MONTE CARLO STUDY ON THE BIRTH OF OUR UNIVERSE BY A LORENTZIAN MATRIX MODEL FOR SUPERSTRING THEORY

SANG-WOO KIM (OSAKA UNIVERSITY)

LATTICE 2012 @ CAIRNS

BASED ON 1108. 1540 (PRL 108 (2012) 011601)

BY SWK, JUN NISHIMURA, ASATO TSUCHIYA

Motivation

- Cosmology is another frontier for high energy particle physics.

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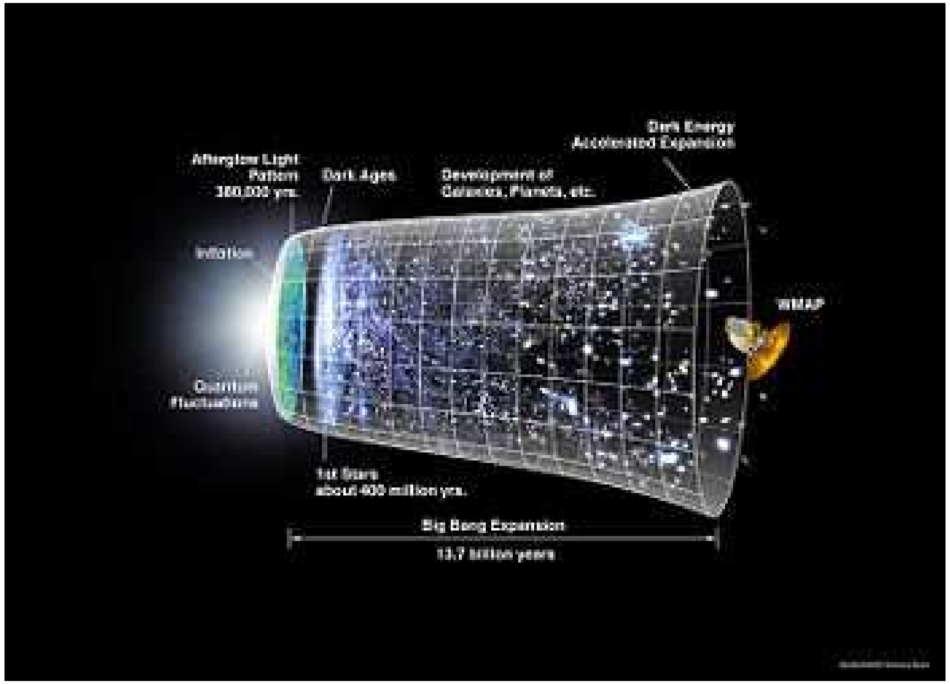
- ➢ Many interesting questions in the early universe :



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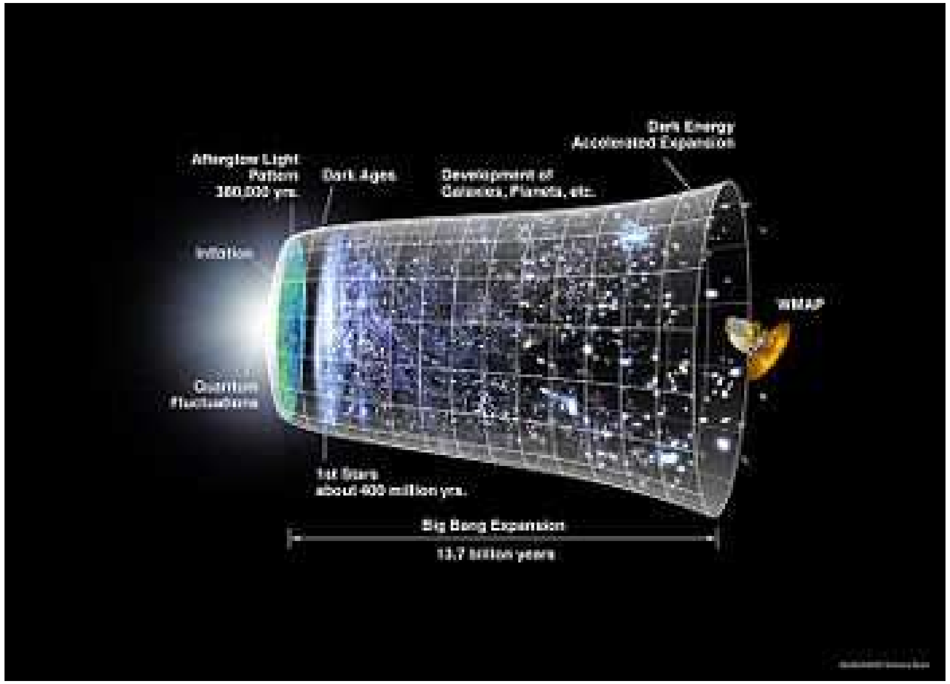

Initial singularity problem

Spacetime dimensionality

Inflation, effective model

CMB spectrum

Dark energy, etc

- Many works so far :

  Quantum cosmology based on Wheeler-DeWitt eq,

  Vilenkin ('82), Hartle, Hawking ('83), …

  String gas cosmology,

  Brandenberger, Vafa ('89), …

  D-brane + Perturbative analysis, etc.

  Herdeiro, Hirano, Kallosh ('01), …

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- Today's talk features :

Based on a matrix model proposal in string theory.
Unique time history is revealed by Monte Carlo method.
Expanding 3d spaces emerge in real time.
Local property for late time is suggested.

Matrix Model

- Our starting point is 0d Matrix Model.

  Ishibashi, Kawai, Kitazawa, Tsuchiya ('96)

A nonperturbative formulation proposed for superstring theory, as lattice QCD is for QCD.

Obtained from Green-Schwarz action in string theory.

N=2 SUSY on matrix eigenvalues.

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  N=2 SUSY on matrix eigenvalues.

- cf. 1d Matrix Quantum Mechanics, 2d Matrix String Theory

  Banks, Fischler, Shenker, Susskind ('96) Dijkraaf, Verlinde, Verlinde ('97)

- Euclidean IKKT by K. Anagnostopoulos at yesterday's parallel.
- More general review by M. Hanada at tomorrow's plenary.

Lorentzian Matrix Model

$$S = -\frac{1}{4g^2}\mathrm{tr}[A_\mu, A_\nu]^2 - \frac{1}{2g^2}\mathrm{tr}\left(\bar{\Psi}\Gamma^\mu[A_\mu, \Psi]\right)$$

A_μ , Ψ_α : $N \times N$ Hermitian matrices

- Let's avoid Wick rotation to study real time evolution.
- Pfaffian is real.

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- Let's avoid Wick rotation to study real time evolution.
- Pfaffian is real.
- Noncompact temporal direction requires a IR cutoff.

$$\frac{1}{N}\mathrm{tr}(A_0)^2 \leq \kappa \frac{1}{N}\mathrm{tr}(A_i)^2$$

- This breaks SUSY and SO(9,1) symmetry.

Lorentzian Matrix Model

- We can regularize oscillating phase by

a) introduce damping term, $\lim_{\epsilon \to 0} \int dA \, \mathrm{Pf}(\mathcal{M}) \, e^{iS_b - \epsilon |S_b|}$

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c) and integrate out scale with $A_\mu \to \sqrt{r} A_\mu$

$$\Rightarrow \int dA \, \delta(\ldots) \, \mathrm{Pf}(\mathcal{M}(A)) \underbrace{\int dr \, r^{18(N^2-1)/2-1} \, e^{-r^2(\epsilon|S_b| - iS_b)}}_{\propto \, |S_b|^{-18(N^2-1)/4}}$$

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d) Need to introduce L : $\int_0^{L^2} dr \, \delta(\dots) \Rightarrow \boxed{\frac{1}{N}\mathrm{tr}(A_i)^2 \leq L^2}$

Lorentzian Matrix Model

- We study following model by Monte Carlo method :

$$Z = \int d\tilde{A}\, \mathrm{Pf}(\mathcal{M})\, \delta\left(\frac{1}{N}\mathrm{tr}F_{\mu\nu}^2\right)$$

$$\begin{cases} d\tilde{A} = dA\ \delta\left(\frac{1}{N}\mathrm{tr}(A_i)^2 - 1\right)\ \theta\left(\kappa - \frac{1}{N}\mathrm{tr}(A_0)^2\right) \\ \int_0^{L^2} dr\, r^{18(N^2-1)/2-1}\, e^{-r^2(\epsilon|x|-ix)} \quad \propto \quad \delta(x) \quad \text{in the large } N,\ L. \end{cases}$$

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- Note that

Lorentzian

$$\mathrm{tr}F^2 = 0 \quad \rightarrow \quad 2\mathrm{tr}F_{0i}^2 = \mathrm{tr}F_{ij}^2$$

noncommutative : Lie algebraic, ...

Euclidean

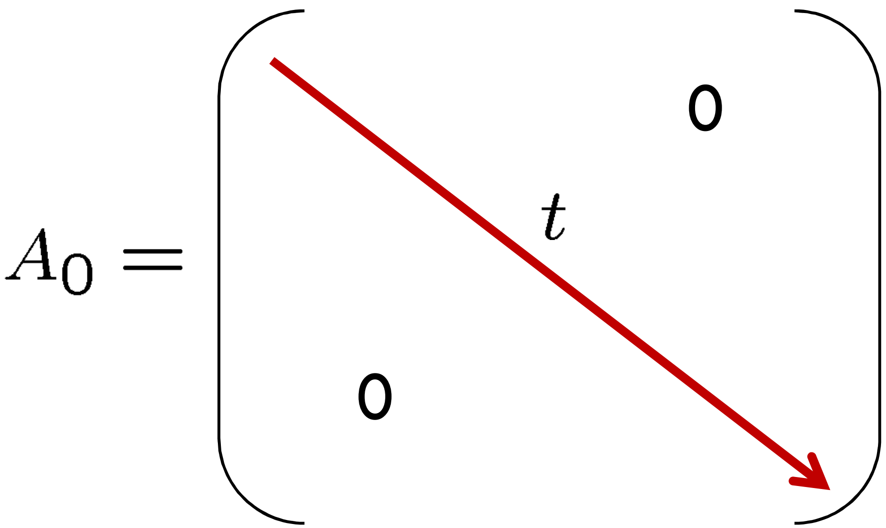
$$\mathrm{tr}F^2 = 0 \quad \rightarrow \quad 2\mathrm{tr}F_{0i}^2 = \mathrm{tr}F_{ij}^2 = 0$$

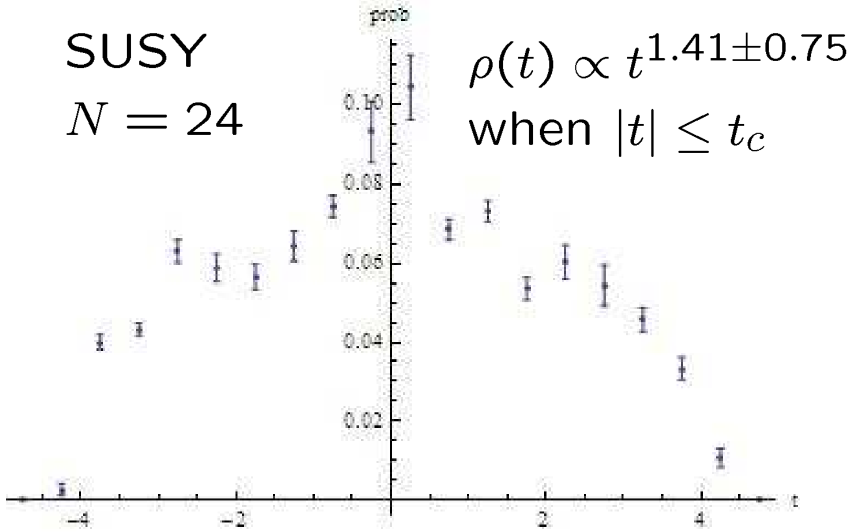
commutative

Results : Time Eigenvalues

- Let $t_1 \le t_2 \le \ldots t_N$ be eigenvalues of A_0.

  Thanks to SUSY, they are smoothly extended as temporal cutoff increases.

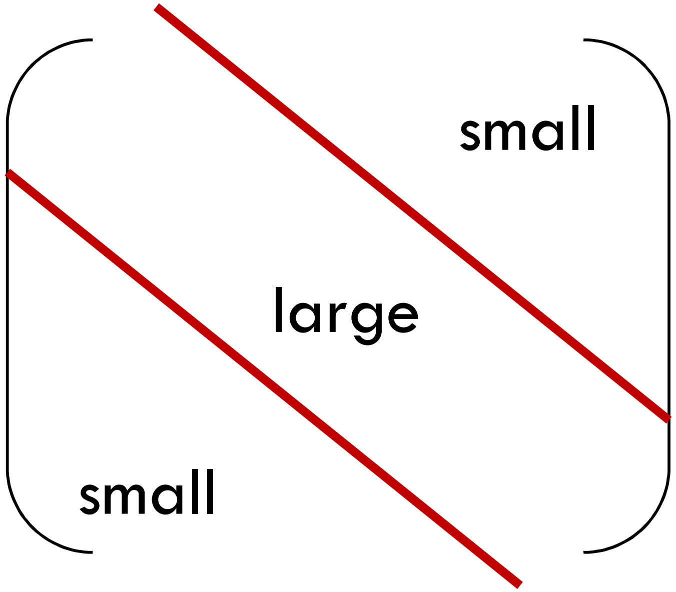


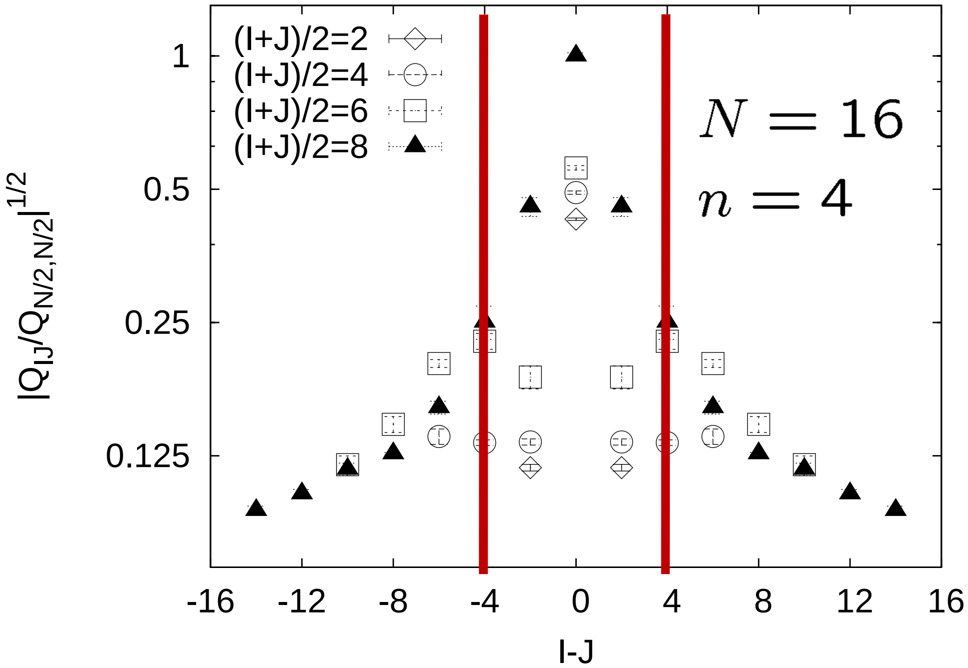


Results : Band Diagonal Structure

- Band diagonal structure appears dynamically for A_i in A_0's diagonal basis.

$A_i =$





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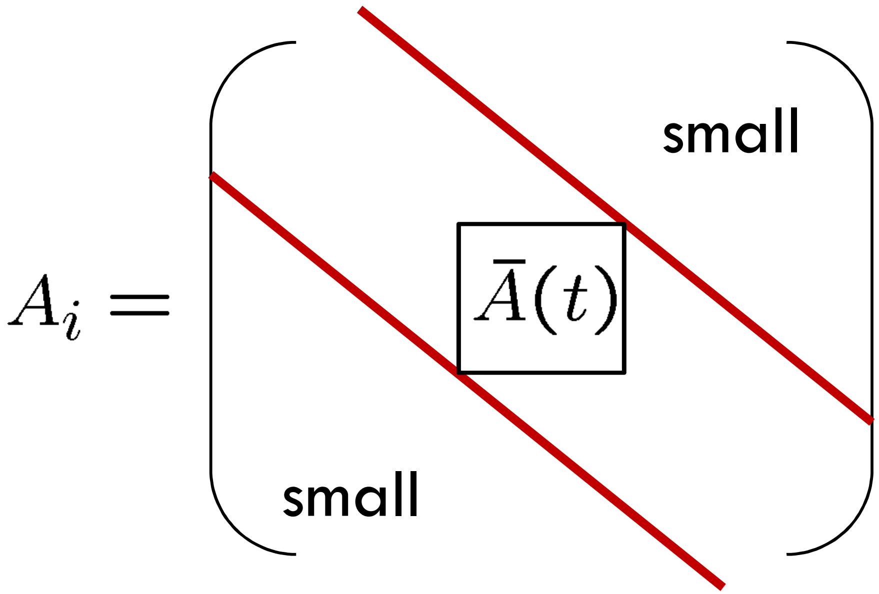

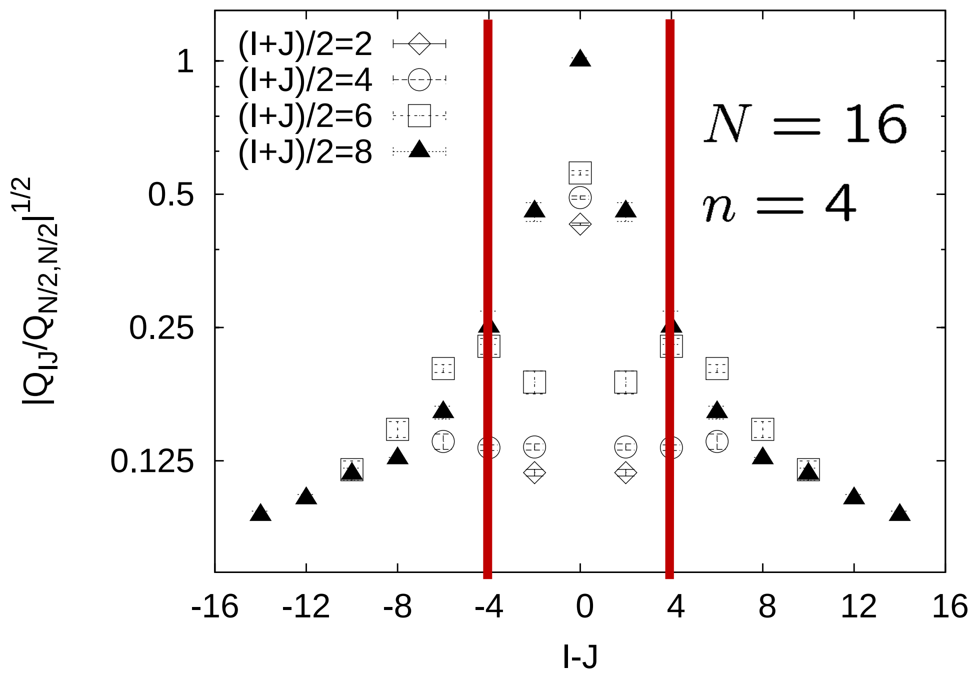

$n \times n$ subblock matrix represents space structure at given time.

$$[\bar{A}_i(t)]_{ab} = \langle t_{\nu+a}|A_i|t_{\nu+b}\rangle \quad , \quad t = \frac{1}{n}\sum_{k=1}^{n} t_{\nu+k}$$
$$(\nu = 1, ..., N-n \ , \quad a, b = 1, ..., n)$$

Results : SSB of SO(9) symmetry

order parameter : $T_{ij}(t) = \frac{1}{n}\mathrm{tr}(\bar{A}_i(t)\bar{A}_j(t))$ 9x9 real sym.

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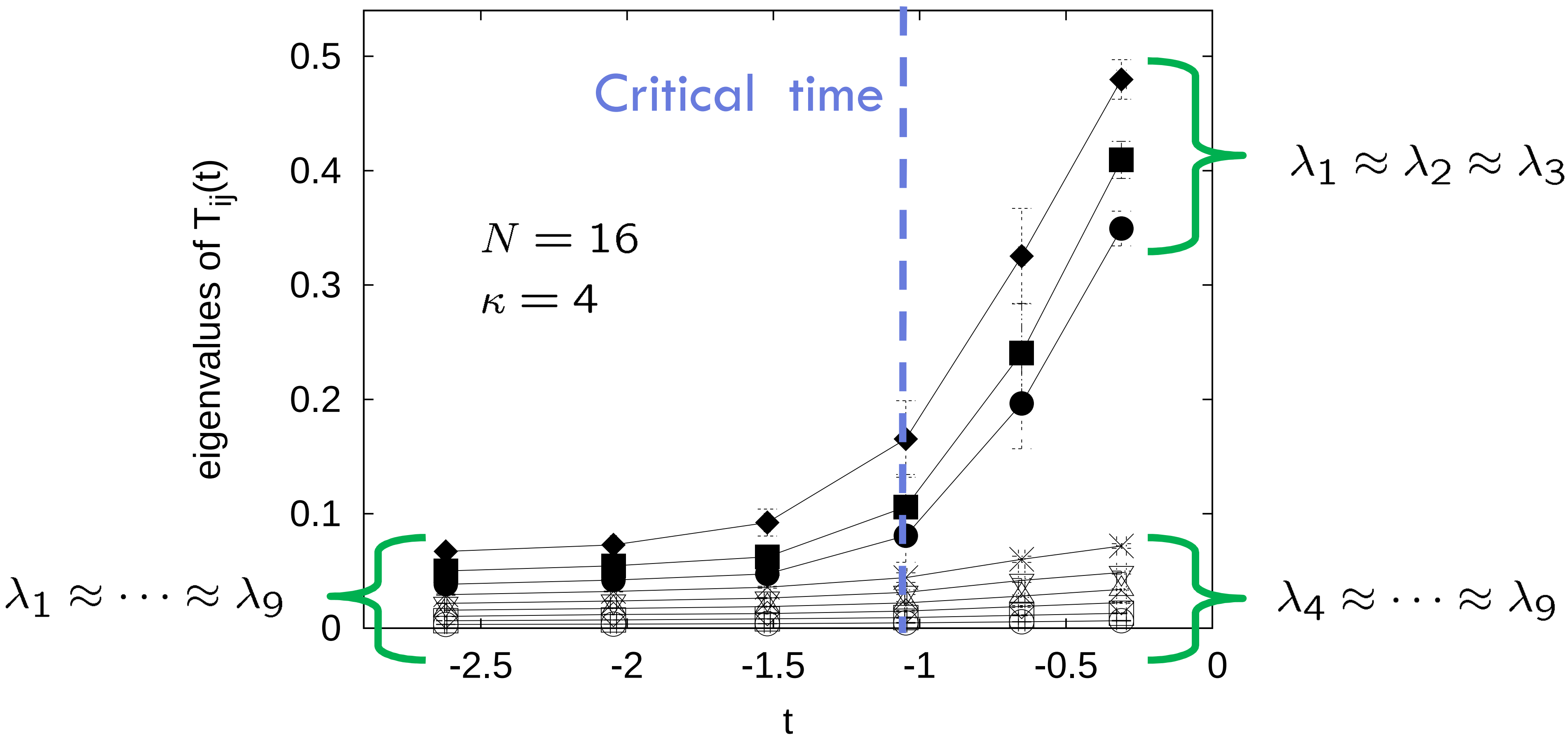

Mechanism of SSB

- Large kappa for fixed N is described by

$$L = -\frac{1}{4N}\mathrm{tr}(F_{ij})^2 + \frac{\lambda}{2}\left(\frac{1}{N}\mathrm{tr}(A_i)^2 - 1\right)$$

- EOM is $[A_j, [A_j, A_i]] = \lambda A_i$

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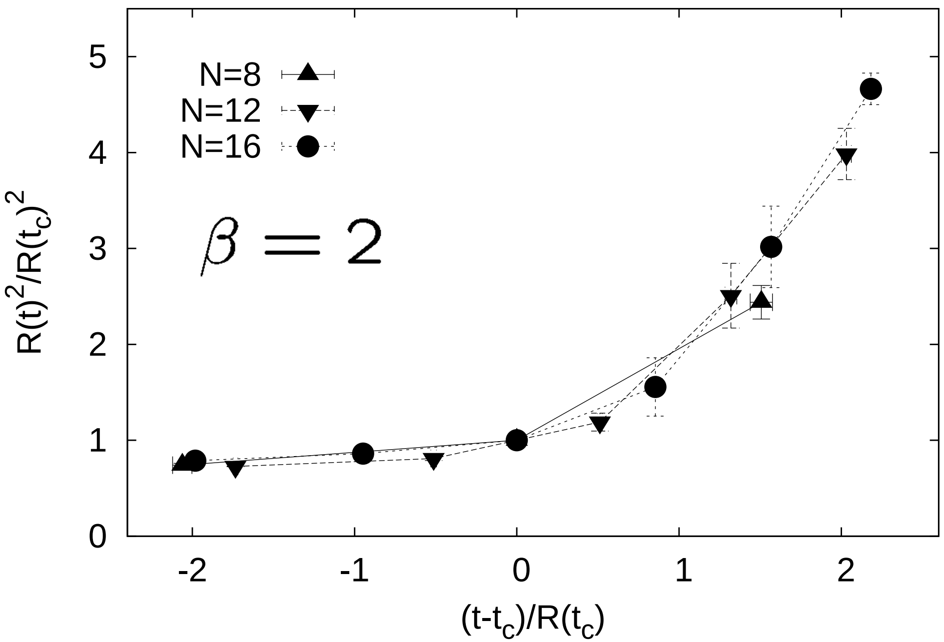
- Let's try an Ansatz

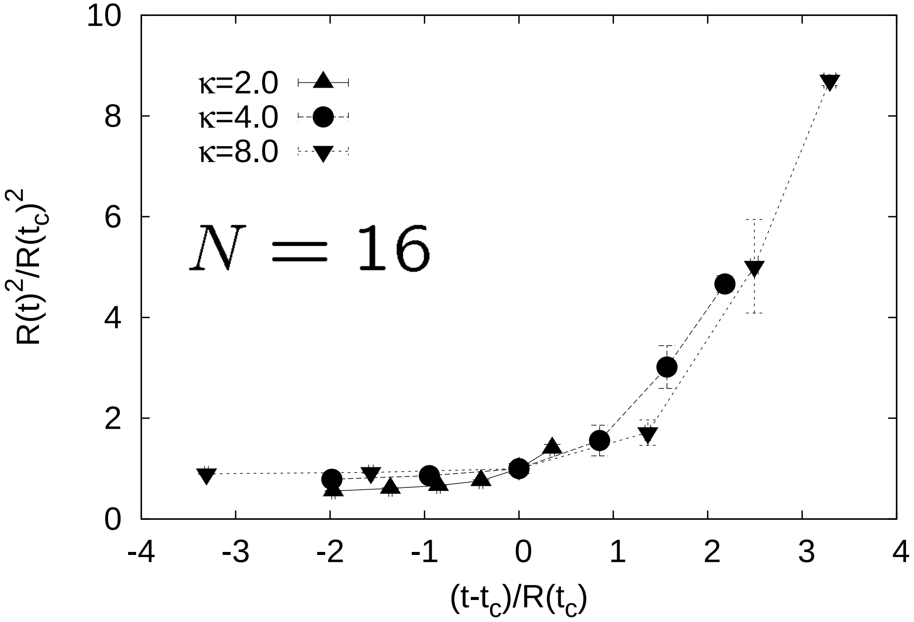
$$A_i = \chi L_i \qquad \text{for } i = 1,...,d$$
$$A_i = 0 \qquad \text{for } i = d+1,...9$$
$$[L_i, L_j] = i f_{ijk} L_k \qquad \text{compact, semisimple}$$

$$\begin{cases} \frac{\chi^2}{N}\mathrm{tr}(L_i)^2 = 1 \quad \rightarrow \quad \chi = \sqrt{\frac{N}{\mathrm{tr}(L_i)^2}} \\ \mathrm{tr}F_{ij}^2 = \chi^4 \mathrm{tr}(f_{ijk}L_k)^2 \propto \frac{1}{\mathrm{tr}(L_i)^2} \leq \frac{2}{3} \end{cases}$$

⇨ $su(2)$ ⇨ 3d

Continuum / Infinite volume limit

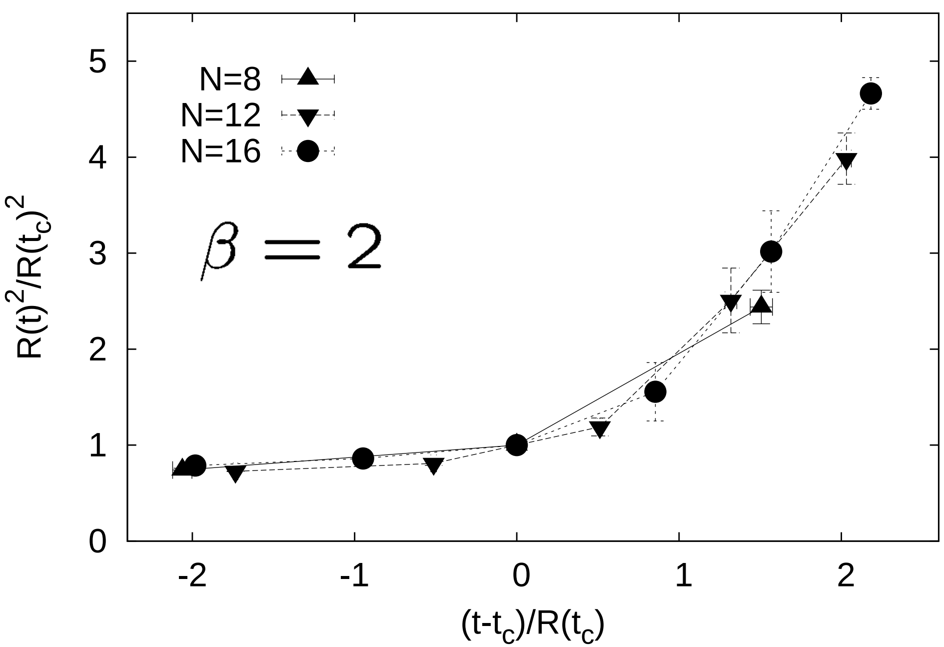


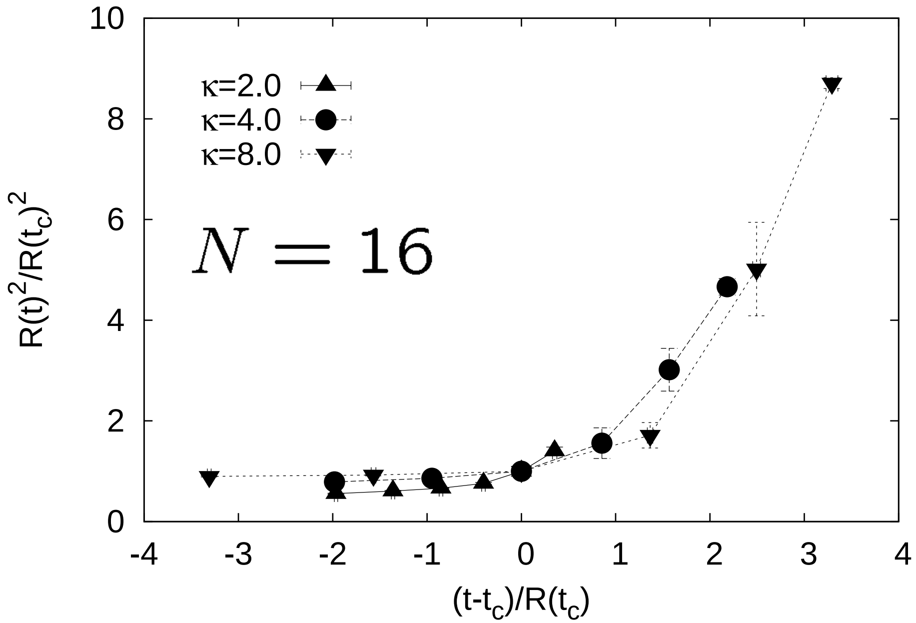


Continuum / Infinite volume limit

$N \to \infty$ with $\kappa = \beta N^p$, $p \sim 1/4$

$L, \ \beta \to \infty$





They seem to converge to a single curve.

VDM model

- In A_0's diagonal basis,

$$Z = \int dt dA_i \, \Delta(t) \, \mathrm{Pf}(\mathcal{M}) \, e^{iS_b}$$

$$\Delta(t) = \prod_{i<j} (t_i - t_j)^2$$

$$\Rightarrow \quad Z_{VDM} = \int dt dA_i \, \Delta(t)^d \, e^{iS_b}$$

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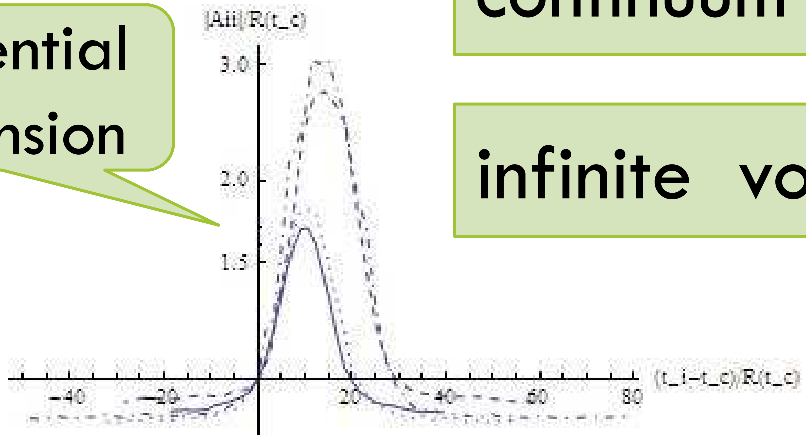
- Bosonic model with fermionic interactions on temporal eigenvalues.

- Interesting properties such as SSB to 3d, expansion are kept.

- It is like quenched QCD for QCD.
  Much faster than full SUSY model.

Preliminary results on VDM model

Exponential expansion



continuum $N \to \infty$ with $\kappa \sim N^{1/4}$

infinite vol $N \to \infty$ with $\kappa \sim N^{5/4}$

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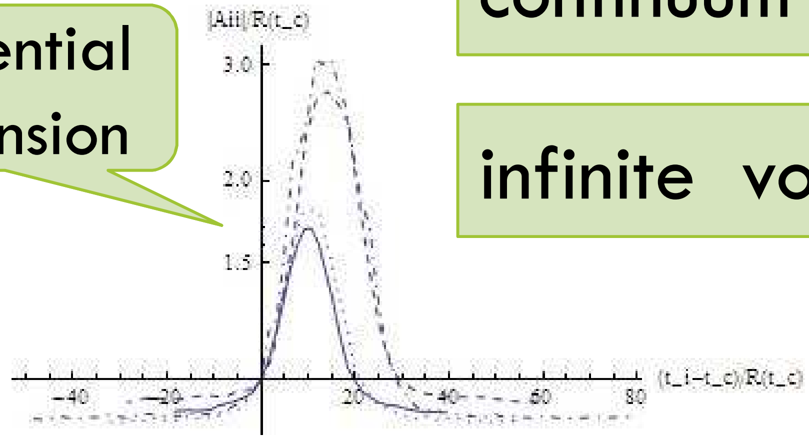
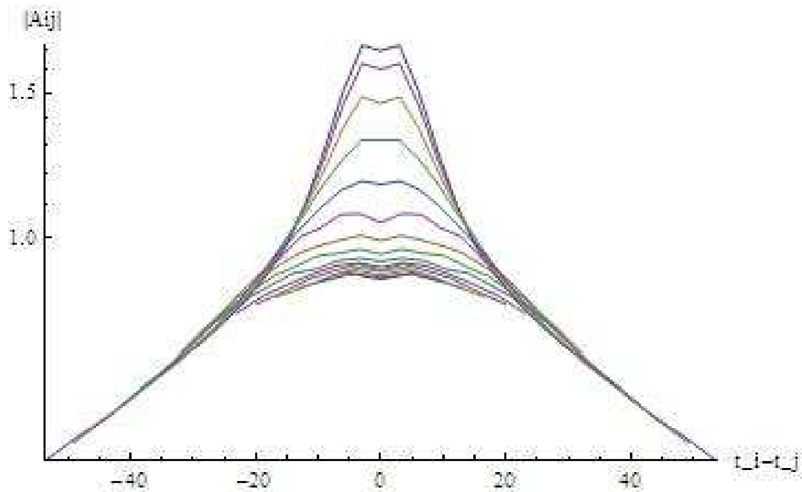
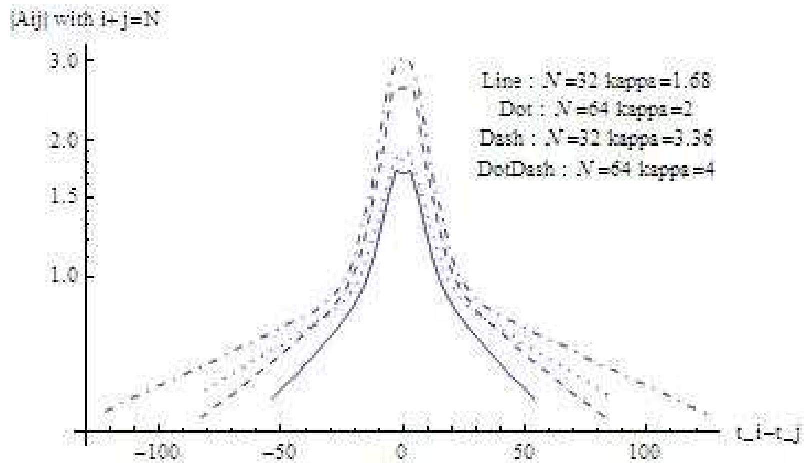






Effective band size decreases for late time.

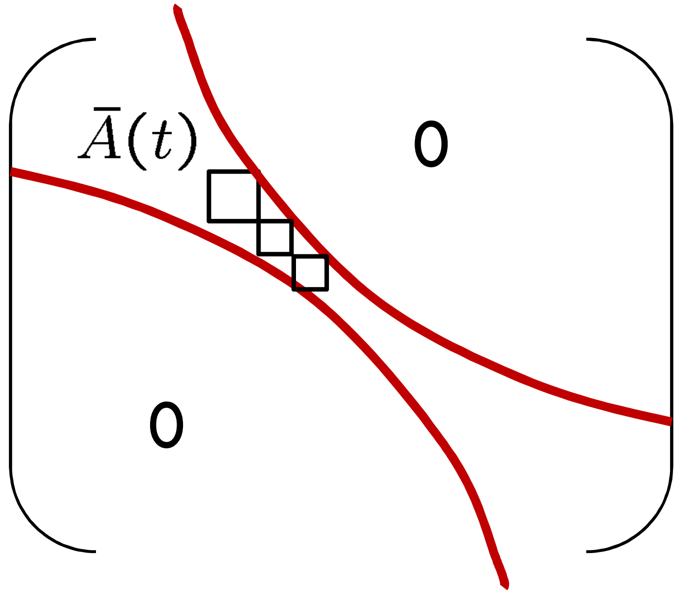
More effective methods

- Let's test whether we can ignore off-diagonal elements with

$$(A_i)_{IJ} = 0 \quad \text{for } |I - J| \geq B \; , \qquad \Delta(t) = \prod_{1 \leq i-j < B} (t_i - t_j)^2$$

- At very late time, the interaction for different time block is likely to be ignored.

$A_i \sim$



- Physics for $\bar{A}(t)$ is just quantum mechanics, which emerges from Lorentzian matrix model.

This QM will be very effective for studying late time.

Summary

0-d matrix model with SO(9,1)

$$S = -\frac{1}{4g^2}\mathrm{tr}[A_\mu, A_\nu]^2 + S_f$$

+

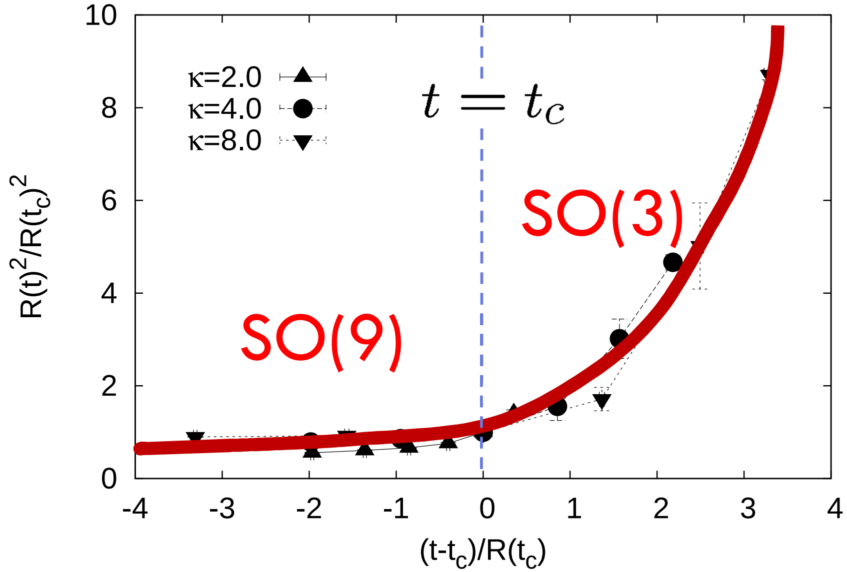
Two IR cutoffs

$$\frac{1}{N}\mathrm{tr}A_0^2 \le \kappa L^2 \ , \qquad \frac{1}{N}\mathrm{tr}A_i^2 \le L^2$$

$$N \ , \ \kappa \ , \ L \quad \to \quad \infty$$

- Unique time history
- SSB from 9d to 3d spaces
- Exponential expansion
- Noncommutative mechanism
- Local property for late time

Early Universe



Backup

Alternative Approach

- The SSB and expansion relies on space-space noncommutativity.
- Does our model allow commutative spacetime in late time ?
- Direct numerical study become more difficult for future.
- We look for classical solutions consistent with previous result.

Equation of Motion

Two IR cutoffs

$$\frac{\delta}{\delta A_\mu}\left(\frac{1}{2}\mathrm{tr}[A_\mu, A_\nu]^2 + \lambda\,\mathrm{tr}A_i^2 + \tilde{\lambda}\,\mathrm{tr}A_0^2\right) = 0$$

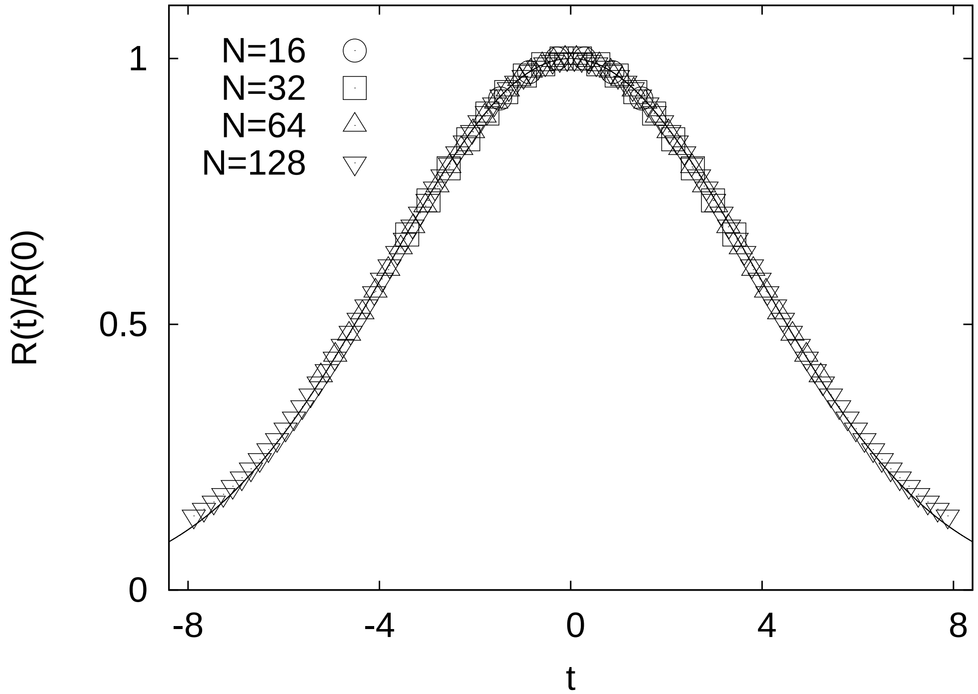
$$\Rightarrow \quad -[A_0,[A_0,A_i]] + [A_j,[A_j,A_i]] = \lambda A_i$$

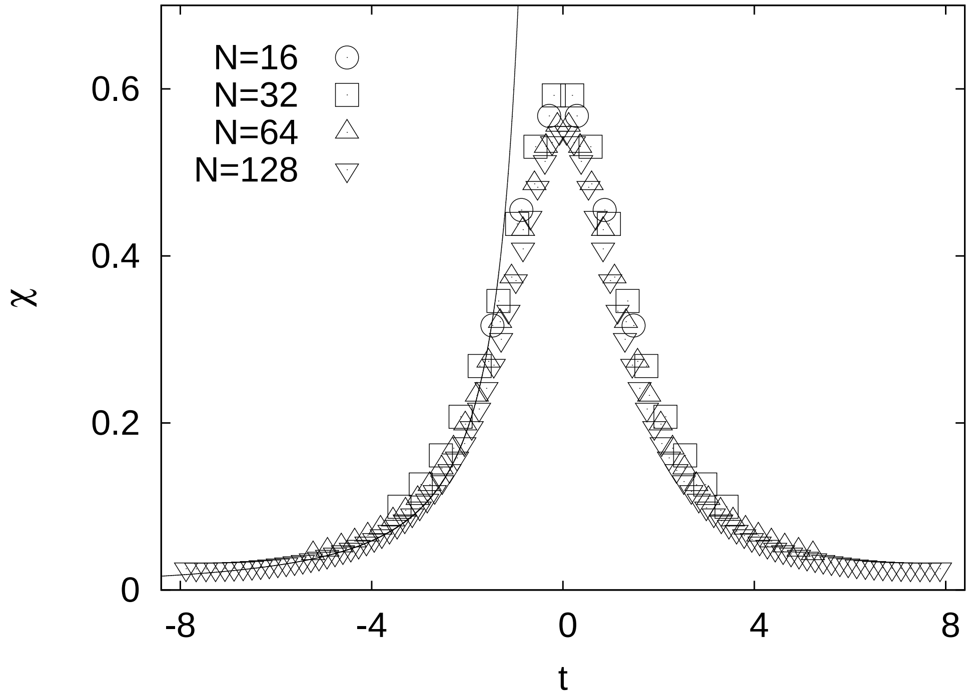
$$-[A_j,[A_j,A_0]] = \tilde{\lambda} A_0$$

Classical solution 1

$$A_0 = -i\sqrt{\lambda}\,\frac{d}{dx} \quad , \qquad A_i = a\,exp(x)$$

$$[A_0, A_i] = -i\sqrt{\lambda}\,A_i \quad , \qquad [A_i, A_j] = 0$$





$$R(t) = \frac{1}{n}\mathrm{tr}\bar{K}^2(t)$$

$$\chi(t) = \frac{-\frac{1}{n}\mathrm{tr}[\bar{P},\bar{K}]^2}{\frac{1}{n}\mathrm{tr}\bar{P}^2\cdot\frac{1}{n}\mathrm{tr}\bar{K}^2}$$

Classical solution 2

$$A_0 = \sqrt{-\lambda}\,T_0,\quad A_1 = c_1\sqrt{-\tilde{\lambda}}\,T_1,\quad A_2 = c_2\sqrt{-\tilde{\lambda}}\,T_1,\quad A_3 = c_3\sqrt{-\tilde{\lambda}}\,T_1$$

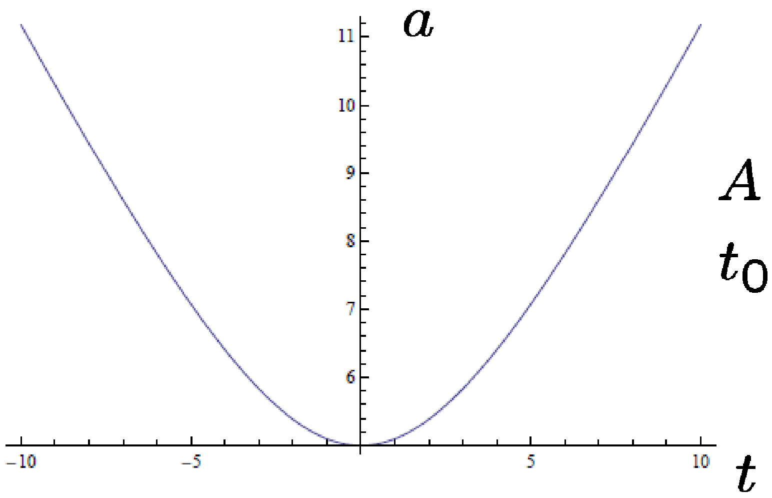
$$[T_0, T_1] = iT_2 \quad [T_0, T_2] = -iT_1 \quad [T_1, T_2] = -iT_0$$

$$\Rightarrow \quad a(t) = A\sqrt{t^2 + t_0^2}$$

$$H = \frac{\dot{a}}{a} \sim a^{-\frac{3}{2}(1+w)} \qquad w = -\frac{1}{3}\left(\frac{2t_0^2}{t^2} + 1\right)$$

$$t = t_0 \longrightarrow w = -1$$

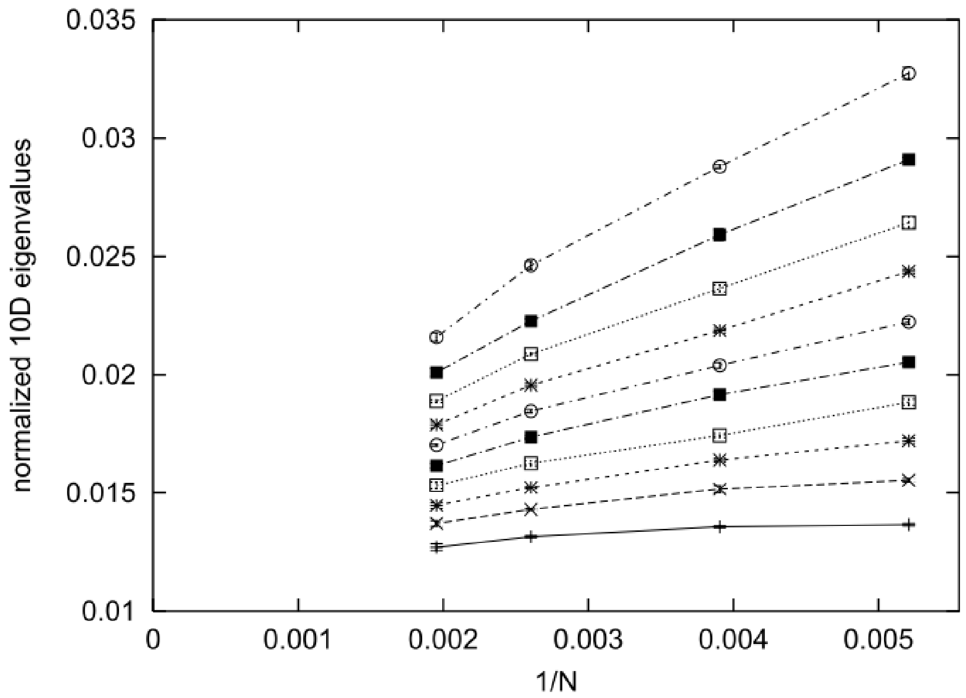
$$t \to \infty \longrightarrow w = -\frac{1}{3}$$



$$A = 1$$
$$t_0 = 5$$

Phase quenched Euclidean model

- Pfaffian is complex in Euclidean signature, which give rise to the sign problem.



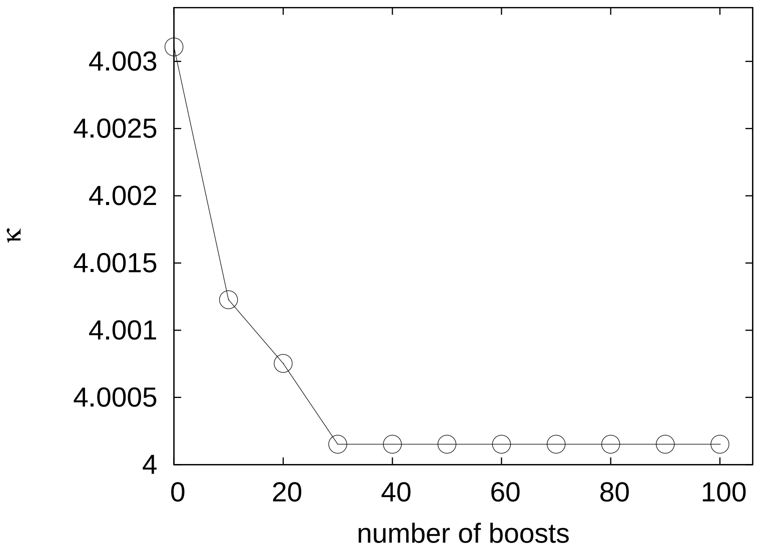
[Ambjorn, Anagnostopoulos, Bietenholz, Hotta, Nishimura 2000]

- Without complex phase, there is no SSB.

(Origin of Euclidean SSB is fermionic)

Lorentz symmetry

- The cutoff restricts boost symmetry and we found that the thermalized configurations have minimum $\frac{1}{N}\mathrm{tr}\,(A_0)^2$ under Lorentz transformation.



- Therefore we may equivalently use Lorentz invariant cutoff, which act on configurations in "minimum $\frac{1}{N}\mathrm{tr}\,(A_0)^2$ frame".

$$\frac{1}{N}\mathrm{tr}(\tilde{A}_0)^2 \leq \kappa \frac{1}{N}\mathrm{tr}(\tilde{A}_i)^2$$

IIB Matrix Model

$$Z = \int dA d\Psi e^{-S_b - S_f}$$

$$S_b = -\frac{1}{4g^2}\mathrm{tr}[A_\mu, A_\nu]^2$$

$$S_f = -\frac{1}{2g^2}\mathrm{tr}\left(\bar{\Psi}\Gamma^\mu[A_\mu, \Psi]\right)$$

NxN hermitian matrices (N→∞)

A_μ : 10d Lorentz vector

Ψ : 10d Majorana-Weyl spinor

[Ishibashi, Kawai, Kitazawa, Tsuchiya 96]

$\mathcal{N} = 2$ SUSY

$$\begin{cases} \delta^{(1)} A_\mu = i\bar{\epsilon}\Gamma_\mu \Psi \\ \delta^{(1)}\Psi = \frac{i}{2}[A_\mu, A_\nu]\Gamma^{\mu\nu}\epsilon \end{cases} \qquad \begin{cases} \delta^{(2)} A_\mu = 0 \\ \delta^{(2)}\Psi = \xi \mathbf{1} \end{cases}$$

Gauge symmetry

10d Lorentz symmetry

Bosonic shift symmetry

$$\begin{cases} \delta A_\mu = c_\mu \mathbf{1} \\ \delta \Psi = 0 \end{cases}$$

Interpretation of Matrix

$$\begin{cases} \tilde{Q}^{(1)} = Q^{(1)} + Q^{(2)} \\ \tilde{Q}^{(2)} = i(Q^{(1)} - Q^{(2)}) \end{cases} \Rightarrow [\bar{\epsilon}\tilde{Q}^{(i)}, \bar{\xi}\tilde{Q}^{(j)}] = \delta^{(ij)}(\text{shift sym.})$$

If eigenvalues of bosonic matrix = spacetime coordinate,

$\mathcal{N}=2$ matrix SUSY	⇒	10d $\mathcal{N}=2$ spacetime SUSY
Bosonic shift symmetry		Translational symmetry

- Note that bosonic action is positive definite in Euclidean signature, and prefers commuting configurations.

$$S_b \propto \mathrm{tr}(F_{\mu\nu}^2) \geq 0 \qquad \text{with} \qquad F_{\mu\nu} = -i[A_\mu, A_\nu]$$

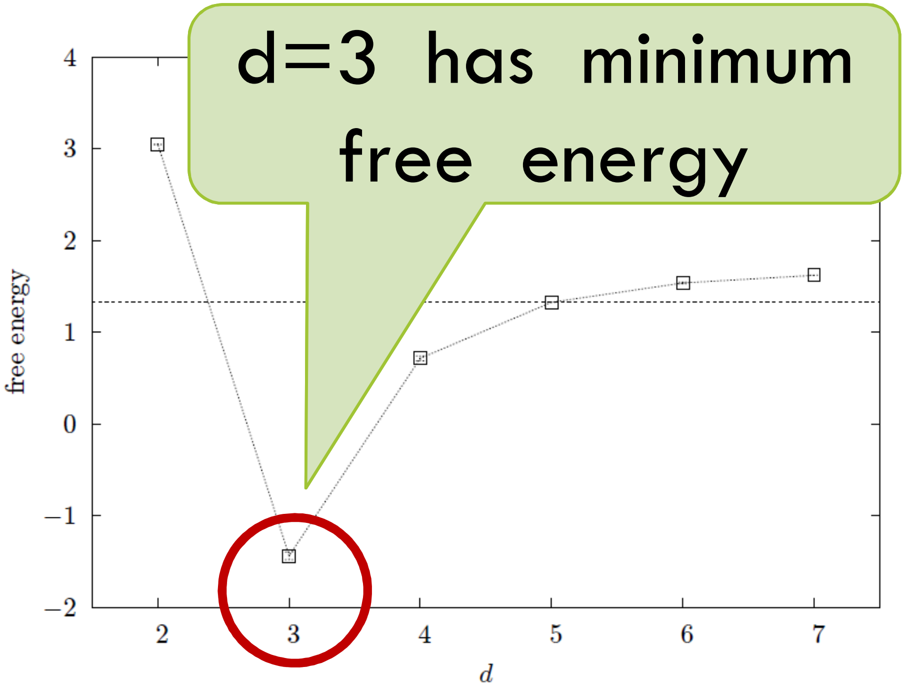
$$A_\mu = \begin{pmatrix} x_\mu & & 0 \\ & \ddots & \\ 0 & & \end{pmatrix}$$

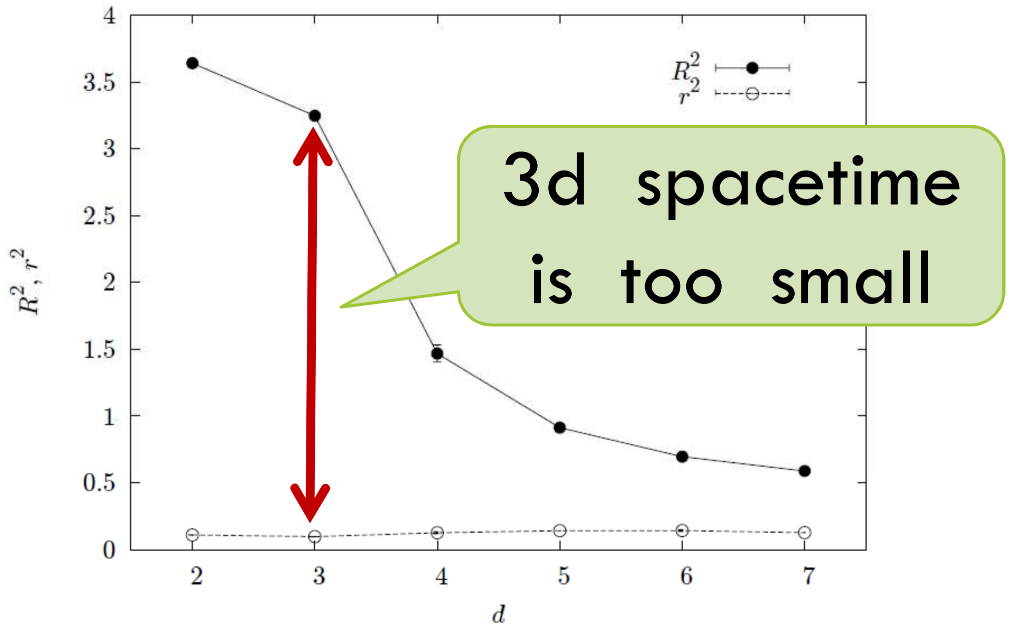
Dynamically generated N discrete spacetime points

Problem of Euclidean model

- A study by GEM for Euclidean IIB matrix model.

[Nishimura, Okubo, Sugino 2011]





- Free energy prefers $SO(10) \to SO(3)$,
  and spacetime is too small compared to extra dimensions.

IR cutoff in temporal direction

- Bosonic part of the action is problematic due to the indefinite signature.

$$\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) = -2\mathrm{tr}(F_{0i})^2 + \mathrm{tr}(F_{ij})^2 \qquad \text{with } F_{\mu\nu} = -i[A_\mu, A_\nu]$$

- Note that the Euclidean model is well defined and temporal direction is the source of the problem. We need to mod out boost transformation from integration measure.

- Let's introduce a cutoff in temporal direction, which "gauge fix" SO(9,1) to SO(9) in general.

$$\frac{1}{N}\mathrm{tr}(A_0)^2 \leq \kappa \frac{1}{N}\mathrm{tr}(A_i)^2$$

- Important question is whether we can remove this constraint in the large N limit.

IR cutoff in spatial direction

$$Z = \int dA\ Pf(\mathcal{M})\ e^{iS_b}\ \theta\left(-\frac{1}{N}\mathrm{tr}(A_0)^2 + \kappa\frac{1}{N}\mathrm{tr}(A_i)^2\right)$$

$$= \int dA\ Pf(\mathcal{M}) \lim_{\epsilon\to 0} e^{-\epsilon|S_b|} e^{iS_b}\ \theta(\ldots)$$

$$= \int dA\ Pf(\mathcal{M}) \lim_{\epsilon\to 0} \int_0^\infty dr\ \delta\left(\frac{1}{N}\mathrm{tr}(A_i)^2 - r\right)\ e^{-\epsilon|S_b| + iS_b}\ \theta(\ldots)$$

identity

rescale $A_\mu \to \sqrt{r}A_\mu$

$$= \int dA\ Pf(\mathcal{M}) \lim_{\epsilon\to 0} \int_0^\infty dr\ r^{\frac{18}{2}(N^2-1)}\ \frac{1}{r}\delta\left(\frac{1}{N}\mathrm{tr}(A_i)^2 - 1\right)\ e^{r^2(-\epsilon|S_b| + iS_b)}\ \theta(\ldots)$$

$$= \int dA\ \delta(\ldots)\ \theta(\ldots)\ Pf(\mathcal{M}) \lim_{\epsilon\to 0} \int_0^\infty dr\ r^{9(N^2-1)}\ \frac{1}{r}\ e^{r^2(-\epsilon|S_b| + iS_b)}$$

diverges when $S_b \to 0$ $\qquad c_1|S_b|^{-\frac{9}{2}(N^2-1)}$

$$= \int dA\ \delta(\ldots)\ \theta(\ldots)\ Pf(\mathcal{M}) \lim_{L\to\infty} \lim_{\epsilon\to 0} \int_0^{L^2} dr\ r^{9(N^2-1)-1}\ e^{r^2(-\epsilon|S_b| + iS_b)}$$

$d\tilde{A}$: scale fixed, boost sym fixed

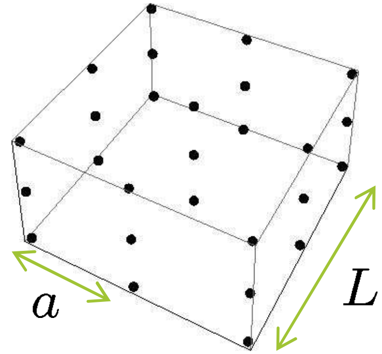
$$c_1\left(\frac{c_2}{L^4} + |S_b|\right)^{-\frac{9}{2}(N^2-1)}$$

Comparison with lattice regularization

Lattice

lattice spacing $a \to 0$

volume of the box $L^d \to \infty$

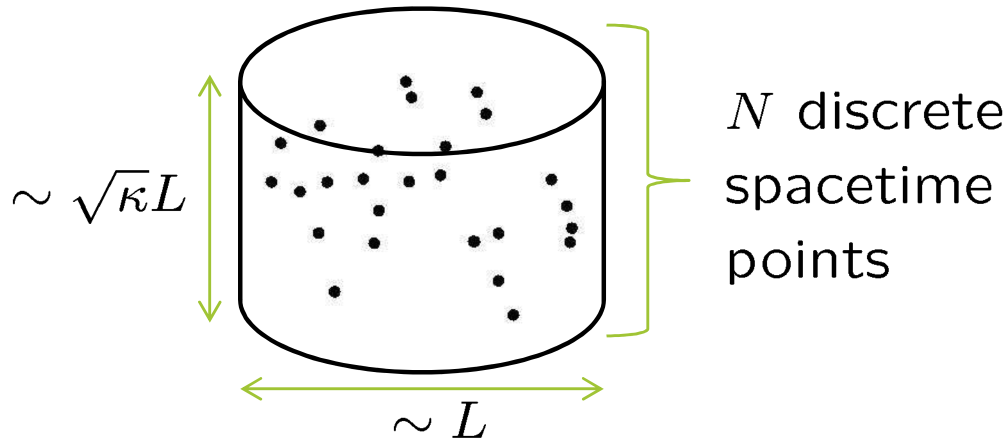


Lorentzian matrix model

matrix size $N \to \infty$

$\frac{1}{N}\mathrm{tr}A_i^2 \leq L^2 \to \infty$

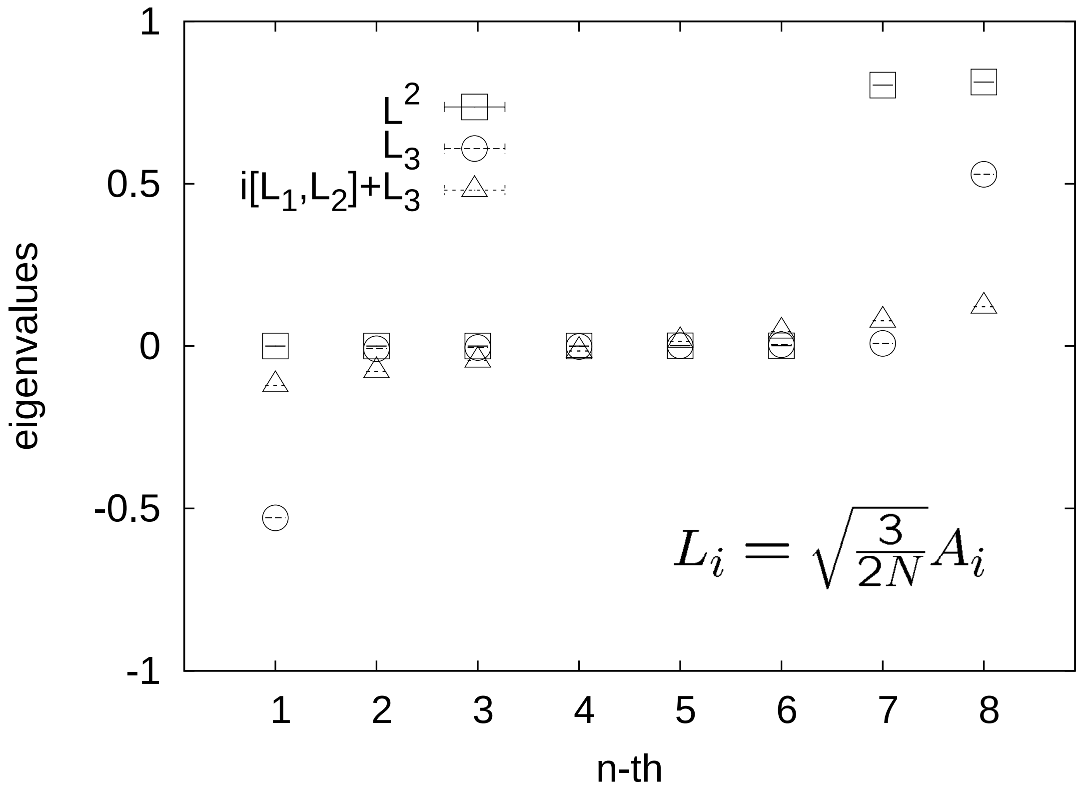
$\frac{1}{N}\mathrm{tr}A_0^2 \leq \kappa L^2 \to \infty$



- In contrast to lattice, SUSY is broken only by IR cutoffs.
- After continuum limit (~N) and infinite volume limit (~L), only one parameter (~kappa) remains.

Mechanism of SSB

- Without any Ansatz



- 2x2 representation of SU(2) algebra gives the maximum, which explains 3 expanding spaces.

Lorentzian vs Euclidean

- Let's consider a solution to simple equatiion.

$$X_\mu^2 = 0$$

- Euclidean : $X_\mu = 0$
- Lorentzian : light-like solutions

- In our case,

Lorentzian

$$\mathrm{tr}F^2 = 0 \quad \rightarrow \quad 2\mathrm{tr}F_{0i}^2 = \mathrm{tr}F_{ij}^2$$

noncommutative : Lie algebraic, …

Euclidean

$$\mathrm{tr}F^2 = 0 \quad \rightarrow \quad 2\mathrm{tr}F_{0i}^2 = \mathrm{tr}F_{ij}^2 = 0$$

commutative

- Wick rotation can not reproduce these solutions !

Mechanism of SSB

$$Z = \int d\tilde{A}\ Pf(\mathcal{M})\ f_{N,L}\left(\frac{1}{N}\mathrm{tr}F_{\mu\nu}^2\right)$$

$$\begin{cases} d\tilde{A} = dA\ \delta\left(\frac{1}{N}\mathrm{tr}(A_i)^2 - 1\right)\ \theta\left(\kappa - \frac{1}{N}\mathrm{tr}(A_0)^2\right) \\ \lim_{N,L\to\infty} f_{N,L}(x) = \delta(x) \end{cases}$$

$$\Rightarrow \quad \frac{1}{N}\mathrm{tr}(A_0)^2 \leq \kappa\ , \quad \frac{1}{N}\mathrm{tr}(A_i)^2 = 1\ , \quad -2\mathrm{tr}F_{0i}^2 + \mathrm{tr}F_{ij}^2 = 0$$

- Can we understand SSB in the large kappa ?

$\kappa \uparrow \qquad \mathrm{tr}A_0^2 \uparrow \qquad \mathrm{tr}F_{0i}^2 \uparrow \qquad \mathrm{tr}F_{ij}^2 \uparrow$

$\Rightarrow$ **Maximize** $\mathrm{tr}F_{ij}^2$ **with** $\frac{1}{N}\mathrm{tr}(A_i)^2 = 1$