Charmonium Spectral Functions and Potentials at Finite Temperature

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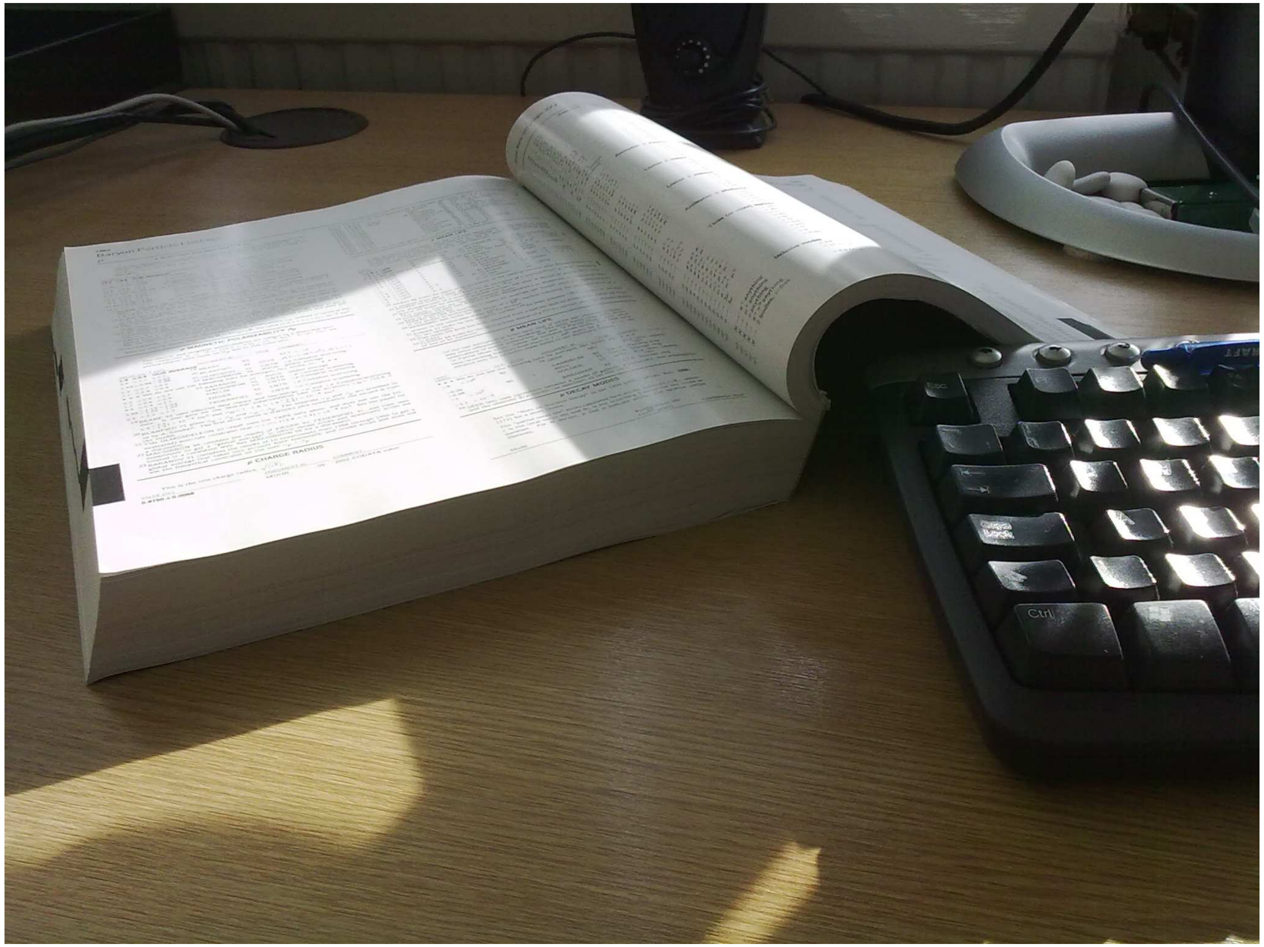
Outline

There is a large body of theoretical work studying the interquark potential in charmonium as a function of temperature, often from models.

Our aim is to determine this potential from first-principles.

- Schrödinger Equation Approach
  - Bethe-Salpeter wavefunction
- Conventional Exponential Fits
- Maximum Entropy Method

Particle Data Book

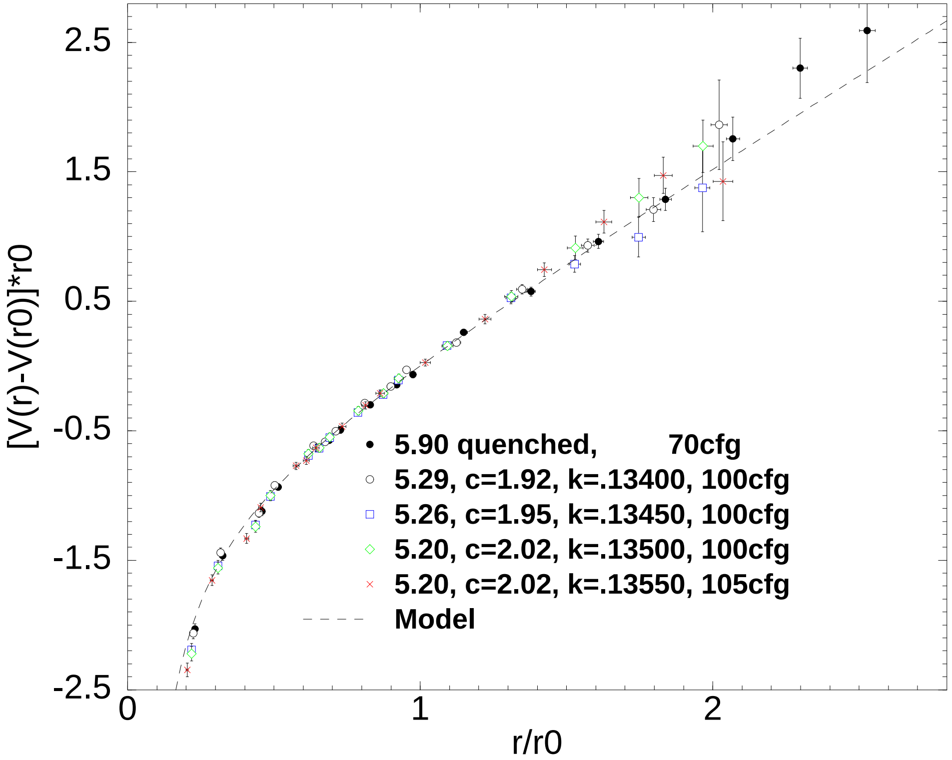


$\sim 1,500$ pages

zero pages on Quark-Gluon Plasma...

Static Quark Potential ($T = 0$)

UKQCD Collaboration [pre-history]



$$r_0 \approx 0.5\ fm$$

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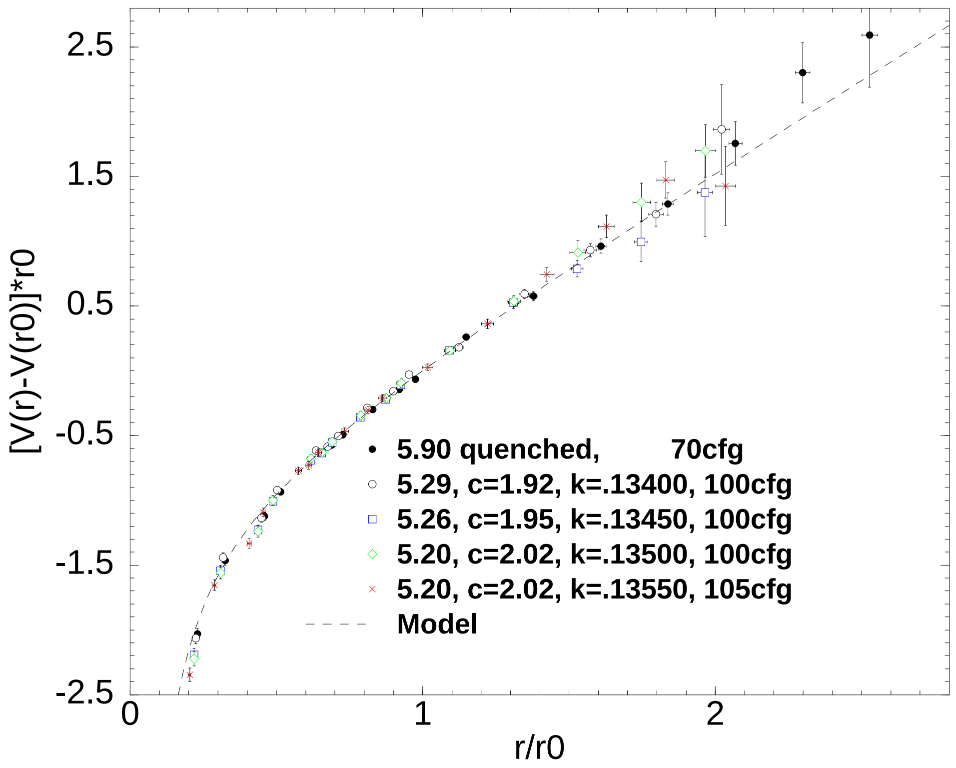
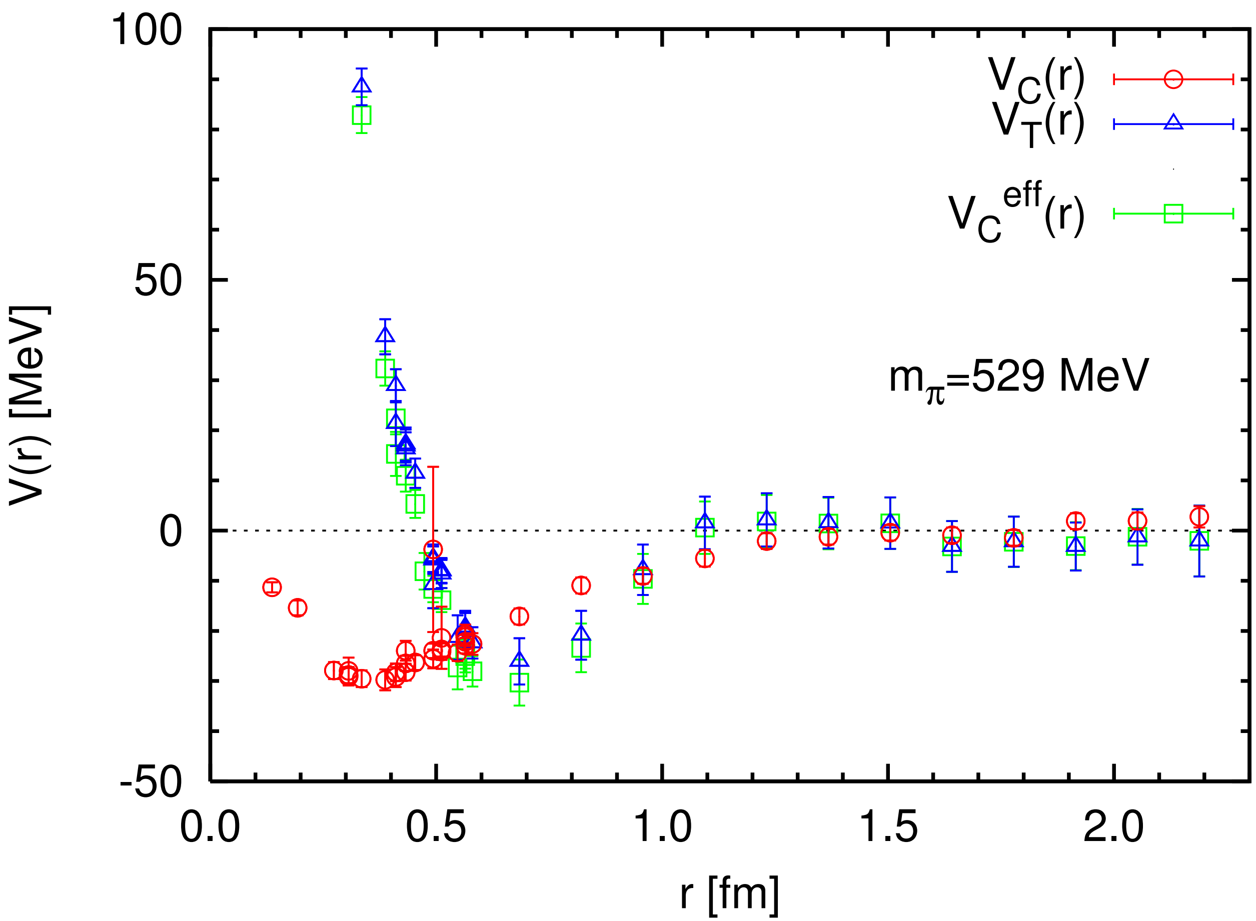


$$r_0 \approx 0.5\ fm$$

From Polyakov Loop $\leftrightarrow \quad m_Q = \infty$

Lattice goes Nuclear

N-N potential



HAL QCD Collaboration, Aoki, Doi, Hatsuda, Ikeda, Inoue, Ishii, Murano, Nemura, Sasaki

Inter-quark potential from the Lattice

Kawanai, Sasaki, arXiv:1111.025
Ikeda, Iida, arXiv:1102.2097

- finite-quark mass
- quenched

We extend this by using:

- $2+1$ flavour
- finite temperature
- anisotropic lattices

Schrödinger Equation Approach

Hatsuda, PoS CD09 (2009) 068 use the Schrödinger equation to "reverse engineer" the potential, $V(r)$, given the Bethe-Salpeter wavefunction, $\psi(r)$:

$$\left(\frac{p^2}{2M} + \underbrace{V(r)}_{\text{output}}\right) \overbrace{\psi(r)}^{\text{input}} = \overbrace{E}^{\text{input}}\ \overbrace{\psi(r)}^{}$$

$\psi(r)$ is determined from a lattice simulation from correlators of *non-local* (point-split) operators, $J(x;\vec{r}) = q(x)\,\Gamma\,U(x, x+\vec{r})\overline{q}(x+\vec{r})$

$$C(\vec{r},t) = \sum_{\vec{x}} < J(0;\vec{r})\ J(x;\vec{r}) >$$

$$\longrightarrow |\psi(r)|^2\ e^{-Et}$$

Lattice Parameters

Dublin-Maynooth $N_f = 2$ configurations

N_s	N_τ	T(MeV)	T/T_c	$N_{\rm cfg}$
12	80	90	0.42	250
12	32	230	1.05	1000
12	28	263	1.20	1000
12	24	306	1.40	500
12	20	368	1.68	1000
12	18	408	1.86	1000
12	16	458	2.09	1000

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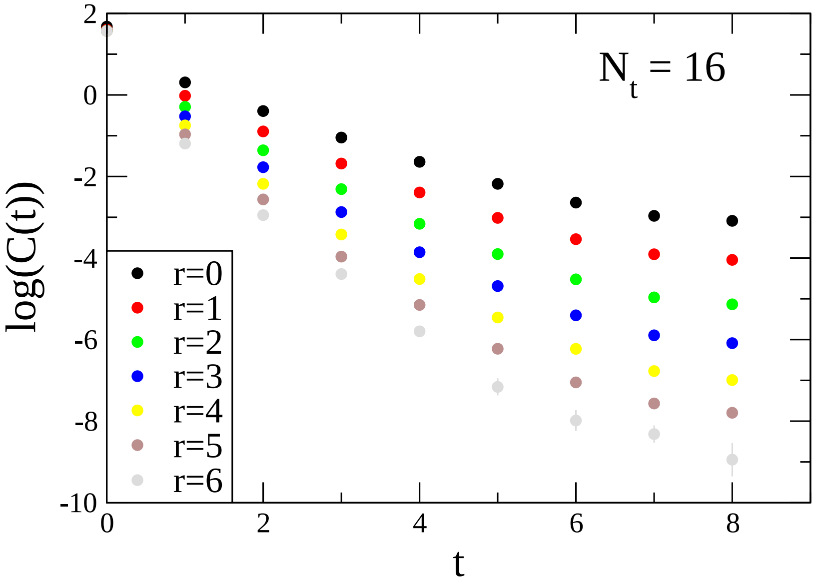
anisotropic lattice with $\xi = a_s/a_\tau \approx 6$

$a_s = 0.167$ fm

Vector and Pseudoscalar Channels

Correlation Functions (PS)



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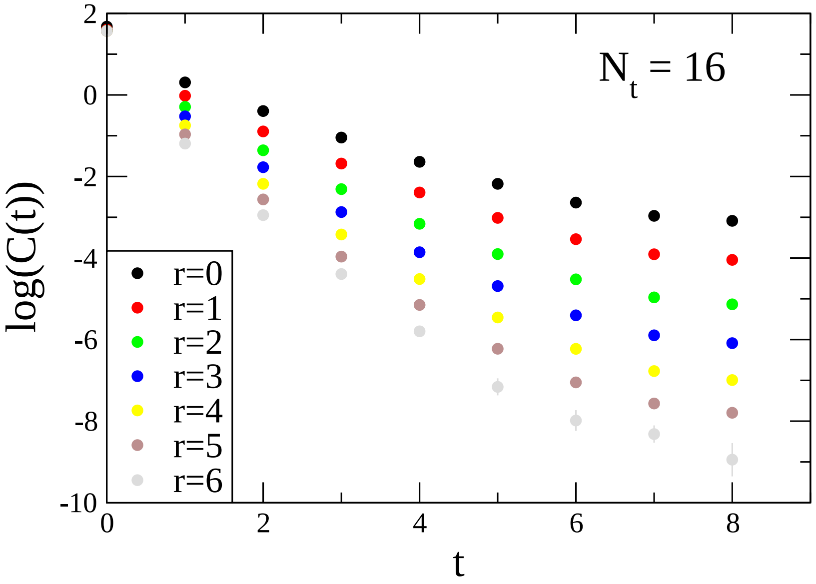

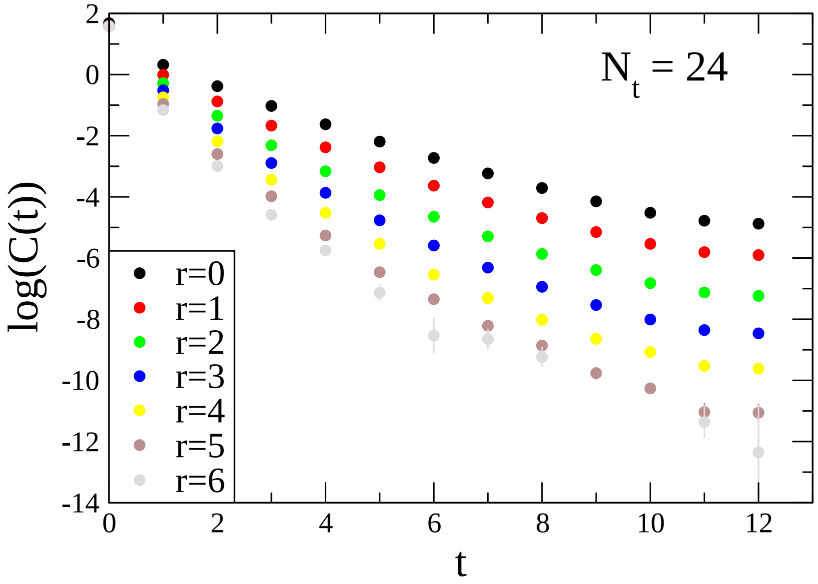

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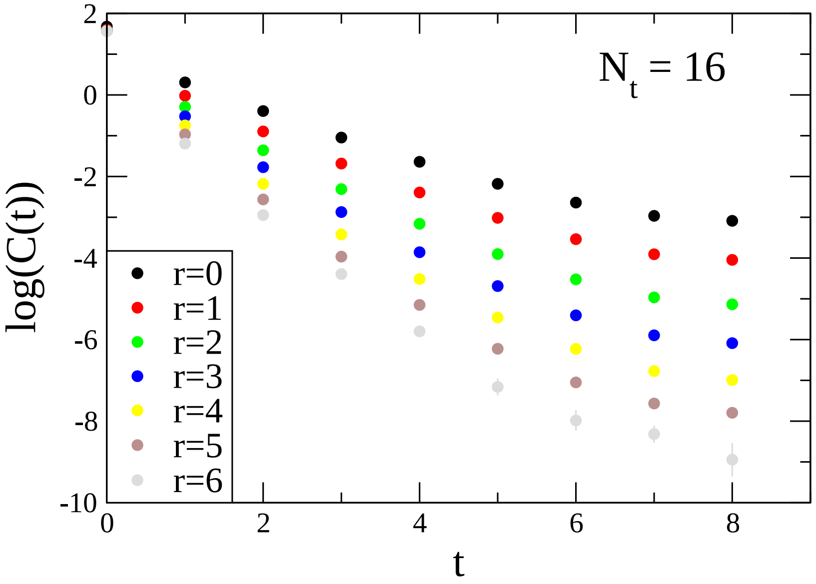

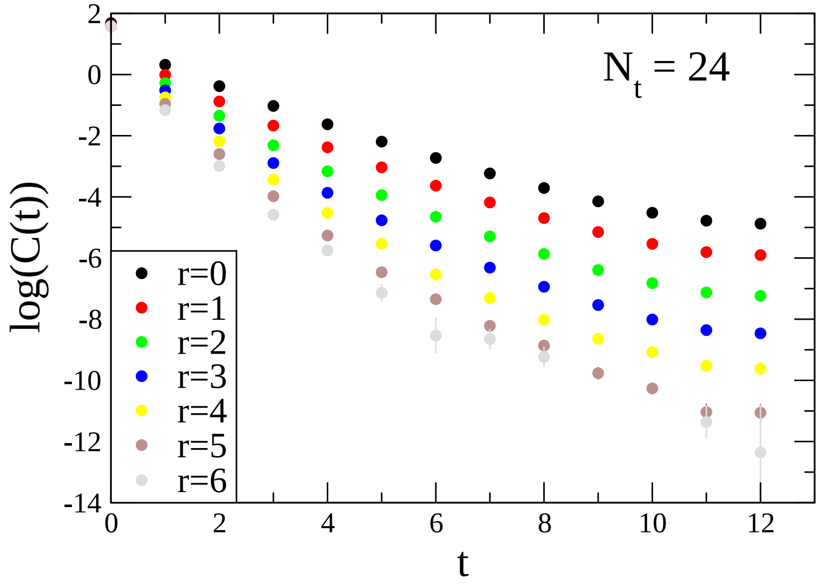

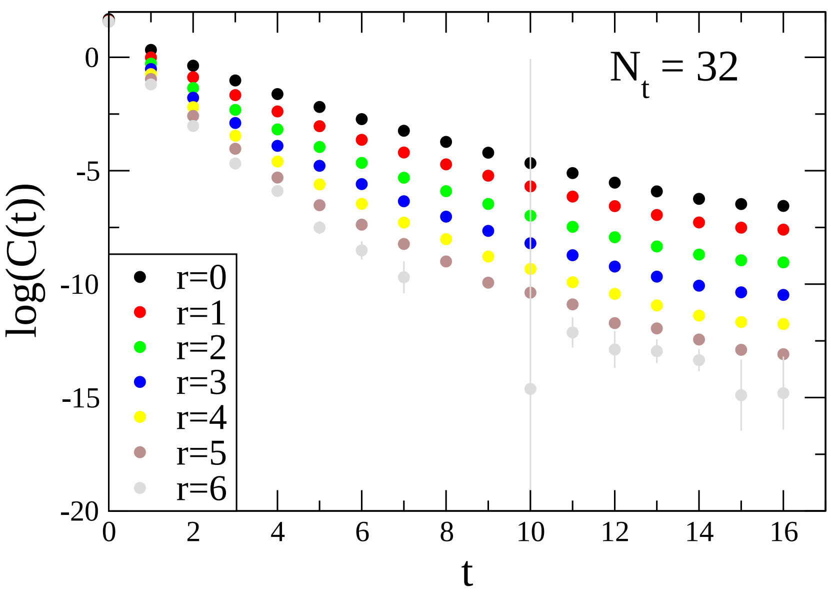

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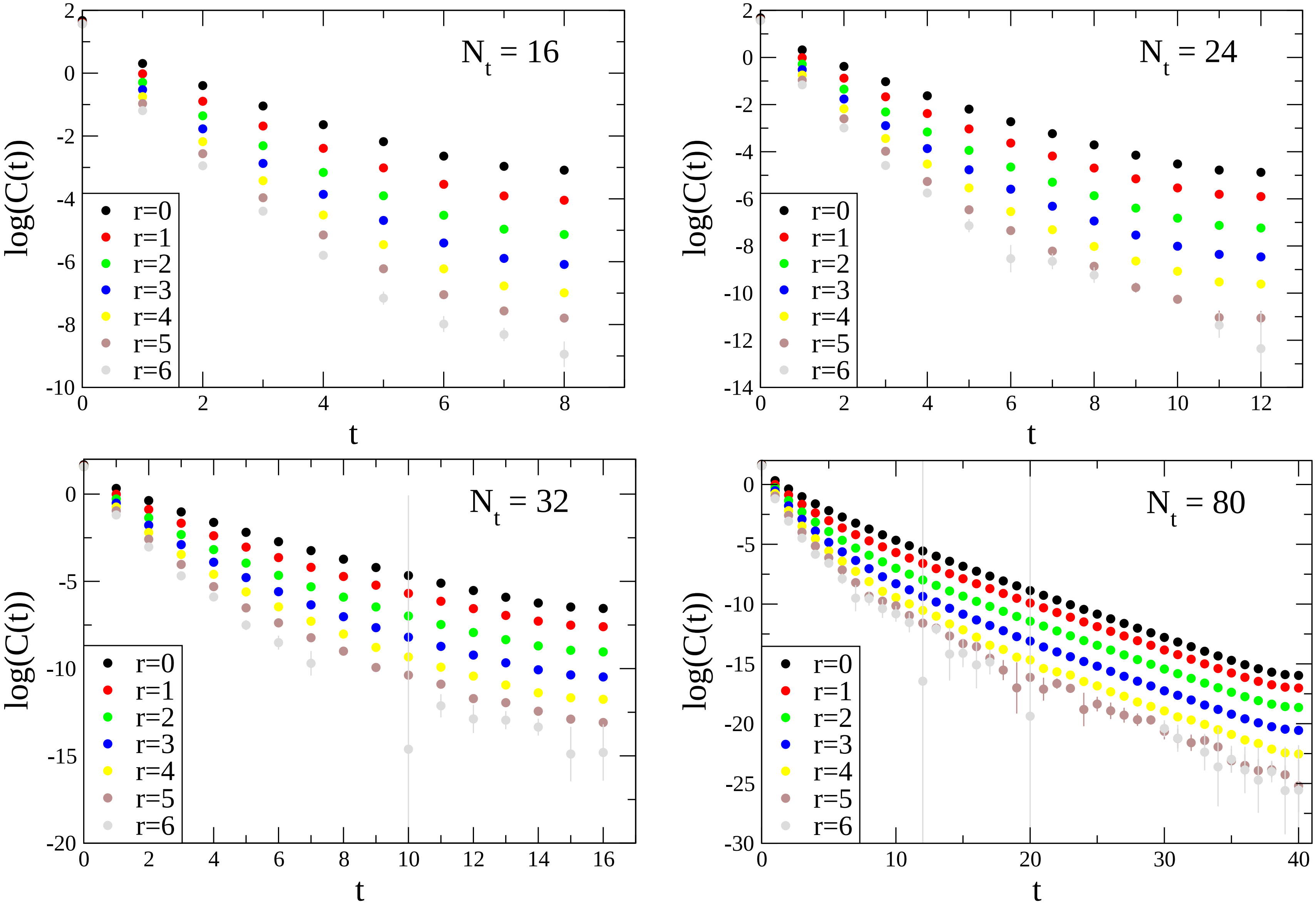
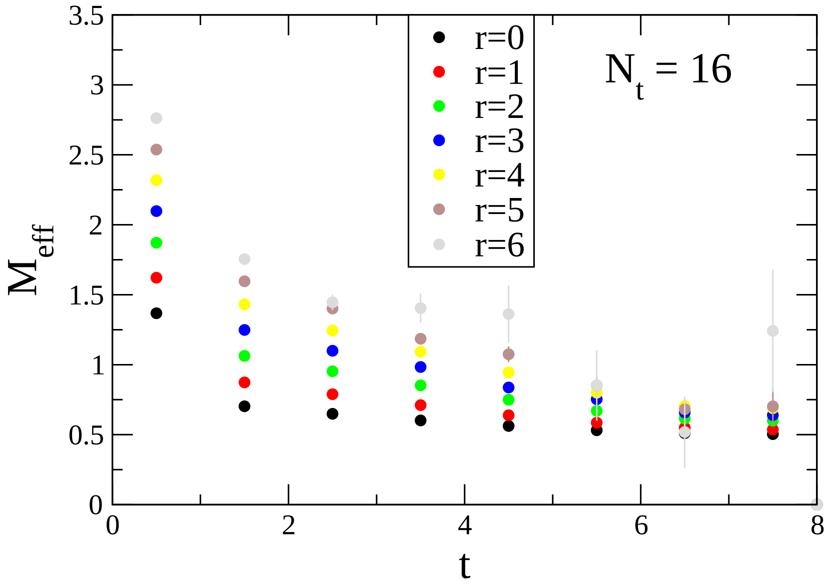


Effective Masses (PS)



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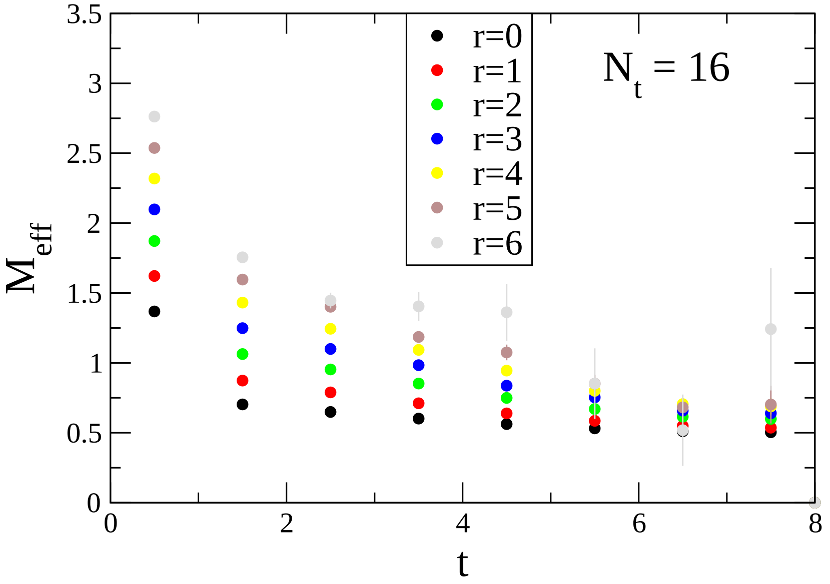

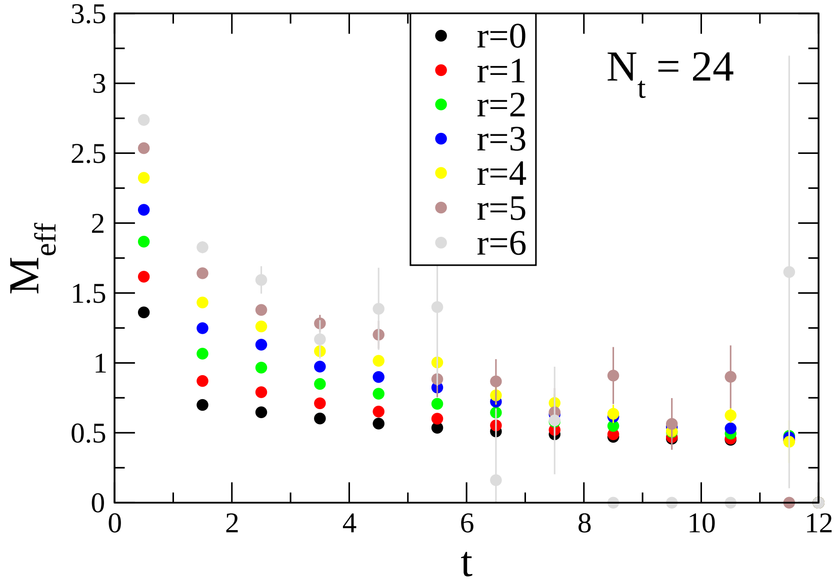

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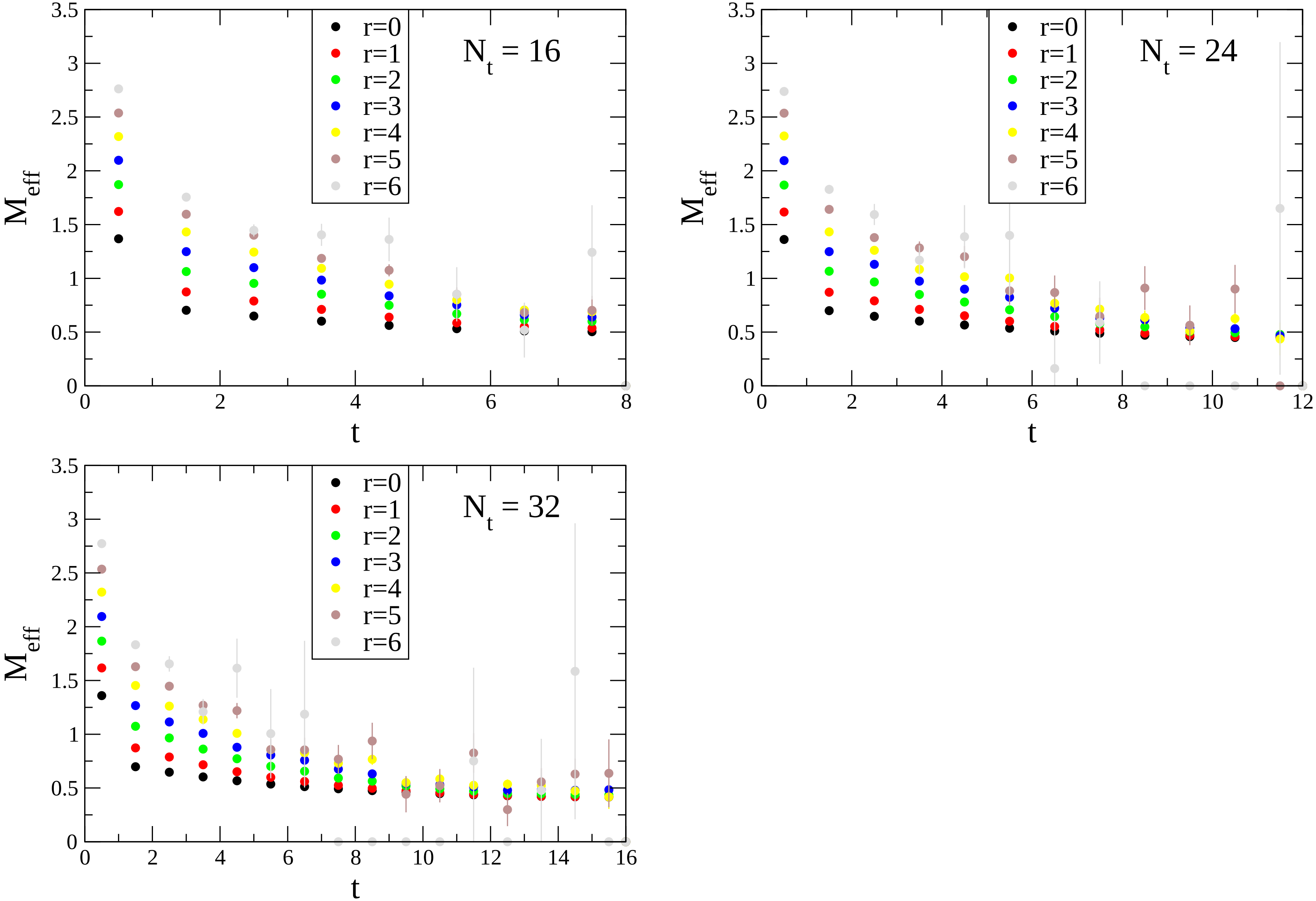

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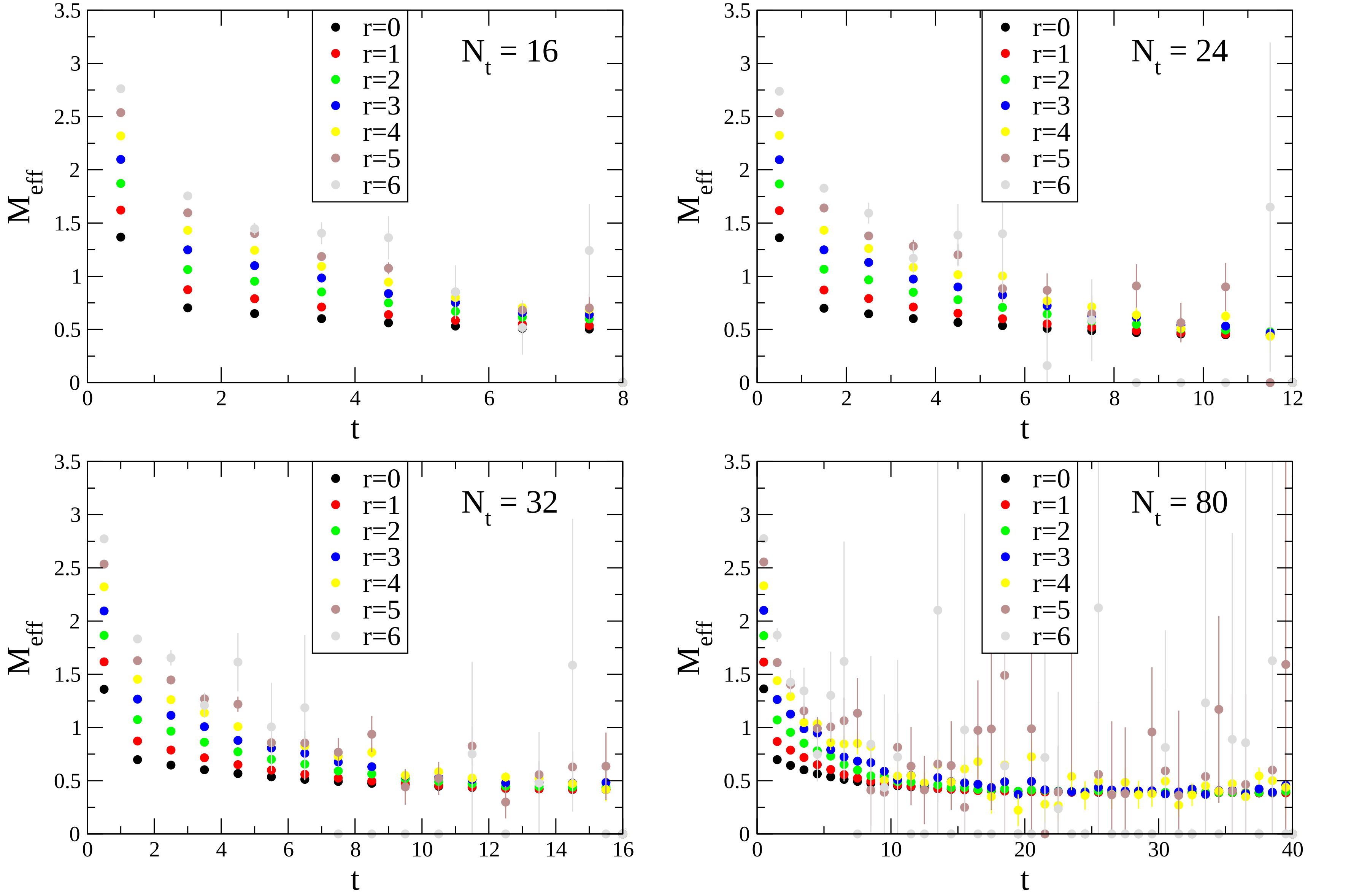
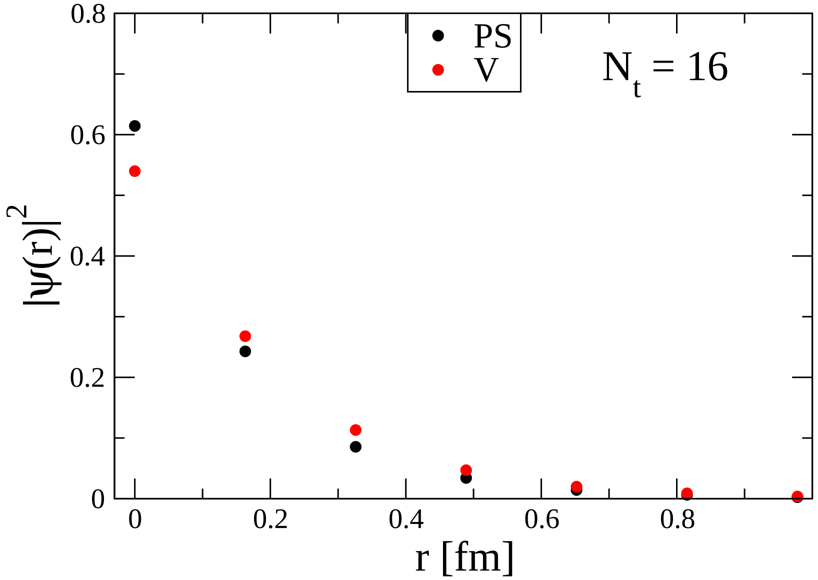


Wavefunctions (exp fitting) $C(t) \to |\psi(r)|^2 e^{-MT}$



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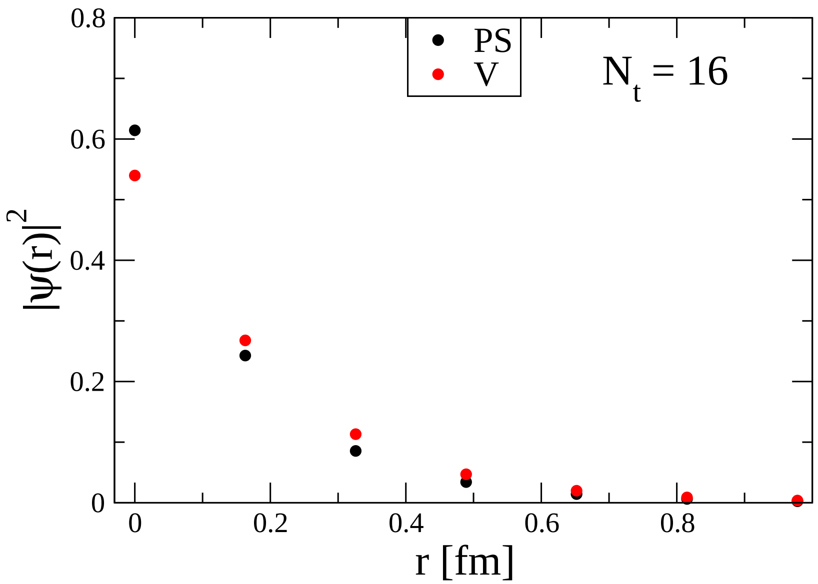

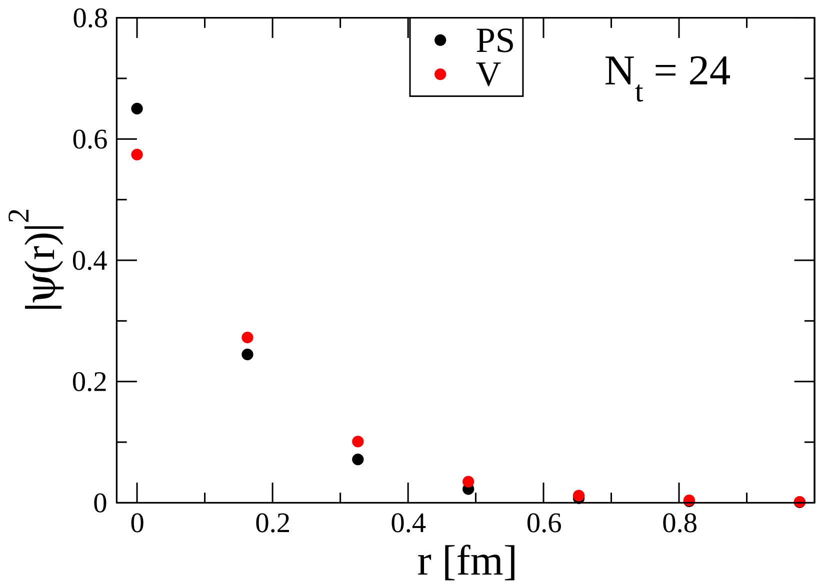

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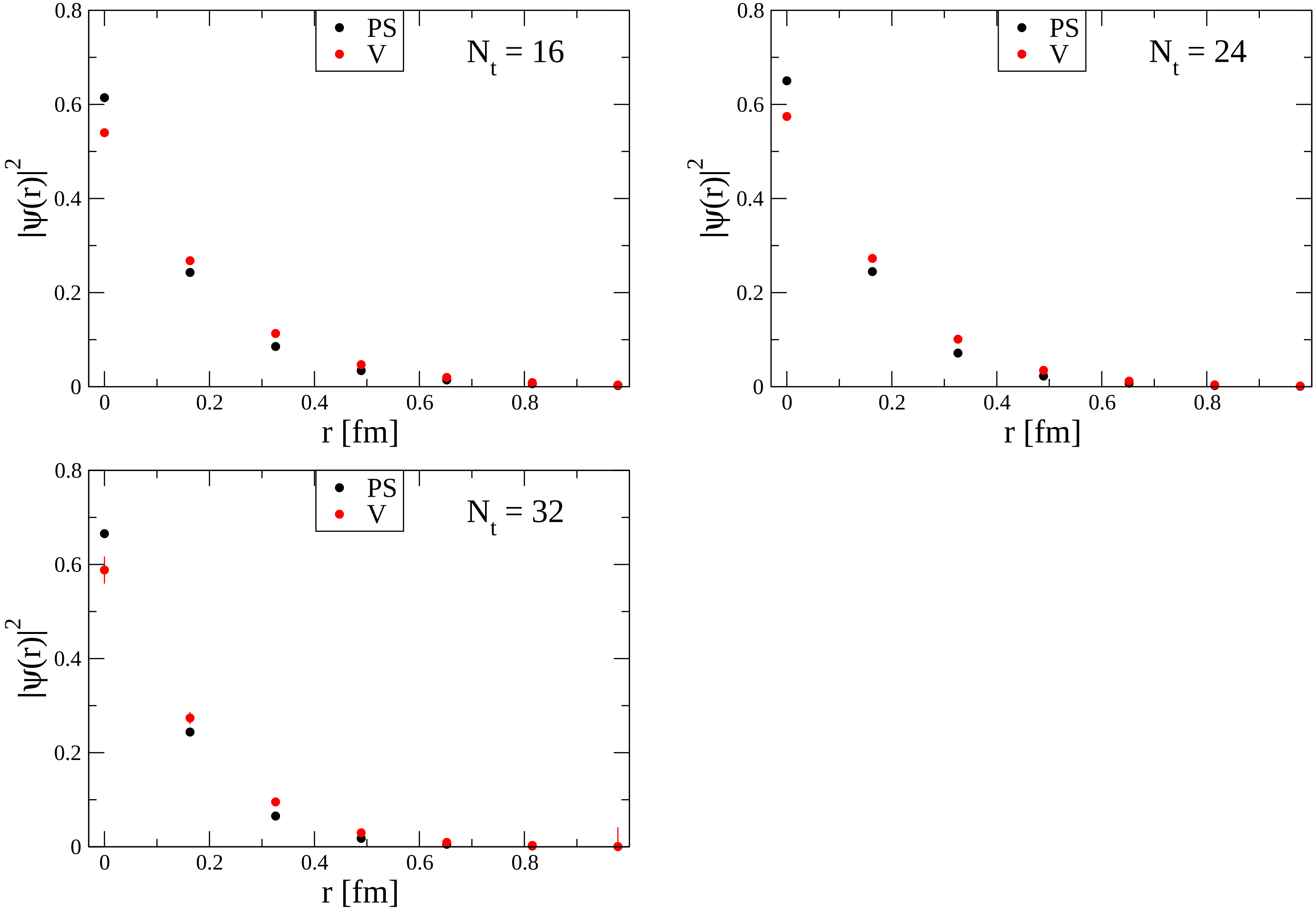

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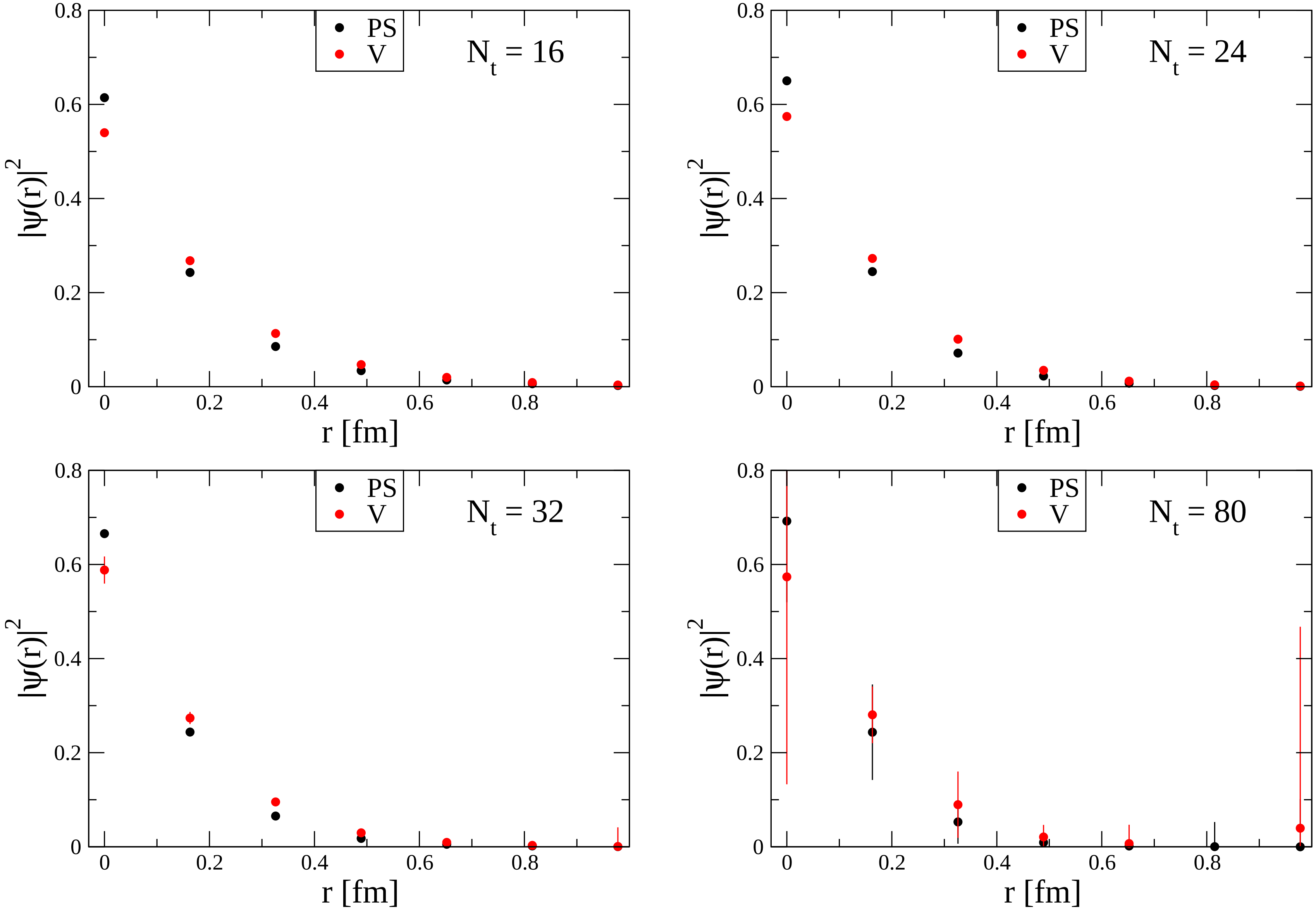


Spin Dependent Potential

$$V_{q\overline{q}} = \frac{1}{4}[V_{\text{PS}}(r) + 3V_V(r)]$$

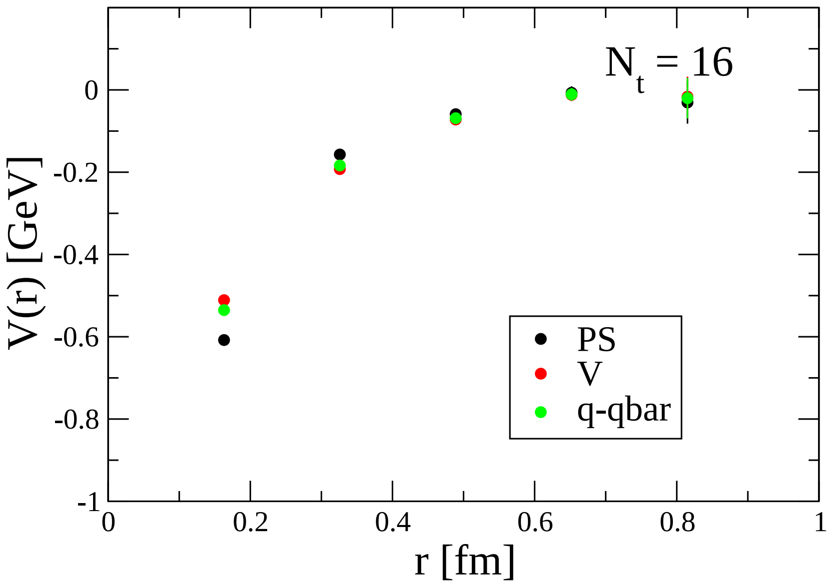
with the Schrödinger Eq'n used to define $V(r)$:

$$V_\Gamma(r) = E + \frac{1}{\psi(r)}\frac{\nabla^2}{2\mu}\psi(r)$$

where μ is the reduced mass:

$$\mu = \frac{1}{2}m_Q \qquad \text{where} \qquad m_Q \approx M_H/2$$

Potential (exp fitting) [Preliminary]



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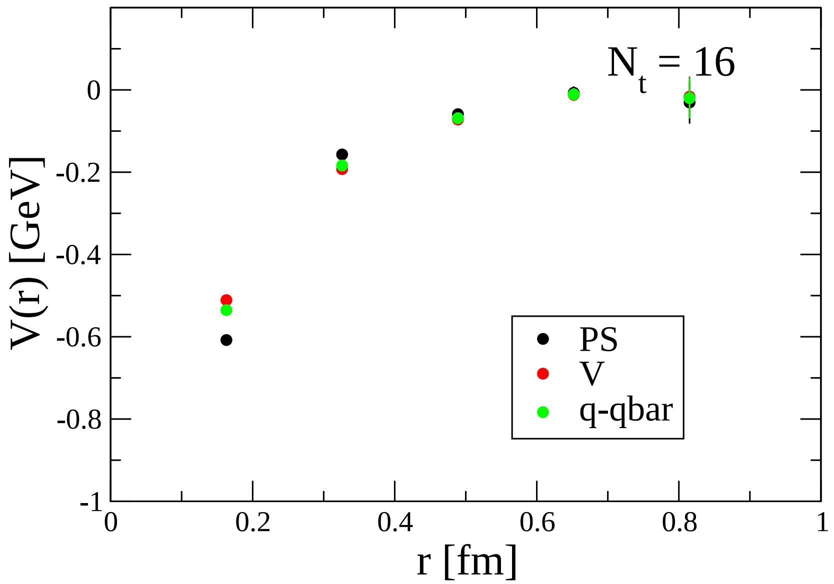

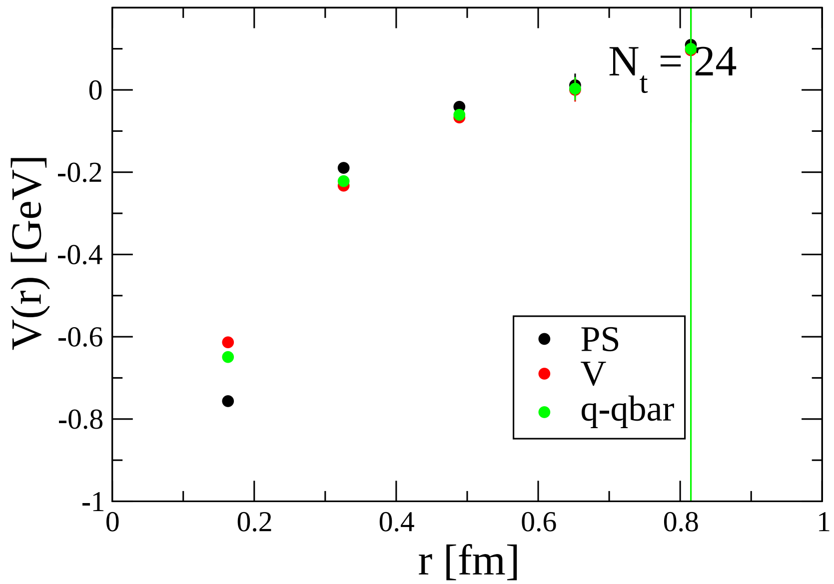

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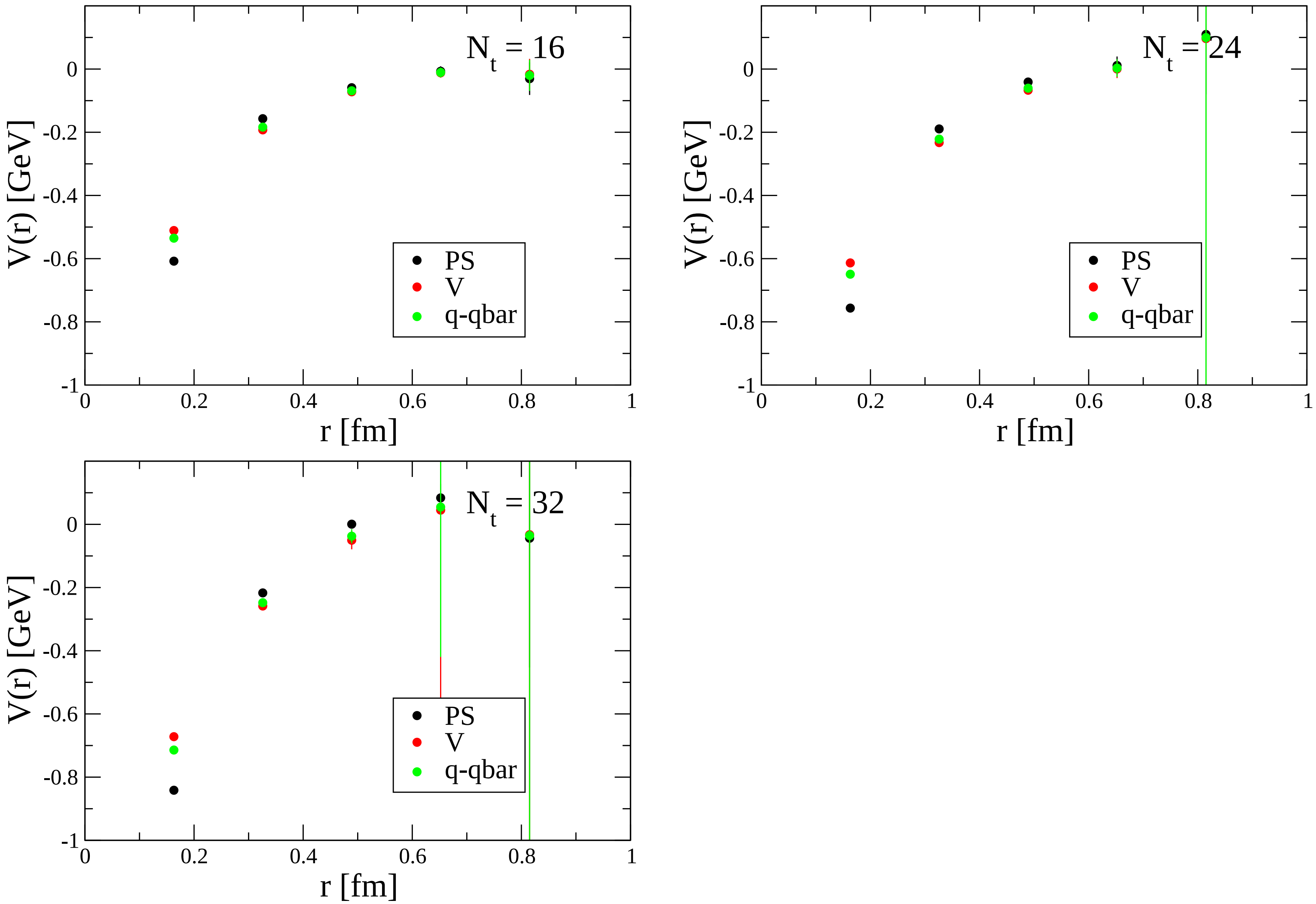

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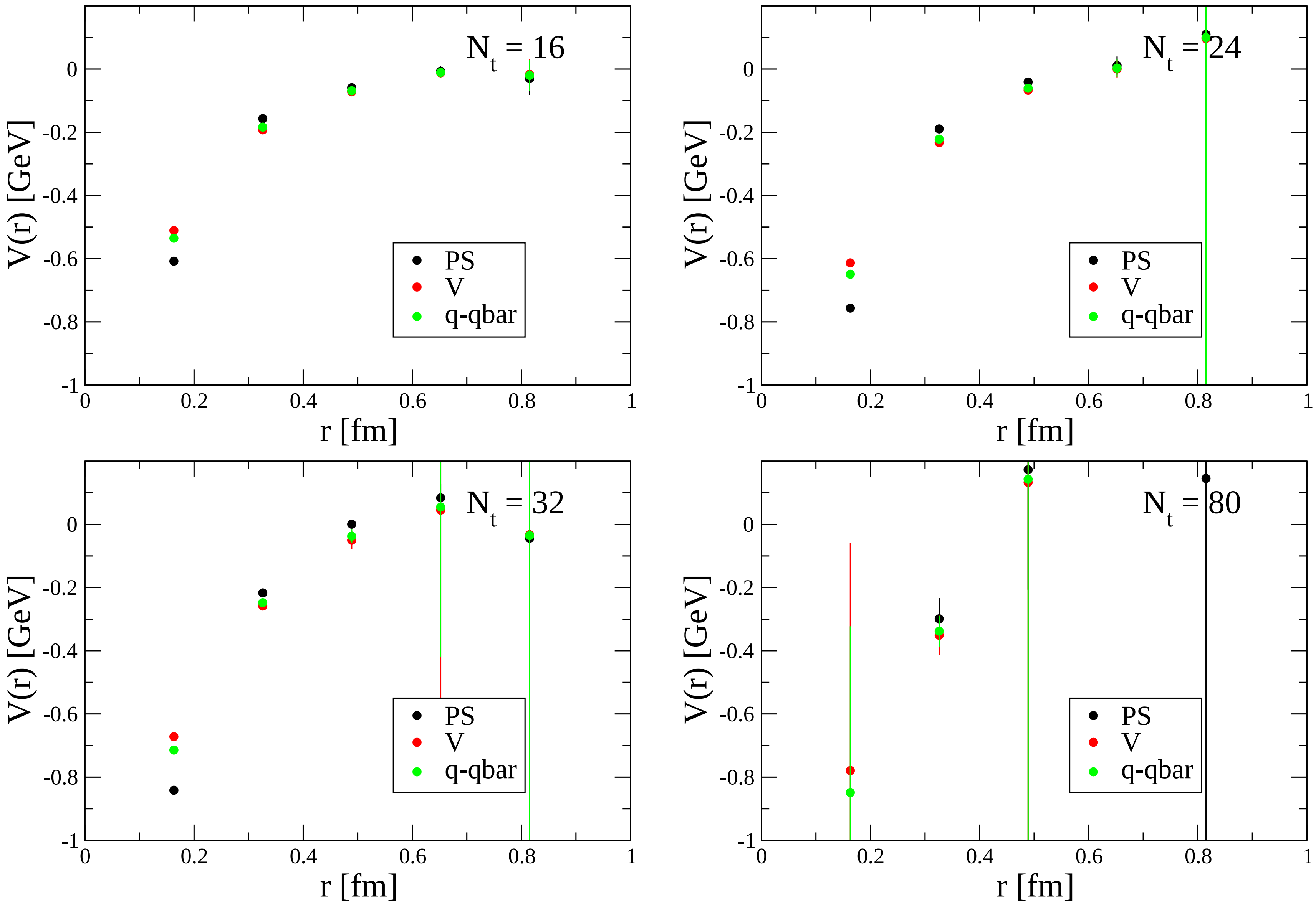
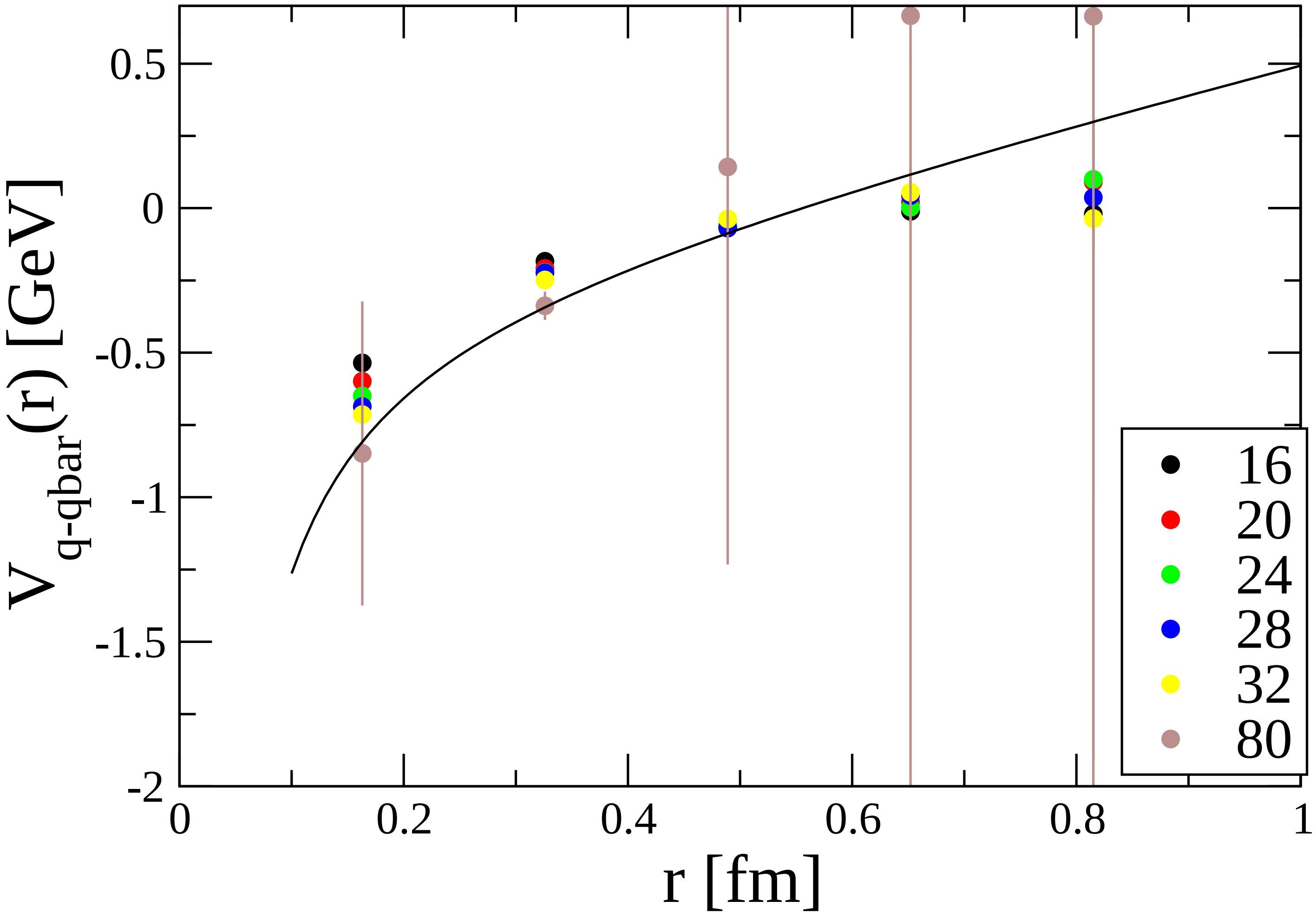


$V_{q-\bar{q}}$ Potential (exp fitting)



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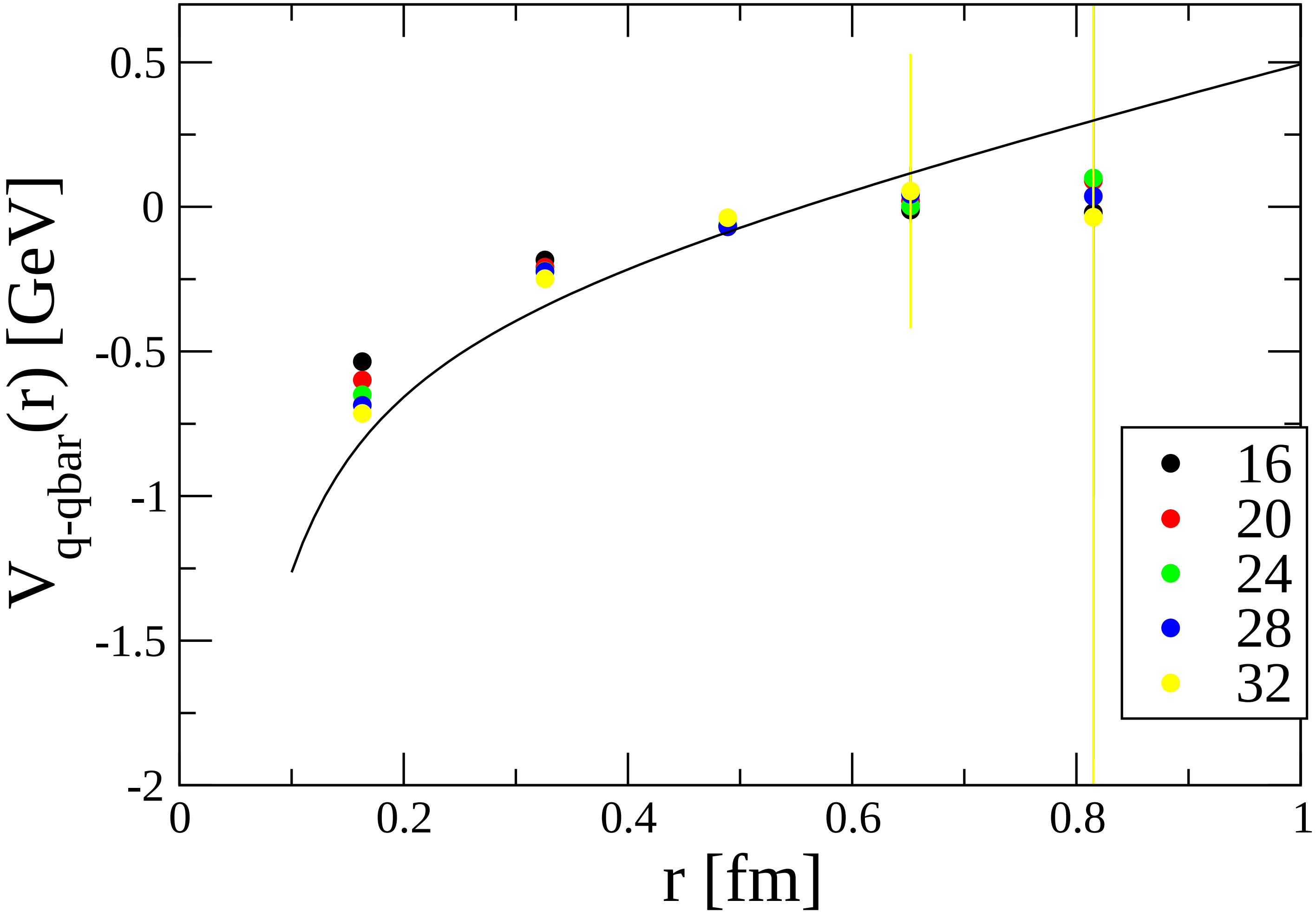


MEM

Motivation

Do bound hadronic states persist into the “quark-gluon” plasma phase?
How can we extract transport coefficients?

- *Spectral functions* can answer this!

$$C(t,\vec{p}) \quad = \quad \int \rho(\omega,\vec{p})\; K(t,\omega)\; d\omega$$

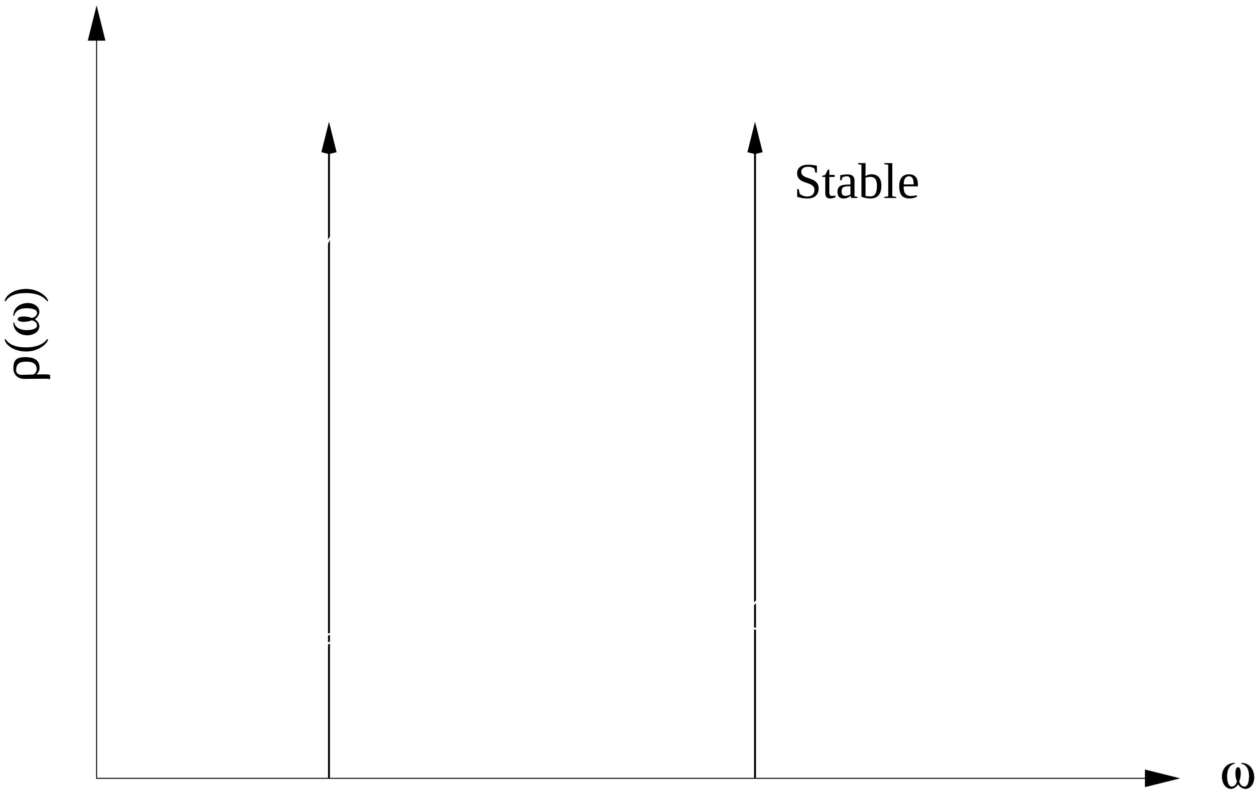
↑	↓	↖
Euclidean Correlator	Spectral Function	(Lattice) Kernel

where the (lattice) Kernel is:

$$\begin{aligned} K(t,\omega) &= \frac{\cosh[\omega(t-N_t/2)]}{\sinh[\omega/(2T)]} \\ &\sim \exp[-\omega t] \end{aligned}$$

Example Spectral Functions

$$C(t) \sim \int \rho(\omega)\; e^{-\omega t}\; d\omega$$



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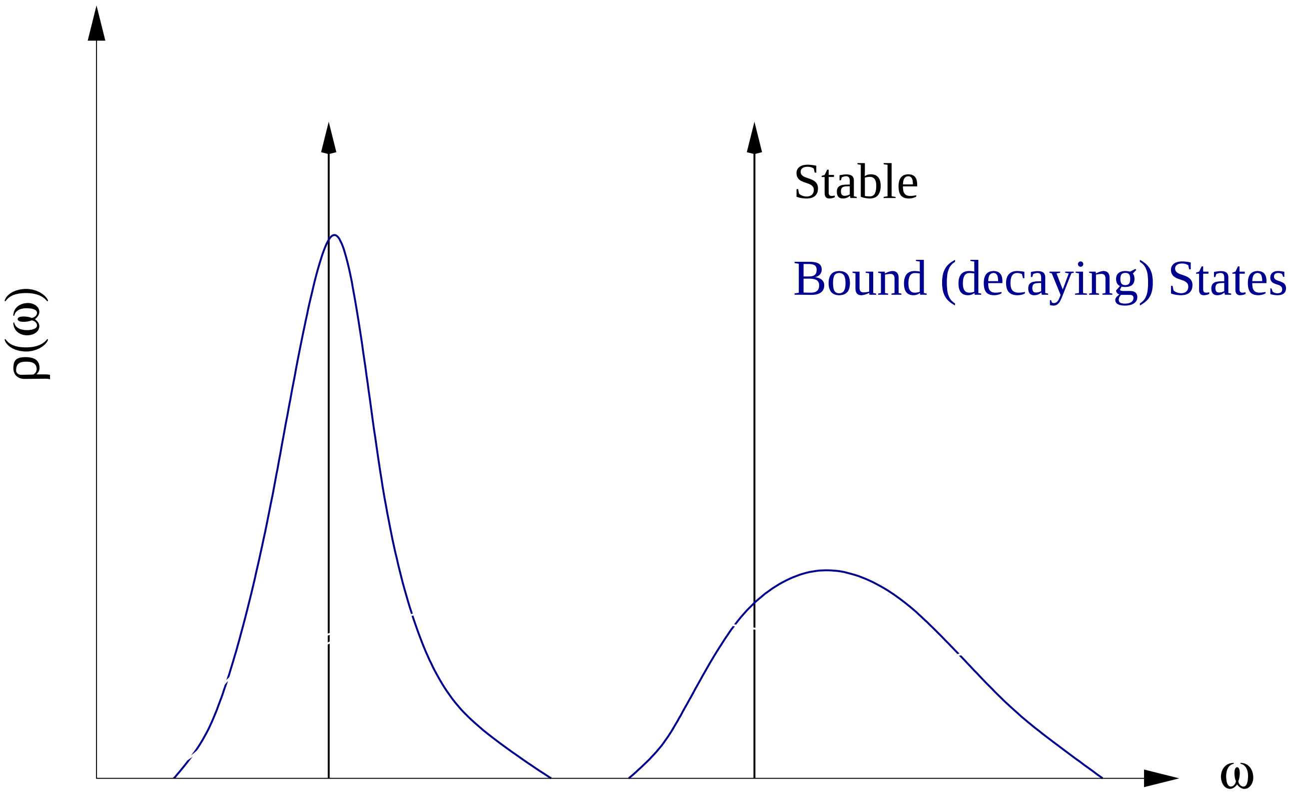

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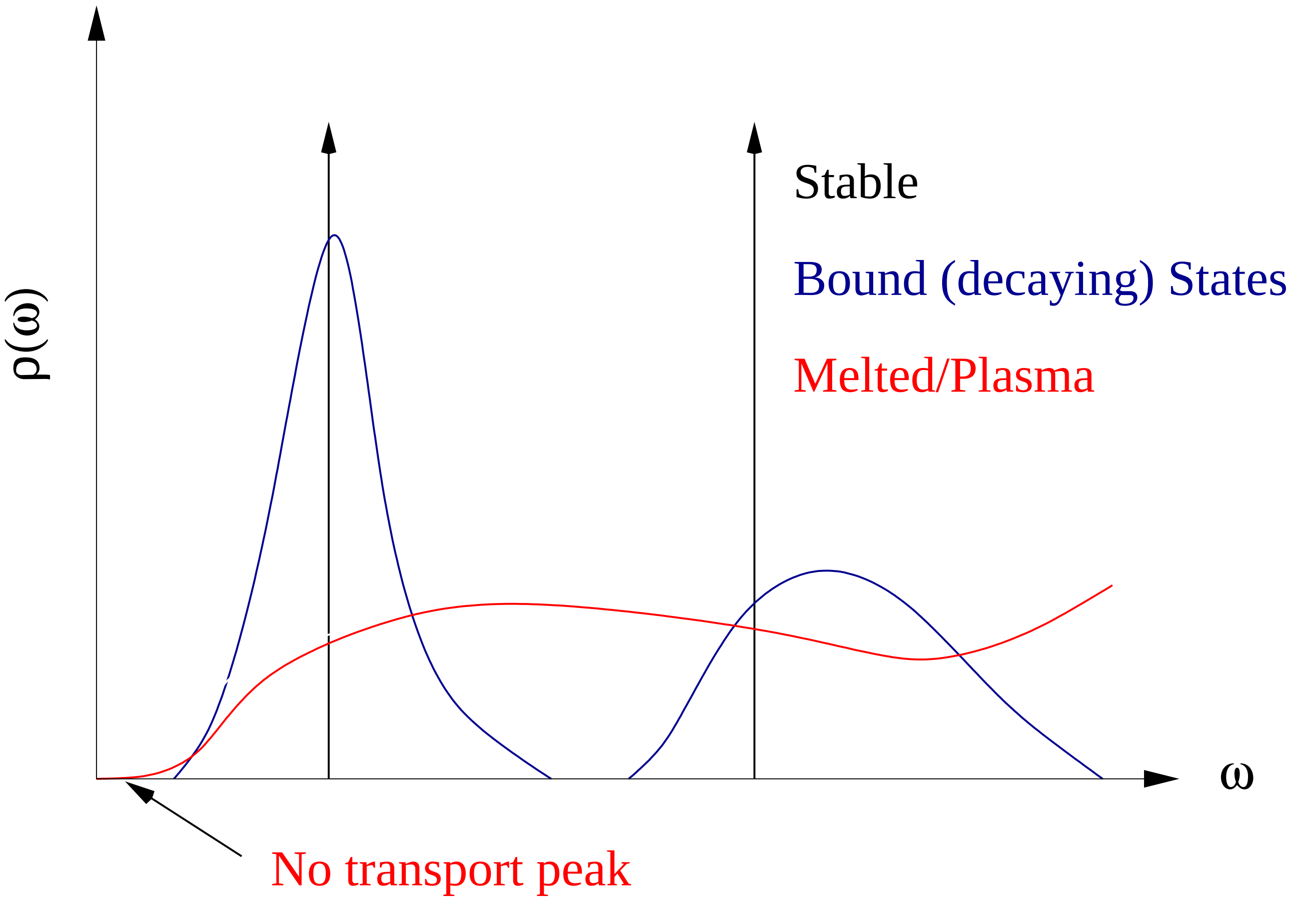

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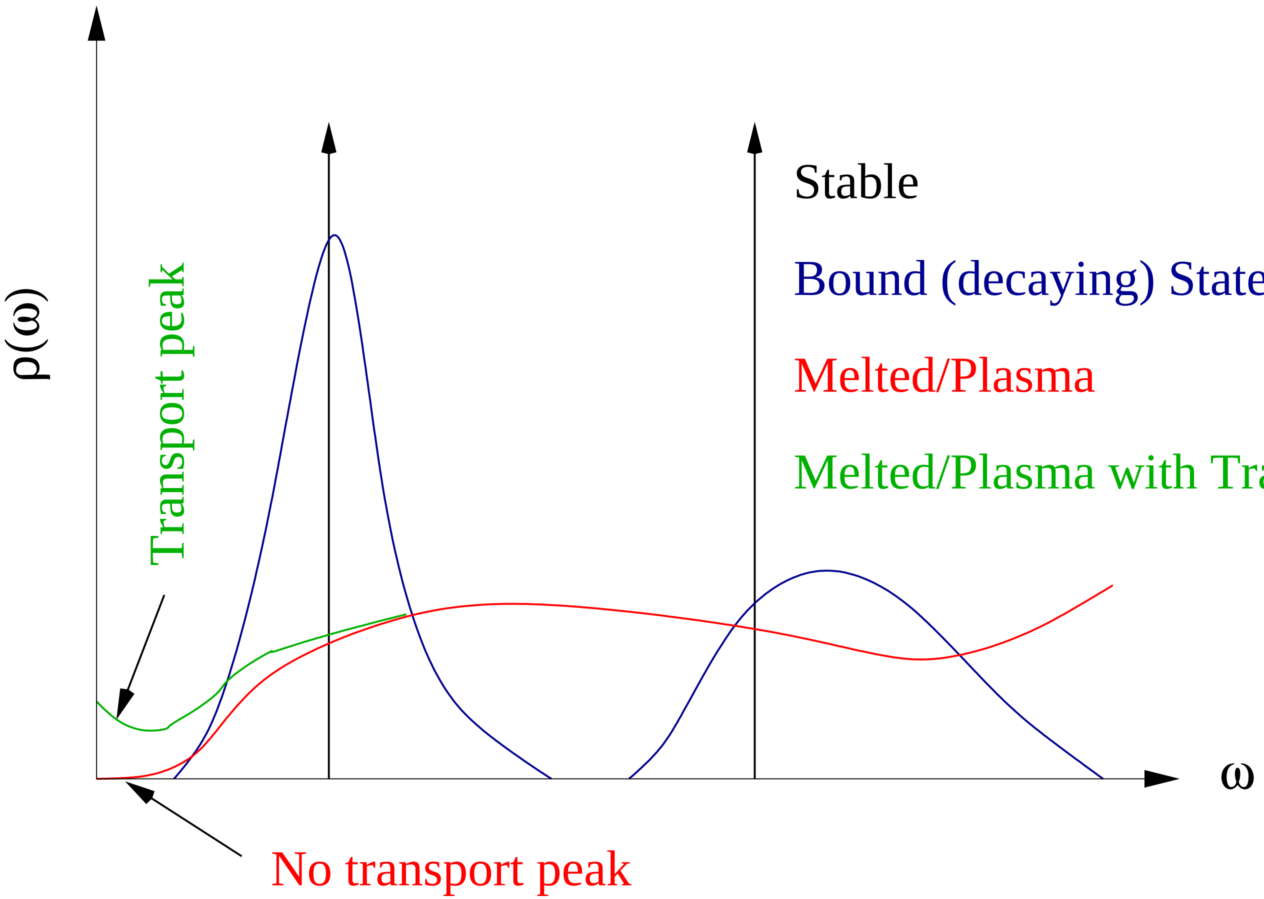
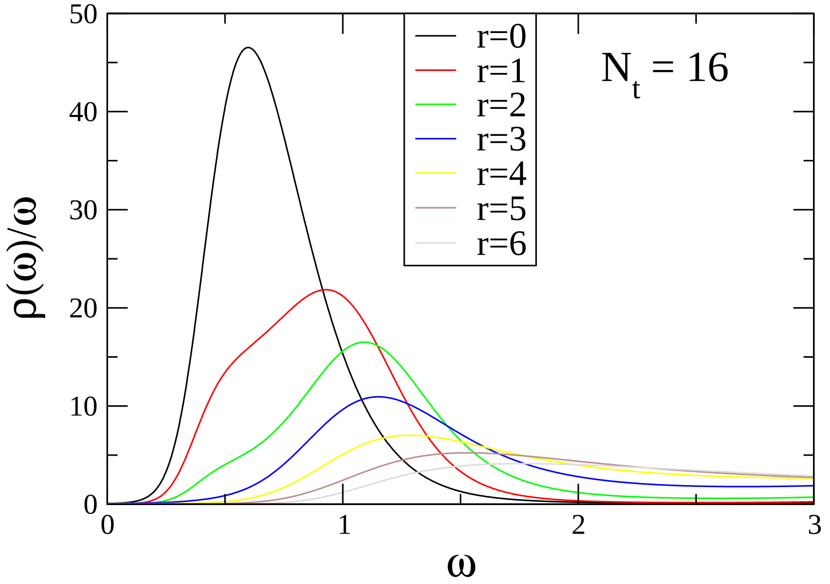


Spectral Functions via MEM

- Extraction of a spectral density from a lattice correlator is an ill-posed problem:
  - *Given* $C(t)$ *derive* $\rho(\omega)$
  - *More* ω *data points then* t *data points!*
- Requires the use of Bayesian analysis - Maximum Entropy Method (MEM)
  - Hatsuda, Asakawa et al
  - Commonly used in other areas...
- Need to check MEM output w.r.t. choice of:
  - Default model
  - Statistics
  - Energy range
  - Euclidean time range

Spectral Functions (PS)



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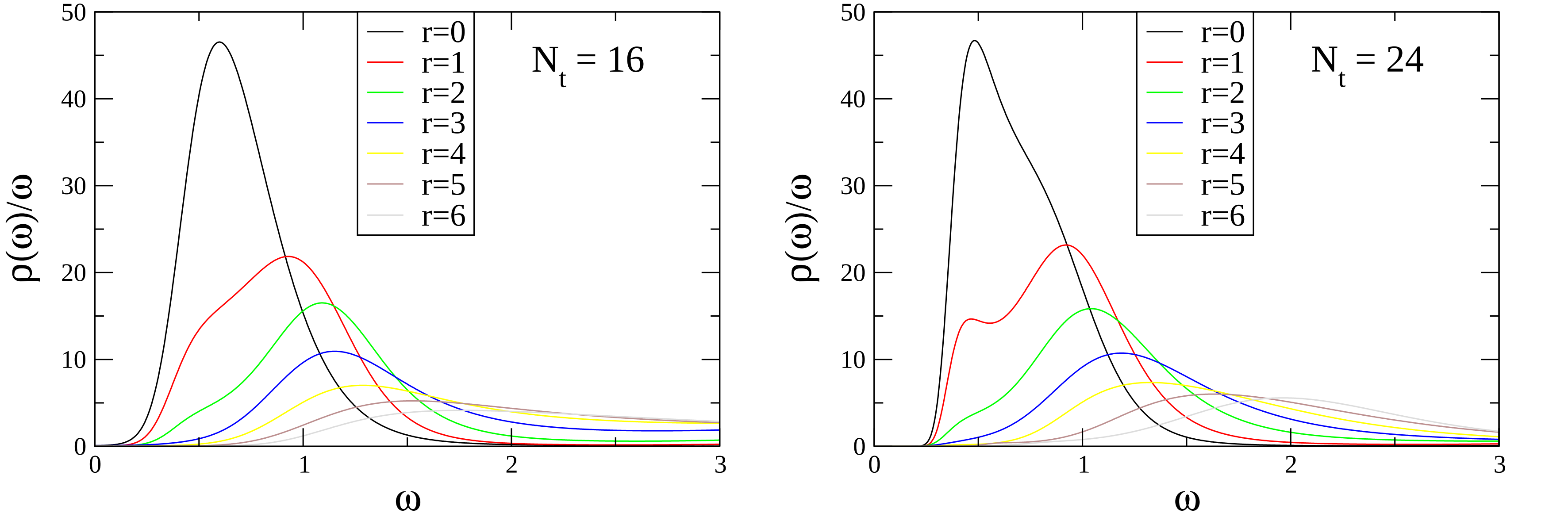

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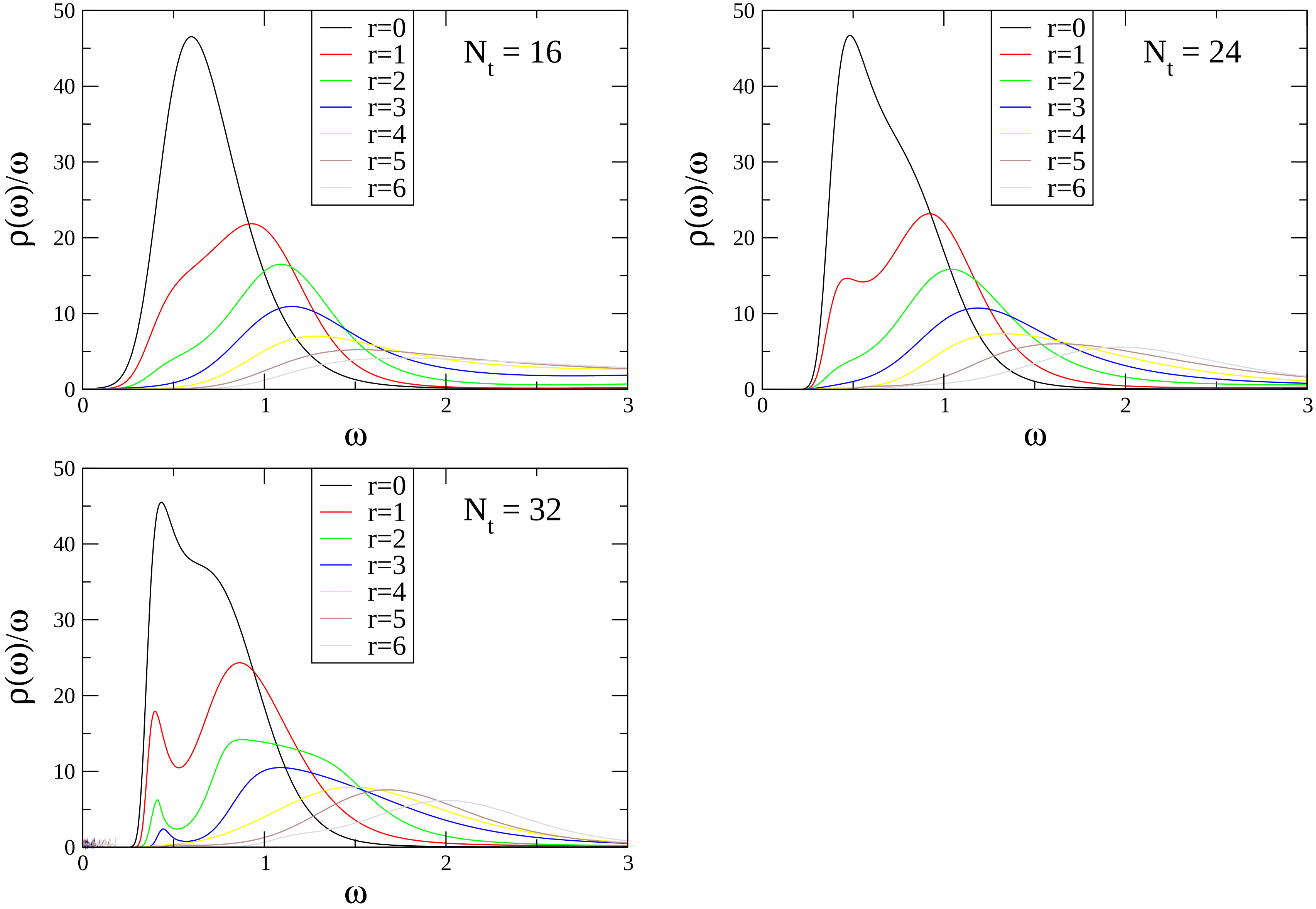

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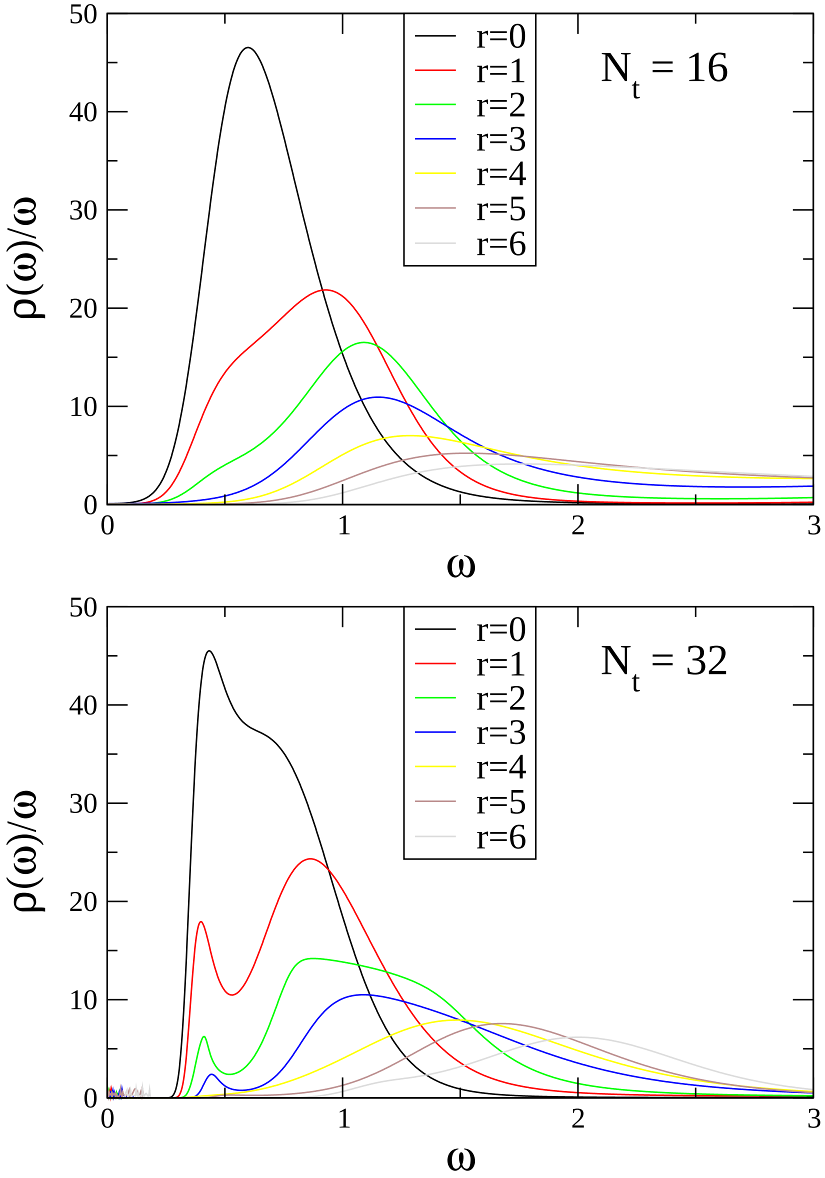
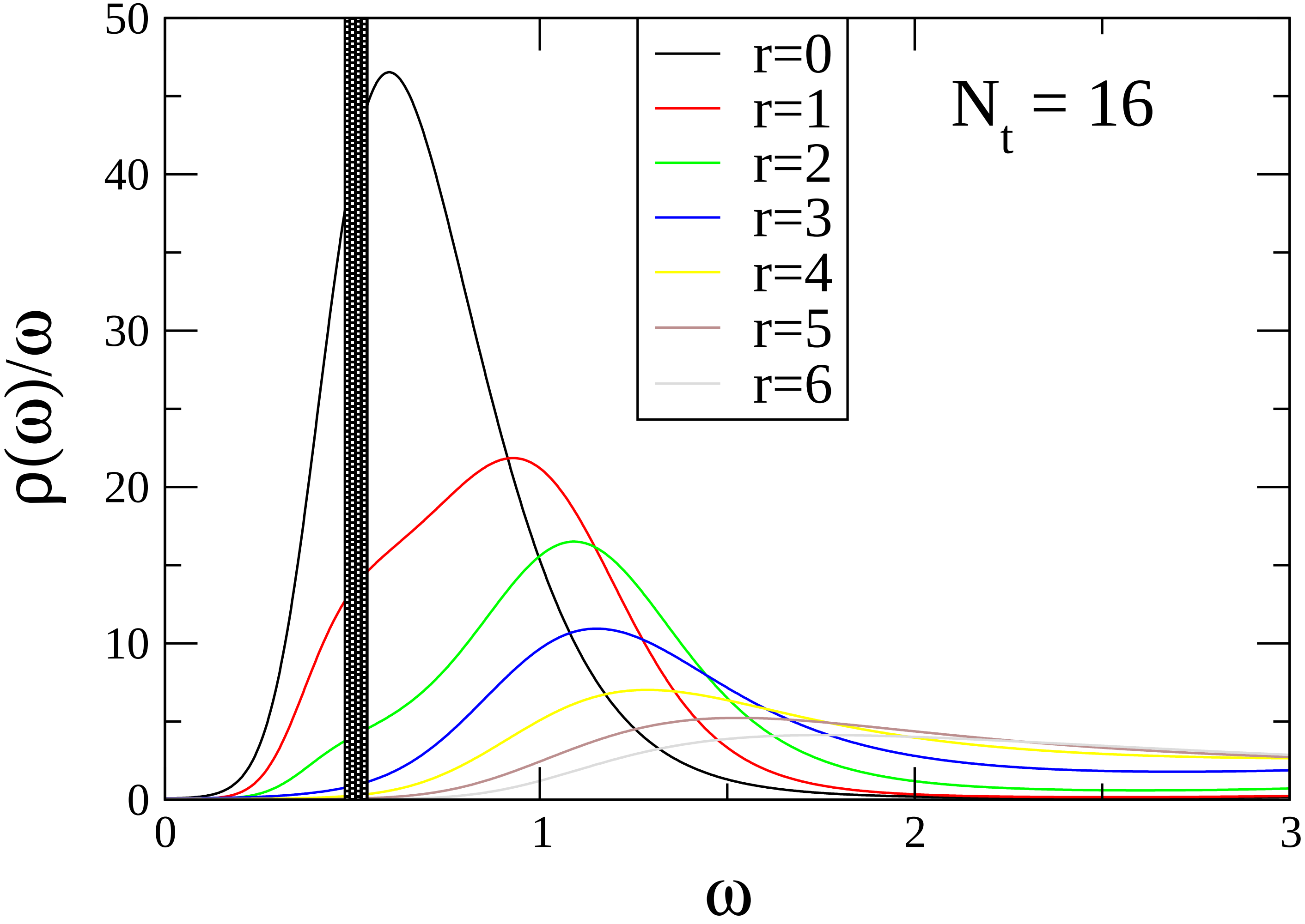


$$C(t) \sim \int \rho(\omega)\, e^{-\omega t}\, d\omega$$

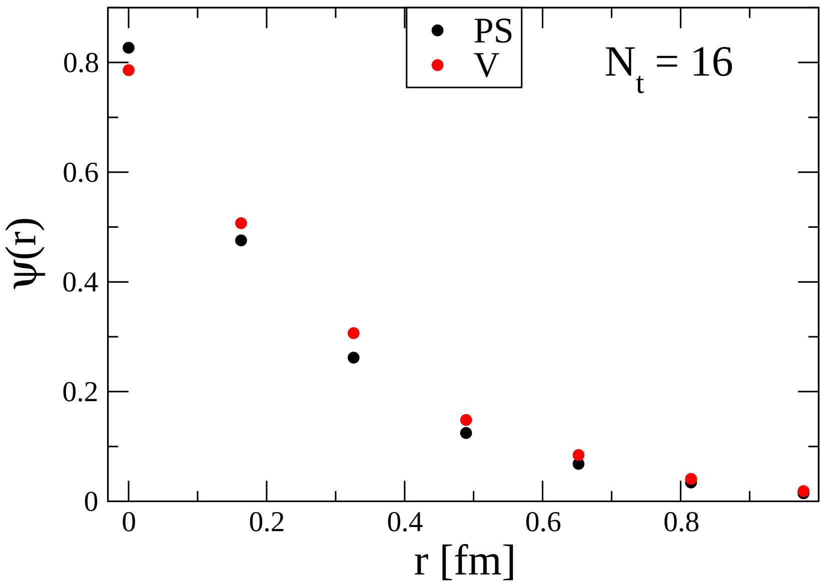
But $\rho(\omega) \sim |\psi(r)|^2$

Spectral Functions (PS)



Range in ω spans the ground state mass from exp fit

Wavefunctions (MEM)



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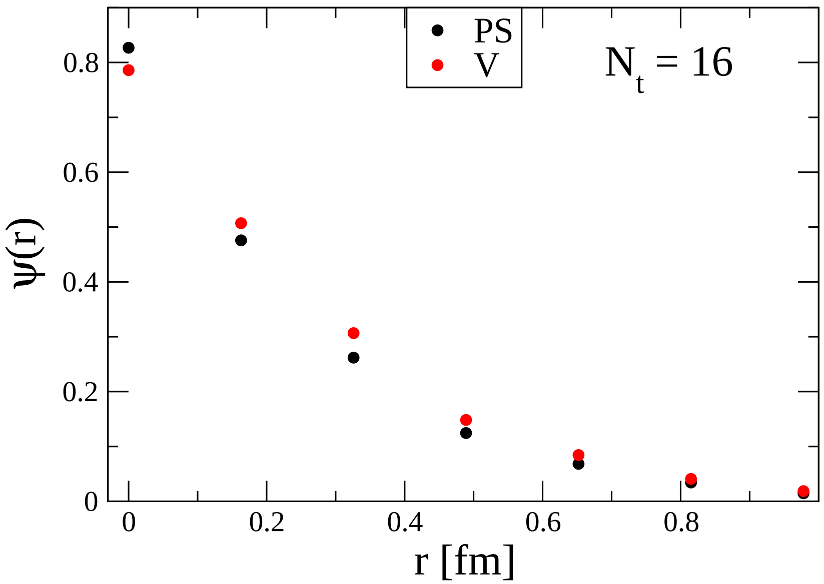

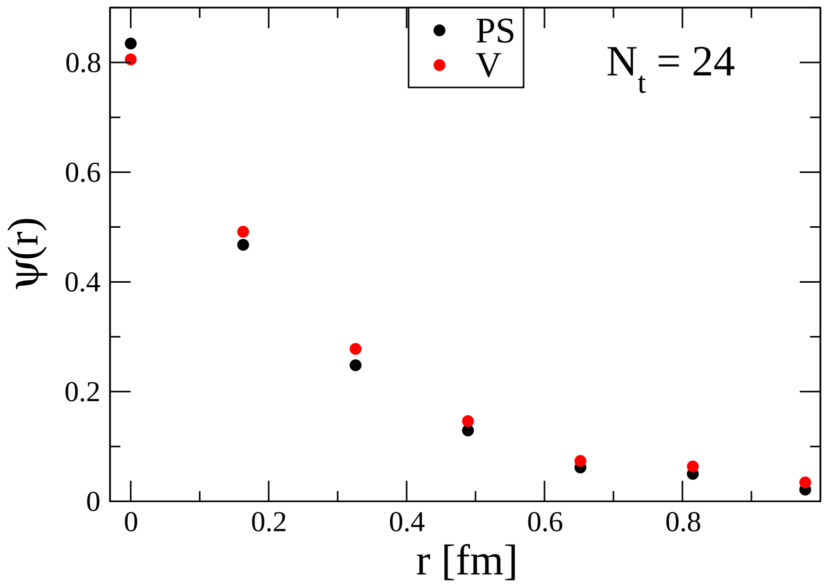

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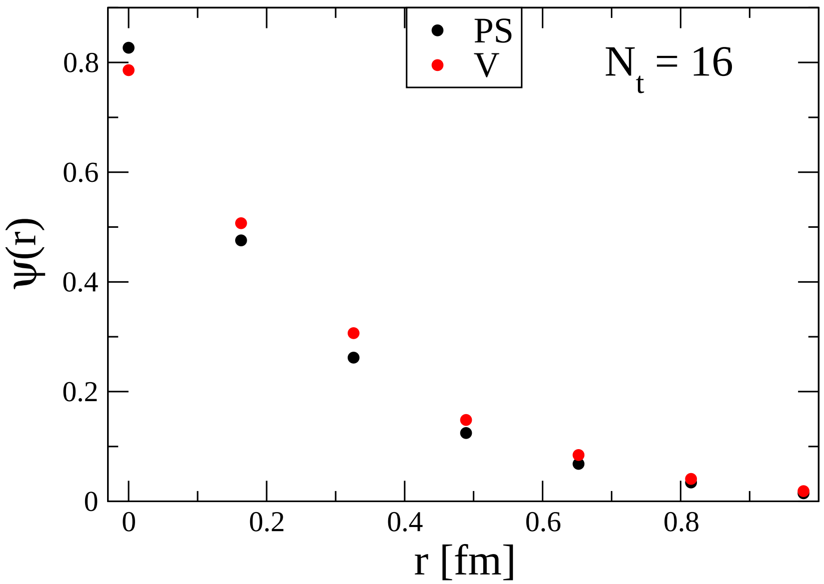

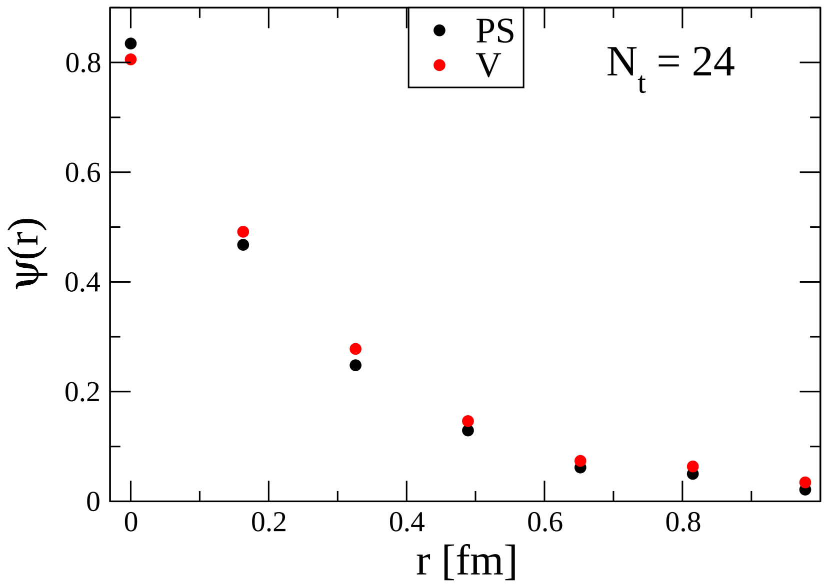

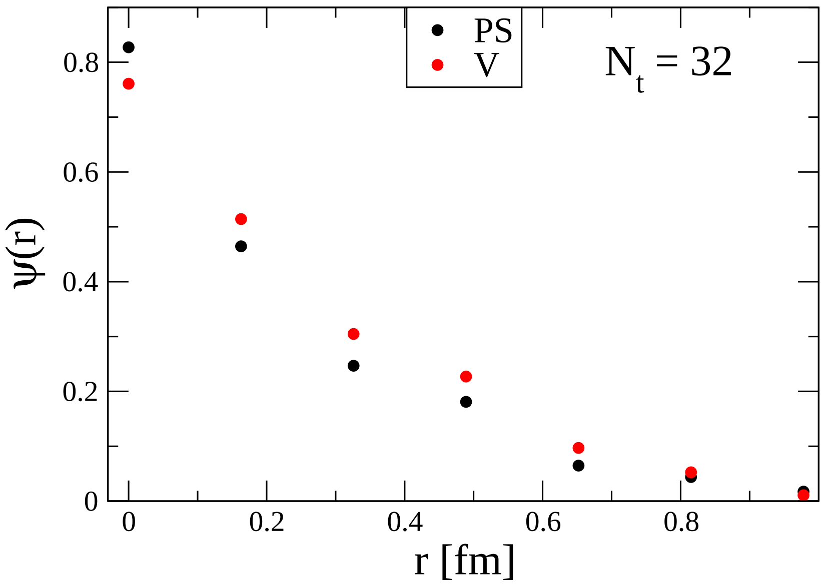

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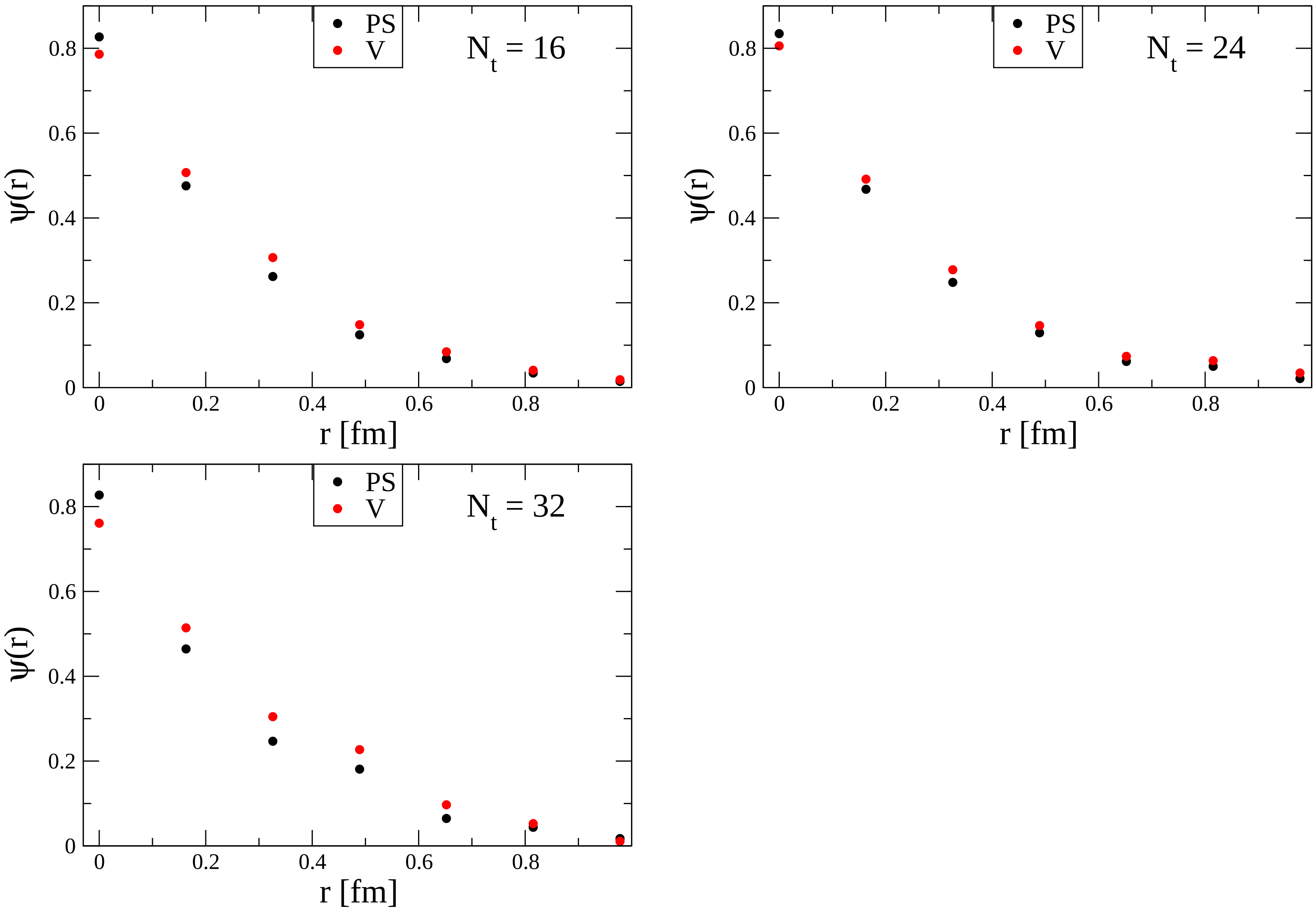
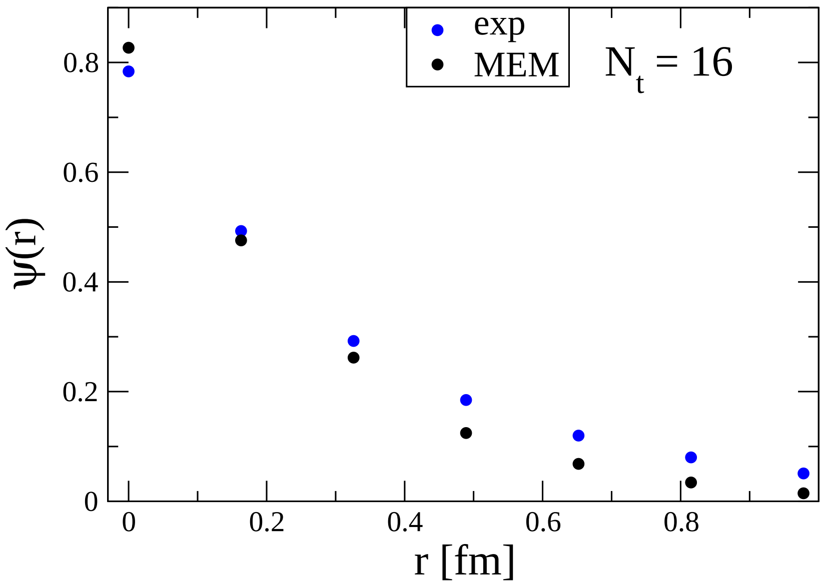


Wavefunctions: MEM v exp (PS)



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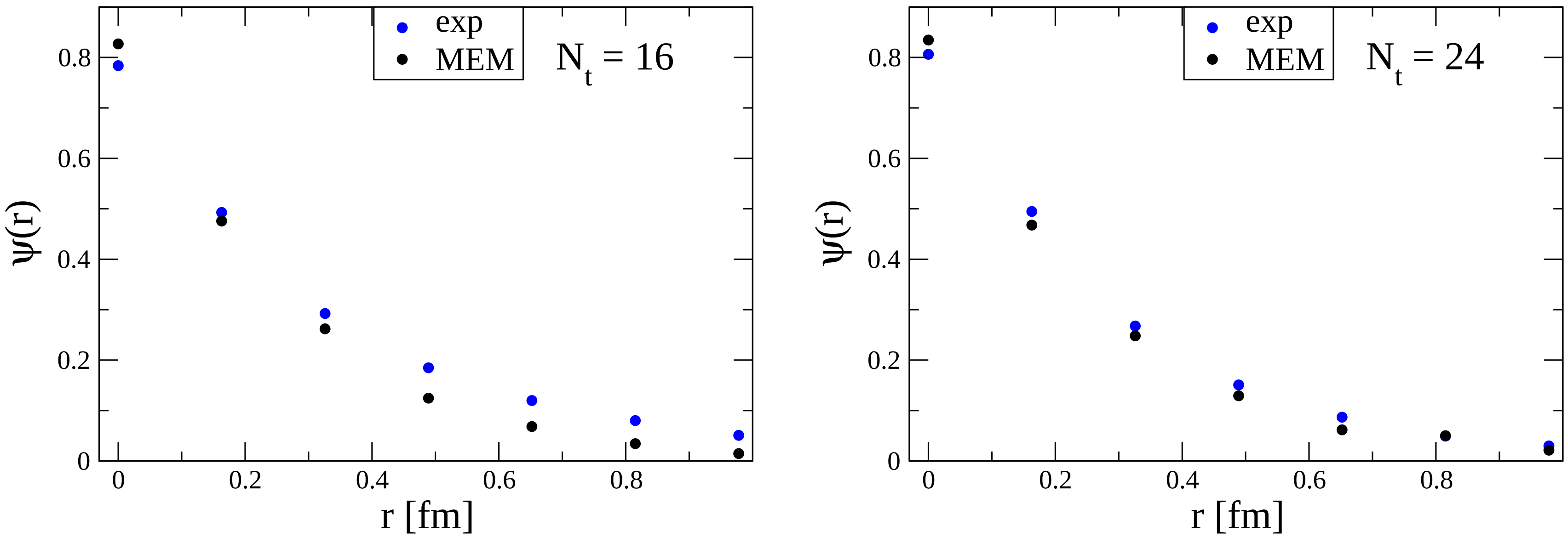

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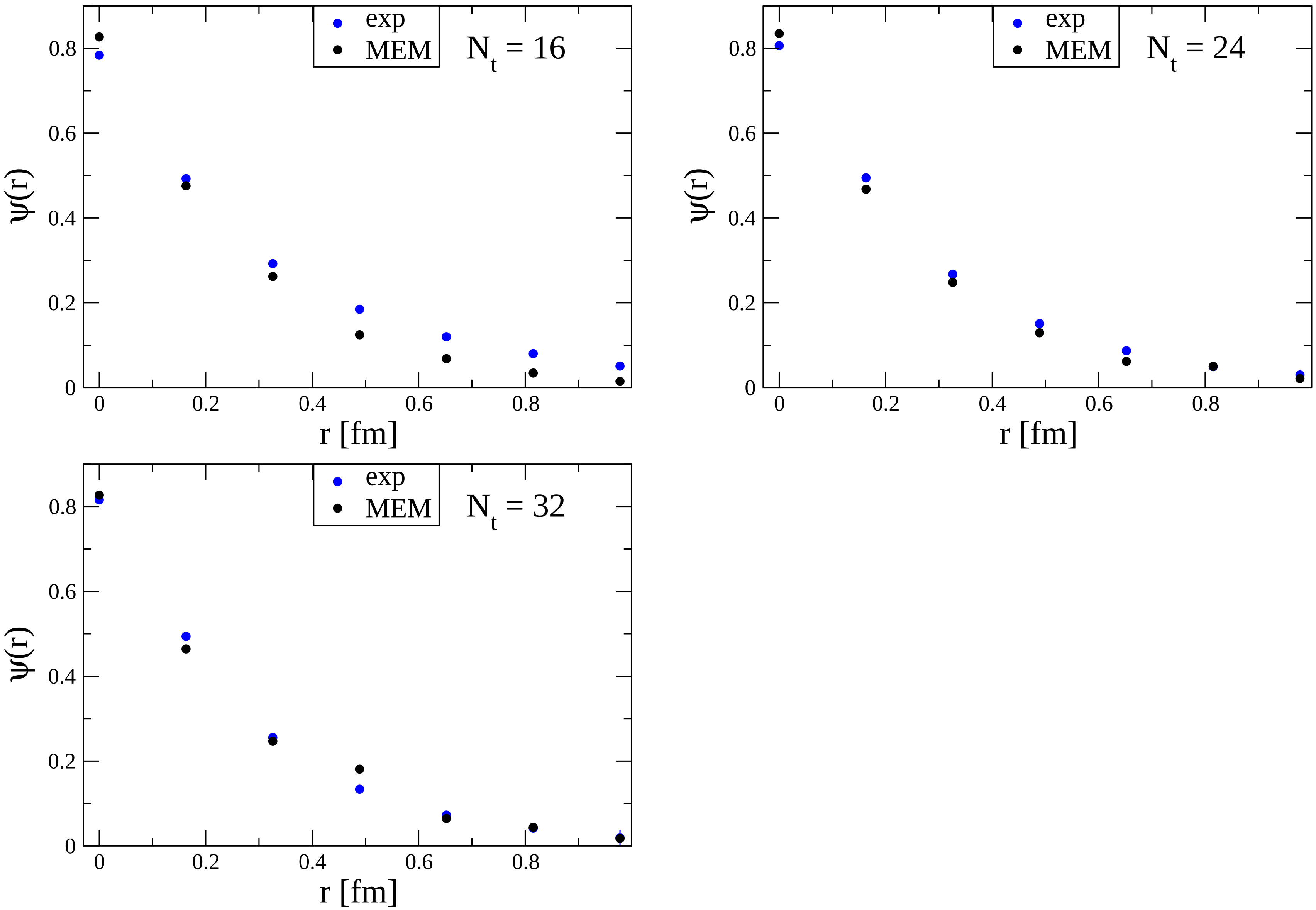

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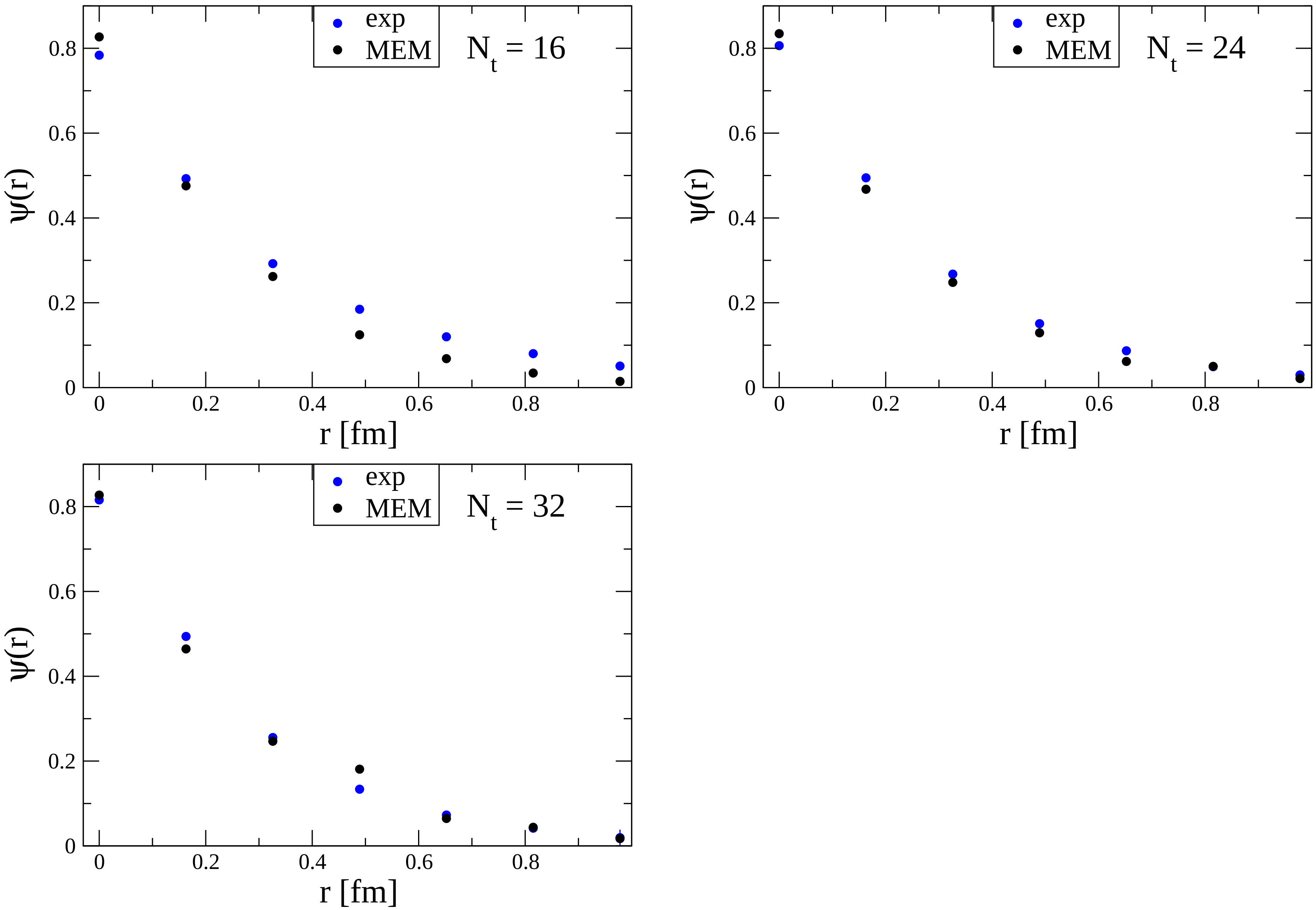
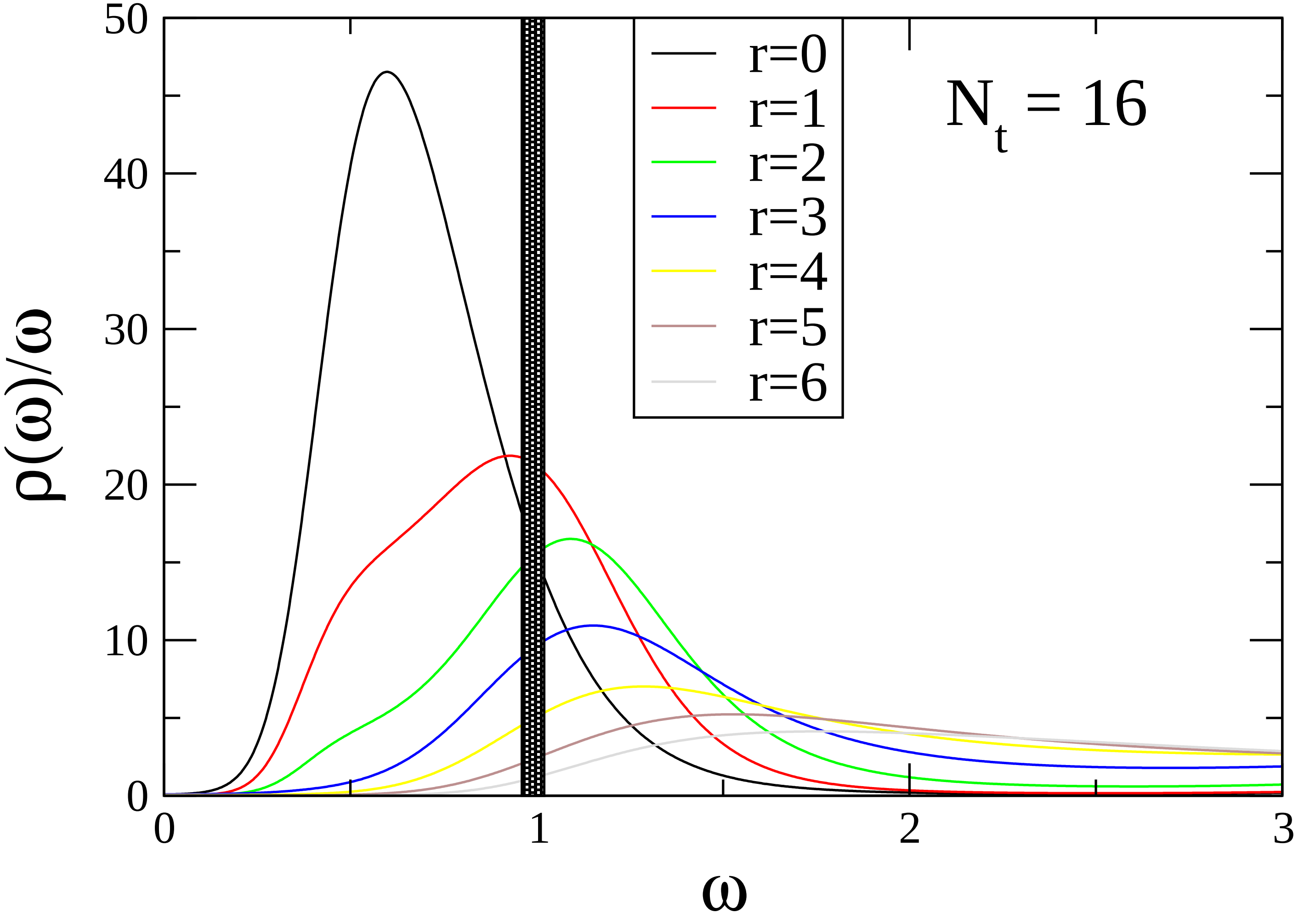


Spectral Functions: Excited State (PS)



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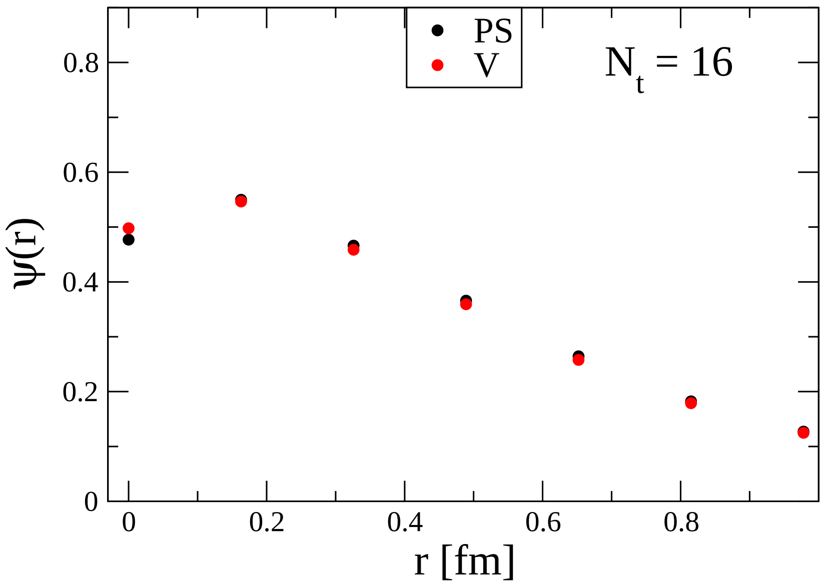

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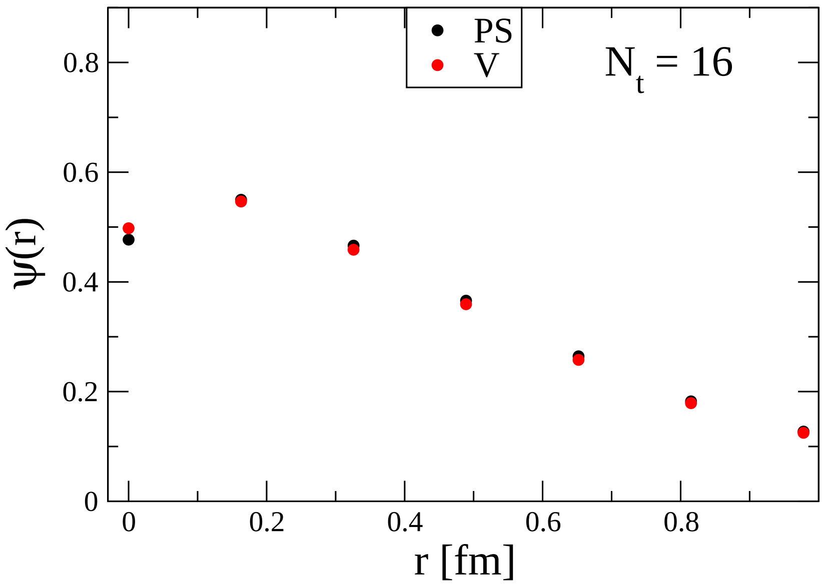

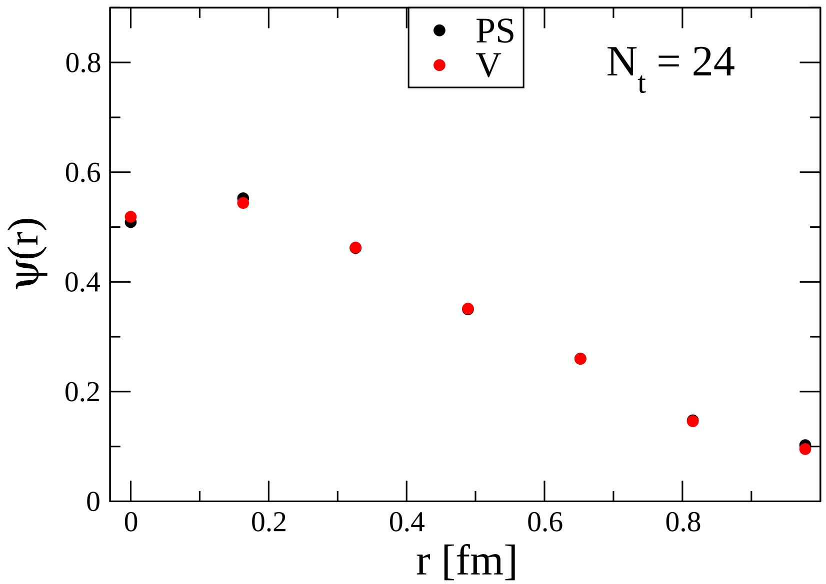

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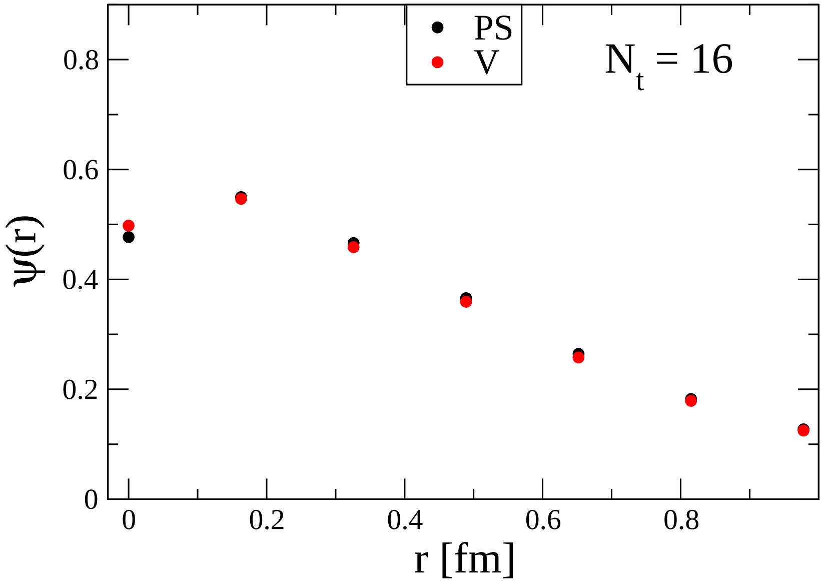

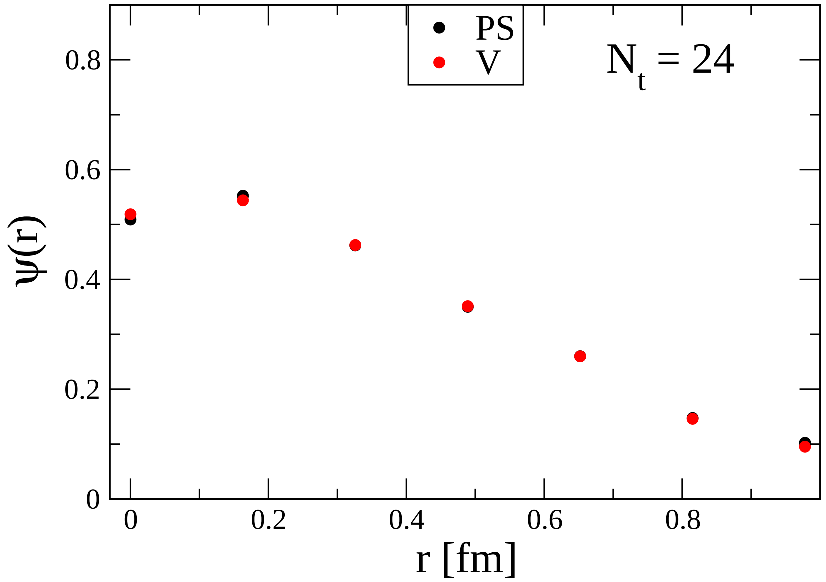

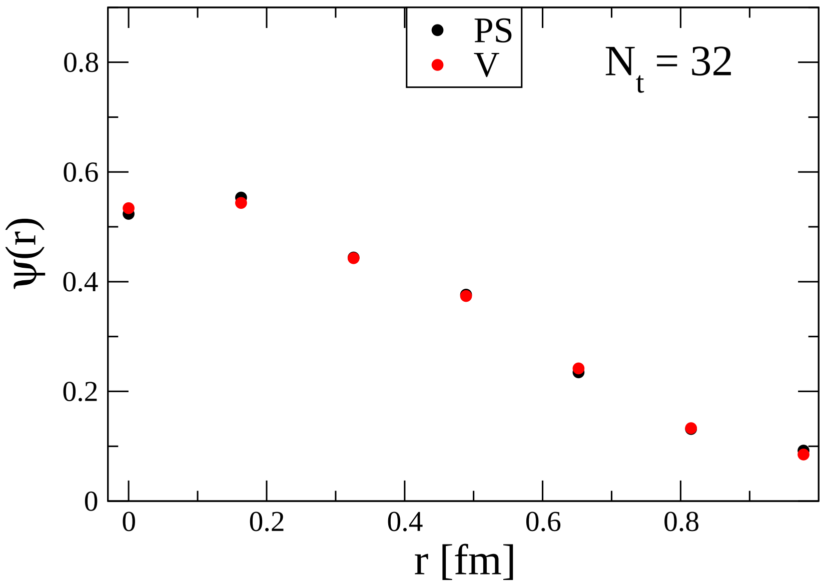

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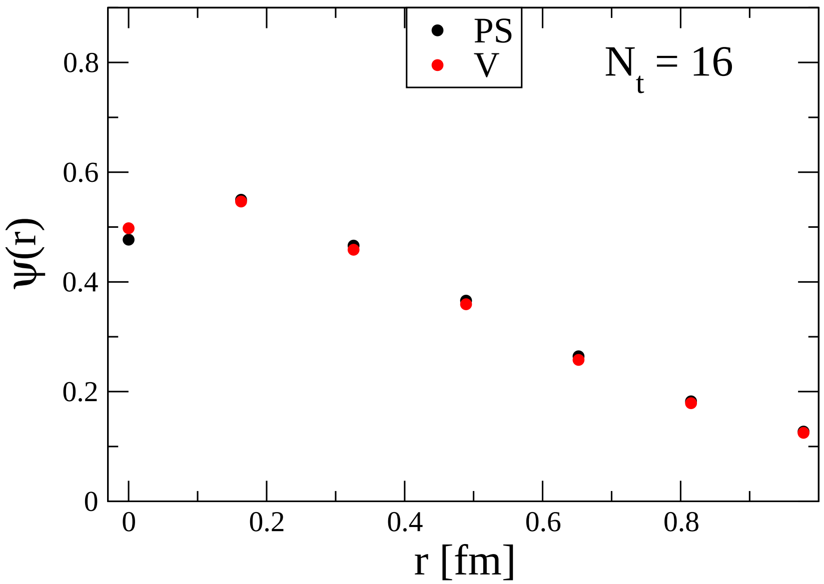

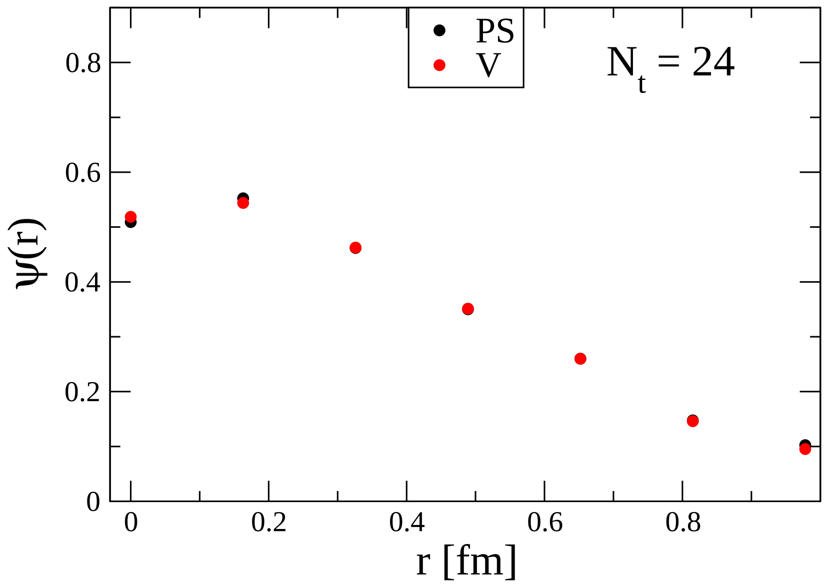

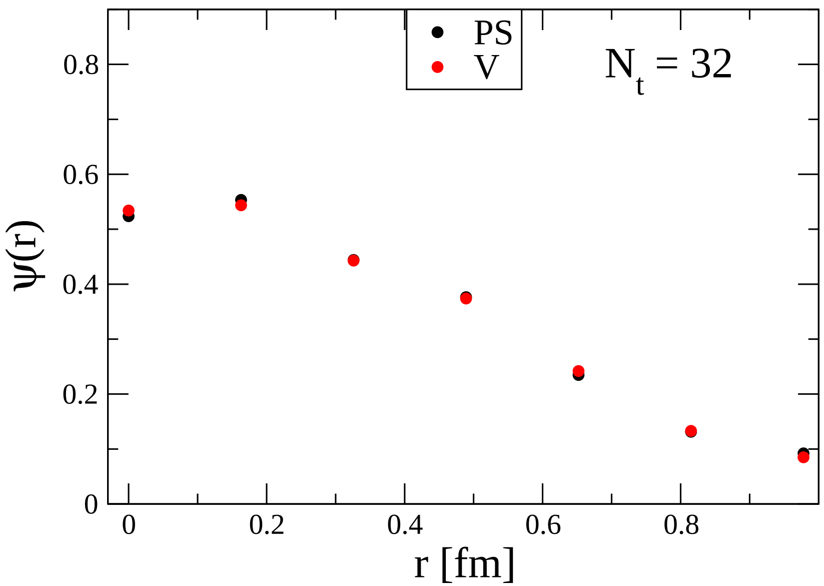

Outline & Future Plans

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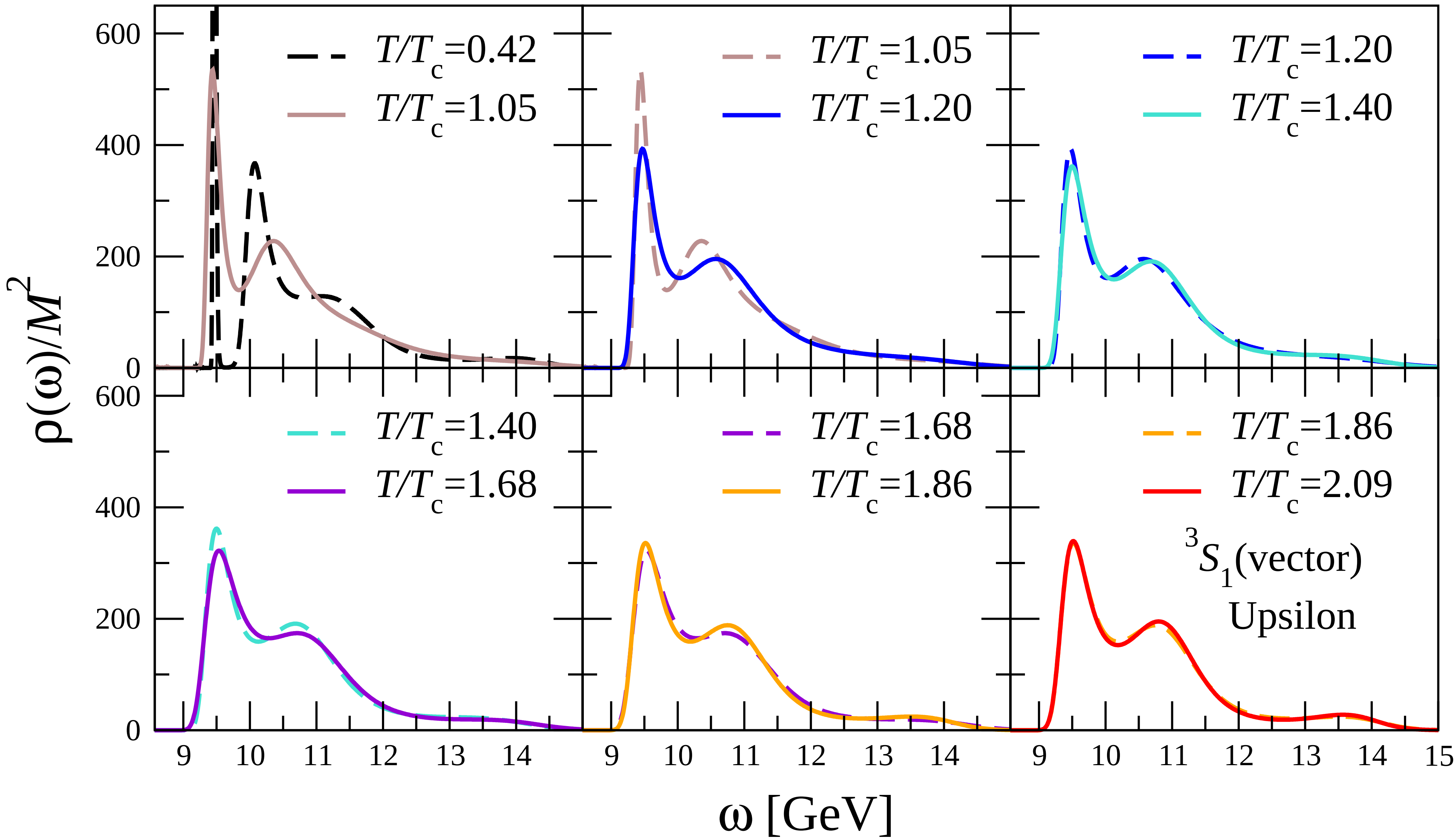
- Maximum Entropy Method

Future Plans:

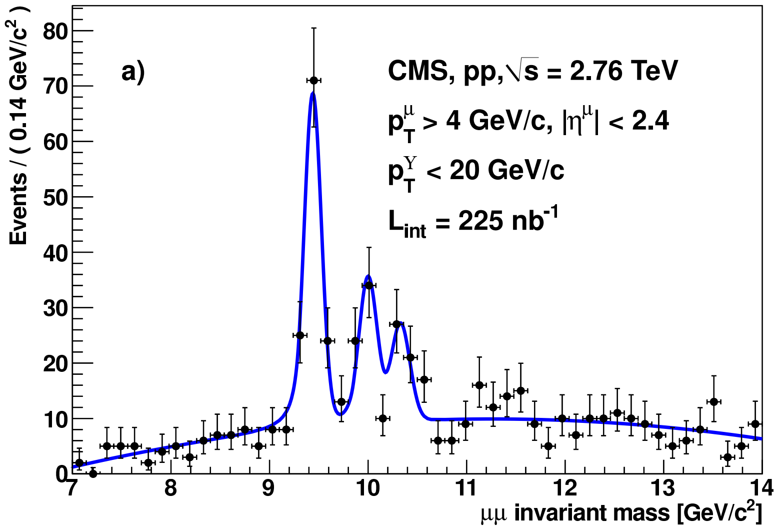
- Increase from 16^3 to 24^3 and 32^3 volumes with $N_f = 2 + 1$

Slides to help me answer difficult questions

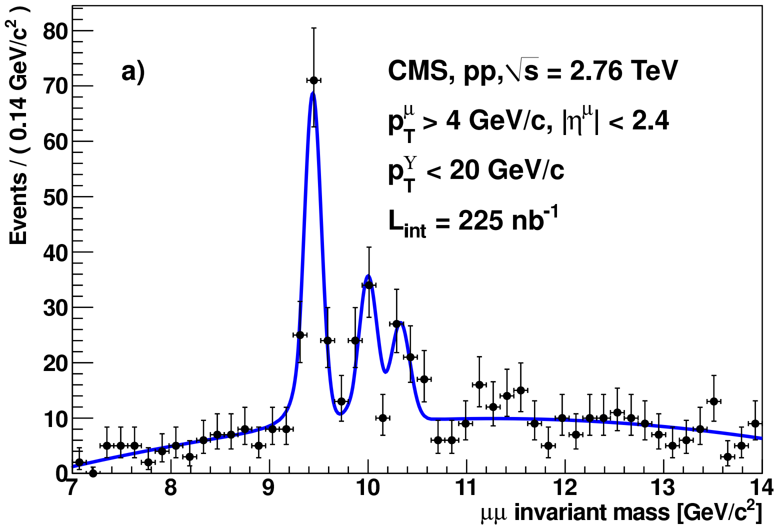
$T \neq 0$ Upsilon Spectral Function



CMS Results arXiv:1105.4894

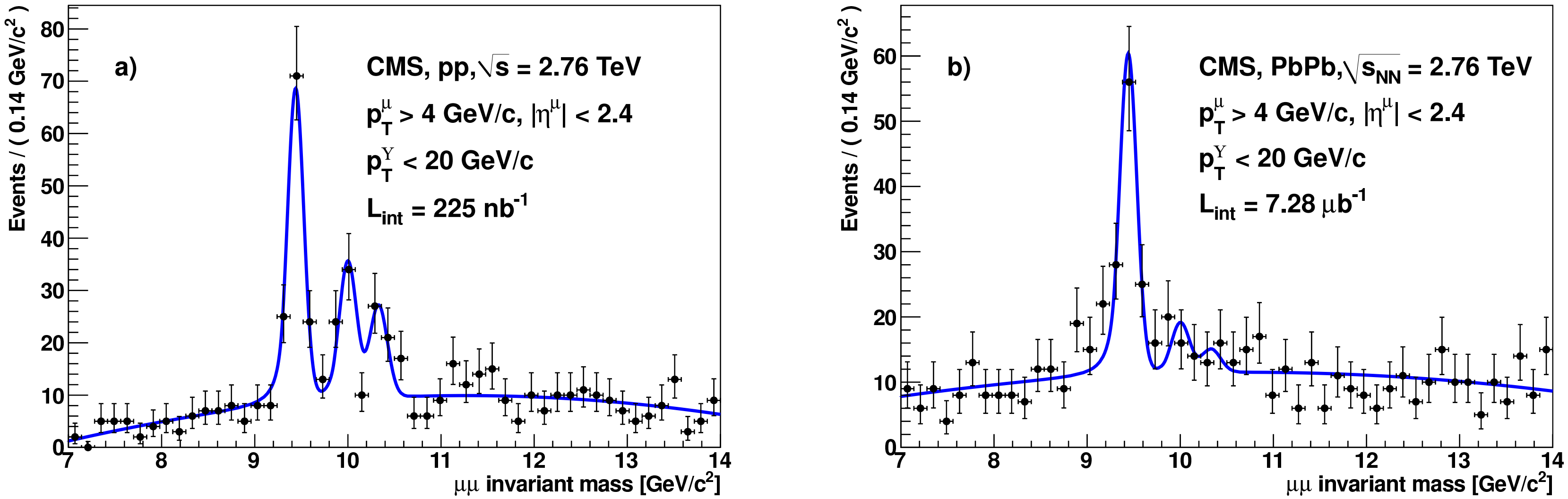


CMS Results arXiv:1105.4894



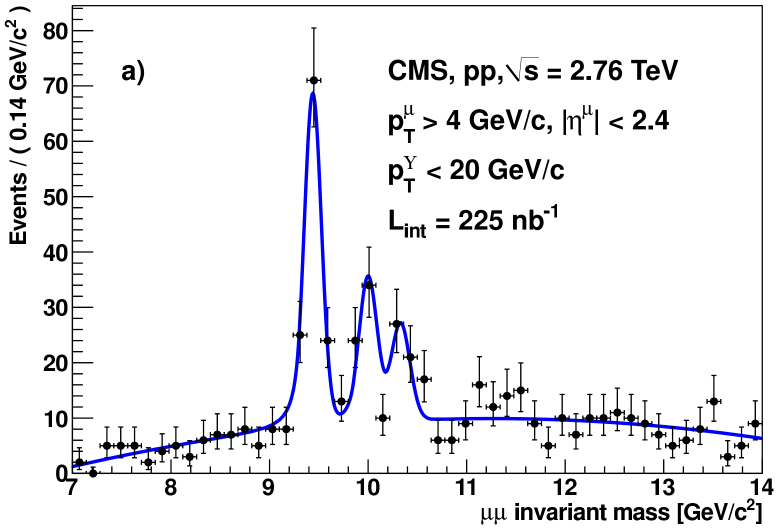
p-p collisions

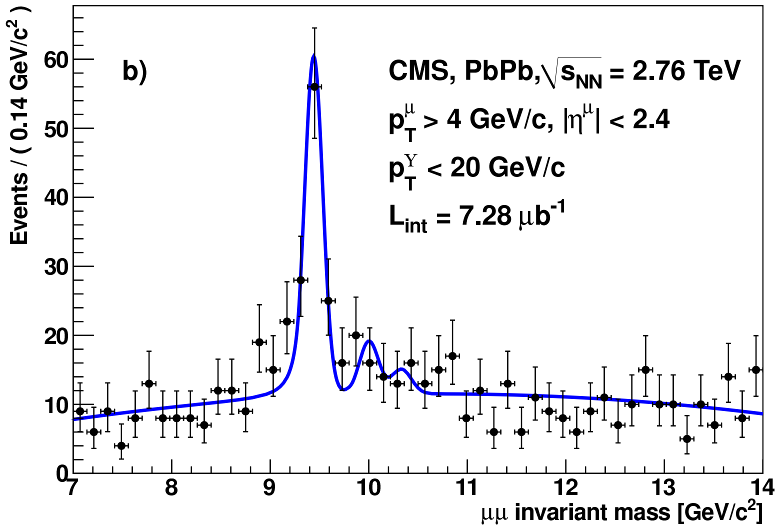
CMS Results arXiv:1105.4894



p-p collisions

CMS Results arXiv:1105.4894





p-p collisions

Pb-Pb collisions

Co-incidence?!