Properties of G2 gauge theories

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Cairns
Australia

DFG

Overview

- Why G2?

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- G2 Yang-Mills theory

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  - Details: Next talk by Björn Wellegehausen
- Summary

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- QCD is a gauge theory

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- QCD is a gauge theory

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g

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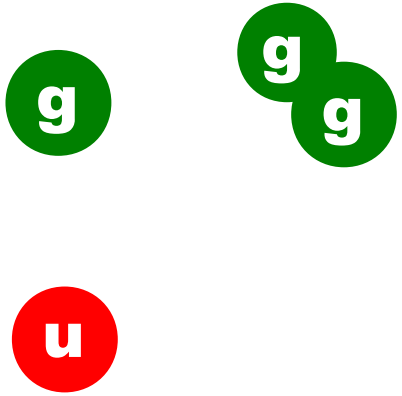

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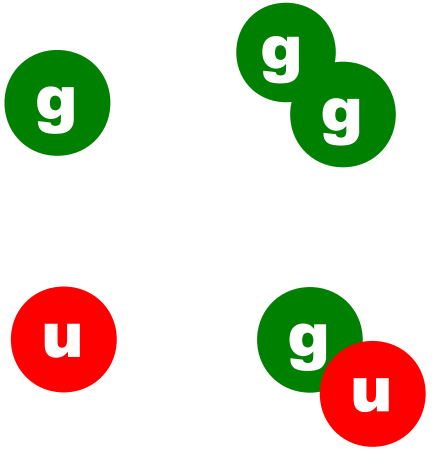

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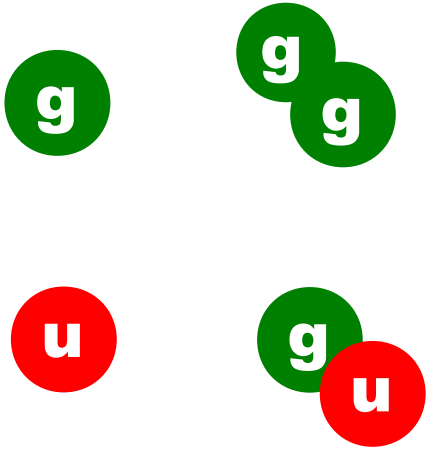

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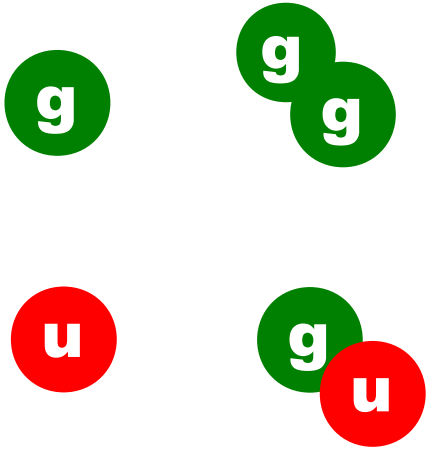

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    - Test of methods and models
    - Qualitative insights

G2 facts sheet

- G2 is an exceptional Lie group
  - Rank 2 (like SU(3))
  - Subgroup of SO(7)
  - Can be formulate as a product of SU(3) and the 6-sphere
  - All representations are equivalent to real representations
  - Fundamental representation 7 dimensional
  - Adjoint representation 14 dimensional

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- Qualitatively similar glueball spectrum
  - Similar gluon-gluon potential [Wellegehausen et al. PRD 11]

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- No center symmetry: What about the phase diagram?

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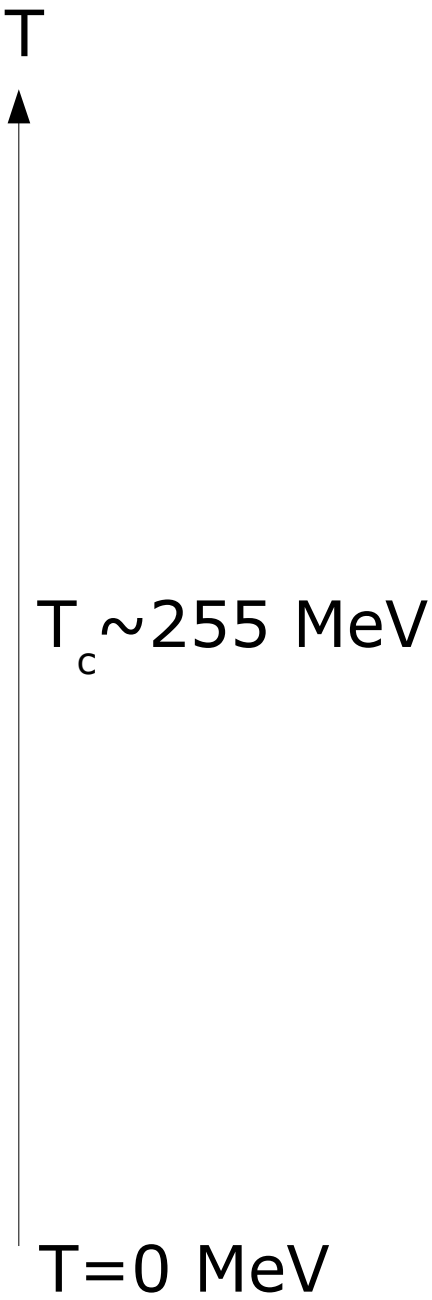

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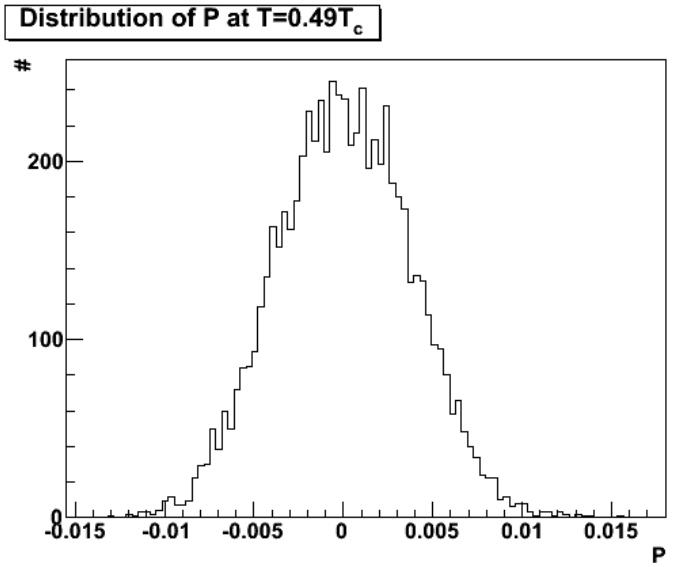
    - Observed in thermodynamic observables
      - Free energy, heat capacity
- Complicated by a bulk transition
  - Requires fine lattice [Cossu et al. JHEP 07]
  - Remains with quarks

Polyakov loop

[Danzer, Gattringer, Maas, JHEP'09
Pepe et al. NPA'07]



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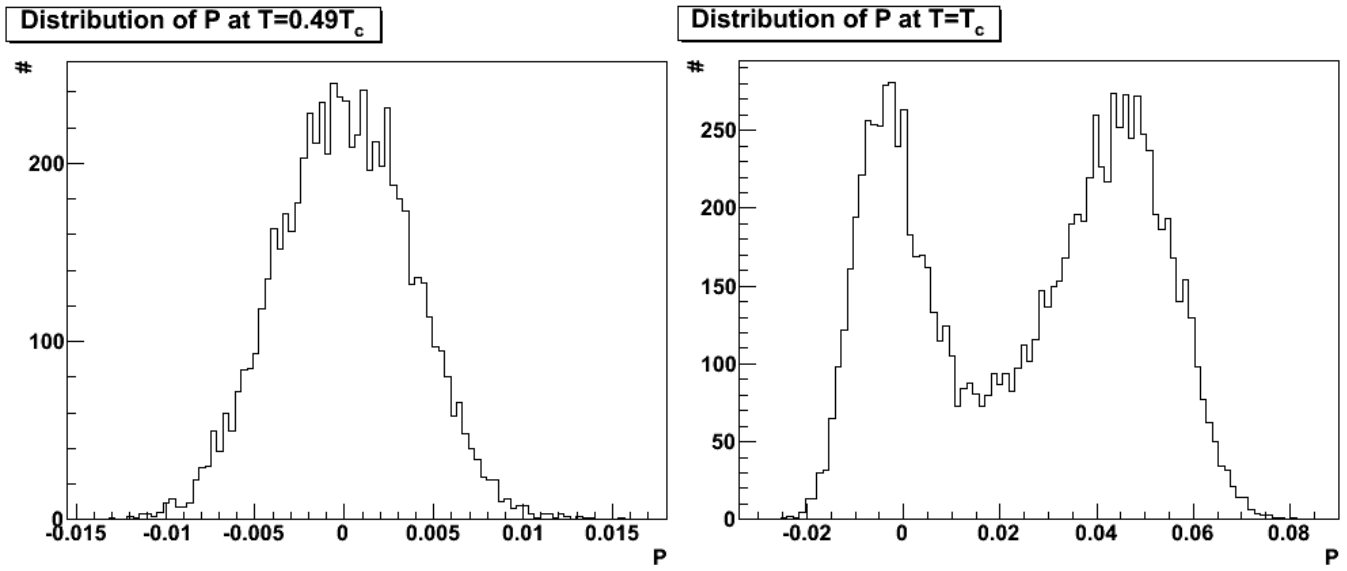

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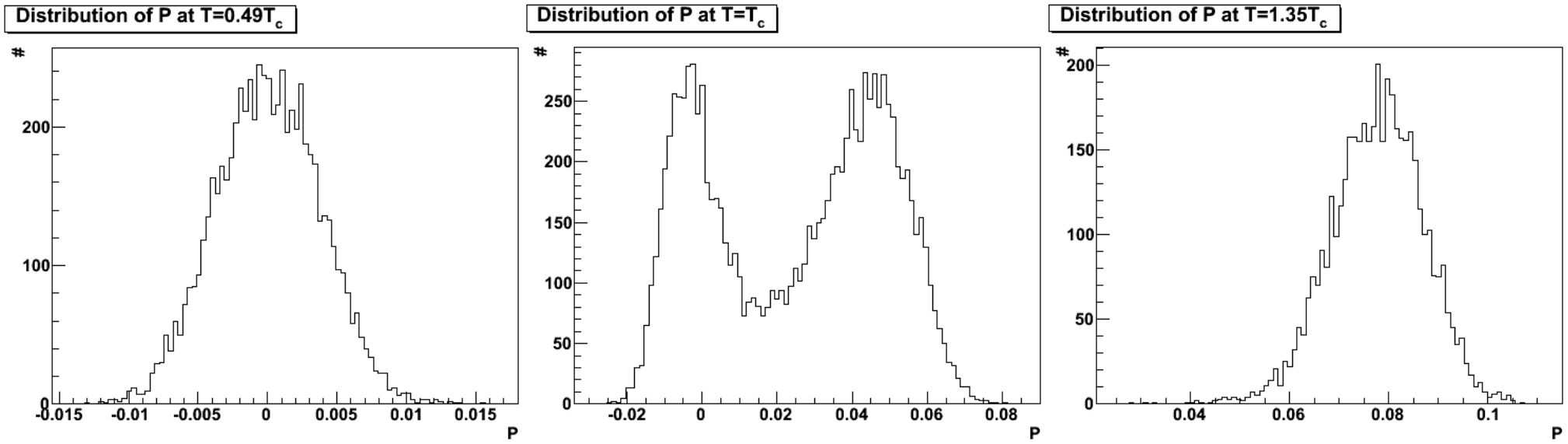

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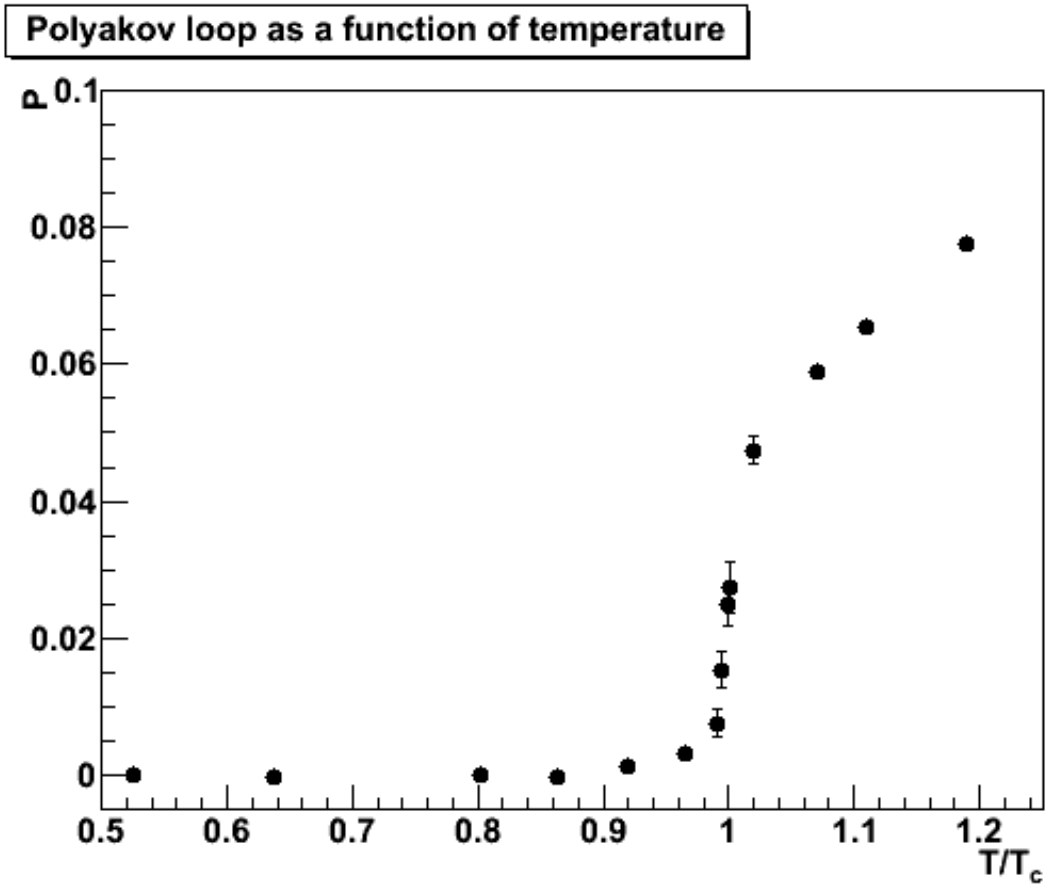

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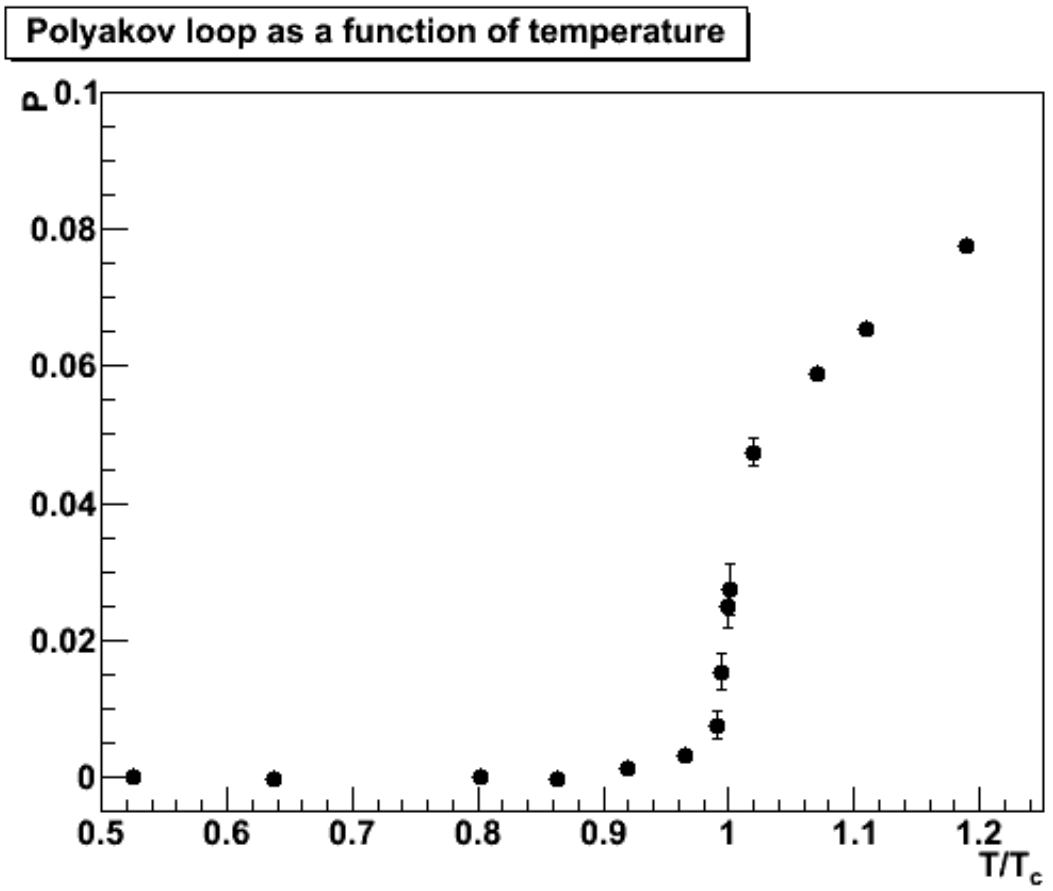

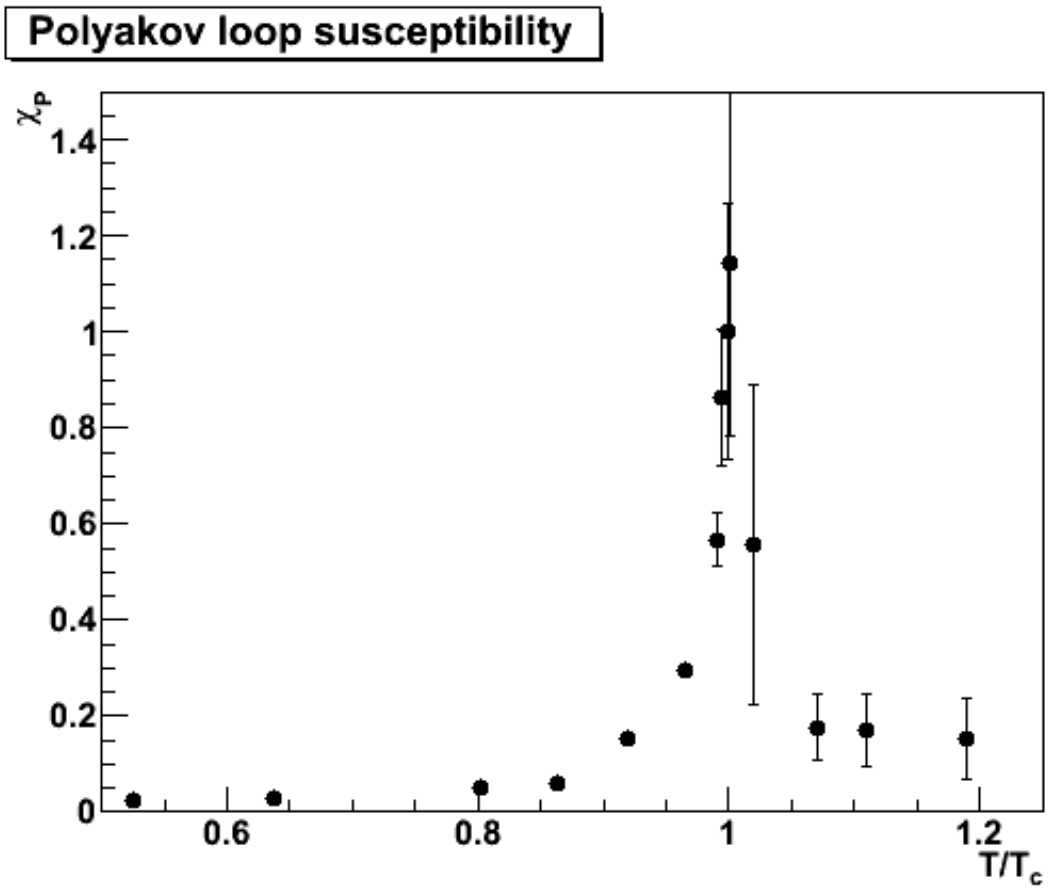

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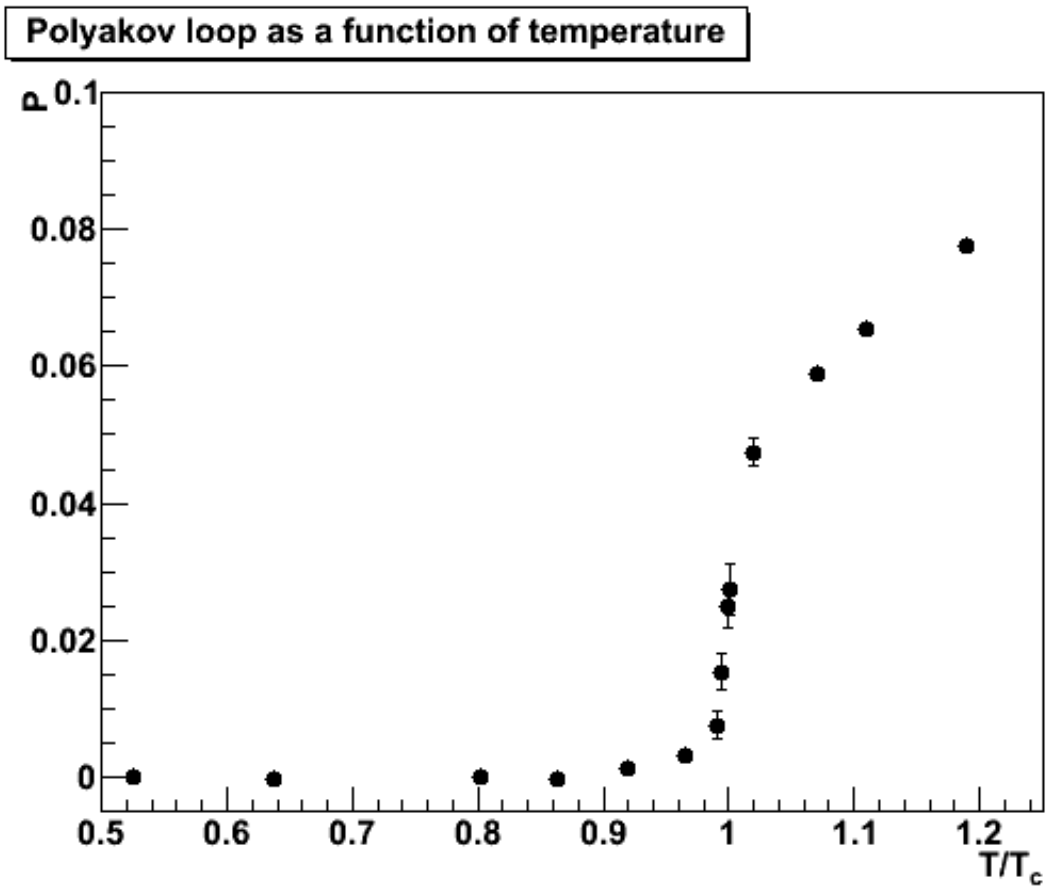

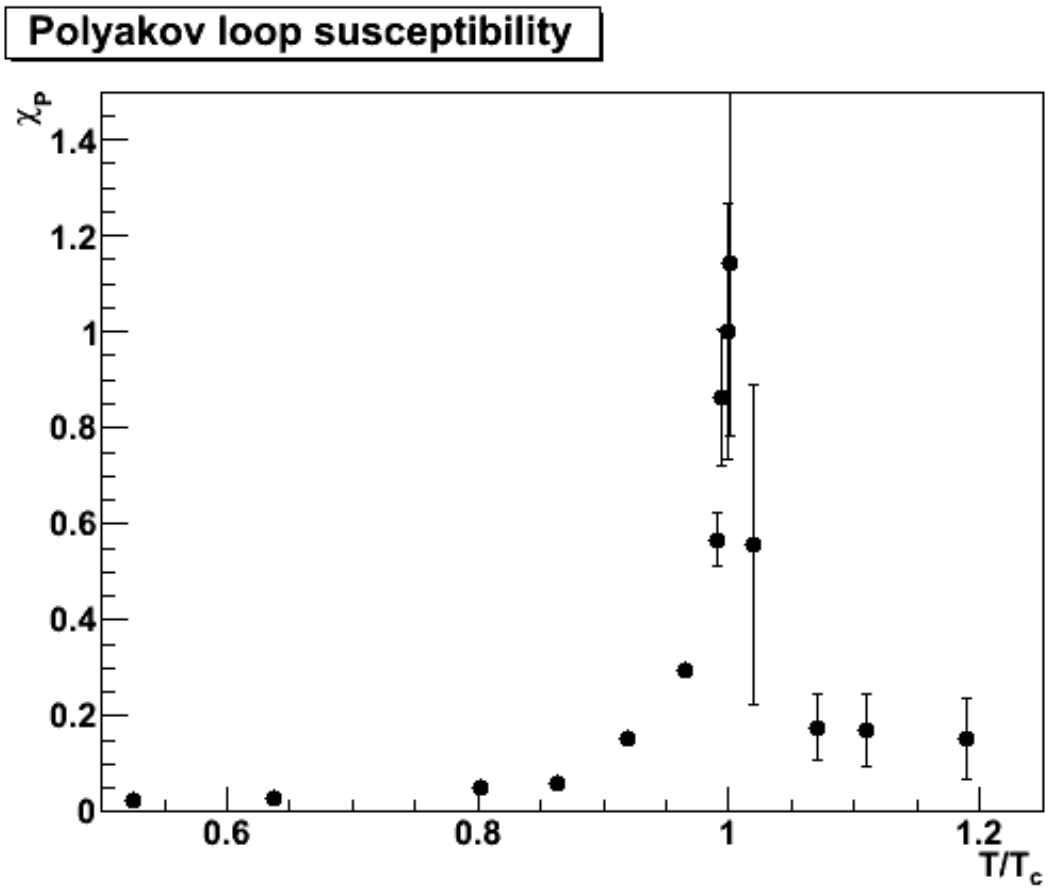

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- Non-anomalous chiral symmetry breaking for 1 flavor possible

Quenched G2 QCD

[Danzer, Gattringer, Maas, JHEP09]

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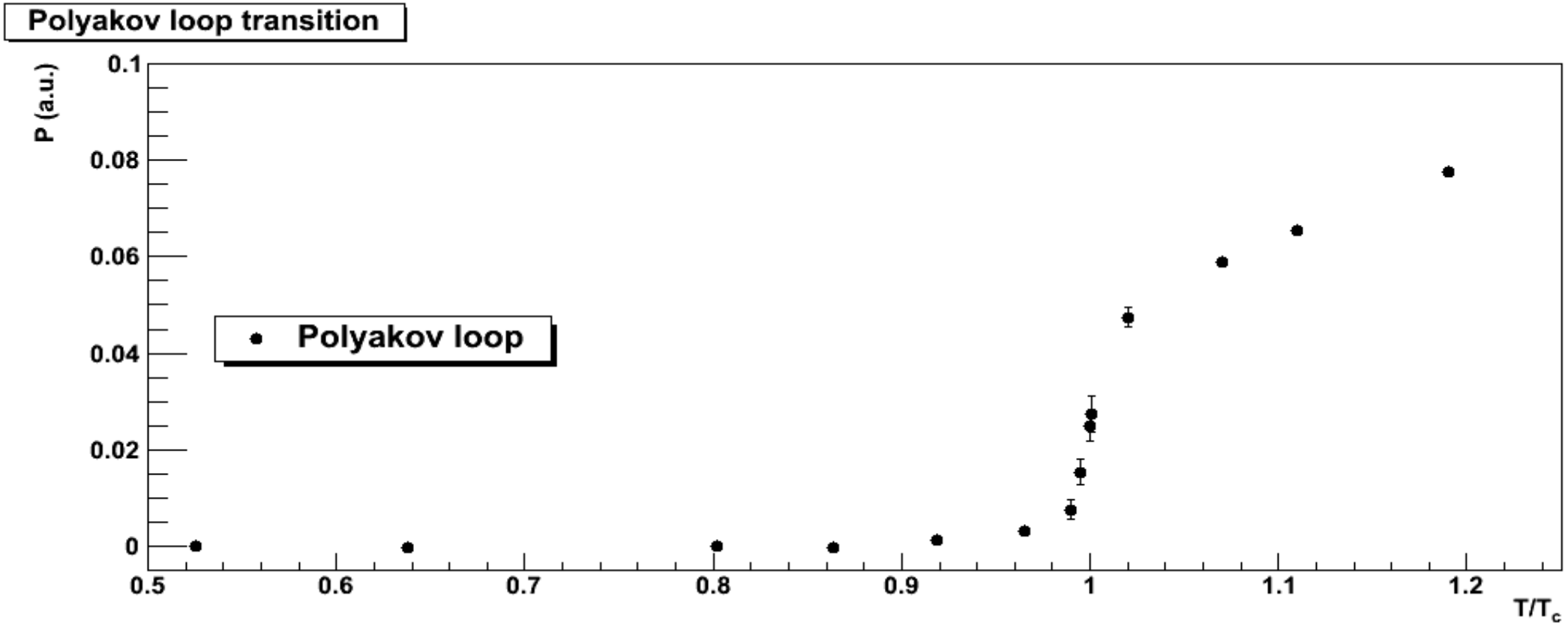

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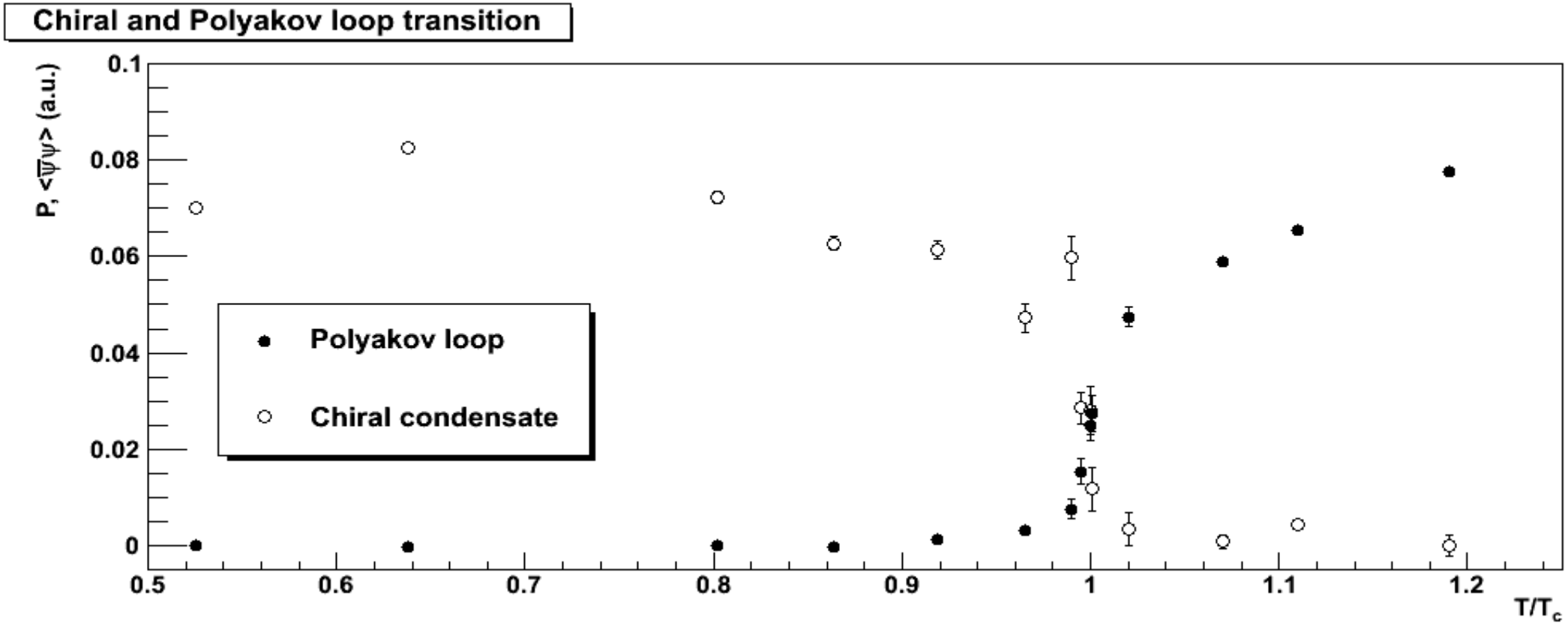

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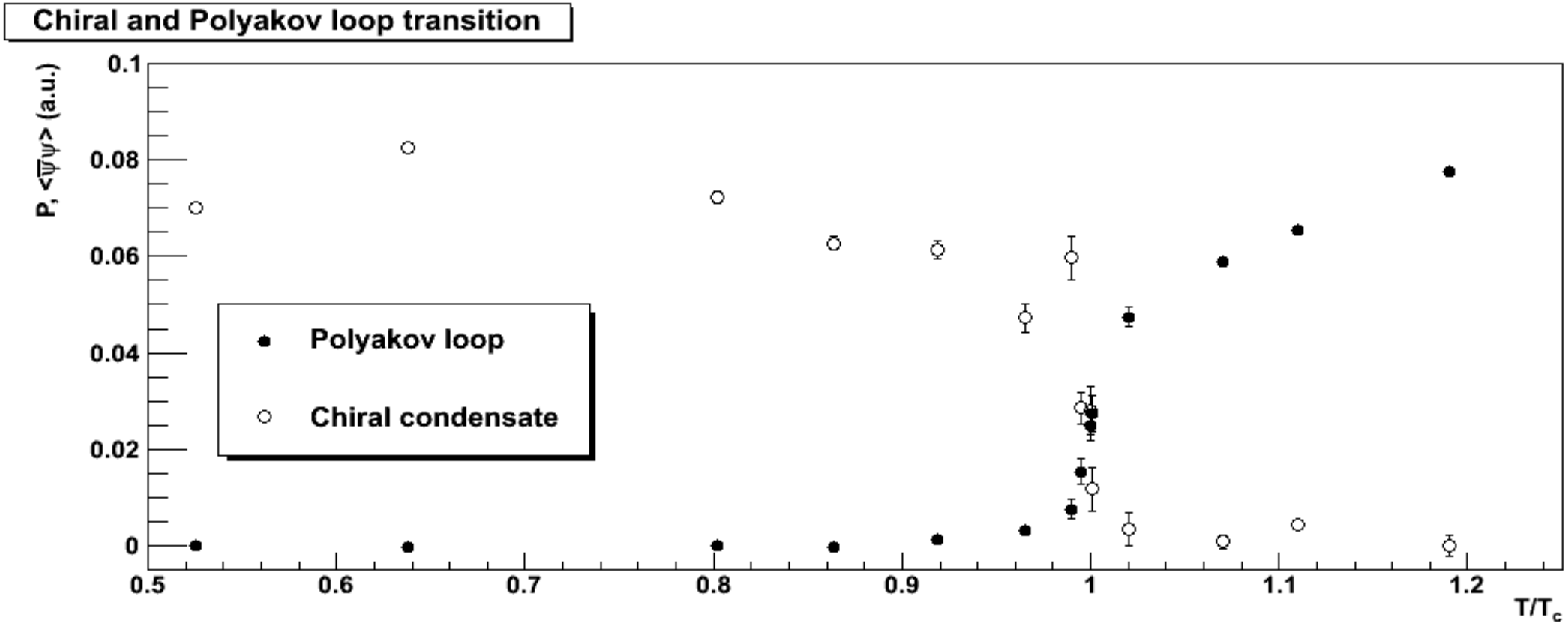

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- 'Restoration' at the phase transition
  - Like in QCD
  - Unlike adjoint QCD [Bilgici, Ilgenfritz, Gattringer, Maas JHEP 09]

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- Change of topological properties at the phase transition?

Picture of G2 topological lumps

- Identified by cooling

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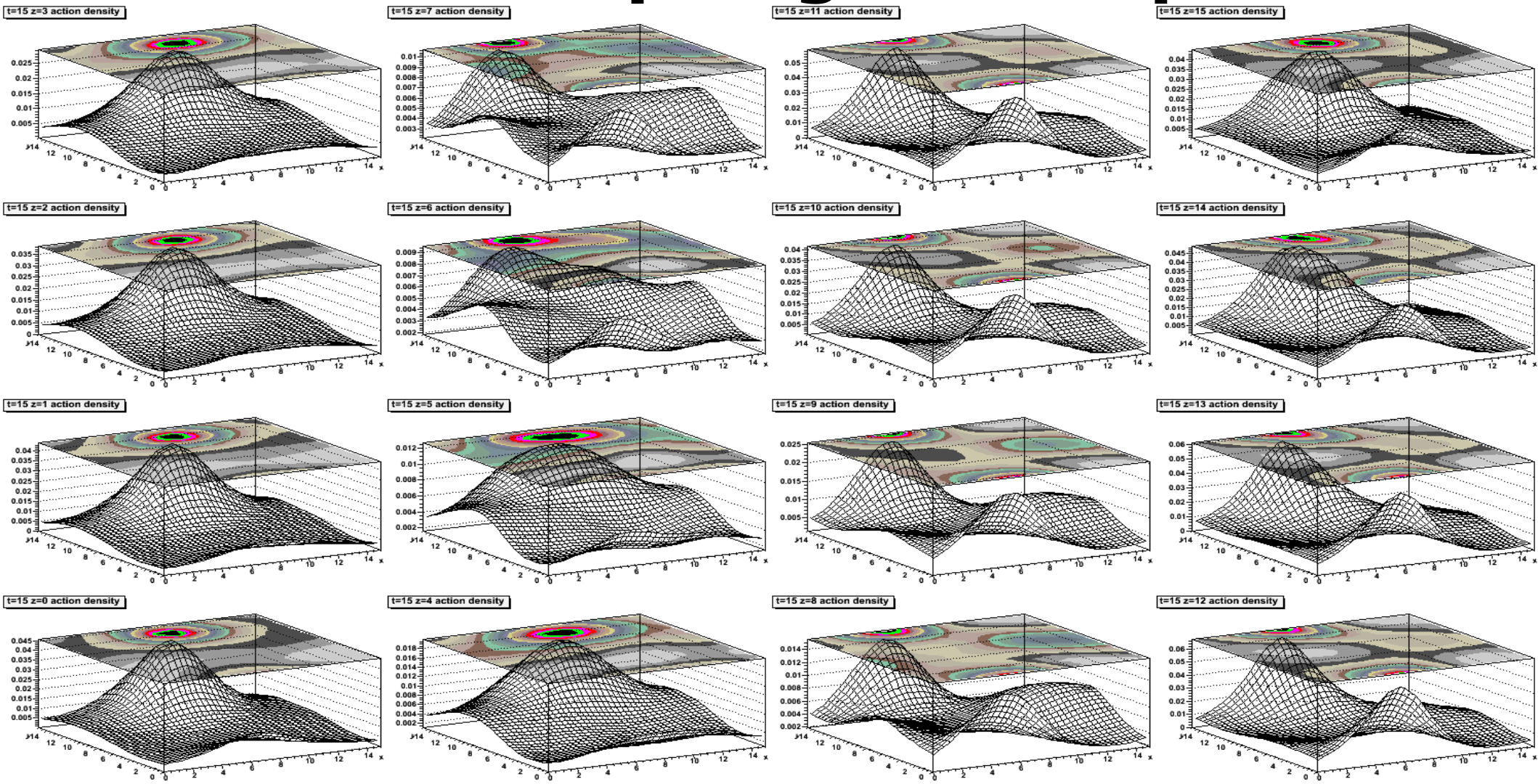

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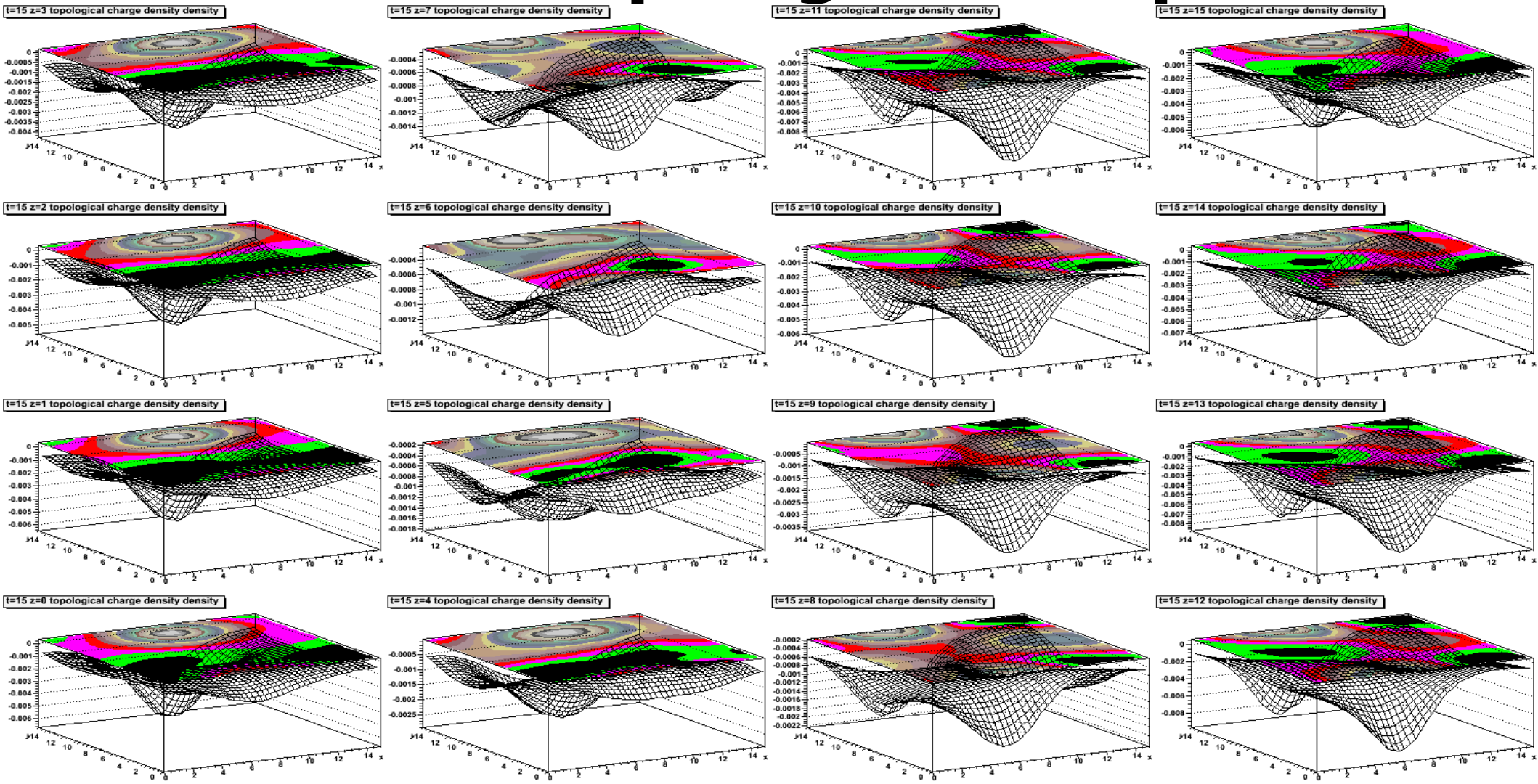

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[Ilgenfritz & Maas, unpublished]

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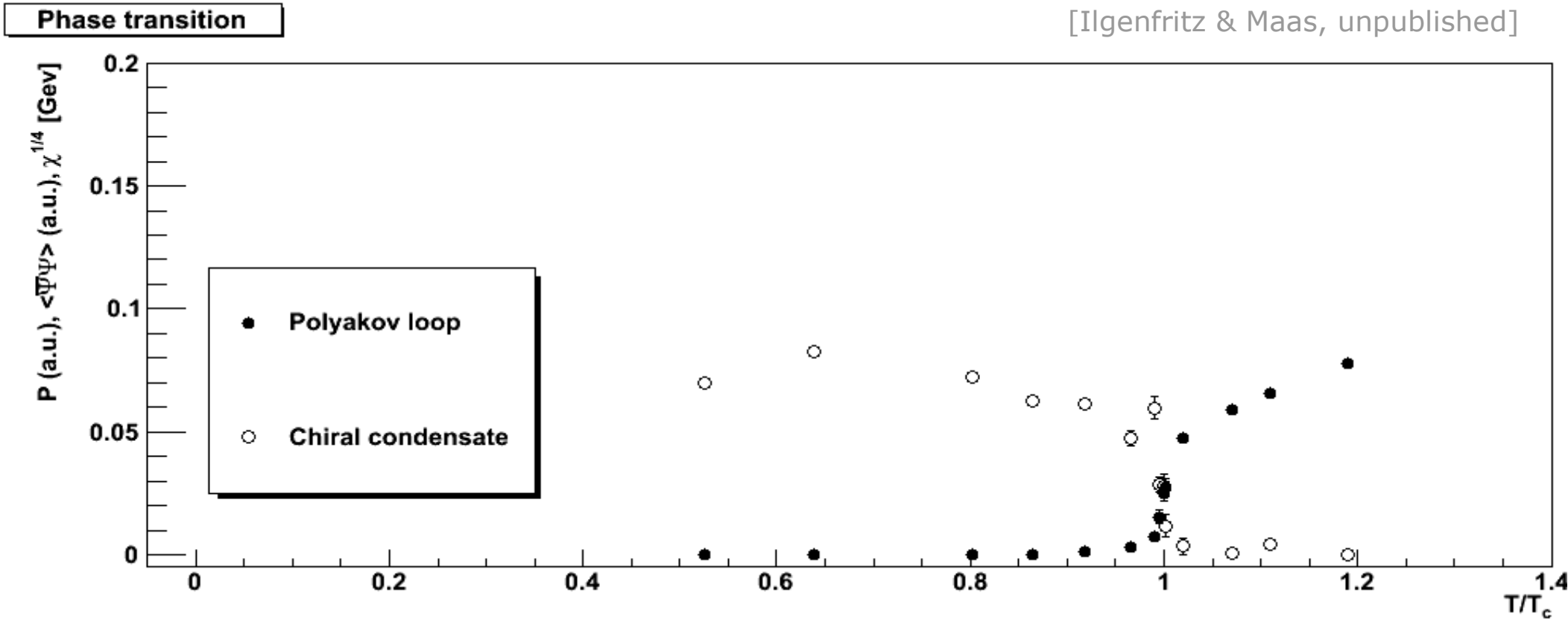

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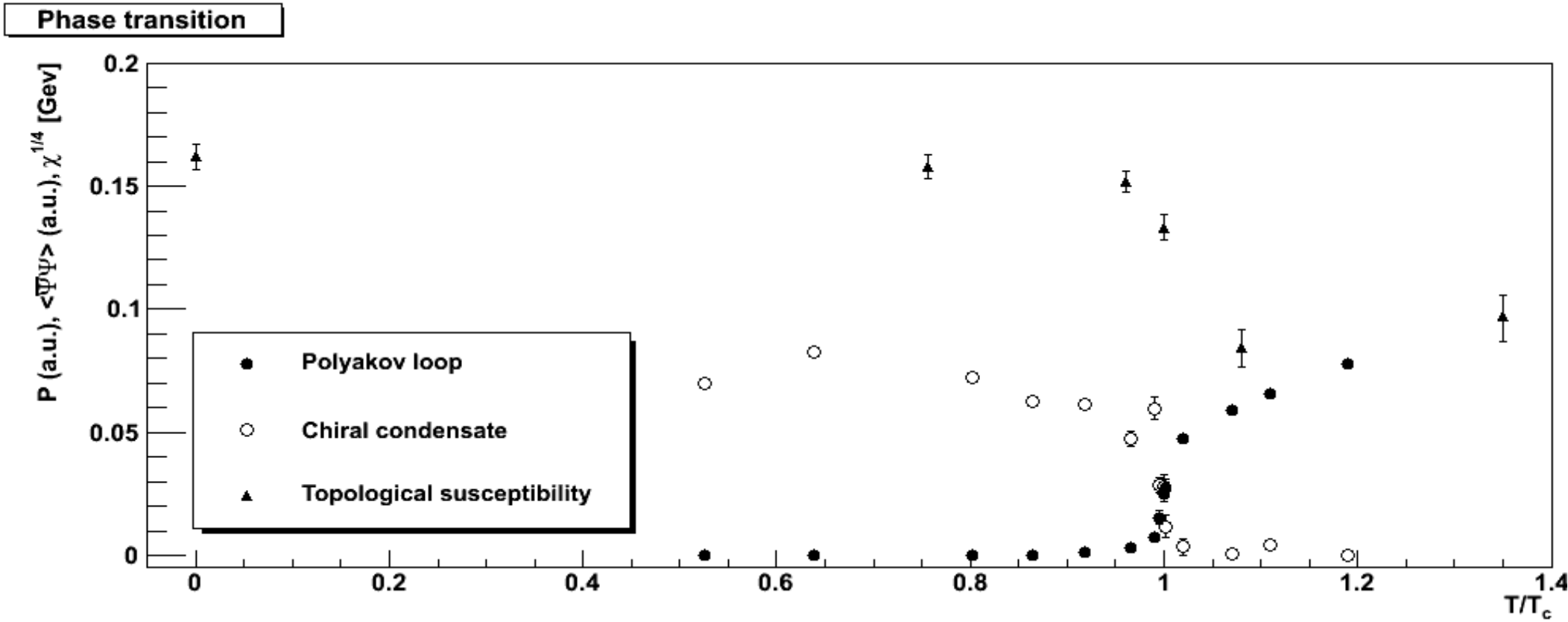

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- Topology reflects phase transition

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[Maas, von Smekal, Wellegehausen, Wipf '12]

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- G2 has fermionic baryons
  - Fermi effects at finite density correctly included
  - Important for compact stellar objects

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    - No sign problem for odd number of flavors
  - Full phase diagram accessible
- G2 has fermionic baryons
  - Fermi effects at finite density correctly included
  - Important for compact stellar objects
- 'Simplest' QCD with all these properties

Finite density

[Maas, von Smekal, Wellegehausen, Wipf '12]

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- G2 has only real representations
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- **Unquenched** 1 flavor calculation

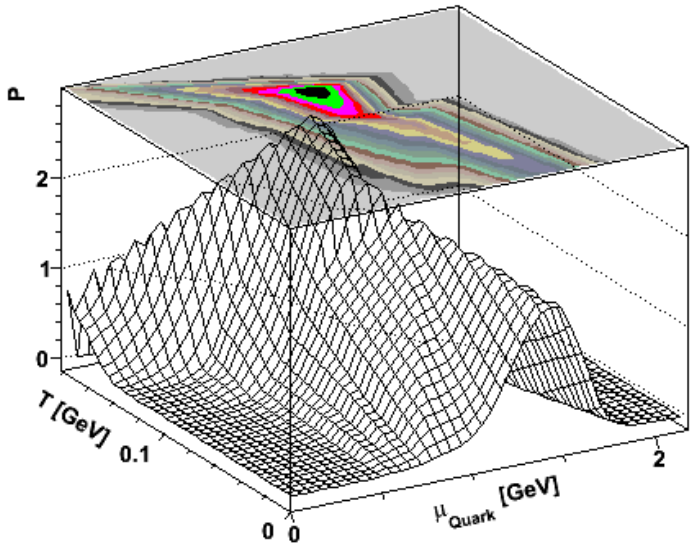
Phase diagram

[Maas, von Smekal, Wellegehausen, Wipf '12]

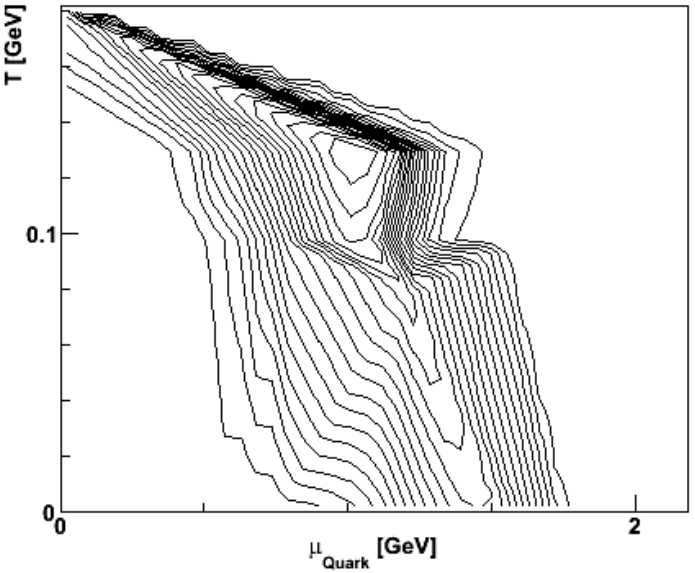
Phase diagram

[Maas, von Smekal, Wellegehausen, Wipf '12]

Polyakov loop



Polyakov loop



Phase diagram

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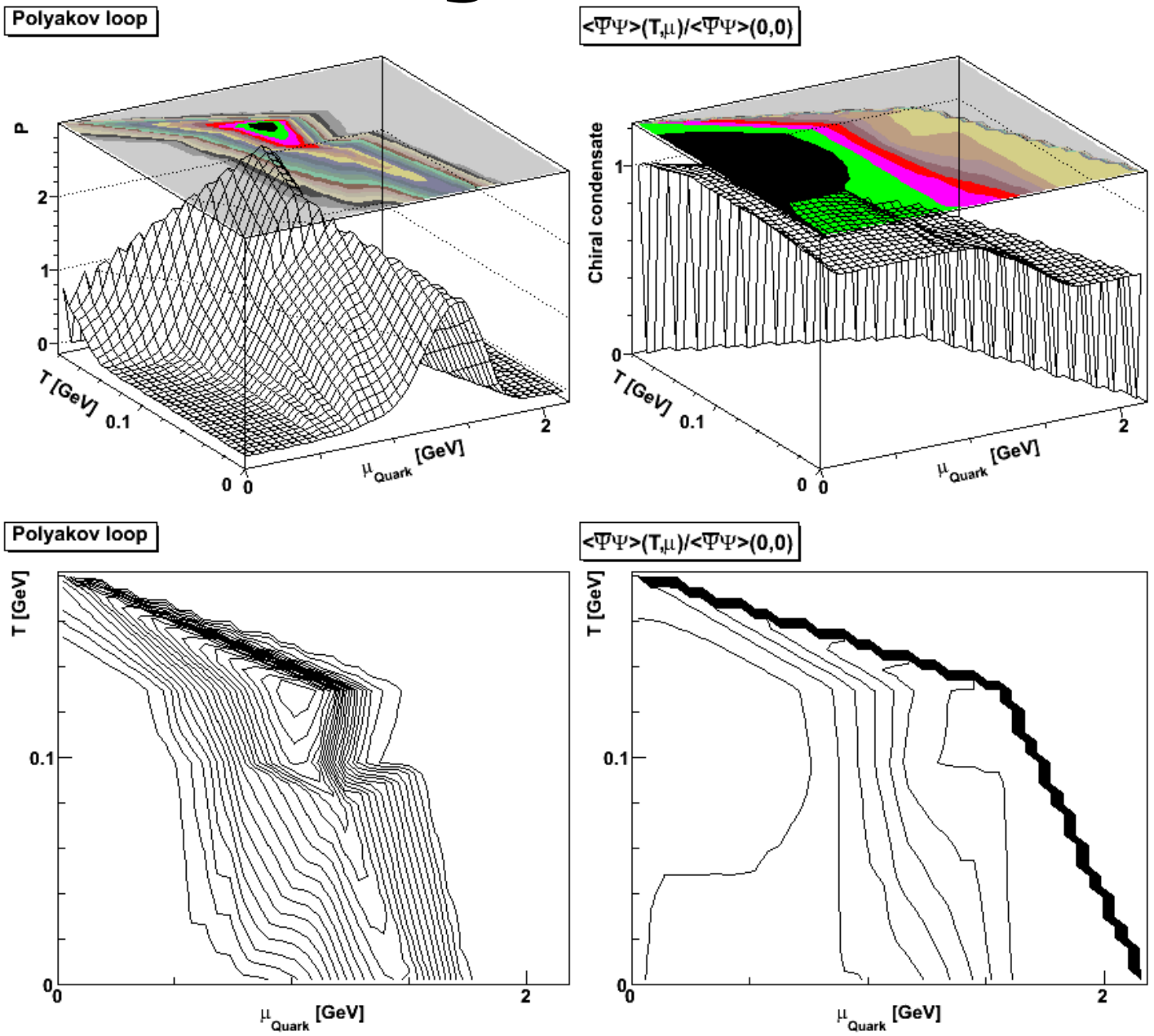

Phase diagram

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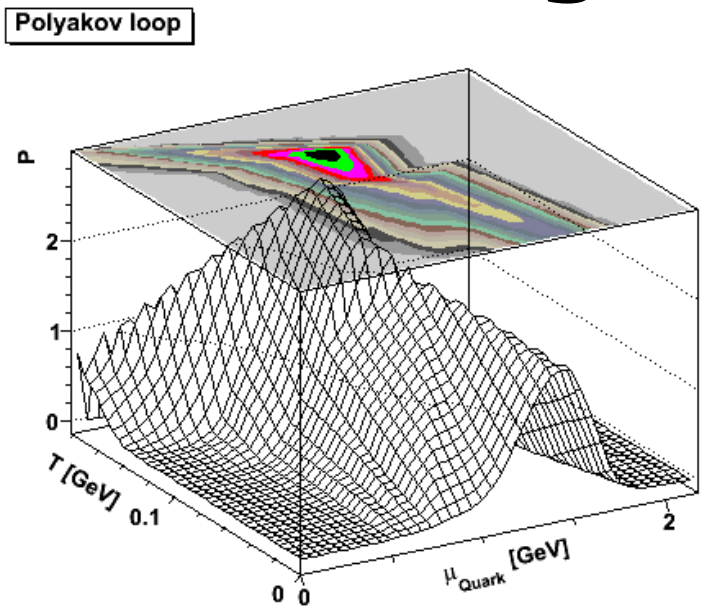

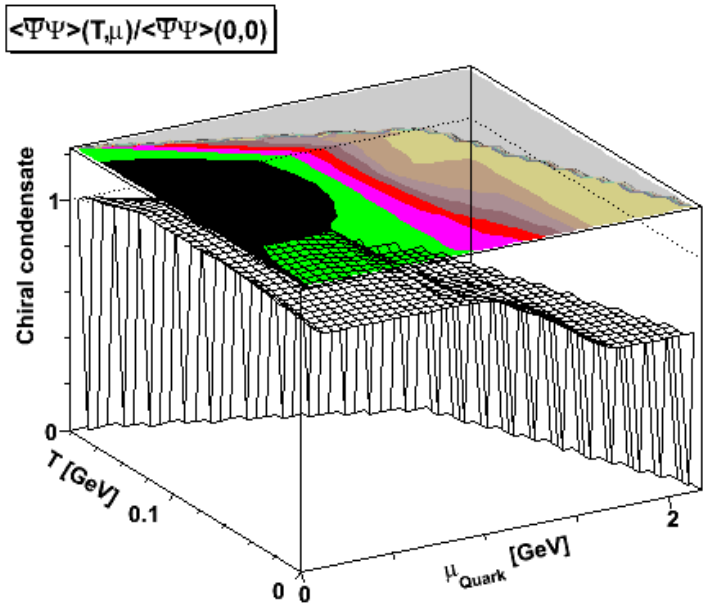

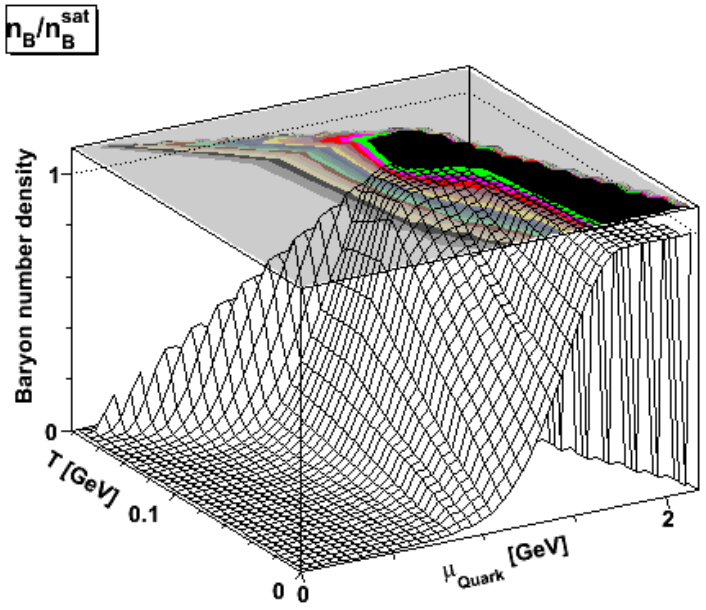

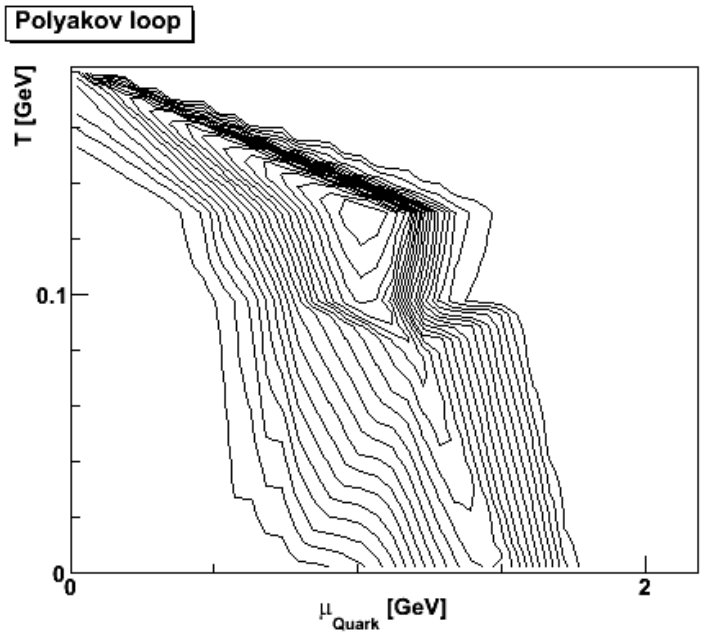

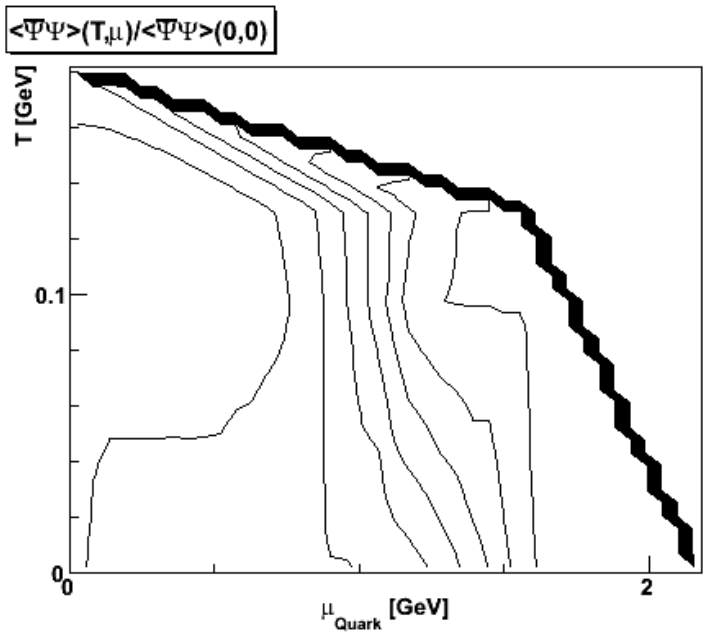

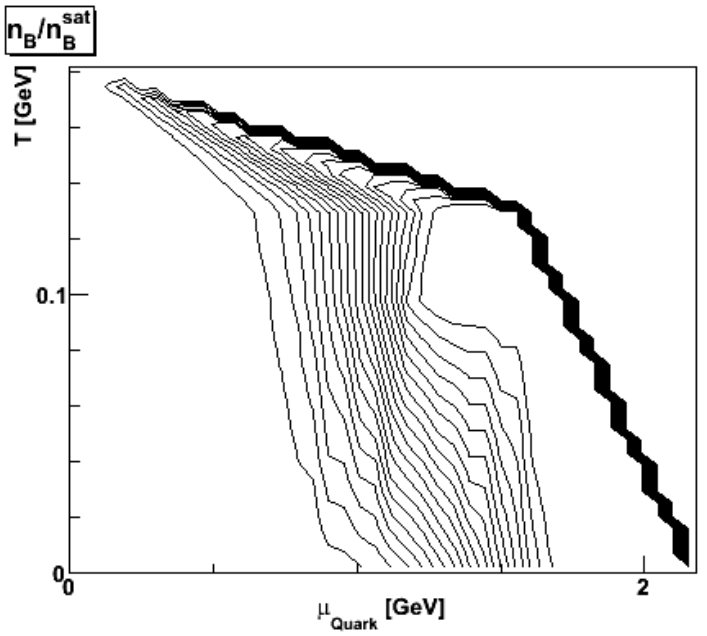

Phase diagram

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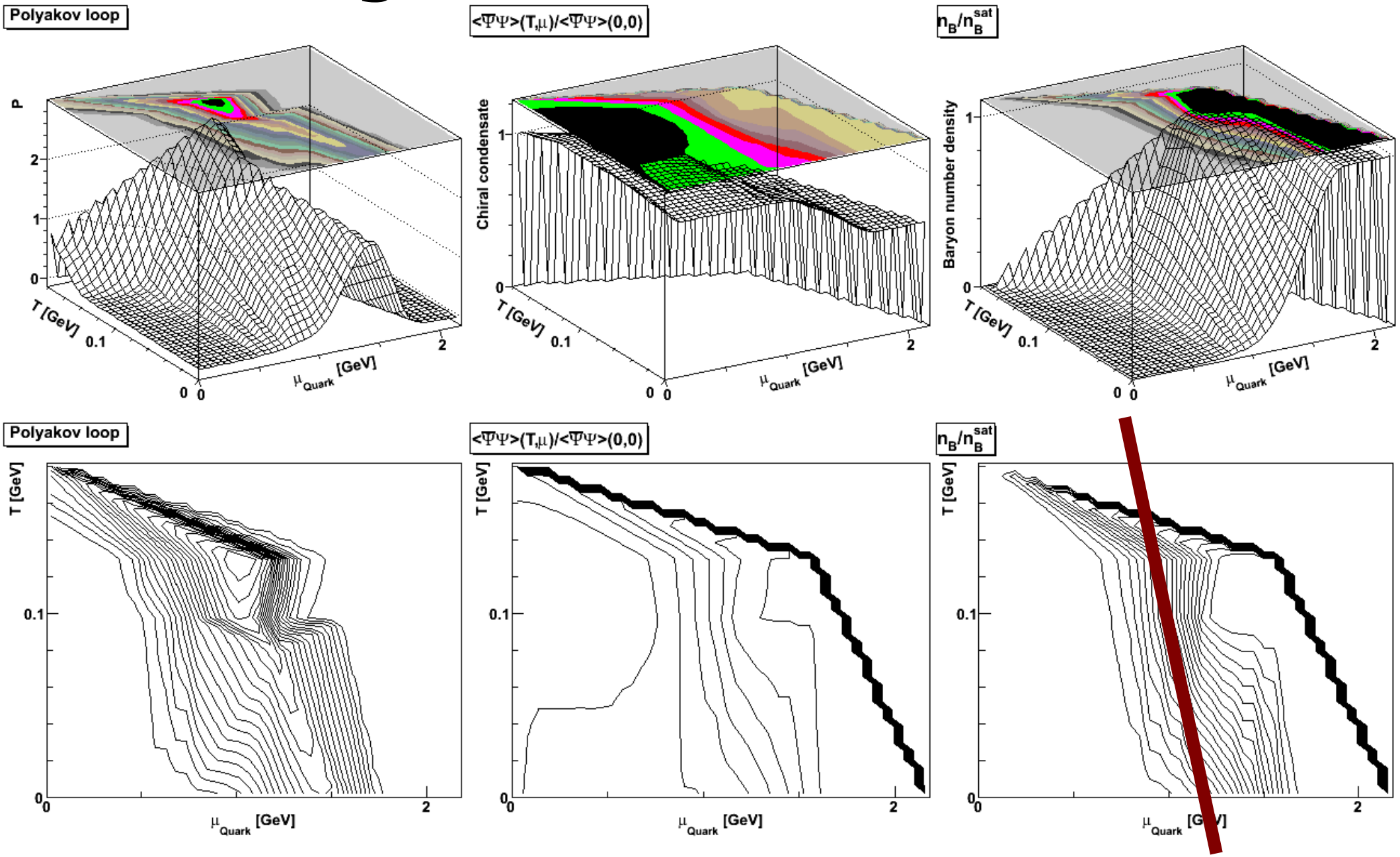

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    - Coincidence of chiral and 'deconfinement' phase transition

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Summary

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    - Topological properties
  - Quenched G2 QCD is almost the same as quenched QCD, up to the static potential
- Practical insights: Phase diagram
  - Rough shape of the phase diagram of a gauge theory is similar to the expected one
  - More details: Next talk