Electric Dipole Moment of Light Nuclei

Iraj R. Afnan
(The Flinders University of South Australia)

Happy Birthday Tony

Collaborator: Benjamin F. Gibson, Los Alamos National Laboratory.

How I met Tony

Date: September 1970 at Flinders University

Occasion: Seminar on the role of D-state of deuteron on saturation properties on nuclei

Subject: Low Energy $pp \to \pi d$

How: Sensitivity of this reaction to deuteron properties?

Question:
How sensitive is the Electric Dipole Moment of Nuclei to the NN interaction

First Joint Publication

CITY WEATHER

Cold. Occasional showers.

Estimated max., low 60s.

Yesterday's temperatures: Min., 56.5; max., 61.0.

April 28 last year: Min., 52.5; max., 64.4.

Weather map, Page 39.

The Advertis

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Vol. 113, No. 35,098 44 Pages 7c ADELAIDE, THURSDAY, APRIL 29, 1971

World first for S.A. physicists

Examining a copy of the formula yesterday at Flinders University are (from left) research student Mr. A. W. Thomas, the professor of physics (Professor I. E. McCarthy) and physics lecturer Dr. I. R. Afnan.

By Science Writer
BARRY HAILSTONE

Two physicists at Flinders University have made a discovery in nuclear physics that is regarded as being of such importance that it will be made available to physicists throughout the world within the next three weeks.

The physicists, Dr. I. R. Afnan, 32, and Mr. A. W. Thomas, 21, a graduate student, have submitted a calculation to the American Institute of Physics, which has undertaken to publish it as a work of urgency.

Their formula will improve the accuracy of calculations in nuclear physics.

Both are members of the university's nuclear theory group.

The group's leader, Professor I. E. McCarthy, said yesterday, "It is a very nice bit of theoretical physics — pretty significant stuff."

$$T_{d,p} \cong \langle \chi_d | V | \chi_p^{(+)} \rangle + \langle \chi_d | V \frac{1}{E-H} V | \chi_p^{(+)} \rangle$$

Greek chi

The formula

Their discovery concerns heavy hydrogen, the characteristic ingredient of heavy water.

Reasoning behind the calculation was developed by Mr. Thomas last year when he was an honors undergraduate.

"He is the most brilliant student I have seen anywhere," Professor McCarthy said.

The idea was expounded during a seminar on nuclear force and lecturer Dr. Afnan and Mr. Thomas have worked on it for the past two months.

By checking quantities in the formula against known reactions, the scientists have been able to check that their calculations are correct.

But because Australia does not have the modern nuclear physics equipment necessary, the experiments cannot be performed here.

However, it is expected that members of the theoretical group will visit accelerators being constructed in the US, Canada and Switzerland to collaborate with experimental physicists.

Their work will be circulated around the world by the American institute in a special publication "Physical Review Letters," within three weeks.

Some History

1949 Purcell & Ramsey:
non-zero electric dipole moment of n implied parity violation in strong interaction:

$$H = -\mu\ \vec{B} \cdot \hat{S} - d\ \vec{E} \cdot \hat{S}$$

$$P[\vec{B} \cdot \hat{S}] = \vec{B} \cdot \hat{S} \quad \text{and} \quad T[\vec{B} \cdot \hat{S}] = \vec{B} \cdot \hat{S}$$

$$P[\vec{E} \cdot \hat{S}] = -\vec{E} \cdot \hat{S} \quad \text{and} \quad T[\vec{E} \cdot \hat{S}] = -\vec{E} \cdot \hat{S}$$

1950 Purcell & Ramsey: $d_n < 3 \times 10^{-18} e\ cm$ from n scattering.

Present limit: $d_n < 0.29 \times 10^{-25} e\ cm$.

1956 Lee & Yang: Parity violation in Weak Interaction.

1957 Landau: CP invariance implies particles have **NO** Electric Dipole Moment (EDM) if CPT is valid.

Therefore: Measurement of EDM is a test for **flavour-conserving** CP violation.

Why deuteron EDM?

The deuteron EDM is the sum of a one- and two-body contribution

$$d_D = d_D^{(1)} + d_D^{(2)} = (d_n + d_p) + d_D^{(2)}$$

Experiment: Proposed experiment to measure Deuteron EDM in a storage ring at the level of (Y.K. Semertzidis et al, hep-ex/0308063)

$$d \approx 10^{-27} e\ cm.$$

Theoretical Estimate: Based on pion exchange model (Liu & Timmermans, PRC **70**, 055501 (2004))

$$d_D^{(2)} \approx 0.20 \bar{g}_\pi^{(1)}$$

$$d_D^{(1)} \approx 0.03 \bar{g}_\pi^{(1)} + 0.09 \bar{g}_\pi^{(0)}$$

with $\bar{g}_\pi^{(1)}/\bar{g}_\pi^{(0)} \approx 10$. This suggests that the dominant contribution to d_D is the two-body contribution $d_D^{(2)}$ which we will now consider.

Evaluation of $d_D^{(2)}$

The Hamiltonian, including PT-violation component, is of the form

$$H = H^S + H^{PT} \quad \text{where} \quad H^S = H_0 + v \quad \text{and} \quad H^{PT} = V$$

Since H^{PT} will mix parity states, *e.g.* for the deuteron we get a coupling between ${}^3\mathrm{S}_1$-${}^3\mathrm{D}_1$ (the large component $|\Psi_L\rangle$) and ${}^3\mathrm{P}_1$ (the small component $|\Psi_S\rangle$), and we can write the coupled channel equations

$$(E - H_0)|\Psi_L\rangle = v\,|\Psi_L\rangle + V\,|\Psi_S\rangle$$

$$(E - H_0)|\Psi_S\rangle = v\,|\Psi_S\rangle + V\,|\Psi_L\rangle \ .$$

Since $V \ll v$, $V\,|\Psi_S\rangle \ll v\,|\Psi_L\rangle$, and we have that $|\Psi_L\rangle$ satisfies

$$(E - H_0)|\Psi_L\rangle = v|\Psi_L\rangle \ .$$

On the other hand the small component $|\Psi_S\rangle$is given by

$$|\Psi_S\rangle = G(E)\,V\,|\Psi_L\rangle$$

where

$$G(E) = (E - H_0 - v)^{-1} = G_0(E) + G_0(E)\,T(E)\,G_0(E)$$

Evaluation of $d_D^{(2)}$ cont.

The two-body electric dipole moment is now given by

$$d_D^{(2)} = \langle \Psi | \, O_d \, | \Psi \rangle = \langle \Psi_L | \, O_d \, | \Psi_S \rangle + \langle \Psi_S | \, O_d \, | \Psi_L \rangle \ ,$$

where O_d is the usual electric dipole operator given by

$$O_d = \frac{e}{2} \sum_i \ \vec{r}_i \, \tau_z(i) \ .$$

Making use of the expression for $|\Psi_S\rangle$ we can write

$$\begin{aligned} \langle \Psi_L | \, O_d \, | \Psi_S \rangle \quad &= \quad \langle \Psi_L | \, O_d \, G_0(E) \, V \, | \Psi_L \rangle + \langle \Psi_L | \, O_d \, G_0(E) \, T(E) \, G_0(E) \, V \, | \Psi_L \rangle \\ &\equiv \quad \frac{e}{2} \left[\, d_{PW} + d_{MS} \, \right] A \quad \text{with} \quad A = \frac{g_{\pi NN} \, \bar{g}^{(1)}_{\pi NN}}{16\pi} \end{aligned}$$

where $T(E)$ is the $^3\mathrm{P}_1$ amplitude calculated at the deuteron energy.

Note:

- d_{PW} invloves taking plane wave intermediate state (no $^3\mathrm{P}_1$),
- while d_{MS} is the contribution from multiple scattering in the $^3\mathrm{P}_1$ partial wave via $T(E)$.

Previous Results

Avishai, 1985:

- Solved the coupled channel problem with separable potentials in both ^{3}S$_1$-^{3}D$_1$ and ^{3}P$_1$ partial waves.
- For PT violating interaction V, he took one pion exchange.
- The EDM $d_D^{(2)} = -0.91\, A\ e\ fm$ with $A = g_{\pi NN}\, \bar{g}_{\pi NN}^{(1)}/16\pi$??.

Khriplovich & Korkin, 2000

- Use zero range theory – independent of ^{3}P$_1$ interaction.
- The EDM $d_D^{(2)} = -0.92\ A\ efm$.

Liu & Timmermans, 2004

- Used Argonne v_{18} and Nijmegen models Reid93 and Nijm II in a coupled channel calculations.
- The EDM $d_D^{(2)} = -0.73 \pm 0.01\ A\ e\ fm$.

Aim of Present Study

- To understand the difference between the previous three calculations specially since Avishai's results might be off by a factor of 2, *i.e.* $d_D^{(2)} = -0.46\ A\ e\ fm$.
- The relative contribution of d_{PW} and d_{MS} with the hope of being able to neglect d_{MS} or treat it perturbativly when going to heavier nuclei, *e.g.* ^{3}He.
- How sensitive are the results to choice of $^3\mathrm{S}_1$-$^3\mathrm{D}_1$ and $^3\mathrm{P}_1$ interaction, and in particular, can one use separable potentials to represent these interaction as one goes beyond the two-nucleon system.
- Finally, how complex a calculation do we need at this stage, considering the experimental limit has not yet been set.
- Ultimately, we hope a measurement of the deuteron EDM will shed some light on flavour-conserving CP violation, and a test of theories beyond the Standard Model.

The Input Interaction

The PT-violating Potential V

In the present analysis we will use only the one pion exchange potential with one vertex having the strong πNN coupling constant $g_{\pi NN}$, while the other has the isospin one PT-violating πNN coupling constant $\bar{g}^{(1)}_{\pi NN}$. These correspond to the Lagrangians

$$\mathcal{L}^{(I=1)}_{P.T} = \bar{g}^{(1)}_{\pi NN}\ \bar{N}\, N\, \pi^0 \qquad \mathcal{L}_S = g_{\pi NN}\ \bar{N}\, i\gamma_5 \vec{\tau}\cdot\vec{\pi}\, N$$

The Strong Nucleon-Nucleon Interactions v

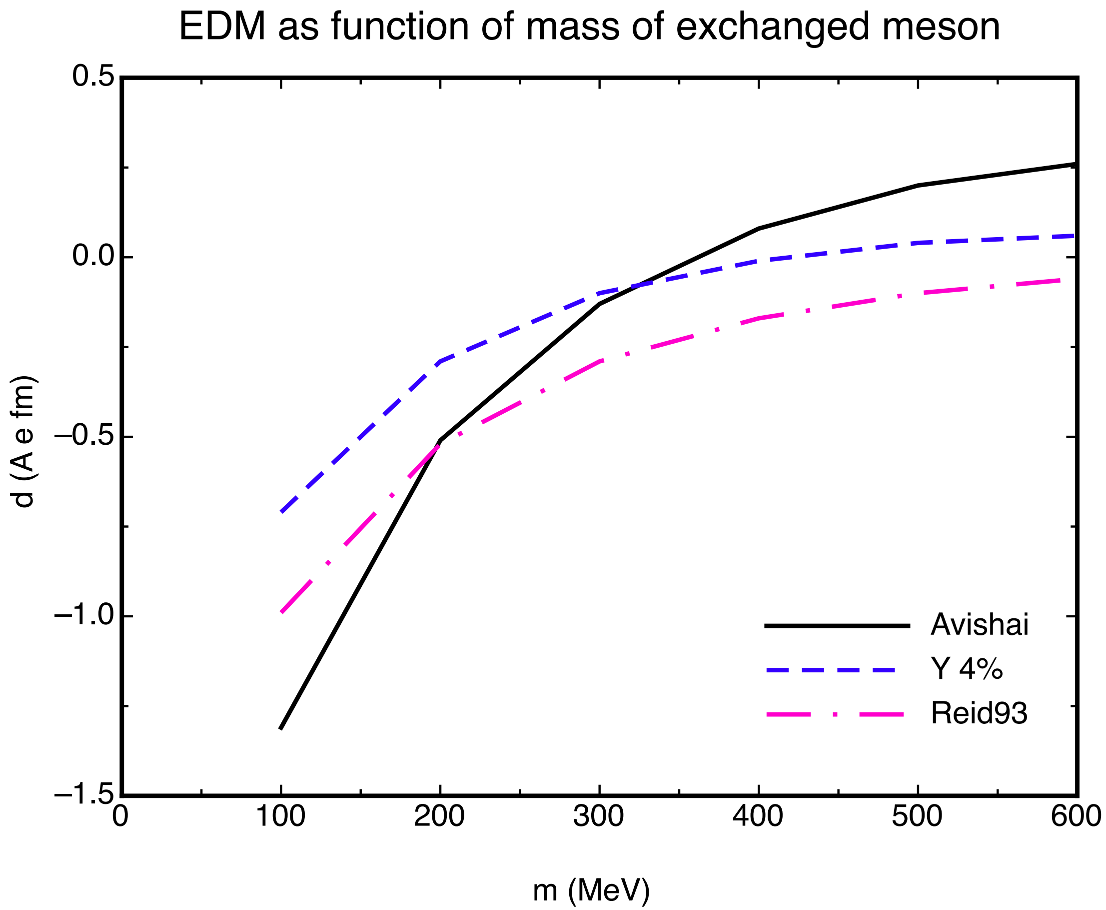
- For the $^3\mathrm{P}_1$ interaction we use separable potentials of the form used by Mongan in the late 60's and adjusted to fit the new np phases from the Nijmegen group.

- For the $^3\mathrm{S}_1$-$^3\mathrm{D}_1$ interaction we use either Yamaguchi rank one potentials or the Unitary Pole Approximation (UPA) to the original Reid(1968) or Reid(1993) potential. The Reid(1993) fits the latest Nijmegen np phases.

  Note: The UPA gives the identical bound state wave function to the original potential.

Sensitivity to PT-Violating Potential

Since I am considering only π-exchange, I will first examine the dependence of EDM $d_D^{(2)}$ on the mass of the exchanged meson.



Note: Contribution to $d_D^{(2)}$ is suppressed for heavy meson exchange.

Importance of the Deuteron Wave Function

We now turn to the sensitivity of the EDM to the deuteron wave function. For the ^{3}P$_1$ potential we take a rank two separable potential that gives the optimum fit the Nijmegen *np* phase shifts at low energies.

^{3}S$_1$-^{3}D$_1$	P_d	$d_{PW}(A\,e\,fm)$	$d_{MS}(A\,e\,fm)$	$d_D^{(2)}(A\,e\,fm)$
YY 4%	4%	-1.035	0.4155	-0.6234
Reid93	5.7%	-0.9715	0.2009	-0.7706
Reid68	6.5%	-0.9620	0.1718	-0.7902
YY 7%	7%	-0.1083	0.4271	-0.6564
Khriplovich *et al.*		-0.92		

Note:

- For d_{PW} the variation with potential is less than 5%. and differes from the zero range results by less that 10%.
- The d_{MS} is sensitive to the short range behaviour of the deuteron wave.

Importance of the ^{3}P$_1$ Interaction

Here we consider the contribution to the EDM from d_{MS} for several ^{3}P$_1$ separable potential that fit the latest Nijmegen np data, with the deuteron wave function from the Reid93 potential. Here $d_{PW} = -0.9715 A\, e\, fm$

Case	Rank	χ^2	$d_{MS}(A\, e\, fm)$	$d_D^{(2)}(A\, e\, fm)$
I	1	0.62	0.2583	-0.7132
I	2	0.02	0.2009	-0.7706
II	1	0.81	0.2229	-0.7486
III	1	0.19	0.3075	-0.6640
III	2	0.12	0.3805	-0.5910
IV	1	0.78	0.2153	-0.7562

Note:

- With the exception of the Case III, results are not sensitive to the ^{3}P$_1$.
- Excluding Case III, the contribution from d_{MS} is about 20%.

Importance of the np data

Q: How important is it to fit the latest np data for the ^{3}P$_1$ channel?

Here we compare results for the same rank one potential as defined by Mongan (1969) and a refited to the latest Nijmegen np phases.

^{3}S$_1$-^{3}D$_1$			Reid68		YY 4%	
			$d_{PW} = -0.96$		$d_{PW} = -1.04$	
Case	χ^2	$g(k)$	d_{MS}	$d_D^{(2)}$	d_{MS}	$d_D^{(2)}$
I (New)	0.62	$k/(k^2+\beta^2)$	0.21	-0.75	0.57	-0.47
I (Old)	1.90	$k/(k^2+\beta^2)$	0.31	-0.66	0.78	-0.26
III (New)	0.19	$[Q_1(1+\frac{\beta^2}{2k^2})/k^2\pi]^{1/2}$	0.25	-0.71	0.77	-0.27
III (Old)	6.67	$[Q_1(1+\frac{\beta^2}{2k^2})/k^2\pi]^{1/2}$	0.42	-0.54	1.16	0.12

Note:

- Sensitivity of $d_D^{(2)}$ to ^{3}P$_1$ is more for YY than the Reid potential.
- The effect is more pronounced for Case III potential than Case I.

Conclusions

- If we ignore the multiple scattering via the $^3\mathrm{P}_1$ ($i.e\ d_{PW}$), the variation due to different deuteron wave function is less than 5%, and consistent with zero range approximation of Khripovich & Korkin.
- The contribution from the $^3\mathrm{P}_1$ ($i.e\ d_{MS}$) is sensitive to the choice of deuteron wave function. With Reid type potentials that have short range repulsion this uncertainty can be as little as 20%.
- The contribution of the $^3\mathrm{P}_1$ via d_{MS} depend on the phase shifts the potentials fit, and the off-shell behaviour of the $^3\mathrm{P}_1$ amplitude.
- Considering the contribution of d_{MS}, we think we can treat the $^3\mathrm{P}_1$ perturbativly in ^{3}He EDM, *i.e.* replace the off-shell three-body amplitude by the two-body sub-amplitude.
- For Reid93 our results are consistent with Liu & Timmermans, suggesting that separable potentials approach could be used for the ^{3}He EDM.
- Finally, we need to understand why the Case III $^3\mathrm{P}_1$ potentials give drastically different results by examining the $^3\mathrm{P}_1$ scattering wave function at the deuteron pole.