Confinement in high-temperature lattice gauge theories

Michael Ogilvie
Washington University in St. Louis

Lattice 2012
Cairns, Australia

High-T confinement on $R^3 \times S^1$ (1)

- Coupling gets weak as T gets large

$$g^2(T) \to 0$$

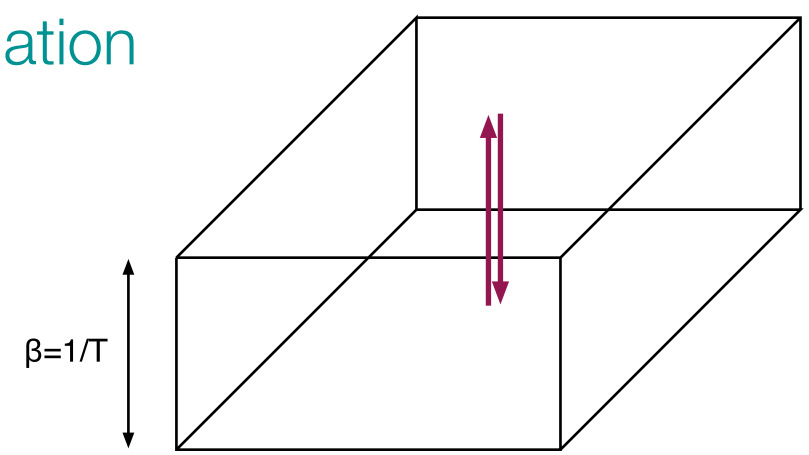
- Modify action to restore Z(N) symmetry **and** force the theory to be Abelian at large distances
  - double trace deformation (Meyers and mco, 2008)

$$S \to S - \int d^3x \, H_A \left| Tr_F P \right|^2$$

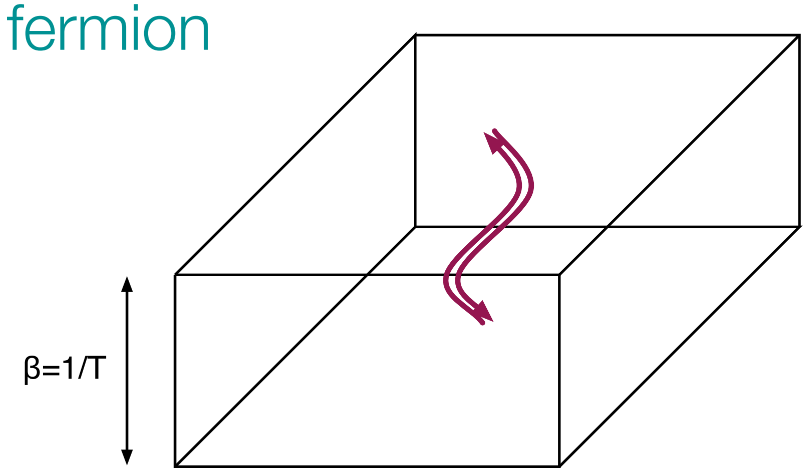
  - adjoint fermions (Unsal, 2008)

- A_4 behaves as a 3d scalar with a center-symmetric expectation value; Euclidean monopoles solutions!

deformation



adjoint fermion



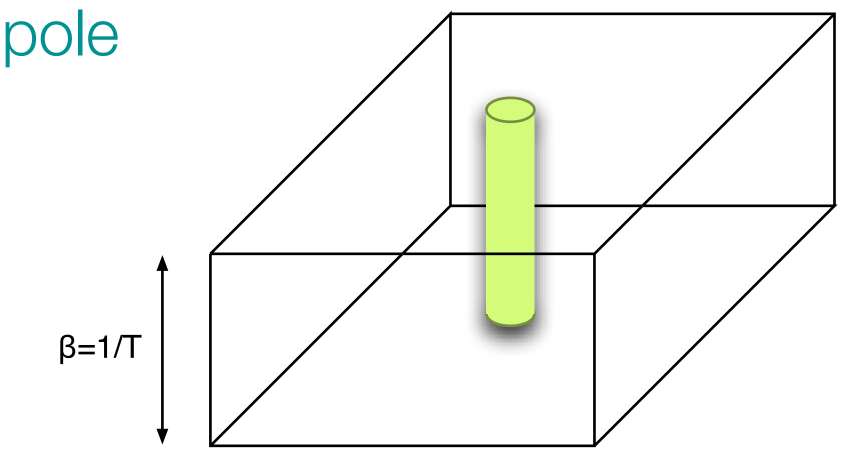
High-T confinement on $R^3 \times S^1$ (2)

- Euclidean monopoles are constituents of instantons (Lee & Yi, 1997, Kraan & van Baal 1998) and confine (Unsal 2008; Unsal and Yaffe, 2008)
- Dimensional reduction
- Confinement as in 3d Georgi-Glashow model (Polyakov 1976) by monopole gas
- Monopole gas is represented by a sine-Gordon

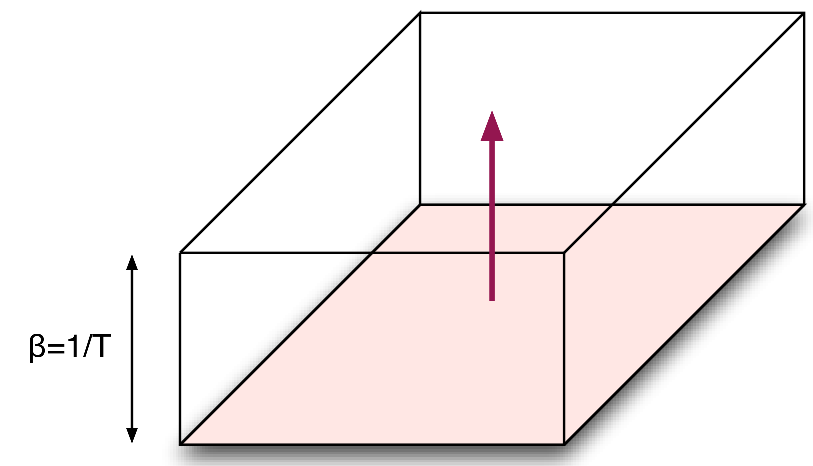
$$S_{eff} = \int d^3x \left[\frac{g^2(T)T}{32\pi^2} (\partial_j \sigma)^2 - 4y\cos(\sigma) \right]$$

$$y \propto T^3 \left(\Lambda/T\right)^{11/3}$$

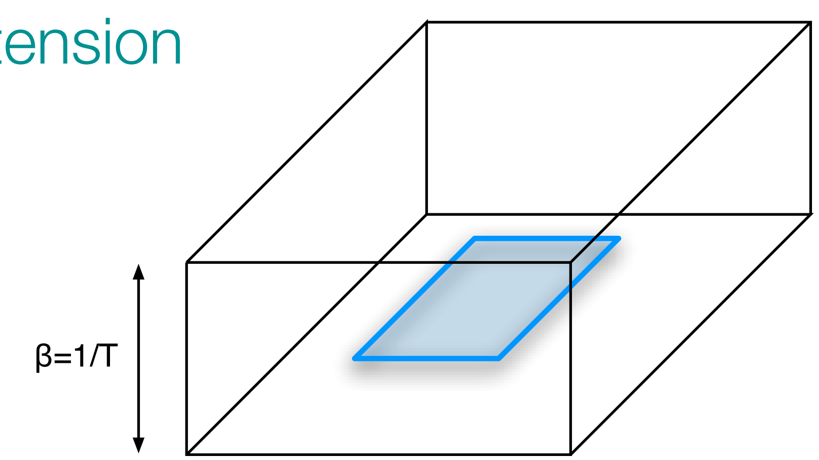
monopole



dimensional reduction



string tension



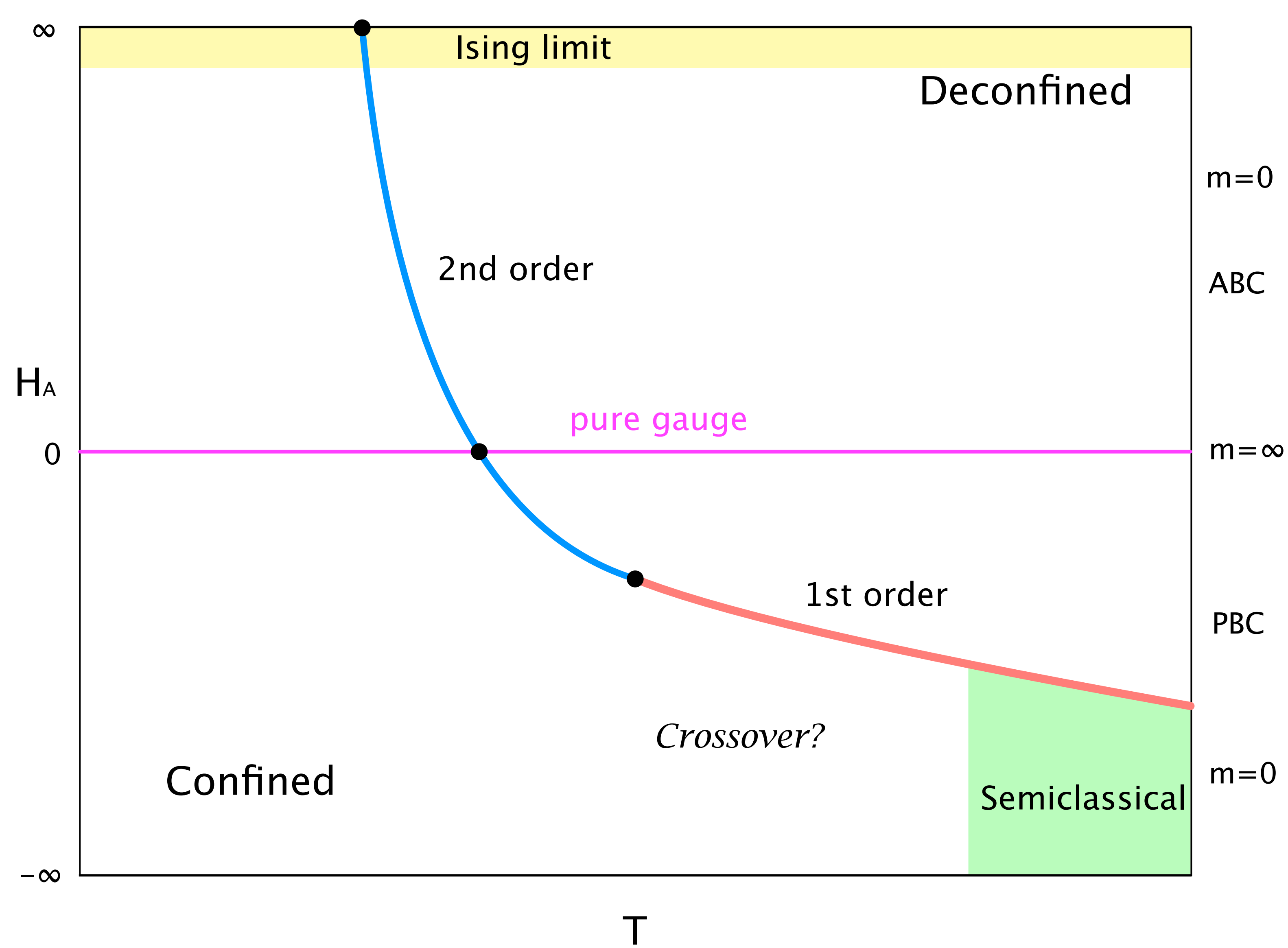
High-T confinement on $R^3 \times S^1$ (3)

- Positive H_A promotes Z(2) breaking and decreases the deconfinement temperature
- Negative H_A increases the deconfinement temperature
- Deconfinement transition changes from 2nd-order to 1st at tricritical point (non-universal- H. Nishimura & mco, 2012)
- Reach region of high-T semiclassical confinement

$$T \gg \Lambda$$

SU(2) phase diagram

$$S \to S - \int d^3x \, H_A \, |Tr_F P|^2$$



O(3) model in d=2 (1)

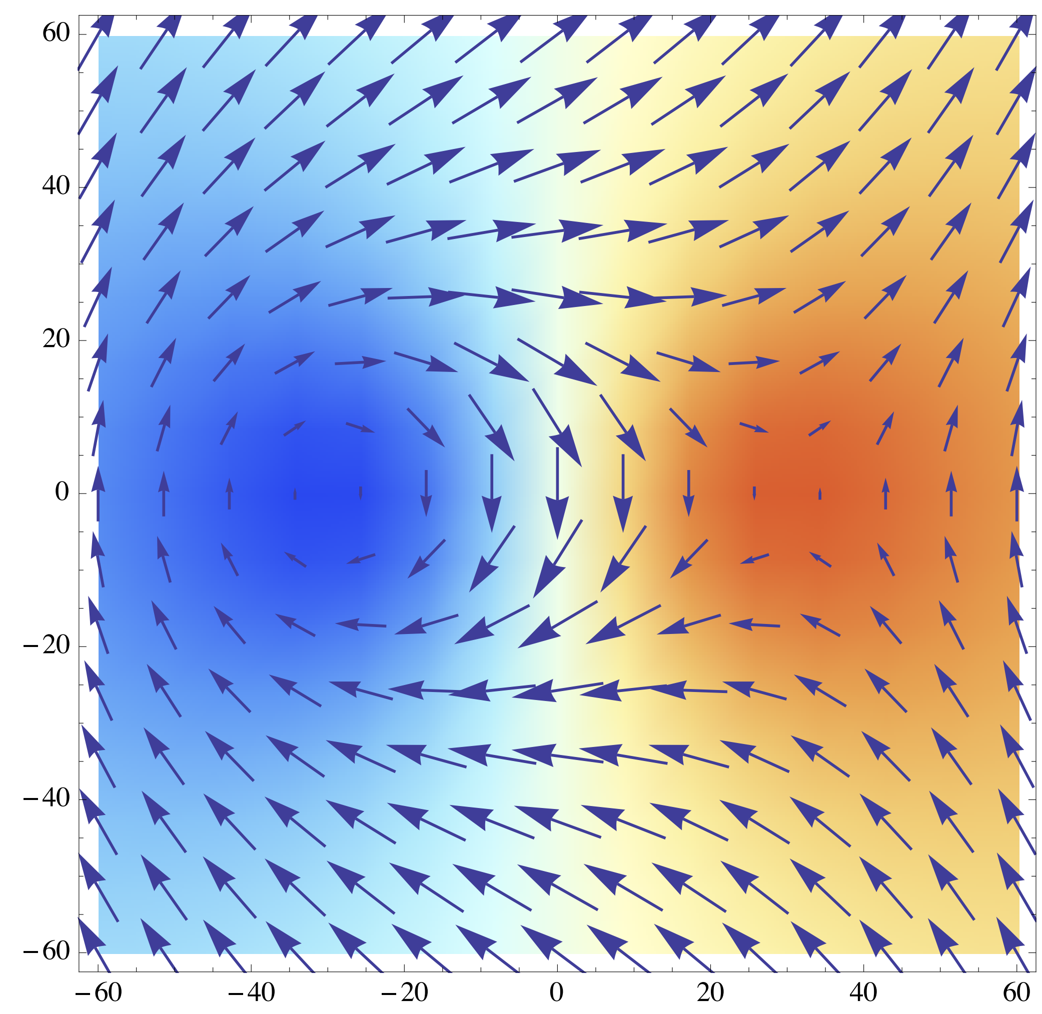
- Asymptotically free (like QCD)
- instantons (like QCD): $\pi_2\left(S^2\right) = Z$
- XY model vortices emerge as constituents of instantons (like monopoles in QCD)
- Can deform O(3) model into an XY model

$$S \to S + \int d^2x\, \frac{1}{2} h \sigma_3^2$$

mco and Guralnik, 1981

$$S = \int d^2x \frac{1}{2g^2} \left(\nabla \vec{\sigma}\right)^2$$

$$\vec{\sigma}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 1$$



O(3) model in d=2 (2)

Setup

$$S = \sum_{x,\mu} K\sigma_a(x)\sigma_a(x+\mu) - \sum_x \frac{1}{2} h\sigma_3^2(x)$$

$$\sigma = \left(\sqrt{1-\sigma_3^2}\cos\theta, \sqrt{1-\sigma_3^2}\sin\theta, \sigma_3\right)$$

$$S = \sum_{x,\mu} K_{eff}\left(x,\mu\right)\cos\left[\theta\left(x\right)-\theta\left(x+\mu\right)\right] + S_3$$

$$S_3 = \sum_{x,\mu} K\sigma_3(x)\sigma_3(x+\mu) - \sum_x \frac{1}{2} h\sigma_3^2(x)$$

$$K_{eff}\left(x,\mu\right) \equiv K\sqrt{1-\sigma_3^2\left(x\right)}\sqrt{1-\sigma_3^2\left(x+\mu\right)}$$

Lattice duality (Jose *et al.* 1977)

$$Z = \int_{S^2}\left[d\sigma\right]e^S = \int_{-1}^{+1}\left[d\sigma_3\left(x\right)\right]e^{S_3}\int_{S^1}\int_{S^1}\left[d\theta\right]\prod_{x,\mu} e^{K_{eff}(x,\mu)\cos(\nabla_\mu\theta(x))}$$

Character expansion:

$$Z = \int_{-1}^{+1}\left[d\sigma_3\left(x\right)\right]e^{S_3}\int_{S^1}\left[d\theta\right]\prod_{x,\mu}\sum_{n_\mu(x)\in Z} I_{n_\mu(x)}(K_{eff}\left(x,\mu\right))e^{in_\mu(x)\nabla_\mu\theta(x)}$$

O(3) model in d=2 (3)

Villain approximation: $K_{eff} \gg 1$

$$Z = \int_{-1}^{+1} [d\sigma_3(x)]\, e^{S_3} \int_{S^1} [d\theta] \prod_{x,\mu} \sum_{n_\mu(x)\in Z} \frac{1}{\sqrt{2\pi K_{eff}(x,\mu)}} e^{K_{eff}(x,\mu) - n_\mu^2(x)/2K_{eff}(x,\mu)} e^{i n_\mu(x) \nabla_\mu \theta(x)}$$

Integration over θ's: $\nabla_\mu n_\mu(x) = 0$ implies $n_\mu(x) = \epsilon_{\mu\nu} \nabla_\nu m(X)$ X on dual lattice

$$Z = \int_{-1}^{+1} [d\sigma_3(x)]\, e^{S_3'} \sum_{\{m(X)\}\in Z} e^{-\sum_{X,\nu} (\nabla_\nu m(X))^2 / 2K_{eff}(x,\mu)}$$

$$S_3' = S_3 + \sum_{x,\mu} \left[K_{eff}(x,\mu) - \frac{1}{2} \log\left(2\pi K_{eff}(x,\mu)\right) \right]$$

Poisson resummation:

$$Z = \int_{-1}^{+1} [d\sigma_3(x)]\, e^{S_3'} \int_R [d\phi(X)]\, e^{-\sum_{X,\nu} (\nabla_\nu \phi(X))^2 / 2K_{eff}(x,\mu)} \sum_{\{m(X)\}\in Z} e^{2\pi i m(X) \phi(X)}$$

O(3) model in d=2 (4)

sine-Gordon approximation:

Keep only m=1 contributions with y=1

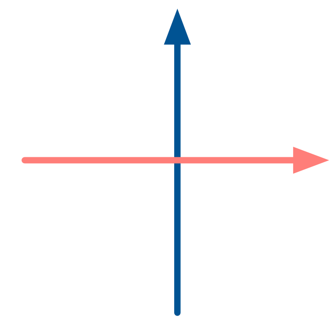
$$Z = \int_{-1}^{+1} [d\sigma_3(x)] \int_R [d\phi(X)] \exp\left[-\sum_{X,\mu} \frac{1}{2K_{eff}(x,\mu)} (\nabla_\mu \phi(X))^2 + \sum_X 2y \cos(2\pi\phi(X)) + S_3'\right]$$

Comments:

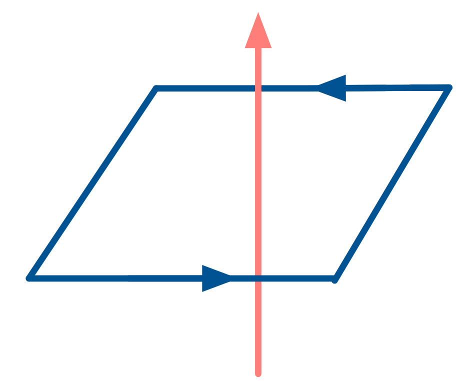
- Continuous path between O(3) model and XY model vortex-dominated phase
- Large h: $\sigma_3 = 0$ recovers XY model and set $K_{eff} = K$
- Intermediate h: h sets scale of vortex core
- Behavior of σ_3 at vortex core doubles number of vortices as with instantons
- Works for O(N) as well as O(3)

High-T confinement for lattice SU(2) in d=4 (1)

- Double-trace deformation forces center-symmetry. A_4 behaves as a 3d scalar
- Non-Abelian degrees of freedom get large masses due to H_A. Obtain U(1) effective lattice gauge theory in d=3
- Abelian lattice duality gives a 3d sine-Gordon theory (Banks *et al* 1977)



- d=2: links are dual to links



- d=3: plaquettes are dual to links

Lattice

$$Z = \int_R [d\sigma(X)] \exp\left[-\sum_{X,\mu} \frac{g^2}{2N_t} (\nabla_\mu \sigma(X))^2 + \sum_X 4y \cos(2\pi\sigma(X)) \right]$$

Continuum

$$S_{eff} = \int d^3x \left[\frac{g^2(T)T}{32\pi^2} (\partial_j \sigma)^2 - 4y \cos(\sigma) \right]$$

Conclusions

- Non-Abelian lattice theories deformed to Abelian effective theories work in the same way as their continuum counterparts
- Lattice duality closely related to semiclassical continuum duality
- Continuous path between confined phase of SU(2) and the monopole-dominated phase of lattice U(1) gauge theory.
- Lattice theories know something about topology, but not about BPS bounds and correct RG scaling
- Vortices and monopoles may be important features even when no stable instantons exist as in 2d O(N) models (*cf*. Argyres and Unsal 2012)
- Many interesting problems to explore, on and off the lattice!