Two topics from lattice NRQCD at non-zero temperature: heavy quark mass dependence and S-wave bottomonium states moving in a thermal bath

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Outline

1. Results
2. Method
3. Conclusion
Last year’s result

- JHEP 11 (2011) 103

1S peak survives, 2S and higher peaks merge and become a broad peak as $T$ increases (melting)

- last year, CMS collaboration observed disappearance of 2S and 3S upsilon state in Pb-Pb collisions
two different ways of quarkonium production in QGP:

1) production through hard process $\rightarrow$ comoving $b - \bar{b}$ pair (moving in a thermal bath)

2) production through recombination (not likely) $\rightarrow$ not moving in a thermal bath
(1) Upsilon moving in a thermal bath

NRQCD_20n sonia_20n_ spp_i_000 K=.00000,.00000 # 2

\[ t = 1-19 \text{ Err=J Sym=N #cfgs=1000 #cfg/clus= 1} \]

\[ \rho(\omega) \]

-0.2 0 0.2 0.4 0.6 0.8 1

-0.2 0 0.2 0.4 0.6 0.8 1

\[ \omega \]
(1) Upsilon moving in a thermal bath

- Observable heavy quarkonium velocity \( \left( \frac{v_{\Upsilon}^2}{c^2} \right) \sim 0.03 \) effect on the S-wave state mass (NR dispersion \( \sim \frac{\vec{p}^2}{2M_{\Upsilon}} \))
(1) Upsilon moving in a thermal bath

- no observable $\nu^2_{\text{Upsilon}}$ effect on the S-wave state “width” (Escobedo et al., PRD84 (2011) 016008, $\Gamma_\nu/\Gamma_0 \sim 1 - \frac{2}{3} \nu^2_{\text{Upsilon}}$)

![Graph showing the effect of Upsilon moving in a thermal bath with data points and a trend line.](image-url)
(2) Heavy quark mass dependence

- At a given $\tau$, the ratio $R(M,T)(\tau) \sim e^{-(E(M,T)-E(M,T=0))\tau}$ since $G(\tau) \sim e^{-\Delta E \tau}$.

Define $\delta_1(\tau = 14) = R(M_2, T_1) - R(M_1, T_1)$ and $\delta_2(\tau = 14) = R(M_1, T_1) - R(M_1, T_2)$. Then $\delta_2 > \delta_1$.

Temperature effect ($\Delta T$) is larger than mass effect ($\Delta M$).
Lattice NRQCD

- NRQCD is an effective field theory and is expansion in $v$, the heavy quark velocity in heavy quarkonium

- used for accurate calculation of quarkonium property at zero temperature

- $\frac{T}{M}$ is small for $T \sim 170$ (MeV) and $M \sim 5000$ (MeV) for bottomonium at the range of non-zero temperature we are interested in

- calculate bottomonium correlator in the background color gauge field which has dynamical light quark effect and finite temperature effect

- usually the number of lattice sites in the time direction is smaller than that that in the space direction since $T = \frac{1}{N_t a}$ $\rightarrow$ we use anisotropic lattice
Lattice NRQCD

- Anisotropic lattice on $12^3 \times N_t$ (ref. G. Aarts et al, PRD 76 (2007) 094513)

<table>
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<th>$N_s$</th>
<th>$N_t$</th>
<th>$a_t^{-1}$</th>
<th>T(MeV)</th>
<th>$T/T_c$</th>
<th>No. of Conf.</th>
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Table: summary for the lattice data set

- $N_f = 2$, two-plaquette Symanzik improved gauge action, fine-Wilson, coarse-Hamber-Wu fermion action with stout-link smearing
Lattice NRQCD

- Non-relativistic QCD in FT

\[ G(\vec{x}, t = 0) = S(x) \]  \hspace{1cm} (1)

\[ G(\vec{x}, t = 1) = \left[ 1 + \frac{1}{2n} \frac{\vec{D}^2}{2m_b^0} \right]^n U^\dagger_4(\vec{x}, t) \left[ 1 + \frac{1}{2n} \frac{\vec{D}^2}{2m_b^0} \right]^n G(\vec{x}, 0) \]  \hspace{1cm} (2)

\[ G(\vec{x}, t + 1) = \left[ 1 + \frac{1}{2n} \frac{\vec{D}^2}{2m_b^0} \right]^n U^\dagger_4(\vec{x}, t) \left[ 1 + \frac{1}{2n} \frac{\vec{D}^2}{2m_b^0} \right]^n [1 - \delta H] G(\vec{x}, t) \]  \hspace{1cm} (3)
where $S(x)$ is the source and

$$\delta H = -\frac{(\vec{D}(2))^2}{8(m_b^0)^3} + \frac{ig}{8(m_b^0)^2}(\vec{D} \cdot \vec{E} - \vec{E} \cdot \vec{D})$$

$$- \frac{g}{8(m_b^0)^2}\vec{\sigma} \cdot (\vec{D} \times \vec{E} - \vec{E} \times \vec{D}) - \frac{g}{2m_b^0}\vec{\sigma} \cdot \vec{B}$$

$$+ \frac{a^2 \vec{D}(4)}{24m_b^0} - \frac{a(\vec{D}(2))^2}{16n(m_b^0)^2}$$

(1)

• momentum injection to quarkonium in $S(x)$
MEM for NRQCD

\[ G_\Gamma(\tau) = \sum \langle \bar{\psi}(\tau, \vec{x}) \Gamma \psi(\tau, \vec{x}) \bar{\psi}(0, \vec{0}) \Gamma \psi(0, \vec{0}) \rangle \tag{2} \]

\[ = \int \frac{d^3 \rho}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \rho_\Gamma(\omega, \vec{p}) \tag{3} \]

and

\[ K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)} \tag{4} \]

With \( \omega = 2M + \omega' \) and \( T/M << 1 \),

\[ G(\tau) = \int_{-2M}^\infty \frac{d\omega'}{2\pi} \exp(-\omega'\tau) \rho(\omega') \tag{5} \]

- unlike QCD case, the NRQCD kernel is independent of \( T \)
- Scale \( M \) is absent \( \rightarrow \) smaller range of \( \omega \) to consider
- obtaining spectral function in NRQCD is equivalent to inverse Laplace
MEM for NRQCD

NRQCD_20n sonia_20n_ spp_i_000 K=0.00000,0.00000 # 2

t = 1-19 Err=J Sym=N #cfgs=1000 #cfg/clus= 1
Using NRQCD formalism in non-zero temperature, we see observable $\nu^2$ effect on the energy of S-wave state moving in thermal bath but no observable effect on the width of S-wave state moving in thermal bath for $\nu^2_{\text{upsilon}} \sim 0.3$

Temperature effect is more important than the heavy quark mass effect in S-wave bottomonium at the temperature around a few $T_c$.

$\rightarrow$ NRQCD as an effective theory for bottomonium in non-zero temperature is a consistent theory.