Towards a QCD equation of state with 2+1+1 flavors using the HISQ action

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Introduction

The EoS with the HISQ action for 2+1 flavors is being computed by the HotQCD collaboration – see Peter Petreczky's talk in the previous session. Why do we want to include dynamical charm, going to 2+1+1 flavors?

- Charm was present in the early Universe.
  It might not be important for heavy ion collisions, because charm does does not thermalize quickly enough.
- A perturbative calculation indicates that the effects of charm become visible around $T \sim 350\mathrm{MeV}$.
  M. Laine, Y. Schröder , Phys. Rev. D74 (2006) 085009 [arXiv:hep-ph/0603048].
- We previously considered charm partially quenched on 2+1 flavor asqtad sea quarks, giving similar results.
  C. DeTar *et al.* (MILC), Phys. Rev. D81 (2010) 114504 [arXiv:1003.5682].

The Budapest-Wuppertal collaboration has shown preliminary 2+1+1 flavor results with the stout action:

S. Borsanyi *et al.*, PoS(Lattice 2011)201 [arXiv:1204.0995].

Lattice action

The simulations for the present study use the same actions as MILC's zero temperature 2+1+1 flavor HISQ project
A. Bazavov *et al.* (MILC), Phys. Rev. D82 (2010) 074501 [arXiv:1004.0342]; and in preparation.
Only two zero-temperature ensembles, on coarser lattices than considered there, were generated specifically for the current thermodynamics project so far.

The gauge action is the one-loop Symanzik action with tadpole improvement. It thus depends on the tadpole factor $u_0(\beta) = \langle \mathrm{Tr} U_p / 3 \rangle^{1/4}$ which is determined during thermalization.

For the fermions we use the highly improved staggered quark (HISQ) action. For the charm quark, with am_c not very small, this includes a mass (and therefore gauge coupling) depended correction factor ϵ_N for the so-called Naik term in the action. This term insures that the charm quark has the correct, relativistic dispersion relation, to good accuracy.

Line of constant physics

Here we compute the EoS along a line of constant physics (LCP) defined by $m_l = m_s/5$, with m_s and m_c roughly tuned to the physical strange and charm quark masses, using the pion, kaon and spin-averaged charmonium ground state.
A posteriori, the guessed simulation masses where not quite correct, as shown in the Table collecting some of the parameters.

β	am_s	am_c	a/r_1	size	am_s^{corr}	am_c^{corr}
5.4	0.091	1.339	0.708(12)	$16^3 \times 48$	0.1058(30)	1.3386(22)
5.6	0.0785	1.080	0.5828(27)	$16^3 \times 48$	0.0799(6)	1.1021(13)
5.8	0.065	0.838	0.4857(54)	$16^3 \times 48$	0.0668(15)	0.8472(16)
6.0	0.0509	0.635	0.3883(26)	$24^3 \times 64$	0.0516(7)	0.6363(9)
6.3	0.037	0.440	0.2858(20)	$32^3 \times 96$	0.0367(5)	0.4396(7)
6.72	0.024	0.286	0.1872(6)	$48^3 \times 144$	0.0225(1)	0.2768(3)
7.0	0.0158	0.188	0.1433(5)	$64^3 \times 192$	0.0169(4)	0.1944(3)

$r_1 \approx 0.31$ fm is used to set the lattice scale.

Line of constant physics

The values were then fit to the form

$$am_q(\beta) = \frac{c_q^0 f(\beta) + c_q^2 (10/\beta) f^3(\beta)}{1 + d_q^2 (10/\beta) f^2(\beta)} \left(\frac{20b_0}{\beta}\right)^{4/9}$$

for $q = s, c$, where

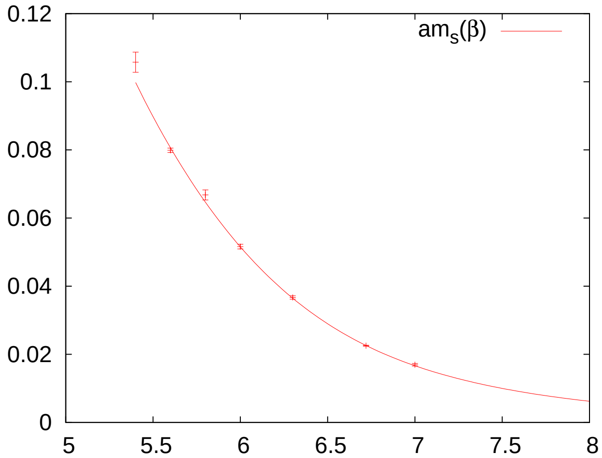
$$f(\beta) = \left(\frac{10b_0}{\beta}\right)^{-b_1/(2b_0^2)} \exp\left(-\beta/20b_0\right).$$

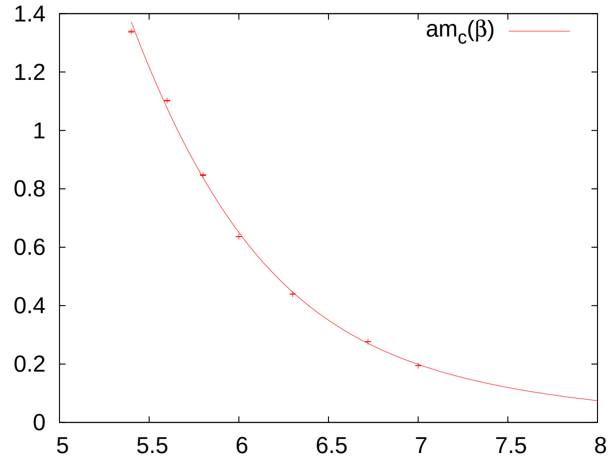
is the perturbative two-loop β-function.
The lattice scale is set using the scale r_1 from the heavy quark potential, fit to a similar form,

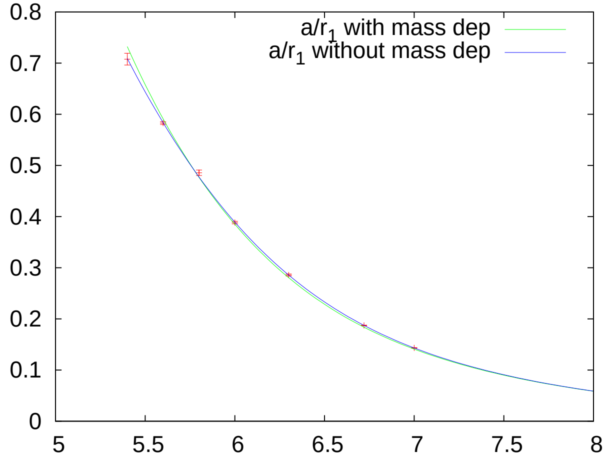
$$(r_1/a)(\beta) = \frac{c_r^0 f(\beta) + c_r^2 (10/\beta) f^3(\beta)}{1 + d_r^2 (10/\beta) f^2(\beta)}.$$

Line of constant physics

We show the data for am_s, am_c and r_1/a together with the fits as a function of β:







Ingredients for the EoS

The quantity convenient to compute on the lattice is the trace anomaly, or interaction measure, in units of T^4

$$\frac{\Theta^{\mu\mu}(T)}{T^4} \equiv \frac{\epsilon - 3p}{T^4} = T\frac{\partial}{\partial T}(p/T^4)$$

with the pressure given by $p/T = V^{-1}\ln Z$.

Changes in the temperature are related to changes in the gauge coupling given by the β-function

$$R_\beta(\beta) = T\frac{\mathrm{d}\beta}{\mathrm{d}T} = -a\frac{\mathrm{d}\beta}{\mathrm{d}a} = (r_1/a)(\beta)\left(\frac{\mathrm{d}(r_1/a)(\beta)}{\mathrm{d}\beta}\right)^{-1}.$$

We introduce similar β-functions describing the changes of bare quark masses, tadpole factor $u_0(\beta)$ and Naik-term correction factor for the charm quark, $\epsilon_N(\beta)$, with the gauge coupling.

Ingredients for the EoS

$$R_{m_q}\beta) = \frac{1}{am_q(\beta)} \frac{\mathrm{d}am_q(\beta)}{\mathrm{d}\beta} \qquad \text{for } q = l, s, c\,,$$

with $R_{m_l} = R_{m_s}$ along the considered LCP, and

$$R_u\beta) = \beta \frac{\mathrm{d}u_0(\beta)}{\mathrm{d}\beta}\,, \qquad R_\epsilon\beta) = \frac{\mathrm{d}\epsilon_N(\beta)}{\mathrm{d}\beta}\,.$$

To evaluate $R_u(\beta)$ we fitted u_0 as $u_0(\beta) = c_1 + c_2\mathrm{e}^{-\mathrm{d}_1\beta}$. R_ϵ is not used yet.

To normalize the interaction measure to zero at zero temperature we subtract observables computed on zero-temperature lattices from those computed on lattices with temporal extend N_τ, defining

$$\Delta(X) = \langle X \rangle_\tau - \langle X \rangle_0$$

where the latter is the expectation value on the zero-temperature lattices at the same (bare) parameters.

Ingredients for the EoS

$$\frac{\Theta^{\mu\mu}(T)}{T^4} = N_\tau^4 R_\beta(\beta) \left\{ -\Delta(s_G) - R_u(\beta)\Delta(\frac{\mathrm{d}s_G}{\mathrm{d}u_0}) + \mathcal{I}_F \right\} ,$$

where s_G denotes the gauge action density and $\mathcal{I}_F$ indicates the "fermion terms":

$$\begin{aligned} \mathcal{I}_F &= R_{m_s}(\beta) \left[2am_l \Delta(\bar{\psi}_l \psi_l) + am_s \Delta(\bar{\psi}_s \psi_s) \right] \\ &\quad + R_{m_c}(\beta) am_c \Delta(\bar{\psi}_c \psi_c) + R_\epsilon \Delta(\bar{\psi}_c [\mathrm{d}M_c/\mathrm{d}\epsilon_N] \psi_c) . \end{aligned}$$

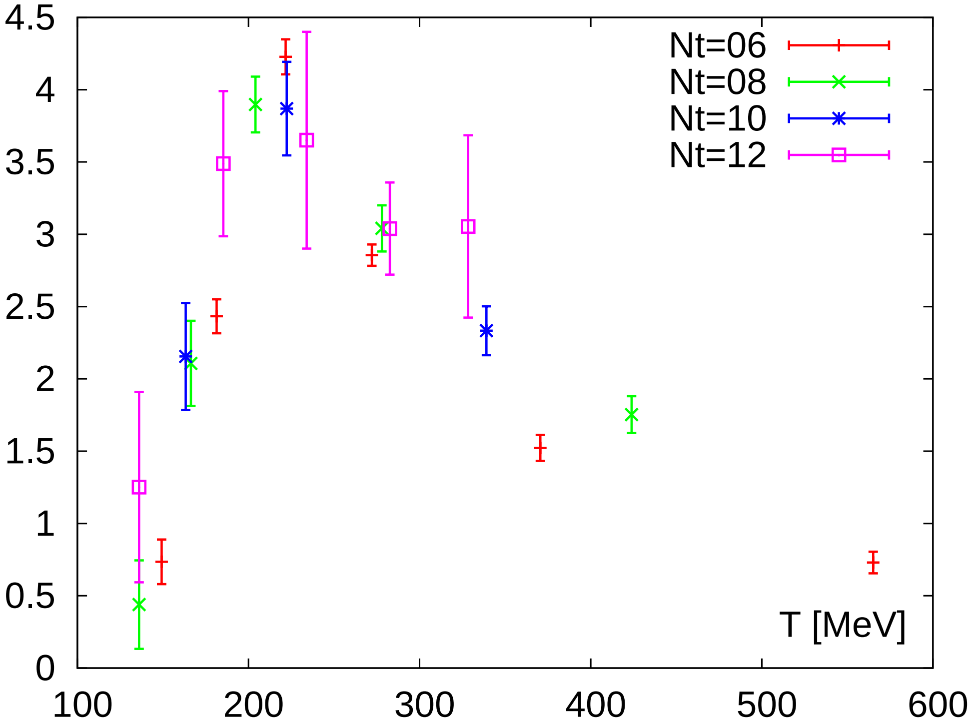
In the last term, M_c denotes the charm-quark fermion matrix.

The needed $\left\langle \bar{\psi}_q \psi_q \right\rangle_{0,\tau}$ for $q = l, s, c$ and $\left\langle \bar{\psi}_c [\mathrm{d}M_c/\mathrm{d}\epsilon_N] \psi_c \right\rangle_{0,\tau}$ are evaluated using stochastic estimators. Because of the "rooting procedure" they are the contribution from a single taste, *i.e.*, with a factor 1/4 implied.

$\left\langle \bar{\psi}_c [\mathrm{d}M_c/\mathrm{d}\epsilon_N] \psi_c \right\rangle_{0,\tau}$ has not been computed yet, but will soon be.

Gauge action contribution

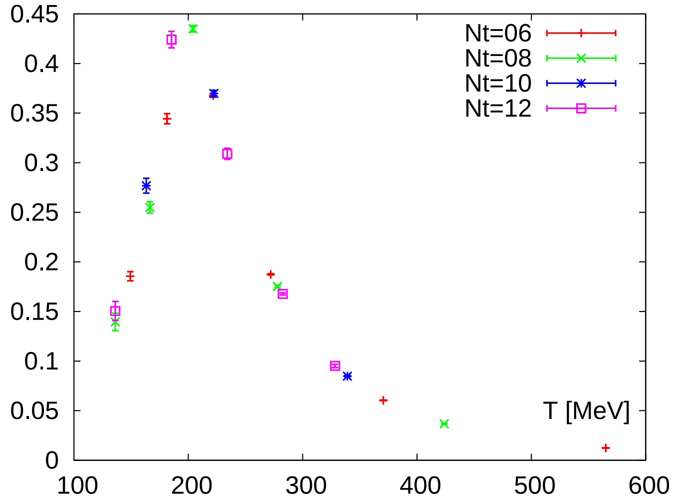
The contribution to the trace anomaly from the gauge action, including from the variation of the tadpole factor u_0.

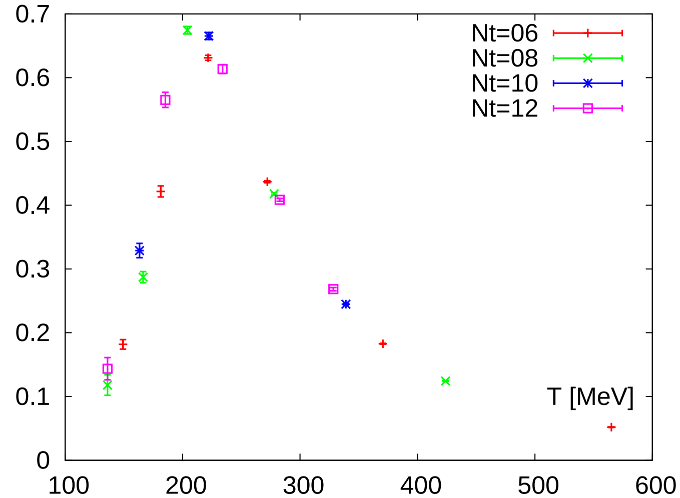


We have data from computations with $N_\tau = 6$, 8, 10, and 12.

Fermion contribution

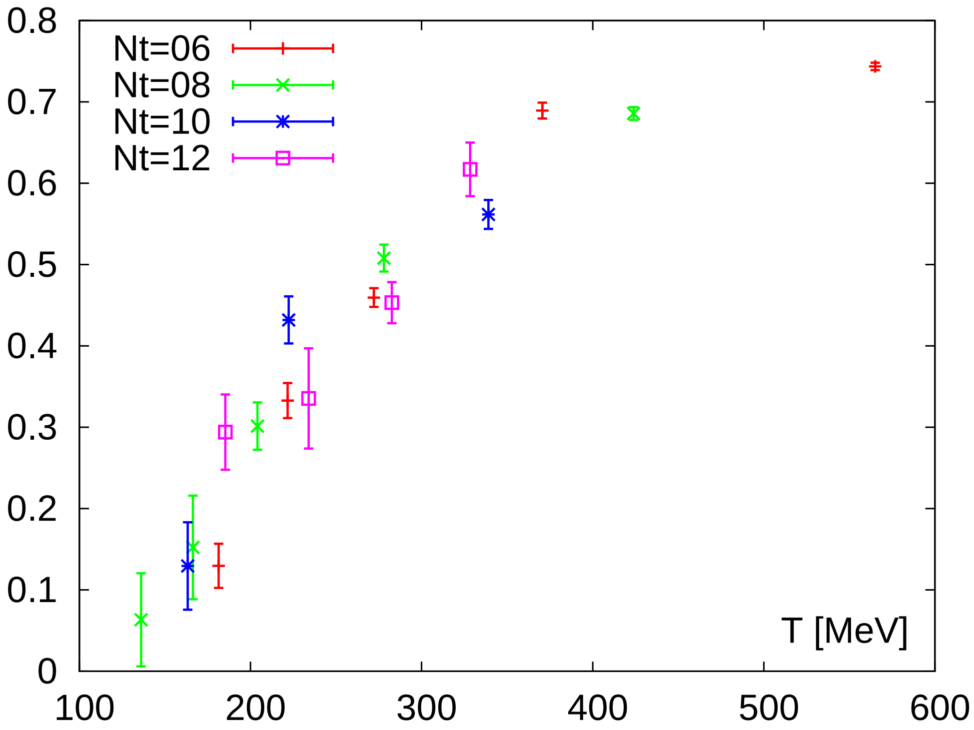
Contribution to the trace anomaly from the light-quark (up/down) condensate (left) and from the strange-quark condensate (right).





Fermion contribution

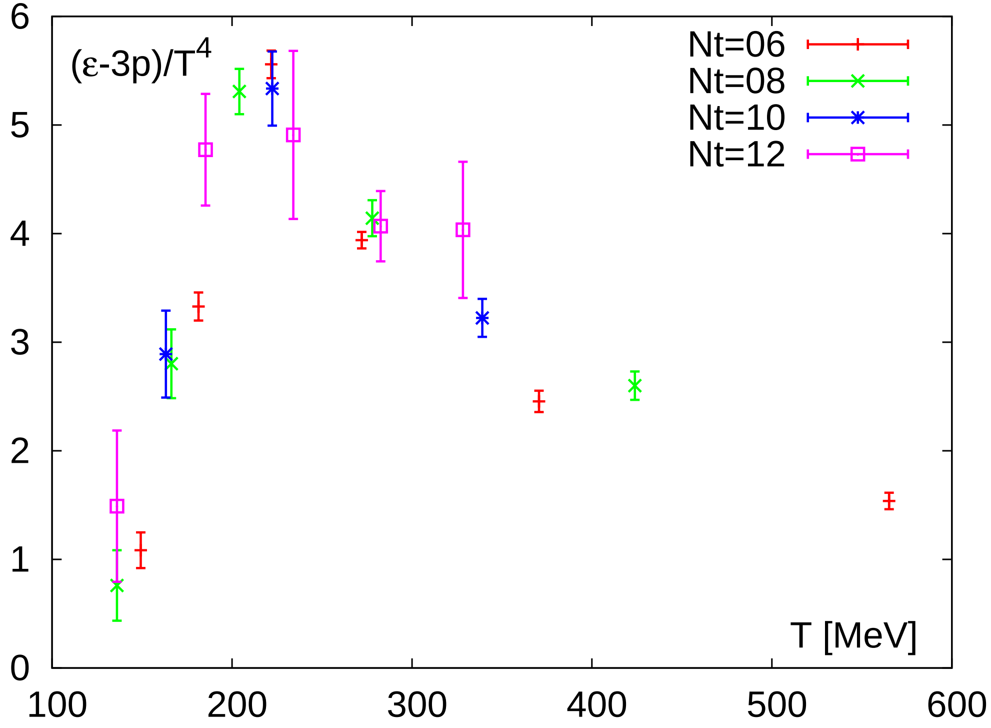
Contribution to the trace anomaly from the charm-quark condensate.



The contribution due to the variation of the charm-quark Naik term has not been computed yet. It is expected to be small compared to the charm-quark condensate contribution.

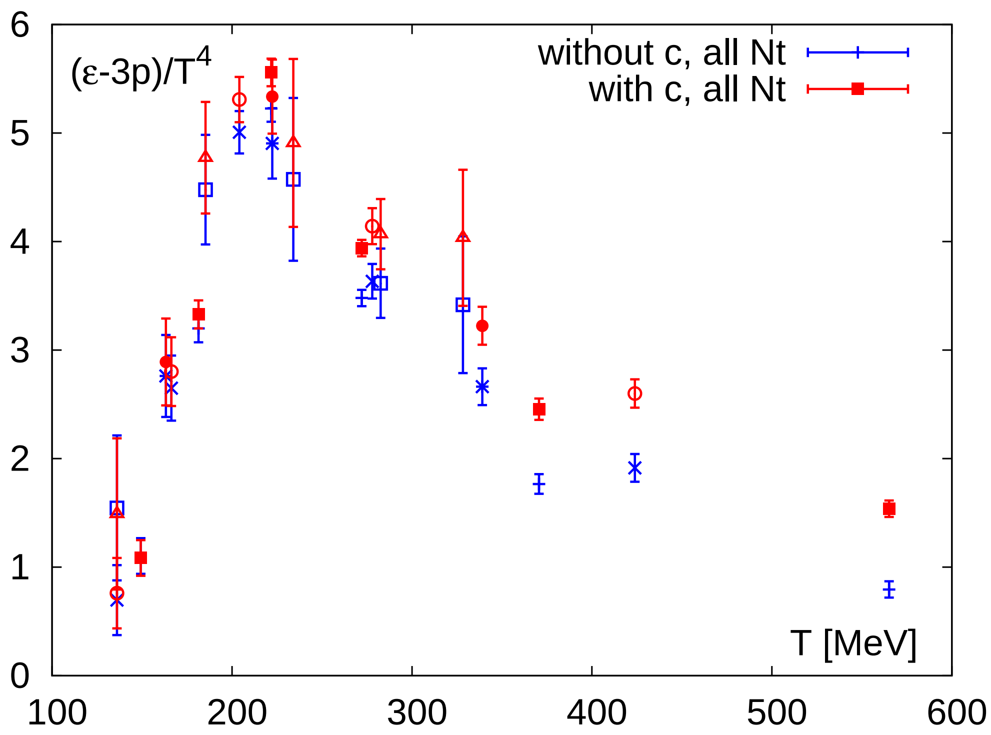
The current EoS

Here we show the current, preliminary status of the trace anomaly with 2+1+1 flavors (except for the contribution due to the variation of the charm-quark Naik term, expected to be small).



The current EoS

Since we are particularly interested in the influence of the charm quark, we show the preliminary results for the trace anomaly with and without the contribution of the charm-quark valence condensate.



The charm sea-quark effects are, of course, included in both versions.

Summary and outlook

- The MILC collaboration has started a study of the QCD EoS wit 2+1+1 flavors, including a dynamical charm quark, using the HISQ action.
- Our first study uses an LCP with $m_l/m_s = 1/5$ with physical strange and charm sea-quark masses.
- We have preliminary results for finite temperature lattices with $N_\tau = 6$, 8, 10 and 12, but have not attempted a continuum extrapolation yet.
- The effect of the charm quark on the EoS appears to set in for temperatures above the peak in the trace anomaly. At those temperatures, the value of the light quark mass has negligible influence, and the results for the LCP with $m_l/m_s = 1/5$ should be close to the case of physical light quark masses.
- Our study will be completed, with a proper continuum limit, in the near future.