Electric Dipole Moment of Light Nuclei

Iraj R. Afnan
(The Flinders University of South Australia)

Happy Birthday Tony

Collaborator: Benjamin F. Gibson, Los Alamos National Laboratory.
How I met Tony

Date: September 1970 at Flinders University
Occasion: Seminar on the role of $D$-state of deuteron on saturation properties on nuclei
Subject: Low Energy $pp \rightarrow \pi d$
How: Sensitivity of this reaction to deuteron properties?

Question:
How sensitive is the Electric Dipole Moment of Nuclei to the $NN$ interaction
World first for S.A. physicists

By Science Writer
BARRY HAILSTONE

Two physicists at Flinders University have made a discovery in nuclear physics that is regarded as being of such importance that it will be made available to physicists throughout the world within the next three weeks.

The physicists, Dr. I. R. Afnan, 32, and Mr. A. W. Thomas, 21, a graduate student, have submitted a calculation to the American Institute of Physics, which has undertaken to publish it as a work of urgency.

Their formula will improve the accuracy of calculations in nuclear physics.

Both are members of the university's nuclear theory group.

The group's leader, Professor I. E. McCarthy, said yesterday, "It is a very nice bit of theoretical physics—pretty significant stuff."

Examining a copy of the formula yesterday at Flinders University are (from left) research student Mr. A. W. Thomas, the professor of physics (Professor I. E. McCarthy) and physics lecturer Dr. I. R. Afnan.

The formula:

\[ t_{1/2} = \left( \frac{1}{t} \right) \frac{1}{(1 - \frac{1}{e^{\frac{V}{kT}}})} \]

Their discovery concerns heavy hydrogen, the characteristic ingredient of heavy water.

Reasoning behind the calculation was developed by Mr. Thomas last year when he was an honors undergraduate.

"He is the most brilliant student I have seen anywhere," Professor McCarthy said.

The idea was expanded during a seminar on nuclear force and lecturer Dr. Afnan and Mr. Thomas have worked on it for the past two months.

By checking quantities in the formula against known reactions, the scientists have been able to check that their calculations are correct.

But because Australia does not have the modern nuclear physics equipment necessary, the experiment cannot be performed here.

However, it is expected that members of the theoretical group will visit accelerators being constructed in the US, Canada and Switzerland to collaborate with experimental physicists.

Their work will be circulated around the world by the American Institute in a special publication "Physical Review Letters," within three weeks.
Some History

1949 Purcell & Ramsey:
non-zero electric dipole moment of $n$ implied parity violation in strong interaction:

$$H = -\mu \vec{B} \cdot \hat{S} - d \vec{E} \cdot \hat{S}$$

$$P[\vec{B} \cdot \hat{S}] = \vec{B} \cdot \hat{S} \quad \text{and} \quad T[\vec{B} \cdot \hat{S}] = \vec{B} \cdot \hat{S}$$

$$P[\vec{E} \cdot \hat{S}] = -\vec{E} \cdot \hat{S} \quad \text{and} \quad T[\vec{E} \cdot \hat{S}] = -\vec{E} \cdot \hat{S}$$

1950 Purcell & Ramsey: $d_n < 3 \times 10^{-18} e \text{ cm}$ from $n$ scattering.

Present limit: $d_n < 0.29 \times 10^{-25} e \text{ cm}$.

1956 Lee & Yang: Parity violation in Weak Interaction.

1957 Landau: $CP$ invariance implies particles have NO Electric Dipole Moment (EDM) if $CPT$ is valid.

Therefore: Measurement of EDM is a test for flavour-conserving $CP$ violation.
Why deuteron EDM?

The deuteron EDM is the sum of a one- and two-body contribution

\[ d_D = d_D^{(1)} + d_D^{(2)} = (d_n + d_p) + d_D^{(2)} \]

**Experiment:** Proposed experiment to measure Deuteron EDM in a storage ring at the level of (Y.K. Semertzidis et al, hep-ex/0308063)

\[ d \approx 10^{-27} e \text{ cm}. \]

**Theoretical Estimate:** Based on pion exchange model (Liu & Timmermans, PRC 70, 055501 (2004))

\[ d_D^{(2)} \approx 0.20 \bar{g}^{(1)}_\pi \]

\[ d_D^{(1)} \approx 0.03 \bar{g}^{(1)}_\pi + 0.09 \bar{g}^{(0)}_\pi \]

with \( \bar{g}^{(1)}_\pi / \bar{g}^{(0)}_\pi \approx 10 \). This suggests that the dominant contribution to \( d_D \) is the two-body contribution \( d_D^{(2)} \) which we will now consider.


Evaluation of $d_D^{(2)}$

The Hamiltonian, including $PT$-violation component, is of the form

$$H = H^S + H^{PT} \text{ where } H^S = H_0 + \nu \text{ and } H^{PT} = V$$

Since $H^{PT}$ will mix parity states, e.g. for the deuteron we get a coupling between $^3S_1$-$^3D_1$ (the large component $|\Psi_L\rangle$) and $^3P_1$ (the small component $|\Psi_S\rangle$), and we can write the coupled channel equations

$$(E - H_0)|\Psi_L\rangle = \nu |\Psi_L\rangle + V |\Psi_S\rangle$$

$$(E - H_0)|\Psi_S\rangle = \nu |\Psi_S\rangle + V |\Psi_L\rangle.$$  

Since $V \ll \nu$, $V |\Psi_S\rangle \ll \nu |\Psi_L\rangle$, and we have that $|\Psi_L\rangle$ satisfies

$$(E - H_0)|\Psi_L\rangle = \nu |\Psi_L\rangle.$$  

On the other hand the small component $|\Psi_S\rangle$ is given by

$$|\Psi_S\rangle = G(E) V |\Psi_L\rangle$$

where

$$G(E) = (E - H_0 - \nu)^{-1} = G_0(E) + G_0(E) T(E) G_0(E)$$
Evaluation of $d_{D}^{(2)}$ cont.

The two-body electric dipole moment is now given by

$$d_{D}^{(2)} = \langle \Psi | O_{d} | \Psi \rangle = \langle \Psi_{L} | O_{d} | \Psi_{S} \rangle + \langle \Psi_{S} | O_{d} | \Psi_{L} \rangle,$$

where $O_{d}$ is the usual electric dipole operator given by

$$O_{d} = \frac{e}{2} \sum_{i} \vec{r}_{i} \tau_{z}(i).$$

Making use of the expression for $|\Psi_{S}\rangle$ we can write

$$\langle \Psi_{L} | O_{d} | \Psi_{S} \rangle = \langle \Psi_{L} | O_{d} G_{0}(E) V | \Psi_{L} \rangle + \langle \Psi_{L} | O_{d} G_{0}(E) T(E) G_{0}(E) V | \Psi_{L} \rangle$$

$$\equiv \frac{e}{2} [d_{PW} + d_{MS}] A \quad \text{with} \quad A = \frac{g_{\pi NN} g_{\pi NN}^{(1)}}{16\pi}$$

where $T(E)$ is the $^{3}P_{1}$ amplitude calculated at the deuteron energy.

**Note:**

- $d_{PW}$ involves taking plane wave intermediate state (no $^{3}P_{1}$),
- while $d_{MS}$ is the contribution from multiple scattering in the $^{3}P_{1}$ partial wave via $T(E)$. 
Previous Results

Avishai, 1985:

- Solved the coupled channel problem with separable potentials in both $^3S_1$-$^3D_1$ and $^3P_1$ partial waves.
- For $PT$ violating interaction $V$, he took one pion exchange.
- The EDM $d_D^{(2)} = -0.91\ A\ e\ fm$ with $A = g_{\pi NN} \bar{g}^{(1)}_{\pi NN}/16\pi$.

Khriplovich & Korkin, 2000

- Use zero range theory – independent of $^3P_1$ interaction.
- The EDM $d_D^{(2)} = -0.92\ A\ efm$.

Liu & Timmermans, 2004

- Used Argonne $v_{18}$ and Nijmegen models Reid93 and Nijm II in a coupled channel calculations.
- The EDM $d_D^{(2)} = -0.73\pm0.01\ A\ efm$. 
Aim of Present Study

- To understand the difference between the previous three calculations specially since Avishai’s results might be off by a factor of 2, i.e. $d_D^{(2)} = -0.46 \, A \, e \, fm$.

- The relative contribution of $d_{PW}$ and $d_{MS}$ with the hope of being able to neglect $d_{MS}$ or treat it perturbativly when going to heavier nuclei, e.g. $^3$He.

- How sensitive are the results to choice of $^3S_1$-$^3D_1$ and $^3P_1$ interaction, and in particular, can one use separable potentials to represent these interaction as one goes beyond the two-nucleon system.

- Finally, how complex a calculation do we need at this stage, considering the experimental limit has not yet been set.

- Ultimately, we hope a measurement of the deuteron EDM will shed some light on flavour-conserving $CP$ violation, and a test of theories beyond the Standard Model.
The Input Interaction

The $PT$-violating Potential $V$

In the present analysis we will use only the one pion exchange potential with one vertex having the strong $\pi NN$ coupling constant $g_{\pi NN}$, while the other has the isospin one $PT$-violating $\pi NN$ coupling constant $\bar{g}_{\pi NN}^{(1)}$. These correspond to the Lagrangians

$$\mathcal{L}_{P.T}^{(I=1)} = \bar{g}_{\pi NN}^{(1)} \bar{N} N \pi^0 \quad \mathcal{L}_S = g_{\pi NN} \bar{N} i\gamma_5 \vec{\tau} \cdot \vec{\pi} N$$

The Strong Nucleon-Nucleon Interactions $v$

- For the $^3P_1$ interaction we use separable potentials of the form used by Mongan in the late 60’s and adjusted to fit the new $np$ phases from the Nijmegen group.

- For the $^3S_1$-$^3D_1$ interaction we use either Yamaguchi rank one potentials or the Unitary Pole Approximation (UPA) to the original Reid(1968) or Reid(1993) potential. The Reid(1993) fits the latest Nijmegen $np$ phases.

**Note:** The UPA gives the identical bound state wave function to the original potential.
Sensitivity to $PT$-Violating Potential

Since I am considering only $\pi$-exchange, I will first examine the dependence of EDM $d^{(2)}_D$ on the mass of the exchanged meson.

![Graph showing EDM as function of mass of exchanged meson](image)

**Note:** Contribution to $d^{(2)}_D$ is suppressed for heavy meson exchange.
Importance of the Deuteron Wave Function

We now turn to the sensitivity of the EDM to the deuteron wave function. For the $^3P_1$ potential we take a rank two separable potential that gives the optimum fit the Nijmegen $np$ phase shifts at low energies.

<table>
<thead>
<tr>
<th>$^3S_1^{-3}D_1$</th>
<th>$P_d$</th>
<th>$d_{PW}(Ae fm)$</th>
<th>$d_{MS}(Ae fm)$</th>
<th>$d^{(2)}_D(Ae fm)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>YY 4%</td>
<td>4%</td>
<td>-1.035</td>
<td>0.4155</td>
<td>-0.6234</td>
</tr>
<tr>
<td>Reid93</td>
<td>5.7%</td>
<td>-0.9715</td>
<td>0.2009</td>
<td>-0.7706</td>
</tr>
<tr>
<td>Reid68</td>
<td>6.5%</td>
<td>-0.9620</td>
<td>0.1718</td>
<td>-0.7902</td>
</tr>
<tr>
<td>YY 7%</td>
<td>7%</td>
<td>-0.1083</td>
<td>0.4271</td>
<td>-0.6564</td>
</tr>
<tr>
<td>Khriplovich et al.</td>
<td></td>
<td>-0.92</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note:

- For $d_{PW}$ the variation with potential is less than 5%. and differs from the zero range results by less that 10%.
- The $d_{MS}$ is sensitive to the short range behaviour of the deuteron wave.
Importance of the $^3P_1$ Interaction

Here we consider the contribution to the EDM from $d_{MS}$ for several $^3P_1$ separable potential that fit the latest Nijmegen $np$ data, with the deuteron wave function from the Reid93 potential. Here $d_{PW} = -0.9715\, A\, e\, fm$

<table>
<thead>
<tr>
<th>Case</th>
<th>Rank</th>
<th>$\chi^2$</th>
<th>$d_{MS}(A, e, fm)$</th>
<th>$d_{D}^{(2)}(A, e, fm)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>0.62</td>
<td>0.2583</td>
<td>-0.7132</td>
</tr>
<tr>
<td>I</td>
<td>2</td>
<td>0.02</td>
<td>0.2009</td>
<td>-0.7706</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
<td>0.81</td>
<td>0.2229</td>
<td>-0.7486</td>
</tr>
<tr>
<td>III</td>
<td>1</td>
<td>0.19</td>
<td>0.3075</td>
<td>-0.6640</td>
</tr>
<tr>
<td>III</td>
<td>2</td>
<td>0.12</td>
<td>0.3805</td>
<td>-0.5910</td>
</tr>
<tr>
<td>IV</td>
<td>1</td>
<td>0.78</td>
<td>0.2153</td>
<td>-0.7562</td>
</tr>
</tbody>
</table>

Note:

- With the exception of the Case III, results are not sensitive to the $^3P_1$.
- Excluding Case III, the contribution from $d_{MS}$ is about 20%.
**Importance of the np data**

**Q:** How important is it to fit the latest np data for the $^3P_1$ channel?

Here we compare results for the same rank one potential as defined by Mongan (1969) and a refitted to the latest Nijmegen np phases.

<table>
<thead>
<tr>
<th></th>
<th>$^3S_1-^3D_1$</th>
<th>Reid68 $d_{PW}$</th>
<th>YY 4% $d_{PW}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$g(k)$</td>
<td>$d_{MS}$</td>
<td>$d_D^{(2)}$</td>
</tr>
<tr>
<td>Case I</td>
<td>$\chi^2$ 0.62 $k/(k^2 + \beta^2)$</td>
<td>0.21</td>
<td>-0.75</td>
</tr>
<tr>
<td>Case I</td>
<td>1.90 $k/(k^2 + \beta^2)$</td>
<td>0.31</td>
<td>-0.66</td>
</tr>
<tr>
<td>Case III</td>
<td>0.19 $[Q_1(1 + \frac{\beta^2}{2k^2})/k^2\pi]^{1/2}$</td>
<td>0.25</td>
<td>-0.71</td>
</tr>
<tr>
<td>Case III</td>
<td>6.67 $[Q_1(1 + \frac{\beta^2}{2k^2})/k^2\pi]^{1/2}$</td>
<td>0.42</td>
<td>-0.54</td>
</tr>
</tbody>
</table>

**Note:**
- Sensitivity of $d_D^{(2)}$ to $^3P_1$ is more for YY than the Reid potential.
- The effect is more pronounced for Case III potential than Case I.
Conclusions

- If we ignore the multiple scattering via the $^3P_1$ (i.e. $d_{PW}$), the variation due to different deuteron wave function is less than 5%, and consistent with zero range approximation of Khripovich & Korkin.

- The contribution from the $^3P_1$ (i.e. $d_{MS}$) is sensitive to the choice of deuteron wave function. With Reid type potentials that have short range repulsion this uncertainty can be as little as 20%.

- The contribution of the $^3P_1$ via $d_{MS}$ depend on the phase shifts the potentials fit, and the off-shell behaviour of the $^3P_1$ amplitude.

- Considering the contribution of $d_{MS}$, we think we can treat the $^3P_1$ perturbatively in $^3$He EDM, i.e. replace the off-shell three-body amplitude by the two-body sub-amplitude.

- For Reid93 our results are consistent with Liu & Timmermans, suggesting that separable potentials approach could be used for the $^3$He EDM.

- Finally, we need to understand why the Case III $^3P_1$ potentials give drastically different results by examining the $^3P_1$ scattering wave function at the deuteron pole.