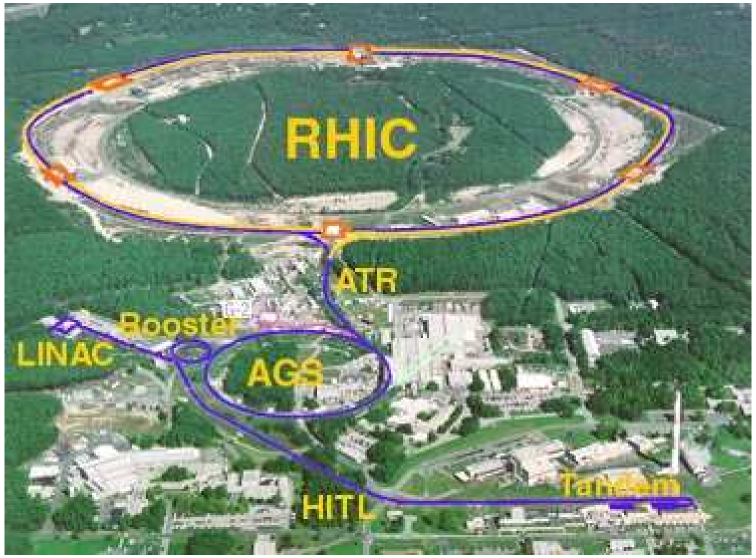

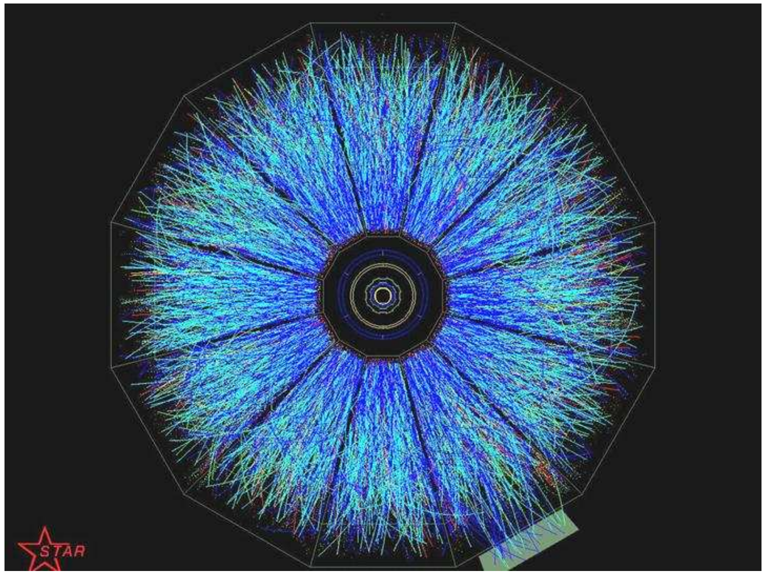

- Naive: quarks and gluons virtually free
- Expt: (relatively) strongly interacting
  - Almost instantaneous equillibration
  - Low viscosity

Viscosity of QCD at $T \neq 0$

- *NOT* weakly coupled $\longrightarrow$ very low viscosity

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RHIC creates the Perfect Fluid

ig-Noble aside

2005 Ig-Nobel Prize for Physics awarded to the “Pitch Drop” experiment by:

Profs. Mainstone and Parnell from the University of Queensland

Pitch has viscosity 10^{11} times water’s...

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Quantitative features of QCD at $T \neq 0$

Weak coupling [Arnold, Moore and Yaffe]:

$$\eta/s \sim 1/g^4$$

i.e. predicts large η *(shear viscosity)*

$\mathcal{N} = 4$ SYM $\Leftrightarrow$ AdS$_5 \times$S^5 [Son, Starinets, Policastro, Kovtun, ...]

$$\eta/s \geq \frac{1}{4\pi} \qquad N_c,\ g^2 N_c \to \infty$$

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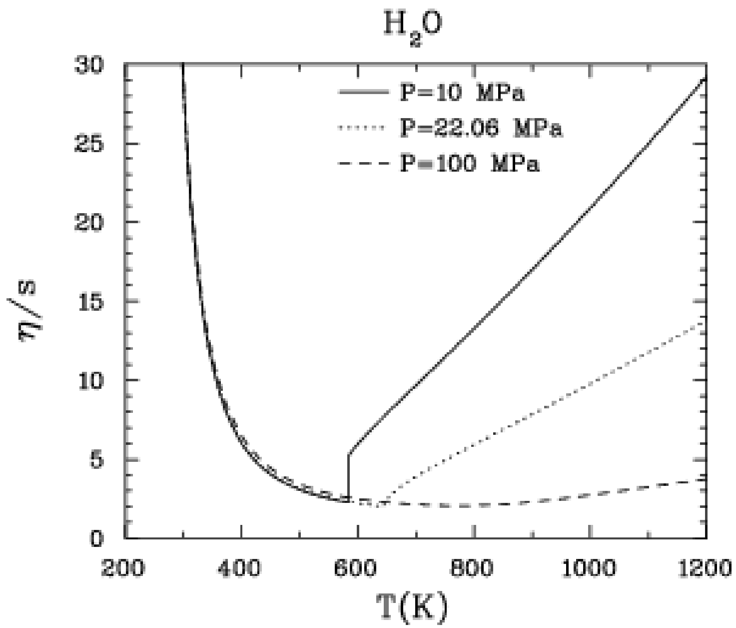
Finally string theory makes contact with nature...

Physical values for η: [Csernai, Kapusta, McLerran, 2006]



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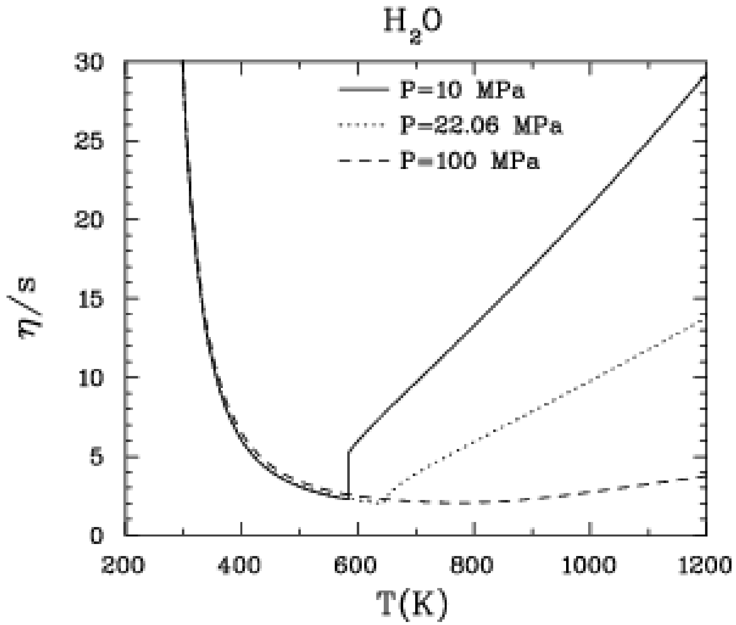

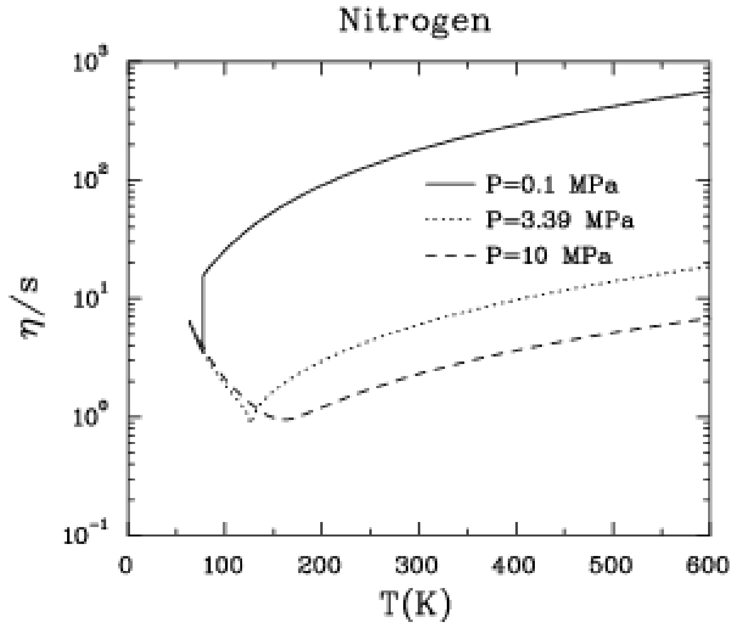

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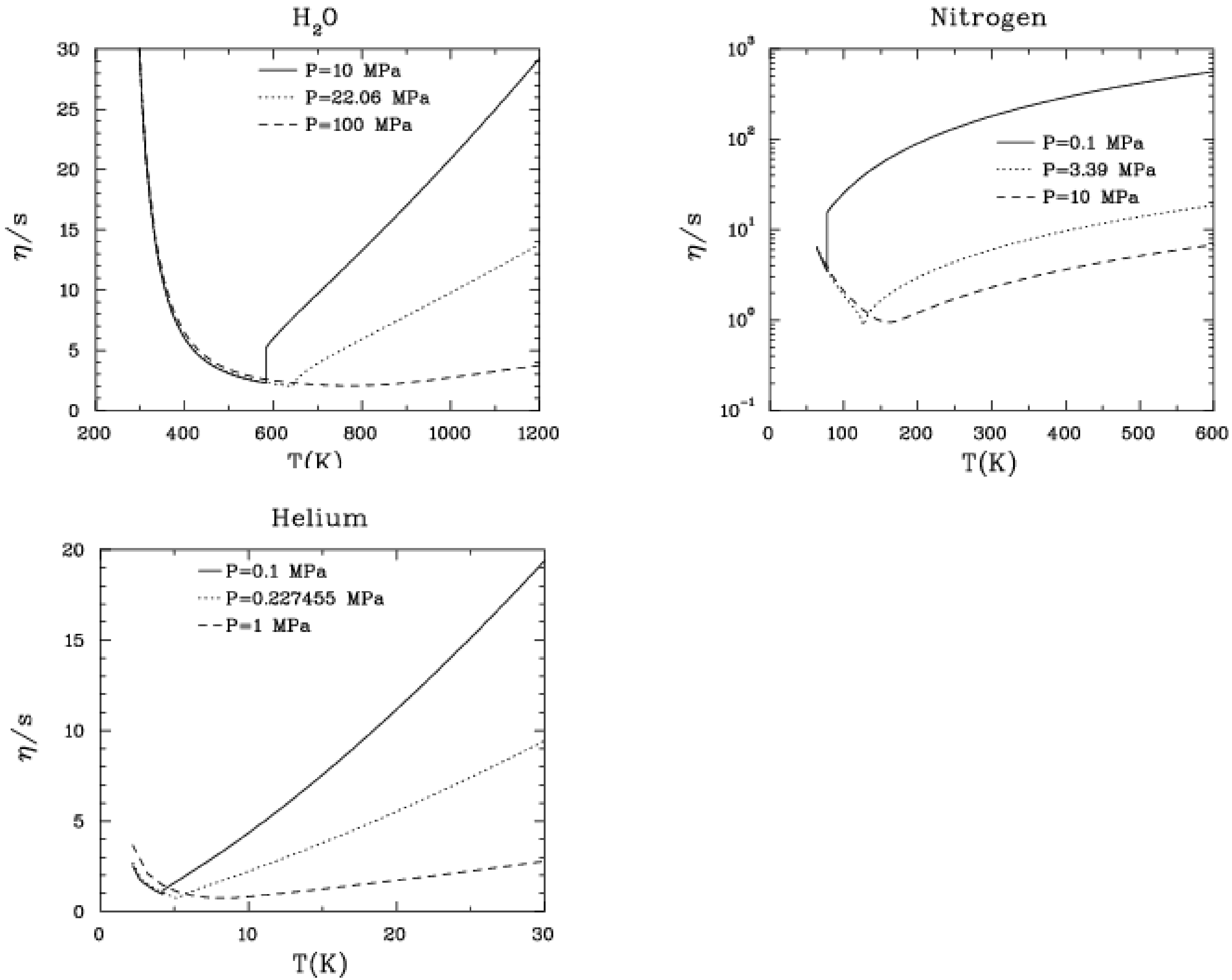

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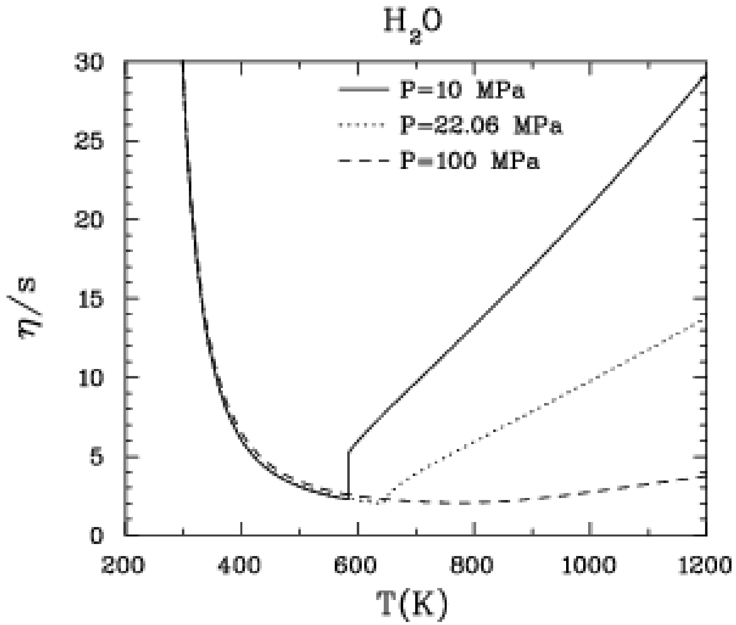
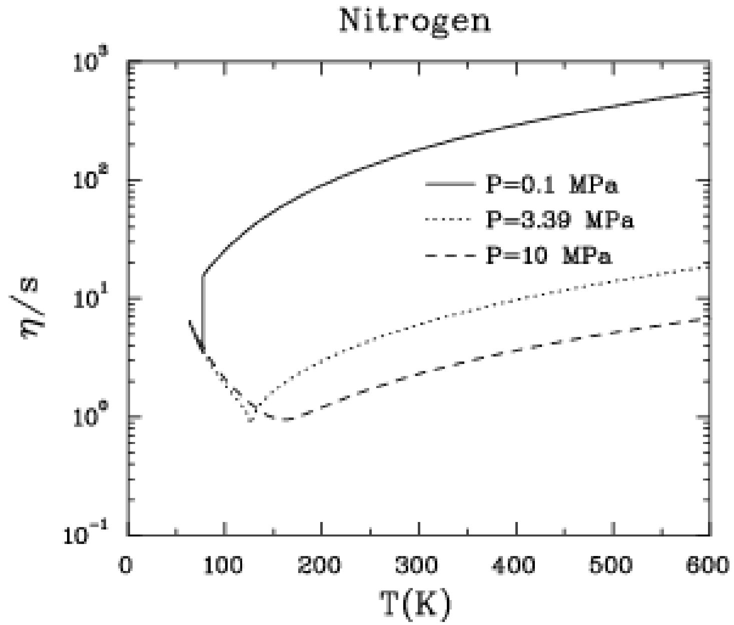
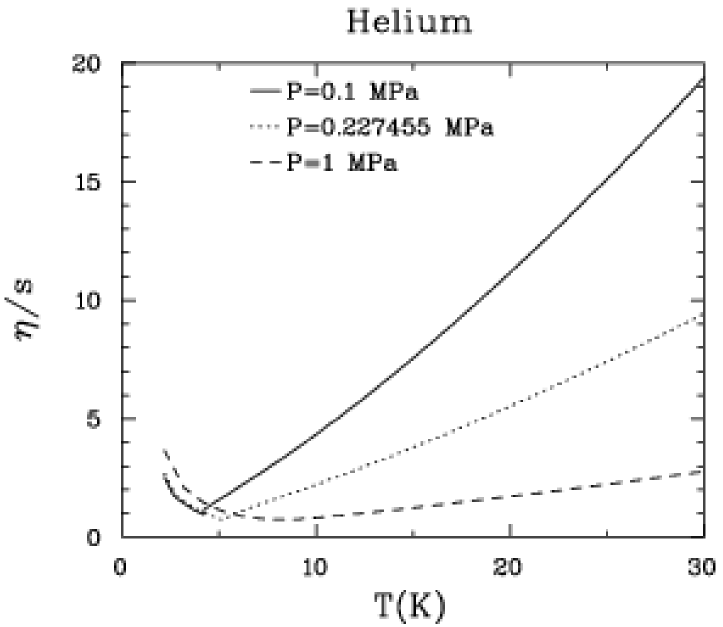
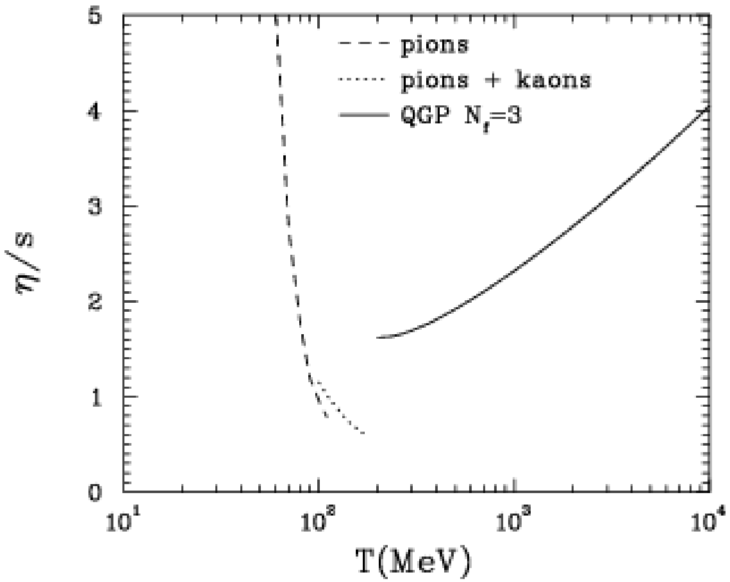






QCD(?)



Transport coefficients

transport coefficients from derivatives of *spectral functions* at $\omega \to 0$.

- shear viscosity η Meyer
  - off-diagonal gluonic correlators
- bulk viscosity ξ
  - diagonal gluonic correlators
- electrical conductivity σ Aarts, CRA, Foley & Hands
  - vector correlators
- Diffusivity D
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$\eta, \xi, \sigma\, D \quad \sim$ *LOW ENERGY CONSTANTS*

SUMMARY SO FAR

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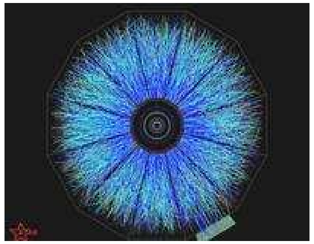
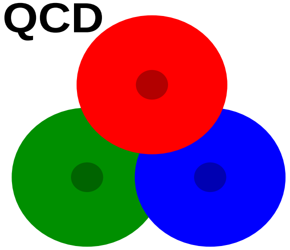
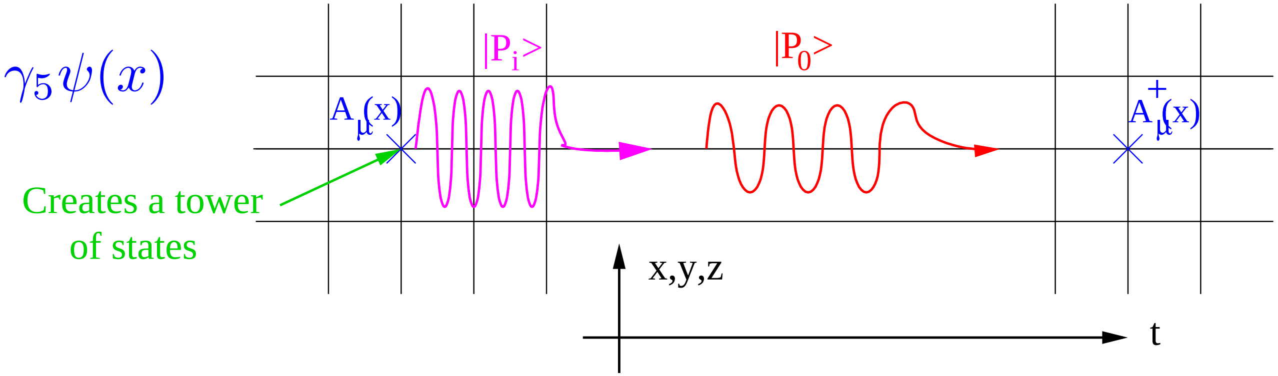
$T \neq 0$	**Extreme QCD** 	
$T = 0$	**Ordinary QCD** 	

Lattice Overview (Correlation F'ns)

$$A_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)$$



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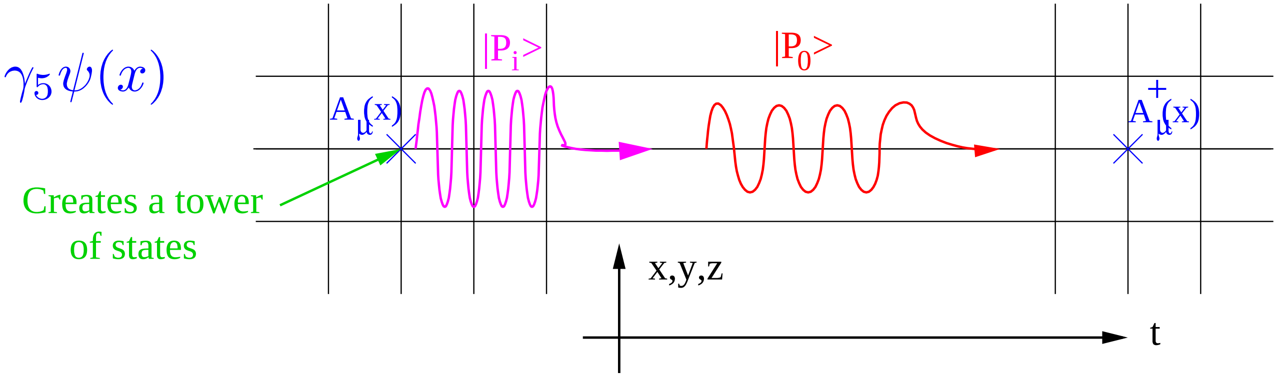

$$\begin{aligned}
\langle\Omega\rangle &= G_2(t) = \sum_{\{U\}}\sum_{\vec{x}}\langle 0|A_\mu(\vec{x},t)A^\dagger_\mu(\vec{0},0)|0\rangle \\
&= \sum_{\{U\}}\sum_{\vec{x}}\sum_{i}\int\frac{d^3k}{2E}\langle 0|A_\mu(\vec{x},t)|P_i(\vec{k})\rangle\langle P_i(\vec{k})|A^\dagger_\mu(\vec{0},0)|0\rangle \\
&= \sum_{\{U\}}\sum_{i}\frac{1}{2M_i}\langle 0|A_\mu(0)|P_i(0)\rangle\langle P_i(0)|A^\dagger_\mu(0)|0\rangle e^{-M_i t}
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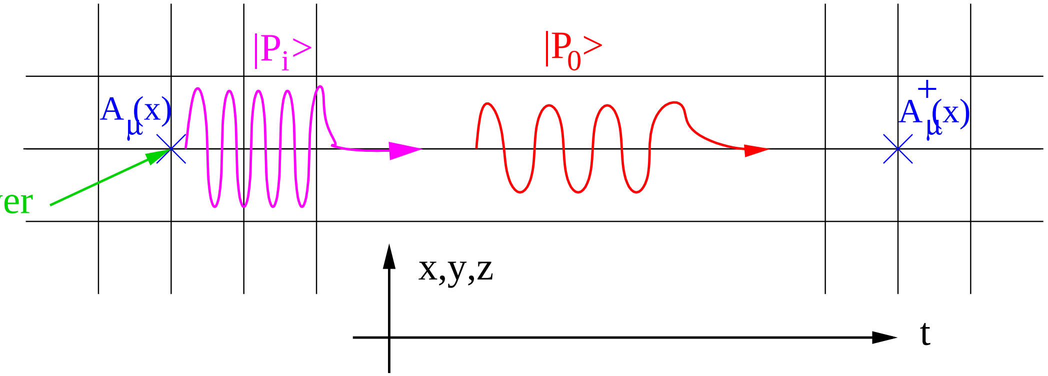

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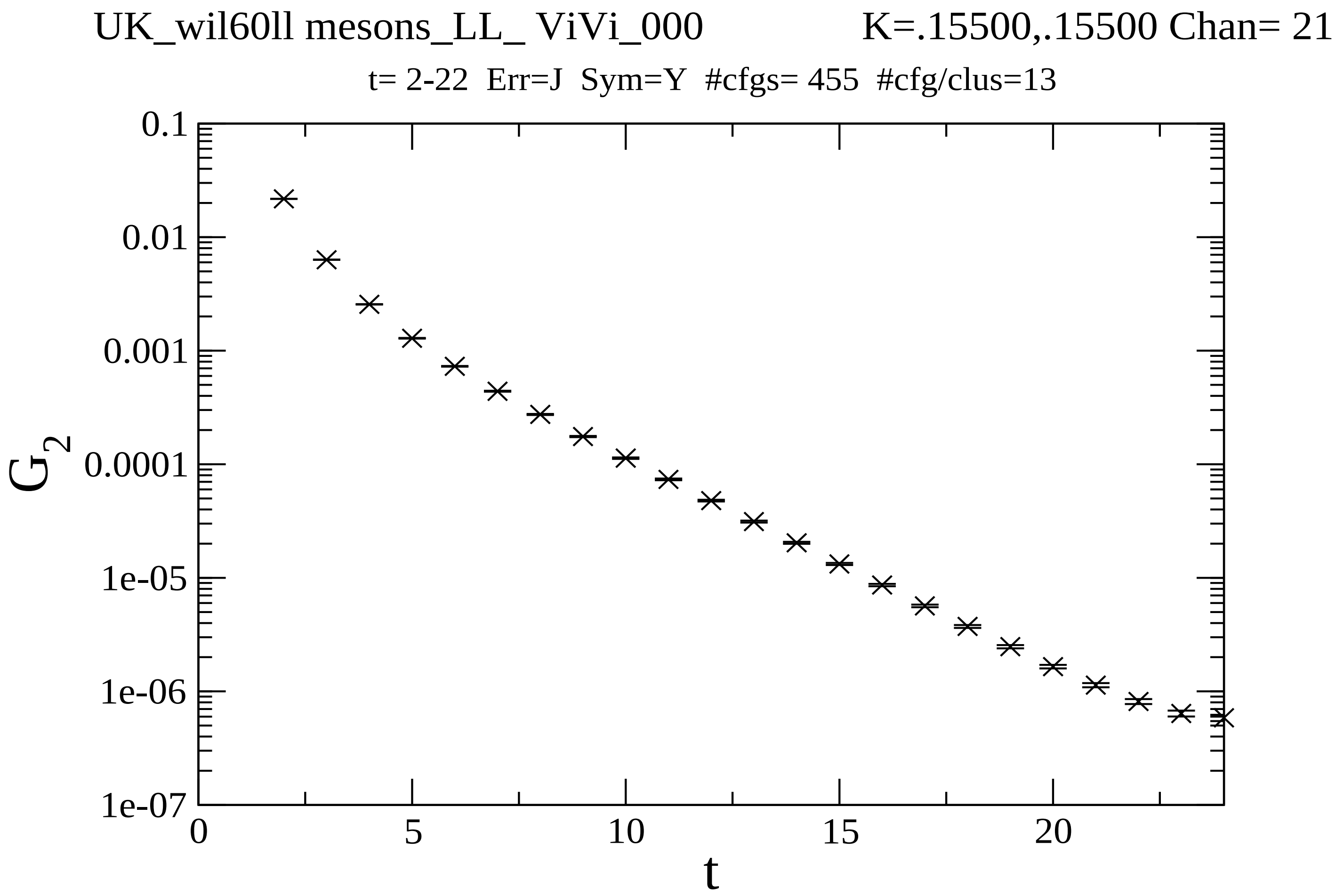
t large: $\rightarrow \dfrac{|\langle 0|A_\mu(0)|P(0)\rangle|^2}{2M_0}\, e^{-M_0 t} \equiv Z\, e^{-M_0 t}$

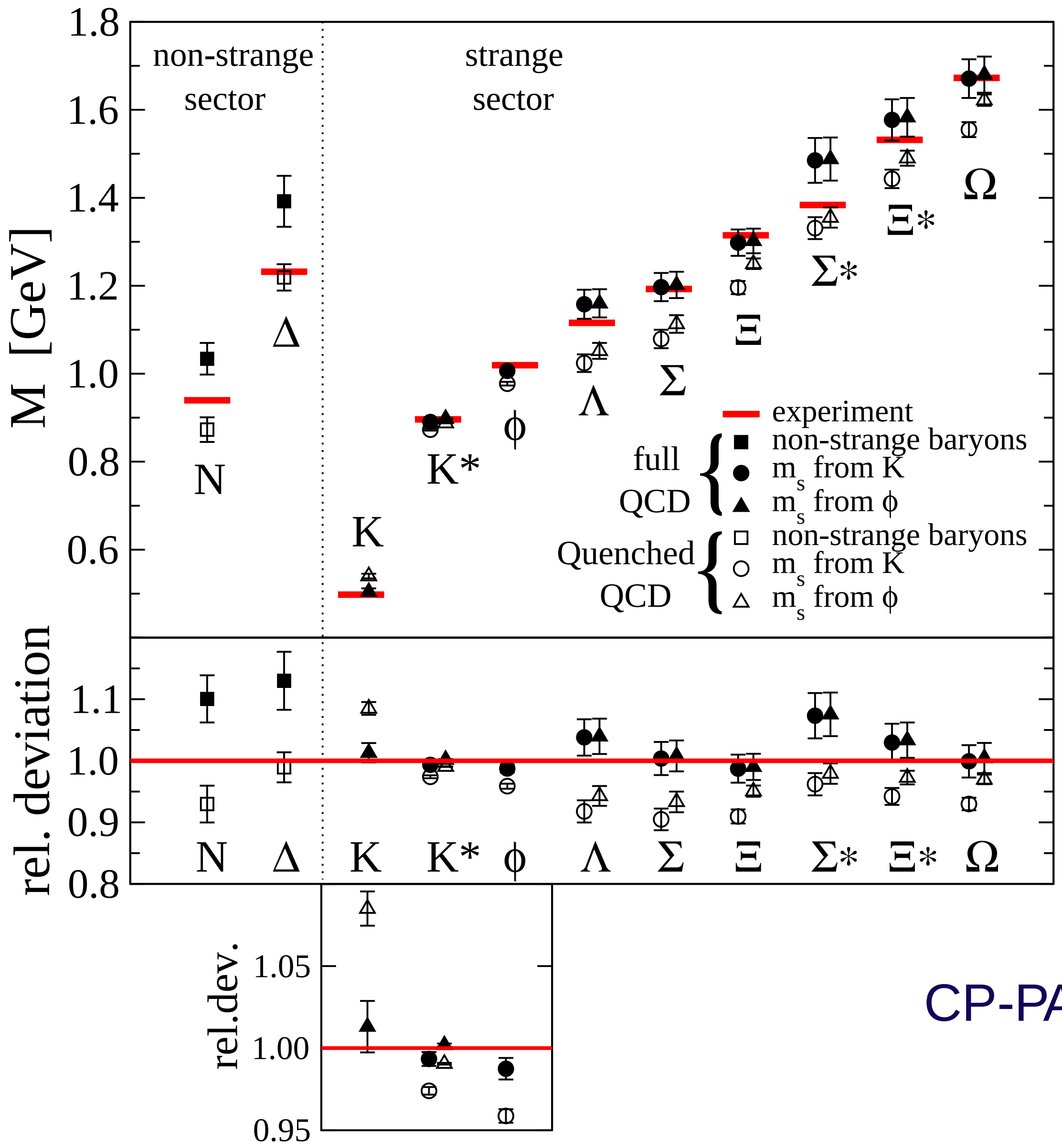
Lightest state!

Example Effective Mass

$$G_2(t) = \sum_{\{U\}} \sum_{\vec{x}} \langle 0 | A_\mu(\vec{x},t) A^\dagger_\mu(\vec{0},0) | 0 \rangle \quad \rightarrow \quad Z\, e^{-M_0 t}$$



Hadronic Spectrum



CP-PACS Collaboration

Extrapolations Required

Lattice simulations don't solve real QCD:

$$< \mathcal{O} > = \; f(g_0, m, \mu, L, N_f, N_{\{U\}})$$

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$L \approx 3$ fm

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But . . . it is systematically improvable

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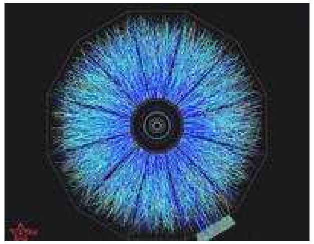
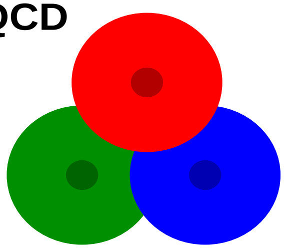
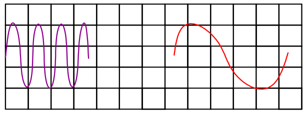
$T \neq 0$	Extreme QCD 	
$T = 0$	Ordinary QCD 	Bound States 

Motivation

Do bound hadronic states persist into the “quark-gluon” plasma phase?

- *Spectral functions* can answer this!

$$G(t,\vec{p}) \;=\; \int \rho(\omega,\vec{p})\; K(t,\omega)\; d\omega$$

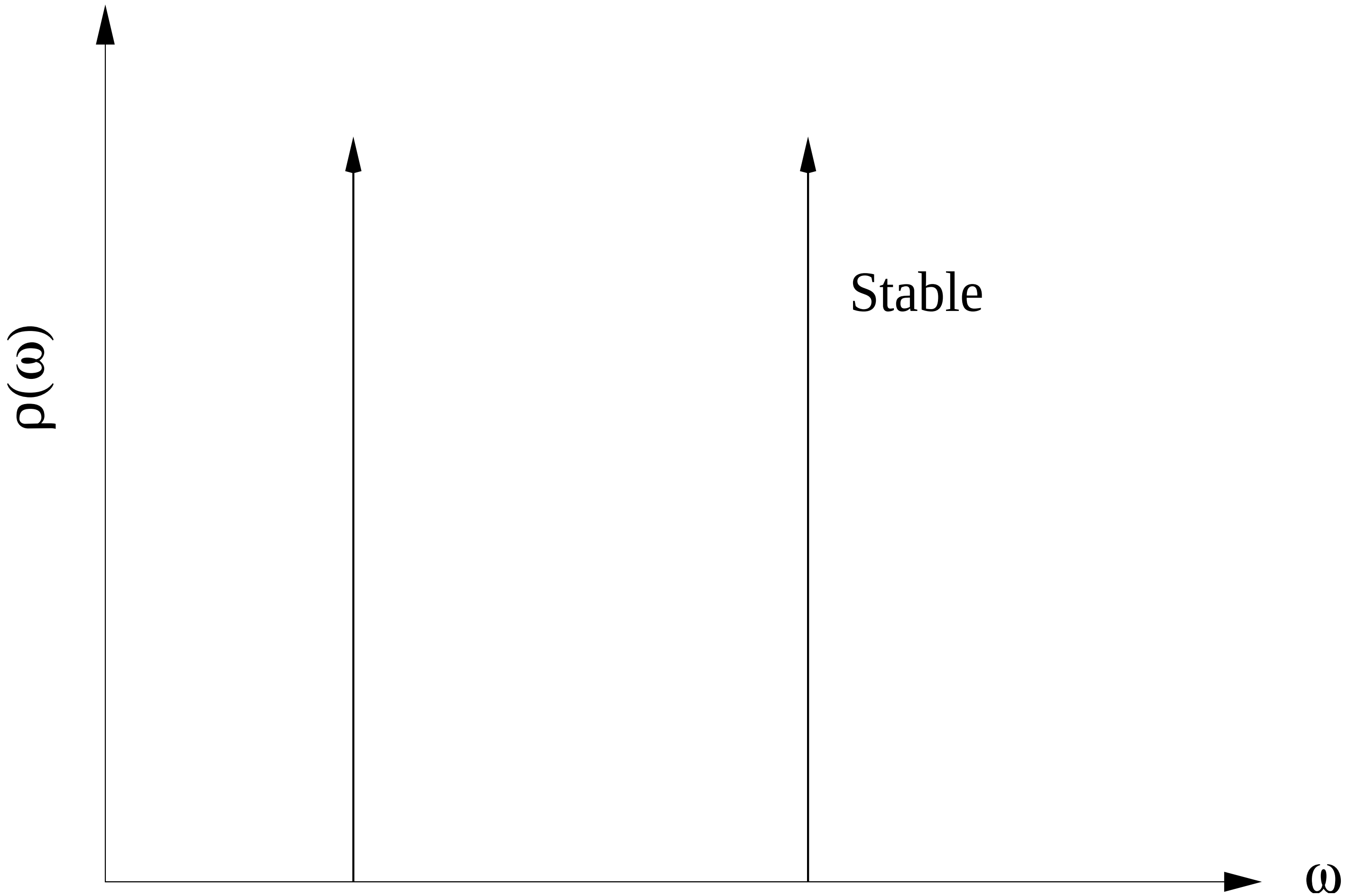
$\uparrow$	$\downarrow$	$\nwarrow$
Euclidean Correlator	Spectral Function	(Lattice) Kernel

where the (Lattice) Kernel is:

$$\begin{aligned} K(t,\omega) &= \frac{\cosh[\omega(t - N_t/2)]}{\sinh[\omega/(2T)]} \\ &\sim \exp[-\omega t] \end{aligned}$$

Example Spectral Functions

$$G(t) \sim \int \rho(\omega)\, e^{-\omega t}\, d\omega$$



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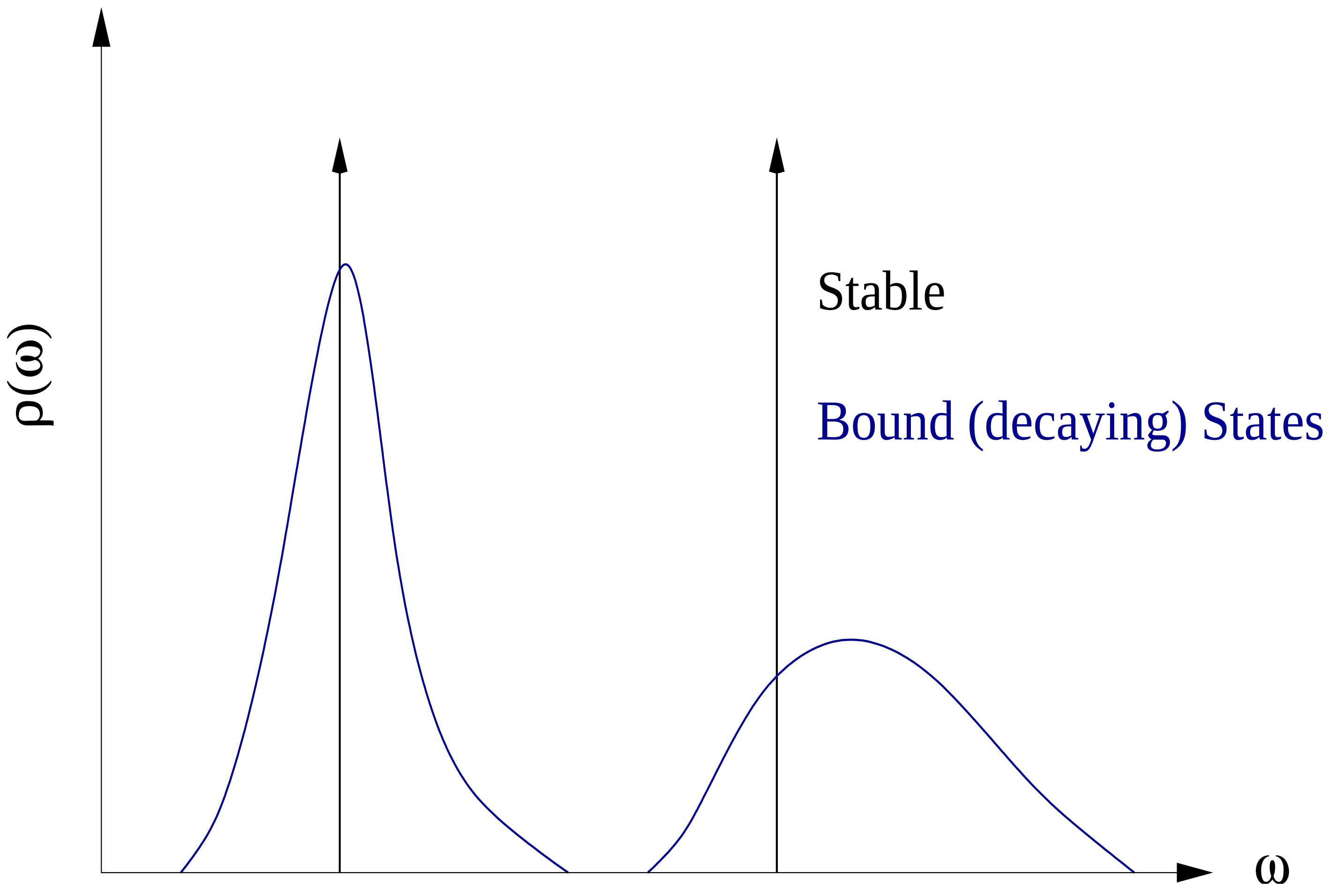

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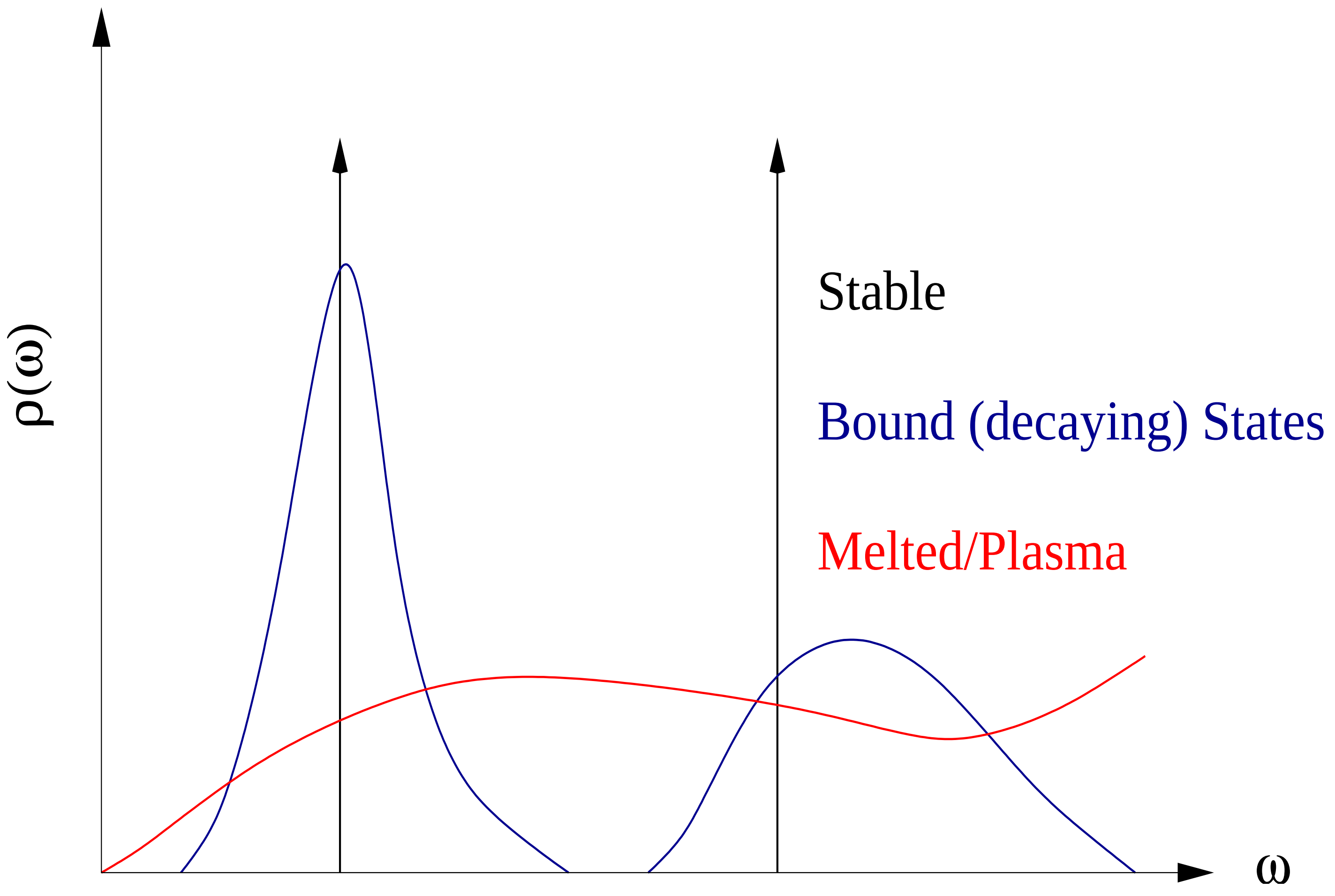

What's special about the Spectral Function?

- $\rho(\omega, \vec{p})$ contains info on
  - (in)stability of hadrons
  - transport coefficients
  - dilepton production . . .

- Extraction of a spectral density from a lattice correlator is an ill-posed problem:
  - *Given* $G(t)$ *derive* $\rho(\omega)$
  - *More* ω *data points then* t *data points!*

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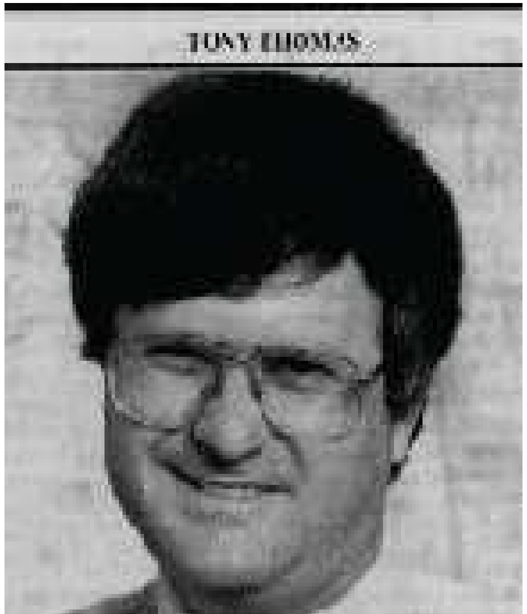

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Maximum Entropy Method

- Extraction of a spectral density from a lattice correlator is an ill-posed problem:
  - *Given* $G(t)$ *derive* $\rho(\omega)$
  - *More* ω *data points then* t *data points!*

- Requires the use of Bayesian analysis - Maximum Entropy Method (MEM)
  - Hatsuda et al

Two lattice studies

Dublin-Swansea

- Dynamical
- Anistropic
- Zero momentum

Swansea

- Quenched
- Isotropic
- Non-zero momentum

Lattice Parameters (Dublin-Swansea)

- Gluon Action:
  - Improved anisotropic
- Fermion Action:
  - Wilson+Hamber-Wu + stout links

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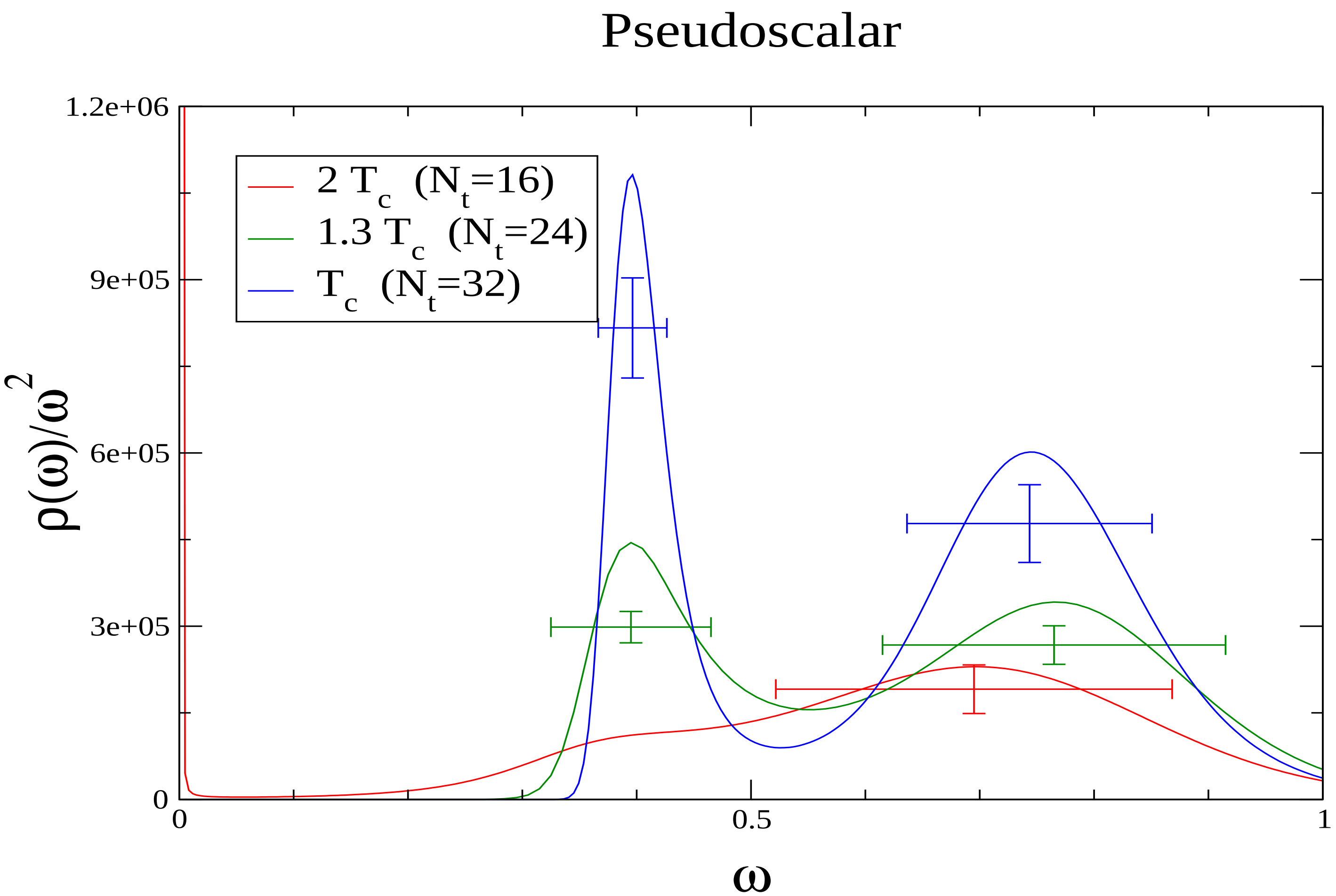
Light quarks	M_π/M_ρ	~ 0.5	
Anisotropy	ξ	6	
Lattice spacings	a_t	~ 0.025 fm	
	a_s	~ 0.15 fm	
Spatial Volume	N_s^3	$8^3 \quad (\&12^3)$	
$1/T$	N_t	16	$\rightarrow T \sim 2T_c$
		24	$\rightarrow T \sim 1.3T_c$
		32	$\rightarrow T \sim T_c$
Statistics	N_{cfg}	~ 500	

[Aarts, Oktay, Peardon, Skullerud, CRA]

Varying **Temperature** (Dublin-Swansea)

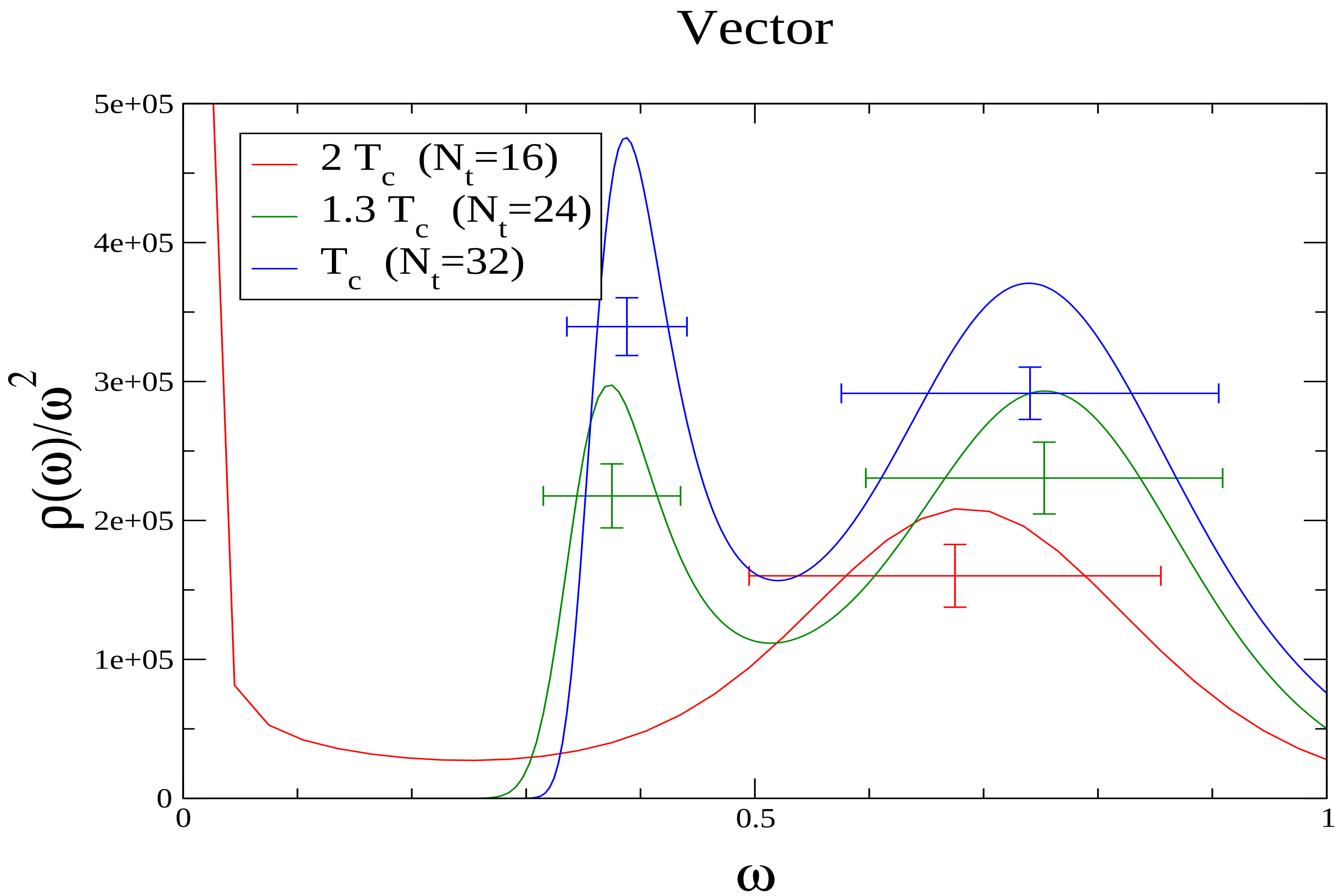
η_C Pseudoscalar ($am_c = 0.080$, $N_s = 8$)



$\longrightarrow$ Can see it melting!

Varying Temperature (Dublin-Swansea)

J/ψ Vector ($am_c = 0.080$, $N_s = 8$)



$\longrightarrow$ Can see it melting!

Note the singularity at $\omega \sim 0$

Lattice Action and Parameters (Swansea) I

- Gluon Action:
  - Wilson
- Quenched
- Twisted Boundary Conditions
  - large range of momenta available
- Singularity at $\omega \sim 0$ traced to $K(\omega, t)$ and corrected

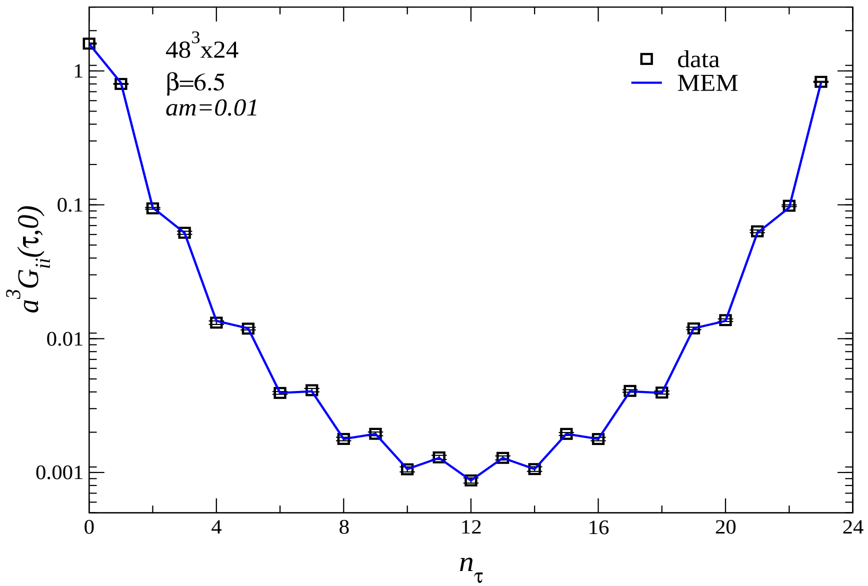
[Aarts, Foley, Hands, Kim, CRA]

Lattice Action and Parameters (Swansea) II

COLD		
Lattice spacings	a^{-1}	~ 4 GeV
Spatial Volume	$N_s^3 \times N_t$	$48^3 \quad \times 24$
T	$1/(aN_t)$	$T \sim 160 MeV \sim \frac{1}{2} T_c$
Statistics	N_{cfg}	~ 100

HOT		
Lattice spacings	a^{-1}	~ 10 GeV
Spatial Volume	$N_s^3 \times N_t$	$64^3 \quad \times 24$
T	$1/(aN_t)$	$T \sim 420 MeV \sim \frac{3}{2} T_c$
Statistics	N_{cfg}	~ 100

Staggered Correlators



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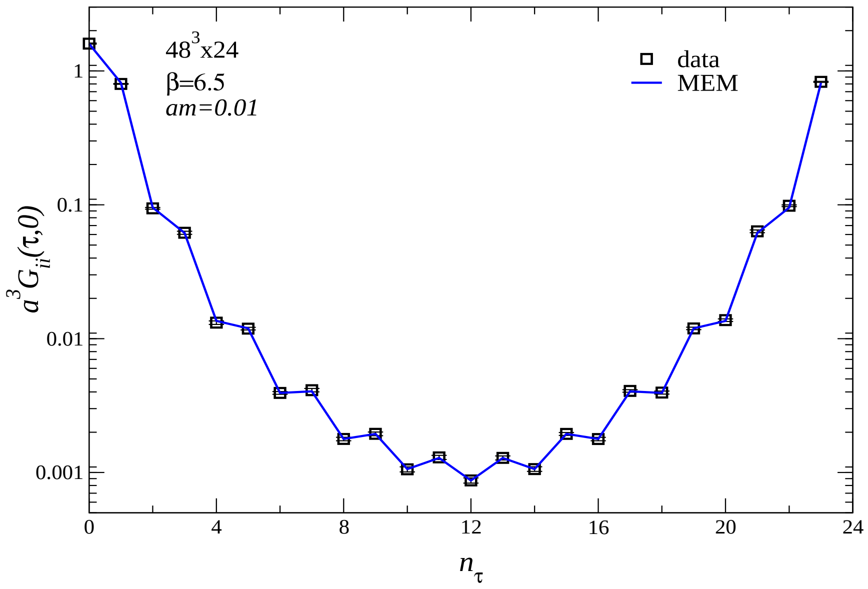


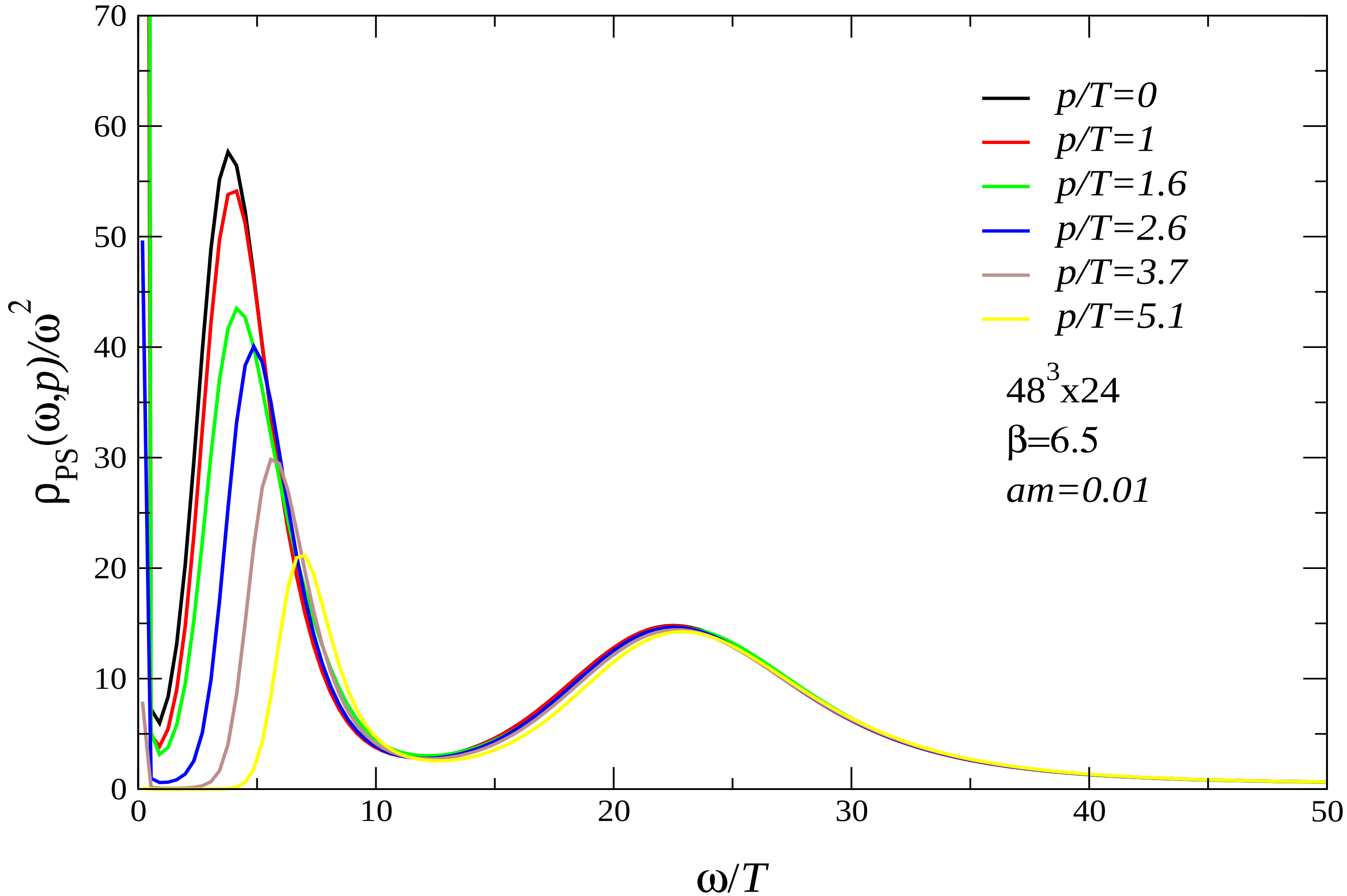
$$G(t) = 2\int \frac{d\omega}{2\pi} K(t,\omega)\left(\rho(\omega) - (-1)^t\,\tilde{\rho}(\omega)\right)$$

$\longrightarrow$ Have to fit to even & odd times separately, then use

$$\rho^{\text{phys}} = \frac{1}{2}\left(\rho^{\text{even}} + \rho^{\text{odd}}\right)$$

MEM results below T_c (Swansea)

Momentum dependence of spectral function (below T_c)



$\longrightarrow$ Can see it moving

Electrical Conductivity

$$\frac{\sigma}{T} = \lim_{\omega \to 0} \frac{\rho(\omega)}{6\omega T}$$

$\sigma =$ Conductivity

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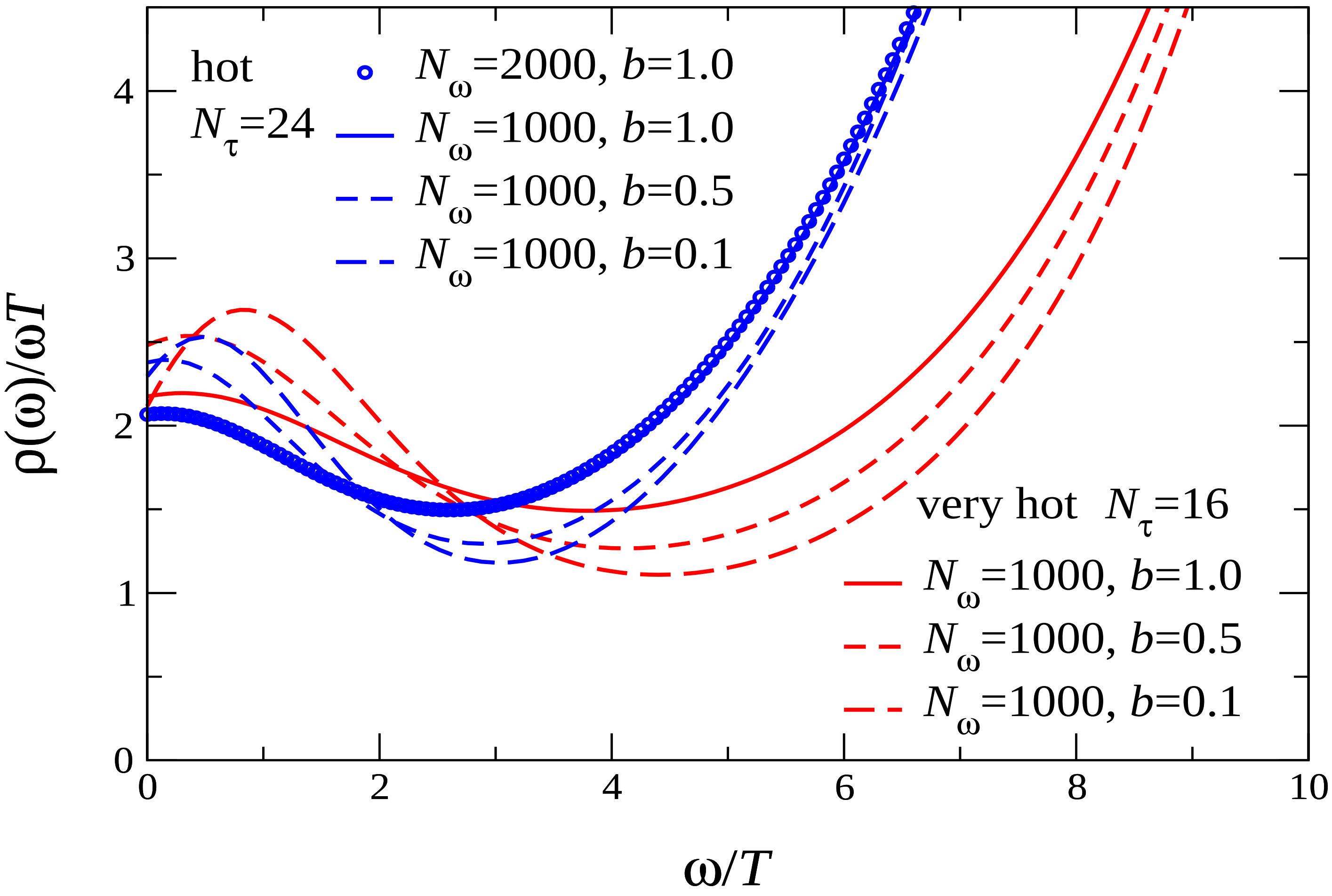

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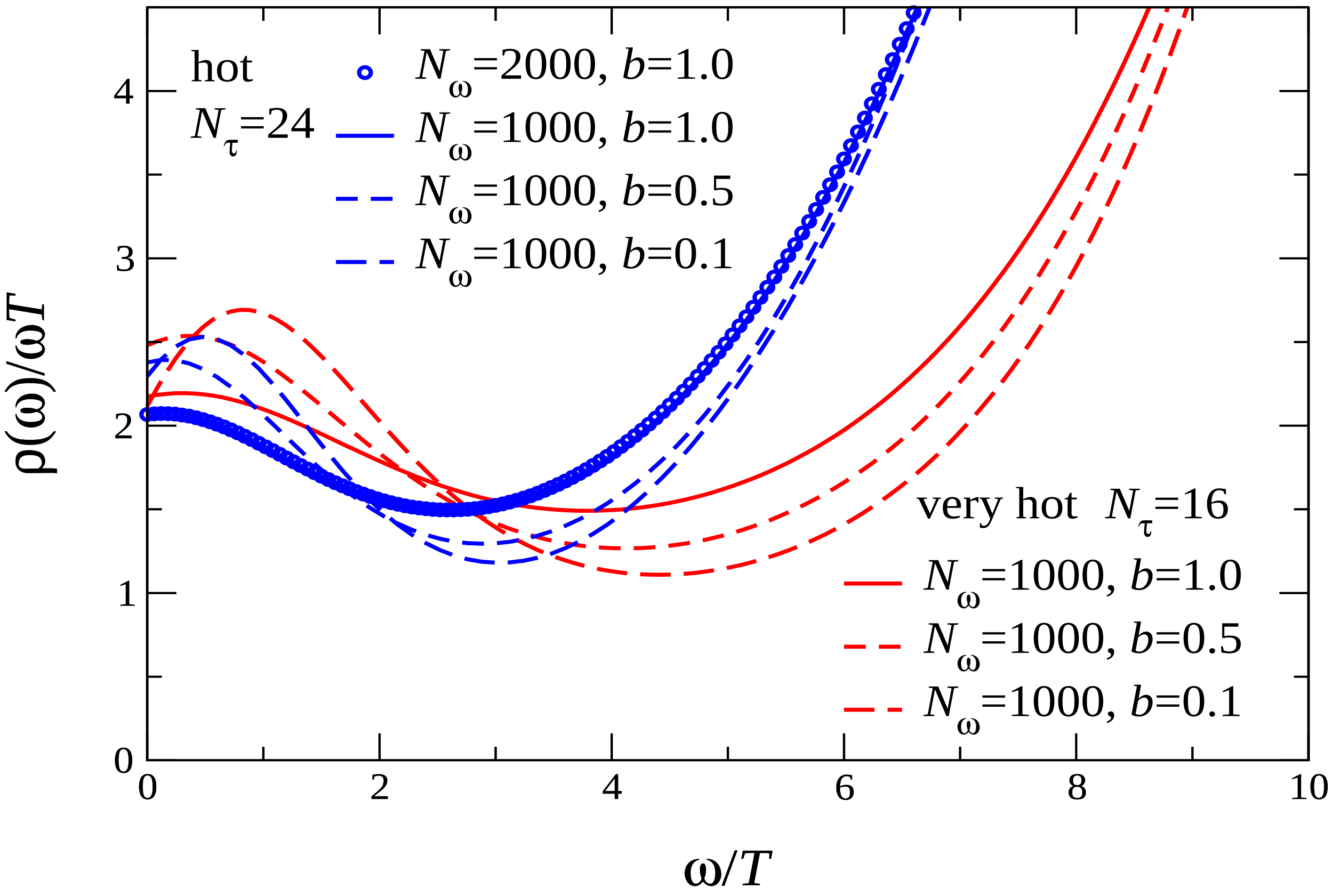
$$\frac{\sigma}{T} = \lim_{\omega\to 0} \frac{\rho(\omega)}{6\omega T}$$

$\sigma =$ Conductivity

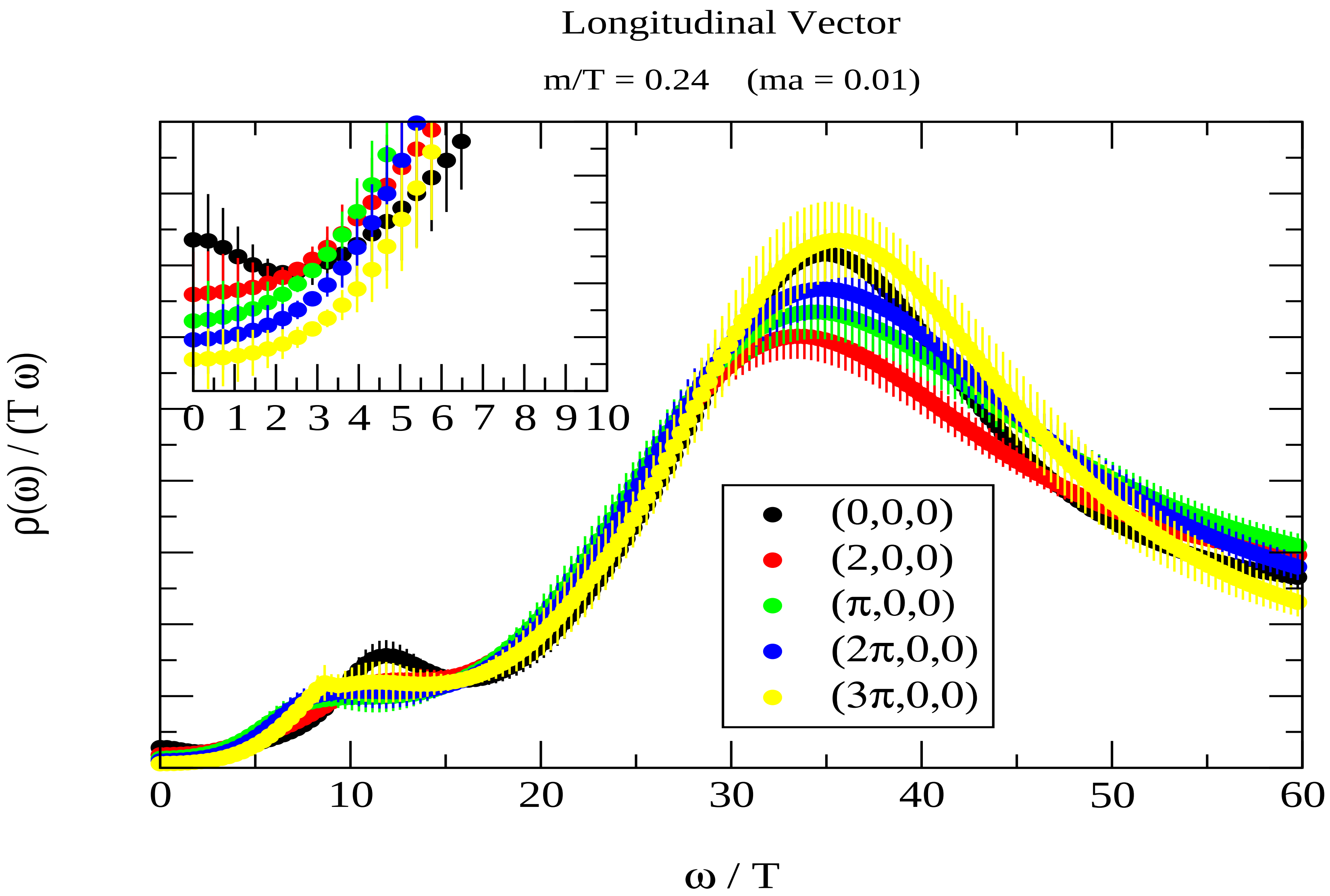


$\longrightarrow \quad \sigma/T = 0.4 \pm 0.1$ Aarts,CRA,Foley & Hands

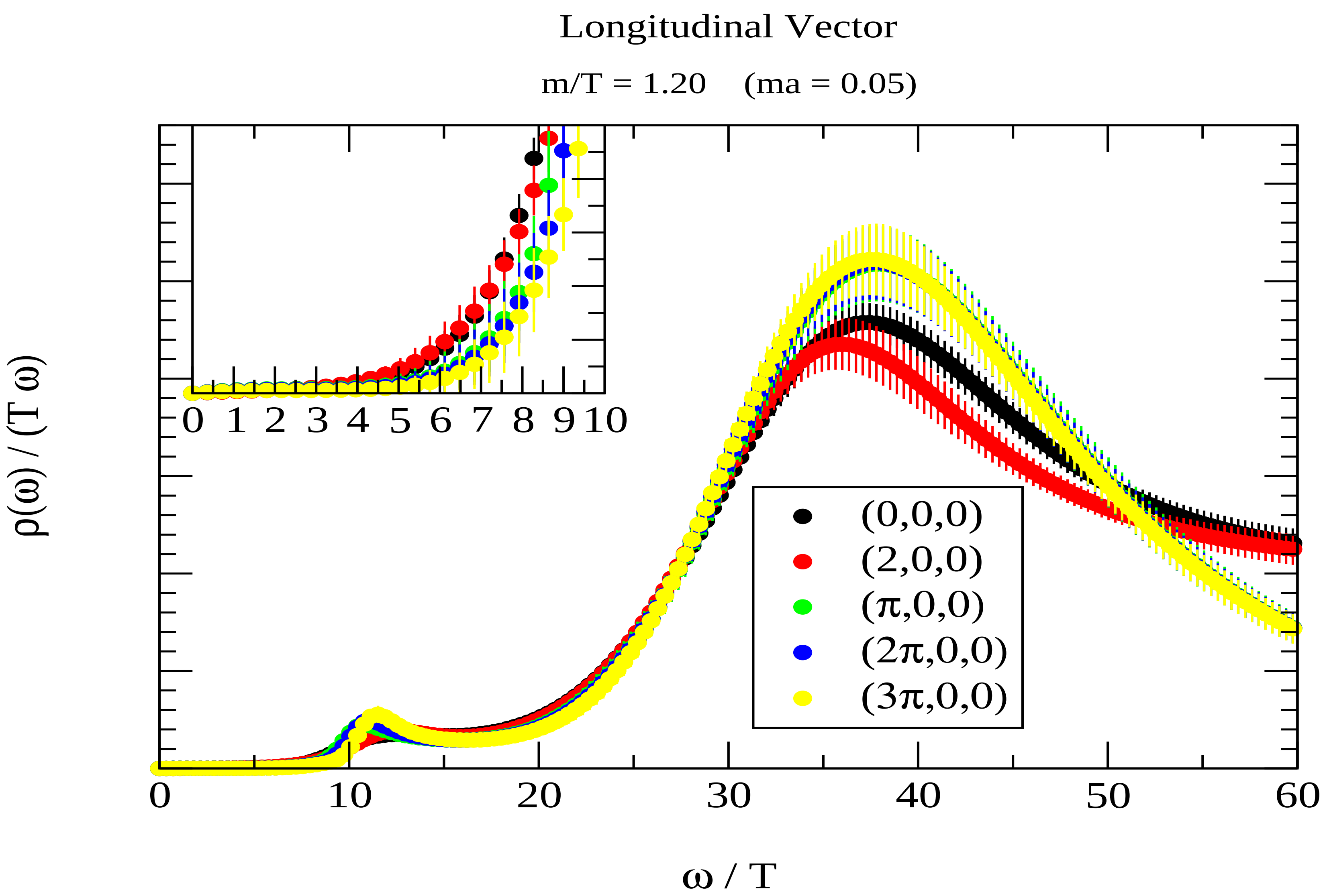
Diffusivity

D obtained from the momentum dependency of $\rho^{\text{Long}}_{\text{Vector}}$ (at small mass)

Diffusivity [Preliminary] $m/T = 0.24$ Hot

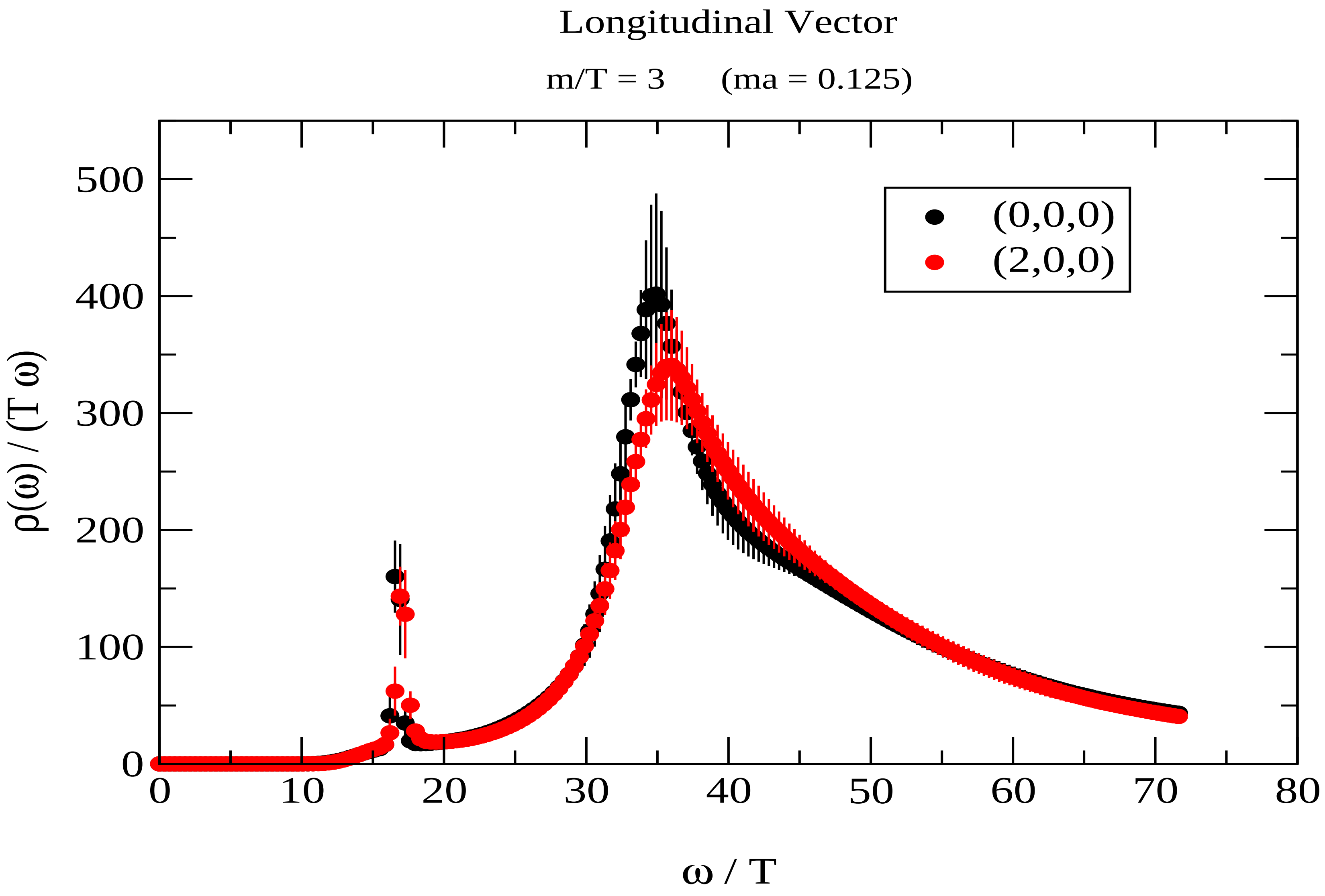


Diffusivity [Preliminary] $m/T = 1.2$ Hot



Lack of Melting? [Preliminary]

$m/T = 3$ Hot



MEM Limitations

If the correlation function has a large dynamic range then there are numerical instabilities in MEM.

$$G(t) \;=\; \int \rho(\omega)\; K(t,\omega)\; d\omega$$

where

$$\begin{aligned} K(t,\omega) &= \frac{\cosh[\omega(t - N_t/2)]}{\sinh[\omega/(2T)]} \\ &\sim \exp[-\omega t] \end{aligned}$$

i.e. it is *almost* a Laplace transform:

$$G(t) \sim \int \rho(\omega)\; e^{-\omega t}\; d\omega$$

Convolution Rules

Aim is to get $G(t)$ expressed as an exact Laplace transform, then can use convolution rules:

$$G(t) \times e^{+\Omega t} \;=\; \int \rho(\omega + \Omega)\; e^{-\omega t}\, d\omega$$

$\uparrow$

smaller
dynamic
range

Possible Solution

$$
\begin{aligned}
G(t) &= e^{-Mt} + e^{-M(T-t)} \\
&= e^{-MT/2}(e^{-M(t-T/2)} + e^{M(t-T/2)}) \\
&= e^{-MT/2}(x^i + x^{-i})
\end{aligned}
$$

where $x = e^{-M(t-T/2)}$ and $i = t - T/2$.

Define

$$
\begin{aligned}
\widetilde{G}(t) &= e^{-MT/2}(x + x^{-1})^i \\
&= e^{-MT/2} \sum_{j=0}^{i} {}^iC_j \; x^{2j-i} \\
&= e^{-MT/2} \sum_{j=0}^{[i/2]} {}^iC_j \; (x^{2j-i} + x^{i-2j}) \\
&= \sum_{j=0}^{[i/2]} {}^iC_j \; G(2j-i)
\end{aligned}
$$

Possible Solution contd.

We can now write $x + x^{-1} \equiv e^y \quad \longrightarrow$

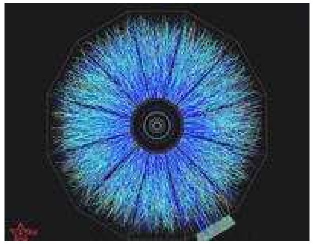
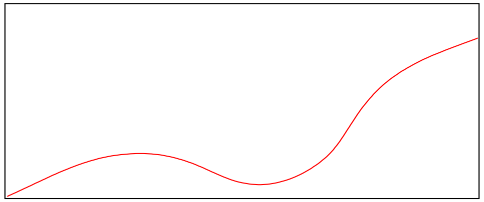
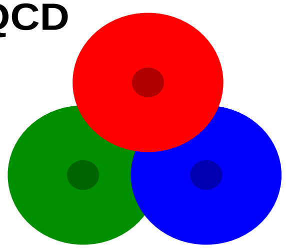
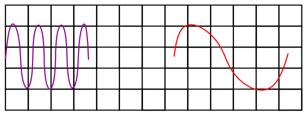
$$\begin{aligned} \widetilde{G}(t) &= e^{-MT/2}(x + x^{-1})^i \\ &= e^{-MT/2} e^{yi} \\ &\equiv \text{linear combination of } G(t) \end{aligned}$$

Fleming et al, Phys.Rev.D80 (2009) 074506

i.e. by defining a linear combination of $G(t)$ correlators, the lattice kernel is transposed into a pure exponential.

$\longrightarrow$ Laplace Transform Convolution Rule is now viable

SUMMARY

	Continuum	Lattice
T ≠ 0	Extreme QCD 	Spectral F’ns 
T = 0	Ordinary QCD 	Bound States 

Happy 60th A.W.T.