Excited States of the Nucleon in Lattice QCD

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Outline

- Two point correlation function:

$$G_{ij}(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle \Omega | T\{\chi_i(x)\bar{\chi}_j(0)\} | \Omega \rangle. \qquad (1)$$

- Inserting completeness

$$\sum_{B,\vec{p'},s} |B,\vec{p'},s\rangle\langle B,\vec{p'},s| = I$$

- Then

$$G_{ij}(t,\vec{p}) = \sum_{B^+} \lambda_{B^+}\bar{\lambda}_{B^+} e^{-E_{B^+}t} \frac{\gamma.p_{B^+} + M_{B^+}}{2E_{B^+}} + \sum_{B^-} \lambda_{B^-}\bar{\lambda}_{B^-} e^{-E_{B^-}t} \frac{\gamma.p_{B^-} - M_{B^-}}{2E_{B^-}}. \qquad (2)$$

- $\lambda_{B^\pm}$, $\bar{\lambda}_{B^\pm}$ are the couplings of $\chi(0)$ and $\bar{\chi}(0)$ with $|B^\pm\rangle$ defined by

$$\langle\Omega|\chi(0)|B^+,\vec{p},s\rangle = \lambda_{B^+}\sqrt{\frac{M_{B^+}}{E_{B^+}}}u_{B^+}(\vec{p},s),$$

$$\langle B^+,\vec{p},s|\bar{\chi}(0)|\Omega\rangle = \bar{\lambda}_{B^+}\sqrt{\frac{M_{B^+}}{E_{B^+}}}\bar{u}_{B^+}(\vec{p},s),$$

and for the negative parity states,

$$\langle\Omega|\chi(0)|B^-,\vec{p},s\rangle = \lambda_{B^-}\sqrt{\frac{M_{B^-}}{E_{B^-}}}\gamma_5 u_{B^-}(\vec{p},s),$$

$$\langle B^-,\vec{p},s|\bar{\chi}(0)|\Omega\rangle = -\bar{\lambda}_{B^-}\sqrt{\frac{M_{B^-}}{E_{B^-}}}\bar{u}_{B^-}(\vec{p},s)\gamma_5.$$

- At $\vec{p} = 0$

$$G^{\pm}_{ij}(t,\vec{0}) = \mathrm{Tr_{sp}}[\Gamma_{\pm} G_{ij}(t,\vec{0})]$$
$$= \sum_{B^{\pm}} \lambda^{\pm}_{i} \bar{\lambda}^{\pm}_{j} e^{-M_{B^{\pm}} t}. \qquad (3)$$

- Parity projection operator,

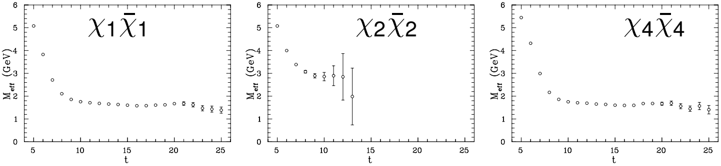
$$\Gamma_{\pm} = \frac{1}{2}(1 \pm \gamma_0).$$

- And

$$G^{\pm}_{ij}(t,\vec{0}) \stackrel{t\to\infty}{=} \lambda^{\pm}_{i0} \bar{\lambda}^{\pm}_{j0} e^{-M_{0^{\pm}} t}. \qquad (4)$$

- Interpolators:

$$\chi_1(x) = \epsilon^{abc}(u^{Ta}(x)C\gamma_5 d^b(x))u^c(x)\,,$$
$$\chi_2(x) = \epsilon^{abc}(u^{Ta}(x)Cd^b(x))\gamma_5 u^c(x)\,,$$
$$\chi_4(x) = \epsilon^{abc}(u^{Ta}(x)C\gamma_5\gamma_4 d^b(x))u^c(x).$$



Variational Method

- Consider N interpolating fields, then

$$\bar{\phi}^{\alpha} = \sum_{i=1}^{N} u_i^{\alpha} \bar{\chi}_i,$$

$$\phi^{\alpha} = \sum_{i=1}^{N} v_i^{\alpha} \chi_i,$$

such that,

$$\langle B_\beta, p, s | \bar{\phi}^{\alpha} | \Omega \rangle = \delta_{\alpha\beta} \bar{z}^{\alpha} \bar{u}(\alpha, p, s),$$

$$\langle \Omega | \phi^{\alpha} | B_\beta, p, s \rangle = \delta_{\alpha\beta} z^{\alpha} u(\alpha, p, s),$$

- Then a two point correlation function matrix for $\vec{p} = 0$,

$$G_{ij}(t)u_j^\alpha = (\sum_{\vec{x}} \mathrm{Tr}_{\mathrm{sp}}\{\Gamma_\pm \langle\Omega|\chi_i\bar{\chi}_j|\Omega\rangle\})u_j^\alpha$$
$$= \lambda_i^\alpha \bar{z}^\alpha e^{-m_\alpha t}. \tag{5}$$

- There is no sum over α
- t dependence only in the exponential term

- Then one can have a recurrence relation at time $(t + \triangle t)$,

$$G_{ij}(t + \triangle t)u_j^\alpha = e^{-m_\alpha \triangle t} G_{ij}(t)u_j^\alpha .$$

- Multiplying by $[G_{ij}(t)]^{-1}$ from left,

$$[(G(t))^{-1} G(t + \triangle t)]_{ij} u_j^\alpha = c^\alpha u_i^\alpha , \tag{6}$$

- where $c^\alpha = e^{-m_\alpha \triangle t}$ is the eigenvalue.
- Similarly, it can also be solved for the left eigenvalue equation for v^α eigenvector,

$$v_i^\alpha [G(t + \triangle t)(G(t))^{-1}]_{ij} = c^\alpha v_j^\alpha . \tag{7}$$

- The vectors u_j^α and v_i^α diagonalize the correlation matrix at time t and $t + \triangle t$ making the projected correlation matrix,

$$v_i^\alpha G_{ij}(t) u_j^\beta = \delta^{\alpha\beta} z^\alpha \bar{z}^\beta e^{-m_\alpha t}. \tag{8}$$

- The projected correlator, is then analyzed to obtain masses of different states,

$$v_i^\alpha G_{ij}^{\pm}(t) u_j^\alpha \equiv G_{\pm}^{\alpha}, \tag{9}$$

- We construct the effective mass

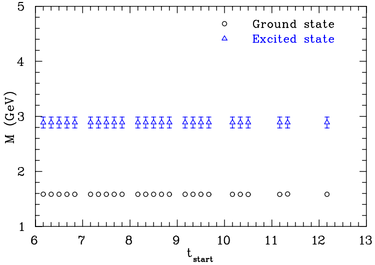
$$M_{\rm eff}^{\alpha}(t) = \ln \left(\frac{G_{\pm}^{\alpha}(t, \vec{0})}{G_{\pm}^{\alpha}(t+1, \vec{0})} \right). \tag{10}$$

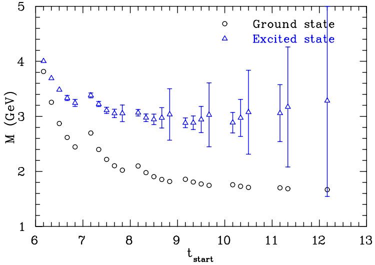
Simulation Details

- lattice volume $16^3 \times 32$
- lattice spacing 0.127 fm
- We use FLIC fermion action and quenched QCD
- Analysis is performed for 10 different pion masses: 797,729,641,541, 430,380,327,295,249,224 MeV.
- We use varieties of Gaussian smearing sweeps (number of sweeps 1,3,7,12,16,26,35,48,65)
- 2×2, 3×3, 4×4, 6×6 and 8×8 correlation matrices are analyzed
- To analyze data we use fitting robot

2x2, for point source, for $\chi_1\chi_2$

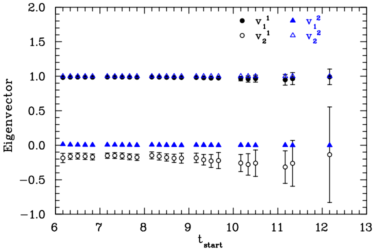
Projected Mass Vs Mass From Eigenvalue



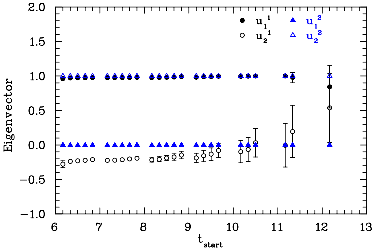


Eigenvectors - Point Source, for $\chi_1\chi_2$

Left Eigenvectors

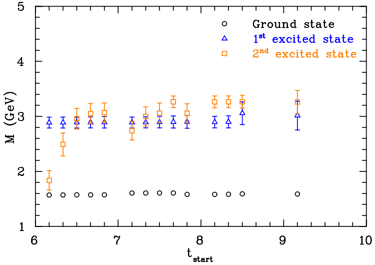


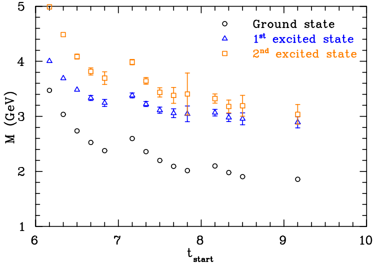
Right Eigenvectors



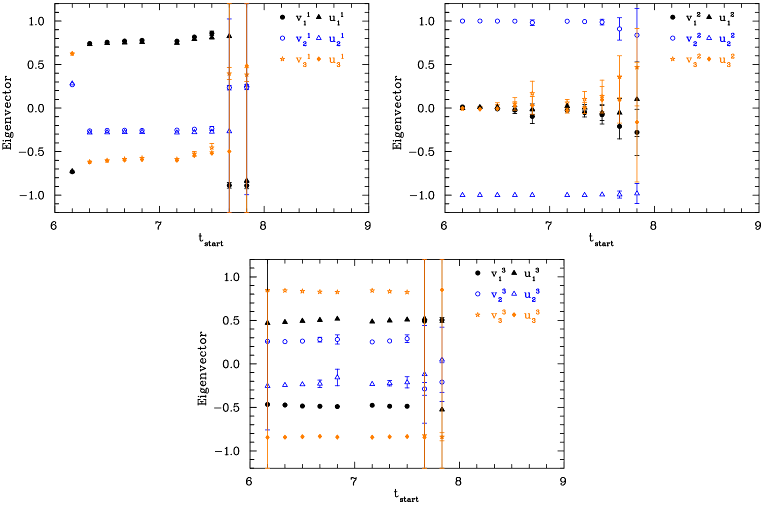
For 3×3, of $\chi_1\chi_2\chi_4$

Projected Mass Vs Mass From Eigenvalue





Eigenvectors - 3 $\times$ 3



Smearing

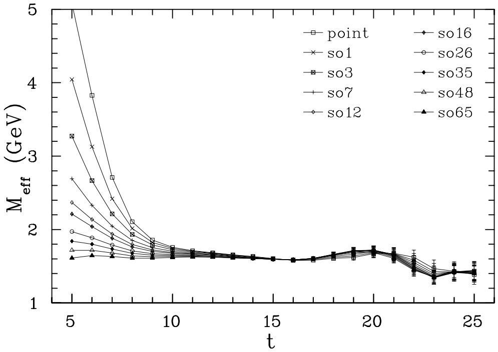
To create a comprehensive basis of interpolating fields we consider source smearing,

$$\psi_i(x,t) = \sum_{x'} F(x,x')\psi_{i-1}(x',t), \tag{11}$$

where,

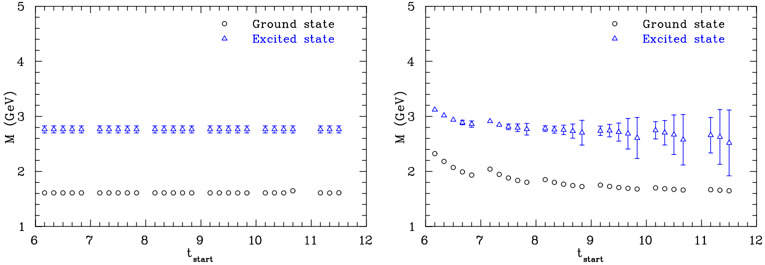
$$F(x,x') = (1-\alpha)\delta_{x,x'} + \frac{\alpha}{6}\sum_{\mu=1}^{3}[U_\mu(x)\delta_{x',x+\hat{\mu}} + U_\mu^\dagger(x-\hat{\mu})\delta_{x',x-\hat{\mu}}], \tag{12}$$

Fixing $\alpha = 0.7$, the procedure is repeated N_{sm} times.


point
so1
so3
so7
so12
so16
so26
so35
so48
so65
M_{eff} (GeV)
t
1
2
3
4
5
5
10
15
20
25

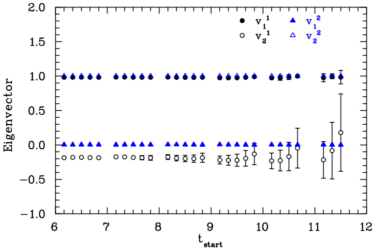
2x2, for smeared source, for $\chi_1\chi_2$

Projected Mass Vs Mass From Eigenvalue

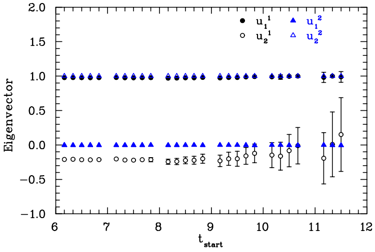


Eigenvectors - Smeared source, for $\chi_1\chi_2$

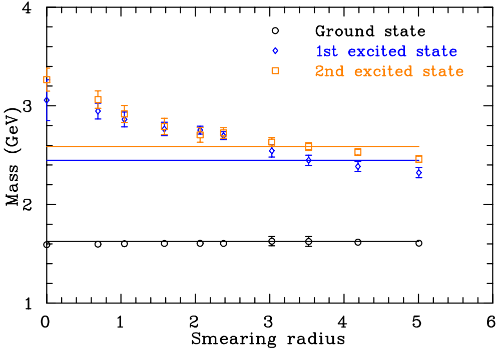
Left Eigenvectors



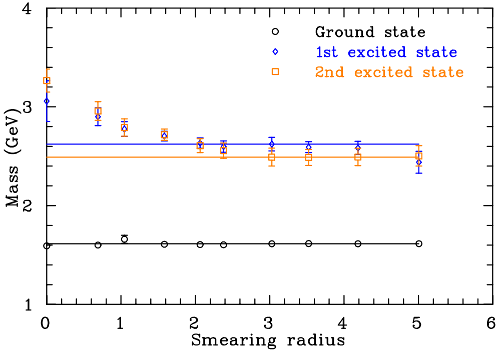
Right Eigenvectors



Smeared Source



Smeared-Smeared



M.S. Mahbub *et al.*, Phys. Rev. D **80**, 054507 (2009), [arXiv:hep-lat/0905.3616].

Roper Resonance

- *Roper resonance* (P_{11}) is the first positive parity excited state of the nucleon
- Observed in 1960's from πN scattering
- The state is puzzling due to its lower mass (1440 MeV) from its nearest negative parity (S_{11}) excited state (1535 MeV).
- In constituent quark model, Roper state is ≈100 MeV heavier than the S_{11} (1535 MeV) state.
- This state appeared too high in all previous attempts using variational method in lattice QCD.

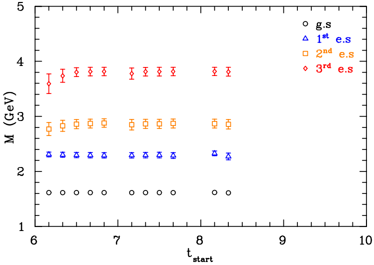
4x4 bases of $\chi_1\chi_1$

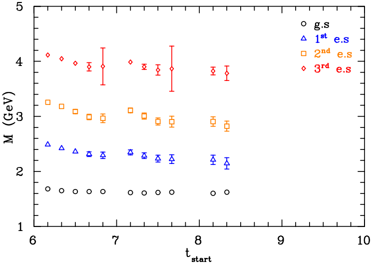
- We use smeared–smeared correlation functions
- Varieties of smearing sweeps

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	-	7	-	16	-	35	-
2	-	3	7	-	16	-	35	-
3	1	-	-	12	-	26	-	48
4	-	3	-	12	-	26	35	-
5	-	3	-	12	-	26	-	48
6	-	-	-	12	16	26	35	-
7	-	-	7	-	16	-	35	48

4x4, For 4^{th} basis (3, 12, 26, 35)

Projected Mass Vs Mass From Eigenvalue

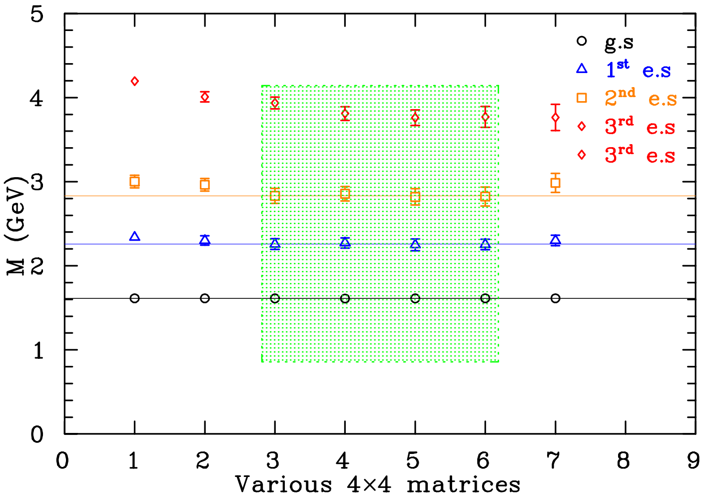


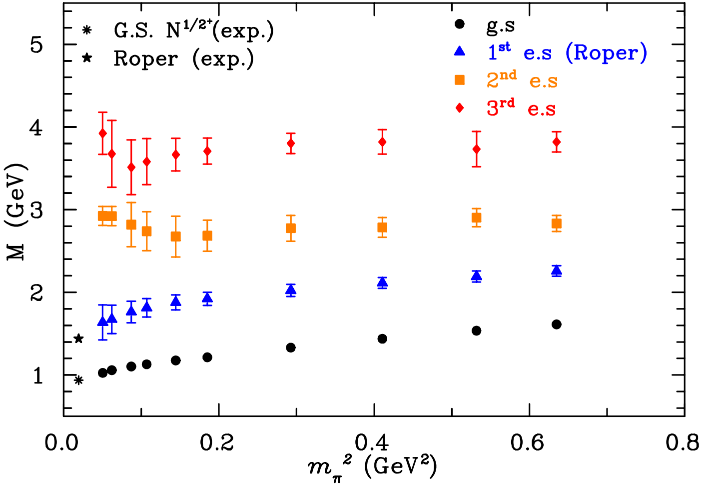


4x4 bases of $\chi_1\chi_1$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	-	7	-	16	-	35	-
2	-	3	7	-	16	-	35	-
3	1	-	-	12	-	26	-	48
4	-	3	-	12	-	26	35	-
5	-	3	-	12	-	26	-	48
6	-	-	-	12	16	26	35	-
7	-	-	7	-	16	-	35	48

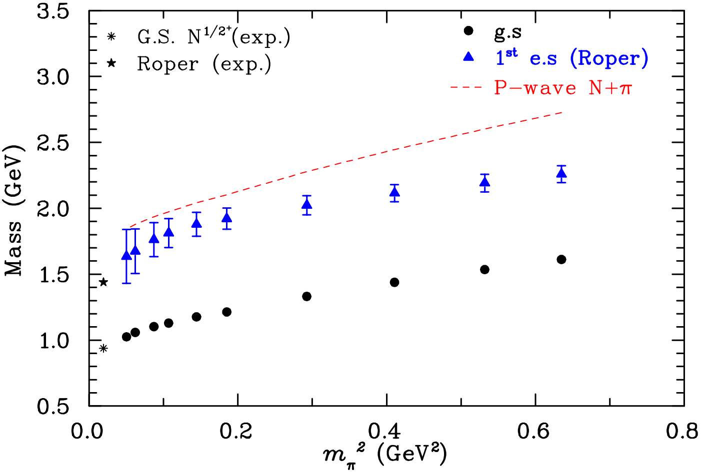
For all 4 × 4 bases




G.S. N1/2+(exp.)
Roper (exp.)
g.s
1st e.s (Roper)
2nd e.s
3rd e.s
M (GeV)
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
mπ2 (GeV2)

3rd basis (1,12,26,48)				4th basis (3,12,26,35)				5th basis (3,12,26,48)				6th basis (12,16,26,35)			
t_1	t_2	aM (Roper)	$\frac{\chi^2}{\mathrm{dof}}$	t_1	t_2	aM (Roper)	$\frac{\chi^2}{\mathrm{dof}}$	t_1	t_2	aM (Roper)	$\frac{\chi^2}{\mathrm{dof}}$	t_1	t_2	aM (Roper)	$\frac{\chi^2}{\mathrm{dof}}$
7	12	1.456(41)	0.58	7	12	1.465(39)	0.63	7	12	1.451(44)	0.51	7	12	1.454(40)	0.57
7	12	1.411(43)	0.55	7	12	1.419(41)	0.62	7	12	1.405(46)	0.48	7	12	1.417(39)	0.60
7	12	1.368(39)	0.54	7	12	1.361(45)	0.60	7	12	1.364(40)	0.53	7	11	1.363(42)	0.68
7	12	1.307(44)	0.57	7	11	1.298(51)	0.60	7	12	1.305(45)	0.57	7	10	1.308(46)	0.54
7	11	1.235(50)	0.43	7	11	1.245(51)	0.57	7	11	1.233(51)	0.37	7	11	1.244(52)	0.38
7	11	1.210(60)	0.42	7	11	1.211(55)	0.58	7	11	1.206(57)	0.38	7	11	1.220(60)	0.49
7	10	1.163(69)	0.60	7	11	1.165(67)	0.56	7	10	1.164(71)	0.53	7	10	1.184(75)	0.56
7	10	1.129(82)	0.61	7	10	1.127(81)	0.84	7	10	1.136(82)	0.58	7	10	1.155(85)	0.54
7	10	1.07(10)	0.56	7	10	1.06(10)	0.95	7	10	1.07(11)	0.68	7	10	1.11(11)	0.63
7	9	1.04(13)	0.85	7	10	1.01(12)	0.97	7	9	1.05(13)	0.79	7	9	1.10(13)	0.70

Roper from 4 × 4

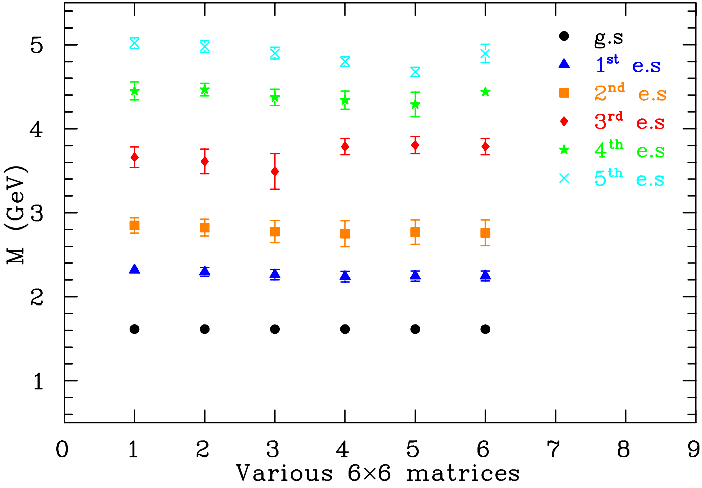


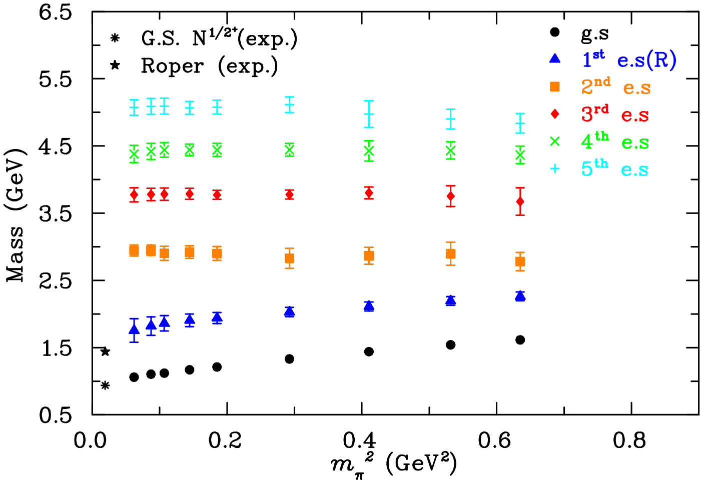
Mahbub *et al*., Phys. Lett. B **679**, 418 (2009), [arXiv:hep-lat/0906.5433].

6x6 bases of $\chi_1\chi_1$

Sweeps →	1	3	7	12	16	26	35	48
Basis No. ↓	Bases							
1	1	3	7	12	16	26	-	-
2	1	3	7	12	16	-	35	-
3	1	3	7	-	16	26	35	-
4	1	3	-	12	16	26	-	48
5	1	-	7	12	16	26	35	-
6	-	3	7	12	16	26	35	-

For all 6x6 bases

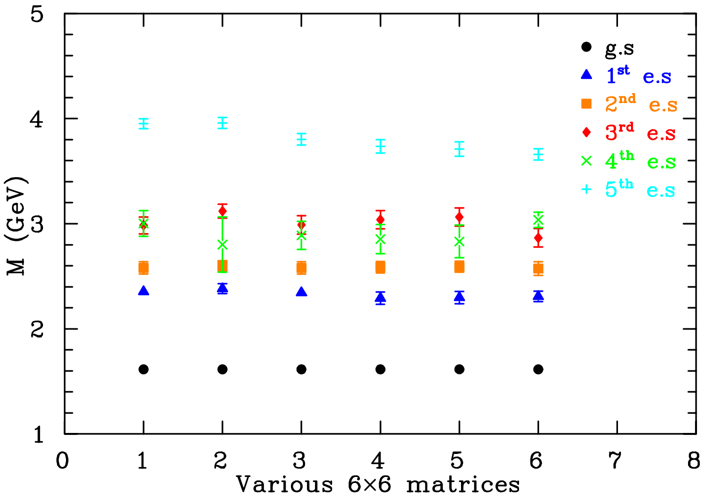


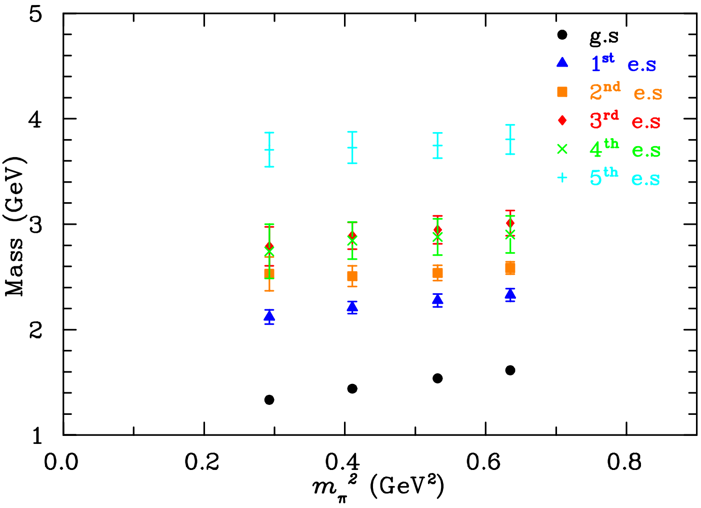

G.S. N1/2+(exp.)
Roper (exp.)
g.s
1st e.s(R)
2nd e.s
3rd e.s
4th e.s
5th e.s
Mass (GeV)
0.5
1.5
2.5
3.5
4.5
5.5
6.5
0.0
0.2
0.4
0.6
0.8
mπ2 (GeV2)

6x6 bases of $\chi_1\chi_2$

Sweeps →	1	3	7	12	16	26	35	48
Basis No. ↓	Bases							
1	1	-	-	-	16	-	-	48
2	-	3	-	12	-	26	-	-
3	-	3	-	-	16	-	-	48
4	-	-	7	-	16	-	35	-
5	-	-	-	12	16	26	-	-
6	-	-	-	-	16	26	35	-

For all 6x6 bases

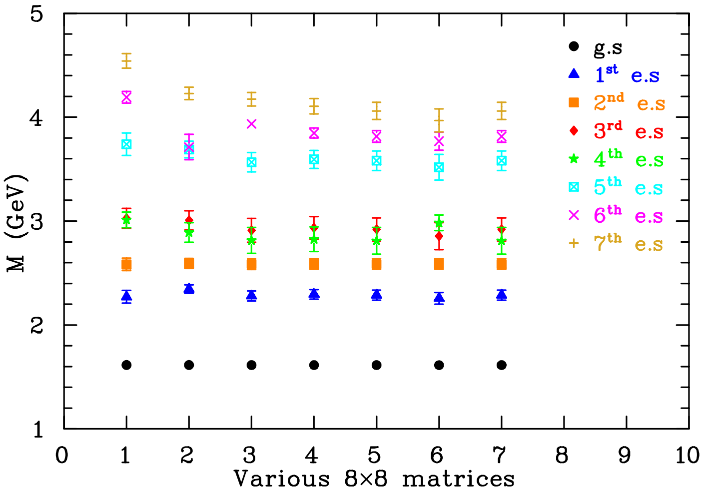


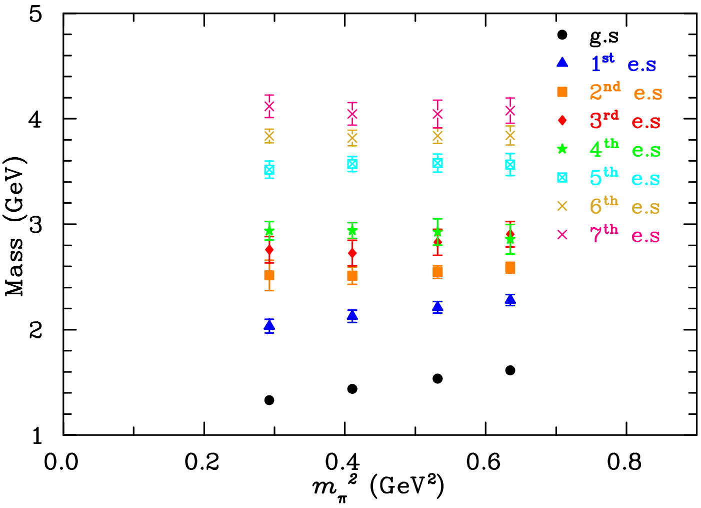

5
4
3
2
1
Mass (GeV)
0.0
0.2
0.4
0.6
0.8
m_π^2 (GeV^2)
g.s
1st e.s
2nd e.s
3rd e.s
4th e.s
5th e.s

8x8 bases of $\chi_1\chi_2$

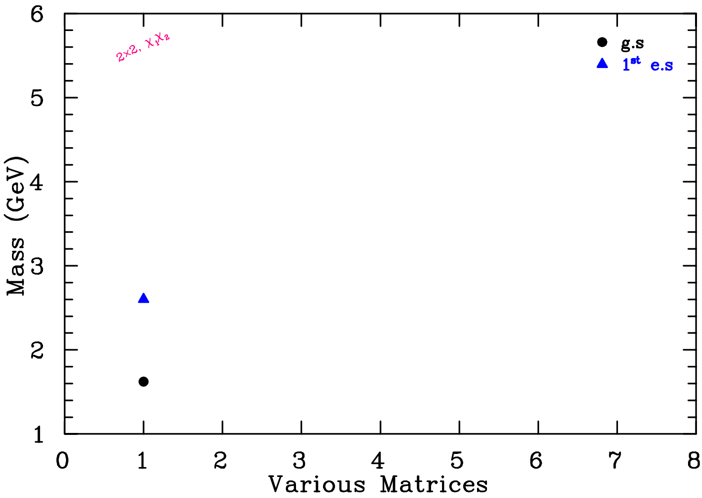
Sweeps →	1	3	7	12	16	26	35	48
Basis No. ↓	Bases							
1	1	-	7	-	16	-	35	-
2	-	-	7	12	16	26	-	-
3	-	3	-	12	-	26	-	48
4	-	-	7	12	-	26	35	-
5	-	-	7	-	16	26	35	-
6	-	-	7	-	16	-	35	48
7	-	-	-	12	16	26	35	-

For all 8x8 bases

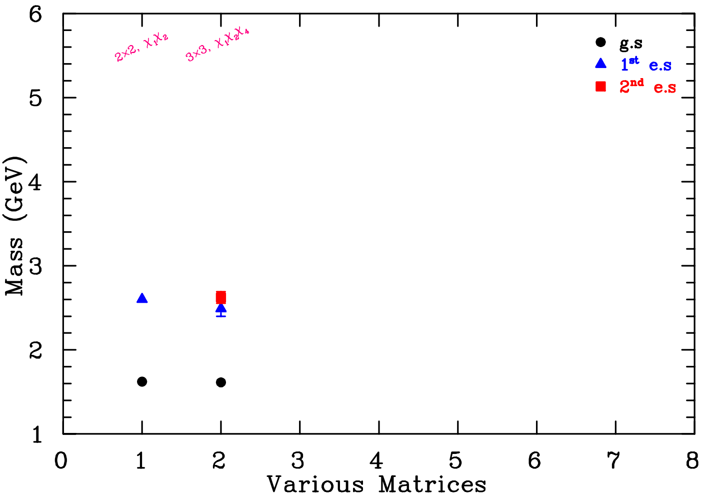



5
4
3
2
1
Mass (GeV)
0.0
0.2
0.4
0.6
0.8
m_π^2 (GeV²)
g.s
1st e.s
2nd e.s
3rd e.s
4th e.s
5th e.s
6th e.s
7th e.s

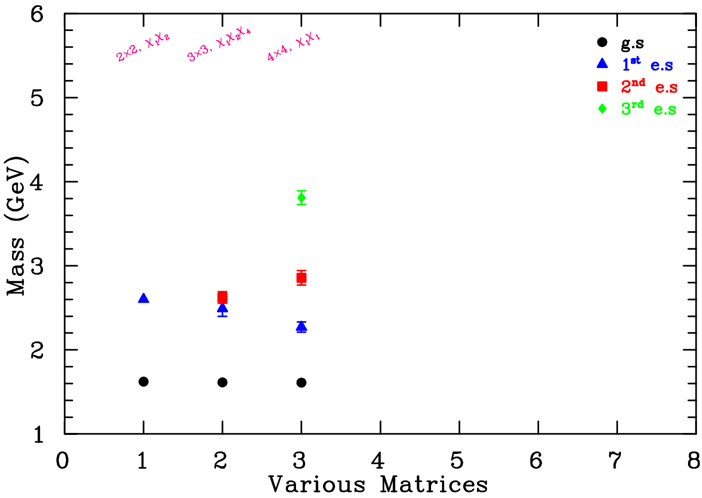
Story of excited states



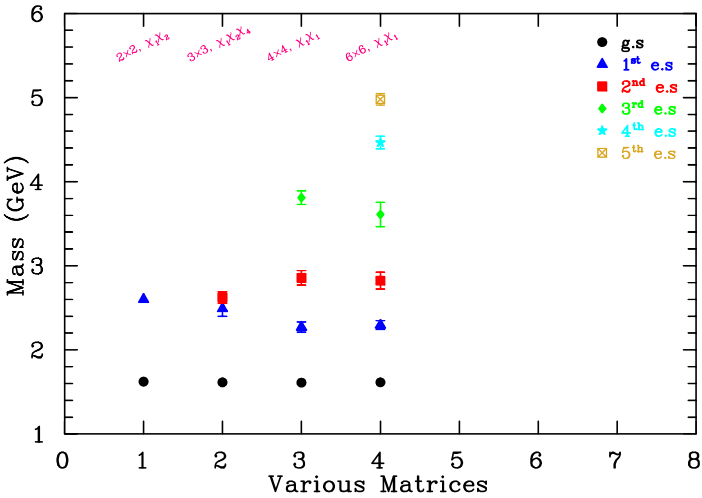
Story of excited states



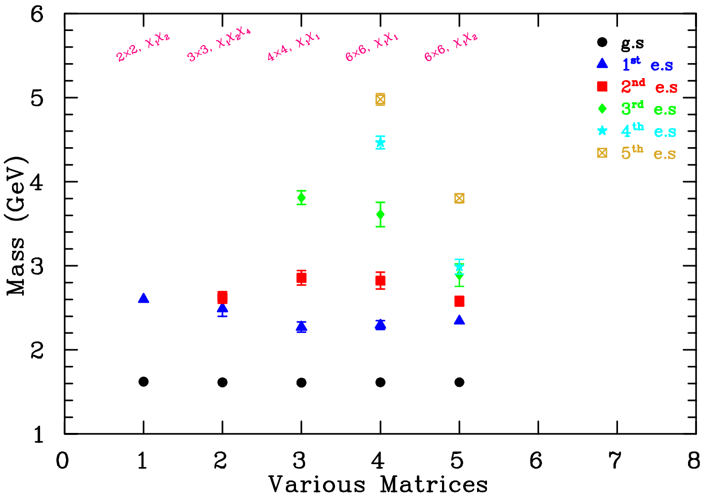
Story of excited states



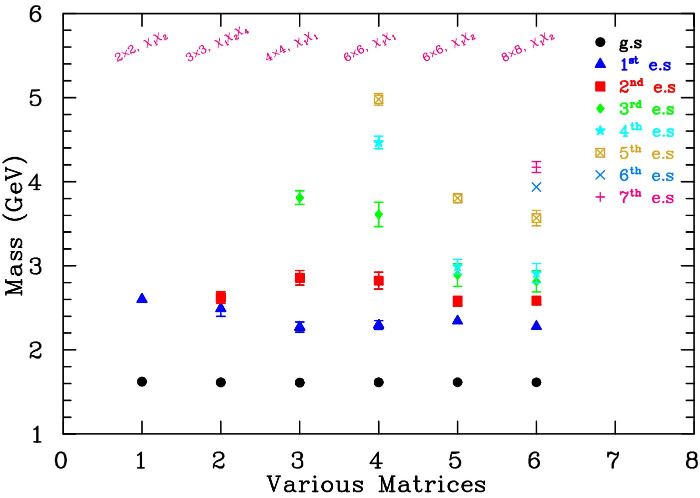
Story of excited states



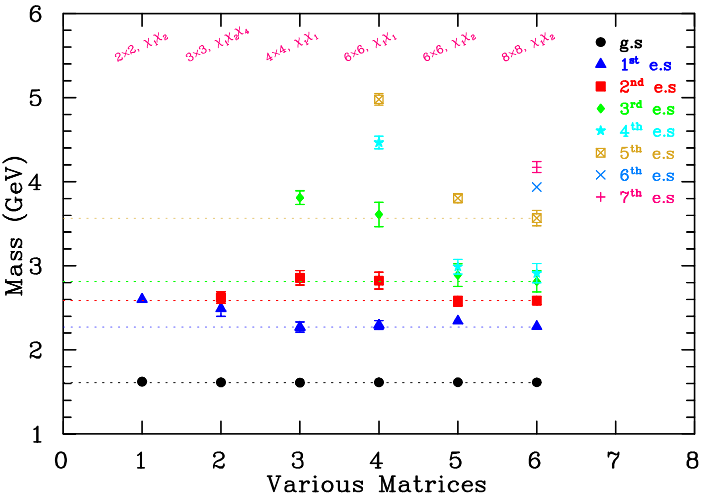
Story of excited states



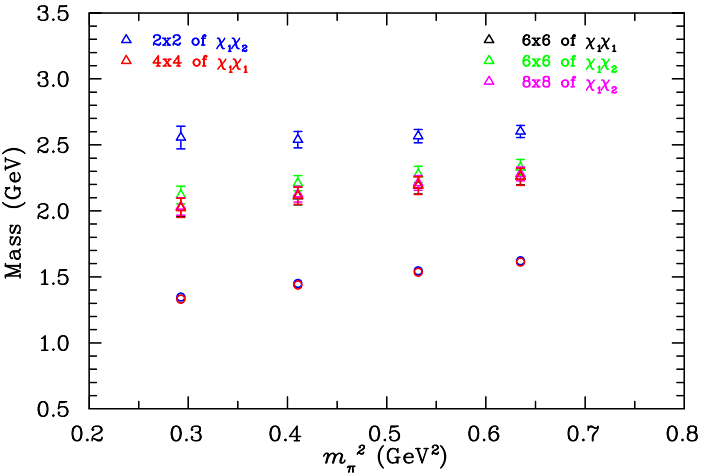
Story of excited states



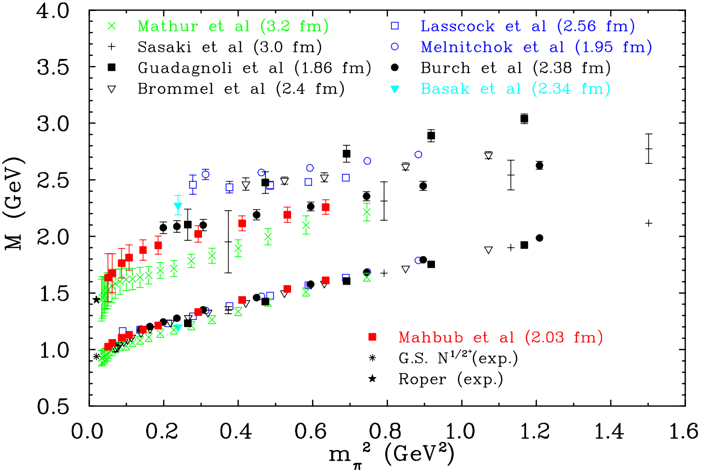
Story of excited states



2 states for 2x2,4x4,6x6,8x8



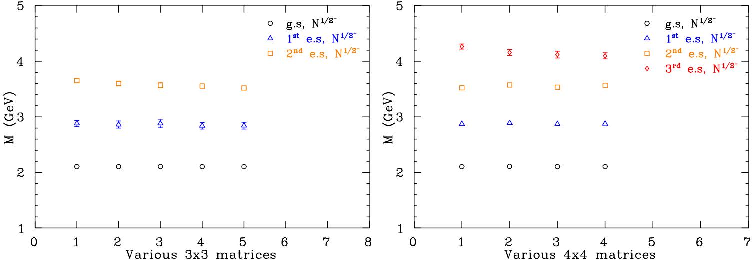
Roper state: Compilation of existing results



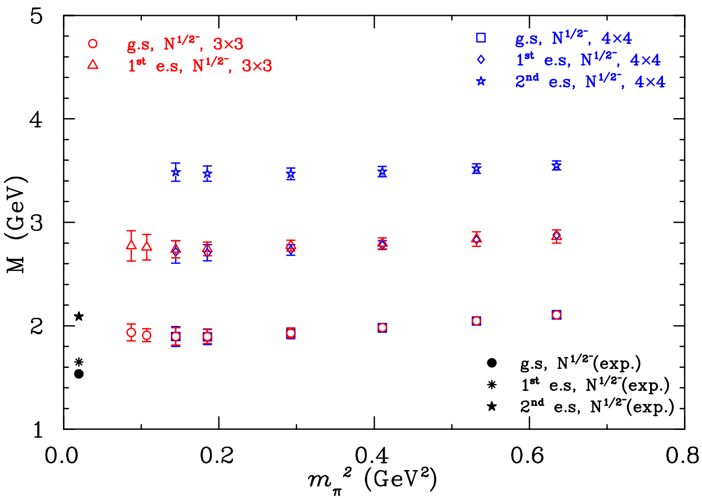
- Level crossing between Roper (1440 MeV) P_{11} and $N^{\frac{1}{2}^-}$ (1535 MeV) S_{11} states.

Projected Mass

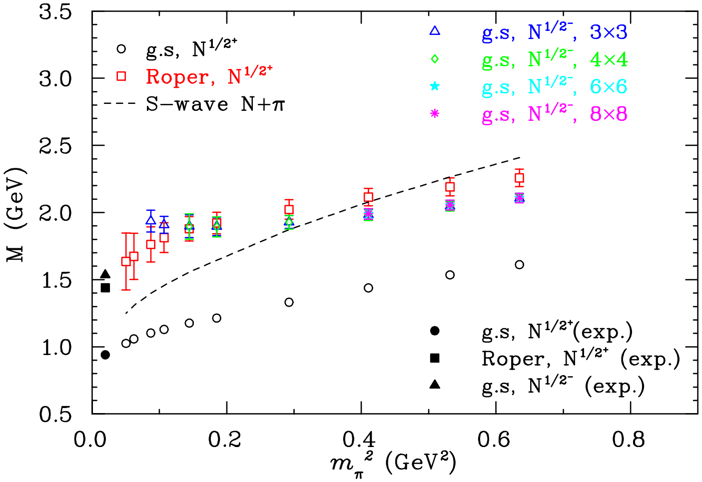
3×3 Vs 4×4



3 × 3 and 4 × 4 results



Roper (1440 MeV) and $N\frac{1}{2}^{-}$ (1535 MeV) states

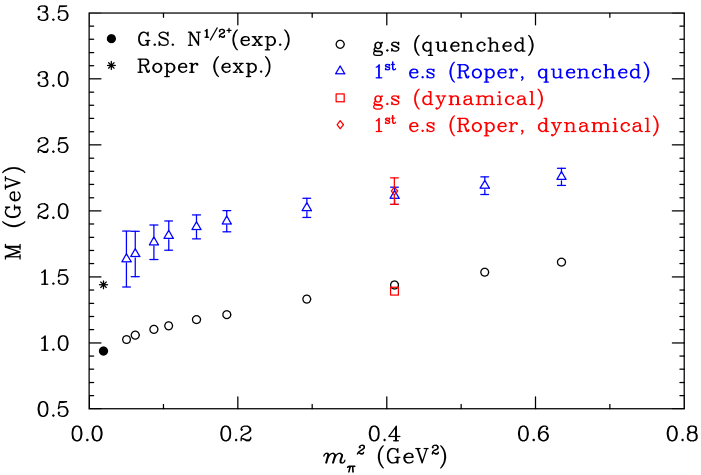


Roper in dynamical QCD: Simulation details

- Lattice volume: $20^3 \times 40$
- $a =$0.125 fm
- 200 configurations, nf = 2, pion mass = 634 MeV.
- FLIC fermion action

Collaborators: $\cdots$, Peter Moran, $\cdots$

Roper in dynamical QCD



Conclusion

- Various dimensions of the correlation matrices have been analyzed.
- Varieties of smearing sweeps have been used in constructing correlation matrices.
- We observed smearing dependency of the excited states given that the ground state is independent on smearing.
- A low-lying Roper state has been identified for the first time using variational method.
- For consistency and reliability check we considered several 4×4, 6×6, 8×8 matrices.

Conclusion

- We have shown how excited states are split up with the dimension of the correlation matrices.
- We have shown the importance of using smeared-smeared correlation functions and larger correlation matrices for the reliable extraction of excited states mass.
- A level crossing between the Roper (1440 MeV) and $N\frac{1}{2}^{-}$ (1535 MeV) states has been observed for the first time in variational approach.
- The Roper results in quenched and dynamical QCD are in very good agreement.

Thanks