

Is there a gap in the QCD Dirac spectrum above T_c ?

Tamás G. Kovács
Institute for Nuclear Research, Debrecen



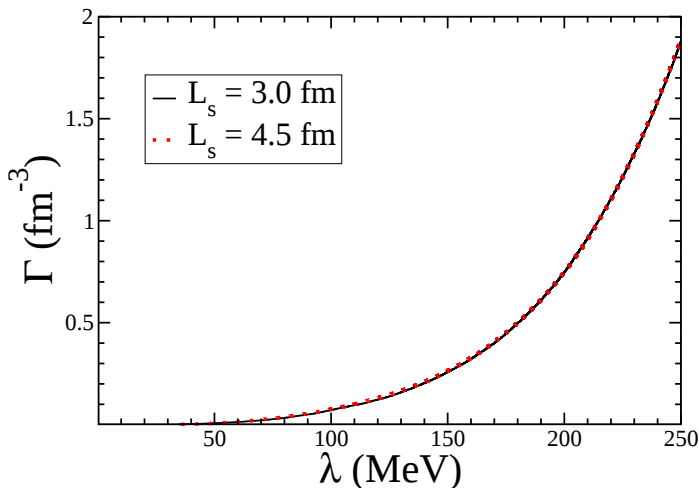
Ferenc Pittler
University of Pécs

June 26, 2012

The Dirac spectrum above T_c

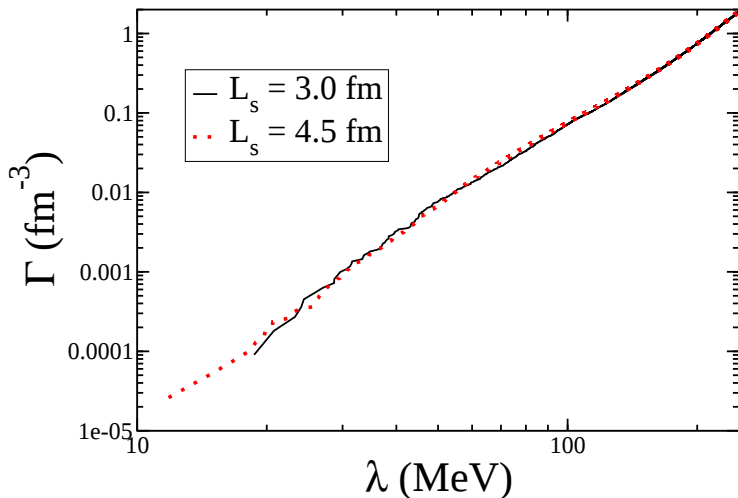
Integrated spectral density, $T = 1.7T_c$ 2+1 flavor staggered, physical quark masses

Other talks at this conference: [S. Aoki](#), [T-H. Hsieh](#), [Z. Lin](#), [H. Ohno](#) (mostly around T_c)



The Dirac spectrum above T_c

Integrated spectral density, $T = 1.7T_c$

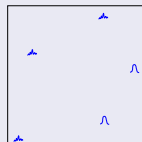


- $N_f = 2 + 1$ staggered, stout, physical quark masses
- Action and parameters from Budapest-Wuppertal group
Borsányi et al. JHEP '10
- Lattice spacings $a = 0.125\text{ fm}, 0.082\text{ fm}, 0.062\text{ fm}$
- Temperature range $1.7T_c \leq T \leq 5T_c$
- Spatial box sizes $2\text{ fm} \leq L_s \leq 6\text{ fm}$

Poisson to RMT transition in the Dirac spectrum

Lowest part of the Dirac spectrum

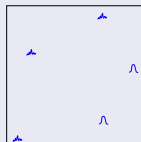
- Sparse spectrum ($\rho(0) = 0$)
- Localized eigenmodes
- Poisson statistics (eigenvalues independent)



Poisson to RMT transition in the Dirac spectrum

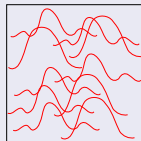
Lowest part of the Dirac spectrum

- Sparse spectrum ($\rho(0) = 0$)
- Localized eigenmodes
- Poisson statistics (eigenvalues independent)



Higher up in the spectrum

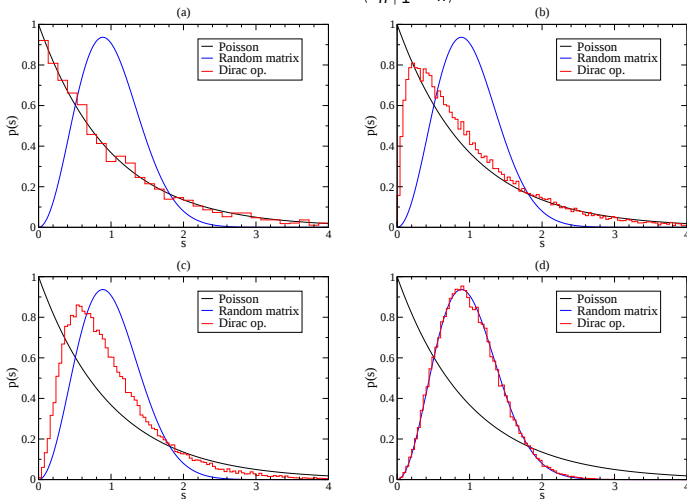
- Spectral density increases
- Eigenmodes become delocalized
- Eigenvalue statistics: random matrix



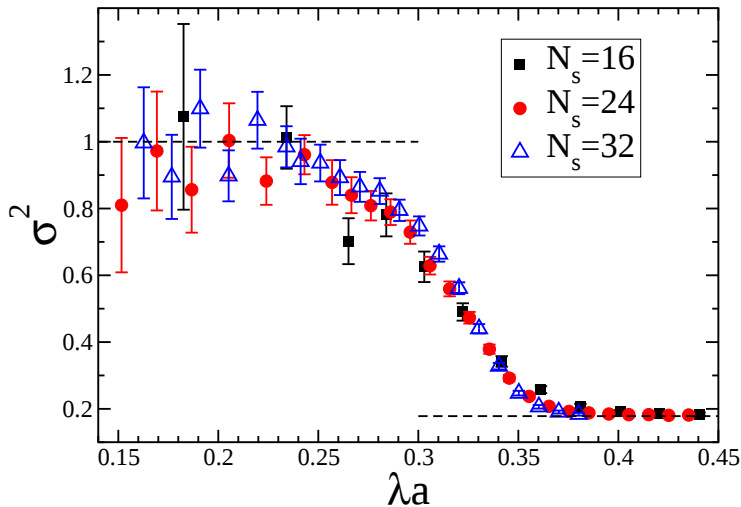
Statistics of Dirac eigenvalues above T_c

unfolded level spacing distribution (take out spectral density)

$$p(s) \quad s = \frac{\lambda_{n+1} - \lambda_n}{\langle \lambda_{n+1} - \lambda_n \rangle}$$



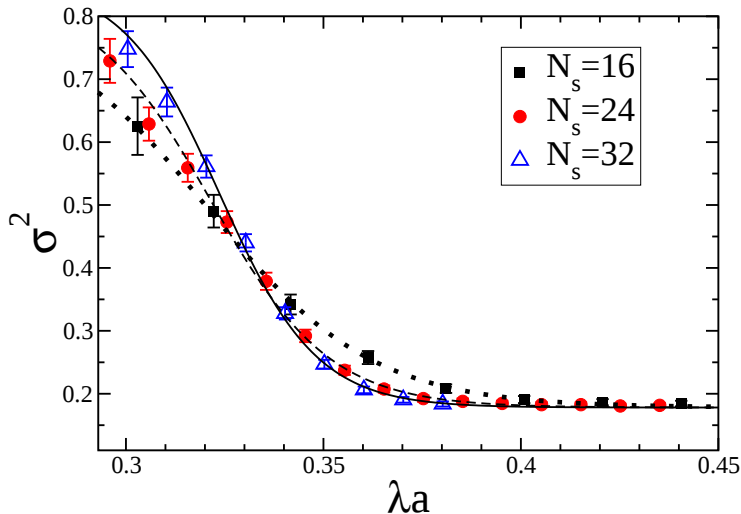
Variance of the unfolded level spacing distribution



Is it a real transition?

Probably yes.

3-parameter tanh fit $\rightarrow \lambda_c$, “susceptibility” $\propto N_s = V_s^{1/3}$



Physical impact of localization

- Why should you care about low modes being localized?
- Quark propagator

$$(D + m)^{-1} = \sum_i \frac{|\psi_i\rangle\langle\psi_i|}{\lambda_i + m}$$

- $\frac{1}{\lambda_i + m} \rightarrow$ for light quarks low modes can dominate
- But if $\psi(x)$ localized to scale $d \ll L$
 $\rightarrow$ no contribution to correlators on scale $\geq L$

Physical impact of localization

- Why should you care about low modes being localized?
- Quark propagator

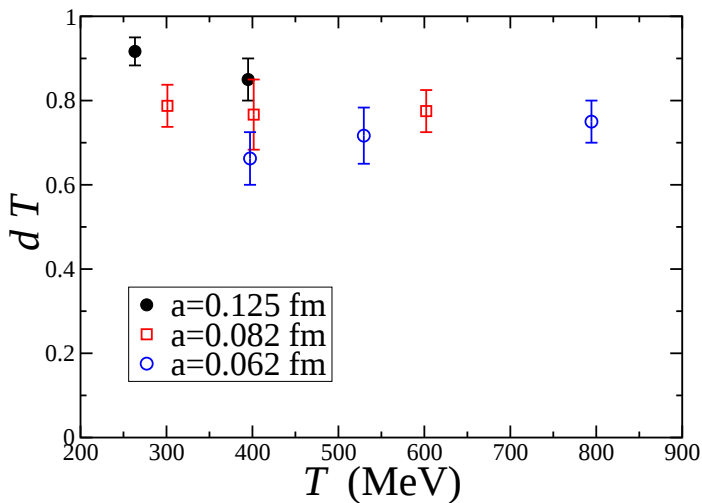
$$(D + m)^{-1} = \sum_i \frac{|\psi_i\rangle\langle\psi_i|}{\lambda_i + m}$$

- $\frac{1}{\lambda_i + m} \rightarrow$ for light quarks low modes can dominate
- But if $\psi(x)$ localized to scale $d \ll L$
 $\rightarrow$ no contribution to correlators on scale $\geq L$
- ① How localized are they? $\rightarrow d$
- ② How far up in the spectrum are states localized? $\rightarrow \lambda_c$
- Need physical answer in the continuum limit.

Localization length of low Dirac modes

in units of the inverse temperature

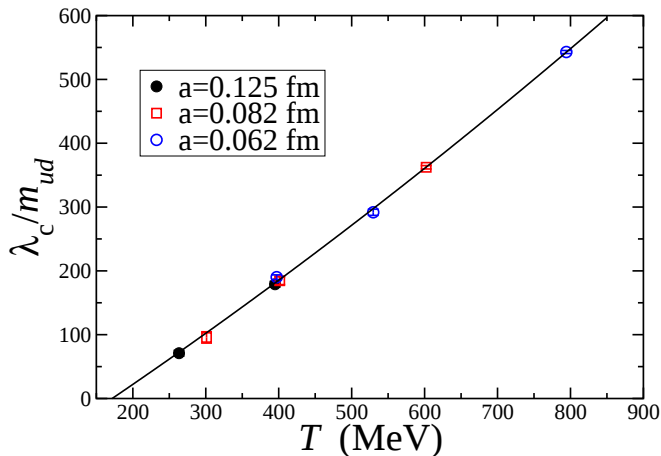
$d = a \cdot \text{IPR}^{-1/4}$, IPR is the inverse participation ratio



- Low modes do not propagate quarks to long distances
- For correlators on scales $L \gg \frac{1}{T}$ λ_c acts as a gap
- Plays a role similar to the quark mass
- λ_c renormalizes in the same way as the quark mass
- a good quantity to measure the gap in the continuum limit

$$\frac{\lambda_c}{m_{ud}}$$

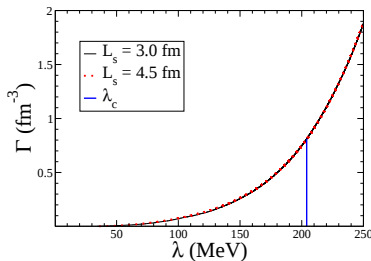
Temperature dependence of λ_c



2nd order polynomial fit: λ_c vanishes at $T = 172\text{MeV}$.

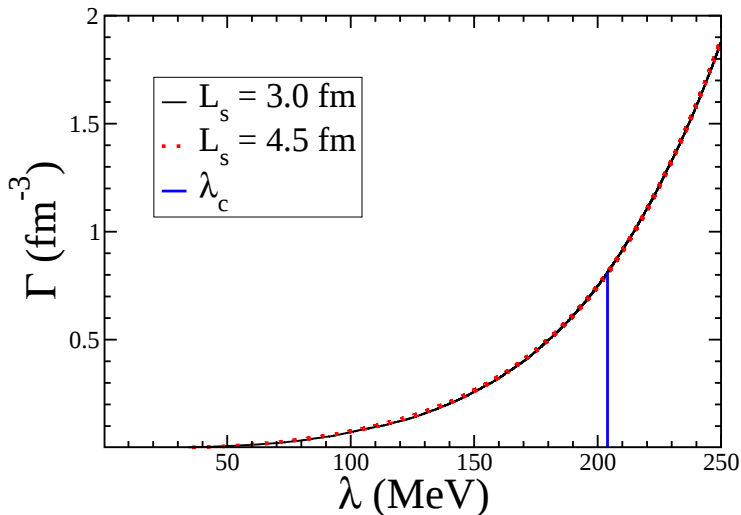
Conclusions

- There is no evidence of a real gap in the spectrum
- Low eigenmodes localized on the scale $1/T_c$
- Transition to delocalized modes at λ_c
- λ_c : *effective gap* for long distance ($L \gg 1/T$) quark propagation



The Dirac spectrum above T_c

Integrated spectral density, $T = 1.7T_c$



Simulation details

	$T(\text{MeV})$	$a(\text{fm})$	N_s	N_t	Nconf	Nevs
A1	263	0.125	24	6	430	512
A2			36		420	256
B	300	0.082	32	8	434	256
C1	394	0.125	16	4	1622	512
C2			24		1600	512
C3			32		900	512
C4			48		604	128
D1	401	0.082	24	6	440	512
D2			36		440	256
E	397	0.062	32	8	593	256
F	530			6	420	512
G	601	0.082	24	4	396	512
H	794	0.062	32	4	417	512