An estimate of heavy quark momentum diffusion coefficient in gluon plasma

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Langevin description

Energy loss and flow of heavy quarks: important probe of QGP created in RHIC.

For thermal heavy quark, $M \gg T, \quad p \sim \sqrt{MT}$
Takes $\mathcal{O}(M/T)$ hard collisions to change momentum by $\mathcal{O}(1)$
Scattering with thermal quarks: uncorrelated momentum kicks

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D p_i, \qquad \langle \xi_i(t) \xi_j(t') \rangle = \kappa \delta_{ij} \delta(t - t')$$

$$\langle p^2 \rangle = 3MT \quad \to \eta_D = \frac{\kappa}{2MT}$$

$$\langle x_i(t) x_j(t) \rangle = 2Dt\delta_{ij} \quad \to \quad D = \frac{2T^2}{\kappa}$$

Svetitsky '88; Moore & Teaney '05; Rapp & van Hees '05; Mustafa '05

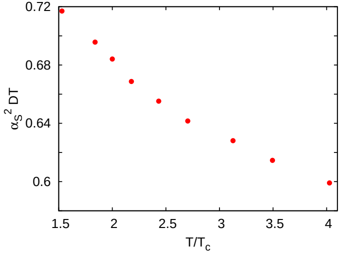
Perturbative estimate

Energy loss dominated by $qQ \to qQ$ and $gQ \to gQ$ collisions.

$$\kappa \approx \frac{2\alpha_S^2}{27\pi}\left[N_c\left(\ln\frac{2T}{m_D}+c\right) + \frac{N_f}{2}\left(\ln\frac{4T}{m_D}+c\right)\right]$$

Moore & Teaney '05

$DT = \frac{2T^3}{\kappa} \sim 14$ at 1.5 T_c for $N_f = 0$



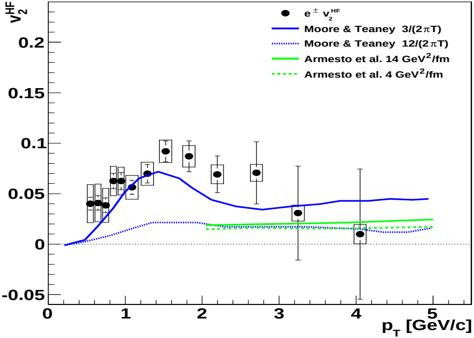
Adapted from Moore & Teaney '05, using m_D of Kaczmarek & Zantow '02

For N_f=3, with $\alpha_s = 0.23$, DT at LO ~ 10, while NLO takes it down by a factor ~ 7
Caron-Huot & Moore JHEP '08

Data

RHIC found huge elliptic flow of open heavy flavors

While the Langevin formalism gives correct trend for the p_T distribution of v_2, it needs a diffusion coefficient *very different* from the leading order perturbative estimate.



Phenix Collab., PRC(1005.1627)

Nonperturbative determination

Nonperturbative estimate of D essential for Langevin understanding of heavy quark flow.
Very difficult to obtain from lattice.
Retarded correlator of vector current $\Leftarrow$ analytic continuation.
Extraction from $\bar{\psi}\gamma_i\psi$ Matsubara correlator: highly nontrivial.

Petreczky & Teaney '06

$$G_{jj}(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_{jj}(\omega) \frac{\cosh \omega(\tau - 1/2T)}{\sinh \frac{\omega}{2T}}, \qquad \frac{\rho_{jj}(\omega)}{\omega}\Big|_{\omega\to 0} \sim 3\chi \frac{D\eta^2}{\eta^2+\omega^2}$$

Result contaminated by structure at $\omega \sim M$
Early analysis was consistent with $\kappa \approx 0$

Umeda '07; Datta & Petreczky '08

Recently, D extracted using very fine ($a = 1/48T$) lattices

Ding et al. 1204.4945

Still, systematics difficult to control.

Static limit and AdS/CFT calculation

$m_Q \to \infty$: heavy quark propagation replaced by Wilson line

$$\kappa = \frac{1}{3} \int dt \ \langle F_i(t) F_i(0) \rangle$$

where the static quark experiences only the color electric field
κ obtained from correlator of $E_a(\vec{x}, t)$ fields joined by Wilson line
Standard use of fluctuation-dissipation theorem $\Rrightarrow$

$$\kappa \ = \lim_{\omega \to 0} \frac{2T}{\omega} \ \rho(\omega)$$

Calculated for $\mathcal{N} = 4$ SYM in the limit $g^2 N_c \to \infty$
using AdS/CFT correspondence
$\rho(\omega) = \frac{\pi}{2} \sqrt{g^2 N_c} T^2 \omega$ Teaney & Cassalderrey-Solana '06
Naively putting $N_c = 3$ and $\frac{g^2}{4\pi} \sim 0.23 \quad \Rrightarrow DT \sim 0.2$

Euclidean version

Can analytically continue the real-time correlator in the heavy quark limit to the Euclidean correlator

Caron-Huot, Laine & Moore, 0901.1195

can be calculated non-perturbatively on the lattice
Has been calculated in (HTL) perturbation theory
Smooth diffusion part $\propto \omega$ at small ω
However, very small κ/T^3, even negative at small temperatures.

Burnier, Laine, Langelage, Meher, 1006.0867

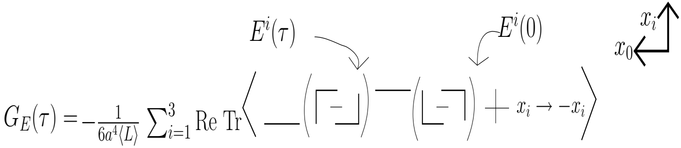
Here, we calculate the Euclidean electric field correlator on lattice and attempt to extract the momentum diffusion coefficient κ
Aim is to see if the nonperturbative estimate is in the right ballpark for understanding the experiments.

Lattice determination

Electric field discretization: use

$E_a((\vec{x},t)) \;=\; D_0\; D_i((\vec{x},t)) \;-\; D_i\; D_0((\vec{x},t))$

Formalism of Caron-Huot et al.:



$$G_E^{\rm ren}(\tau) \;=\; Z^2_{E,\rm Lat}\; G_E^{\rm Lat}(\tau)$$

We plan to calculate $Z_E^{\rm Lat}$ nonperturbatively.

Now use LO tadpole factor.

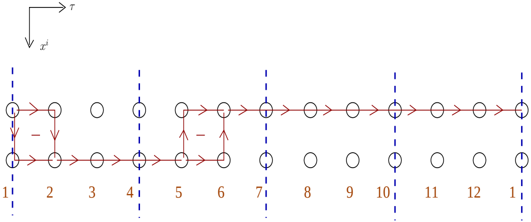
Was found to be very close to nonperturbative renormalization factor of electric field at coarser lattices.

Y. Koma and M. Koma, NP B 769 (2007) 79.

Calculational difficulty and algorithm

In order to extract κ from $G_E(\tau)$ we need large extent in τ : very difficult to calculate $G_E(\tau)$ for large τ with Naive Monte Carlo.

Used multilevel algorithm Luscher & Weisz '01



Turns out to be absolutely essential:

$\sim$ 2000 times more efficient for $G_E(\tau = 10)$

$\beta = 6.9$ lattices at $T = 1.09\,T_c$

Calculational details

Temperatures $1-2T_c$ covered with lattices with $N_\tau = 12-24$
$LT = 2, 3, 4$ to look for finite volume effects
(Larger LT only for smaller N_τ)
No significant volume effect seen.

Reliable extraction of κ possible for $N_\tau \geq 20$ lattices.

β	N_τ	T/T_c	# sublattice	# update
6.76	20	1.04	5	4000
6.80	20	1.09	5	3000
6.90	20	1.24	5	2000
7.192	24	1.50	4	2000
7.255	20	1.96	5	2000

Analysis

To extract κ, used ansatz for the spectral function

$$\rho(\omega) \;=\; \frac{\kappa}{T}\omega\;\Theta(\Lambda-\omega) \;+\; b\omega^3$$

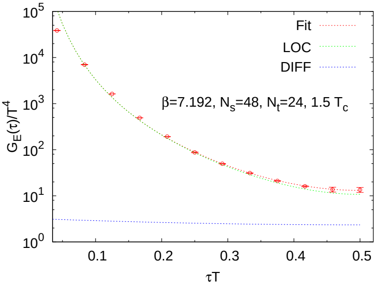
motivated by AdS/CFT and (HTL resummed) perturbation theory.
Stable fit for fine lattices
finite volume effect not large for $LT > 2$
result does not change significantly if one takes the infrared form

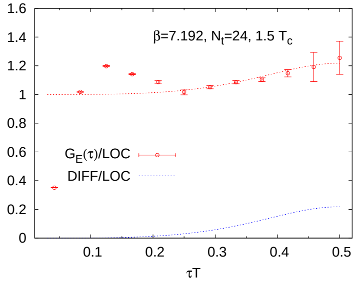
$$2\kappa \tanh\frac{\omega}{2T}$$

motivated by classical lattice gauge theory (Laine et al. '09).

Diffusive part

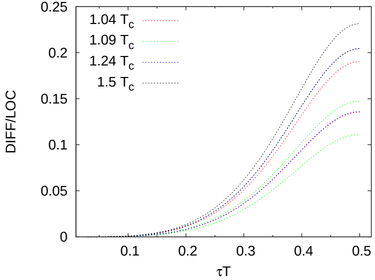
In all cases, stable fit is obtained with the full covariance matrix.

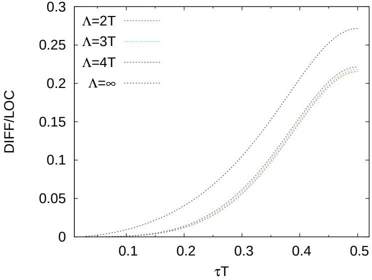




Diffusive part is very flat, and reaches $\sim 20\%$ of leading order value near center.

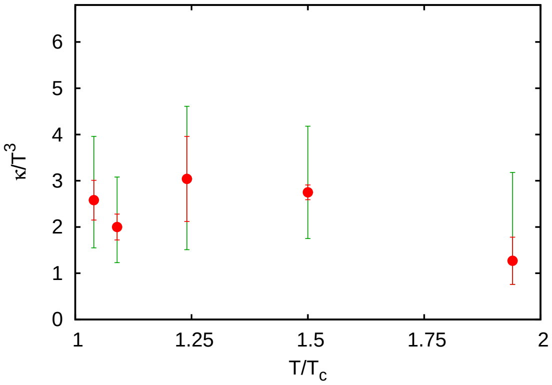
Diffusive part(contd.)





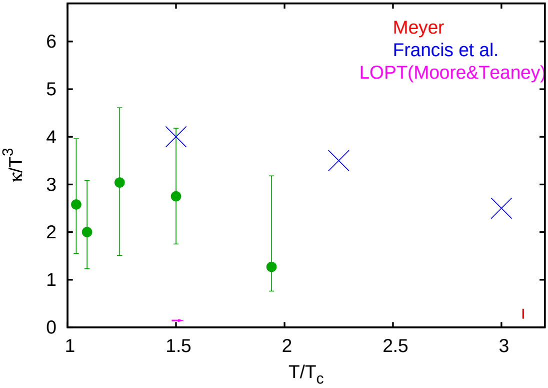
We fit for fixed Λ, and investigate change in κ by varying Λ.

Main Result



Central value quoted for $\Lambda = 3T$, where diffusive part $\approx \; \omega^3$ part.
Systematic band includes variation with Λ and fit form.
Finite volume effect much smaller than the other systematics.

Comparison

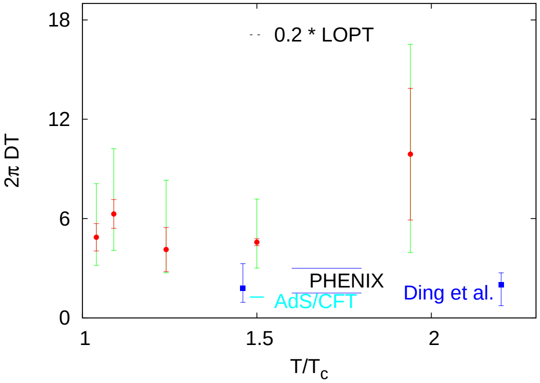


Obtained κ/T^3 order-of-magnitude larger than leading order perturbation theory.

In agreement with similar calculation by Francis et al. (1109.3941)

An earlier calculation at higher temperature (Meyer, 1012.0234, New J. Phys. 13, 035008) quoted a much smaller value.

Diffusion coefficient and PHENIX



D much smaller than leading order perturbation theory
Near-flat in the temperature range 1.06-2 T_c
Rough agreement with Ding et al. (1204.4945)
(added their errors in quadrature)
In the right ballpark for Phenix (apples and oranges?)

Summary

- The momentum diffusion coefficient, κ, of a heavy quark in a gluon plasma studied using $m_Q \to \infty$ limit.
- The nonperturbative estimate of $\kappa/T^3 \sim 1.5 - 4$ in the temperature range $T_c \lesssim T \lesssim 2T_c$ is much larger than perturbation theory estimates.
- In agreement with other recent lattice determinations.
- The estimated diffusion coefficient provides strong support for understanding of Phenix data using Langevin dynamics.

- Calculation of the electric field correlator on larger and finer lattices in progress.
- Plan to calculate nonperturbative renormalization constant for the electric field correlator.

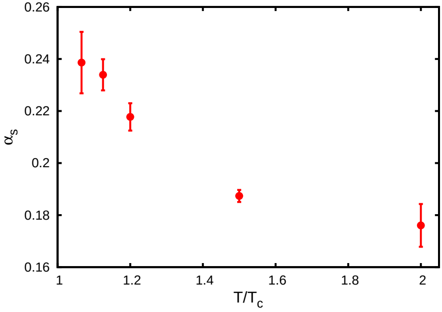
Coupling

Can use LO result for a nonperturbative definition of α_S

$$G_E^{\mathrm{LO}}(\tau) \;=\; \frac{8\alpha_S}{9}\,\omega^3$$

In perturbation theory, to $\mathcal{O}(\alpha_S^2)$

$$\begin{aligned}\alpha_S(\mu) &= \alpha_S^{\overline{MS}}(\mu)(1-\frac{\alpha_S^{\overline{MS}}(\mu)}{4\pi}\cdot \\ &\quad (\frac{8\pi^2}{3}-\frac{149}{9})) \\ &\sim \alpha_S^{\overline{MS}}(\mu)-0.78\alpha_S^{\overline{MS}}(\mu)^2\end{aligned}$$



In agreement with Ding et al., PR D 83 (2011) 034504, Kaczmarek & Zantow, PR D 71 (2005) 114510.

Coupling not large near T_c!