QCD phase diagram with two-flavor fermion formulations

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collaboration with
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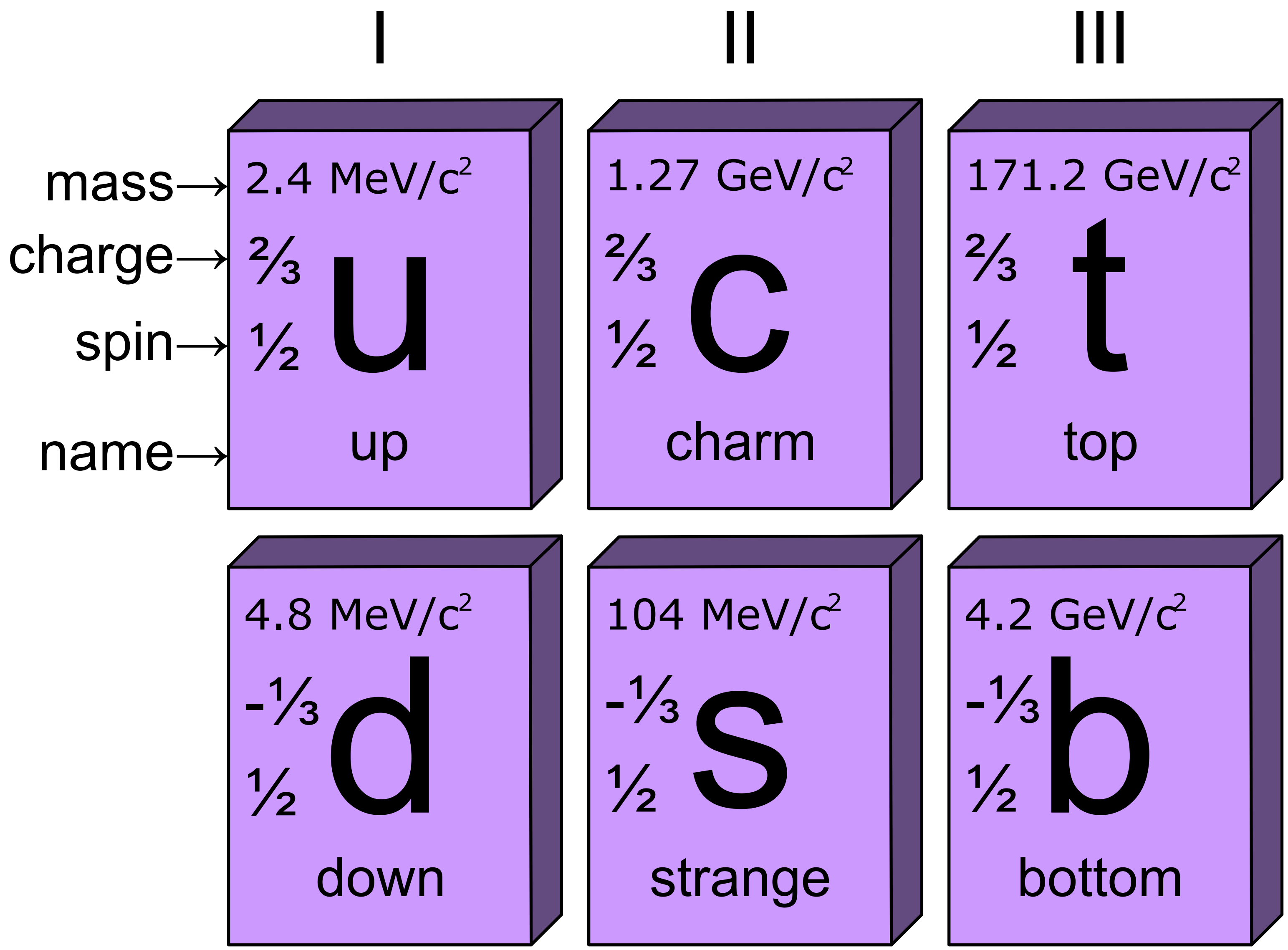
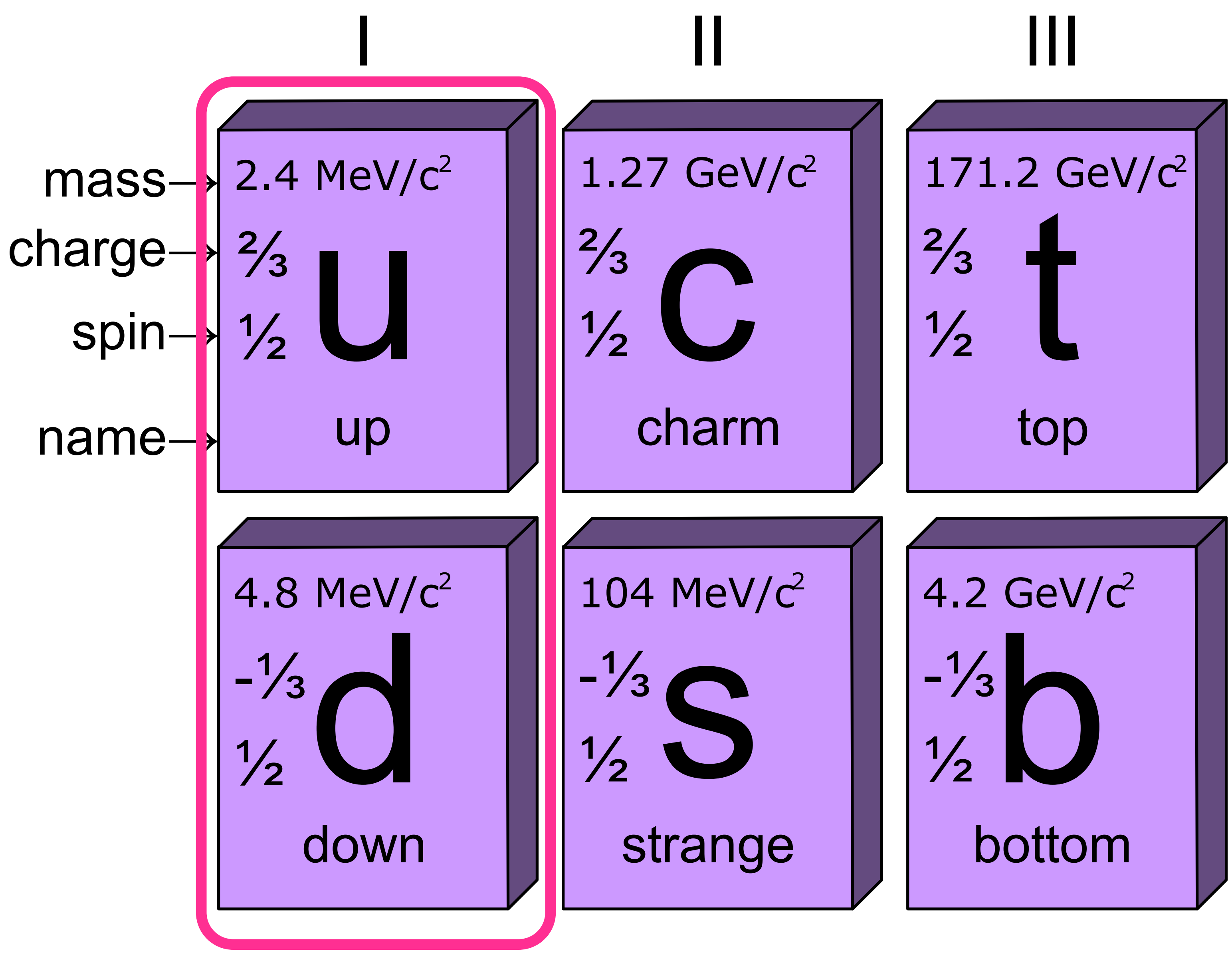
based on
[arXiv:1206.1977]

Why two-flavor QCD?

Quarks


I
II
III
mass
charge
spin
name
2.4 MeV/c2
⅔
½
u
up
1.27 GeV/c2
⅔
½
c
charm
171.2 GeV/c2
⅔
½
t
top
4.8 MeV/c2
-⅓
½
d
down
104 MeV/c2
-⅓
½
s
strange
4.2 GeV/c2
-⅓
½
b
bottom

Quarks



Effectively two massless quarks

Lattice fermions

	#doublers	chiral	spinor
naive	16	exact	4
Wilson	1	none	4
staggered	4	exact	1
min double	2	exact	4

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Minimal-doubling fermion

- #doublers = 2
- exact chiral symmetry
- Ultra-locality

[Karsten '81] [Wilczek '87]
[Creutz '08] [Borici '08]
[Creutz-Misumi '10]

$$D_{\rm KW}(p) = i\sum_{\mu=1}^{4}\gamma_\mu \sin p_\mu + ir\gamma_4\sum_{j=1}^{3}(1-\cos p_j)$$

Wilson-like term:

not mass, but (img) chemical potential

[Misumi '12]

Minimal-doubling fermion

- Weak point :

Symmetry is not enough to restore Lorentz symmetry in the continuum lim.

- KW fermion:
    - P & CT
    - Cubic symmetry

Minimal-doubling fermion

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What's the meaning of this symmetry?

Symmetry of KW fermion

- P & CT
- Cubic symmetry

$$S_{\mathrm{KW}} = \sum_x \left[\frac{1}{2} \sum_{\mu=1}^{4} \bar{\psi}_x \gamma_\mu (U_{x,x+\hat{\mu}} \psi_{x+\hat{\mu}} - U_{x,x-\hat{\mu}} \psi_{x-\hat{\mu}}) \right.$$
$$\left. + \frac{r}{2} \sum_{j=1}^{3} \bar{\psi}_x i\gamma_4 (2\psi_x - U_{x,x+\hat{j}} \psi_{x+\hat{j}} - U_{x,x-\hat{j}} \psi_{x-\hat{j}}) \right]$$

Symmetry of KW fermion

- P & CT
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specifying temporal direction

Symmetry of finite density

- P & CT
- Cubic symmetry

$$S_{\rm naive} = \sum_x \left[\sum_{j=1}^{3} \bar{\psi}_x \gamma_j (U_{x,x+\hat{j}} \psi_{x+\hat{j}} - U_{x,x-\hat{j}} \psi_{x-\hat{j}}) \right.$$

$$\left. + \bar{\psi}_x \gamma_4 (e^{\mu} U_{x,x+\hat{4}} \psi_{x+\hat{4}} - e^{-\mu} U_{x,x-\hat{4}} \psi_{x-\hat{4}}) \right]$$

Symmetry of finite density

- P & CT
- Cubic symmetry

$$S_{\rm naive} = \sum_x \Big[\sum_{j=1}^{3} \bar{\psi}_x \gamma_j (U_{x,x+\hat{j}} \psi_{x+\hat{j}} - U_{x,x-\hat{j}} \psi_{x-\hat{j}})$$

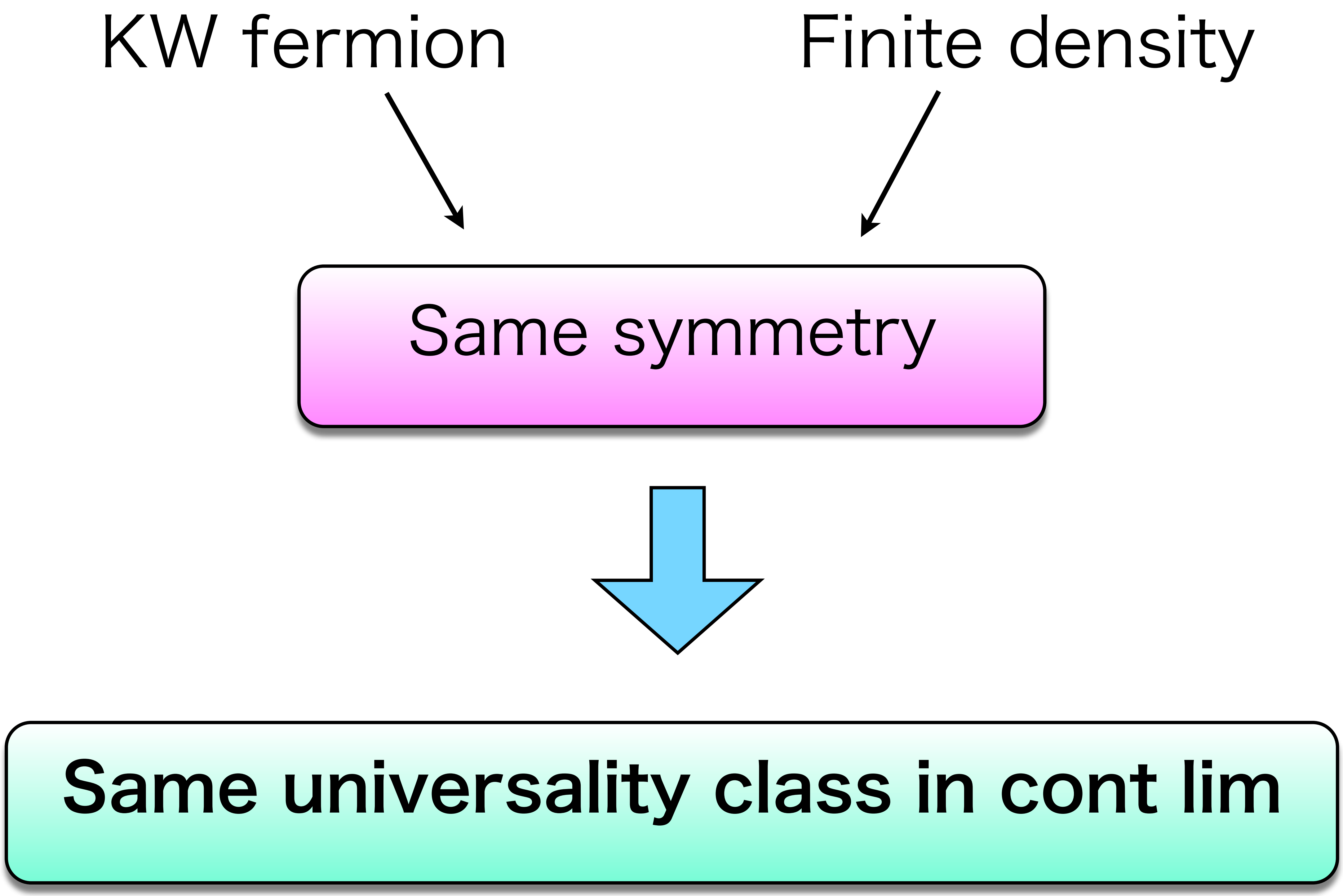
$$+ \underline{\bar{\psi}_x \gamma_4 (e^{\mu} U_{x,x+\hat{4}} \psi_{x+\hat{4}} - e^{-\mu} U_{x,x-\hat{4}} \psi_{x-\hat{4}})} \Big]$$

specifying temporal direction

KW fermion

Finite density

Same symmetry


KW fermion
Finite density
Same symmetry
Same universality class in cont lim

Renormalization effect

- KW term -> flavored ***chemical pot***

[Misumi '12]

$$S_{\mathrm{KW}} = i\gamma_4 \sum_{j=1}^{3} (1 - \cos p_j)$$

⇨ additive (img) chem pot renormalization

cf. Wilson term

- Counter term : $\mu_3 \, \bar{\psi}_x i\gamma_4 \psi_x$

tuning this μ_3

Strong-coupling analysis in finite density

Strong-coupling lattice QCD

1. Link variable integral
2. Bosonization & fermion integral
3. Determine the vacuum from the effective potential

⇒ Finite temperature & density case

[Nishida-Fukushima-Hatsuda '04]

Meson fields

- Chiral : $\langle \bar{\psi}\psi \rangle = \sigma$
- Vector (imag density): $\langle \bar{\psi} i\gamma_4 \psi \rangle = \pi_4$

$$\rightarrow \langle i\psi^\dagger \psi \rangle$$

Derive the effective potential in terms of these meson fields

Effective potential

$$\mathcal{F}_{\text{eff}}(\sigma,\pi_4;m,T,\mu,\mu_3) = \frac{N_c D}{4}\left((1+r^2)\sigma^2+(1-r^2)\pi_4^2\right) - N_c\log A$$
$$-\frac{T}{4}\log\left(\sum_{n\in\mathbb{Z}}\det\left(Q_{n+i-j}\right)_{1\le i,j\le N_c}\right).$$

for Nc = 3

$$\begin{aligned}
&\sum_{n\in\mathbb{Z}}\det\left(Q_{n+i-j}\right)_{1\le i,j\le N_c}\\
&= 8\left(1+12\cosh^2\frac{E}{T}+8\cosh^4\frac{E}{T}\right)\left(15-60\cosh^2\frac{E}{T}+160\cosh^4\frac{E}{T}-32\cosh^6\frac{E}{T}+64\cosh^8\frac{E}{T}\right)\\
&+64\cosh\frac{\mu_B}{T}\cosh\frac{E}{T}\left(-15+40\cosh^2\frac{E}{T}+96\cosh^4\frac{E}{T}+320\cosh^8\frac{E}{T}\right)\\
&+80\cosh\frac{2\mu_B}{T}\left(1+6\cosh^2\frac{E}{T}+24\cosh^4\frac{E}{T}+80\cosh^6\frac{E}{T}\right)\\
&+80\cosh\frac{3\mu_B}{T}\cosh\frac{E}{T}\left(-1+\cosh^2\frac{E}{T}\right)+2\cosh\frac{4\mu_B}{T},
\end{aligned} \tag{19}$$

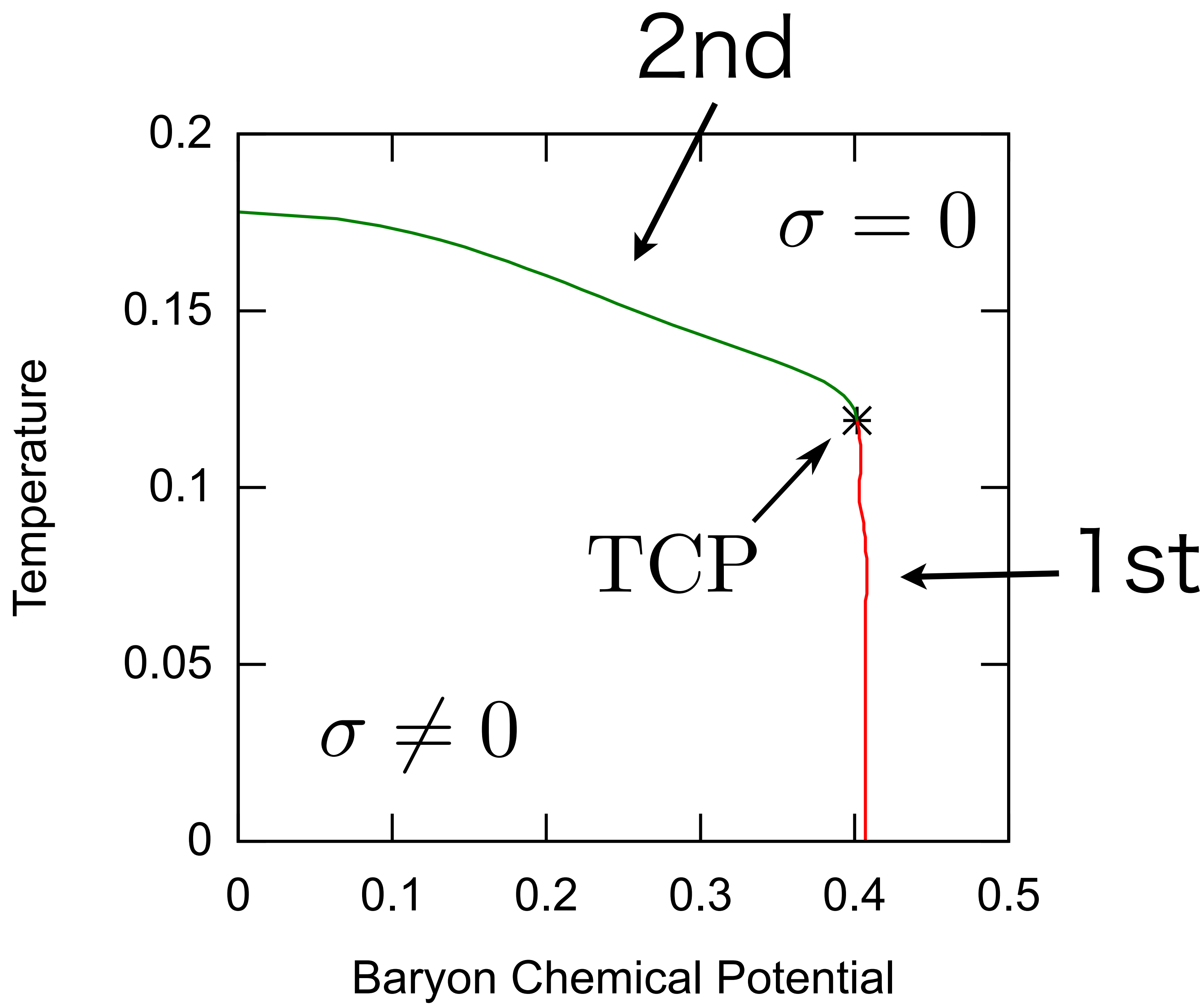
Effective potential

with

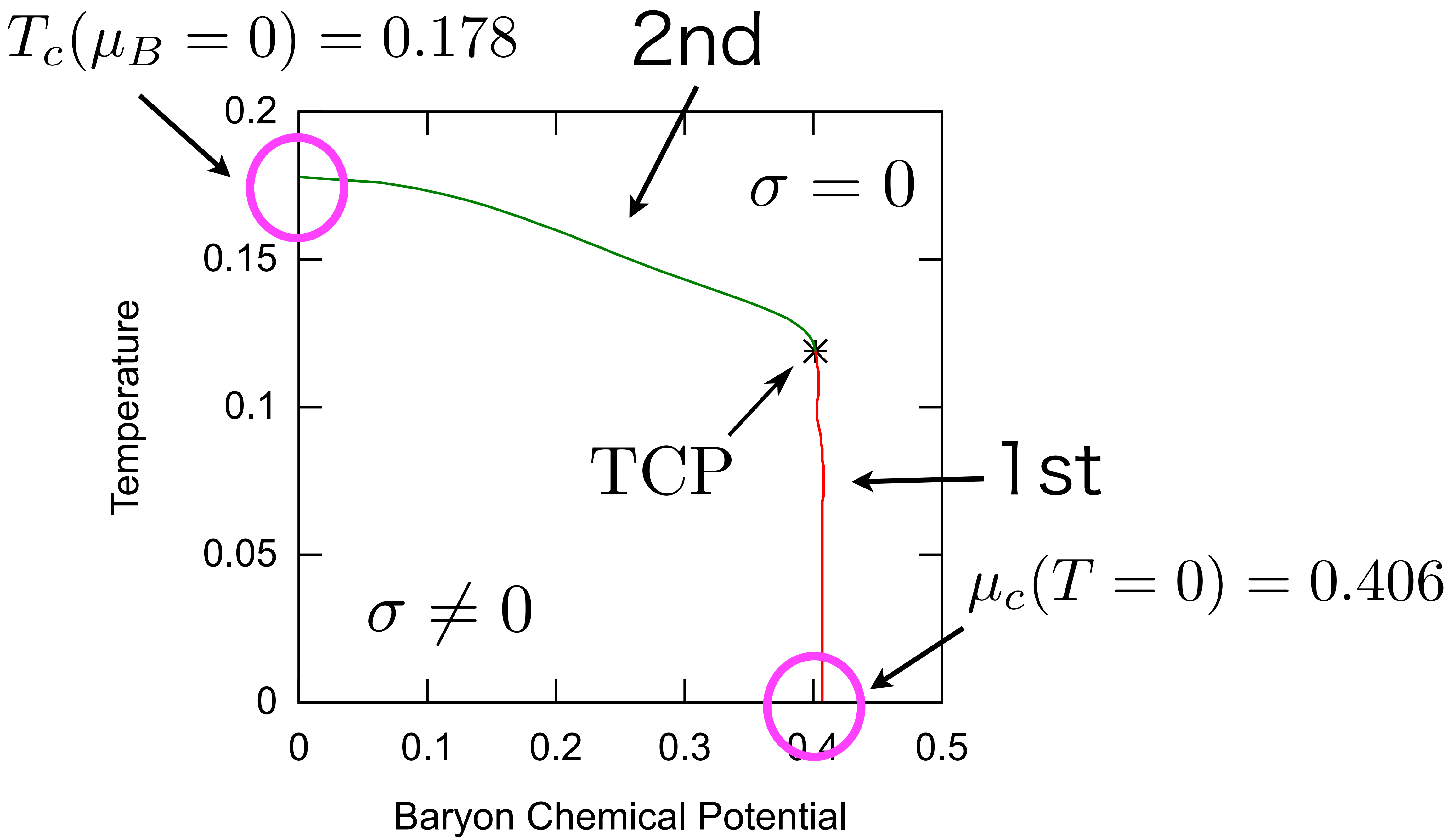
$$A^2 = 1 + \left(\mu_3 + Dr - \frac{D}{2}\sqrt{1-r^2}\pi_4\right)^2, \qquad B = m + \frac{D}{2}\sqrt{1+r^2}\sigma,$$

$$E = \operatorname{arcsinh}\left(\frac{B}{A}\right) = \log\left[\frac{B}{A} + \sqrt{1+\left(\frac{B}{A}\right)^2}\right],$$

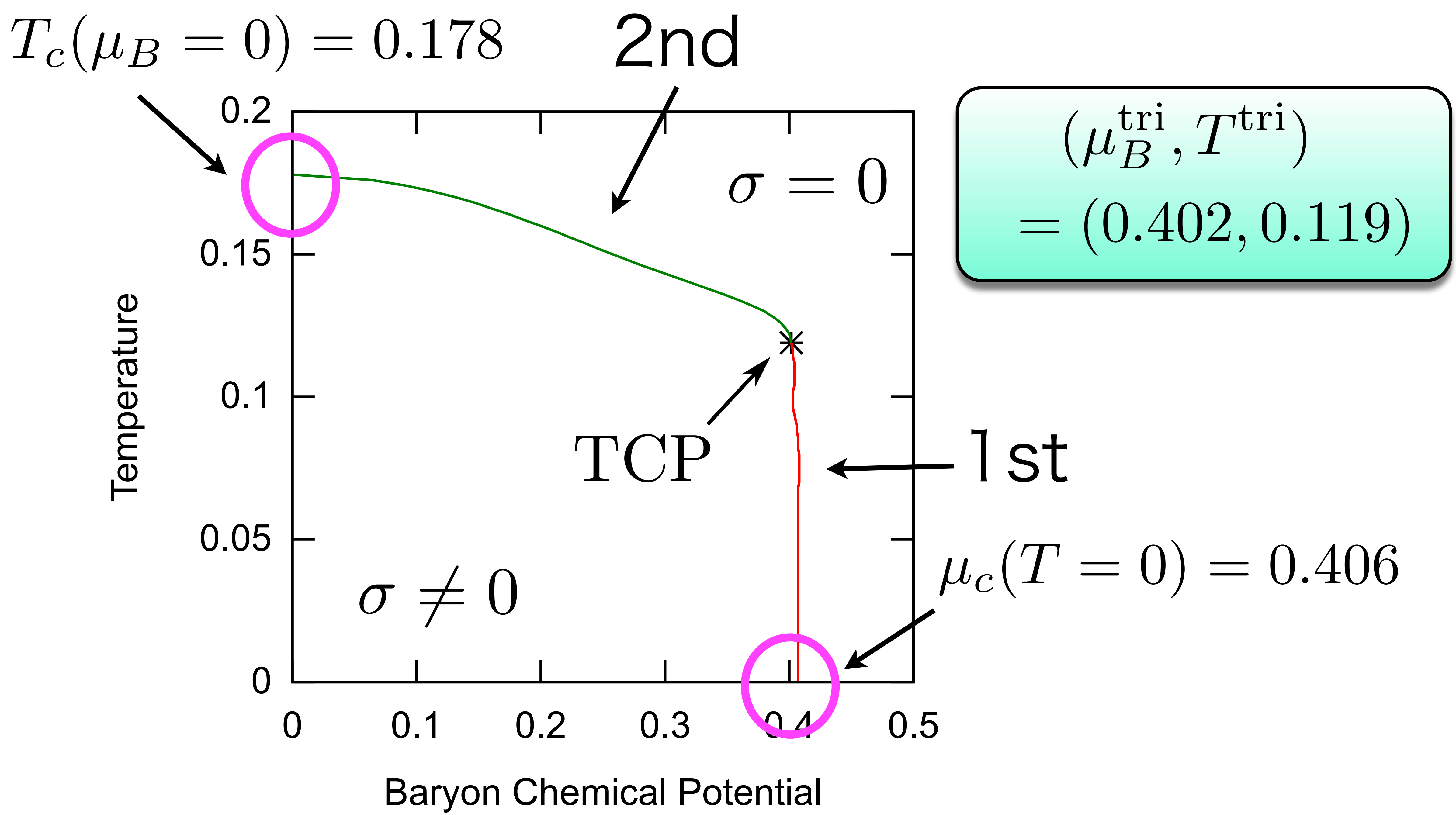
Phase diagram



Phase diagram



Phase diagram



Phase diagram

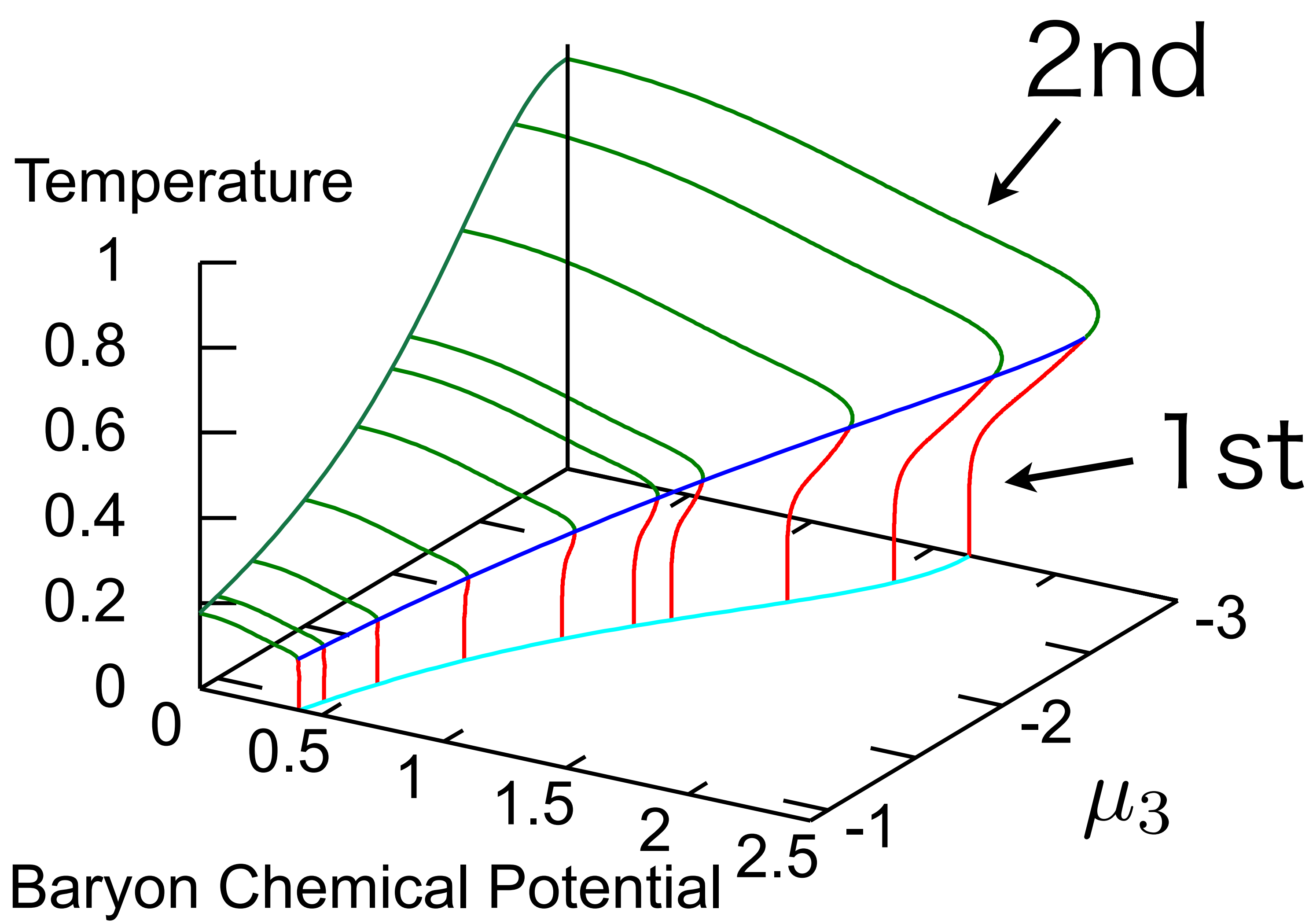
- Critical density/temp ratio
  - KW fermion : $R^0_{\rm KW} = \dfrac{\mu_c(T=0)}{T_c(\mu_B=0)} \sim 2.3$
  - Staggered : $R^0_{\rm st} \sim 1$
  - Phenomenology : $R^0_{\rm ph} \gtrsim 5.5$

Phase diagram

- Tricritical point ratio
  - KW fermion : $R^{\rm tri}_{\rm KW} = \dfrac{\mu_B^{\rm tri}}{T^{\rm tri}} \simeq 3.4$
  - Staggered : $R^{\rm tri}_{\rm st} \simeq 2.0$
  - Monte-Calro simulation : $R^{\rm tri}_{\rm MC} \gtrsim 3$

Phase diagram

- 3-dim diagram : (μ_B, T, μ_3)



Summary

Summary

- KW-type minimal doubling fermion
  - Availability in finite density two-flavor QCD with exact chiral sym
- QCD phase diagram
  - close to phenomenological result