QCD Phase Transition and the Distribution of Low-lying Eigenvalues with $2+1$ Flavors of DWF

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LATTICE 2012,
June 24-29, 2012

Outline

Introduction

QCD phase transition and the symmetries associated with it has been an intriguing topic for a long time both experimentally and theoretically.

- Chiral symmetry $SU(2)_L \times SU(2)_R$ is spontaneously broken to $SU(2)_V$ below critical temperature(T_c).
- Above T_c, chiral symmety is restored and chiral condensate will vanish in the chiral limit.
- The fate of $U(1)_A$ anomolous symmetry is yet unclear. Whether it is effectively restored and when does this happen remain open questions.
- Domain wall fermions serve as an optimum choice:
  - Exact chiral symmetry even with finite lattice spacing
  - $U(1)_A$ not broken by lattice artifacts

Control of Chiral Symmetry with DWF

Residual chiral symmetry breaking is characterized by an additive renormorlization to quark mass. $\widetilde{m} = m_{\mathrm{input}} + m_{\mathrm{res}}$

$$m_{\mathrm{res}}(L_s) = c_1 \frac{e^{-\lambda_c L_s}}{L_s} + c_2 \frac{1}{L_s} \tag{1}$$

- First term: standard 5-D states, exponentially suppressed.
- Second term: localized states created by changing topology.
- Dislocation suppressing determinant ratio (DSDR)

  [Vranas 00,Fukaya*et.al*.06]

$$\mathcal{W}(M_0, \epsilon_b, \epsilon_f) = \frac{\det\left[D_W^\dagger(-M_0)D_W(-M_0) + \epsilon_f^2\right]}{\det\left[D_W^\dagger(-M_0)D_W(-M_0) + \epsilon_b^2\right]} \tag{2}$$

- Suppress the configurations with near-zero modes while allowing enough topological tunneling.
- Even smaller residual mass with Möbius DWF (Hantao Yin)

Simulation Details

- Natural extension of our previous $16^3 \times 8$ lattice with a larger volume of $32^3 \times 8$
- DSDR + Iwasaki gauge action
- $2 + 1$ flavors of DWF
- 7 temperatures in the range $T = 139 - 195$MeV
- Input quark masses adjusted so that $m_\pi \approx 200$MeV
- $\tilde{m}_l / \tilde{m}_s = 0.088$ so that m_K almost physical

Ensembles

T (MeV)	β	L_s	m_{res}	m_l	m_s	N_{traj}
139	1.633	48	0.00588(39)	−0.00136	0.0519	1209
149	1.671	32	0.00643(9)	−0.00189	0.0464	2074
159	1.707	32	0.00377(11)	0.000551	0.0449	2943
168	1.740	32	0.00209(9)	0.00175	0.0427	2519
177	1.771	32	0.00132(6)	0.00232	0.0403	2708
186	1.801	32	0.00076(3)	0.00258	0.0379	2400
195	1.829	32	0.00047(1)	0.00265	0.0357	3015

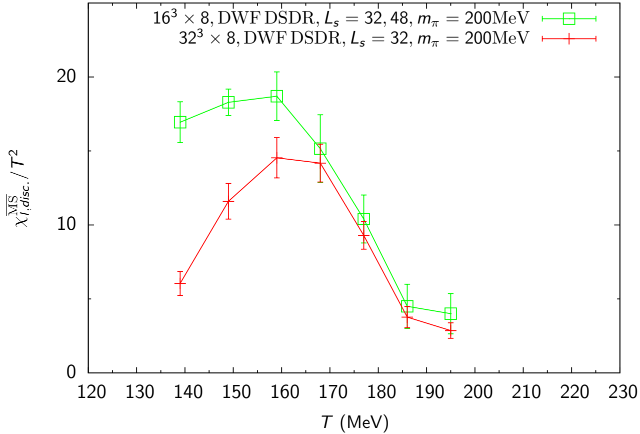
- Negative input quark mass tested with $16^3 \times 8$. Safer with larger volume.
- Residual masses are from $16^3 \times 8$ ensembles.

Chiral Observables

T	β	$\langle\bar{\psi}\psi\rangle_l / T^3$	$\langle\bar{\psi}\psi\rangle_s / T^3$	$\Delta\bar{\psi}\psi / T^3$	$\chi_{l,disc.} / T^2$	$\chi^{\overline{\rm MS}}_{l,disc.} / T^2$
139	1.633	11.30(7)	37.47(3)	12.29(7)	13(2)	6.0(8)
149	1.671	6.93(6)	36.45(2)	8.41(6)	26(3)	11(1)
159	1.707	5.80(6)	33.73(2)	5.39(6)	31(3)	15(1)
168	1.740	4.16(8)	30.73(3)	2.90(8)	33(3)	14(1)
177	1.771	3.18(6)	27.95(2)	1.57(6)	22(2)	9.3(9)
186	1.801	2.40(4)	25.37(2)	0.67(4)	9(2)	3.8(7)
195	1.829	2.15(3)	23.21(2)	0.43(3)	7(1)	2.9(5)

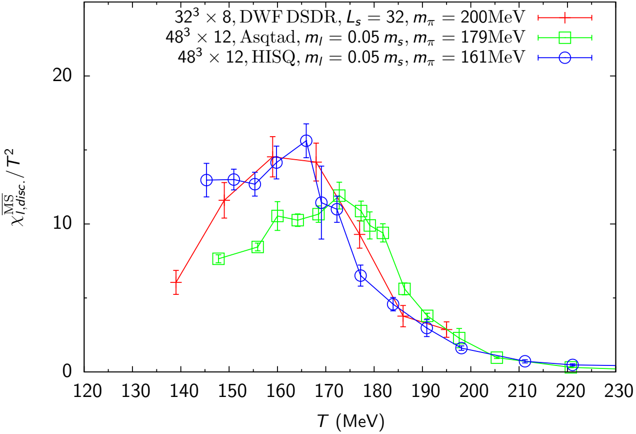
- Subtracted chiral condensate is defined as $\Delta_{l,s} = \langle\bar{\psi}\psi\rangle_l - m_l/m_s\langle\bar{\psi}\psi\rangle_s$ so that the divergence proportional to the quark mass is removed.
- In the last column, the disconnected susceptibilies are renormalized to $\overline{\rm MS}(\mu = 2{\rm GeV})$ so that a comparison with other schemes are possible (*e.g.* staggered).

Disconnected Susceptibilies



- Finite volume effect: below T_c, $\chi_{disc.}$ decreases significantly when volume gets bigger. [J. Braun, B. Klein, P. Piasecki 2011]
- $T_c \in (160\mathrm{MeV}, 170\mathrm{MeV})$?

Disconnected Susceptibilies



- HISQ and Asqtad disagree, scaling errors?
- $N_\tau = 12,\ m_{\rm Goldstone} = 161$MeV HISQ
  $\stackrel{?}{=} N_\tau = 8,\ m_\pi = 200$MeV DWF

Dirac Spectrum and Symmetries

The low-lying eigenmodes of Dirac operator are directly related to the symmetries and phase transition of QCD.

- Chiral Condensate

$$-\langle\bar{\psi}\psi\rangle_q = \int \mathrm{d}\lambda\,\rho(\lambda)\frac{2m_q}{m_q^2+\lambda^2}\,, \qquad q = l, s \tag{3}$$

- $U(1)_A$ symmetry

$$\Delta_{\pi-\delta} \equiv \chi_\pi - \chi_\delta = \int \mathrm{d}\lambda\ \rho(\lambda)\frac{4m_l^2}{\left(m_l^2+\lambda^2\right)^2}\,. \tag{4}$$

- Above T_c: $\rho(\lambda \to 0) \sim \delta(\lambda)$? λ? m?

Renormalization of Eigenvalue Spectrum

- Lowest 100 eigenvalues (Λ) of the hermitian version of DWF Dirac operator are calculated with the unitary light quark mass, $D_H \equiv R_5 \gamma_5 D_{\text{DWF}}$.
- R_5: the reflection operator in the fifth dimension.
- Renormalized with Giusti and Lüscher's approach and non-perturbative renormalization [HotQCD hep-lat/1205.3535]
- $\Lambda = \sqrt{\lambda^2 + \widetilde{m}^2}$. Only after proper renormalization can we remove the mass from the eigenvalue Λ.

T (MeV)	β	N_{cfg}
159	1.707	99
168	1.740	38
177	1.771	68
186	1.801	104
195	1.829	76

Eigenvalue Distrubition at 159 and 168 MeV

Figure: 159MeV

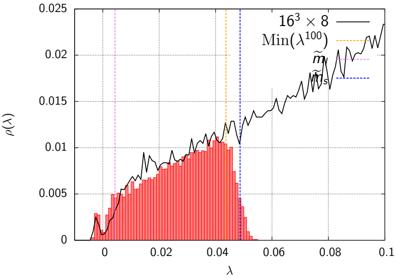
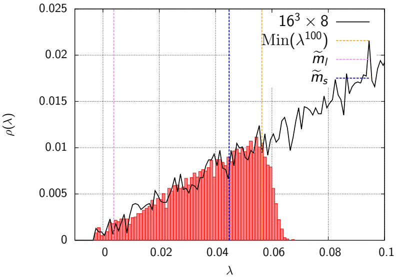


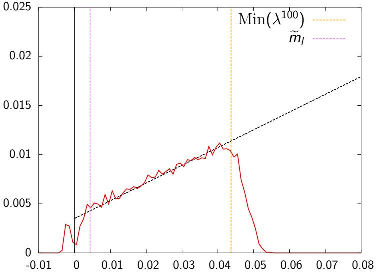
Figure: 168MeV



- Larger volume contains more modes and lowest 100 eigenvalues are more condensed.
- Distribution from both volumes agree.
- $\widetilde{m}_l$ is not well defined on a single configuration. When the mass is removed from the eigenvalue, some "unphysical" modes will appear. They are plotted as $-\sqrt{\left|\Lambda^2 - \widetilde{m}_l^2\right|}$

Linear Fit for Eigenvalue Distrubition

Figure: 159MeV



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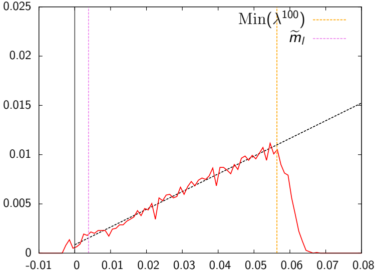
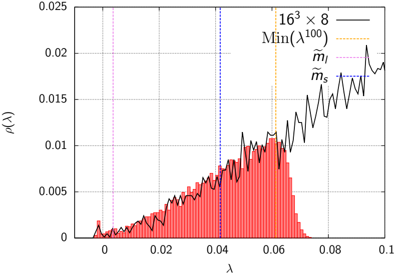
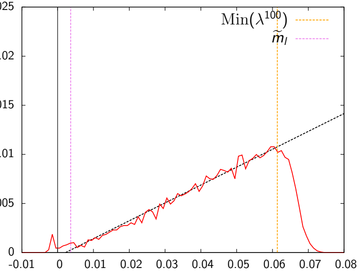


- With less fluctuation of larger volume, one can recognize that linear relation describes the distribution quite well.
- At 159MeV, the intercept is positive.
- At 168MeV, the intercept decreases dramatically.

EIGENVALUE DISTRUBITION AT 177 MEV





- Intercept has become 0.
- Assume $\rho(\lambda)$ is composed of two parts, $\rho(\lambda) = c_1\delta(\lambda) + c_2\lambda$
- c_1: number of "zero modes", renormalization invariant
- c_2: slope of the distribution

$$\chi_\pi - \chi_\delta = \int_0^\infty \mathrm{d}\lambda \frac{4\widetilde{m}^2\rho(\lambda)}{(\widetilde{m}^2 + \lambda^2)^2}$$

$$\frac{\chi_\pi - \chi_\delta}{T^2} = 48.72{+}9.70 = 36(14)(\text{Correlator measurement of } 16^3{\times}8)$$

- Axial symmetry breaking: $\sim$ 83% from near zero modes; $\sim$ 17% from linear part

Eigenvalue Distrubition at 186 and 195MeV

Figure: 186MeV

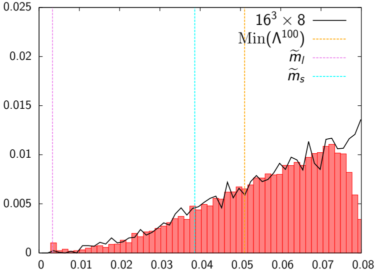
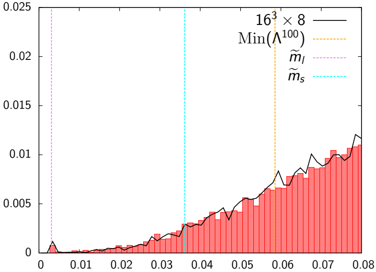


Figure: 195MeV



- Here the eigenvalue distribution without removing the mass.
- A small peak around $\widetilde{m}_l$
  - Exact zero modes: $\# \sim \sqrt{V}$
  - Dilute instanton gas model: $\# \sim V$

# of near zero mode/conf.

T	16^3	32^3
177	0.15(2)	1.6(2)
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Eigenvalue Distrubition at 186 and 195MeV

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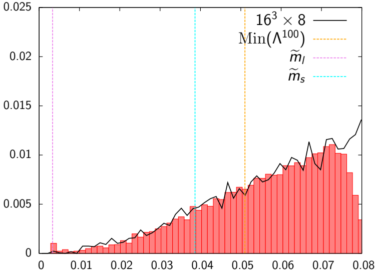
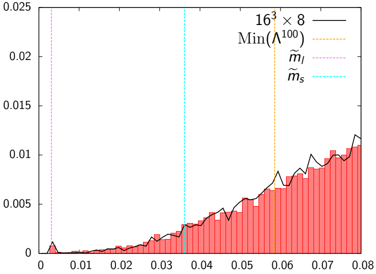


Figure: 195MeV



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More about Near Zero Modes

Table: Distribution of near zero modes at 180 MeV.

N_+	0	1	2	3	4	5
$N_0 = 1$	10	6	–	–	–	–
$N_0 = 2$	6	8	7	–	–	–
$N_0 = 3$	0	6	3	0	–	–
$N_0 = 4$	0	1	1	1	0	–
$N_0 = 5$	0	1	2	0	1	0

- N_0: # of total near zero modes.
- N_+: # of total near zero modes with positive chirality.
- For each configuration, if the near zero modes from:

  Topology : exact zero mode, all have the same chirality
  Dilute instanton: + and − chirality equally possible

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Fit for Eigenvalue Distrubition at 186 and 195MeV

Figure: 186MeV

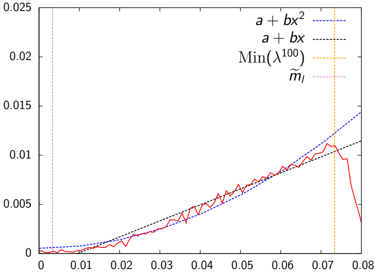
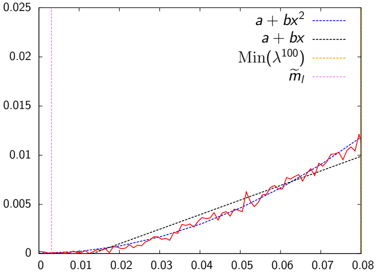


Figure: 195MeV



- Now again with the mass removed, we try to fit the eigenvalue distributions into linear and quadratic forms.
- Linear function works poorly yet a quadratic form gives a satisfying description.

Concluding Remarks and Outlook

- $32^3 \times 8$ ensembles give satisfying results. And certain finite volume effects are observed.
- Dirac eigenvalue distributions contain rich physics, providing a unique perspective to many quantities associated with QCD phase transitions.
- We are planning to extend our study to a larger volume ($64^3 \times 8$) and with physical pion mass.