Two topics from lattice NRQCD at non-zero temperature: heavy quark mass dependence and S-wave bottomonium states moving in a thermal bath

Seyong Kim

Sejong University

G. Aarts, C. Allton(Swansea), M.P. Lombardo(Frascati), M.B. Oktay(Utah), S.M. Ryan (Trinity), D.K. Sinclair(Argonne), J.I. Skullerud(NUIM)

Outline

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2 Method

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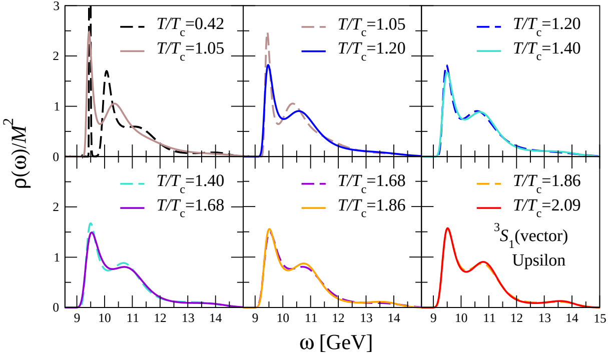
Last year's result

- JHEP 11 (2011) 103



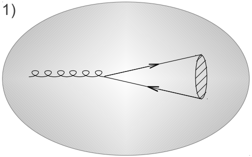
- 1S peak survives, 2S and higher peaks merge and become a broad peak as T increases (melting)

- last year, CMS collaboration observed disappearance of 2S and 3S upsilon state in Pb-Pb collisions

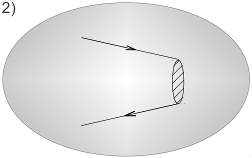
effects on quarkonium production

- two different ways of quarkonium production in QGP:
  (1) production through hard process $\rightarrow$ comoving $b - \overline{b}$ pair (moving in a thermal bath)
  (2) production through recombination (not likely) $\rightarrow$ not moving in a thermal bath

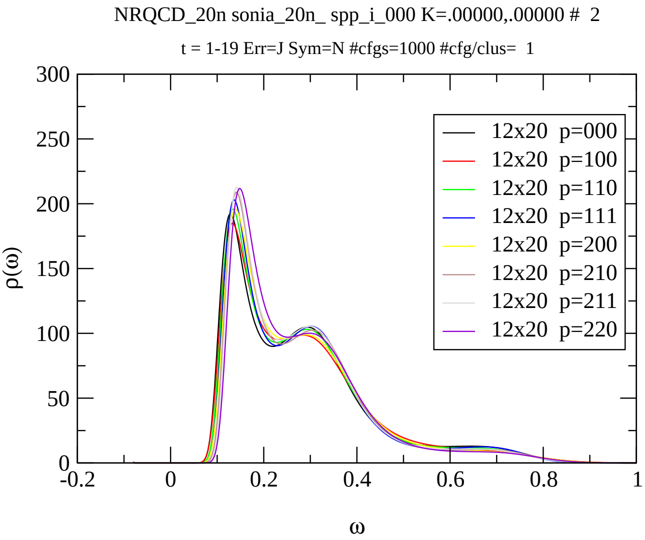
1)



2)



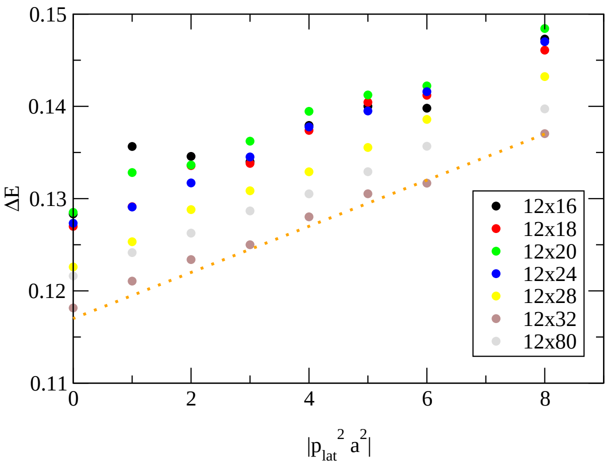
(1) Upsilon moving in a thermal bath



(1) Upsilon moving in a thermal bath

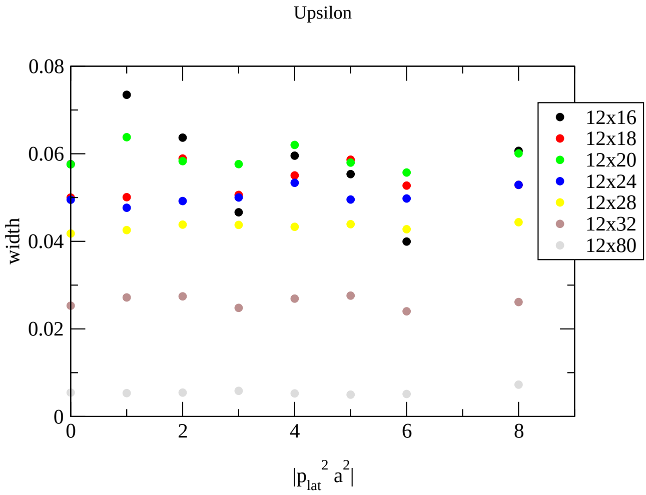
- observable heavy quarkonium velocity ($\frac{v^2_{\mathrm{Upsilon}}}{c^2} \sim 0.03$) effect on the S-wave state mass (NR dispersion $\sim \frac{\vec{p}^2}{2M_{\mathrm{Upsilon}}}$)

Upsilon



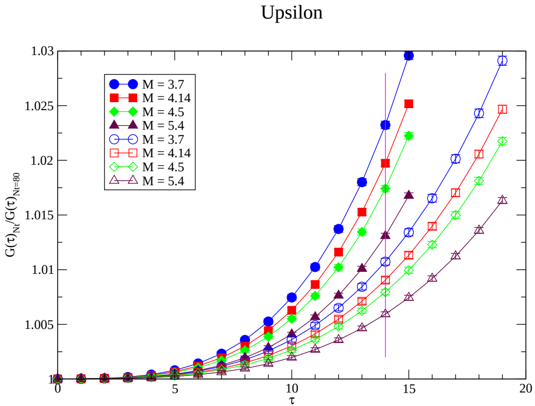
(1) Upsilon moving in a thermal bath

• no observable v^2_{Upsilon} effect on the S-wave state "width" (Escobedo et al., PRD84 (2011) 016008, $\Gamma_v/\Gamma_0 \sim 1 - \frac{2}{3}v^2_{\text{Upsilon}}$)



(2) Heavy quark mass dependence

- At a given τ, the ratio $R(M,T)(\tau) \sim e^{-(E(M,T)-E(M,T=0))\tau}$ since $G(\tau) \sim e^{-\Delta E \tau}$.
Define $\delta_1(\tau = 14) = R(M_2,T_1) - R(M_1,T_1)$ and $\delta_2(\tau = 14) = R(M_1,T_1) - R(M_1,T_2)$. then $\delta_2 > \delta_1$
temperature effect (ΔT) is larger than mass effect (ΔM)



Lattice NRQCD

- NRQCD is an effective field theory and is expansion in v, the heavy quark velocity in heavy quarkonium

- used for accurate calculation of quarkonium property at zero temperature

- $\frac{T}{M}$ is small for $T \sim 170$ (MeV) and $M \sim 5000$ (MeV) for bottomonium at the range of non-zero temperature we are interested in

- calculate bottomonium correlator in the background color gauge field which has dynamical light quark effect and finite temperature effect

- usually the number of lattice sites in the time direction is smaller than that that in the space direction since $T = \frac{1}{N_t a} \rightarrow$ we use anisotropic lattice

Lattice NRQCD

• Anisotropic lattice on $12^3 \times N_t$ (ref. G. Aarts et al, PRD 76 (2007) 094513)

N_s	N_t	a_τ^{-1}	T(MeV)	T/T_c	No. of Conf.
12	80	7.35GeV	90	0.42	250
12	32	7.35GeV	230	1.05	1000
12	28	7.35GeV	263	1.20	1000
12	24	7.35GeV	306	1.40	500
12	20	7.35GeV	368	1.68	1000
12	18	7.35GeV	408	1.86	1000
12	16	7.35GeV	458	2.09	1000

Table: summary for the lattice data set

• $N_f = 2$, two-plaquette Symanzik improved gauge action, fine-Wilson, coarse-Hamber-Wu fermion action with stout-link smearing

Lattice NRQCD

- Non-relativistic QCD in FT

$$G(\vec{x},t=0) = S(x) \tag{1}$$

$$G(\vec{x},t=1) = \left[1+\frac{1}{2n}\frac{\vec{D}^2}{2m_b^0}\right]^n U_4^\dagger(\vec{x},t)\left[1+\frac{1}{2n}\frac{\vec{D}^2}{2m_b^0}\right]^n G(\vec{x},0) \tag{2}$$

$$G(\vec{x},t+1) = \left[1+\frac{1}{2n}\frac{\vec{D}^2}{2m_b^0}\right]^n U_4^\dagger(\vec{x},t)\left[1+\frac{1}{2n}\frac{\vec{D}^2}{2m_b^0}\right]^n [1-\delta H]\,G(\vec{x},t) \tag{3}$$

Lattice NRQCD

where $S(x)$ is the source and

$$\begin{aligned}
\delta H &= -\frac{(\vec{D}^{(2)})^2}{8(m_b^0)^3} + \frac{ig}{8(m_b^0)^2}(\vec{D}\cdot\vec{E} - \vec{E}\cdot\vec{D}) \\
&- \frac{g}{8(m_b^0)^2}\vec{\sigma}\cdot(\vec{D}\times\vec{E} - \vec{E}\times\vec{D}) - \frac{g}{2m_b^0}\vec{\sigma}\cdot\vec{B} \\
&+ \frac{a^2\vec{D}^{(4)}}{24m_b^0} - \frac{a(\vec{D}^{(2)})^2}{16n(m_b^0)^2} \qquad (1)
\end{aligned}$$

- momentum injection to quarkonium in $S(x)$

MEM for NRQCD

$$G_\Gamma(\tau) = \sum_{\vec{x}} \langle \overline{\psi}(\tau,\vec{x})\Gamma\psi(\tau,\vec{x})\overline{\psi}(0,\vec{0})\Gamma\psi(0,\vec{0})\rangle \quad (2)$$

$$= \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho_\Gamma(\omega,\vec{p}) \quad (3)$$

and

$$K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}. \quad (4)$$

With $\omega = 2M + \omega'$ and $T/M << 1$,

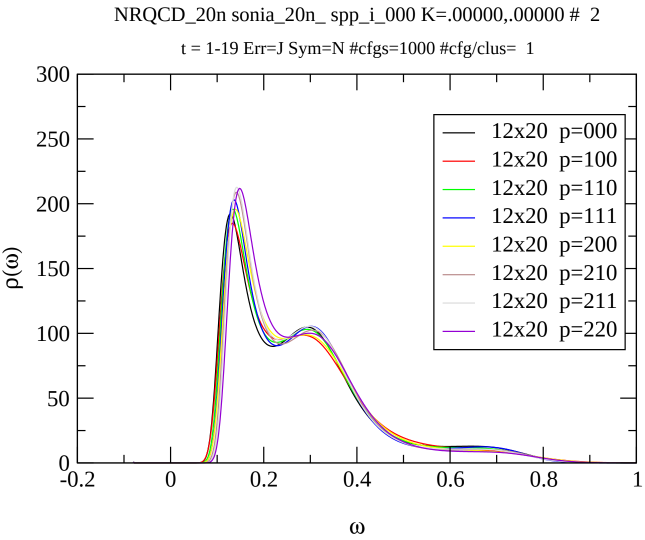
$$G(\tau) = \int_{-2M}^\infty \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega') \quad (5)$$

- unlike QCD case, the NRQCD kernel is independent of T

- Scale M is absent $\rightarrow$ smaller range of ω to consider

- obtaing spectral function in NRQCD is equivalent to inverse Laplace

MEM for NRQCD



Conclusion

• Using NRQCD formalism in non-zero temperature, we see observable v^2 effect on the energy of S-wave state moving in thermal bath but no observable effect on the width of S-wave state moving in thermal bath for $v^2_{\text{upsilon}} \sim 0.3$

• Temperature effect is more important than the heavy quark mass effect in S-wave bottomonium at the temperature around a few T_c
$\rightarrow$ NRQCD as an effective theory for bottomonium in non-zero temperature is a consistent theory