Phase structure of QC_2D at high temperature and density

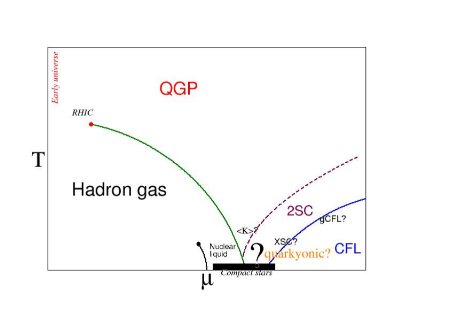
Jon-Ivar Skullerud
with Simon Hands, Seyong Kim, Seamus Cotter

NUI Maynooth

Lattice 2012, Cairns, 28 June 2012

Outline

Background



- A plethora of phases at high μ, low T
- Based on models and perturbation theory

Indirect approach

Study QCD-like theories without a sign problem

- Generic features of strongly interacting systems at $\mu \neq 0$
- Check on model calculations

Global symmetries of QC_2D

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Anti-unitary symmetry: $KMK^{-1} = \mathcal{M}^*$ with $K \equiv C\gamma_5\tau_2$

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$m = \mu = 0$:
global $SU(2N_f)$ symmetry $\longrightarrow$ $Sp(2N_f)$ by $\langle\overline{\psi}\psi\rangle \neq 0$.

$\Rightarrow N_f(2N_f - 1) - 1$Goldstone modes

$N_f = 2$: 5 modes

$\overline{\psi}\vec{\sigma}\gamma_5\psi$ pion $\psi^T\epsilon\tau_2 C\gamma_5\psi$, $\overline{\psi}\epsilon\tau_2 C\gamma_5\overline{\psi}^T$ scalar diquark

Diquark condensation

Diquarks are colour singlets in QC_2D

$\rightarrow$ superfluidity rather than colour superconductivity

$\rightarrow$ exact Goldstone mode from breaking of $U(1)_B$ symmetry

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Bose–Einstein Condensation:

Condensation of tightly bound diquarks (Goldstone baryons)

$\leftrightarrow$ Chiral perturbation theory

$$\langle\psi\psi\rangle \propto \sqrt{1-(\mu/\mu_o)^4}$$

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→ exact Goldstone mode from breaking of $U(1)_B$ symmetry

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↔ Chiral perturbation theory

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Bardeen–Cooper–Schrieffer:

Pairing of quarks near the Fermi surface

$$\langle\psi\psi\rangle \propto \Delta\mu^2$$

Lattice formulation

We use Wilson fermions:

- Correct symmetry breaking pattern, Goldstone spectrum
- $N_f < 4$ needed to guarantee continuum limit
- No problems with locality, fourth root trick
- Chiral symmetry buried at bottom of Fermi sea

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$$S = \bar{\psi}_1 M(\mu)\psi_1 + \bar{\psi}_2 M(\mu)\psi_2 - J\bar{\psi}_1(C\gamma_5)\tau_2\bar{\psi}_2^T + \bar{J}\psi_2^T(C\gamma_5)\tau_2\psi_1$$

$$\gamma_5 M(\mu)\gamma_5 = M^\dagger(-\mu), \quad C\gamma_5\tau_2 M(\mu) C\gamma_5\tau_2 = -M^*(\mu)$$

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Diquark source $J \equiv \kappa j$ introduced to

- lift low-lying eigenmodes in the superfluid phase
- study diquark condensation without uncontrolled approximations

Simulation Parameters

β	κ	a	am_π	m_π/m_ρ
1.9	0.168	0.18fm	0.65	0.80

N_s	N_τ	T (MeV)	μa	ja
12	24	47	0.25–1.10	0.02, 0.04 (0.03)
16	24	47	0.30–0.90	0.04
12	16	70	0.30–0.90	0.04
16	12	94	0.20–0.90	0.02, 0.04
16	8	141	0.10–0.90	0.02, 0.04

250–500 trajectories used for each μ.

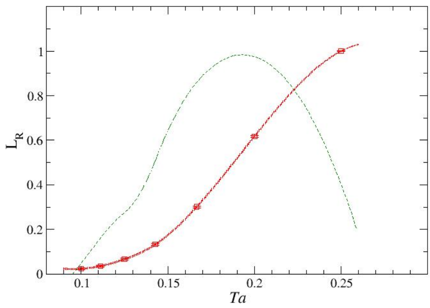
$j \to 0$ extrapolation

- Used linear extrapolation in j
- Check using power law, power+const, quadratic

Phase structure and order parameters

$\mu = 0$ deconfinement transition

Performed temperature scan with $N_\tau = 4 - 10$ at $\mu = 0$



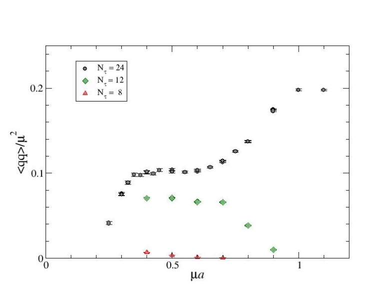
L renormalised s.t. $L(N_\tau = 4) = 1$

Inflection point from spline interpolation

$T_d a = 0.193(20)$ gives $T_d = 217(23)$MeV

Diquark condensate

Results shown are for linear extrapolation
Power law $\langle qq \rangle = Aj^{\alpha}$ works for $\mu a \lesssim 0.4$, with $\alpha = 0.85 - 0.5$.



- BCS scaling $\langle qq \rangle \sim \mu^2$ for $0.35 \lesssim \mu a \lesssim 0.7$
- Melted at $T = 141\text{MeV}$ ($N_\tau = 8$)

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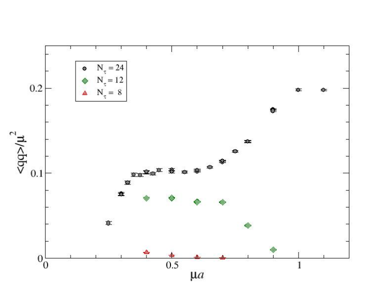
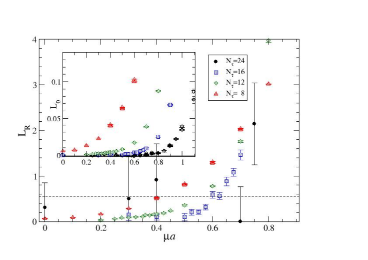


- BCS scaling $\langle qq \rangle \sim \mu^2$ for $0.35 \lesssim \mu a \lesssim 0.7$
- Melted at $T = 141\text{MeV}$ ($N_\tau = 8$)
- New transition for $\mu a \gtrsim 0.7$?
- Melting for $N_\tau = 12, \mu a \gtrsim 0.7$?

$N_\tau = 16$ results are very close to $N_\tau = 24$ results.

Polyakov loop, static quark potential



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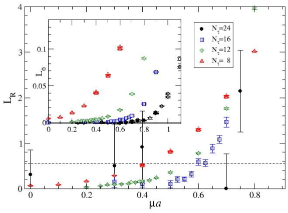

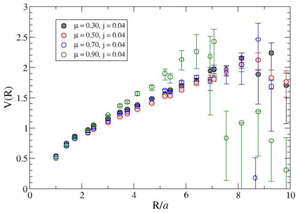

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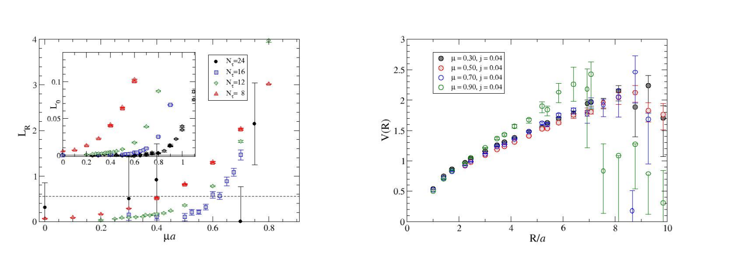

- Results suggest deconfinement at higher μ as T increases
- T-scan at fixed μ required to locate transition
- Crossing of lines at $\mu a \approx 0.75$ — lattice artefact?
- Potential is slightly screened at intermediate μ
- Anti-screening at large μ?

Chiral condensate

Computed $\langle\overline{\psi}\psi\rangle$ with naive fermions ($r = 0$),
$ma = 1/2\kappa = 0.0625, 0.0313, 0.0125$.
$ma = 0.0625(\kappa = 8.0)$ gives $m_\pi a = 0.66$

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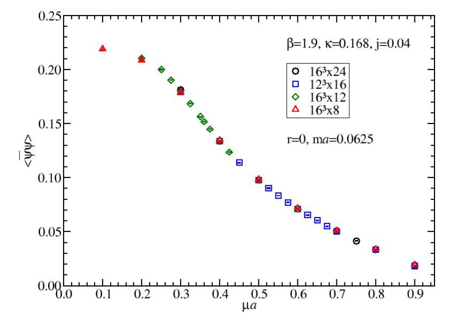

- Chiral symmetry restoration at $\mu a \approx 0.45$?
- At all temperatures?

Chiral symmetry

Chiral symmetry: $\langle\overline{\psi}\psi\rangle \propto m_q$

$$\implies \frac{m_2\langle\overline{\psi}\psi(m_1)\rangle}{m_1\langle\overline{\psi}\psi(m_2)\rangle} = 1$$

$$\iff \frac{\kappa_1^2\langle\overline{\psi}\psi(\kappa_1)\rangle_W}{\kappa_2^2\langle\overline{\psi}\psi(\kappa_2)\rangle_W} = 1$$

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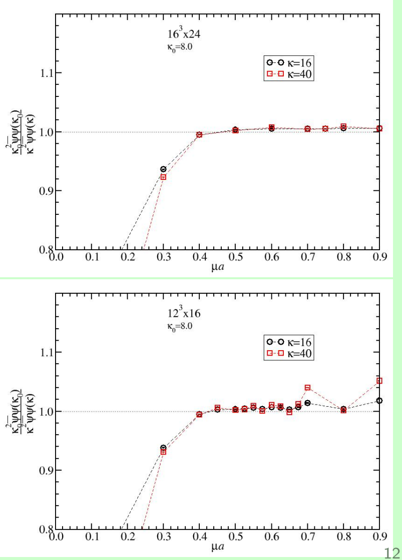
"Chiral symmetry" restored for $\mu a \gtrsim 0.4$
(but broken by $\langle qq\rangle$)



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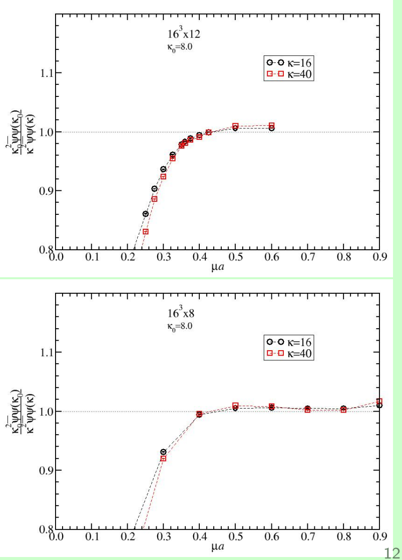
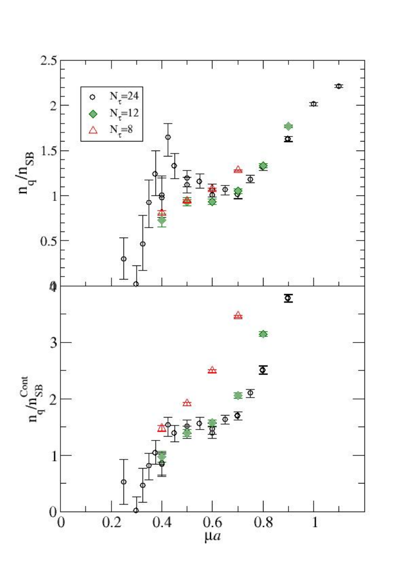


Number density

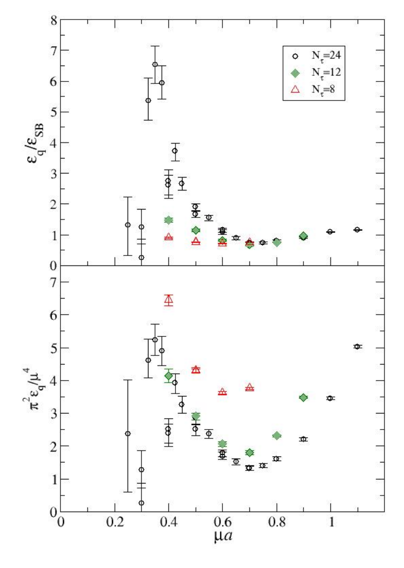


- Normalised to lattice SB or

$$n_{SB}^{\text{Cont}} = \frac{4\mu T^2}{3} + \frac{4\mu^3}{3\pi^2}$$

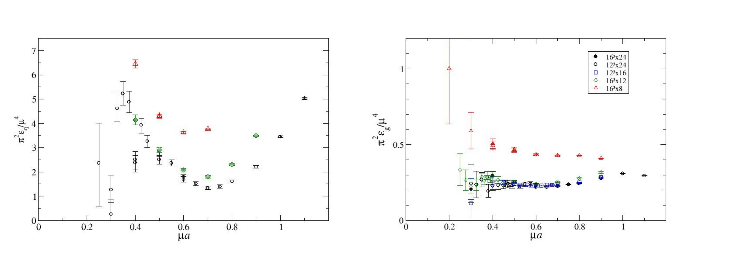
- Close to SB scaling for $0.4 \lesssim \mu a \lesssim 0.7$, $N_\tau = 24, 12$.
- Little T-dependence for $T \lesssim 100\text{MeV}$.
- Strong thermal effects for $N_\tau = 8$
- $N_\tau = 16, 24$ almost identical

Energy density



- Normalised to lattice SB, continuum SB at $T = 0$
- Close to SB scaling for intermediate μ
- Big peak at $\mu a \approx 0.35 - 0.40$
- Strong T-dependence, hard to disentangle from lattice artefacts
- Strong thermal effects for $N_\tau = 8$
- $N_\tau = 16, 24$ almost identical

Quark and gluon energy density



- Big peak in ε_q in intermediate region?
- $\varepsilon_q \sim 2\varepsilon_{SB} \to k_F > E_F \implies$ binding energy?
- 10–20% of total energy from gluons?
- Renormalisation of energy densities in progress

Conformal anomaly

Conformal anomaly $\Theta = T_{\mu\mu} = \varepsilon - 3p$ is given by

$$\Theta = \Theta_g + \Theta_q = -a\frac{\partial\beta}{\partial a}\frac{3\beta}{N_c}\,\mathrm{Tr}\langle\Box\rangle + a\frac{\partial\kappa}{\partial a}\kappa^{-1}(4N_f N_c - \langle\overline{\psi}\psi\rangle)$$

Beta functions $a\frac{\partial\beta}{\partial a}, a\frac{\partial\kappa}{\partial a}$ can be estimated from parameters of two matched lattices (coarse and fine)

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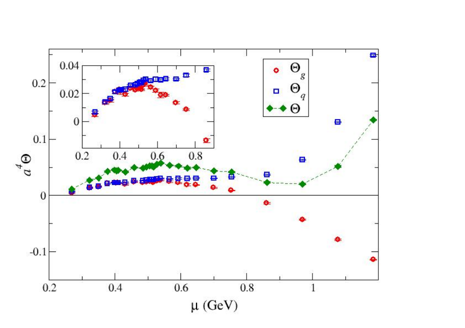
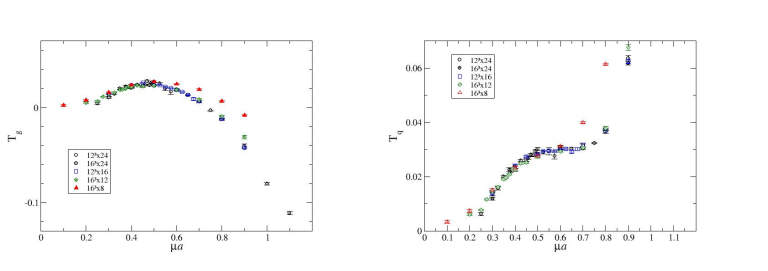


- $\Theta_q \approx \Theta_g$ up to $\mu = 500$MeV
- Θ_g becomes negative for $\mu \gtrsim 800$MeV
- Θ_q increases rapidly for $\mu \gtrsim 800$MeV
- $\Theta > 0$ for all μ

Conformal anomaly — temperature dependence



Very weak T-dependence except for $N_\tau = 8$!

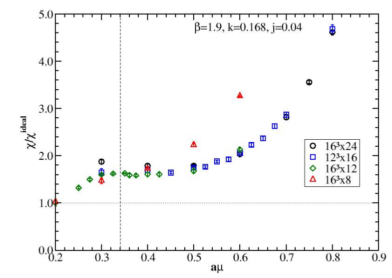
Quark number susceptibility

[See also P.Giudice, Lat11]

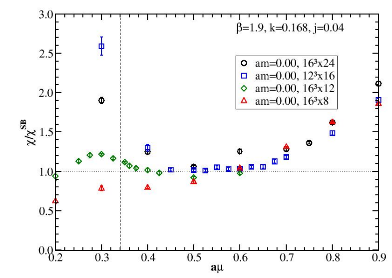
$\chi_q \equiv \frac{\partial n_q}{\partial \mu}$ — measures fluctuations in quark number

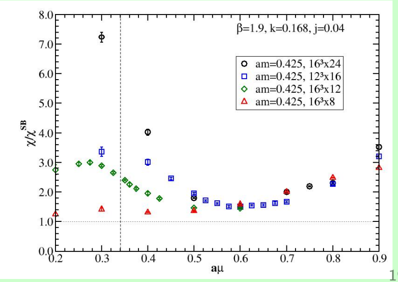
- Fluctuation peak may signal 2nd order critical point
- Used as measure of deconfinement of light quarks
- QC_2D has light (pseudo-Goldstone) baryons
  $\longrightarrow$ fluctuations may be large even in confined phase

Susceptibility results



- Smooth behaviour at all T, μ
- Plateau in BCS region
- $N_\tau = 8$ qualitatively different



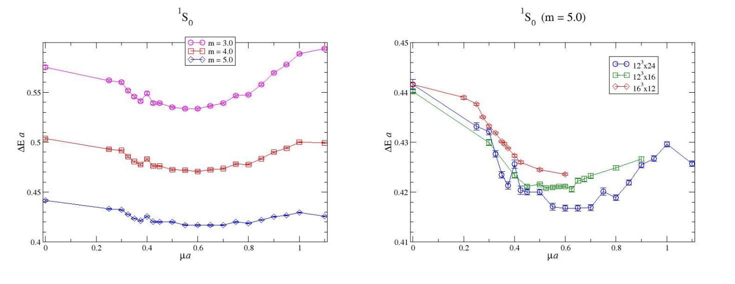


Heavy quarkonium

[PLB 711 (2012) 199, arXiv:1202.4353]

- Heavy quarkonia are much studied at high T
- Can be treated as probes (not in thermal equilibrium)
- $M_q \gg T, Mv \quad \Longrightarrow \quad$ can use effective theory: NRQCD
- We use lattice NRQCD to $\mathcal{O}(v^4)$
- Computed S and P wave correlators
- Note that $Q\bar{Q}$ is equivalent to QQ in QC_2D

NRQC$_2$D correlators and energies



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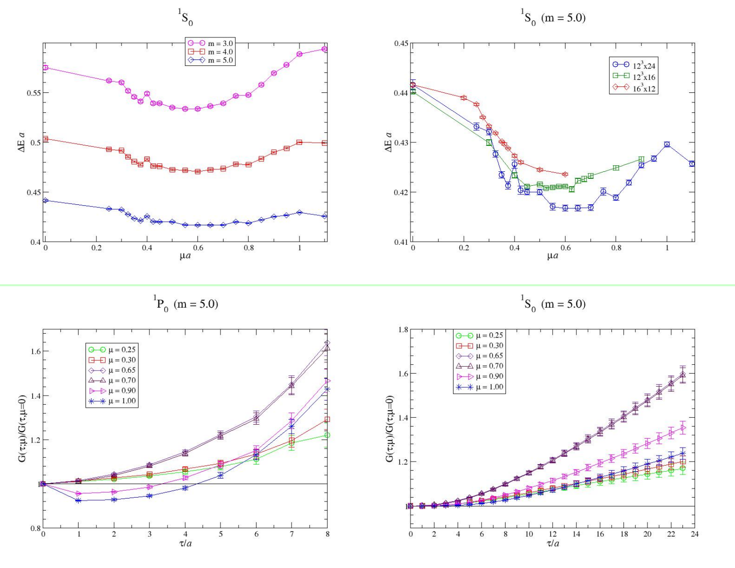
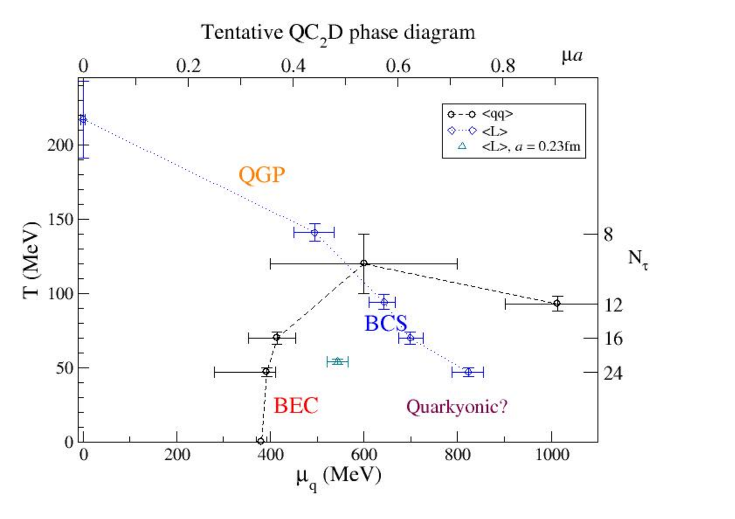


Summary


Tentative QC$_2$D phase diagram
μa
0
0.2
0.4
0.6
0.8
<qq>
<L>
<L>, a = 0.23fm
QGP
BCS
BEC
Quarkyonic?
T (MeV)
0
50
100
150
200
8
12
16
24
N$_\tau$
0
200
400
600
800
1000
μ$_q$ (MeV)

Summary

Evidence for three phases/regions

- Vacuum/hadronic phase below $\mu_o = m_\pi/2$, low T
- BCS/quarkyonic for intermediate μ, low T
  - $\rightarrow$ BCS scaling of $n_q, \langle qq \rangle, \chi_q \rightarrow$ quark degrees of freedom?
  - $\rightarrow$ $\langle \overline{\psi}\psi \rangle \approx 0$ in chiral limit
  - $\rightarrow$ Quark conformal anomaly constant
- Deconfined/QGP matter at high T
  - $\rightarrow$ Evidence of $T_d(\mu)$ decreasing with μ
  - $\rightarrow$ Situation at very high μ unclear
- Possible deconfined BCS region?
- BEC region narrow/washed out — peak in ε_q a possible signal?

Outlook

- Renormalise energy densities imminent
- Temperature scan to determine $T_d(\mu)$
- Gluon and quark propagators in progress
- Smaller quark mass at fixed lattice spacing in progress
- Finer lattice at same quark mass in progress