Model approach to the sign problem on lattice QCD with theta vacuum

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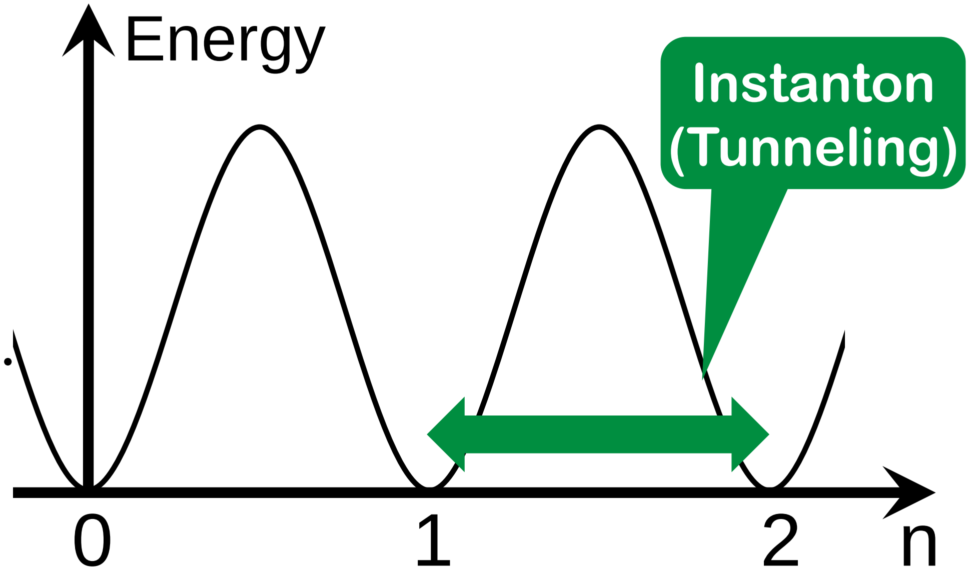
Theta vacuum

The QCD vacuum is
a superposition of vacua
characterized by winding number n.



$$|\theta\rangle = \sum_n e^{in\theta}|n\rangle$$

Then QCD effective Lagrangian is required an extra term.

$$\mathcal{L} = \sum_f \bar{q}_f(\gamma_\nu D_\nu + m_f)q_f + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - i\theta\frac{g^2}{64\pi^2}\epsilon_{\mu\nu\sigma\rho}F^a_{\mu\nu}F^a_{\sigma\rho}$$

At zero temperature, experimental measurement of
neutron dipole moment gives the upper limit : $|\theta| < 10^{-10}$

There is no theoretical interpretation for this property
(Strong CP problem).

At finite temperature, the behavior of theta is nontrivial.

Sign problem vs. Theta vacuum

$$\mathcal{L}_{\mathrm{QCD}} = \bar{q}_f(\gamma_\nu D_\nu + m_f)q_f + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - i\theta\frac{1}{64\pi^2}\epsilon_{\mu\nu\sigma\rho}F^a_{\mu\nu}F^a_{\sigma\rho}$$

$\mathrm{U_A}(1)$ transformation

- Complex
- Topological effect

$$\mathcal{L}'_{\mathrm{QCD}} = \bar{q}'_f(\gamma_\nu D_\nu + m_f(\theta))q'_f + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$$

This parity-odd term
makes fermion determinant complex.

Lattice QCD simulation

- Estimate the probability distribution of the topological number
- Taylor expansion around $\theta = 0$

Ref: M. D'Elia and F. Negro, arXiv:1205.0538 [hep-lat] (2012)

Sign problem vs. Theta vacuum

$$\mathcal{L}_{QCD} = \bar{q}_f(\gamma_\nu D_\nu + m_f)q_f + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - i\theta\frac{g^2}{64\pi^2}\epsilon_{\mu\nu\sigma\rho}F^a_{\mu\nu}F^a_{\sigma\rho}$$

<u>$\mathrm{SU_A}(3) \otimes \mathrm{U_A}(1)$ transformation</u>

$$q_u = e^{i\gamma_5\frac{\theta}{4}}q'_u$$

$$q_d = e^{i\gamma_5\frac{\theta}{4}}q'_d$$

$$q_s = q'_s$$

$$\mathcal{L}_{QCD} = \sum_{l=u,d}\bar{q}'_l\mathcal{M}_l(\theta)q'_l + \bar{q}'_s\mathcal{M}_s q'_s + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$$

$$\mathcal{M}_l(\theta) \equiv \gamma_\nu D_\nu + m_l\cos(\theta/2) + m_l i\gamma_5\sin(\theta/2),$$

$$\mathcal{M}_s \equiv \gamma_\nu D_\nu + m_s.$$

Much smaller than the QCD scale

Suggestion

$$\mathcal{L}_{QCD} = \sum_{l=u,d} \bar{q}'_l \mathcal{M}_l(\theta) q'_l + \bar{q}'_s \mathcal{M}_s q'_s + \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

$$\mathcal{M}_l(\theta) \equiv \gamma_\nu D_\nu + m_l \cos(\theta/2) + m_l i\gamma_5 \sin(\theta/2),$$

$$\mathcal{M}_s \equiv \gamma_\nu D_\nu + m_s.$$

$$\Rightarrow \mathcal{M}'_l(\theta) \equiv \gamma_\nu D_\nu + m_l \cos(\theta/2)$$

Neglect the P-odd mass

Reweighting Method

$$\langle \mathcal{O} \rangle = \int \mathcal{D}A\, \mathcal{O}' \underbrace{(\det \mathcal{M}'_l(\theta))^2 \det \mathcal{M}_s e^{-S_g}}_{\text{Free from the sign problem}}$$

$$\mathcal{O}' \equiv \mathcal{O} \frac{(\det \mathcal{M}_l(\theta))^2}{(\det \mathcal{M}'_l(\theta))^2} \approx \mathcal{O}$$

$\det \mathcal{M}'_l(\theta)$: Fermion determinant without P odd mass

Entanglement PNJL model

2+1 flavor Polyakov-loop extended Nambu-Jona-Lasinio model

$$\mathcal{L} = \bar{q}_f(\gamma_\nu D_\nu + \hat{m}_f)q_f - G_{\rm S}(\Phi)\sum_{a=0}^{8}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2\right]$$
$$+G_{\rm D}\left[\det\bar{q}_f(1-\gamma_5)q_{f'} + \det\bar{q}_f(1+\gamma_5)q_{f'}\right] + \mathcal{U}(T,\Phi[A],\Phi^*[A])$$

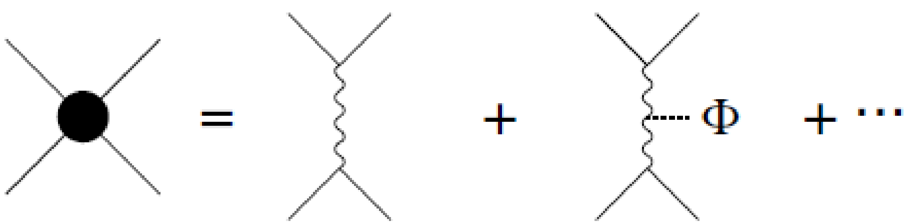
$$q = (q_u, q_d, q_s)$$
$$\hat{m}_0 = {\rm diag}(m_u, m_d, m_s)$$

Kobayashi-Maskawa-'t Hooft interaction

- breaks the $U_{\rm A}(1)$ symmetry explicitly
- Determinant is taken in the flavor space.

Entanglement vertex

$$G_S(\Phi) = G_S\left[1 - \alpha_1\Phi\Phi^* - \alpha_2(\Phi^3 + \Phi^{*3})\right]$$



Ref : Y. Sakai, T. S. , H. Kouno, and M. Yahiro, Phys. Rev. D**82,** 076003 (2010)
K. –I. Kondo, Phys. Rev. D**82**, 065024 (2010)

Entanglement PNJL model

Properties of EPNJL and PNJL model

	PNJL model	EPNJL model
Transition temperature T_c	○ (Deconfinement) △ (Chiral)	○
Equation of state	○ (Qualitatively)	○ (Qualitatively)
Roberge-Weiss periodicity (for imaginary μ)[1]	○	○
Quark mass dependence of RW end point [2,3]	×	○

[1] A. Roberge and N. Weiss, Nucl. Phys. B**275**, 734 (1986)
[2] P. de Forcrand and O. Philipsen, Phys. Rev. Lett. **105**, 152001 (2010)
[3] T. S. ,Y. Sakai, H. Kouno, and M. Yahiro, Phys. Rev. D**84**, 091901 (2011)

EPNJL model with theta vacuum

EPNJL model with theta vacuum

$$\mathcal{L} = \bar{q}_f(\gamma_\nu D_\nu + \hat{m}_f)q_f - G_S(\Phi)\sum_{a=0}^{8}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2\right]$$
$$+G_D\left[\underline{\underline{e^{i\theta}}}\mathrm{det}\bar{q}_f(1-\gamma_5)q_{f'} + \underline{\underline{e^{-i\theta}}}\mathrm{det}\bar{q}_f(1+\gamma_5)q_{f'}\right] + \mathcal{U}(T,\Phi[A],\Phi^*[A])$$

$\mathrm{SU_A}(3)\otimes \mathrm{U_A}(1)$ transformation

$$\mathcal{L} = \bar{q}'_f(\gamma_\nu D_\nu + \underline{\underline{m_f(\theta)}})q'_f - G_S(\Phi)\sum_{a=0}^{8}\left[(\bar{q}'\lambda_a q')^2 + (\bar{q}'i\gamma_5\lambda_a q')^2\right]$$
$$+G_D\left[\mathrm{det}\bar{q}'_f(1-\gamma_5)q'_{f'} + \mathrm{det}\bar{q}'_f(1+\gamma_5)q'_{f'}\right] + \mathcal{U}(T,\Phi[A],\Phi^*[A])$$

$$\begin{cases} m_u(\theta) &= m_l\,\cos(\theta/2) + i\gamma_5 m_l\,\sin(\theta/2) \\ m_d(\theta) &= m_l\,\cos(\theta/2) + i\gamma_5 m_l\,\sin(\theta/2) \\ m_s(\theta) &= m_s \end{cases}$$

Expected : These effects are much smaller than the scale of chiral symmetry breaking.

EPNJL model with theta vacuum

$$\mathcal{L} = \bar{q}'_f(\gamma_\nu D_\nu + m_f(\theta))q'_f - G_{\rm S}(\Phi)\sum_{a=0}^{8}\left[(\bar{q}'\lambda_a q')^2 + (\bar{q}' i\gamma_5\lambda_a q')^2\right]$$
$$+G_{\rm D}\left[\det\bar{q}'_f(1-\gamma_5)q'_{f'} + \det\bar{q}'_f(1+\gamma_5)q'_{f'}\right] + \mathcal{U}(T,\Phi[A],\Phi^*[A])$$

Meanfield approximation

$$\sigma'_f = \langle\bar{q}'_f q'_f\rangle \ , \ \eta'_f = \langle\bar{q}'_f i\gamma_5 q'_f\rangle$$
$$\langle\bar{q}'\lambda_a q'\rangle = \langle\bar{q}' i\gamma_5\lambda_a q'\rangle = 0 \quad (a = 0,\cdots,7)$$

Φ : Traced Polyakov loop

Partition function : Z

Thermodynamic potential : $\Omega = -\frac{1}{\beta}\ln Z$

Stationary condition

Meanfields are determined by the stationary condition.

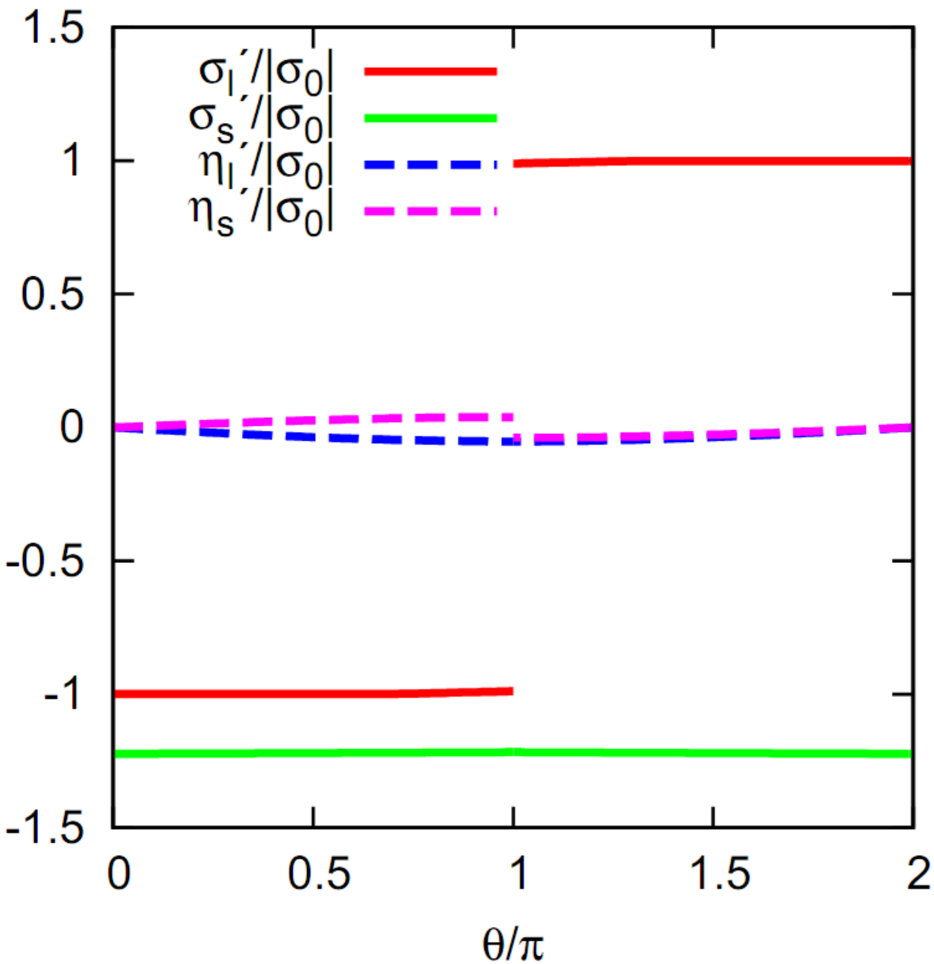
$$\frac{\partial\Omega}{\partial X} = 0 \quad (X = \sigma'_l, \sigma'_s, \eta'_l, \eta'_s, \Phi, \Phi^*)$$

Sign problem vs. Theta vacuum

With parity odd mass

$$T = \mu = 0$$

$$m_l(\theta) = m_l \cos(\theta/2) + i\gamma_5 m_l \sin(\theta/2)$$



$$\sigma_0 \equiv \sigma_l(T = \mu = \theta = 0)$$

P even condensate

$\sigma'_f = \langle \bar{q}'_f q'_f \rangle$: Red and Green lines

Dominant part

P odd condensate

$\eta'_f = \langle \bar{q}'_f i\gamma_5 q'_f \rangle$: Blue and Purple lines

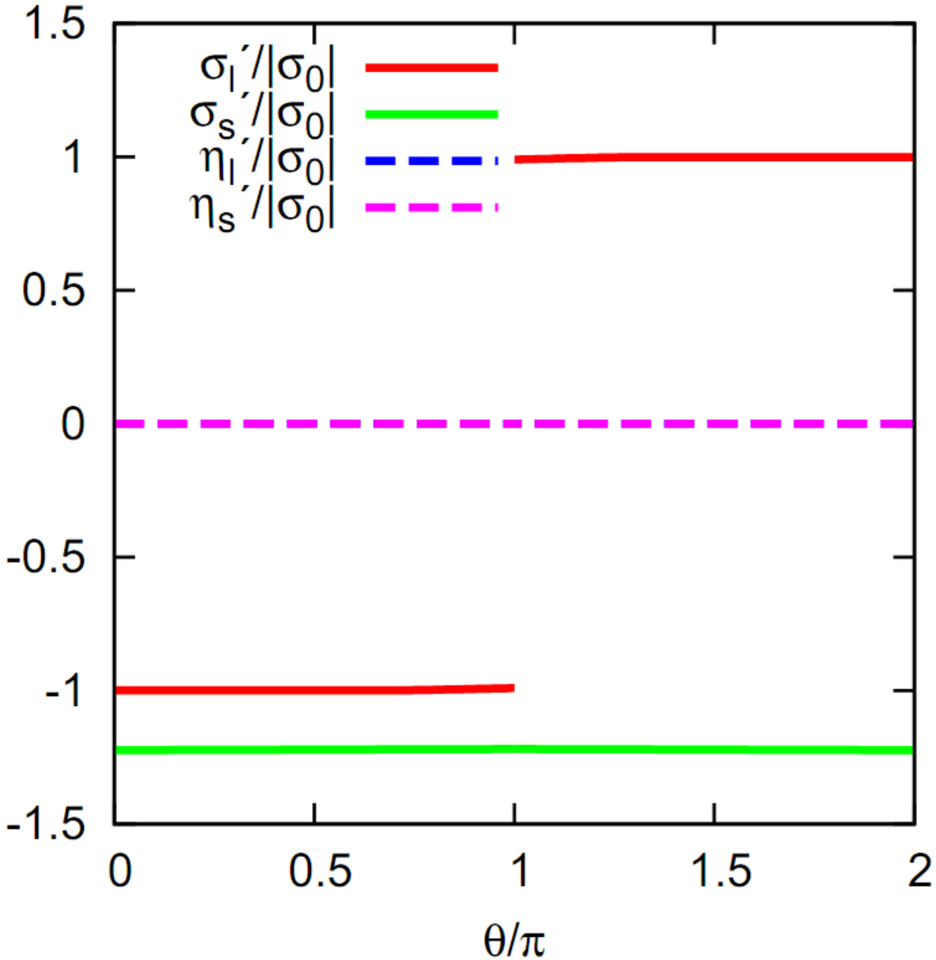
Always small

Sign problem vs. Theta vacuum

Without parity odd mass

$$T = \mu = 0$$

$$m_l(\theta) = m_l \cos(\theta/2) + \cancel{i\gamma_5 m_l \sin(\theta/2)}$$



$$\sigma_0 \equiv \sigma_l(T = \mu = \theta = 0)$$

P even condensate

$\sigma'_f = \langle \bar{q}'_f q'_f \rangle$: **Red** and **Green** lines

Not affected

P odd condensate

$\eta'_f = \langle \bar{q}'_f i\gamma_5 q'_f \rangle$: **Blue** and **Purple** lines

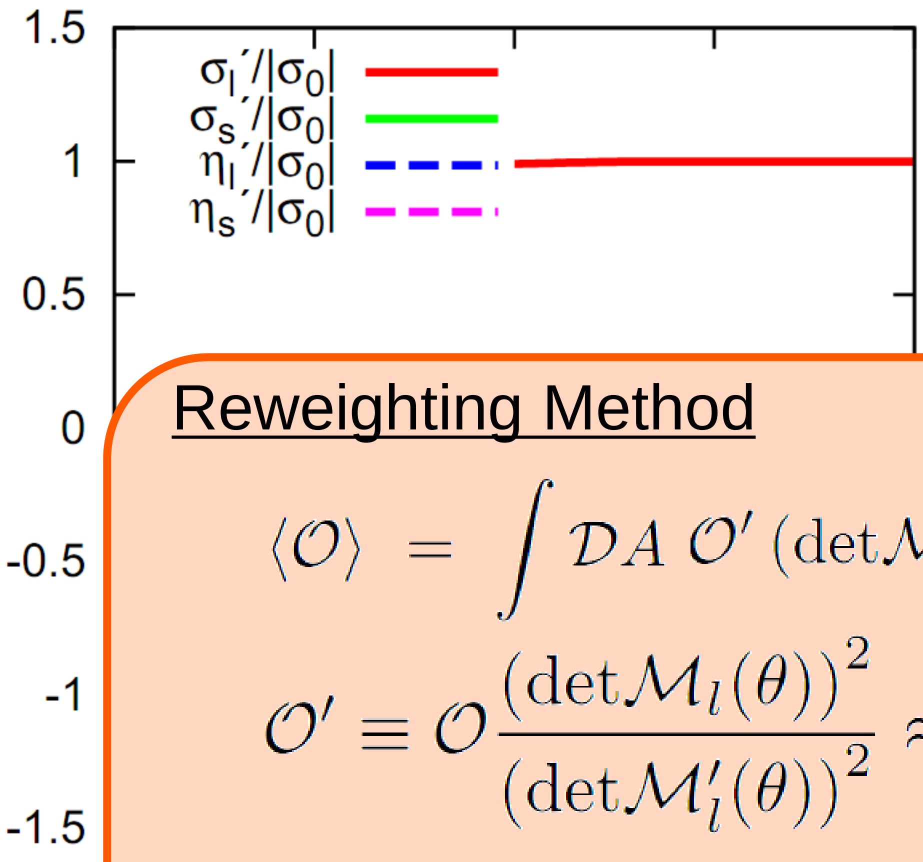
Become zero

Sign problem vs. Theta vacuum

<u>Without parity odd mass</u>

$$T = \mu = 0$$

$$m_l(\theta) = m_l \cos(\theta/2) + \cancel{i\gamma_5 m_l \sin(\theta/2)}$$



<u>P even condensate</u>

$\sigma'_f = \langle \bar{q}'_f q'_f \rangle$: **Red** and **Green** lines

➡ Not affected

e lines

<u>Reweighting Method</u>

$$\langle \mathcal{O} \rangle = \int \mathcal{D}A \; \mathcal{O}' \left(\det \mathcal{M}'_l(\theta)\right)^2 \det \mathcal{M}_s e^{-S_g}$$

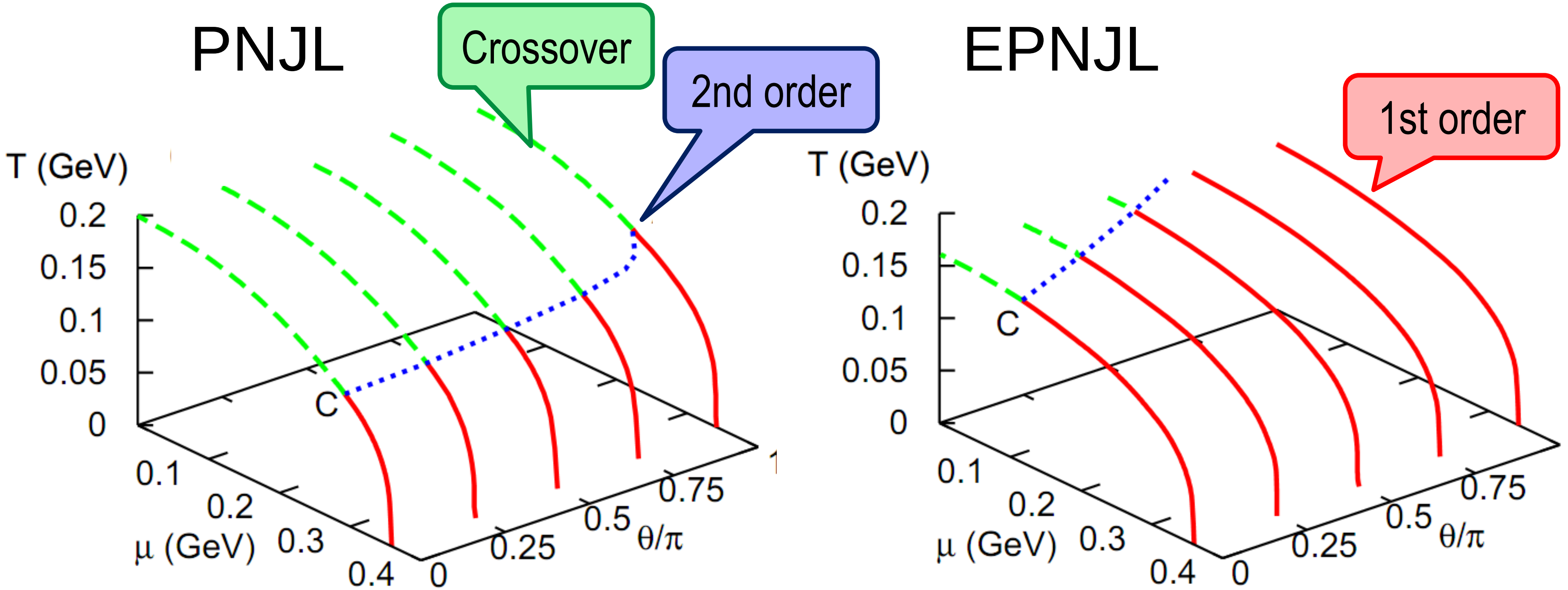
$$\mathcal{O}' \equiv \mathcal{O} \frac{\left(\det \mathcal{M}_l(\theta)\right)^2}{\left(\det \mathcal{M}'_l(\theta)\right)^2} \approx \mathcal{O}$$

$\det \mathcal{M}'_l(\theta)$: Fermion determinant without P odd mass

$$\sigma_0 \equiv \sigma_l(T = \mu = \theta = 0)$$

Phase structure

Transition order of chiral transition



C : Critical endpoint at $\theta = 0$

There is a possibility that the cosmic evolution is changed at QCD epoch if theta is large

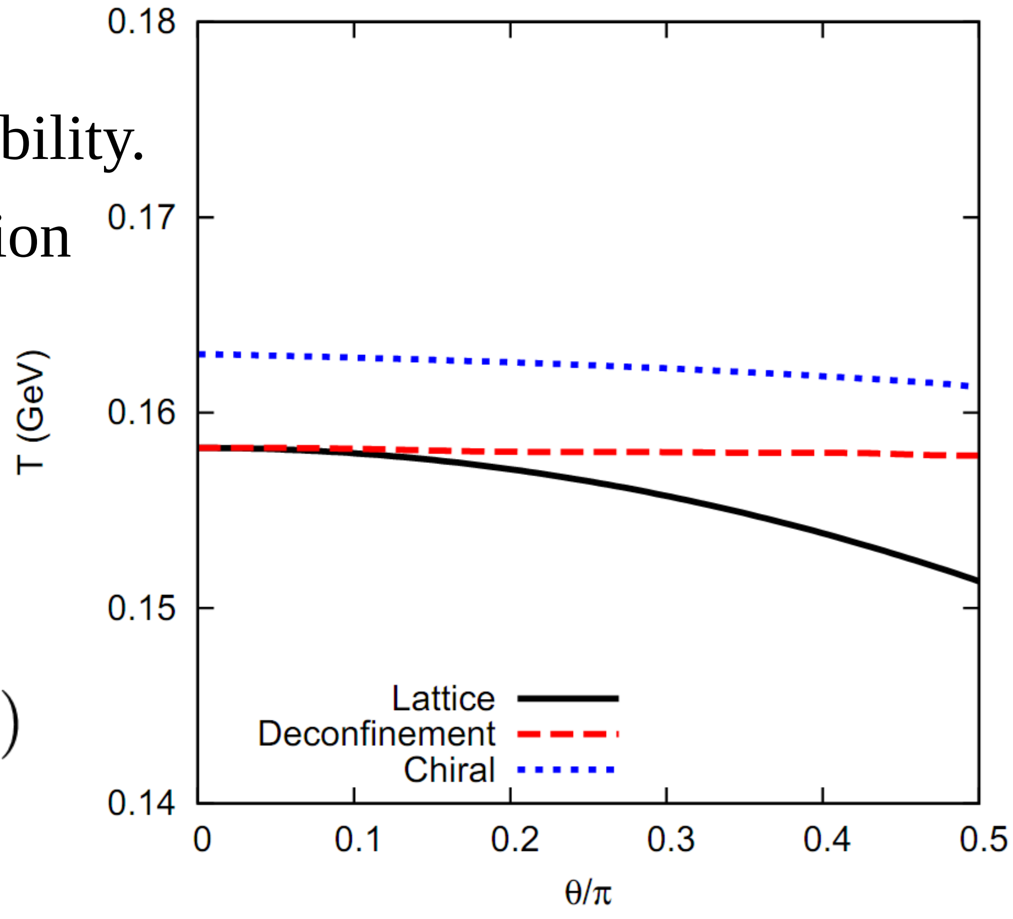
Theta dependence of transition temperatures

EPNJL Model

Transition temperatures are defined as the peak of susceptibility.

Red : Deconfinement transition

Blue : Chiral transition



Lattice data

$$\frac{T_c(\theta)}{T_c(0)} = 1 - R_\theta\ \theta^2 + O(\theta^4)$$

$$R_\theta = 0.0175(7)$$

Lattice data :

M. D'Elia and F. Negro, arXiv:1205.0538 [hep-lat] (2012) ***The next speaker***

Summary

Ordinal representation → **Transformed Lagrangian**

$$i\theta\tilde{F}F$$

$$\boxed{\begin{aligned} q_l &= e^{i\gamma_5 \frac{\theta}{4}} q_l' \\ q_s &= q_s' \end{aligned}}$$

$$m_l \cos(\theta/2) + i\gamma_5 m_l \sin(\theta/2)$$

Parity odd mass is negligible.

Reweighting Method

$$\langle \mathcal{O} \rangle = \int \mathcal{D}A\, \mathcal{O}' \left(\det \mathcal{M}_l'(\theta)\right)^2 \det \mathcal{M}_s e^{-S_g}$$

$$\mathcal{O}' \equiv \mathcal{O} \frac{\left(\det \mathcal{M}_l(\theta)\right)^2}{\left(\det \mathcal{M}_l'(\theta)\right)^2}$$

$\det \mathcal{M}_l'(\theta)$: Fermion determinant without P odd mass

EPNJL model prediction

QCD transition at zero chemical potential may be 1st order when theta is large.