Finite density phase transition of 3 flavor QCD

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in collaboration with

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Finite density study

Overview

- rich phase structure in $T-\mu$ plane
- heavy-ion collision experiments, the early universe, neutron star
- on LQCD study, sign problem, many studies by KS fermion
- on LQCD study, to find the critical endpoint is as hard as looking for a small island in the ocean on a boat
- on LQCD study, many simulation parameters ($L_s, L_t, \beta, am_u, am_d, am_s, a\mu$, we are interested in physics in the limits of $V \to \infty$ and $a \to 0$ at the physical point)
- on LQCD study, so far lattice size is small, lattice spacing is very coarse

Motivation

- we test phase quenched generation + reweighting with exact phase method on larger lattice
- finally we want to determine the critical endpoint precisely by pushing L_s, β, κ with Wilson type fermion

Simulation at finite chemical potential

Integrating out the Grassmann variables in the partition function,

$$Z = \int DUe^{-S_g}(\det D)^{N_f}$$

$$D = 1 + \frac{i}{2}\kappa\, c_{\mathrm{SW}}\, \sigma_{\mu\nu} F_{\mu\nu} \delta_{y,x} - \kappa\Big[\sum_{i=1}^{3}(T_i^+ + T_i^-) + e^{a\mu}T_4^+ + e^{-a\mu}T_4^-\Big]$$

$$T_\mu^\pm = (1 \mp \gamma_\mu) U_\pm \delta_{y,x\pm\mu}$$

probabilistic interpretation is necessary for Monte Carlo simulations,

$$\det D(\mu) \in \mathbb{C} \quad \text{for} \quad \mu \in \mathbb{R}$$

Proposed method

- Taylor expansion method: Allton et al. (2002)
- Analytic continuation Imaginary chemical potential: de Forcrand & Philipsen (2002)
- Reweighting: Fodor & Katz (2001)
- Canonical approach: de Forcrand (2006), Kentucky (2010)
- Density of state method: Fodor et al. (2007)
- Histgram method: WHOT-QCD (2011)

Phase quenched simulation and reweighting

Our approach

$$Z = \int DUe^{-S_g}(\det D(\mu))^{N_f} = \int DUe^{-S_g}|\det D(\mu)|^{N_f} e^{iN_f\theta}$$
$$= \int DUe^{-S_g} \det D^{\dagger}(\mu)^{N_f/2} \det D(\mu)^{N_f/2} e^{iN_f\theta}$$
$$= \int DUe^{-S_g}[\det D^{\dagger}(\mu) D(\mu)]^{N_f/2} e^{iN_f\theta}$$

- generate gauge field configurations with weight:

phase quenched action is invariant under $\mathcal{T}$ transformation

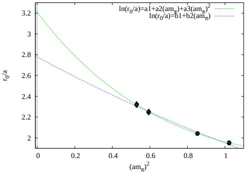
$$e^{-S_g + \ln \det[D^{\dagger}(\mu) D(\mu)]^{N_f/2}}$$

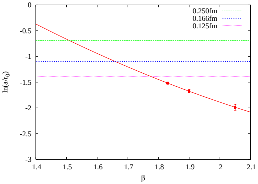
- reweight with exact phase: $\theta(U) = -\theta(\mathcal{T}U)$
  - phase calculation by reduction technique [Danzer & Gabringer (2008)]

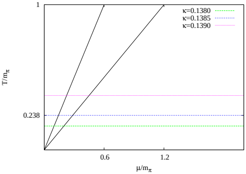
$$\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O} e^{iN_f\theta} \rangle}{\langle \cos(N_f\theta) \rangle}$$

Simulations

- $L_s = 10, 8, 6$ (statistics: 2000, 3000, 8000), $L_t = 6$
- $\beta = 1.77$ (Iwasaki)
- $\kappa = 0.1371$ to $\kappa = 0.13975$ (np clover)
- $a\mu = 0.1$ and $a\mu = 0.2$
- algorithm: mass preconditioning for 2 of 3 flavors and RHMC for 1 flavor, multiple time scale, Omelyan integrator
- computer: K & GPU-cluster@AICS, HA-PACS@Tsukuba, FX10@Tokyo
- physical scale $a^{-1} \sim 1.28$ GeV, $m_\pi \sim 400 - 1200$ MeV (very preliminary)





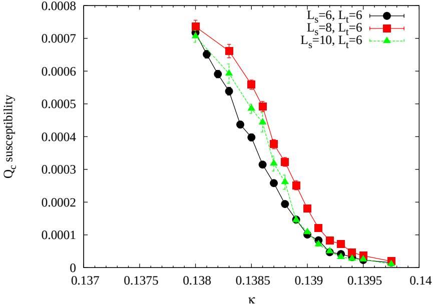


m_π is too heavy

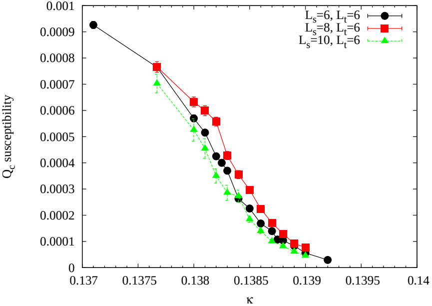
no finite size correction

Topological charge susceptibility

$a\mu = 0.1$



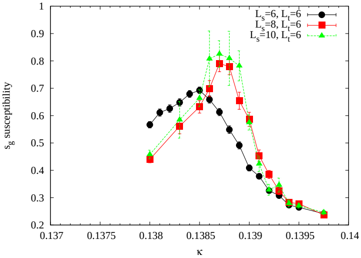
$a\mu = 0.2$

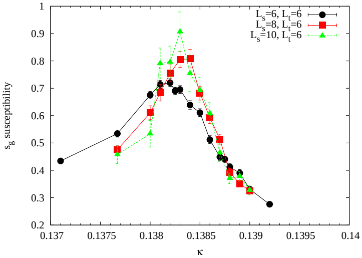


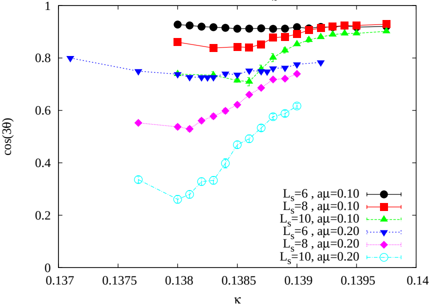
- topological charge is measured with the cooling method
- typical behavior around transition point is observed at $\mu \neq 0$ as well as at $\mu = 0$
- phase changes at larger quark masses by increasing μ
- L_s dependence is small

Reweighting factor near transition

ex) Action density $a\mu = 0.1$ $a\mu = 0.2$







- all reweighting factors are finite at all simulating points
- smaller reweighting factor for larger lattice size, phase increases as L_s^3
- reweighting factor drops near transition

Higher moments

i-th derivative of weight with respect to control parameter c:

$$Q_i = \frac{\partial^i}{\partial c^i} e^{-S_g} (\det D)^{N_f}$$

- Variance

$$V = \frac{\partial^2 \ln Z}{\partial c^2} = \langle Q_2 \rangle - \langle Q_1 \rangle^2$$

- Skewness

e.g. right-skewed $\rightarrow S > 0$, left-skewed $\rightarrow S < 0$

$$S = \frac{1}{V^{\frac{3}{2}}} \frac{\partial^3 \ln Z}{\partial c^3} = \frac{\langle Q_3 \rangle - 3\langle Q_2 \rangle \langle Q_1 \rangle + 2\langle Q_1 \rangle^3}{V^{\frac{3}{2}}}$$

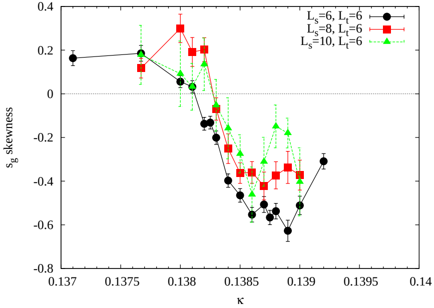
- Kurtosis

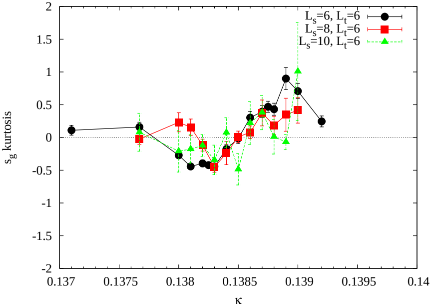
e.g. Gaussian $\rightarrow K = 0$, uniform $\rightarrow K = -1$, 2δ func. $\rightarrow K = -2$

$$K = \frac{1}{V^2} \frac{\partial^4 \ln Z}{\partial c^4} = \frac{\langle Q_4 \rangle - 4\langle Q_3 \rangle \langle Q_1 \rangle - 3\langle Q_2 \rangle^2 + 12\langle Q_2 \rangle \langle Q_1 \rangle^2 - 6\langle Q_1 \rangle^4}{V^2}$$
$$= B_4 - 3$$

Skewness and kurtosis

of action density at $a\mu = 0.2$

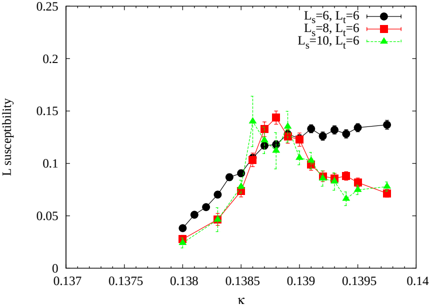




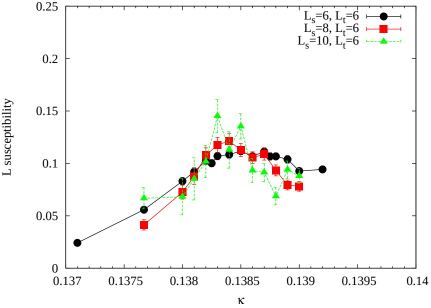
- typical $\mathcal{N}$-shape skewness and $\mathcal{M}$-shape kurtosis appear around transition
- 1^{st} order phase transition may be close

Polyakov loop susceptibility

$a\mu = 0.1$

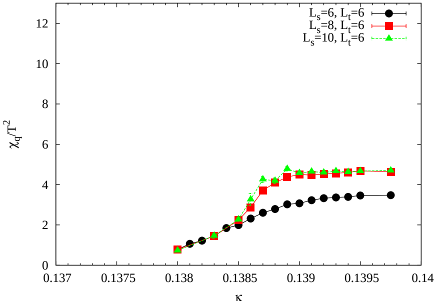


$a\mu = 0.2$

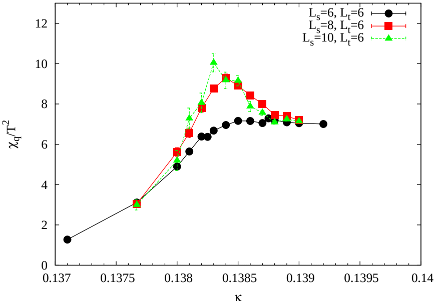


Normalized quark number susceptibility

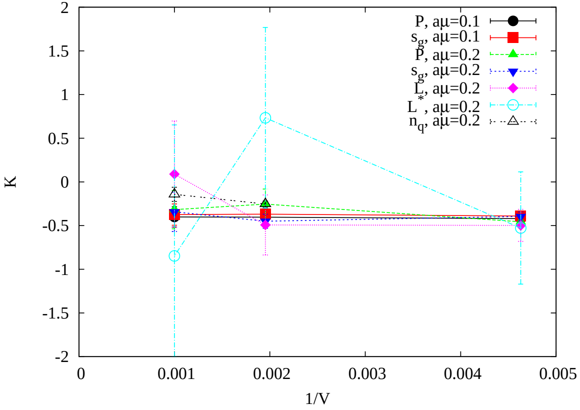
$a\mu = 0.1$



$a\mu = 0.2$



Order of transition



- $K \sim -0.5$, small V dependence up to $L_s = 10$
- K is approaching to 0 for some observables as increasing V

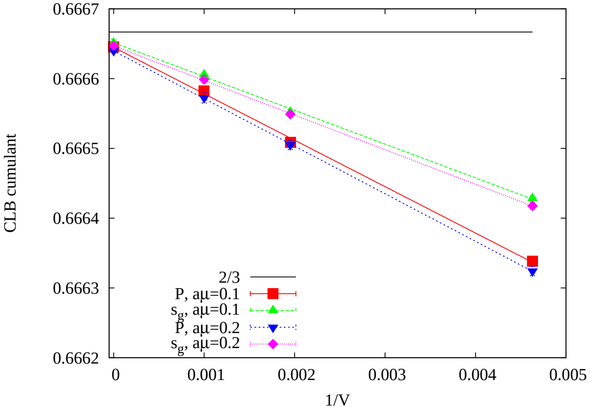
Summary

- we reported preliminary results on finite density phase transition of 3 flavor QCD at $\beta = 1.77$, $L_t = 6$ $a\mu = 0.1$ and 0.2
- in finite size study up to $L_s = 10$, no clear sign of 1^{st} or 2^{nd} order phase transition for $T \sim 200$ MeV $m_\pi \sim 900$ MeV, more statistics/larger volume?
- probably, better/easier to decrease T and m_π to find 1^{st} order phase transition
- future direction: decreasing $T(L_t = 8, 10...)$, we expect
  - m_π^{crit} decreases (hopefully ~ 400 MeV)
  - 1^{st} order phase transition becomes closer because of smaller m_π
  - phase transition occur at smaller $a\mu$
  - even at smaller m_π phase could be controllable by larger L_t

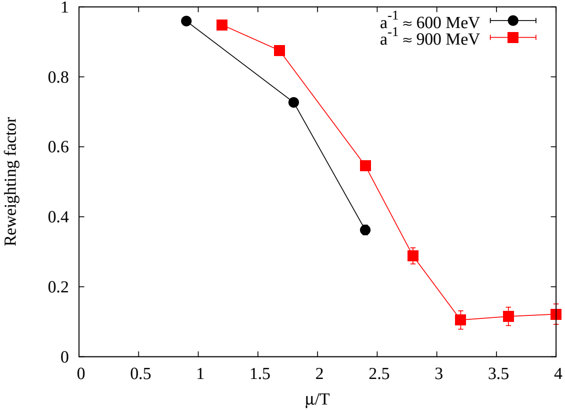
$$|\theta| \leq 12 L_s^3 (2\kappa)^{L_t} \sinh(a\mu L_t)$$

Takeda, Kuramashi, Ukawa (2011)

backup slides

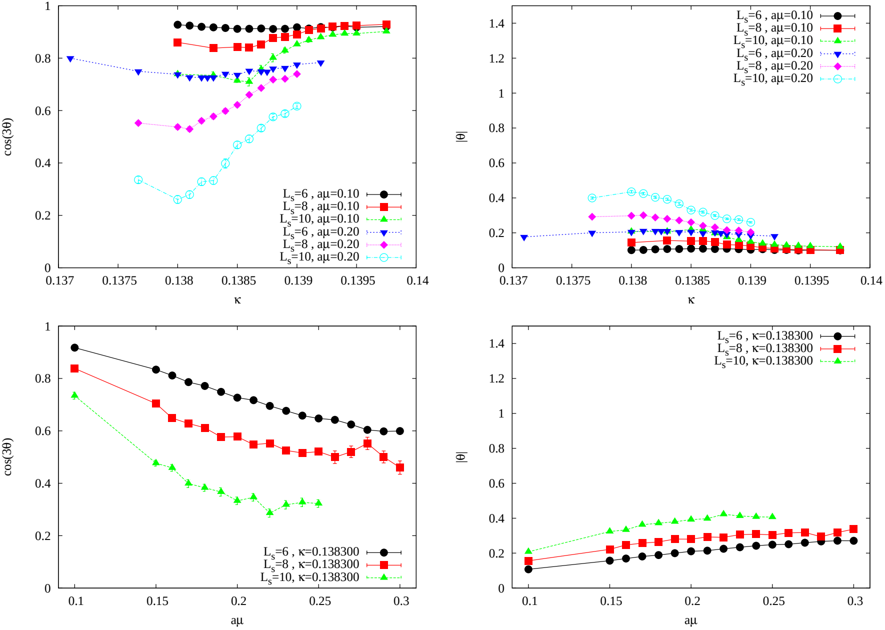


reweighting factor at different lattice spacing



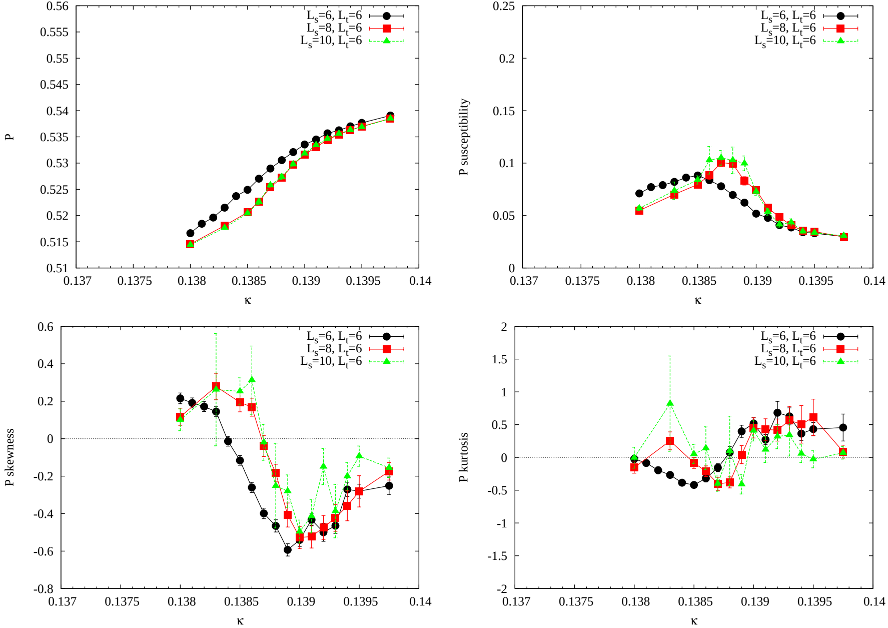
$\boldsymbol{\beta = 1.70}$, $\boldsymbol{\kappa = 0.1371}$, $\boldsymbol{6^4}$ and $\boldsymbol{\beta = 1.77}$, $\boldsymbol{\kappa = 0.13767}$, $\boldsymbol{8^4}$

Phase

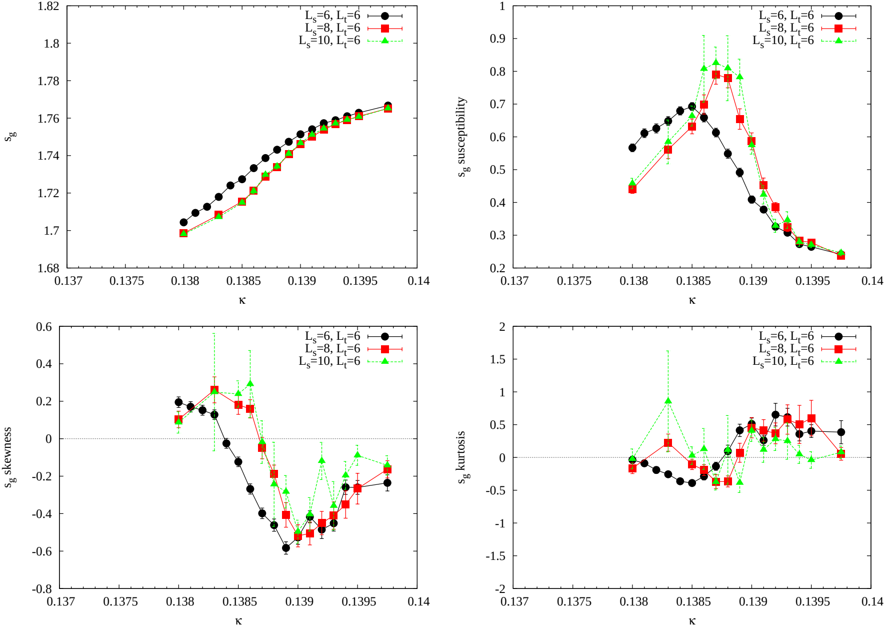


cos(3θ)
κ
L_s=6 , aμ=0.10
L_s=8 , aμ=0.10
L_s=10, aμ=0.10
L_s=6 , aμ=0.20
L_s=8 , aμ=0.20
L_s=10, aμ=0.20
|θ|
aμ
L_s=6 , κ=0.138300
L_s=8 , κ=0.138300
L_s=10, κ=0.138300

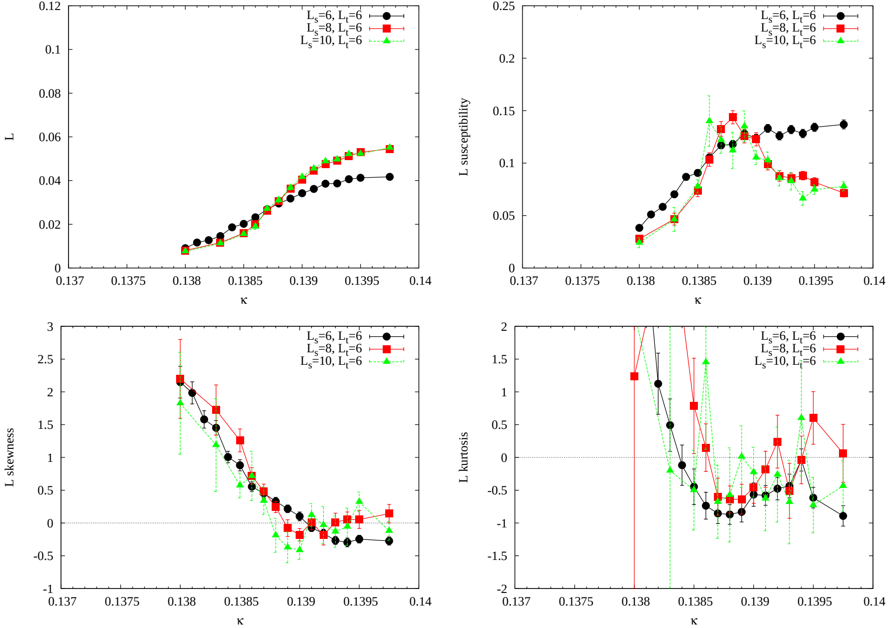
Plaquette at $a\mu = 0.1$



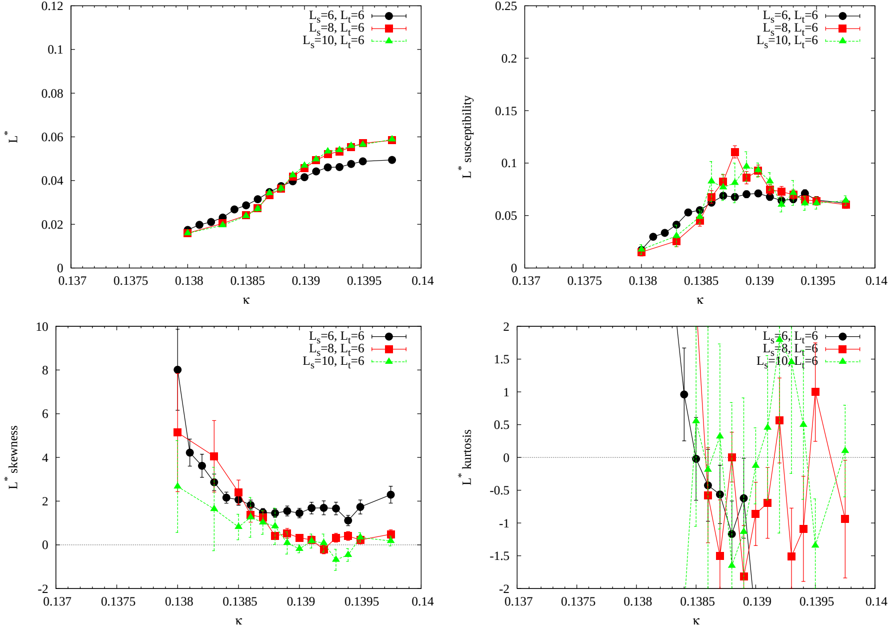
Action density at $a\mu = 0.1$



Polyakov loop at $a\mu = 0.1$

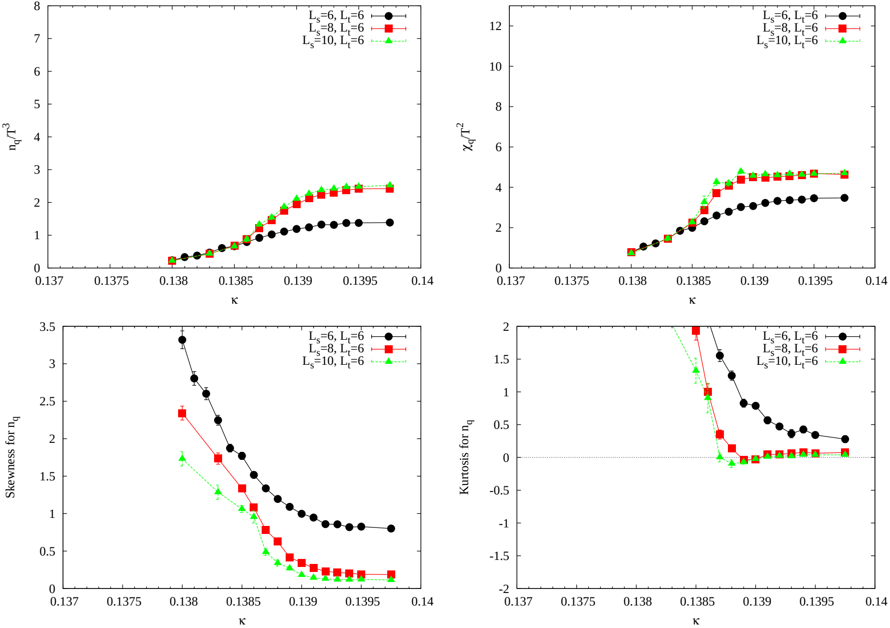


L^* at $a\mu = 0.1$

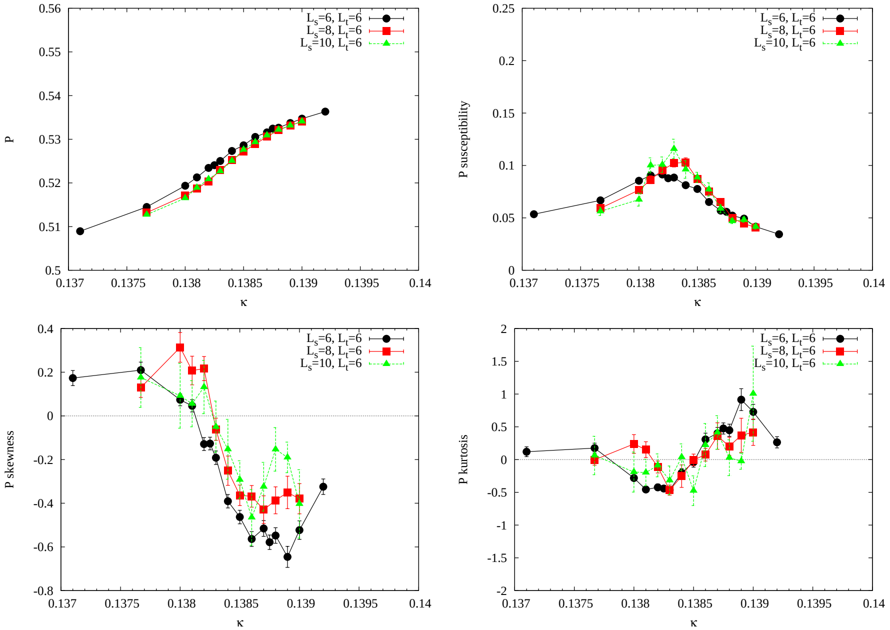


L_s=6, L_t=6
L_s=8, L_t=6
L_s=10, L_t=6
L^*
L^* susceptibility
L^* skewness
L^* kurtosis
κ

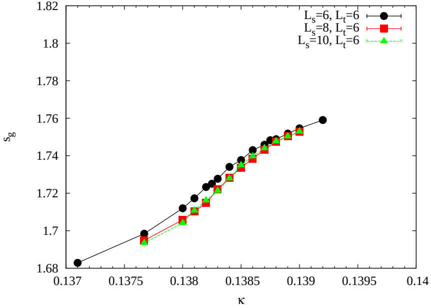
Quark number at $a\mu = 0.1$

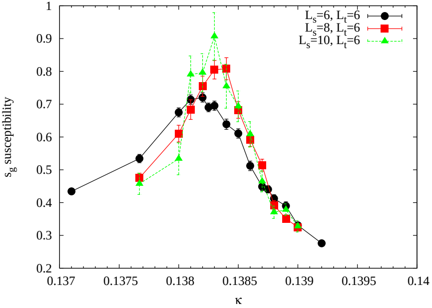


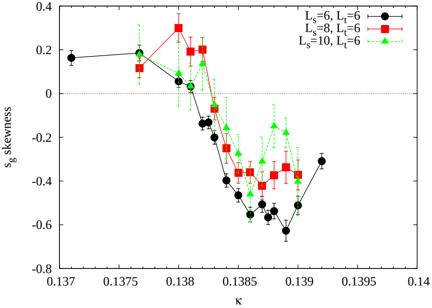
Plaquette at $a\mu = 0.2$

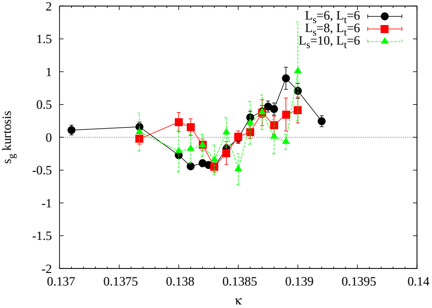


Action density at $a\mu = 0.2$

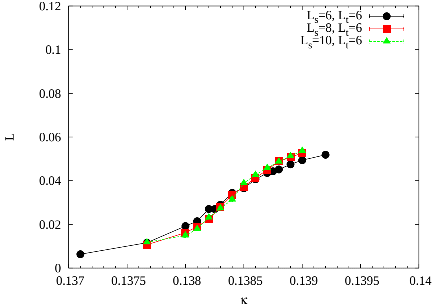


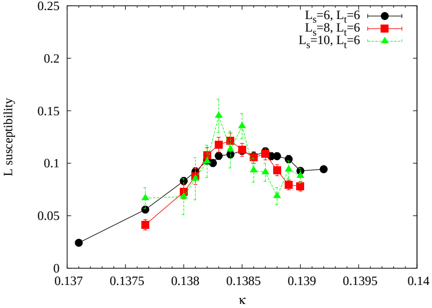


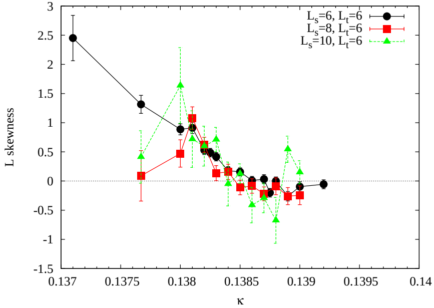


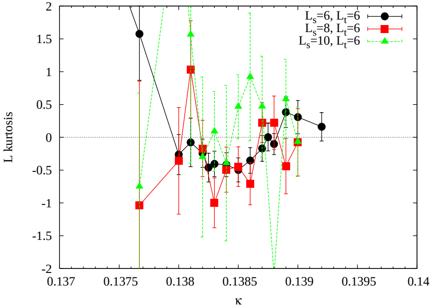


Polyakov loop at $a\mu = 0.2$

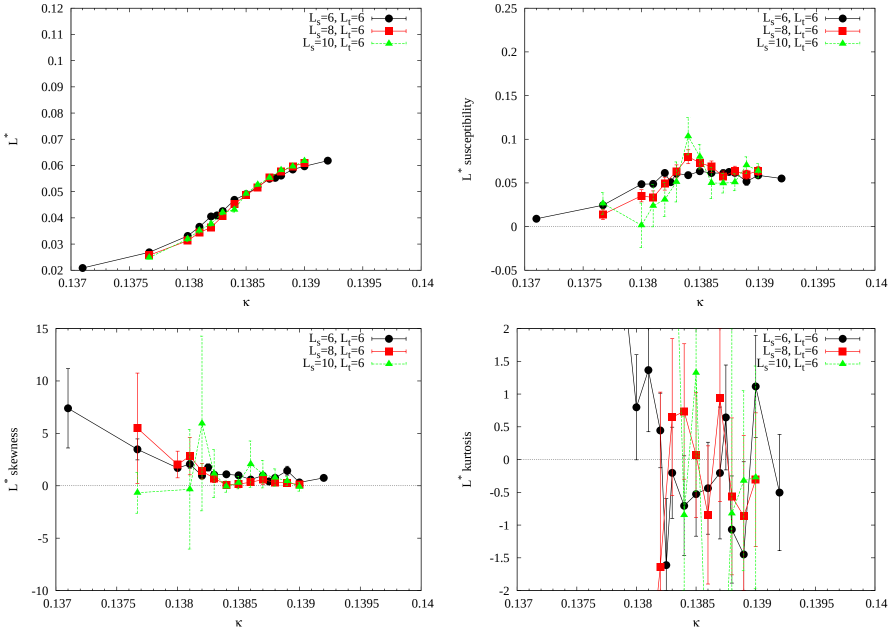






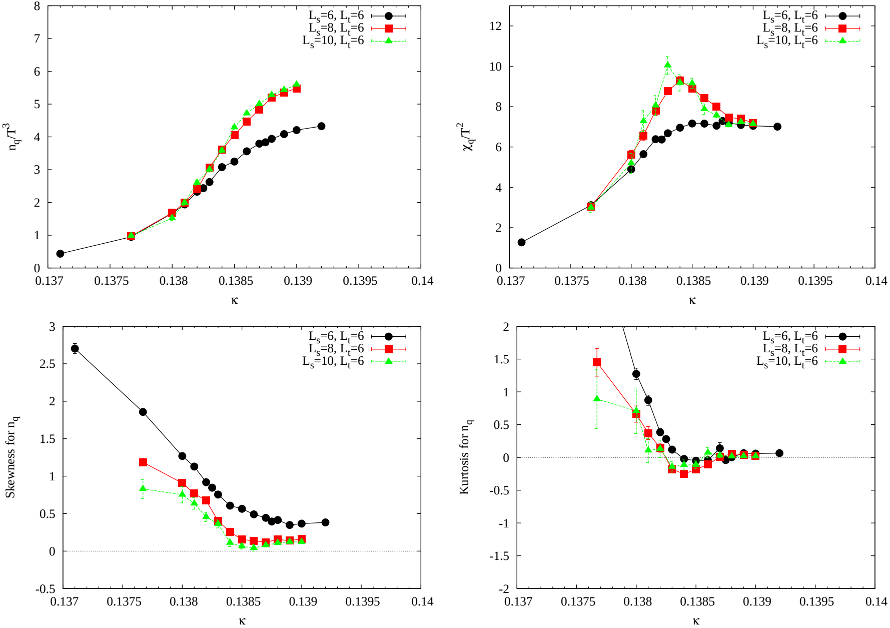


L^* at $a\mu = 0.2$

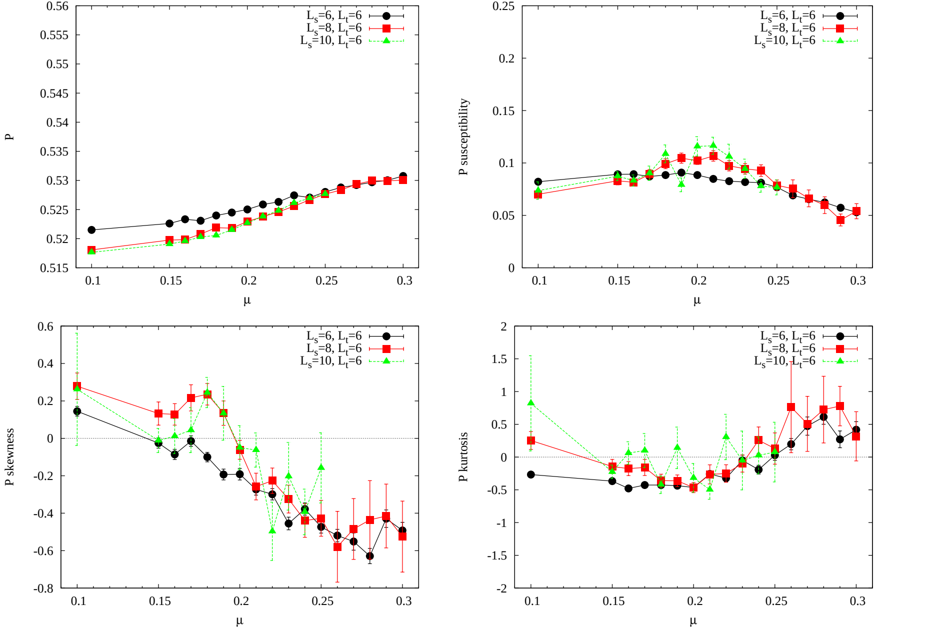


L*
κ
L* susceptibility
L* skewness
L* kurtosis
Ls=6, Lt=6
Ls=8, Lt=6
Ls=10, Lt=6

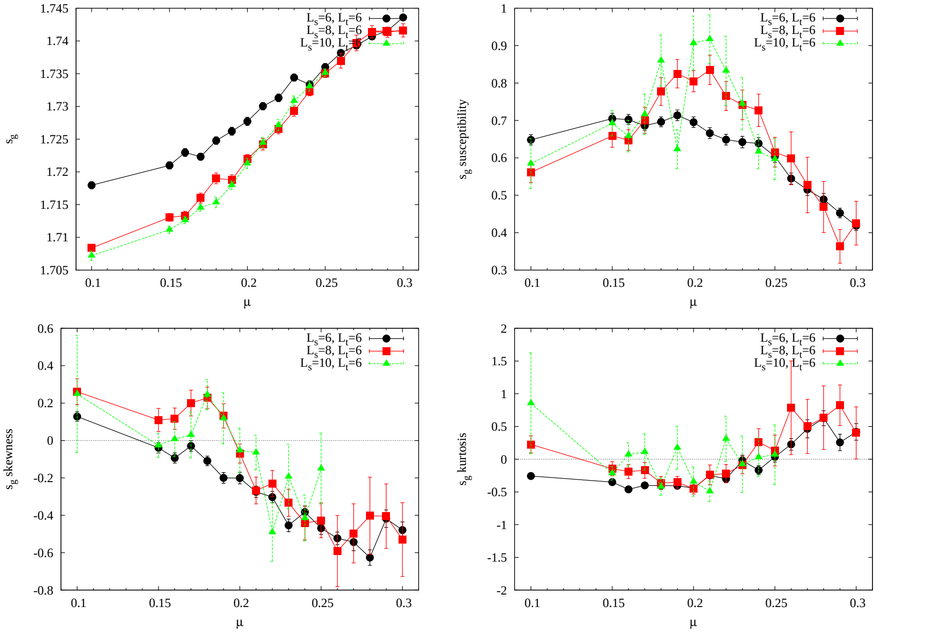
Quark number at $a\mu = 0.2$



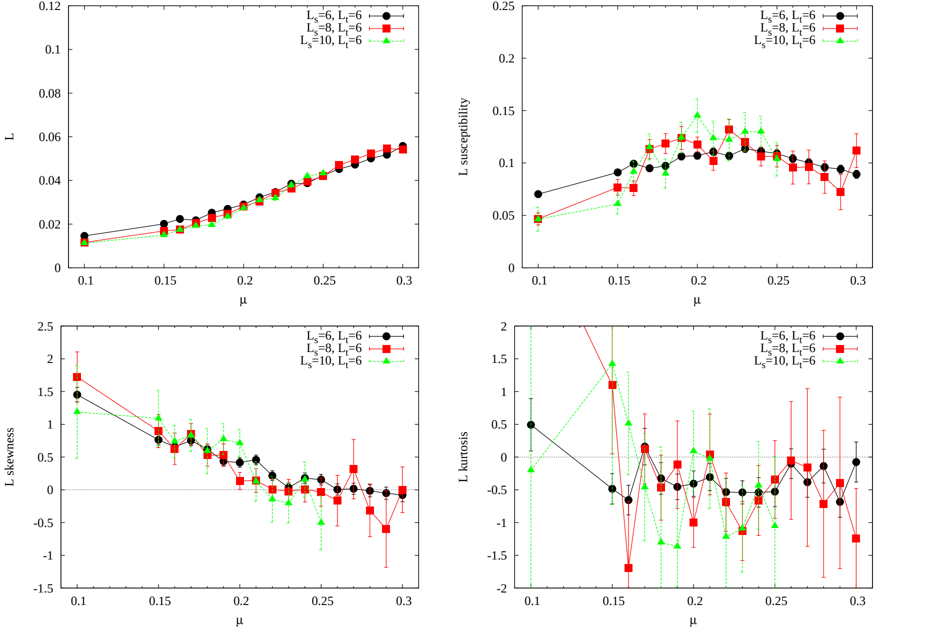
Plaquette at $\kappa = 0.1383$



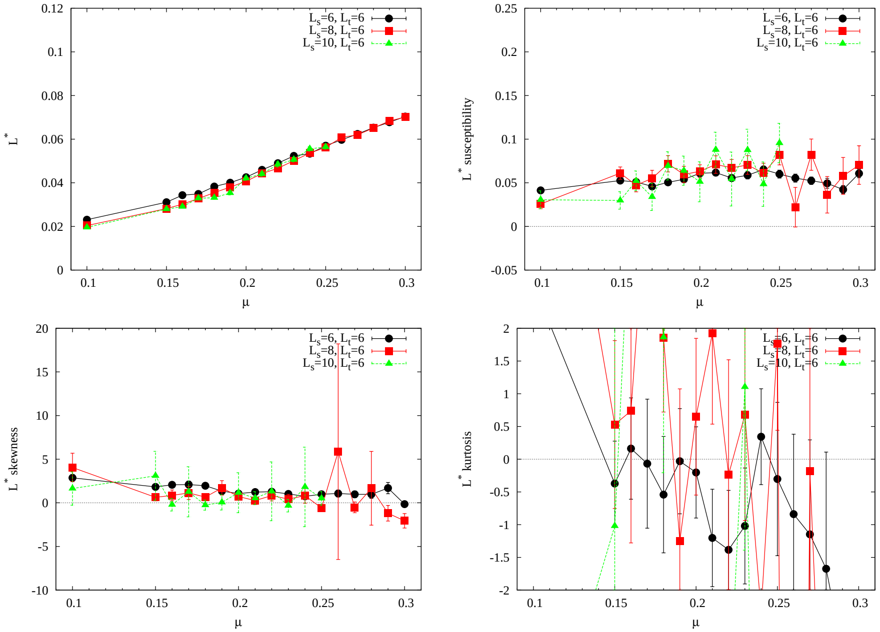
Action density at $\kappa = 0.1383$



Polyakov loop at $\kappa = 0.1383$



L^* at $\kappa = 0.1383$



Quark number at $\kappa = 0.1383$