The phase diagram of G_2-QCD

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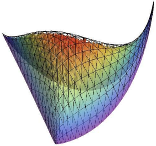

Research Training Group
Quantum and Gravitational Fields

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Introduction

- G_2 is the smallest Lie-group which is simply connected and has a trivial center
- The group has rank 2 and hence possesses two fundamental representations

$$(7) \sim \text{quark}, \quad (14) \sim \text{gluons}$$

- Similar as in $SU(3)$ two or three quarks can build a colour singlet

$$(7) \otimes (7) = (1) \oplus \cdots \quad , \quad (7) \otimes (7) \otimes (7) = (1) \oplus \cdots$$

- In contrast gluons can screen the colour charge of a single static quark

$$(7) \otimes (14) \otimes (14) \otimes (14) = (1) \oplus \cdots$$

- All representations of G_2 are real

- The Polyakov loop is not an order parameter for confinement.
- The flux tube between two static quarks can break due to dynamical gluons.
- No linear rising potential up to arbitrary long distances.

Confinement in G_2 gluodynamic really means as in QCD

linear rising potential only at intermediate scales

...and on a finite lattice

- The Polyakov loop is an approximate order parameter which changes rapidly at the phase transition and is small (but non-zero) in the confining phase
- First order deconfinement phase transition

K. Holland, P. Minkowski, M. Pepe and U. J. Wiese, Nucl. Phys. **B668** (2003) 207

- Casimir scaling at short and intermediate scales
- String breaking in the fundamental and adjoint representation at larger distances

B. Wellegehausen, A. Wipf, C. Wozar, Casimir Scaling and String Breaking in G(2) Gluodynamics, Phys.Rev.D83:016001,2011

B. Wellegehausen, A. Wipf, C. Wozar, Phase diagram of the lattice G_2 Higgs model, Phys. Rev. D 83, 114502 (2011)

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Fundamental constituents and colorless bound states of G_2-QCD compared to QCD

	G_2	$SU(3)$	
quark	(7)	(3)	fermion
antiquark	(7)	$(\bar{3})$	fermion
gluon	(14)	(8)	boson
meson	$(7) \otimes (7)$	$(3) \otimes (\bar{3})$	boson
(bosonic) baryon	$(7) \otimes (7)$	-	boson
(fermionic) baryon	$(7) \otimes (7) \otimes (7)$	$(3) \otimes (3) \otimes (3)$	fermion
glueballs	$(14) \otimes (14)$	$(8) \otimes (8)$	boson
	$(14) \otimes (14) \otimes (14)$	$(8) \otimes (8) \otimes (8)$	boson
hybrid	$(7) \otimes (14) \otimes (14) \otimes (14)$	-	fermion

- With respect to the color representation there is no difference between quarks and antiquarks
- Additional bound states of quarks and gluons (hybrids) exist
- The most important difference is the existence of diquarks in G_2-QCD

G_2-QCD in the continuum

Lagrange density for N_f (Dirac) flavour G_2-QCD

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi} \left(\mathrm{i}\, \not{D} - m + \mathrm{i}\, \gamma_0\, \mu \right) \Psi$$

- Decompose the Dirac spinor $\Psi = \chi + \mathrm{i}\, \eta$ into Majorana spinors

$$\mathcal{L}_{\text{matter}} = \bar{\Psi} \left(\mathrm{i}\, \not{D} - m + \mathrm{i}\, \gamma_0\, \mu \right) \Psi = \begin{pmatrix} \bar{\chi} \\ \bar{\eta} \end{pmatrix} \begin{pmatrix} \mathrm{i}\, \not{D} - m & \mathrm{i}\, \gamma_0\, \mu \\ -\mathrm{i}\, \gamma_0\, \mu & \mathrm{i}\, \not{D} - m \end{pmatrix} \begin{pmatrix} \chi \\ \eta \end{pmatrix}$$

- For vanishing baryon chemical potential $\mu = 0$

$$\mathcal{L}_{\text{matter}} = \bar{\lambda} \left(\mathrm{i}\, \not{D} - m \right) \lambda,$$

where $\lambda = (\chi, \eta)$ is a $2N_f$ component Majorana spinor.

- Vector chiral transformation

$$\lambda \mapsto e^{\beta \otimes \mathbb{1}} \lambda \quad \Longrightarrow \quad SO(2N_\mathrm{f}), \mathbb{Z}(2)^{2N_\mathrm{f}}$$

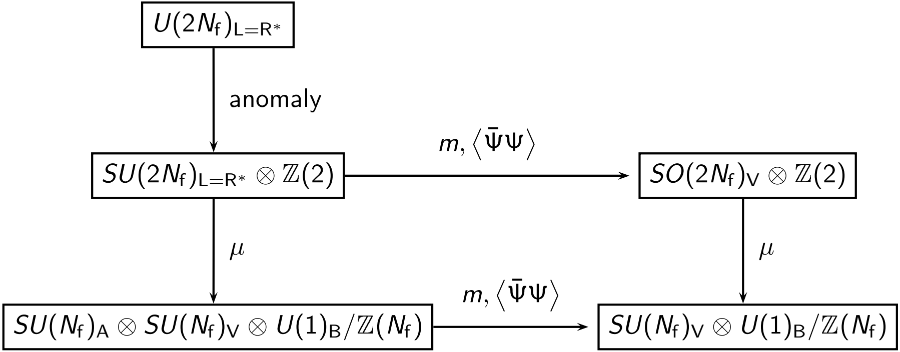
- Axial chiral transformation

$$\lambda \mapsto e^{\mathrm{i}\,\alpha \otimes \gamma_5} \lambda \quad \Longrightarrow \quad SO(2N_\mathrm{f}), U(1)^{2N_\mathrm{f}}$$

- Due to the Majorana constraint, left and right-handed transformations are not independent

Chiral symmetry of G_2-QCD

$$U(2N_\mathrm{f})_{\mathrm{L}=\mathrm{R}^*} = SU(2N_\mathrm{f})_{\mathrm{L}=\mathrm{R}^*} \otimes U(1)_\mathrm{A}/\mathbb{Z}(2N_\mathrm{f})$$


$U(2N_f)_{L=R^*}$
anomaly
$SU(2N_f)_{L=R^*} \otimes \mathbb{Z}(2)$
$m, \langle \bar{\Psi}\Psi \rangle$
$SO(2N_f)_V \otimes \mathbb{Z}(2)$
μ
μ
$SU(N_f)_A \otimes SU(N_f)_V \otimes U(1)_B/\mathbb{Z}(N_f)$
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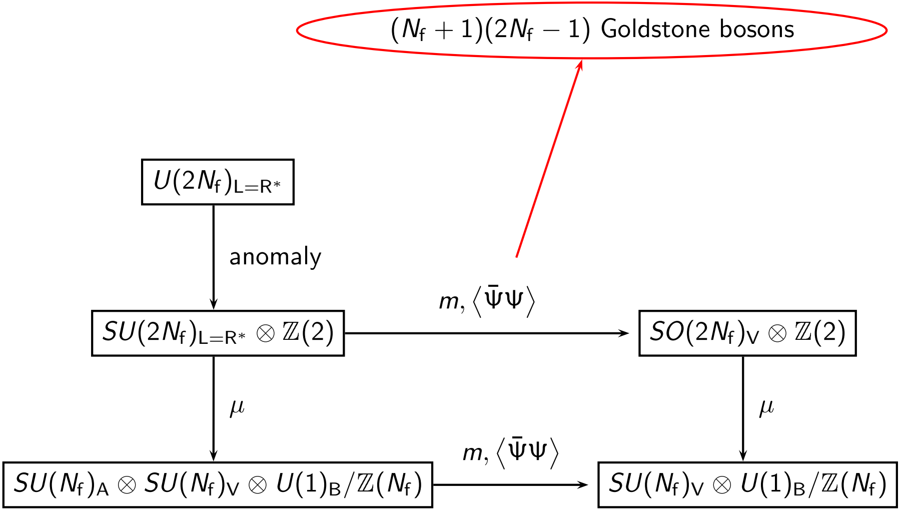
$(N_f + 1)(2N_f - 1)$ Goldstone bosons
$U(2N_f)_{L=R^*}$
anomaly
$m, \langle \bar{\Psi}\Psi \rangle$
$SU(2N_f)_{L=R^*} \otimes \mathbb{Z}(2)$
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μ
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$SU(N_f)_V \otimes U(1)_B/\mathbb{Z}(N_f)$

- For $N_f = 1$ we have a nontrivial chiral symmetry
- Chiral symmetry $SU(2)_{\mathrm{L=R}^*} \otimes \mathbb{Z}(2) \longrightarrow U(1)_{\mathrm{B}} \otimes \mathbb{Z}(2)$
- 2 (would-be) Goldstone bosons (pions)

$$\pi_+ = \pi_- = \pi_\pm = \bar{\chi}\gamma_5\eta = \bar{\Psi}^{\mathrm{C}}\gamma_5\Psi - \bar{\Psi}\gamma_5\Psi^{\mathrm{C}}$$

$$\pi_0 = \frac{1}{\sqrt{2}}\left(\bar{\chi}\gamma_5\chi - \bar{\eta}\gamma_5\eta\right) = \bar{\Psi}^{\mathrm{C}}\gamma_5\Psi + \bar{\Psi}\gamma_5\Psi^{\mathrm{C}}$$

- Pions carry baryon charge $n_{\mathrm{B}} = 2$, i.e. they couple to baryon chemical potential

In contrast to QCD ...

...the Goldstone bosons of chiral symmetry breaking are scalar baryons instead of pseudoscalar mesons

Unitary op. T, $T^\dagger = T^{-1}$ and Dirac-Op. $D[A,m,\mu]$, $\mu \in \mathbb{R}$

$$D^* T = T D \quad \Longrightarrow \quad \det D \in \mathbb{R}$$

If additionally $T^* T = -\mathbb{1}$, then $\det D \geq 0$

$$D[A,m,\mu] = \gamma^\mu(\partial_\mu - gA_\mu) - m + \gamma_0 \mu$$

$$T = F \otimes \Gamma \quad \Longrightarrow \quad F A_\mu F^\dagger = A_\mu^* \quad \text{and} \quad \Gamma \gamma_\mu \Gamma^\dagger = \gamma_\mu^*$$

A_μ and γ_μ have to be unitary equivalent to real representations.

Euclidean representation for γ-Matrices: $\Gamma = C\gamma_5$ and $\Gamma^*\Gamma = -\mathbb{1}$

For G_2 every representation is real: $F = \mathbb{1} \quad \Longrightarrow \quad T^* T = -\mathbb{1}$

$$\det D[A,m,\mu] \geq 0$$

- Coset space decomposition $G_2/SU(3) \sim SO(7)/SO(6) \sim S_6$

$$\Longrightarrow \quad \mathcal{U} = \mathcal{S} \cdot \mathcal{V} \quad \text{with} \quad \mathcal{S} \in G_2/SU(3) \quad \text{and} \quad \mathcal{V} \in SU(3)$$

- With a massive scalar field ϕ in the (7) representation G_2 ($SO(7)$) can be broken down to its $SU(3)$ subgroup ($m \sim \langle \phi^2 \rangle \to \infty$)

$$(7) \longrightarrow (3) \oplus (\bar{3}) \oplus (1)$$

$$\mathcal{L} = \begin{pmatrix} \bar{\Psi}_3 \\ \bar{\Psi}_{\bar{3}} \\ \bar{\Psi}_1 \end{pmatrix} \begin{pmatrix} \mathrm{i}\,\not{D}_3 - m + \mathrm{i}\,\gamma_0\,\mu & 0 & 0 \\ 0 & \mathrm{i}\,\not{D}_{\bar{3}} - m + \mathrm{i}\,\gamma_0\,\mu & 0 \\ 0 & 0 & \mathrm{i}\,\not{D}_1 - m + \mathrm{i}\,\gamma_0\,\mu \end{pmatrix} \begin{pmatrix} \Psi_3 \\ \Psi_{\bar{3}} \\ \Psi_1 \end{pmatrix}$$

In this limit, QCD with isospin chemical potential is obtained

$$\mathcal{L} = \bar{\Psi}_u \left(\mathrm{i}\,\not{D} - m + \mathrm{i}\,\gamma_0\,\mu\right) \Psi_u + \bar{\Psi}_d \left(\mathrm{i}\,\not{D} - m - \mathrm{i}\,\gamma_0\,\mu\right) \Psi_d$$

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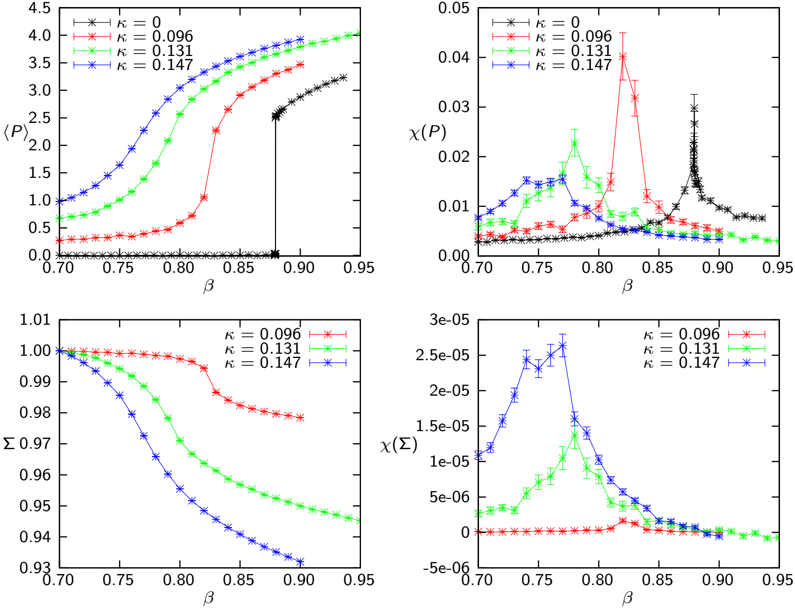
Lattice results

- Lattice simulations with Wilson fermions and $N_f = 1$ Dirac flavour
- Symanzik improved gauge action with inverse gauge coupling $\beta \sim g^{-2}$
- Different lattices from $V = 8^3 \times 2$ up to $V = 16^4$ and different values of κ

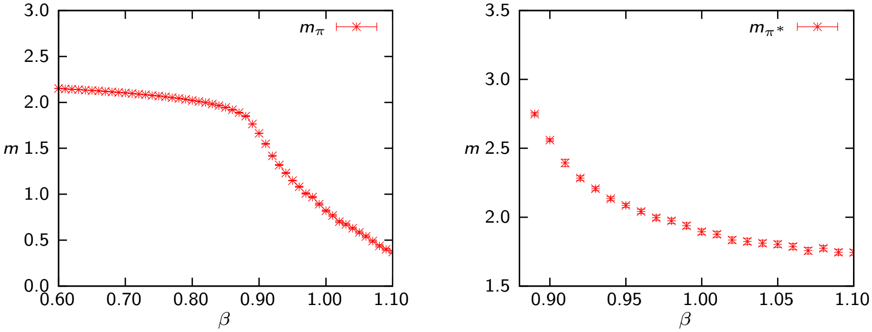
Observables considered here are ...

- ... the Polyakov loop $P(T, \mu, m)$
- ... the chiral condensate $\Sigma(T, \mu, m) = \frac{1}{V} \frac{\partial \ln Z}{\partial m}$
- ... the quark number density $n_q(T, \mu, m) = \frac{1}{V} \frac{\partial \ln Z}{\partial \mu}$
- ... its susceptibilities $\chi(\mathcal{O})$
- ... and the pion correlation function (pion mass)

$$C_\pi(x, y) = \left\langle \pi_0(x)\, \pi_0^\dagger(y) \right\rangle = \left\langle \pi_\pm(x)\, \pi_\pm^\dagger(y)) \right\rangle = \left\langle \bar{\chi}(x)\gamma_5\chi(x)\, \bar{\chi}(y)\gamma_5\chi(y) \right\rangle$$



Volume $V = 8^3 \times 2$ $\mu = 0.0$

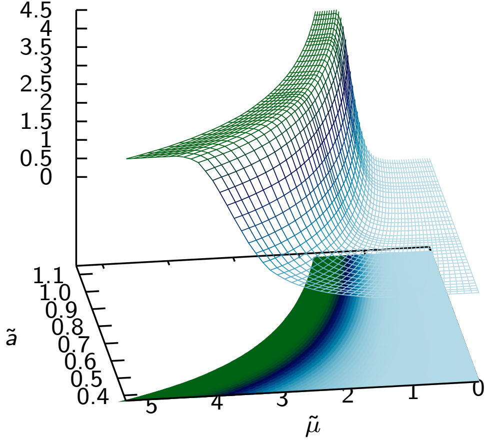


$$\tilde{a} \equiv m_\pi(\beta) = m_{\pi,\mathrm{phys}}\, a(\beta)$$

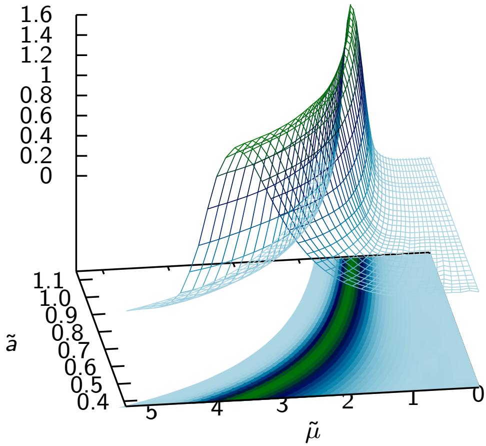
$$\tilde{T} = \frac{T}{m_{\pi,\mathrm{phys}}} = \frac{1}{N_t\, m_\pi(\beta)} \quad , \quad \tilde{\mu} = \frac{\mu}{m_\pi} = \frac{\mu_{\mathrm{phys}}}{m_{\pi,\mathrm{phys}}} \quad , \quad \tilde{\mu}/\tilde{T} = \mu\, N_t$$

Zero temperature

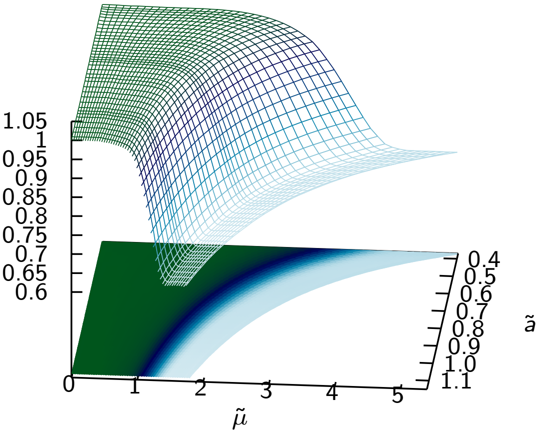
Quark number density $n_q = \frac{1}{V}\frac{\partial \ln Z}{\partial \mu}$



Polyakov loop P



Chiral condensate $\Sigma = \frac{1}{V}\frac{\partial \ln Z}{\partial m}$



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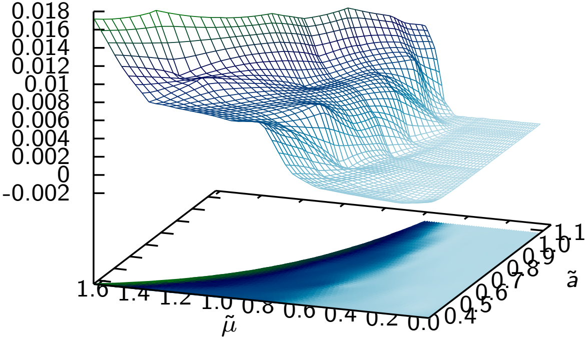
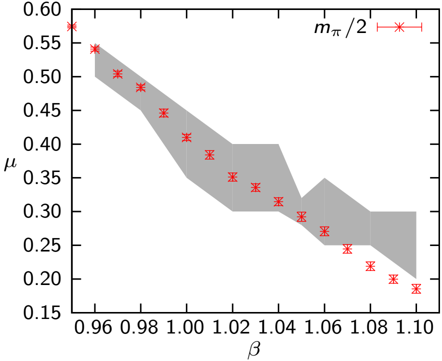
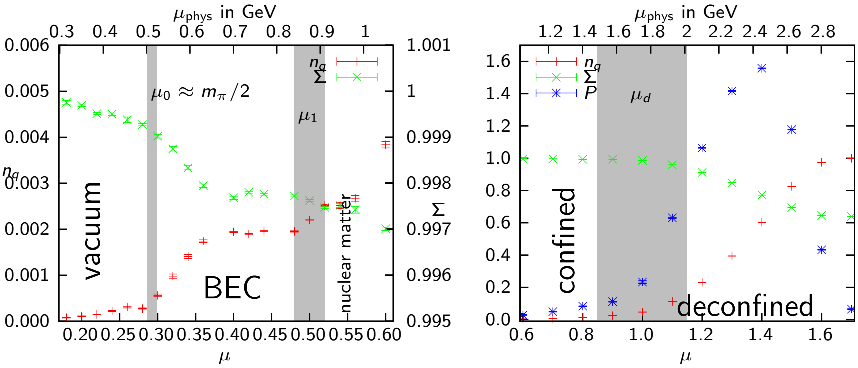


Onset transition to baryonic matter compared to pion mass



- Onset transition to (bosonic) baryonic matter at $\mu_0 \approx m_\pi/2$
- Silver blaze property known from QCD
- Diquarks condensate for $\mu > \mu_0$

- Physical units set by $T_c(\mu = 0) = 160\,\text{MeV} \implies m_\pi \approx 1024\,\text{MeV}$

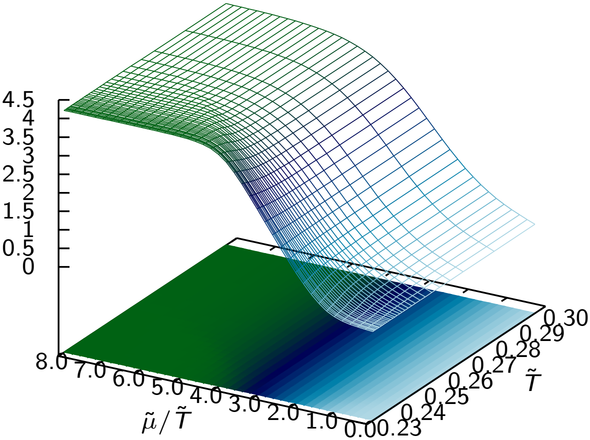


Quark number density n_q Chiral condensate Σ Polyakov loop P

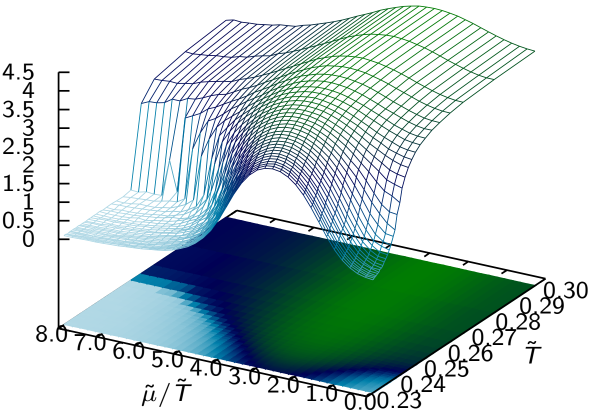
Volume $V = 8^3 \times 16$ $\kappa = 0.147$ $\beta = 1.05$

Finite temperature

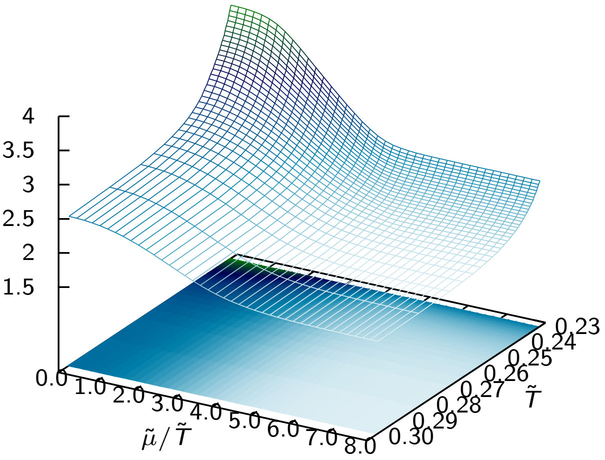
Quark number density $n_q = \frac{1}{V}\frac{\partial \ln Z}{\partial \mu}$



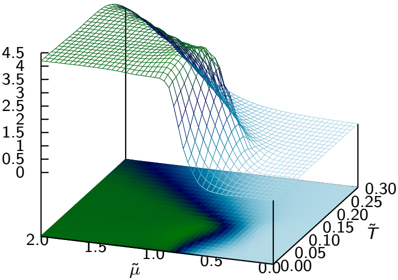
Polyakov loop P



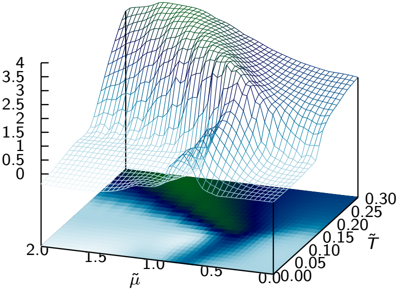
Chiral condensate $\Sigma = \frac{1}{V}\frac{\partial \ln Z}{\partial m}$



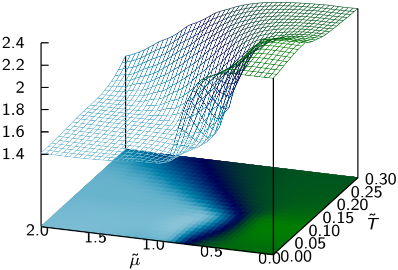
Quark number density



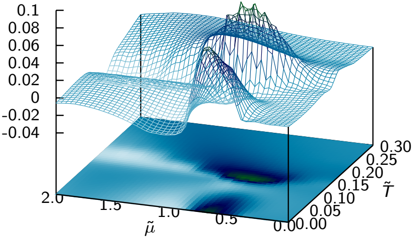
Polyakov loop



Chiral condensate

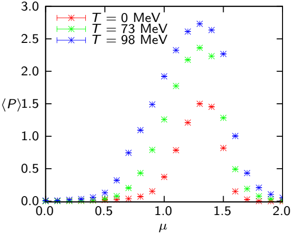


Plaquette density

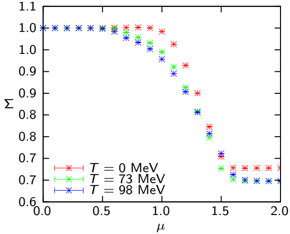


Volume $V = 8^3 \times 4$ and $V = 8^3 \times 16$ $\kappa = 0.147$ $\tilde{a} = m_\pi,\ \tilde{\mu} = \mu/m_\pi,\ \tilde{T} = T/m_\pi$

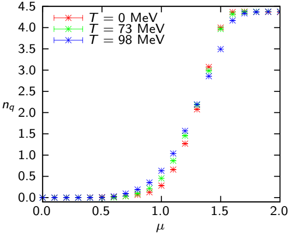
Polyakov loop



Chiral condensate



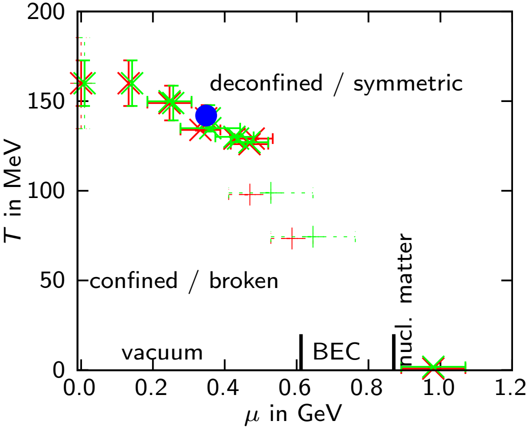
Quark number density



- Physical units set by $T_c(\mu = 0) = 160\,\text{MeV} \implies m_\pi \approx 785\,\text{MeV}$
- Qualitatively the same results as on the smaller lattices

Conclusions

The G_2-QCD phase diagram



- G_2 gauge theories share many important features with $SU(3)$ gauge theories
- There is no sign problem in G_2-QCD: It is possible to investigate the phase diagram of a theory with fundamental quarks and fermionic baryons even at low temperatures and high densities with lattice simulations
- G_2-QCD possesses the silver blaze property
- Various transitions at zero temperature: diquark condensation, onset of nuclear matter and deconfinement/chiral restoration

Outlook

- Smaller pion masses and larger lattices in order to verify the transitions at zero temperature
- Computation of neutron/proton masses
- Location of chiral and deconfinement transition
- Order of the phase transitions at zero and finite temperature
- 2 flavour G_2-QCD
- ...

A. Maas, L. von Smekal, B. H. Wellegehausen and A. Wipf, *The phase diagram of a gauge theory with fermionic baryons*, arXiv:1203.5653 [hep-lat], 2012.