Charmonium melting in the quark-gluon plasma phase of QCD

Chris Allton

Swansea University
H$_2$O phase diagram
QCD phase diagram

early universe

ALICE
quark-gluon plasma

RHIC
quark-gluon plasma

SPS
crossover

hadronic fluid

Tc ~ 170 MeV

<ψψ> > 0

<ψψ> ~ 0

n_B = 0

n_B > 0

vacuum

nuclear matter

μ_o ~ 922 MeV

μ

<ψψ> > 0

2SC

CFL

phases?

superfluid/superconducting

neutron star cores
<table>
<thead>
<tr>
<th></th>
<th>( T = \mu = 0 )</th>
</tr>
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<tr>
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~ $1.5 \times 10^3$ pages

zero pages on Quark-Gluon Plasma...
Experiments of QCD at $T \neq 0$

RHIC Experiment @ BNL
Experiments of QCD at $T \neq 0$

- Naive: quarks and gluons virtually free
- Expt: (relatively) strongly interacting
  - Almost instantaneous equilibration
  - Low viscosity
Viscosity of QCD at $T \neq 0$

- NOT weakly coupled $\rightarrow$ very low viscosity
Viscosity of QCD at $T \neq 0$

- NOT weakly coupled $\longrightarrow$ very low viscosity
Viscosity of QCD at $T \neq 0$

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Viscosity of QCD at $T \neq 0$

- *NOT* weakly coupled $\rightarrow$ very low viscosity

*RHIC creates the Perfect Fluid*
2005 Ig-Nobel Prize for Physics
awarded to the “Pitch Drop” experiment by:

Profs. Mainstone and Parnell
from the University of Queensland

Pitch has viscosity $10^{11}$ times water’s...
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Pitch has viscosity $10^{11}$ times water’s...
Quantitative features of QCD at $T \neq 0$

Weak coupling [Arnold, Moore and Yaffe]:

$$\eta/s \sim 1/g^4$$

i.e. predicts large $\eta$ (shear viscosity)

$\mathcal{N} = 4$ SYM $\leftrightarrow$ AdS$_5 \times$ S$^5$ [Son, Starinets, Policastro, Kovtun, ...]

$$\eta/s \geq \frac{1}{4\pi} \quad N_c, \ g^2 N_c \rightarrow \infty$$

i.e. predicts small $\eta$

(Conjectured lower bound for all matter)
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Finally string theory makes contact with nature...
Physical values for $\eta$: [Csernai, Kapusta, McLerran, 2006]
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![Graphs showing physical values for $\eta$ for different substances and pressures.](image)
Transport coefficients

transport coefficients from derivatives of spectral functions at $\omega \rightarrow 0$.

- shear viscosity $\eta$ — Meyer
  - off-diagonal gluonic correlators

- bulk viscosity $\xi$
  - diagonal gluonic correlators

- electrical conductivity $\sigma$ — Aarts, CRA, Foley & Hands
  - vector correlators

- Diffusivity $D$
  - energy dependence of vector correlators
Transport coefficients

Transport coefficients from derivatives of *spectral functions* at $\omega \to 0$.

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- **Diffusivity** $D$
  - energy dependence of vector correlators

$\eta, \xi, \sigma \sim LOW ENERGY CONSTANTS$
SUMMARY SO FAR

Continuum

Extreme QCD

T ≠ 0

Lattice

Ordinary QCD

T = 0
\[ A_\mu(x) = \bar{\psi}(x) \gamma_\mu \gamma_5 \psi(x) \]

Creates a tower of states
Lattice Overview (Correlation F’ns)

\[ A_\mu(x) = \bar{\psi}(x) \gamma_\mu \gamma_5 \psi(x) \]

\[ \langle \Omega \rangle = G_2(t) = \sum_{\{U\}} \sum_{\vec{x}} \langle 0 | A_\mu(\vec{x}, t) A_\mu^\dagger(\vec{0}, 0) | 0 \rangle \]

\[ = \sum_{\{U\}} \sum_{\vec{x}} \sum_i \int \frac{d^3k}{2E} \langle 0 | A_\mu(\vec{x}, t) | P_i(\vec{k}) \rangle \langle P_i(\vec{k}) | A_\mu^\dagger(\vec{0}, 0) | 0 \rangle \]

\[ = \sum_{\{U\}} \sum_i \frac{1}{2M_i} \langle 0 | A_\mu(0) | P_i(0) \rangle \langle P_i(0) | A_\mu^\dagger(0) | 0 \rangle e^{-M_i t} \]
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\[ t \text{ large:} \quad \rightarrow \quad \frac{|\langle 0 | A_\mu(0) | P(0) \rangle|^2}{2M_0} e^{-M_0 t} \equiv Z e^{-M_0 t} \]

Lightest state!
$G_2(t) = \sum_{\{U\}} \sum_{\vec{x}} \langle 0|A_\mu(\vec{x}, t)A_{\mu}^\dagger(\vec{0}, 0)|0 \rangle \quad \rightarrow \quad Z \ e^{-M_0 t}$

UK_wil60ll mesons_LL_ ViVi_000 \quad K=.15500,.15500 Chan= 21

$t= 2-22$ \ Err=J \ Sym=Y \ #cfgs= 455 \ #cfg/clus=13
Lattice simulations don’t solve real QCD:

\[ \langle \mathcal{O} \rangle = f(g_0, m, \mu, L, N_f, N_U) \]
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\[ m_q \approx 50 \text{ MeV} \]

\[ L \approx 3 \text{ fm} \]

\[ N_f = 0, 2 \text{ or } 2+1 \]

\[ N_U = \mathcal{O}(100) \]
Extrapolations Required

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But ... it is systematically improvable
SUMMARY SO FAR

Continuum

Ordinary QCD

Extreme QCD

T ≠ 0

Lattice

Bound States

T = 0
Do bound hadronic states persist into the “quark-gluon” plasma phase?

- **Spectral functions** can answer this!

\[
G(t, \vec{p}) = \int \rho(\omega, \vec{p}) K(t, \omega) \, d\omega
\]

\[
K(t, \omega) = \frac{\cosh[\omega(t - N_t/2)]}{\sinh[\omega/(2T)]} \sim \exp[-\omega t]
\]
Example Spectral Functions

\[ G(t) \sim \int \rho(\omega) \ e^{-\omega t} \ d\omega \]
Example Spectral Functions

$$G(t) \sim \int \rho(\omega) e^{-\omega t} d\omega$$

Stable

Bound (decaying) States
Example Spectral Functions

\[ G(t) \sim \int \rho(\omega) e^{-\omega t} \, d\omega \]
What’s special about the Spectral Function?

- $\rho(\omega, \vec{p})$ contains info on
  - (in)stability of hadrons
  - transport coefficients
  - dilepton production . . .

- Extraction of a spectral density from a lattice correlator is an ill-posed problem:
  - Given $G(t)$ derive $\rho(\omega)$
  - More $\omega$ data points then $t$ data points!
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Requires the use of Bayesian analysis - Maximum Entropy Method (MEM)

- Hatsuda et al
Two lattice studies

Dublin-Swansea

- Dynamical
- Anistropic
- Zero momentum

Swansea

- Quenched
- Isotropic
- Non-zero momentum
Lattice Parameters (Dublin-Swansea)

- **Gluon Action:**
  - Improved anisotropic

- **Fermion Action:**
  - Wilson+Hamber-Wu + stout links
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<th>Light quarks</th>
<th>( M_\pi / M_\rho )</th>
<th>( \sim 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anisotropy</td>
<td>( \xi )</td>
<td>6</td>
</tr>
<tr>
<td>Lattice spacings</td>
<td>( a_t )</td>
<td>( \sim 0.025 \text{ fm} )</td>
</tr>
<tr>
<td></td>
<td>( a_s )</td>
<td>( \sim 0.15 \text{ fm} )</td>
</tr>
<tr>
<td>Spatial Volume</td>
<td>( N_s^3 )</td>
<td>8(^3) (&amp;12(^3))</td>
</tr>
<tr>
<td>1/T</td>
<td>( N_t )</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>32</td>
</tr>
<tr>
<td>Statistics</td>
<td>( N_{cfg} )</td>
<td>( \sim 500 )</td>
</tr>
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</table>

[Aarts, Oktay, Peardon, Skullerud, CRA]
$\eta_C$ Pseudoscalar ($am_c = 0.080, N_s = 8$)

Can see it melting!
**J/ψ Vector** ($am_c = 0.080$, $N_s = 8$)

![Graph showing temperature variations](image)

- Can see it melting!
- Note the singularity at $\omega \sim 0$
Gluon Action:
- Wilson

Quenched

Twisted Boundary Conditions
- large range of momenta available

Singularity at $\omega \sim 0$ traced to $K(\omega, t)$ and corrected

[Aarts, Foley, Hands, Kim, CRA]
### COLD

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lattice spacings</td>
<td>$a^{-1} \sim 4 \text{ GeV}$</td>
</tr>
<tr>
<td>Spatial Volume</td>
<td>$N_s^3 \times N_t$</td>
</tr>
<tr>
<td></td>
<td>$48^3 \times 24$</td>
</tr>
<tr>
<td>Temperature $T$</td>
<td>$T \sim 160\text{MeV} \sim \frac{1}{2}T_c$</td>
</tr>
<tr>
<td>Statistics</td>
<td>$N_{cfg}$</td>
</tr>
<tr>
<td></td>
<td>$\sim 100$</td>
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</table>

### HOT

<table>
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<tbody>
<tr>
<td>Lattice spacings</td>
<td>$a^{-1} \sim 10 \text{ GeV}$</td>
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<tr>
<td>Spatial Volume</td>
<td>$N_s^3 \times N_t$</td>
</tr>
<tr>
<td></td>
<td>$64^3 \times 24$</td>
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<tr>
<td>Temperature $T$</td>
<td>$T \sim 420\text{MeV} \sim \frac{3}{2}T_c$</td>
</tr>
<tr>
<td>Statistics</td>
<td>$N_{cfg}$</td>
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<td></td>
<td>$\sim 100$</td>
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</table>
Staggered Correlators

\[ a^3 G_{ii}(\tau,0) \]

Data MEM

\( \beta = 6.5 \)

\( am = 0.01 \)

48\(^3\) x 24
Staggered Correlators

\[ G(t) = 2 \int \frac{d\omega}{2\pi} K(t,\omega) \left( \rho(\omega) - (-1)^t \tilde{\rho}(\omega) \right) \]

→ Have to fit to even & odd times separately, then use

\[ \rho_{\text{phys}} = \frac{1}{2} \left( \rho_{\text{even}} + \rho_{\text{odd}} \right) \]
MEM results below $T_c$ (Swansea)

Momentum dependence of spectral function (below $T_c$)

Can see it moving
Electrical Conductivity

\[ \frac{\sigma}{T} = \lim_{\omega \to 0} \frac{\rho(\omega)}{6\omega T} \]

\[ \sigma = \text{Conductivity} \]
Electrical Conductivity

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\]

\[\sigma = \text{Conductivity}\]

\[\begin{array}{c}
\text{hot} \\
N_{\tau} = 24 \\
N_{\omega} = 1000, b = 1.0 \\
- - - N_{\omega} = 1000, b = 0.5 \\
- - - N_{\omega} = 1000, b = 0.1
\end{array}\]

\[\begin{array}{c}
\text{very hot} \\
N_{\tau} = 16 \\
N_{\omega} = 1000, b = 1.0 \\
- - - N_{\omega} = 1000, b = 0.5 \\
- - - N_{\omega} = 1000, b = 0.1
\end{array}\]

\[\sigma / T = 0.4 \pm 0.1\] Aarts, CRA, Foley & Hands
$D$ obtained from the *momentum* dependency of $\rho_{\text{Long Vector}}$ (at small mass)
Diffusivity [Preliminary] \( m/T = 0.24 \) Hot

Longitudinal Vector

\( m/T = 0.24 \) (ma = 0.01)
Diffusivity [Preliminary] $m/T = 1.2$ Hot

Longitudinal Vector

$m/T = 1.20$ (ma = 0.05)
Lack of Melting? [Preliminary]

\( m/T = 3 \) Hot

Longitudinal Vector

\[ m/T = 3 \quad (ma = 0.125) \]

\[ \rho(\omega) / (T \omega) \]

- \( (0,0,0) \)
- \( (2,0,0) \)
MEM Limitations

If the correlation function has a large dynamic range then there are numerical instabilities in MEM.

\[ G(t) = \int \rho(\omega) K(t, \omega) \, d\omega \]

where

\[ K(t, \omega) = \frac{\cosh[\omega(t - N_t/2)]}{\sinh[\omega/(2T)]} \]
\[ \sim \exp[-\omega t] \]

i.e. it is almost a Laplace transform:

\[ G(t) \sim \int \rho(\omega) e^{-\omega t} \, d\omega \]
Aim is to get $G(t)$ expressed as an exact Laplace transform, then can use convolution rules:

$$G(t) \times e^{+\Omega t} = \int \rho(\omega + \Omega) \, e^{-\omega t} \, d\omega$$

↑

smaller
dynamic
range
Possible Solution

\[
G(t) = e^{-Mt} + e^{-M(T-t)} = e^{-MT/2}(e^{-M(t-T/2)} + e^{M(t-T/2)}) = e^{-MT/2}(x^i + x^{-i})
\]

where \( x = e^{-M(t-T/2)} \) and \( i = t - T/2 \).

Define \( \tilde{G}(t) = e^{-MT/2}(x + x^{-1})^i \)

\[
\begin{align*}
\tilde{G}(t) &= e^{-MT/2} \sum_{j=0}^{i} i C_j x^{2j-i} \\
&= e^{-MT/2} \sum_{j=0}^{[i/2]} i C_j (x^{2j-i} + x^{i-2j}) \\
&= \sum_{j=0}^{[i/2]} i C_j G(2j - i)
\end{align*}
\]
We can now write \( x + x^{-1} \equiv e^y \rightarrow \)

\[
\tilde{G}(t) = e^{-MT/2}(x + x^{-1})^i = e^{-MT/2}e^{yi} \equiv \text{linear combination of } G(t)
\]


i.e. by defining a linear combination of \( G(t) \) correlators, the lattice kernel is transposed into a pure exponential.

\[\rightarrow \text{ Laplace Transform Convolution Rule is now viable}\]
SUMMARY

Continuum

T = 0

Lattice

T ≠ 0

Extreme QCD

Spectral F'ns

Ordinary QCD

Bound States
Happy 60th A.W.T.